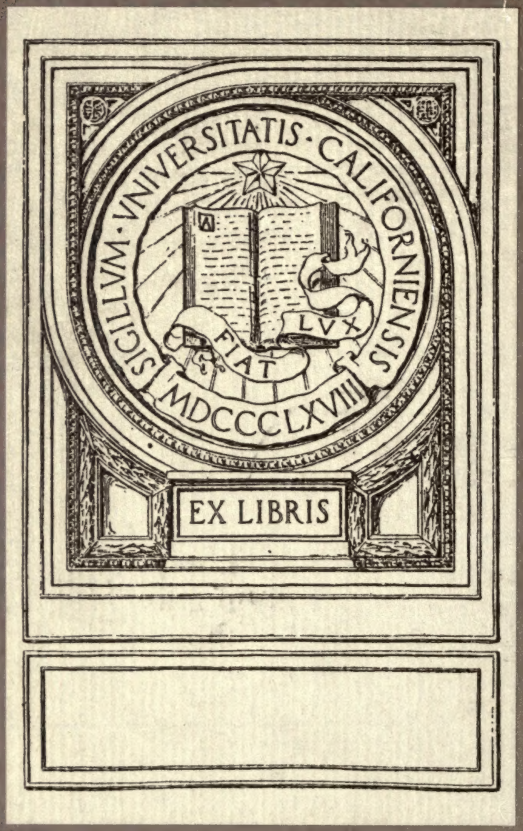

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ENGINEERING THERMODYNAMICS

BY

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PREFACE

CALCULATIONS about heat as a form of energy, and about work, another related form, both of them in connection with changes in the condition of all sorts of substances that may give or take heat, and perform or receive work while changing condition, constitute the subject matter of this book. The treatment of the subject matter of this text is the result of personal experience in professional engineering practice and teaching students of engineering at Columbia University.

Even a brief examination of the conditions surrounding changes in substances as they gain or lose heat, do work or have work done on them, and of the corresponding relations between heat and work as forms of energy independent of substances, will convince any one that the subject is one of great complexity. Accordingly the simplicity needed for practical use in the industries can be reached only by a consideration of a great mass of sub-topics and data.

That the doing of work, and the changes in heat content of substances were related phenomena, and that these relations when formulated, would constitute a branch of science, was conceived about a half century ago, and the science was named Thermodynamics. The Engineer Rankine, who helped to create it, defined thermodynamics as "the reduction of the laws according to which such phenomena took place to a physical theory or corrected system of principles." Since Rankine's time thermodynamics has become a very highly developed science and has proved of great assistance in the formulation of modern physical chemistry, and to those branches of engineering that are concerned with heat. Unfortunately, as thermodynamics developed as a separate subject it did not render proportionate service to engineering, which itself developed even more rapidly in the same period under the guidance of men whose duty it was to create industrial apparatus and make it work properly, and who had little or no time to keep in touch with purely scientific advances or to interpret such advances for utilitarian ends. Thermodynamics proper is concerned with no numerical quantities nor with any particular substance nor for that matter with any actual substances whatever, but it is a physical theory of energy in relation to matter as a branch of natural philosophy. Engineering, however, is concerned with real substances, such as coal, steam, and gases and with numerical quantities, horse-powers of engines, temperatures of steam, the heats of combustion of oils, so that alone, the principles of thermodynamic philosophy will not yield a solution of a practical problem,

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be it one of design or one of analysis of test performance of actual heat machine or thermal apparatus. It is the province of engineering thermodynamics to guide numerical computation on thermal problems for real substances being treated in real apparatus. Its field, while including some of that of pure thermodynamics, extends far beyond the established provinces of that subject and extends to the interpretation of all pertinent principles and facts for purely useful purposes. Engineering thermodynamics, while using whatever principles of pure thermodynamics may help to solve its problems, must rely on a great mass of facts or relations that may not yet have risen to the dignity of thermodynamic laws. The workers in shops, factories, power plants or laboratories engaged in designing or operating to the best advantage machines and apparatus using heat with all sorts of substances, have developed great quantities of rules, methods and data that directly contribute to the ends sought. While for each class or type of apparatus there has grown up a separate set of data and methods in which much is common to several or all groups, not nearly so much assistance is rendered by one to another as should be by a proper use of engineering thermodynamics, which applies methods, principles and conclusion to all related problems. Classes of apparatus about which such groups of methods of analysis or synthesis, or collections of special data have developed and which it is province of engineering thermodynamics to unify so far as may be, include air compressors, and compressed air engines, reciprocating steam engines, steam turbines, steam boilers, coal-, oil- and gas-fired furnaces, gasifiers of coal and oil, gas producers, gas engines, complete steam or gas power plants, mechanical refrigeration and ice-making plants and chemical factory equipment, or more generally, machinery and apparatus for heating and cooling, evaporating and condensing, melting and freezing, moistening and drying, gasification and combustiom.

The nature of the subject and its division are better indicated by the classes of problems to be solved by its aid or the contributions expected of it than by the kinds of apparatus to which they apply. Probably its broadest contribution is the establishment of limits of possible performance of heat apparatus and machines. These limits will show what might be expected of a steam engine, gas engine or refrigerating machine when its mechanism is quite perfect and thus they become standards of reference with which actual performance can be compared, and a measure of the improvements yet possible. These same methods and practices are applicable to the analysis of the operating performance of separate units and complete plants to discover the amount of energy being lost, how the total amount is divided between the different elements of the apparatus, which of the losses can be prevented and how, and finally which are absolutely unavoidable. This sort of analysis of the performance of thermal apparatus is the first step to be taken by the designer or manufacturer to improve the machine that he is creating for sale, and is essential to the purchaser and user of the machine, who cannot possibly keep it in the best operating condition without continually analyzing its performance and comparing results with thermodynamic possibilities.

The subject naturally divides into three general parts, the first dealing with the conditions surrounding the doing of work without any consideration of heat changes, the second heat gains and losses by substances without reference to work involved and the third, transformation of heat into work or work into heat in conjunction with changes in the condition of substances. The first part applies to the behavior of fluids in the cylinders of compressors and engines. The second part is concerned with the development of heat by combustion, its transmission from place to place, and the effect on the physical condition of solids, liquids, gases with their mixtures, solutions and reactions. The third part is fundamental to the efficient production of power by gases in internal combustion gas engines or compressed-air engines and by steam or other vapors in steam engines and turbines, and likewise as well to the production of mechanical refrigeration by ammonia, carbon dioxide and other vapors.

Accordingly, the six chapters of the book treat these three parts in order. The first three chapters deal with work without any particular reference to heat, the second two with heat, without any particular reference to work, while the last is concerned with the relation between heat and work. After establishing in the first chapter the necessary units and basic principles for fixing quantities of work, the second chapter proceeds at once to the determination of the work done in compressor cylinders and the third, the available work in engine cylinders, in terms of all the different variables that may determine the work for given dimensions of cylinder or for given quantities of fluid. There is established in these first three chapters a series of formulas directly applicable to a great variety of circumstances met with in ordinary practice. All are derived from a few simple principles and left in such form as to be readily available for numerical substitution. This permits of the solution of numerical problems on engine and compressor horse-power, fluid consumption or capacity with very little labor or time, although it has required the expansion of the subject over a considerable number of pages of book matter. A similar procedure is followed in the succeeding chapters, formulas and data are developed and placed always with a view to the maximum clearness and utility. The essential unity of the entire subject has been preserved in that all the important related subjects are treated in the same consistent manner and at sufficient length to make them clear. When no general principles were available for a particular solution there has been no hesitation in reverting to specific data. The subject could have been treated in a very much smaller space with less labor in book writing but necessitating far greater labor in numerical work on the part of the user. This same aim, that is, the saving of the user's time and facilitating the arrival at numerical answers, is responsible for the insertion of a very considerable number of large tables, numerous original diagrams and charts, all calculated for the purpose and drawn to scale. These, however, take a great deal of room but are so extremely useful in everyday work as to justify any amount of space thus taken up. For the sake of clearness all the steps in the derivation of any formula used are given, and numerical examples are added to illustrate their meaning

and application. This also requires a considerable amount of space but without it the limitations of the formulas would never be clear nor could a student learn the subject without material assistance. Similarly, space has been used in many parts of the book by writing formulas out in words instead of expressing them in symbols. This saves a great deal of time and labor in hunting up the meaning of symbols by one who desires to use an unfamiliar formula involving complex quantities, the meaning of which is often not clear when it is entirely symbolic. Thus, in the discussion of steam boiler capacity and efficiency, a dozen or more pages are taken up with formulas that could have been concentrated in a single page were symbols used entirely, but only at the sacrifice of clearness and utility. Where in the derivation of a new formula or in the treatment of a new subject, reference to an old formula or statement is needed and important, repetition is resorted to, rather than mere reference, so that the new topic may be clear where presented, without constantly turning the pages of the book. It will be found, therefore, that while the size of the book is unusually large it will be less difficult to study than if it were short.

As a text the book may be used for courses of practically any length, but it is not intended that in any course on the subject every page of the book shall be used as assigned text. In the new graduate course in mechanical engineering at Columbia University, about three-fourths of the subject matter of the book will be so used for a course of about one hundred and twenty periods of one hour each. All of the book matter not specifically assigned as text or reference in a course on engineering thermodynamics in any school may profitably be taken up in courses on other subjects, serving more or less as a basis for them. It is therefore adapted to courses on gas power, compressed air, steam turbines, steam power plants, steam engine design, mechanical refrigeration, heating and ventilation, chemical factory equipment, laboratory practice and research. Whenever a short course devoted to engineering thermodynamics alone is desired, the earlier sections of each chapter combined in some cases with the closing sections, may be assigned as text. In this manner a course of about thirty hours may be profitably pursued. This is a far better procedure than using a short text to fit a short course, as the student gets a better perspective, and may later return to omitted topics without difficulty.

The preparation of the manuscript involves such a great amount of labor, that it would never have been undertaken without the assurance of assistance by Mr. E. D. Thurston, Jr., a fellow instructor at Columbia, in checking text and tables, calculating diagrams, writing problems and working examples. This help has been invaluable and is gratefully acknowledged. Recognition is also due for material aid rendered by Mr. T. M. Gunn in checking and in some cases deriving formulas, more especially those of the first three chapters. In spite, however, of all care to avoid errors it is too much to expect complete success in a new work of this character, but it is hoped that readers finding errors will point them out that future editions may be corrected.

C. E. L.

Columbia University, New York, September, 1912.

CONTENTS

Chapter I. Work and Power. General Principles

PAGE

1. Work defined 1
2. Power defined 2
3. Work in terms of pressure and volume 3
4. Work of acceleration and resultant velocity 6
5. Graphical representation of work 8
6. Work by pressure volume change 10
7. Work of expansion and compression 13
8. Values of exponent s defining special cases of expansion or compression 20
9. Work phases and cycles, positive, negative and net work 24
10. Work determination by mean effective pressure 31
11. Relation of pressure-volume diagrams to indicator cards 34
12. To find the clearance 37
13. Measurement of areas of PV diagrams and indicator cards 43
14. Indicated horse-power 44
15. Effective horse-power, brake horse-power, friction horse-power, mechanical efficiency, efficiency of transmission, thermal efficiency 47
16. Specific displacement, quantity of fluid per hour, or per minute per I.H.P. 49
17. Velocity due to free expansion by PV method 52
18. Weight of flow through nozzles by PV method 55
19. Horse-power of nozzles and jets, by PV method 57

Chapter II. Work of Compressors. Horse-power and Capacity of Air, Gas and Vapor Compressors, Blowing Engines and Dry Vacuum Pumps

1. General description of structures and processes 73
2. Standard reference diagrams or PV cycles for compressors and methods of analysis of compressor work and capacity 75
3. Single-stage compressor, no clearance, isothermal compression, Cycle I. Work, capacity, and work per cubic foot in terms of pressures and volumes 81
4. Single-stage compressor with clearance, isothermal compression, Cycle II. Work, capacity, and work per cubic foot in terms of pressures and volumes 85
5. Single-stage compressor, isothermal compression. Capacity, volumetric efficiency, work, mean effective pressure, horse-power and horse-power per cubic foot of substance, in terms of dimensions and cylinder clearance 87
6. Single-stage compressor, no clearance, exponential compression, Cycle III. Work, capacity and work per cubic foot, in terms of pressures and volumes 91
7. Single-stage compressor with clearance, exponential compression, Cycle IV. Work, capacity, and work per cubic foot in terms of pressures and volumes 96
8. Single-stage compressor, exponential compression. Relation between capacity, volumetric efficiency, work, mean effective pressure, horse-power and horse-power per cubic foot of substance and the dimensions of cylinder and clearance .. 98

PAGE

9. Two-stage compressor, no clearance, perfect intercooling, exponential compression, best-receiver pressure, equality of stages. Cycle V. Work and capacity in terms of pressures and volumes 103
10. Two-stage compressor, with clearance, perfect intercooling exponential compression, best-receiver pressure, equality of stages, Cycle VI. Work and capacity in terms of pressures and volumes 109
11. Two-stage compressor, *any receiver pressure*, exponential compression. Capacity, volumetric efficiency, work, mean effective pressure and horse-power, in terms of dimensions of cylinders and clearances 113
12. Two-stage compressor, *with best-receiver pressure*, exponential compression. Capacity, volumetric efficiency, work, mean effective pressure and horse-power in terms of dimensions of cylinders and clearances 120
13. Three-stage compressor, no clearance, perfect intercooling exponential compression, best two receiver pressures, equality of stages, Cycle VII. Work and capacity, in terms of pressures and volumes 125
14. Three-stage compressor with clearance, perfect intercooling exponential compression, best-receiver pressures, equality of stages, Cycle VIII. Work and capacity in terms of pressures and volumes 131
15. Three-stage compressor, *any receiver pressure*, exponential compression. Capacity, volumetric efficiency, work, mean effective pressure, and horse-power in terms of dimensions of cylinders and clearances 135
16. Three-stage compressor *with best-receiver* pressures, exponential compression. Capacity, volumetric efficiency, work, mean effective pressure and horse-power in terms of dimensions of cylinders and clearances 143
17. Comparative economy or efficiency of compressors 148
18. Conditions of maximum work of compressors 151
19. Compressor characteristics 153
20. Work at partial capacity in compressors of variable capacity 160
21. Graphic solution of compressor problems 168

CHAPTER III. WORK OF PISTON ENGINES. HORSE-POWER AND CONSUMPTION OF PISTON ENGINES USING STEAM, COMPRESSED AIR, OR ANY OTHER GAS OR VAPOR UNDER PRESSURE

1. Action of fluid in single cylinders. General description of structure and processes .. 187
2. Simple engines. Standard reference cycles or *PV* diagrams for the work of expansive fluids in a single cylinder 192
3. Work of expansive fluid in single cylinder without clearance. Logarithmic expansion, Cycle I. Mean effective pressure, horse-power and consumption of simple engines 197
4. Work of expansive fluid in single cylinder without clearance. Exponential expansion, Cycle II. Mean effective pressure, horse-power and consumption of simple engines 205
5. Work of expansive fluid in single cylinder with clearance. Logarithmic expansion and compression; Cycle III. Mean effective pressure, horse-power, and consumption of simple engines 208
6. Work of expansive fluid in single cylinder with clearance; exponential expansion and compression, Cycle IV. Mean effective pressure, horse-power and consumption of simple engines 219
7. Action of fluid in multiple-expansion cylinders. General description of structure and processes 225
8. Standard reference cycles or *PV* diagrams for the work of expansive fluids in two-cylinder compound engines 235

PAGE
9. Compound engine with infinite receiver. Logarithmic law. No clearance, Cycle V. General relations between pressures, dimensions, and work 256
0. Compound engine with infinite receiver. Exponential law. No clearance, Cycle VI. General relations between pressures, dimensions and work 268
1. Compound engine with finite receiver. Logarithmic law. No clearance, Cycle. VII. General relations between dimensions and work when H.P. exhaust and L.P. admission are not coincident 274
2. Compound engine with finite receiver. Exponential law, no clearance, Cycle VIII. General relations between pressures, dimensions, and work, when high pressure Exhaust and low-pressure admission are independent 287
3. Compound engine without receiver. Logarithmic law. No clearance, Cycle IX. General relations between dimensions and work when high-pressure exhaust and low-pressure admission are coincident 292
4. Compound engine without receiver. Exponential law. No clearance, Cycle X. General relations between dimensions and work when high-pressure exhaust and low-pressure admission are coincident 301
5. Compound engine with infinite receiver. Logarithmic law, with clearance and compression, Cycle XI. General relations between pressures, dimensions and work 306
6. Compound engine with infinite receiver. Exponential law, with clearance and compression, Cycle XII. General relations between pressures, dimensions and work 319
7. Compound engine with finite receiver. Logarithmic law, with clearance and compression, Cycle XIII. General relations between pressures, dimensions, and work when H.P. exhaust and L.P. admission are independent 325
8. Compound engine with finite receiver. Exponential law, with clearance and compression, Cycle XIV. General relations between pressures, dimensions, and work when H.P. exhaust and L.P. admission are independent 335
9. Compound engine without receiver. Logarithmic law, with clearance and compression, Cycle XV. General relations between pressures, dimensions, and work when H.P. exhaust and L.P. admission are coincident 339
0. Compound engine without receiver. Exponential law, with clearance and compression, Cycle XVI. General relations between pressures, dimension, and work, when H.P. exhaust and L.P. admission are coincident 346
1. Triple-expansion engine with infinite receiver. Logarithmic law. No clearance, Cycle XVII. General relations between pressures, dimensions and work 349
2. Multiple-expansion engine. General case. Any relation between cylinders and receiver. Determination of pressure-volume diagram and work, by graphic methods 357
3. Mean effective pressure, engine horse-power, and work distribution and their variation with valve movement and initial pressure. Diagram distortion and diagram factors. Mechanical efficiency 363
4. Consumption of steam engines and its variation with valve movement and initial pressure. Best cut-off as affected by condensation and leakage 371
5. Variation of steam consumption with engine load. The Willans line. Most economical load for more than one engine and best load division 381
6. Graphical solution of problems on engine horse-power and cylinder sizes 387

CHAPTER IV. HEAT AND MATTER. QUALITATIVE AND QUANTITATIVE RELATIONS BETWEEN HEAT CONTENT OF SUBSTANCES AND PHYSICAL-CHEMICAL STATE

PAGE

1. Substances and heat effects important in engineering 398
2. Classification of heating processes. Heat addition and abstraction with or without temperature change, qualitative relations 401
3. Thermometry based on temperature change, heat effects. Thermometer and absolute temperature scales 407
4. Calorimetry based on proportionality of heat effects to heat quantity. Units of heat and mechanical equivalent 415
5. Temperature change relation to amount of heat for solids, liquids, gases and vapors not changing state. Specific heats 419
6. Volume or density variation with temperature of solids, liquids, gases and vapors not changing state. Coefficients of expansion. Coefficients of pressure change for gases and vapors 435
7. Pressure, volume, and temperature relations for gases. Perfect and real gases 438
8. Gas density and specific volume and its relation to molecular weight and gas constant . 446
9. Pressure and temperature relations for vapor of liquids or solids. Vaporization, sublimation and fusion curves. Boiling- and freezing-points for pure liquids and dilute solutions. Saturated and superheated vapors 451
10. Change of state with amount of heat at constant temperature. Latent heats of fusion and vaporization. Total heats of vapors. Relation of specific volume of liquid and of vapor to the latent heat 467
11. Gas and vapor mixtures. Partial and total gas and vapor pressures. Volume, weight and gas constant relations. Saturated mixtures. Humidity 481
12. Absorption of gases by liquids and by solids. Relative volumes and weights with pressure and temperature. Heats of absorption and of dilution. Properties of aqua ammonia 493
13. Combustion and related reactions. Relative weights and volumes of substances and elements before and after reaction 506
14. Heats of reaction. Calorific power of combustible elements and of simple chemical compounds. B.T.U. per pound and per cubic foot 516
15. Heat transmission processes. Factors of internal conduction, surface resistance, radiation and convection 528
16. Heat transmission between separated fluids. Mean temperature differences, coefficients of transmission 538
17. Variation in coefficient of heat transmission due to kind of substance, character of separating wall and conditions of flow 555

CHAPTER V. HEATING BY COMBUSTION. FUELS, FURNACES, GAS PRODUCERS AND STEAM BOILERS

1. Origin of heat and transformation to useful form. Complexity of fuels as sources of heat. General classification, solid, liquid, gaseous, natural and artificial 644
2. Natural solid fuels, wood peat, lignite, bituminous and anthracite coals. Chemical and physical properties. Classifications based on ultimate and proximate analysis and on behavior on heating 649
3. Calorific power of coals and the combustible of coals. Calculation of calorific power from ultimate and proximate analyses. Calorific power of the volatile 662
4. Mineral oil and natural gas fuel. Chemical and physical properties. Calorific power direct and as calculated for oils from ultimate analysis or from density, and for gas from sum of constituent gases 670

PAGE

5. Charcoal, coke, coke oven and retort coal gas as products of heating wood and coal. Chemical, physical, and calorific properties per pound. Calorific power of gases per cubic foot in terms of constituent gases. Yield of gas and coke per pound coal..... 675
6. Distillate oils, kerosene and gasoline, residue oils, and oil gas as products of heating mineral oils. Chemical, physical, and calorific properties. Calorific power of fractionated oils in terms of, (*a*) carbon and hydrogen; (*b*) density per pound, and estimated value per cubic foot vapor. Calorific power of oil gas per pound and per cubic foot in terms of constituent gases. Yield of distillates and oil gas..... 685
7. Gasification of fixed carbon and coke by air-blast reactions, producing air gas, and blast-furnace gas. Comparative yield per pound coke and air. Sensible heat and heat of combustion of gas. Relation of constituents in gas. Efficiency of gasification..... 695
8. Gasification of fixed carbon, coke, and coal previously heated, by steam-blast reactions, producing water gas. Composition and relation of constituents of water gas, yield per pound of steam and coal. Heat of combustion of gas and limitation of yield by negative heat of reaction..... 710
9. Gasification of coals by steam and air blasts, resulting in producer gas. Composition and relation of constituents of producer gas, yield per pound of fixed carbon, air and steam. Modification of composition by addition of volatile of coal. Heat of combustion of gas, sensible heat, and efficiency of gasification. Horse-power of gas producers..... 719
10. Combustion effects. Final temperature, volume and pressure for explosive and non-explosive combustion. Estimation of air weights and heat suppression due to CO in products from volumetric analysis..... 740
11. Temperature of ignition and its variation with conditions. Limits of proportion air gas neutral, or detonating gas and neutral, for explosive combustion of mixtures. Limits of adiabatic compression for self-ignition of mixtures..... 758
12. Rate of combustion of solid fuels with draft. Propagation rates, uniform and detonating for explosive gaseous mixtures..... 765
13. Steam-boiler evaporative capacity and horse-power. Horse-power units, equivalent rates of evaporation and of heat absorption. Factors of evaporation. Relation between absorption rates and rates of heat generation. Influence of heating and grate surface, calorific power of fuels, rates of combustion and furnace losses.... 773
14. Steam-boiler efficiency, furnace and heating-surface efficiency. Heat balances and variation in heat distribution. Evaporation and losses per pound of fuel..... 796

CHAPTER VI. HEAT AND WORK. GENERAL RELATIONS BETWEEN HEAT AND WORK. THERMAL EFFICIENCY OF STEAM, GAS, AND COMPRESSED-AIR ENGINES. FLOW OF EXPANSIVE FLUIDS. PERFORMANCE OF MECHANICAL REFRIGERATING SYSTEMS

1. General heat and work relations. Thermal cycles. Work and efficiency determination by heat differences and ratios. Graphic method of temperature entropy heat diagram..... 874
2. General energy equation between heat change, intrinsic energy change, and work done. Derived relations between physical constants for gases and for changes of state, solid to liquid, and liquid to vapor..... 882
3. Quantitative relations for primary thermal phases, algebraic, and graphic to PV, and $T\Phi$ coordinates. Constancy of PV, and T for gases and vapors, wet, dry and superheated..... 892
4. Quantitative relations for secondary thermal phases. Adiabatics for gases and vapors. Constant quality, constant total heat, and logarithmic expansion lines for steam..... 904

PAGE

5. Thermal cycles representative of heat-engine processes. Cyclic efficiency. A reference standard for engines and fuel-burning power systems. Classification of steam cycles 927
6. The Rankine cycle. Work, mean effective pressure, jet velocity, water rate, heat consumption and efficiency of steam Cycle I. Adiabatic expansion, constant pressure, heat addition and abstraction, no compression 936
7. Carnot cycle and derivatives. Work, mean effective pressure, water rate, heat consumption and efficiency of steam Cycle II. Adiabatic expansion and compression, constant pressure heat addition and abstraction 957
8. Gas cycles representative of ideal processes and standards of reference for gas engines 970
9. Brown, Lenoir, Otto and Langen non-compression gas-cycles. Work, mean effective pressure, volume and pressure ranges, efficiency, heat and gas consumption 978
10. Stirling and Ericsson gas cycles. Work, efficiency and derived quantities for isothermal compression with and without regenerators 993
11. Otto, Atkinson, Brayton, Diesel, and Carnot gas cycles. Work, efficiency and derived quantities for adiabatic compression gas cycles 1006
12. Comparison of steam and gas cycles with the Rankine as standard for steam, and with the Otto and Diesel, as standard for gas. Relations of Otto and Diesel to Rankine cycle. Conditions for equal efficiency 1031
13. Gas cycle performance as affected by variability of the specific heats of gases, applied to the Otto cycle 1035
14. Actual performance of Otto and Diesel gas engines, and its relation to the cyclic. Diagram factors for mean effective pressure and thermal efficiency. Effect of load on efficiency. Heat balance of gas engines alone and with gas producers 1042
15. Actual performance of piston steam engines and steam turbines at their *best load* and its relation to the cyclic. Effect on efficiency of initial pressure, vacuum, superheat, jacketing, and reheating. Heat balances of steam power plants 1062
16. Flow of hot water, steam and gases through orifices and nozzles. Velocity, weight per second, kinetic energy, and force of reaction of jets. Nozzle friction and reheating and coefficient of efflux. Relative proportions of series nozzles for turbines for proper division of work of expansion 1083
17. Flow of expansive fluids under small pressure drops through orifices, valves, and Venturi tubes. Relation between loss of pressure and flow. Velocity heads and quantity of flow by Pitot tubes 1097
18. Flow of gases and vapors in pipes, flues, ducts, and chimneys. Relation between quantity of flow and loss of pressure. Friction resistances. Draught and capacity of chimneys 1111
19. Thermal efficiency of compressed-air engines alone and in combination with air compressors. Effect of preheating and reheating. Compressor suction heating, and volumetric efficiency. Wall action 1127
20. Mechanical refrigeration, general description of processes and structures. Thermal cycles and refrigerating fluids. Limiting temperatures and pressures 1142
21. Performance of mechanical refrigerating cycles and systems. Quantity of fluid circulated per minute per ton refrigeration, horse-power, and heat supplied per ton. Refrigeration per unit of work done and its relation to thermal efficiency of the system 1157

LIST OF TABLES

NO.		PAGE
1.	Conversion table of units of distance	62
2.	Conversion table of units of surface	62
3.	Conversion table of units of volume	62
4.	Conversion table of units of weights and force	63
5.	Conversion table of units of pressure	63
6.	Conversion table of units of work	64
7.	Conversion table of units of power	64
8.	Units of velocity	64
9.	Barometric heights, altitudes and pressures	65
10.	Values of s in the equation PV^s = constant for various substances and conditions	67
11.	Horse-power per pound mean effective pressure	68
12.	Ratio of cut-offs in the two cylinders of the compound engine to give equal work for any receiver volume	284
13.	Piston positions for any crank angle	395
14.	Values for x for use in Heck's formula for missing water	396
15.	Some actual steam engine dimensions	396
16.	Fixed temperatures	411
17.	Fahrenheit temperatures by hydrogen and mercury thermometers	414
18.	Freezing-point of calcium chloride brine	425
19.	Specific heat of sodium chloride brine	427
20.	Specific heat and gas constants,	431
21.	The critical point	453
22.	Juhlin's data on the vapor pressure of ice	456
23.	Tamman's value on fusion pressure and temperature of water-ice	456
24.	Lowering of freezing-points	465
25.	Berthelot's data on heat for complete dilution of ammonia solutions	500
26.	Air required for combustion of various substances	515
27.	Radiation coefficients	535
28.	Coefficients of heat transfer	550
29.	Temperatures, Centigrade and Fahrenheit	571
30.	Heat and power conversion table	573
31.	Specific heat of solids	574
32.	Specific heats of liquids	576
33.	Baumé-specific gravity scale	577
34.	Specific heats of gases	578
35.	Coefficient of linear expansion of solids	580
36.	Coefficient of cubical expansion of solids	581
37.	Coefficient of volumetric expansion of gases and vapors at constant pressure	582

NO. PAGE

38. Coefficient of pressure rise of gases and vapors at constant volume 583
39. Compressibility of gases by their isothermals 584
40. Values of the gas constant R 584
41. Density of gases 585
42. International atomic weights 586
43. Melting- or freezing-points 586
44. Boiling-points 588
45. Latent heats of vaporization 590
46. Latent heats of fusion 591
47. Properties of saturated steam 592
48. Properties of superheated steam 596
49. Properties of saturated ammonia vapor 603
50. Properties of saturated carbon dioxide vapor 618
51. Relation between pressure, temperature and per cent NH_3 in solution 628
52. Values of partial pressure of ammonia and water vapors for various temperatures and per cents of ammonia in solution 632
53. Absorption of gases by liquids 634
54. Absorption of air in water 635
55. Heats of combustion of fuel elements and chemical compounds 636
56. Internal thermal conductivity 639
57. Relative thermal conductivity 642
58. General classification of fuels 648
59. Comparison of cellulose and average wood compositions 650
60. Classification of coals by composition 652
61. Classification of coals by gas and coke qualities 654
62. Composition of peats 655
63. Composition of Austrian lignites 656
64. Composition of English coking coals 658
65. Wilkesbarre anthracite coal sizes and average ash content 659
66. Density and calorific power of natural gas 673
67. Products of wood distillation 676
68. Products of peat distillation 678
69. Products of bituminous coal distillation 680
70. Gas yield of English cannel coals 682
71. Comparison of coke oven and retort coal gas 682
72. Relation between oxygen in coal and hydrocarbon in gas 684
73. Density and calorific power of coke oven gas 684
74. Average distillation products of crude mineral oils 686
75. American mineral oil products 687
76. U.S. gasolene and kerosene bearing crude oils 688
77. Calorific power of gasolenes and kerosenes 691
78. Properties of oil-gas 693
79. Yield of retort oil gas 694
80. Density and calorific power of oil gas 694
81. Boudouard's equilibrium relations for CO and CO_2 with temperature 697
82. Change of O_2 in air to CO and CO_2 at 1472° F. 699
83. Composition of hypothetical air gas, general 704
84. Composition of hypothetical air gas, no CO_2 and no CO 705
85. Density and calorific power of blast furnace gas 708
86. Water gas characteristics with bed temperature 710
87. Composition of hypothetical water gas, general 714
88. Composition of hypothetical water gas, no CO_2 and no CO 715

NO.		PAGE
89.	Density and calorific power of water gas	718
90.	Composition of hypothetical producer gas from fixed carbon	725
91.	Density and calorific power of producer gas	737
92.	Characteristics of explosive mixtures of oil gas and air	747
93.	Calculated ignition temperatures for producer gas	761
94.	Compressions commonly used in gas engines	762
95.	Ignition temperatures	763
96.	Variation of ignition temperature of charcoal with distillation temperature	763
97.	Per cent detonating mixture at explosive limits of proportion	764
98.	Velocity of detonating or explosive waves	766
99.	Rate of propagation, feet per second, for different proportions in gaseous mixtures	768
100.	Rates of combustion for coal	769
101.	Constants of proportion for rate of coal combustion for use in Eq. (848)	771
102.	Boiler efficiency summaries	799
103.	Three examples of heat balance for boilers	800
104.	Composition and calorific power of characteristic coals	818
105.	Combustible and volatile of coals, lignites and peats	826
106.	Paraffines from Pennsylvania petroleums	835
107.	Calorific power of mineral oils by calorimeter and calculation by density formula of Sherman and Kropff	836
108.	Properties of mineral oils	838
109.	Composition of natural gases	841
110.	Composition of coke oven and retort coal gas	842
111.	Composition of U. S. coke	846
112.	Fractionation tests of kerosenes and petroleums	847
113.	Fractionation tests of gasolenes	851
114.	Composition of blast-furnace gas and air gas	853
115.	Rate of formation of CO from CO_2 and carbon	855
116.	Composition of water gas	857
117.	Composition of producer gas	858
118.	Gas producer tests	864
119.	Composition of oil producer gas	866
120.	Composition of powdered coal producer gas	866
121.	Calorific powers of best air-gas mixtures	867
122.	Composition of boiler-flue gases	868
123.	Limits of proportions of explosive air-gas mixtures	869
124.	Rate of combustion of coal with draft	870
125.	Rate of combustion of coal	871
126.	Values of s for adiabatic expansion of steam	912
127.	Values of s for adiabatic expansion of steam determined from initial and final volumes only	913
128.		1042
129.	Diagram factors for Otto cycle gas engines	1046
130.	Mechanical efficiencies of gas engines	1050
131.	Allowable compression for gas engines	1050
132.	Mean effective pressure factors for Otto cycle engines	1053
133.	Güldner's values of Otto engine real volumetric efficiency with estimated mean suction resistances	1055
134.	Comparative heat balances of gas producer and engine plants	1057
135.	Heat balances of gas producer plants	1060
136.	Heat balances of gas and oil engines	1060
137.	Steam plant heat balances	1063

NO. PAG

138. Efficiency factors for reciprocating steam engines and turbines 106
139. Steam turbine efficiency and efficiency factors with varying vacuum and with steam approximately at constant initial pressure 107
140. Efficiency factors for low-pressure steam in piston engines 107
141. Coefficient of discharge for various air pressure and diameters of orifice (Durley). 110
142. Values of C for air flow (Weisbach) 110
143. Flow change resistance factors F_R (Reitschel) 112

TABLE OF SYMBOLS

A = area in square feet.
= constant, in formula for most economical load of a steam engine, Chapter III.
= constant, in pipe flow formula, Chapter VI.
= excess air per pound of coal, Chapter V.
= pounds of ammonia dissolved per pound of weak liquor, Chapter IV.
a = area in square inches.
= coefficient of linear expansion, Chapter IV.
= constant in equation for the ratio of cylinder sizes for equal work distribution in compound engine, Chapter III.
= constant in equation for change in intrinsic energy, Chapter VI.
= constant in equation for specific heat, Chapter IV.
= cubic feet of air per cubic foot of gas in explosive mixtures, Chapter V.
= effective area of piston, square inches, Chapter I.

B = constant in equation for the most economical load of the steam engine, Chapter III.
= constant in equation for flow in pipes, Chapter VI.
Bé. = Baumé.
B.H.P. = brake horse-power, Chapters III and VI.
= boiler horse-power, Chapter V.
B.T.U. = British thermal unit.
b = constant in equation for change in intrinsic energy, Chapter VI.
= constant in equation for specific heat, Chapter IV.
(bk.pr.) = back pressure in pounds per square inch.

C = Centigrade.
= circumference or perimeter of ducts in equations for flow, Chapter VI.
= constant.
= heat suppression factor, Chapter V.
= ratio of pressure after compression to that before compression in gas engine cycles, Chapter VI.
= specific heat, Chapter IV.
C_C = per cent of ammonia in weak liquor, Chapter VI.
C_p = specific heat at constant pressure.
C_R = per cent of ammonia in rich liquor, Chapter VI.
C_s = specific heat of water, Chapter VI.
C_v = specific heat at constant volume.
Cl = clearance expressed in cubic feet.
c = clearance expressed as a fraction of the displacement
= constant.
cu.ft. = cubic foot.
cu.in. = cubic inch.

D = constant in equations for pipe flow, Chapter VI.
= density, Chapter IV.
= diameter of pipe in feet, Chapter VI.
= displacement in cubic feet.
D_s = specific displacement, Chapter I.

d = constant in equation for change in intrinsic energy, Chapter VI.
= diameter of a cylinder in inches, Chapter I.
= diameter of pipe in inches, Chapter VI.
= differential.
(del.pr.) = delivery pressure in pounds per square inch, Chapter II.

E = constant in equation for pipe flow, Chapter VI.
= external latent heat, Chapter IV.
= thermal efficiency, Chapter VI.
E_B = thermal efficiency referred to brake horse-power, Chapter III.
E_b = boiler efficiency, Chapter V.
E_f = furnace efficiency, Chapter V.
E_I = thermal efficiency referred to indicated horse-power, Chapter III.
E_m = mechanical efficiency, Chapter III.
E_s = heating surface efficiency, Chapter V.
E_V = volumetric efficiency (apparent), Chapter VI.
$E_{V'}$ = volumetric efficiency (true), Chapter VI.
e = as a subscript to log to designate base e.
= constant in equation for change in intrinsic energy, Chapter VI.
e_1 = ratio of true volumetric efficiency to hypothetical, Chapter II.
e_2 = ratio of true volumetric efficiency to apparent, Chapter III.
e_3 = ratio of true indicated horse-power to hypothetical, Chapter II.

F = constant in equation for pipe flow, Chapter VI.
= diagram factor for gas engine indicator cards, Chapter VI.
= Fahrenheit.
= force in pounds.
F_F = friction factor, $F_F \times$ velocity head = loss due to friction, Chapter VI.
F_R = resistance factor, $F_R \times$ velocity head = loss due to resistances, Chapter VI.
F_S = special resistances to flow in equations for chimney draft, Chapter VI.
f = constant in equation for changes in intrinsic energy, Chapter VI.
= function.
ft. = foot.
ft.-lb. = foot-pound.

G = constant in equation for pipe flow, Chapter VI.
= weight of gas per hour in equation for chimney flow, Chapter VI.
G_m = maximum weight of gases in equation for chimney flow, Chapter VI.
G. S. = grate surface.
g = acceleration due to gravity, 32.2 (approx.) feet per second, per second.

H = as a subscript to denote high pressure cylinder.
= heat per pound of dry saturated vapor above 32° F.
= heat per cubic foot gas.
= heat transmitted, Chapter IV.
= height of column of hot gases in feet, Chapter VI.
= pressure or head in feet of fluid, Chapter VI.
H_A = difference in pressure on two sides of an orifice in feet of air, Chapter VI.
H_G = equivalent head of hot gases, Chapter VI.
H_M = pressure in feet of mercury, Chapter VI.
H_W = pressure in feet of water, Chapter VI.
H.P. = high pressure.
= horse-power, Chapter I.

H.S. = heating surface.
H.P.cap.) = high pressure cylinder capacity, Chapter III.
h = heat of superheat.
h_M = difference in pressure on two sides of an orifice in inches of mercury, Chapter VI.
h_W = difference in pressure on two sides of an orifice in inches of water, Chapter VI.

I = as a subscript to denote intermediate cylinder, Chapter III.
I.H.P. = indicated horse-power.
in. = inch.
in.pr.) = initial pressure in pounds per square inch.

J = Joule's equivalent = 778 (approx.) foot-pounds per B.T.U.

K = coefficient of thermal conductivity, Chapter IV.
= constant.
= proportionality coefficient in equation for draft, Chapter VI.
K_e = engine constant $= \frac{Lan}{33000}$ in expression for horse-power. Chapter III.

L = as a subscript to denote low-pressure cylinder.
= distance in feet.
= latent heat, Chapters IV and VI.
= length of stroke in feet, Chapter I.
L = per cent of heat in fuel lost in furnace, Chapter V.
L.P. = low pressure.
(L.P.Cap.) = low-pressure capacity, Chapter II.
l = constant, Chapter III.
= length, Chapter IV.
lb. = pound.
log = logarithm to the base 10.
$\log_e$ = logarithm to the base e.

M = mass.
(M.E.P.) = mean effective pressure, pounds per square foot.
m = constant, Chapter III.
= mean hydraulic radius $= \frac{\text{area}}{\text{perimeter}}$.
= molecular weight, Chapter IV.
= ratio of initial pressure to that end of expansion in Otto and Langen gas cycle, Chapter VI.
(m.b.p.) = mean back pressure in pounds per square inch.
(m.e.p.) = mean effective pressure in pounds per square inch.
(m.f.p.) = mean forward pressure in pounds per square inch.

N = constant, Chapter III.
= revolutions per minute.
n = cycles per minute.
= constant, Chapter III.
= cubic foot of neutral per cubic foot of gaseous mixture, Chapter V.
= number of degrees exposed on thermometer stem, Chapter IV.
= ratio of volume after expansion to volume before in Atkinson gas cycle, Chapter IV.
= specific volume of dry saturated steam, Chapter VI.

O = volume of receiver of compound engine in cubic feet, Chapter III.
P = draft in pounds per square foot, Chapter VI.
= load in kilowatts, Chapter III.
= pressure in pounds per square foot.
P_F = static pressure in pounds per square foot lost in wall friction, Chapter VI.
P_R = static pressure in pounds per square foot lost in changes of cross-section, etc., Chapter VI.
P_V = velocity head in pounds per square foot.
p = pressure in pounds per square inch.
p_e = mean exhaust pressure, Chapter VI.
p_s = mean suction pressure, Chapter VI.
p_V = partial pressure of water vapor in air, Chapter VI.

Q = quantity of heat or energy in B.T.U. gained or lost by a body passing from one state to another.
Q_1' = heat added from fire in Stirling and Ericsson cycles, Chapter VI.
Q_1'' = heat added from regenerator in Stirling and Ericsson cycles, Chapter VI.
Q_2' = heat abstracted by water jacket in Stirling and Ericsson cycles, Chapter VI.
Q_2'' = heat abstracted by regenerator in Stirling and Ericsson cycles, Chapter VI.
q = quantity of heat per pound of liquid above 32° F.

R = ratio of heating surface to grate surface, Chapter V.
= gas constant.
R_C = ratio of cylinder sizes in two-stage air compressor or compound engine, Chapters II and III.
R_H = ratio of expansion in high-pressure cylinder, Chapter III.
R_L = ratio of expansion in low-pressure cylinder, Chapter III.
R_P = ratio of initial to back pressure, Chapters III and VI.
R_p = ratio of delivery to supply pressure, Chapter II.
R_V = ratio of larger volume to smaller volume.
r = rate of flame propagation in explosive mixtures, Chapter V.
r_P = pressure differences (maximum − minimum) in gas cycles, Chapter VI.
r_V = volume differences (maximum—minimum) in gas cycles, Chapter VI.
(rec.pr.) = receiver pressure in pounds per square inch, Chapter III.
(rel.pr.) = release pressure in pounds per square inch, Chapter III.

S = per cent of ammonia in solution, Chapter IV.
= piston speed, Chapter I.
= pounds of steam per pound of air in producer blast, Chapter V.
= specific heat, Chapter IV.
= specific heat of superheated steam, Chapter VI.
(Sup.Vol.) = volume of steam supplied to the cylinder per stroke, Chapter III.
s = general exponent of V in expansion or compression of gases.
sp.gr. = specific gravity.
sp.ht. = specific heat.
sq.ft. = square foot.
sq.in. = square inch.
(sup.pr.) = supply pressure, in pounds per square inch.

T = temperature, degrees absolute.
T_C = temperature of air, Chapter VI.
T_H = temperature of gases in chimney, Chapter VI.
t = temperature in degrees scale.

J = rate of heat transfer in B.T.U. per square foot per hour per degree difference in temperature, Chapter IV.
$\bar{J}$ = intrinsic energy, Chapter VI.
u = velocity in feet per second.
m = velocity in feet per minute, Chapter VI.

V = volume in cubic feet.
v_A = cubic feet per pound air, Chapter VI.
v_G = cubic feet per pound, gas, Chapter VI.
v_L = volume of liquid in cubic feet per pound.
v_S = volume of solid in cubic feet per pound.
v_V = volume of vapor in cubic feet per pound.
v = volume, Chapter IV.

W = work in foot-pounds.
W.R. = water rate.
w = pounds of water per pound of ammonia in solution, Chapter IV.
= weight in pounds.
w_R = pounds of rich liquor per pound of ammonia, Chapter VI.

$\mathfrak{X}$ = compression in the steam engine as a fraction of the stroke, Chapter III.
$= 1 + \frac{\text{heat added}}{\text{temperature at beginning of addition} \times \text{specific heat at constant volume}}$, Chapter VI.
x = constant in the expression for missing water, Chapter III.
= fraction of liquid made from solid or vapor made from liquid, Chapter VI.
= per cent of carbon burned to CO_2, Chapter V.
= per cent of nozzle reheat, Chapter VI.
= per cent of steam remaining in high-pressure cylinder of compound engine at any point of the exhaust stroke, Chapter III.
= quantity of heat added in generator of absorption system in addition to the amount of heat of absorption of 1 lb. of ammonia, Chapter VI.
= ratio of low-pressure admission volume to high-pressure admission volume, Chapter III.

Y = total steam used per hour by an engine, Chapter III.
$= 1 + \frac{\text{heat added}}{\text{temperature at beginning of addition} \times \text{specific heat at constant pressure}}$, Chapter VI.
y = per cent of vane reheat Chapter VI.
= ratio of the volume of receiver to that of the high-pressure cylinder of the compound engine, Chapter III.

Z = fraction of the stroke of the steam engine completed at cut-off, Chapter III.
$= 1 + \frac{\text{heat added from regenerator}}{\text{temperature at beginning of addition} \times \text{specific heat at constant volume}}$, Chapter VI.
Z' = hypothetical best value of Z.
$= 1 + \frac{\text{heat added from regenerator}}{\text{temperature at beginning of addition} \times \text{specific heat at constant pressure}}$, Chapter VI.
z = ratio of R.P.M. to cycles per minute.

α = an angle, Chapter I.
= coefficient of cubical expansion, Chapter III.
= constant in the equation for latent heat, Chapter VI.

α = constant in equation for variable specific heat at constant volume, Chapter VI.
α' = constant in equation for variable specific heat at constant pressure, Chapter VI.

β = constant in equation for latent heat, Chapter VI.
= fraction of fuel heat available for raising temperature, Chapter V.

γ = constant in equation for latent heat, Chapter VI.
= ratio of cross-section to perimeter, Chapter IV.
= special value for s for adiabatic expansion or compression = $\frac{\text{sp. ht. at const. press.}}{\text{sp. ht. at const. vol.}}$
γ' = ratio of specific heat at constant pressure to specific heat at constant volume when ea
is a variable, Chapter VI.

Δ = increment.
δ = density in pounds per cubic foot.
δ_C = density in cold gases in equations for chimney draft, Chapter VI.
δ_H = density of hot gases in equations for chimney draft, Chapter VI.

ζ = coefficient of friction, Chapter VI.

μ = material coefficient in heat transfer expression, Chapter IV.

ρ = internal thermal resistance, Chapter IV.

Σ = summation.
σ = surface thermal resistance, Chapter IV.

τ = time in seconds.

Φ = entropy, Chapter VI.
ϕ = entropy, Chapter VI.

NOTE. A small letter when used as a subscript to a capital in general refers to a po
on a diagram, e.g., P_a designates pressure at the point A. Two small letters used as su
scripts together, refer in general to a quantity between two points, e.g., W_{ab} designa
work done from point A to point B.

ENGINEERING THERMODYNAMICS

CHAPTER I

WORK AND POWER. GENERAL PRINCIPLES.

1. Work Defined. Work, in the popular sense of performance of any labor, is not a sufficiently precise term for use in computations, but the analytical mechanics has given a technical meaning to the word which is definite and which is adopted in all thermodynamic analysis. The mechanical definition of work is mathematical inasmuch as work is always a product of forces opposing motion and distance swept through, the force entering with the product being limited to that acting in the direction of the motion. The unit of distance in the English system is the foot, and of force the pound, so that the unit of work is the foot-pound. In the metric system the distance unit is the meter and the force unit the kilogramme, making the work unit the kilogrammeter. Thus, the lifting of one pound weight one foot requires the expenditure of one foot-pound of work, and the falling of one pound through one foot will perform one foot-pound of work. It is not only by lifting and falling weights that work is expended or done; for if any piece of mechanism be moved through a distance of one foot, whether in a straight or curved path, and its movement be resisted by a force of one pound, there will be performed one foot-pound of work against the resistance. It is frequently necessary to transform work from one system of units to the other, in which case the factors given at the end of this Chapter are useful.

Work is used in the negative as well as in the positive sense, as the force considered resists or produces the motion, and there may be both positive and negative work done at the same time; similar distinctions may be drawn with reference to the place or location of the point of application of the force. Consider, for example, the piston rod of a direct-acting pump in which a certain force acting on the steam end causes motion against some less or equal force acting at the water end. Then the work at the steam end of the pump may be considered to be positive and at the water end negative, so far as the movement of the rod is concerned; when, however, this same movement causes a movement of the water, work done at the water end (although negative with reference to the rod motion, since it opposes that motion) is positive with refer-

ence to the water, since it causes this motion. It may also be said that the steam does work on the steam end of the rod and the water end of the rod does work on the water, so that one end receives and the other delivers work, the rod acting as a transmitter or that the work performed at the steam end is the input and that at the water end the output work.

(See the end of Chapter I for Tables I, II, III, IV, and VI, Units of Distance, of Surface, of Volume, of Weight and Force, and of Work.)

Example. An elevator weighing 2000 lbs. is raised 80 ft. How much work is done in foot-pounds?

$$\text{Foot-pounds} = \text{force} \times \text{distance}$$
$$= 2000 \times 80 = 160{,}000 \text{ ft.-lbs.}$$

Ans. 160,000 ft.-lbs.

Prob. 1. A pump lifts 150 gallons of water to a height of 250 ft. How much work does it do?

Prob. 2. By means of a jack a piece of machinery weighing 10 tons is raised $\frac{3}{4}$ in. What is the work done?

Prob. 3. A rifle bullet weighing 2 oz. travels vertically upward $1\frac{1}{4}$ miles. What work was done in foot-pounds?

Prob. 4. A cubic foot of water falls 50 ft. in reaching a water-wheel. How much work can it do?

Prob. 5. A piston of an elevator is 12 ins. in diameter and has acting on it a pressure of 80 lbs. per square inch. What work is done per foot of travel?

Prob. 6. It has been found that a horse can exert 75 lbs. pull when going 7 miles per hour. How much work can be done per minute?

Prob. 7. How much work is done by an engine which raises a 10-ton casting 50 ft.?

Prob. 8. The pressure of the air on front of a train is 50 lbs. per square foot when the speed is 50 miles per hour. If the train presents an area of 50 sq.ft., what work is done in overcoming wind resistance?

Prob. 9. The pressure in a 10-inch gun during the time of firing is 2000 lbs. per square inch. How much work is done in ejecting the projectile if the gun is 33 ft. 4 ins. long?

2. Power Defined. Power is defined as the rate of working or the work done in a given time interval, thus introducing a third unit of mechanics, time, so that power will always be expressed as a quotient, the numerator being a product of force and distance, and the denominator time. This is in opposition to the popular use of the word, which is very hazy, but is most often applied to the capability of performing much work or the exertion of great force, thus, popularly, a powerful man is one who is strong, but in the technical sense a man would be powerful only when he could do much work continuously and rapidly. An engine has large power when it can perform against resistance many foot-pounds per minute. The unit of power in the English system is the horse-power, or the performance of 550 foot-pounds per second or 33,000 foot-pounds per minute, or 1,980,000 foot-pounds per hour. In the metric system the horse-power is termed cheval-vapeur, and is the performance of 75 killogrammeters $= 542\frac{1}{2}$ foot-pounds per second, or 4500 kilogrammeters $= 32{,}549$ foot-pounds

per minute, or 270,000 kilogrammeters = 1,952,932 foot-pounds per hour. Table VII at the end of Chapter I gives conversion factors for power units.

Example. The piston of a steam engine travels 600 ft. per minute and the mean force of steam acting upon it is 65,000 lbs. What is the horse-power?

$$\text{Horse-power} = \frac{\text{foot-pounds per minute}}{33{,}000}$$

$$= \frac{\text{force} \times \frac{\text{distance}}{\text{time}}}{33{,}000}$$

$$= \frac{65{,}000 \times 600}{33{,}000} = 1182 \text{ H.P.}$$

Prob. 1. The draw-bar pull of a locomotive is 3000 lbs. when the train is traveling 50 miles per hour. What horse-power is being developed?

Prob. 2. A mine cage weighing 2 tons is lifted up a 2000-ft. shaft in 40 seconds. What horse-power will be required if the weight of the cable is neglected?

Prob. 3. By direct pull on a cable it is found possible to lift 4 tons 20 ft. per second. With a differential pulley 40 tons may be lifted 3 ft. per second. What is the difference in power required?

Prob. 4. A horse exerts a pull of 100 lbs. on a load. How fast must the load be moved to develop one horse-power?

Prob. 5. The resistance offered to a ship at a speed of 12 knots was 39,700 lbs. What horse-power must be available to maintain this speed? (One knot is a speed of one nautical mile per hour.)

Prob. 6. It is estimated that 100,000 cu. ft. of water go over a fall 60 ft. high every second. What horse-power is going to waste?

Prob. 7. The force acting on a piston of a pump is 80,000 lbs. If the piston speed is 150 ft. per minute, what is the horse-power?

Prob. 8. To draw a set of plows $2\frac{1}{2}$ miles per hour requires a draw-bar pull of 10,000 lbs. What must be the horse-power of a tractor to accomplish this?

Prob. 9. The horse-power to draw a car up a grade is the sum of the power necessary to pull it on a level and that necessary to lift it vertically the same number of feet as it rises on the grade. What will be the horse-power required to draw a car 20 miles per hour up a 12 per cent grade if the car weighs 2500 lbs. and the draw-bar pull on the level is 250 lbs.?

3. Work in Terms of Pressure and Volume. Another of the definitions of mechanics fixes pressure as *force per unit area* so that pressure is always a quotient, the numerator being force and the denominator area, or length to the second power. If, therefore, the pressure of a fluid be known, and according to hydromechanics it acts equally and normally over all surface in contact with it, then the force acting in a given direction against any surface will be the product of the pressure and the projected area of the surface, the projection being on a plane at right angles to the direction considered. In the case of pistons and plungers the line of direction is the axis of the cylinder, and the projected area is the area of the piston less the area of any rod passing completely through the fluid that may be so placed. When this plane area moves in a

direction perpendicular to itself, the product of its area and the distance will be the volume swept through, and if a piston be involved the volume is technically the *displacement* of the piston. Accordingly, work may be expressed in three ways, as follows:

$$\text{Work} = \text{force} \times \text{distance};$$
$$\text{Work} = \text{pressure} \times \text{area} \times \text{distance};$$
$$\text{Work} = \text{pressure} \times \text{volume}.$$

The product should always be in foot-pounds, but will be, only when appropriate units are chosen for the factors. These necessary factors are given as follows:

$$\begin{aligned}\text{Work in foot-pounds} &= \text{force in lbs.} \times \text{distance in ft.} \\ &= \text{pressure in lbs. per sq.ft.} \times \text{area in sq.ft.} \times \text{distance in ft} \\ &= \text{pressure in lbs. per sq.in.} \times \text{area in sq.in.} \times \text{distance in ft} \\ &= \text{pressure in lbs. per sq.ft.} \times \text{volume in cu.ft.} \\ &= \text{pressure in lbs. per sq.in.} \times 144 \times \text{volume in cu.ft.}\end{aligned}$$

As pressures are in practice expressed in terms not only as above, but also in heights of columns of common fluids and in atmospheres, both in English and metric systems, it is convenient for calculation to set down factors of equivalence as in Table V, at the end of the Chapter.

In thermodynamic computations the pressure volume product as an expression for work is most useful, as the substances used are always vapors and gases, which, as will be explained later in more detail, have the valuable property of changing volume indefinitely with or without change of pressure according to the mode of treatment. Every such increase of volume gives, as a consequence, some work, since the pressure never reaches zero, so that to derive work from vapors and gases they are treated in such a way as will allow them to change volume considerably with as much pressure acting as possible.

It should be noted that true pressures are always *absolute*, that is, measured above a perfect vacuum or counted from zero, while most pressure gages and other devices for measuring pressure, such as indicators, give results measured above or below atmospheric pressure, or as commonly stated, above or below atmosphere. In all problems involving work of gases and vapors, the absolute values of the pressures must be used; hence, if a gage or indicator measurement is being considered, the pressure of the atmosphere found by means of the barometer must be added to the pressure above atmosphere in order to obtain the absolute or true pressures. When the pressures are below atmosphere the combination with the barometric reading will depend on the record. If a record be taken by an indicator it will be in pounds per square inch below atmosphere and must be subtracted from the barometric equivalent in the same units to give the absolute pressure in pounds per square inch. When, however, a vacuum gage reads in inches of mercury below atmosphere, as such gages do, the difference between its reading and the barometric gives the absolute

ressure in inches of mercury directly, which can be converted to the desired nits by the proper factors.

While it is true that the barometer is continually fluctuating at every place, frequently happens that standards for various altitudes enter into calculations, nd to facilitate such work, values are given for the standard barometer at various titudes with equivalent pressures in pounds per square inch in Table IX.

Frequently in practice, pressures are given without a definite statement what units are used. Such a custom frequently leads to ambiguity, but it often possible to interpret them correctly from a knowledge of the nature of the roblem in hand. For instance, steam pressures stated by a man in ordinary prac-ce as being 100 lbs. may mean 100 lbs. per square inch gage (above atmosphere), ut may be 100 lbs. per square inch absolute. Steam pressures are then most ommonly stated per square inch and should be designated as either gage or abso-ute. Pressures of compressed air are commonly expressed in the same units as team, either gage or absolute, though sometimes in atmospheres. Steam pressures elow atmosphere may be stated as a vacuum of so many inches of mercury, neaning that the pressure is less than atmosphere by that amount, or may e given as a pressure of so many inches of mercury absolute, or as so many ounds per square inch absolute. The pressures of gases stored in tanks under igh pressure are frequently recorded in atmospheres, due to the convenience f computation of quantities on this basis. Pressures of air obtained by blowers r fans are usually given by the manufacturers of such apparatus in ounces er square inch above (or below) atmosphere. Such pressures and also differ-nces of pressure of air due to chimney draft, or forced draft, and the pressure f illuminating gas in city mains, are commonly stated in inches of water, each nch of water being equivalent to 5.196 lbs. per square foot. The pressure of vater in city mains or other pressure pipes may be stated either in pounds per quare inch or in feet of water head.

Example. A piston on which the mean pressure is 60 lbs. per square inch sweeps through volume of 300 cu.ft. What is the work done?

$$W = P \times V, \text{ where } V = \text{cu.ft. and } P = \text{lbs. per sq.ft.}$$
$$\therefore \quad W = 60 \times 144 \times 300 \quad = 2{,}592{,}000 \text{ ft.-lbs.}$$

Prob. 1. The mean pressure acting per square inch when a mass of air changes in volume from 10 cu.ft. to 50 cu.ft. is 40 lbs. per square inch. How much work is done?

Prob. 2. An engine is required to develop 30 H.P. If the volume swept through per minute is 150 cu.ft., what must the mean pressure be?

Prob. 3. The mean effective pressure in compressing air from one to five atmospheres is 28.7 lbs. per square inch. How many horse-power are required to compress 1000 cu.ft. of free air per minute?

Prob. 4. At an altitude of 4100 ft. a pressure gage showed the pressure on one side of a piston to be 50 lbs. per square inch while the pressure on the opposite side is 3 lbs. per square inch absolute. What pressure was tending to move the piston?

Prob. 5. At an altitude of 1 mile the mean pressure in a gas engine cylinder during the suction stroke was found to be 12 lbs. per square inch absolute. What work was done by the engine to draw in a charge if the cylinder was 5 ins. in diameter and the stroke 6 ins.?

Prob. 6. After explosion the piston of the above engine was forced out so that the ga volume was five times that at the beginning of the stroke. What must the M.E.P. hav been to get 20,000 ft.-lbs. of work?

Prob. 7. On entering a heating oven cold air expands to twice its volume. Wha work is done per cubic foot of air?

Prob. 8. A projectile is forced from a gun by a constant air pressure of 1000 lbs. pe square inch. Before it begins to move there is ½ a cu.ft. of air in the barrel, and at the instan it leaves the barrel the volume is 10 cu.ft. What work was done on the projectile?

Prob. 9. Water is forced from a tank against a head of 75 ft. by filling the tank wit compressed air. How much work is done in emptying a tank containing 1000 cu.ft.?

4. Work of Acceleration and Resultant Velocity. When a force acting on a mass is opposed by an equal resistance there may be no motion at all, o there may be motion of constant velocity. Any differences, however, betwee the two opposing forces will cause a change of velocity so long as the differenc lasts, and this difference between the two forces may be itself considered as th only active force. Observations on unresisted falling bodies show that they increase in velocity 32.16 ft. per second for each second they are free to fall and this quantity is universally denoted by g. If then, a body have an velocity, u_1, and be acted on by a force equal to its own weight in the direction of its motion for a time, τ seconds, it will have a velocity u_2 after that time.

$$u_2 = u_1 + g\tau. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It may be that the force acting is not equal to the weight of the body, in which case the acceleration will be different and so also the final velocity, due to the action of the force, but the force producing any acceleration will be to the weight of the body as the actual acceleration is to the gravitational acceleration. So that

$$\frac{\text{Actual acceleration force}}{\text{Weight of body or gravitational force}} = \frac{\text{actual acceleration}}{\text{gravitational acceleration } (g)},$$

and

$$\text{Actual accelerating force} = \frac{\text{weight of body}}{\text{gravitational acceleration } (g)} \times \text{actual acceleration.}$$

or

$$\text{Force} = \text{mass} \times \text{acceleration} = \text{mass} \times \frac{\text{change of velocity}}{\text{time of change}},$$

$$F = M \times \frac{u_2 - u_1}{\tau}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The work performed in accelerating a body is the product of the resistance met into the distance covered, L, while the resistance, or the above-defined force, acts, or while the velocity is being increased. This distance is the product of the time of action and the mean velocity, or the distance in feet,

$$L = \frac{u_2 + u_1}{2}\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The work is the product of Eqs. (2) and (3), or, work of acceleration is

$$W=\frac{M(u_2-u_1)}{\tau}\times\frac{(u_2+u_1)\tau}{2}$$

$$=\tfrac{1}{2}M(u_2^2-u_1^2)=\frac{w}{2\times 32.16}(u_2^2-u_1^2)=\frac{w}{64.32}(u_2^2-u_1^2), \quad . \quad (4)$$

where w is the weight in pounds. Exactly the same result will be obtained by the calculus when the acceleration is variable, so that Eq. (4) is of universal application.

The work performed in accelerating a body depends on nothing but its mass and the initial and final velocities, and is in every case equal to the product of half the mass and the difference between the squares of the initial and final velocities, or the product of the weight divided by 64.4 and the difference between the squares of the initial and final velocities.

It frequently happens that the velocity due to the reception of work is desired, and this is the case with nozzle flow in injectors and turbines, where the steam performs work upon itself and so acquires a velocity. In all such cases the velocity due to the reception of the work energy is

$$u_2=\sqrt{u_1^2+\frac{64.32W}{w}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where W is work in foot-pounds and w, as before, is weight in pounds. Or if the initial velocity be zero, as it frequently is,

$$u_2=\sqrt{\frac{2gW}{w}}=\sqrt{64.32\frac{W}{w}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For conversion of velocity units, Table VIII, at the end of the Chapter, is useful.

Example. A force of 100 lbs. acts for 5 seconds on a body weighing 10 lbs.; if the original velocity of the body was 5 ft. per second, what will be the final velocity, the distance traveled and the work done?

$$F=\frac{M(u_2-u_1)}{\tau};$$

$$100=\frac{10}{32.2}\,\frac{(u_2-5)}{5};$$

$$U_2=1615 \text{ ft. per second};$$

$$S=\left(\frac{u_2+u_1}{2}\right)\tau=4050 \text{ ft.}$$

$$W=\frac{M(u_2^2-u_1^2)}{2}=405{,}000 \text{ ft.-lbs.}$$

Prob. 1. A stone weighing $\frac{1}{2}$ lb. is dropped from a height of 1 mile. With what velocity and in what length of time will it strike if the air resistance is zero?

Prob. 2. A car moving 20 miles per hour and weighing 25 tons is brought to rest in 500 ft. What is the negative acceleration, the time required to stop, and the work done?

Prob. 3. Steam escapes through an opening with a velocity of half a mile per second. How many foot-pounds of energy were imparted to each pound of it to accomplish this?

Prob. 4. A weight of 100 lbs. is projected upward with a constant force of 200 lbs. How much further will it have gone at the end of 10 seconds than if it had been merely falling under the influence of gravity for the same period of time?

Prob. 5. A projectile weighing 100 lbs. is dropped from an aeroplane at the height of $\frac{1}{2}$ mile. How soon will it strike, neglecting air resistance?

Prob. 6. A water-wheel is kept in motion by a jet of water impinging on flat vanes. The velocity of the vanes is one-half that of the jet. The jet discharges 1000 lbs. of water per minute with a velocity of 200 ft. per second. Assuming no losses, what is amount of the work done?

Prob. 7. With the wind blowing 30 miles per hour, how much work could a 12-ft. windmill perform if 25 per cent of the available work were utilized.

NOTE. The weight of a cubic foot of air may be taken as .075 lb.

Prob. 8. An engine has a piston speed of 600 ft. per minute and runs at 150 R.P.M. If the reciprocating parts weigh 500 lbs., how much work is done in accelerating the piston during each stroke?

Prob. 9. A flywheel with rim 10 ft. in diameter to center of section and weighing 5 tons, revolves at a rate of 150 R.P.M.; 100,000 ft.-lbs. of work are expended on it. How much will the speed change?

5. Graphical Representation of Work. As work is always a product of force and distance or pressure and volume, it may be graphically expressed by

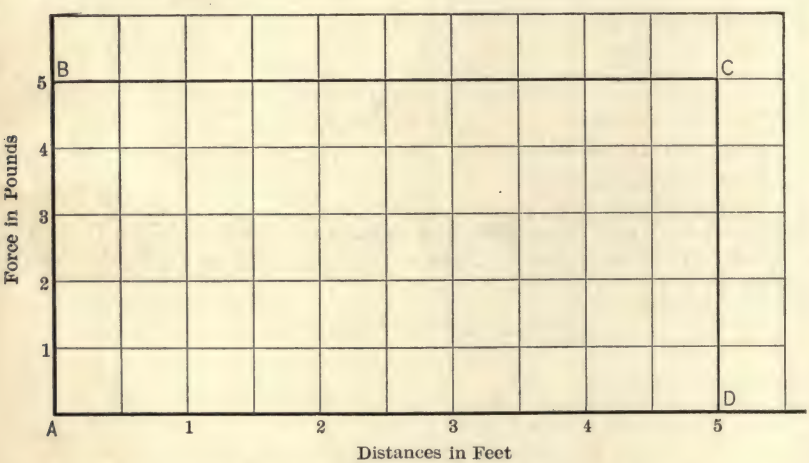


FIG. 1.—Constant Force, Work Diagram, Force-Distance Coordinates.

an area on a diagram having as coordinates the factors of the product. It is customary in such representations to use the horizontal distances for volumes and the vertical for pressures, which, if laid off to appropriate scale and in proper units, will give foot-pounds of work directly by the area enclosed. Thus in Fig. 1, if a force of 5 lbs. (AB) act through a distance of 5 ft. (BC) there will be performed 25 foot-pounds of work as indicated by the area of the

rectangle *ABCD*, which encloses 25 unit rectangles, each representing one foot-pound of work.

If a steam cylinder piston suffers a displacement of 5 cu.ft. under the steam pressure (absolute) of 5 lbs. per square foot then the operation which results in the performance of 25 foot-pounds of work is represented by the diagram Fig. 2, *ABCD*.

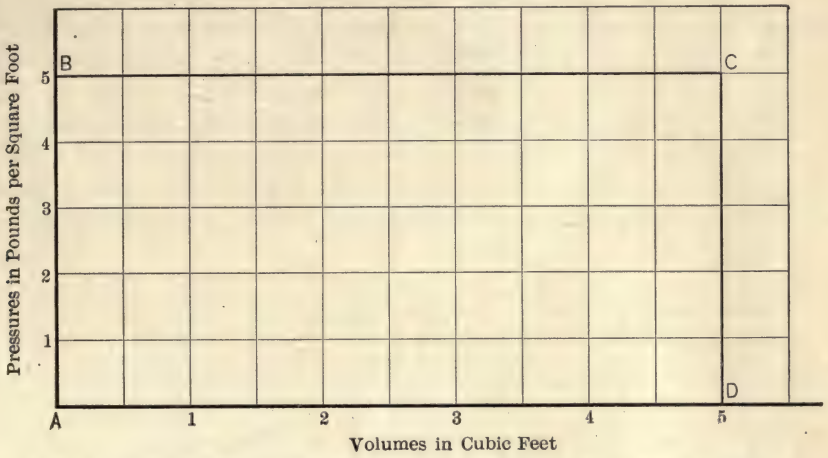


FIG. 2.—Constant Pressure Work Diagram, Pressure-Volume Coordinates.

Prob. 1. Following the method given for Fig. 1, draw a diagram for the example of Section 3

Prob. 2. By means of a diagram, show that the work done by a pressure of 1000 lbs. per square foot traversing a distance of 10 ft. is 10,000 ft.-lbs.

Prob. 3. Draw a diagram for the case of a volume change from 1 to 10 cu.ft. while the pressure acting is 20 lbs. per square inch.

Prob. 4. Draw a pressure volume diagram for the case of forcing a piston out of a cylinder by a water pressure of 15,000 lbs. per square foot, the volume of the cylinder at the start is $\frac{1}{2}$ cu.ft. and at the end 6 cu.ft. Make a diagram to scale and report work per square inch of diagram.

Prob. 5. A pump draws in water at a constant suction pressure of 14 lbs. and discharges it at a constant delivery pressure of 150 lbs. per sq.in. Considering the pump barrel to be empty at beginning of suction and end of delivery and to contain 3 cu.ft. when full, draw the diagram for this case and find the foot-pounds of work done.

Prob. 6. In raising a weight a man pulls on a rope with a constant force of 80 lbs. If the weight is lifted 40 ft., find from a diagram the work done.

Prob. 7. In working a windlass a force of 100 lbs. is applied at the end of a 6-ft. lever, the drum of th windlass being 1 ft. in diameter. Draw a work diagram for work applied and for work done in lifting if there be no loss in the windlass.

Prob. 8. The steam and water pistons of a pump are on the same rod and the area of the former is twice that of the latter, the stroke being 3 ft. Show by a diagram that the work done in the two cylinders is the same if losses be neglected.

Prob. 9. An engine exerts a draw-bar pull of 8000 lbs. at speed of 25 miles an hour. A change in grade occurs and speed increases to 40 miles per hour and the pull decreases to 5000 lbs. Show by a diagram the change in horse-power.

6. Work by Pressure Volume Change. Suppose that instead of being constant the pressure were irregular and, being measured at intervals of 1 cu.ft. displacement, found to be as follows:

Pressure. Lbs. per Sq.Ft.	Displacement Volume. Cu.Ft.
100	0
125	1
150	2
100	3
75	4
50	5

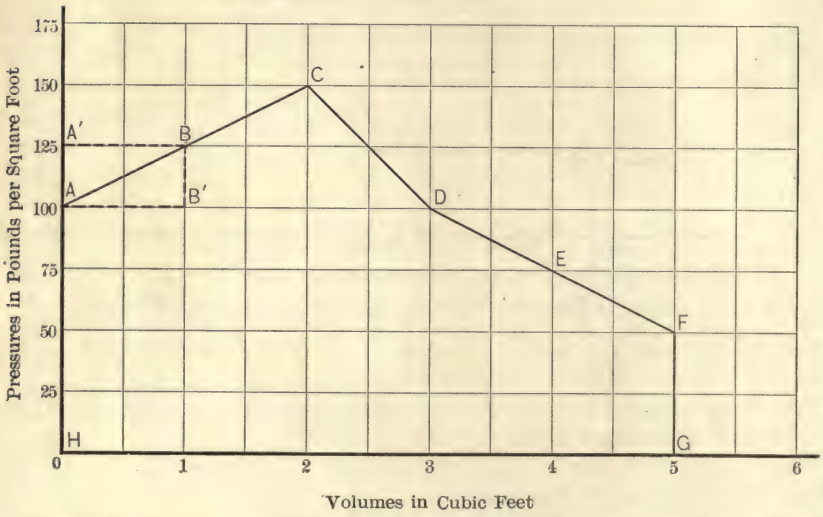


FIG. 3.—Work Diagram, Pressure-Volume Coordinates. Discontinuous Pressure-Volume Relations.

This condition might be plotted as in Fig. 3, A, B, C, D, E, F, G, H. The work done will be the area under the line joining the observation points. In the absence of exact data on the nature of the pressure variations between the two observation points A and B, a variety of assumptions might be made as to the precise evaluation of this area, as follows:

(*a*) The pressure may have remained constant at its original value for the first cubic foot of displacement, as shown dotted A–B', and then suddenly have risen to B. In this case the work done for this step would be 100 foot-pounds.

(*b*) Immediately after the measurement at A the pressure may have risen to A' and remained constant during displacement A' to B, in which case the work done would be 125 foot-pounds.

(*c*) The pressure may have risen regularly along the solid line AB, in which case the work area is a trapezoid and has the value $\frac{100+125}{2}\times 1 = 112.5$ foot-pounds.

It thus appears that for the exact evaluation of work done by pressure volume change, continuous data are necessary on the value of pressure with respect to the volume. If such continuous data, obtained by measurement or otherwise, be plotted, there will result a continuous line technically termed the pressure-volume curve for the process. Such a curve for a pressure volume change starting at 1 cu.ft. and 45 lbs. per square foot, and ending at 7 cu.ft., and 30 lbs. per square foot, is represented by Fig. 4, *A*, *B*, *C*, *D*, *E*.

The work done during this displacement under continuously varying pressure is likewise the area between the curve and the horizontal axis when pressures are laid off vertically, and will be in foot-pounds if the scale of pressures is pounds per square foot and volumes, cubic feet. Such an irregular area can be divided into small vertical rectangular strips, each so narrow that the pressure is sensibly constant, however much it may differ in different strips. The area of the rectangle is $P\Delta V$, each having the width ΔV and the height P, and the work

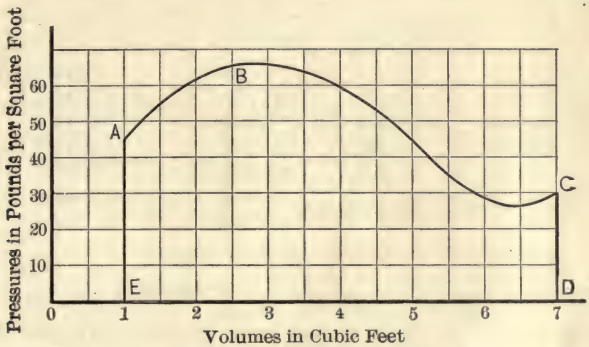


FIG. 4.—Work Diagram, Pressure-Volume Coordinates. Continuous Pressure-Volume Relations.

area will be exactly evaluated if the strips are narrow enough to fulfill the conditions of sensibly constant pressure in any one. This condition is true only for infinitely narrow strips having the width dV and height P, so that each has the area PdV and the whole area or work done is

$$W = \int PdV. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This is the general algebraic expression for work done by any sort of continuous pressure volume change. It thus appears that whenever there are available sufficient data to plot a continuous curve representing a pressure volume change, the work can be found by evaluating the area lying under the curve and bounded by the curve coordinates and the axis of volumes. The work done may be found by actual measurement of the area or by algebraic solution of Eq. (7), which can be integrated only when there is a known algebraic relation between the pressure and the corresponding volume of the expansive fluid, gas or vapor.

Prob. 1. Draw the diagrams for the following cases: (*a*) The pressure in a cylinder 12 ins. in diameter was found to vary at different parts of an 18-in. stroke as follows:

Pressure in Pounds per Sq.In.	Per Cent of Stroke.
100	0
100	10
100	30
100	50
83.3	60
71.5	70
62.5	80
55.5	90
50.0	100

(*b*) On a gas engine diagram the following pressures were found for parts of stroke.

In		Out			
V	*p*	*V*	*p*	*V*	*p*
0.25 cu.ft.	14.7	0.1	45.2	0.13	146.2
0.20 "	19.5	0.102	79.7	0.15	116.7
0.14 "	29.7	0.104	123.2	0.17	95.7
0.10 "	45.2	0.106	157.7	0.19	80.7
		0.108	181.7	0.21	68.7
		0.11	188.2	0.23	58.7
		0.12	166.2		

Prob. 2. Steam at a pressure of 100 lbs. per square inch absolute is admitted to a cylinder containing .1 cu.ft. of steam at the same pressure, until the cylinder contains 1 cu.ft., when the supply valve closes and the volume increases so that the product of pressure and volume is constant until a pressure of 30 lbs. is reached. The exhaust valve is opened, the pressure drops to 10 lbs. and steam is forced out until the volume becomes 1 cu.ft., when the exhaust valve closes and the remaining steam decreases in volume so that product of pressure and volume is constant until the original point is reached. Draw the pressure volume diagram for this case.

Prob. 3. During an air compressor stroke the pressures and volumes were as follows:

Volume in Cu.Ft.	Pressure in Lbs. Sq.In.
2.0	14.0
1.8	15.5
1.6	17.5
1.4	20.0
1.2	23.3
1.0	28.0
0.8	28.0
0.4	28.0
0.0	28.0

Draw the diagram to a suitable scale to give work area in foot-pounds directly.

Prob. 4. Draw the diagrams for last two problems of Section 3.

7. Work of Expansion and Compression. Any given quantity of gas or vapor confined and not subject to extraordinary thermal changes such as explosion, will suffer regular pressure changes for each unit of volume change, or conversely, suffer a regular volume change for each unit of pressure change, so that pressure change is dependent on volume change and vice versa. When the volume of a mass of gas or vapor, V_1, is allowed to increase to V_2 by the movement of a piston in a cylinder, the pressure will regularly increase or decrease from P_1 to P_2, and experience has shown that no matter what the gas or vapor or the thermal conditions, if steady, the volumes and pressures will have the relation for the same mass,.

$$P_1V_1^s = P_2V_2^s = K, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or the product of the pressure and s power of the volume of a given mass will always be the same. The exponent s may have any value, but usually lies between 1 and 1.5 for conditions met in practice.

The precise value of s for any given case depends on

(*a*) The substance.

(*b*) The thermal conditions surrounding expansion or compression, s being different if the substance receives heat from, or loses heat to, external surroundings, or neither receives nor loses.

(*c*) The condition of vapors as to moisture or superheat when vapors are under treatment.

Some commonly used values of s are given in Table X at the end of this chapter for various substances subjected to different thermal conditions during expansion or compression.

Not only does Eq. (8) express the general law of expansion, but it likewise expresses the law of compression for decreasing volumes in the cylinder with corresponding rise in pressure. Expansion in a cylinder fitted with a piston is called balanced expansion because the pressure over the piston area is balanced by resistance to piston movement and the mass of gas or vapor is substantially at rest, the work of expansion being imparted to the piston and resisting mechanism attached to it. On the other hand when the gas or vapor under pressure passes through a nozzle orifice to a region of lower pressure the falling pressure is accompanied by increasing volumes as before, but the work of expansion is imparted not to a piston, because there is none, but to the fluid itself, accelerating it until a velocity has been acquired as a resultant of the work energy received. Such expansion is termed free expansion and the law of Eq. (8) applies as well to free as to balanced expansion. This equation, then, is of very great value, as it is a convenient basis for computations of the work done in expansion or compression in cylinders and nozzles of all sorts involving every gas or vapor substance. Some expansion curves for different values of s are plotted to scale in Fig. 5, and the corresponding compression curves in Fig. 6, in which

Curve A has the exponent $s = 0$
Curve B " " $s = .5$

Curve C has the exponent $s=1.0$
Curve D " " $s=1.1$
Curve E " " $s=1.2$
Curve F " " $s=1.3$
Curve G " " $s=1.4$
Curve H " " $s=1.5$

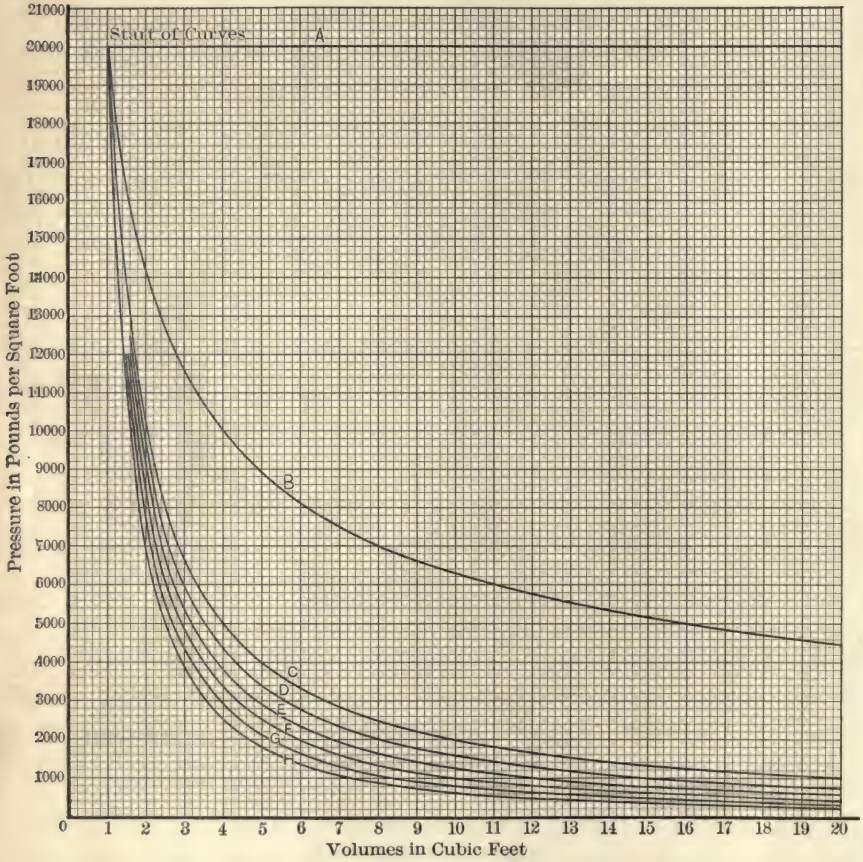


FIG. 5.—Comparison of Expansion Lines having Different Values of s.

The volume after expansion is given by

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{s}}, \quad \ldots \ldots \ldots \quad (9)$$

so that the final volume depends on the original volume, on the ratio of the two pressures and on the value of the exponent. Similarly, the pressure after expansion

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{s}, \quad \ldots \ldots \ldots \quad (10)$$

epends on the original pressure, on the ratio of the two volumes and on he exponent.

The general equation for the work of expansion or compression can now be ntegrated by means of the Eq. (8), which fixes the relation between pressures nd volumes. From Eq. (8),

$$P = \frac{K}{V^s},$$

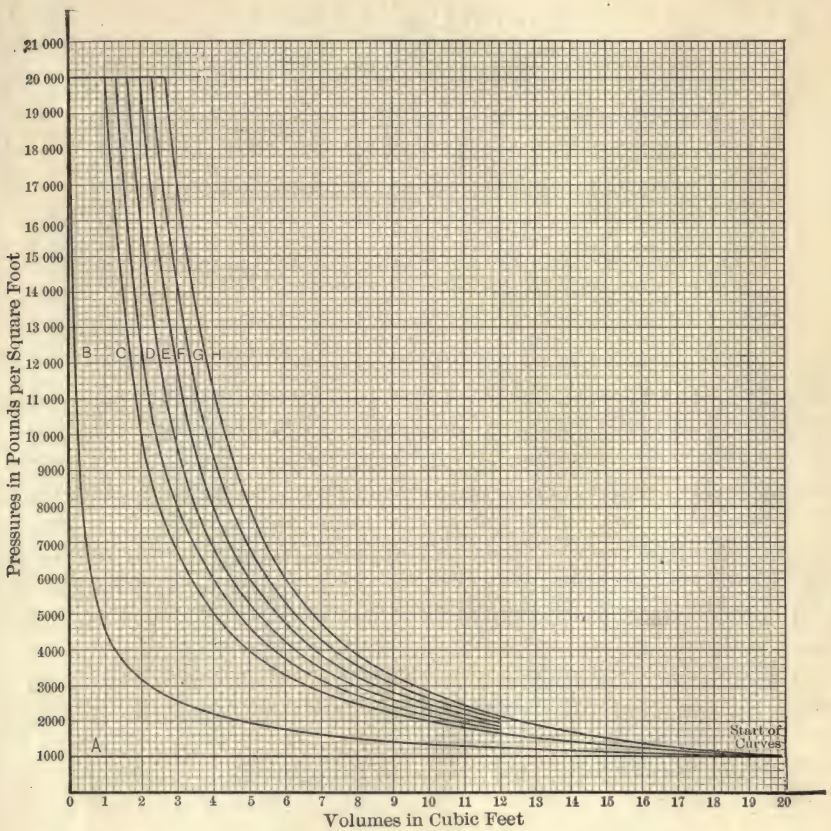


FIG. 6.—Comparison of Compression Curves having Different Values of s.

which, substituted in Eq.(7), gives

$$W = \int \frac{K dV}{V^s},$$

but as K is a constant,

$$W = K \int \frac{dV}{V^s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The integral of Eq. (11) will have two forms:

(1) When s is equal to one, in which case $P_1V_1 = P_2V_2 = K^1$;

(2) When s is not equal to one.

Taking first the case when s is equal to one,

$$W = K^1 \int_{V_1}^{V_2} \frac{dV}{V}$$

Whence

$$\left.\begin{aligned} W &= K^1 \log_e \frac{V_2}{V_1} && (a) \\ &= P_1 V_1 \log_e \frac{V_2}{V_1} && (b) \\ &= P_2 V_2 \log_e \frac{V_2}{V_1} && (c) \\ &= K^1 \log_e \frac{P_1}{P_2} && (d) \\ &= P_1 V_1 \log_e \frac{P_1}{P_2} && (e) \\ &= P_2 V_2 \log_e \frac{P_1}{P_2} && (f) \end{aligned}\right\} \text{When } s = 1 \ldots \quad (12)$$

Eqs. (12) are all equal and set down in different forms for convenience in computation; in them

V_2 = largest volume = initial vol. for compression = final vol. for expansion.

P_2 = smallest pressure = initial pres. for compression = final pres. for expansion.

V_1 = smallest volume = final vol. for compression = initial vol. for expansion.

P_1 = largest pressure = final pres. for compression = initial pres. for expansion.

These Eqs. (12) all indicate that the work of expansion and compression of this class is dependent only on the ratio of pressures or volumes at the beginning and end of the process, and the PV product at either beginning or end, this product being of constant value.

When the exponent s is not equal to one, the equation takes the form,

$$W = K \int_{V_1}^{V_2} \frac{dV}{V^s} = K \int_{V_1}^{V_2} V^{-s} dV$$
$$= \frac{K}{1-s} \left[{V_2}^{1-s} - {V_1}^{1-s}. \right]$$

As s is greater than one, the denominator and exponents will be negative, so changing the form to secure positive values,

$$W = \frac{K}{s-1} \left(\frac{1}{{V_1}^{s-1}} - \frac{1}{{V_2}^{s-1}} \right).$$

This can be put in a still more convenient form. Multiplying and dividing by $\frac{1}{{V_2}^{s-1}}$ or $\frac{1}{{V_1}^{s-1}}$.

$$W = \frac{K}{s-1} \frac{1}{{V_2}^{s-1}} \left[\left(\frac{V_2}{V_1} \right)^{s-1} - 1 \right] = \frac{K}{s-1} \frac{1}{{V_1}^{s-1}} \left[1 - \left(\frac{V_1}{V_2} \right)^{s-1} \right].$$

ubstituting the value of $K=P_2V_2{}^s=P_1V_1{}^s$,

$$W=\frac{1}{s-1}\frac{P_2V_2{}^s}{V_2{}^{s-1}}\left[\left(\frac{V_2}{V_1}\right)^{s-1}-1\right]=\frac{1}{s-1}\frac{P_1V_1{}^s}{V_1{}^{s-1}}\left[1-\left(\frac{V_1}{V_2}\right)^{s-1}\right].$$

Vhence

$$\left.\begin{aligned} W&=\frac{P_2V_2}{s-1}\left[\left(\frac{V_2}{V_1}\right)^{s-1}-1\right] \quad (a)\\ &=\frac{P_2V_2}{s-1}\left[\left(\frac{P_1}{P_2}\right)^{\frac{s-1}{s}}-1\right] \quad (b)\\ &=\frac{P_1V_1}{s-1}\left[1-\left(\frac{V_1}{V_2}\right)^{s-1}\right] \quad (c)\\ &=\frac{P_1V_1}{s-1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{s-1}{s}}\right] \quad (d)\end{aligned}\right\}. \quad \text{When } s\neq 1. \quad . \quad . \quad (13)$$

Eqs. (13) gives the work for this class of expansion and compression in terms of pressure ratios and volume ratios and in them

V_2 = largest volume = initial vol. for compression = final vol. for expansion;
P_2 = smallest pressure = initial pres. for compression = final pres. for expansion;
V_1 = smallest volume = final vol. for compression = initial vol. for expansion;
P_1 = largest pressure = final pres. for compression = initial pres. for expansion.

The work of expansion or compression of this class is dependent according to Eqs. (13), upon the ratio of pressures or volumes at beginning and end of the process, the exponent, and on the pressure volume product appropriately taken. It should be remembered that for the result to be in foot-pounds appropriate units should be used and all pressures taken absolute. Examination of Eqs. (12) and (13), for the work done by expansion or compression of both classes, shows that it is dependent on the initial and final values of pressures and volumes and on the exponent s, which defines the law of variation of pressure with volume between the initial and final states.

Example 1. Method of calculating Diagram, Figs. 5 and 6. Consider the curve for which $s=1.4$ as typical of the group.

Assumed Data. $V_1=1.0$ cu.ft. $P_1=20{,}000$ lbs. per square foot.
$s=1.4$.

Then $P_1V_1{}^s=K=20{,}000\times 1^{1.4}=20{,}000$.

For any other value of P, V was found from the relation,

$$V=\left(\frac{K}{P}\right)^{\frac{1}{s}}.$$

Let $P_x = 6000$,

then $$V_x = \left(\frac{K}{P_x}\right)^{\frac{1}{1.4}} = \left[\frac{20{,}000}{6000}\right]^{.715} = [3.33]^{.715};$$

$$\log 3.33 = .5224$$

or

$$.715 \times .5224 = .373 = \log V_t.$$

$$\therefore \quad V_x = 2.36.$$

A series of points, as shown below, were found, through which the curve was drawn

P	$\frac{20{,}000}{P}$.	$\log \frac{20{,}000}{P}$.	$\frac{1}{s} \log \frac{20{,}000}{P}$.	V
18000	1.111	0.0453	0.032	1.08
14000	1.430	0.1553	0.111	1.30
10000	2.000	0.3010	0.214	1.64
6000	3.330	0.5224	0.373	2.36
2000	10.000	1.0000	0.714	5.18
1000	20.000	1.3010	0.930	8.51

Curves for other values of s were similarly drawn. Starting at a common volume of 20 cu. ft. the compression curves of Fig. 6 were determined by the same methods

Example 2. A pound of air at 32° F. and under atmospheric pressure is compressed to a pressure of five times the original. What will be the final volume and the work done if $s = 1$ and if $s = 1.4$? The volume of 1 lb. of air at 32° F. and one atmosphere is 12.4 cu.ft. approx.

For $s = 1$,

$$\frac{P_1}{P_2} = 5, \qquad V_2 = 12.4 \text{ cu.ft.};$$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{12.4}{V_1} = 5, \quad \text{whence} \quad V_1 = 2.48 \text{ cu.ft.}$$

$$W = P_2 V_2 \log_e \frac{P_1}{P_2} = 2116 \times 12.4 \log_e 5;$$

$$= 2116 \times 12.4 \times 1.61 = 42{,}300 \text{ foot-pounds.}$$

For $s = 1.4$,

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.4}} = \frac{12.4}{V_1} = (5)^{\frac{1}{1.4}} = (5)^{.71}.$$

5 may be raised to the .71 power by means of logarithms as follows: $(5)^{.71}$ is equal to the number whose logarithm is .71 log 5.

Log 5 = .699, .71 × .699 = .4963, and number of which this is the logarithm is 3.13,

hence,

$$V_1 = V_2 \div 3.13 \quad \text{or} \quad V_1 = 3.96;$$

$$W = \frac{P_2V_2}{s-1}\left[\left(\frac{P_1}{P_2}\right)^{\frac{s-1}{s}} - 1\right] = \frac{2116 \times 12.4}{.4}\left[(5)^{\frac{.4}{1.4}} - 1\right].$$

$$= \frac{2116 \times 12.4}{.4} \times .583 = 38{,}200 \text{ ft.-lbs.}$$

The value of W can also be found by any other form of equation (13) such as,

$$W = \frac{P_1V_1}{s-1}\left[1 - \left(\frac{V_1}{V_2}\right)^{s-1}\right].$$

The value of V_1 being found as before, the work expression becomes after numerical substitution

$$W = \frac{10{,}580 \times 3.96}{.4}\left[1 - \left(\frac{3.96}{12.4}\right)^{.4}\right].$$

As the quantity to be raised to the .4 power is less than one, students may find it easier to use the reciprocal as follows:

$$\left(\frac{3.96}{12.4}\right)^{.4} = \frac{1}{\left(\frac{12.4}{3.96}\right)^{.4}} = \frac{1}{(3.13)^{.4}} = \frac{1}{1.58} = .632$$

Hence

$$W = \frac{10{,}580 \times 3.96}{.4}(1 - .632) = 38{,}200 \text{ ft.-lbs.}$$

Prob. 1. Find V_1 and W for Example 2 if $s = 1.2$ and 1.3.

Prob. 2. If a pound of air were compressed from a pressure of 1 lb. per square inch absolute to 15 lbs. per square inch absolute find V_1 and W when $s = 1$ and 1.4. $V_2 = 180$ cu.ft. What would be the H.P. to compress 1 lb. of air per minute?

Prob. 3. Air expands so that $s = 1$. If $P_1 = 10{,}000$ lbs. per square foot, $V_1 = 10$ cu.ft. and $V_2 = 100$ cu.ft. and the expansion takes place in 20 seconds, what is the H.P. developed?

Prob. 4. 100 cu.ft. of air at atmospheric pressure is compressed in a cylinder to a pressure of 8 atmospheres and then expelled against this constant pressure. Find graphically and by calculation the foot-pounds of work done for the case where $s = 1$ and for the case where $s = 1.4$.

Prob. 5. At an altitude of 4000 ft., air is compressed to a pressure of 60 lbs. per sq.in. gage. Find the H.P. required to compress 1000 cu.ft. of free air per minute.

Prob. 6. From the algebraic equation show how much work is done for a volume change of 1 to 4, provided pressure is originally 1000 lbs. per square foot when

(*a*) $PV^0 = K_1$,
(*b*) $PV = K_2$,
(*c*) $PV^2 = K_3$.

Prob. 7. A vacuum pump compresses air from 1 lb. per square inch absolute to 15 lbs. per square inch absolute and discharges it. An air compressor compresses air from atmosphere to 15 atmospheres and discharges it. Compare the work done for equal initial volumes, $s=1.4$.

Prob. 8. For steam expanding according to the saturation law, compare the work done by 1 lb. expanding from 150 lbs. per square inch absolute to 15 lbs. per square inch absolute with. the work of the same quantity expanding from 15 lbs. to 1 lb. per square inch absolute.

NOTE. 1 lb. of steam occupies 3 cu.ft. at 150 lbs. per square inch absolute.

Prob. 9. Two air compressors of the same size compress air adiabatically from atmosphere to 100 lbs. gage and discharge it. One is at sea level, the other at 10,000 ft. elevation. Compare the work in the two cases.

8. Values of Exponent s Defining Special Cases of Expansion or Compression. There are three general methods of finding s for the definition of particular cases of expansion or compression to allow of the solution of numerical problems. The first is experimental, the second and third thermodynamic. If by measurement the pressures and volumes of a series of points on an expansion or compression curve, obtained by test with appropriate instruments, for example, the indicator, be set down in a table and they be compared in pairs, values of s can be found as follows: Calling the points A, B, C, etc., then,

$$P_a V_a^s = P_b V_b^s,$$

and

$$\log P_a + s \log V_a = \log P_b + s \log V_b,$$

or

$$s(\log V_b - \log V_a) = \log P_a - \log P_b,$$

hence

$$s = \frac{\log P_a - \log P_b}{\log V_b - \log V_a} \quad (a)$$

or

$$s = \frac{\log \left(\frac{P_a}{P_b}\right)}{\log \left(\frac{V_b}{V_a}\right)} \quad (b) \qquad \ldots\ldots\ldots \quad (14)$$

According to Eq. (14a), if the difference between the logarithms of the pressures at B and A be divided by the differences between the logarithms of the volumes at A and B respectively, the quotient will be s. According to Eq. (14b), the logarithm of the ratio of pressures, B to A, divided by the logarithm of the ratio of volumes, A to B respectively will also give s. It is interesting to note that if the logarithms of the pressures be plotted vertically and logarithms of volumes horizontally as in Fig. 7, then the line AC equal to the intercept on the horizontal axis represents the difference between the logarithms of volumes or,

$$\overline{CA} = \log V_a - \log V_b,$$

nd similarly

$$\overline{CB} = \log P_b - \log P_a.$$

Hence

$$s = \frac{\overline{CB}}{\overline{CA}} = \tan \alpha,$$

or the slope of the line indicates the value of s. This is a particularly valuable method, as it indicates at a glance the constancy or variability of s, and there are many cases of practice where s does vary. Should s be constant the line will be straight; should it be variable the line will be curved, but can generally

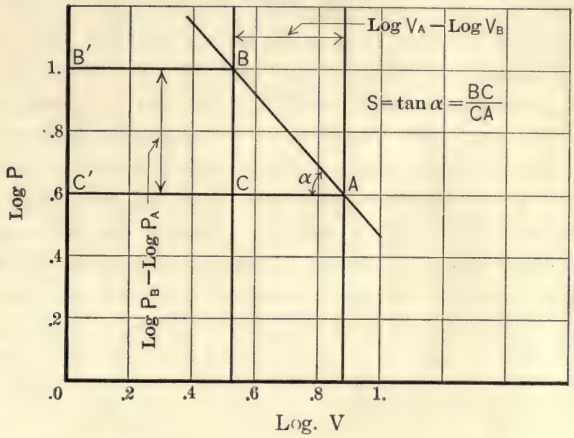


FIG. 7.—Graphic Method of Finding s, from Logarithms of Pressures and Volumes.

be divided into parts, each of which is substantially straight and each will have a different s. It is sometimes most convenient to take only the beginning and end of the curve and to use the value of s corresponding to these points, neglecting intermediate values.

A second method for finding s for a given compression or expansion line by means of areas is indicated in a note in Section 17 of this Chapter that is omitted here because it depends on formulas not yet derived. It is by this sort of study of experimental data that most of the valuable values of s have been obtained. There is, however, another method of finding a value for s by purely thermodynamic analysis based on certain fundamental hypotheses, and the value is as useful as the hypotheses are fair or true to the facts of a particular case.

One of the most common hypotheses of this sort is that the *gas* or *vapor* undergoing expansion or compression shall neither receive any heat from, nor give up any to bodies external to itself during the process, and such a process is given the name *adiabatic*. Whether adiabatic processes are possible in actual cylinders or nozzles does not affect the analysis with which pure thermodynamics is concerned. By certain mathematical transformations, to be carried out later, and based on a fundamental thermodynamic proposition, the adia-

batic hypothesis will lead to a value of s, the use of which gives results valuable as a basis of reference, and which when compared with an actual case will permit of a determination of how far the real case has departed from the adiabatic condition, and how much heat has been received or lost at any part of the process. The particular value of s which exists in an adiabatic change is represented by the symbol γ.

Another common hypothesis on which another value of s can be derived, is that gases in expansion or compression shall remain at a constant temperature, thus giving rise to the name *isothermal*. This is generally confined to gases and superheated vapors, as it is difficult to conceive of a case of isothermal or constant temperature expansion or compression of wet vapors, as will be seen later.

In the study of vapors, which, it must be understood, may be dry or wet that is, containing liquid, a common hypothesis is that during the expansion or compression they shall remain just barely dry or that they shall receive or lose just enough heat to keep any vapor from condensing, or but no more than sufficient to keep any moisture that tends to form always evaporated. Expansion or compression according to this hypothesis is said to follow the saturation law, and the substance to remain saturated. It will appear from this thermal analysis later that the value of s for the isothermal hypothesis is the same for all gases and equal to one, but for the adiabatic hypothesis $s=\gamma$ will have a different value for different substances, though several may have the same value, while for vapors γ will be found to be a variable for any one, its value depending not only on the substance, but on the temperatures, pressures and wetness.

When gases or vapors are suffered to expand in cylinders and nozzles or caused to compress, it is often difficult and sometimes impossible or perhaps undesirable to avoid interference with the adiabatic conditions for vapors and gases, with the isothermal for gases or with the saturation law for vapors yet the work to be done and the horse-power developed cannot be predicted without a known value of s, which for such cases must be found by experience. A frequent cause of interference with these predictions, which should be noted, is leakage in cylinders, which, of course, causes the mass under treatment to vary.

According to these methods those values of s have been found which are given in Table X, at the end of the Chapter. Mixtures of common gases such as constitute natural, producer, blast furnace or illuminating gas, alone or with air or products of combustion, such as used in internal combustion engines have values of s that can be calculated from the elementary gases or measured under actual conditions.

All vapors, except those considerably overheated, have variable exponents for adiabatic expansion and compression. This fact makes the exact solution of problems of work for wet vapors, expanding or compressing, which form the bulk of the practical cases, impossible by such methods as have been described. This class of cases can be treated with precision only by strictly thermal methods, to be described later.

Prob. 1. By plotting the values for the logarithms of the following pressures and volmes, see if the value for s is constant, and if not find the mean value in each case.

(*a*) Gas Engine Compression

V	p	V	p	V	p
10	45.2	13	32.2	18	21.0
11	39.7	14	29.7	20	19.5
12	35.7	16	24.7	25	14.7

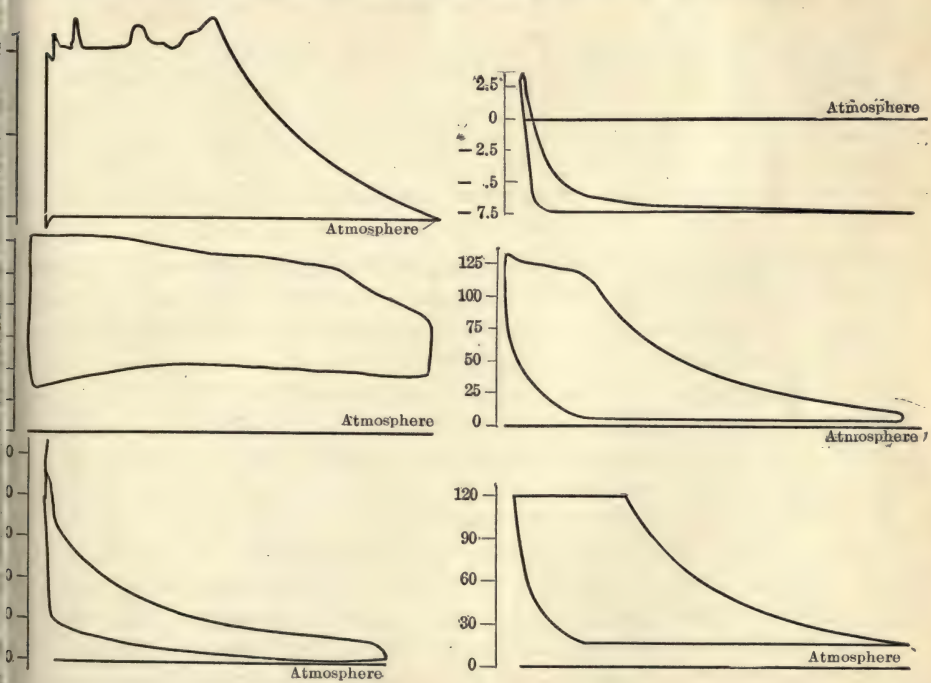
(*b*) Gas Engine Expansion

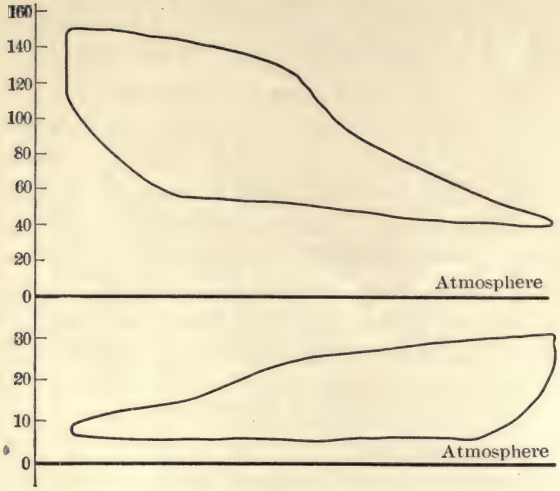
V	p	V	p	V	p
		13	146.2	19	80.7
11	188.2	15	116.7	21	68.7
12	166.2	17	65.7	23	58.7

(*c*) Steam Expansion

V	p	V	p
2.242	203.3	7.338	52.5
2.994	145.8	12.44	28.8
4.556	89.9	22.68	14.7

Prob. 2. By plotting the values for the logarithms of the volumes and pressures on he expansion and compression curves of the following cards, find value for s.





Prob. 3. From the steam tables at the end of Chapter IV. select the pressures ar volumes for dry-saturated steam and find the value of s between

(a) 150 lbs. per square inch and 1 lb. per square inch.
(b) 15 " " "

Prob. 4. Find for superheated steam at 150 lbs. per square inch and with 10 of superheat expanding to 100 lbs. per square inch without losing any superheat, t corresponding value of s, using tabular data.

Prob. 5. From the ammonia table data for dry-saturated vapor find the value o between

(a) 150 lbs. per square inch and 1 lb. per square inch.
(b) 15 " " "

9. Work Phases and Cycles, Positive and Negative and Net Work. Accor ing to the preceding it is easy to calculate or predict numerically the work expansion or compression whenever the conditions are sufficiently definite permit of the selection of the appropriate s. It very seldom happens, howeve that the most important processes are single processes or that the work expansion or compression is of interest by itself. For example, before expansi can begin in a steam cylinder steam must be first admitted, and in air co pressors air must be drawn in before it can be compressed. Similarly, aft expansion in a steam cylinder there must be an expulsion of used vapor befo another admission and expansion can take place, while in the air compress after compression the compressed air must be expelled before more can ent for treatment. The whole series of operations is a matter of more conce than any one alone, and must be treated as a whole. The effect can be m easily found by the summation of the separate effects, and this method summation will be found of universal application.

The whole series of processes taking place and involving pressure volu changes is called a *cycle*, any one of them a *phase*. It is apparent that th can be only a limited number of phases so definite as to permit of the mat

matical treatment necessary for prediction of work, but it is equally clear that there may be a far greater number of combinations of phases constituting cycles. Before proceeding to analyze the action of steam or gas in a cylinder it is necessary first to determine on structural, thermal or any other logical grounds, what series of separate processes will be involved, in what order, and the pressure volume characteristics of each. Then and then only, can the cycle as a whole be treated. These phases or separate and characteristic processes affecting the work done or involving pressure volume changes are divisible into two classes so far as the causes producing them are concerned, the first thermal and the second mechanical. It requires no particular knowledge of thermodynamics to realize that if air be confined in a cylinder with a free piston and is heated, that the volume will increase while pressure remains constant, since the piston will move out with the slightest excess of pressure inside over what is outside. This is a pressure constant, volume increasing, phase, and is thermal since it is a heat effect. If an ample supply of steam be available from a boiler held at a constant pressure by the manipulation of dampers and fires by the fireman and the steam be admitted to a cylinder with a piston, the piston will move out, the pressure remaining constant and volume increasing. This is also a pressure constant, volume increasing phase, exactly as before, but is mechanical because it is due to a transportation of steam from the boiler to the cylinder, although in another sense it may be considered as thermal if the boiler, pipe and cylinder be considered as one part during the admission. A similar constant pressure phase will result when a compressor piston is forcibly drawn out, slightly reducing the pressure and permitting the outside atmosphere to push air in, to follow the piston, and again after compression of air to a slight excess, the opening of valves to storage tanks or pipe lines having a constant pressure will allow the air to flow out or be pushed out of the cylinder at constant pressure. These two constant pressure phases are strictly mechanical, as both represent transmission of the mass. If a cylinder contain water and heat be applied without permitting any piston movement, there will be a rise of pressure at constant volume, a similar constant volume pressure rise phase will result from the heating of a contained mass of gas or vapor under the same circumstances, both of these being strictly thermal.

However much the causes of the various characteristic phases may differ, the work effects of similar ones is the same and at present only work effects are under consideration. For example, all constant volume phases do no work as work cannot be done without change of volume.

The consideration of the strictly thermal phases is one of the principal problems of thermodynamics, for by this means the relation between the work done to the heat necessary to produce the phase changes is established, and a basis laid for determining the ratio of work to heat, or efficiency. For the present it is sufficient to note that the work effects of any phase will depend only on the pressure volume changes which characterize it.

Consider a cycle Fig. 8, consisting of (AB), admission of 2 cu.ft of steam at a constant pressure of 200,000 lbs. per square foot, to a cylinder originally

containing nothing, followed by (BC), expansion with $s=1$, to a pressure o 20,000 lbs. per square foot; (CF), constant volume change of pressure, an (FG), constant pressure exhaust at 10,000 lbs. per square foot. These opera tions are plotted to scale. Starting at zero volume, because the cylinde

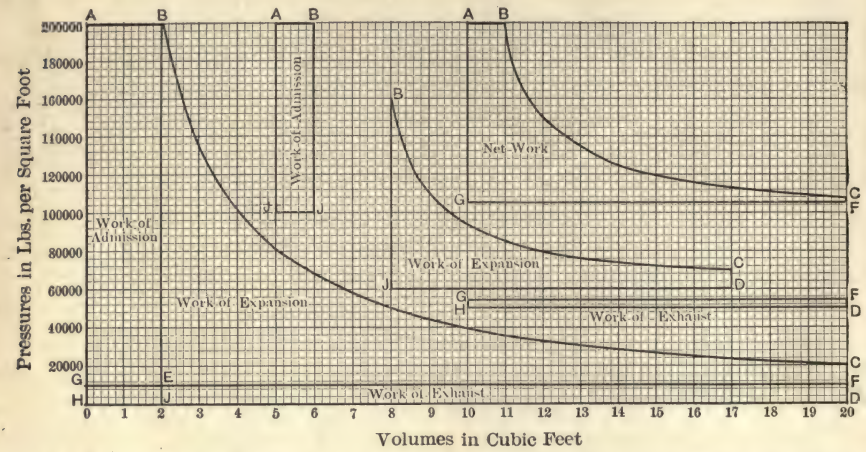


FIG. 8.—Analysis of Work Diagram for Admission Expansion and Exhaust of Engine without Clearance.

originally contains nothing, and at a pressure of 200,000 lbs. per square foot, the line AB, ending at volume 2 cu.ft., represents admission and the cross-hatched area under AB represents the 400,000 ft.-lbs. of work done during admission. At B the admission ceases by closure of a valve and the 2 cu.ft. of steam at the original pressure expands with lowering pressure according to the law

$$P_a V_a = P_b V_b = 200{,}000 \times 2 = 400{,}000 \text{ ft.-lbs.},$$

So that when

$$V = 4 \text{ cu.ft.}, \quad P = \frac{400{,}000}{4} = 100{,}000 \text{ lbs. per sq.ft.};$$

$$V = 5 \text{ cu.ft.}, \quad P = \frac{400{,}000}{5} = 80{,}000 \text{ lbs. per sq.ft.};$$

$$V = 10 \text{ cu.ft.}, \quad P = \frac{400{,}000}{10} = 40{,}000 \text{ lbs. per sq.ft.}$$

This continues until $V=20$ at point C, at which time $P = \frac{400{,}000}{20} = 20{,}000$ lbs. per square foot, and the work done during expansion is the cross-hatched area $JBCD$ under the expansion curve BC, the value of which can be found by measuring the diagram or by using the formula Eq. (12),

$$W_{bc} = P_b V_b \log_e \frac{V_c}{V_b},$$

hich on substitution gives

$$W_{bc} = 400{,}000 \log_e 10 = 400{,}000 \times 2.3;$$

$$= 920{,}000 \text{ ft.-lbs.}$$

his completes the stroke and the work for the stroke can be found by addition f the numerical values,

$$W_{ab} = 400{,}000 \text{ ft.-lbs.};$$

$$W_{bc} = 920{,}000 \text{ ft.-lbs.};$$

$$W_{ab} + W_{bc} = 1{,}320{,}000 \text{ ft.-lbs.}$$

It is often more convenient to find an algebraic expression for the whole, hich for this case will be,

$$W_{ab} = P_a V_a = P_b V_b;$$

$$W_{bc} = P_b V_b \log_e \frac{V_c}{V_b};$$

$$W_{ac} = W_{ab} + W_{be} = P_b V_b \left(1 + \log_e \frac{V_c}{V_b}\right),$$

$$= 400{,}000(1 + \log_e 10) = 400{,}000 \times 3.3 = 1{,}320{,}000 \text{ ft.-lbs.}$$

On the return of the piston it encounters a resistance due to a constant pressure f 10,000 lbs. per square inch, opposing its motion; it must, therefore, do work n the steam in expelling it. Before the return stroke begins, however, the ressure drops by the opening of the exhaust valve from the terminal pressure f the expansion curve to the exhaust or *back* pressure along the constant volume ine, *CF*, of course, doing no work, after which the return stroke begins, the ressure volume line being *FG* and the work of the stroke being represented by he cross-hatched area *DFGH*,

$$W_{fg} = P_f V_f = 10{,}000 \times 20 = 200{,}000 \text{ ft.-lbs.}$$

This is *negative* work, as it is done in opposition to the movement of the piston. The cycle is completed by admission of steam at constant zero volume, raising he pressure along *GA*. The net work is the difference between the positive nd negative work, or algebraically

$$W = P_b V_b \left(1 + \log_e \frac{V_c}{V_b}\right) - P_f V_f,$$

$$= 1{,}320{,}000 - 200{,}000 = 1{,}120{,}000 \text{ ft.-lbs.}$$

Consider now a cycle of an air compressor, Fig. 9. Admission or suction is represented by AB, compression by BC, delivery by CD and constant volume drop in pressure after delivery by DA. The work of admission is represented by the area $ABFE$ or algebraically by

$$W_{ab} = P_b V_b,$$

the work of compression by the area $FBCG$, or algebraically since $s = 1.4$ by

$$W_{bc} = \frac{P_b V_b}{s-1}\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right],$$

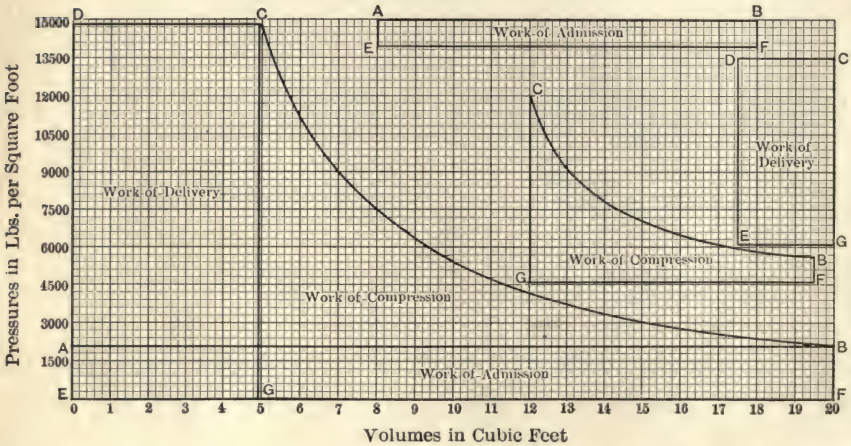


FIG. 9.—Analysis of Work Diagram for Admission Compression and Delivery of Compressor without Clearance.

the work of delivery by the area $CDEG$, or algebraically

$$W_{cd} = P_c V_c.$$

The positive work is that assisting the motion of the piston during suction the area $ABFE$ or algebraically $P_b V_b$. The negative work, that in opposition to the motion, is the sum of the compression and delivery work, the area $FBCDE$, or algebraically,

$$W_{bc} + W_{cd} = P_c V_c + \frac{P_b V_b}{s-1}\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right].$$

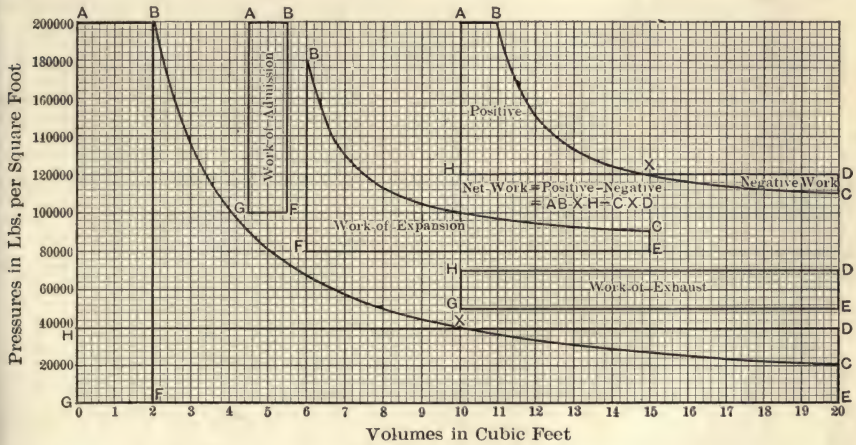
The net work is the difference and is negative, as such a cycle is mainly resistant and to execute it the piston must be driven with expenditure of work on the gas. The value of the net work is,

$$W = W_{bc} + W_{cd} - W_{ab}$$

$$= P_c V_c + \frac{P_b V_b}{s-1}\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right] - P_b V_b,$$

n expression which will be simplified in the chapter on compressors. This et work is represented by the area *ABCD*, which is the area enclosed by the ycle itself independent of the axes of coordinates.

It might seem from the two examples given as if net work could be btained without the tedious problem of summation, and this is in a sense true the cycle is plotted to scale or an algebraic expression be available, but hese processes are practically equivalent to summation of phase results. It ight also seem that the work area would always be that enclosed by the ycle, and this is true with a very important limitation, which enters when he cycle has loops. If, for example, as in Fig. 10, steam admitted *A* to *B*, xpanded along *BC* to a pressure *C*, then on opening the exhaust the pressure nstead of falling to the back pressure or exhaust line as in Fig. 8, would here



IG. 10.—Analysis of Work Diagram for Engine with Over-expansion Negative Work Loop.

ise along *CD*, as the back pressure is higher than the terminal expansion pres- ure, after which exhaust will take place at constant back pressure along *DE*. The forward stroke work is that under *AB* and *BC* or *ABCEG*, the return troke work is the area *DEGH* and the net work is

$$\text{Area } ABCEG - \text{Area } DEGH.$$

As the area *HGECX* is common to both terms of the difference, the net work nay be set down as equal to

$$\text{Area } ABXH - CDX.$$

It may be set down then in general for looped cycles that the net work area s the difference between that of the two loops. If, however, the method laid lown for the treatment of any cycle be adhered to there need not be any dis- inction drawn between ordinary and looped cycles, that is, *in finding the work*

of a cycle divide it into characteristic phases and group them into positive an negative, find the work for each and take the algebraic sum.

Special cases of cycles and their characteristics for steam compressors an gas engine cylinders, as well as nozzle expansion, will be taken up later in mor detail and will constitute the subject matter of the next two chapters.

Example 1. Method of calculating Diagram, Fig. 8.

$$\textit{Assumed data} \begin{cases} V_a = 0 \text{ cu.ft.} \\ V_b = 2 \quad " \\ V_d = V_c \quad " \\ V_e = 2 \quad " \\ s = 1 \end{cases} \qquad \begin{cases} P_a = 200{,}000 \text{ lbs. per square foot.} \\ P_b = P_a \quad " \quad " \\ P_c = 20{,}000 \quad " \quad " \\ P_f = 10{,}000 \quad " \quad " \\ P_e = P_f. \quad " \quad " \end{cases}$$

To obtain point C.

$$P_c V_c = P_b V_b \quad \text{or} \quad V_c = \frac{P_b V_b}{P_c} = \frac{200{,}000 \times 2}{20{,}000} = 20,$$

$$\therefore \quad V_c = 20 \quad \text{and} \quad P_c = 20{,}000.$$

Intermediate points B to C are obtained by assuming various pressures an finding the corresponding volumes as for V_c.

Example 2. Method of calculating Diagram, Fig. 9.

$$\textit{Assumed data} \begin{cases} V_a = 0 \text{ cu.ft.} \\ V_b = 20 \quad " \\ V_d = 0 \quad " \\ s = 1.4 \end{cases} \qquad \begin{cases} P_a = 2116 \text{ lbs. per square foot.} \\ P_b = P_a \quad " \quad " \\ P_c = 14{,}812 \quad " \quad " \\ P_d = P_c \quad " \quad " \end{cases}$$

To obtain point C,

$$P_b V_b^{1.4} = P_c V_c^{1.4} \quad \text{or} \quad V_c = V_b \div \left(\frac{P_c}{P_b}\right)^{\frac{1}{1.4}} = V_b \div \left(\frac{P_c}{P_b}\right)^{.715}$$

$$\frac{P_c}{P_b} = 7, \quad \log 7 = .845 \; ; \quad \text{and} \quad .715 \times .845 = \log \left(\frac{P_c}{P_b}\right)^{\frac{1}{1.4}} = .6105,$$

or

$$V_c = 4.02.$$

Therefore,

$$V_c = 4.02, \quad \text{and} \quad P_c = 14{,}812.$$

Intermediate values B to C may be found by assuming pressures and finding volumes co responding as for V_c.

Prob. 1. Steam at 150 lbs. per square inch absolute pressure is admitted into a cyli der in which the volume is originally zero until the volume is 2 cu.ft., when the valve closed and expansion begins and continues until the volume is 8 cu.ft., then exhau valve opens and the pressure falls to 10 lbs. absolute and steam is entirely swept ou Draw the diagram and find the net work done.

Prob. 2. A piston moving forward in a cylinder draws in 10 cu.ft. of CO_2 at a pressure f .9 of an atmosphere at sea level and then compresses it adiabatically until the pressure ises to 9 atmospheres and discharges it at constant pressure. Draw the diagram and nd net work done.

Prob. 3. A cylinder 18 ins. in diameter and 24 ins. piston stroke receives steam at 00 lbs. per square inch absolute pressure for $\frac{1}{3}$ of the stroke. It then expands to the nd of the stroke and is exhausted at atmospheric pressure. Draw the diagram and nd the H.P. if the engine makes 100 strokes per minute.

Prob. 4. Two compressors without clearance each with a cylinder displacement of cu.ft. draw in air at 14 lbs. per square inch absolute and compress it to 80 lbs. per quare inch absolute before delivery. Find the difference in H.P. per 1000 cu.ft. of ree air per minute if one is compressing isothermally and the other adiabatically. Draw diagram for each case.

Prob. 5. A quantity of air 5 cu.ft. in volume and at atmospheric pressure is compressed n a cylinder by the movement of a piston until the pressure is 50 lbs. per square inch age. If the air be heated the pressure will rise, as in an explosion. In this case the iston remains stationary, while the air is heated until the pressure reaches 200 lbs. per quare inch gage. It then expands adiabatically to the original volume when the ressure is reduced to atmosphere with no change in volume. Draw the diagram, and nd the work done.

Prob. 6. The *Brayton cycle* is one in which gas is compressed adiabatically and then, y the addition of heat, the gas is made to expand without change of pressure. Adiabatic expansion then follows to original pressure and the cycle ends by decrease in volume o original amount without change of pressure. Draw such a cycle starting with 5 cu.ft. air at atmospheric pressure, compressing to 4 atmospheres, expanding at constant ressure to 5 cu.ft., expanding adiabatically to original pressure and finally ending at iginal point. Find also, work done.

Prob. 7. In the *Ericsson cycle* air is expanded at constant temperature, cooled at nstant pressure, compressed at constant temperature and receives heat at constant olume. Draw a diagram for the case where 5 cu.ft. at atmospheric pressure are compressed to 1 cu.ft., heated until volume is 8 cu.ft., expanded to atmosphere and then ooled to original volume. Find the work.

Prob. 8. In the *Stirling cycle* constant volume heating and cooling replace that at nstant pressure in the Ericsson. Draw diagram starting with 5 cu.ft. and atmsopheric essure compressing to 1 cu.ft. and then after allowing the pressure to double, expand original volume and cool to atmosphere. Find the work.

Prob. 9. The *Joule cycle* consists of adiabatic compression and expansion and conant pressure heating and cooling. Assuming data as in last problem draw the agram and find the work.

Prob. 10. The *Carnot cycle* consists of isothermal expansion, adiabatic expansion, othermal compression and adiabatic compression. Draw the diagram for this cycle d find the work.

10. Work Determination by Mean Effective Pressure. While the methods ready described are useful for finding the work done in foot-pounds for a defined cle with *known pressure and volume limits*, they are not, as a rule, convenient r the calculation of the work done in a cylinder *of given dimensions*. As ork done can always be represented by an area, this area divided by its length ll give its mean height. If the area be in foot-pounds with coordinates

pounds per square foot, and cubic feet, then the division of area in foot-pound by length in cubic feet will give the mean height or the mean pressure i pounds per square foot. Again, dividing the work of the cycle into forward stroke work and back-stroke work, or the respective foot-pound areas divide by the length of the diagram in cubic feet, will give the *mean forward pressur* and the *mean back pressure*. The difference between mean forward pressur and mean back pressure will give the *mean effective pressure*, or that averag pressure which if maintained for one stroke would do the same work as th cycle no matter how many strokes the cycle itself may have required for it execution, which is very convenient considering the fact that most gas engine require four strokes to complete one cycle. The mean effective pressure ma also be found directly from the enclosed cycle area, taking proper account loops, as representative of net work by dividing this net work area by the lengt of the diagram in appropriate units. This method is especially convenien when the diagram is drawn to odd scales so that areas do not give foot-pound directly, for no matter what the scale the mean height of the diagram, whe multiplied by the pressure scale factor, represents the mean effective pressur This mean height can always be found in inches for any scale of diagram b finding the area of the diagram in square inches and by dividing by the leng in inches, and this mean height in inches multiplied by the scale of pressur in whatever units may be used will give the mean effective pressure in the sam units.

Mean pressures, forward, back or effective, are found and used in two gener ways; first, algebraically, and second graphically and generally in this ca from test records. By the first method, formulas, based on some assum laws for the phases, can be found, and the mean effective pressure and its val predicted. This permits of the prediction of work that may be done by a giv quantity of gas or vapor, or the work per cycle in a cylinder, or finally the hor power of a machine, of which the cylinder is a part, operating at a given spe and all without any diagram measurement whatever. By the second metho a diagram of pressures in the cylinder at each point of the stroke can be obtain by the indicator, yielding information on the scale of pressures. The net wo area measured in square inches, when divided by the length in inches, giv the mean height in inches, which, multiplied by the pressure scale per in of height, gives the mean effective pressure in the same units, which usually pounds per square inch in practice.

As an example of the algebraic method of prediction, consider the cy represented by Fig. 8. The forward work is represented by

$$\text{Forward work} = P_b V_b \left(1 + \log_e \frac{V_c}{V_e}\right),$$

the length of the diagram representing the volume swept through in the p formance of this work is V_c, hence

$$\text{Mean forward pressure} = \frac{P_b V_b}{V_c} \left(1 + \log_e \frac{V_c}{V_e}\right).$$

ut $P_bV_b = P_cV_c$ by the law of this particular expansion curve, hence

$$\text{Mean forward pressure} = P_c\left(1+\log_e \frac{V_c}{V_e}\right).$$

s the back pressure is constant its mean value is this constant value, hence

$$\text{Constant (mean) back pressure} = P_f.$$

y subtraction

$$\text{Mean effective pressure} = P_c\left(1+\log_e \frac{V_c}{V_e}\right) - P_f$$

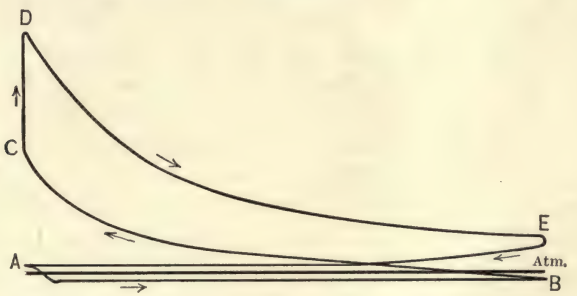
$$= 3.3P_c - P_f;$$

$$= 3.3\times 20{,}000 - 10{,}000;$$

$$= 66{,}000 - 10{,}000 = 56{,}000 \text{ lbs. per sq.ft.}$$

The work done in foot-pounds is the mean effective pressure in pounds per quare foot, multiplied by the displacement in cubic feet.

$$W = 56{,}000\times 20 = 1{,}120{,}000 \text{ ft.-lbs. as before.}$$



IG. 11.—Gas-Engine Indicator Card. For Determination of Mean Effective Pressure without Volume Scale.

As an example of the determination of mean effective pressure from a test indicator diagram of unknown scale except for pressures, and without axes coordinates, consider Fig. 11, which represents a gas engine cycle in four rokes, the precise significance of the lines being immaterial now. The ressure scale is 180 lbs. per square inch, per inch of height.

By measurement of the areas in square inches it is found that

Large loop area *CDEXC*	= 2.6 sq.in.
Small loop area *ABXA*	= 0.5 sq.in.
Net cycle area	= 2.1 sq.in.
Length of diagram	= 3.5 in.
Mean height of net work cycle	= 0.6 in.
Mean effective pressure = $120\times .06$ = 72 lbs. per square inch.	

It is quite immaterial whether this diagram were obtained from a large a small cylinder; no matter what the size, the same diagram might be secure and truly represent the pressure volume changes therein. If this particul cylinder happened to have a diameter of 10 ins. and a stroke of 12 ins. t work per stroke can be found. The area of the cylinder will be 78.54 sq.in hence the average force on the piston is 72 lbs. per square inch $\times$78.54 sq.ins. 5654.88 lbs., and the stroke being 1 ft. the work per stroke is 5654.88 ft.-lb Both of these methods are used in practical work and that one is adopted any particular case which will yield results by the least labor.

Prob. 1. An indicator card from an air compressor is found to have an area of 3. sq. ins., while the length is $2\frac{1}{2}$ ins. and scale of spring is given as 80 lbs. per square in per inch height. What is m.e.p. and what would be the horse-power if the compres ran with a piston speed of 250 ft. per minute and had a piston 9 ins. in diameter?

Prob. 2. For the same machine another card was taken with a 60-lb. spring and h an area of 4.12 sq.ins. How does this compare with first card, the two having the sa length?

Prob. 3. A steam engine having a cylinder 18 ins. in diameter and a stroke of 24 i takes in $\frac{1}{2}$ cu.ft. of steam at 100 lbs. absolute, allows it to expand and exhausts at atm pheric pressure. An indicator card taken from the same engine showed a length o ins., an area of .91 sq.in. when an 80-lb. spring is used. How does the actual m.e compare with the computed?

Prob. 4. Find m.e.p. by the algebraic method of prediction for,

(*a*) Brayton cycle;
(*b*) Carnot cycle;
(*c*) Stirling cycle;
(*d*) Ericsson cycle;
(*e*) Joule cycle.

(See problems following Section 9).

11. Relation of Pressure-Volume Diagrams to Indicator Cards. The Indicator. When a work cycle or diagram of pressure volume changes is dra to scale with pressures and volumes as coordinates, it is termed a press volume or PV diagram, and may be obtained by plotting point by point fr the algebraic expression for the law of each phase or by modifying the indica card. The indicator card is that diagram of pressures and stroke obtai by applying the indicator to a cylinder in operation. This instrument cons essentially of a small cylinder in which a finely finished piston moves fr without appreciable friction, with a spring to oppose its motion, a pencil mech ism to record the extent of the motion, and a drum carrying paper which is mo in proportion to the engine piston movement. The indicator cylinder is o at the bottom and fitted with a ground union joint for attachment to the m cylinder through a special cock, which when open permits all the vary pressures in the main cylinder to act on the indicator piston, and when cl to the main cylinder opens the indicator cylinder to the atmosphere.

upper side of the indicator piston being always open to the atmosphere, its movement will be the result of the difference between the pressure in the main cylinder and atmospheric pressure. A helical spring, carefully calibrated and, therefore, of known scale, is fixed between the indicator piston and open cap or head of its cylinder, so that whenever the pressure in the main cylinder exceeds atmosphere the indicator piston moves toward the open head of the indicator cylinder, compressing the spring. Pressures in the main cylinder if less than atmosphere will cause the indicator piston to move the other way, extending the spring. This compression and extension of the spring is found in the calibration of the spring to correspond to a definite number of pounds per square inch above or below atmosphere per inch of spring distortion, so that the extent of the piston movement measures the pressure above or below atmosphere. A piston rod projects outward through the cylinder cap and moves a series of levers and links carrying a pencil point, the object of the linkage being to multiply the piston movement, but in direct proportion, giving a large movement to the pencil for a small piston movement. A cylinder drum carrying a sheet of paper is pivoted to the cylinder frame so that the pencil movement will draw on the paper a straight line parallel to the axis of the drum, if drum is stationary, or perpendicular to it if drum rotates and pencil is stationary. The height of such lines then above or below a zero or datum line, which is the atmospheric line drawn with the cock closed, measures the pressure of the fluid under study. The springs have scale numbers which give the pressure in pounds per square inch per inch of pencil movement. This paper-carrying drum is not fixed, but arranged to rotate about its axis, being pulled out by a cord attached to the piston or some connecting part through a proportional reducing motion so as to draw out the cord an amount slightly less than the circumference of the drum no matter what the piston movement. After having been thus drawn out a coiled spring inside the drum draws it back on the return stroke. By this mechanism it is clear that, due to the combined movement of the pencil up and down, in proportion to the pressure, and that of the drum and paper across the pencil in proportion to the piston movement, a diagram will be drawn whose ordinates represent pressures above and below atmosphere and abscissæ, piston stroke completed at the same time, or displacement volume swept through. It must be clearly understood that such indicator diagrams or cards do not give the true or absolute pressures nor the true volumes of steam or gas in the cylinder, but only the pressures above or below atmosphere and the changes of volume of the fluid corresponding to the piston movement. Of course, if there is no gas or steam in the cylinder at the beginning of the stroke, the true volume of the fluid will be always equal to the displacement, but no such cylinder can be made.

While the indicator card is sufficient for the determination of mean effective pressure and work per stroke, its lack of axes of coordinates of pressure and volume prevents any study of the laws of its curves. That such study is important must be clear, for without it no data or constants such as the exponent s can be obtained for prediction of results in other similar cases, nor can the

presence of leaks be detected, or the gain or loss of heat during the various processes studied. In short, the most valuable analysis of the operations is impossible.

To convert the indicator card, which is only a diagram of stroke or displacement on which are shown pressures above and below atmosphere into a pressure volume diagram, there must first be found (*a*) the relation of true or absolute pressures to gage pressures, which involves the pressure equivalent of the barometer, and (*b*) the relation of displacement volumes to true volumes of vapor or gas present, which involves the *clearance* or inactive volume of the cylinder. The conversion of gage to absolute pressures by the barometer reading has already been explained, Section 3, while the conversion of displacement volumes to true fluid volumes is made by adding to the displacement volume the constant value in the same units of the clearance, which is usually the result of irregularity of form at the cylinder ends dictated by structural necessities of valves, and of linear clearance or free distance between the piston at the end of its stroke and the heads of the cylinder to avoid any possibility of touching due to wear or looseness of the bearings.

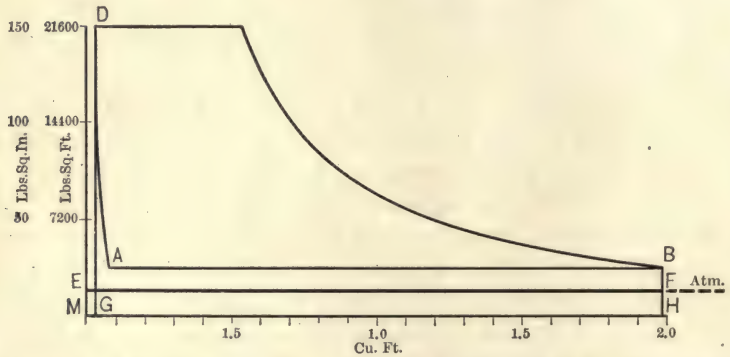


FIG. 12.—Ammonia Compressor Indicator Card with Coordinates of Pressures and Volumes Added to Convert it into a Pressure-Volume Diagram.

Let *ABCD*, Fig. 12, represent an indicator card from an ammonia compressor on which *EF* is the atmospheric line. The cylinder bore is 14 ins., stroke 22 ins., and the scale of the indicator spring 100, barometer 28 ins., and measured clearance 32 cu.in. According to Table IX, 28 ins. of mercury corresponds to 13.753 lbs. per square inch, and as 100 lbs. per square inch, according to the spring scale, corresponds to 1 in. of height on the diagram, 1 lb. per square inch corresponds to 0.01 in. of height, or 13.75 lbs. per square inch atmospheric pressure to .137 in. of height. The zero of pressures then on the diagram must lie .137 in. below the line *EF*. Lay off then a line *MH*, this distance below *EF*. This will be the position of the axis of volume coordinates.

Actual measurement of the space in the cylinder with the piston at the end of its stroke gave the clearance volume of 32 cu. ins. As the bore is 14 ins. the piston area is 153.94 sq. ins. which in connection with the stroke

of 22 ins. gives a displacement volume of $22 \times 153.94 = 3386.68$ cu. ins. Compared with this the clearance volume is $\frac{32}{3386.68} = .94$ per cent of the displacement. It should be noted here that clearance is generally expressed in per cent of displacement volume. Just touching the diagram at the ends drop two lines at right angles to the atmospheric line intersecting the axis of volumes previously found at G and H. The intercept GH then represents the displacement, or 3386.68 cu.in. or 1.96 cu.ft. Lay off to the left of G, .0094, or in round numbers 1/100 of GH, fixing the point M, MG representing the clearance to scale, and a vertical through M the axis of pressures. The axes of coordinates are now placed to scale with the diagram but no scale marked thereon. The pressure scale can be laid off by starting at M and marking off inch points each representing 100 lbs. per square inch. Pounds per square foot can also be marked by a separate scale 144 times as large. As the length of the diagram is 2.94 ins. and displacement 1.96 cu.ft., 1 in. of horizontal distance corresponds to .667 cu.ft. or 1 cu.ft to 1.50 ins. of distance. Lay off then from M distances of 1.50 ins. and mark the first 1 cu.ft. and the second 2 cu.ft., dividing the intervals into fractions. A similar scale of volumes in cubic inches might also be obtained.

By this process any indicator card may be converted into a pressure volume diagram for study and analysis, but there will always be required the two factors of *true atmospheric pressure* to find one axis of coordinates and the *clearance volume* to find the other.

Prob. 1. If in cards Nos. 1 and 2, Section 8, the clearances are 5 per cent and 3 per cent respectively of the displacement, convert the cards to PV diagrams on the same base to scales of 4 ins. to 1 cu.ft. and 1 in. to 1000 lbs. per square foot, for cylinders $9\frac{1}{4}$ ins. and $14\frac{1}{4}$ ins. respectively in diameter and stroke 12 ins.

Prob. 2. Do the same for cards Nos. 3 and 4, if clearances are 7 per cent and 4 per cent respectively, for cylinders 10 ins. and 17 ins. respectively in diameter and stroke 12 ins.

Prob. 3. Do the same for cards Nos. 7 and 8 if clearances are 12 per cent and 8 per cent respectively and cylinders 12 ins. and 19 ins. in diameter and stroke 24 ins.

12. To Find the Clearance. There are two general methods for the finding of clearance, the first a direct volumetric measurement of the space itself by filling with measured liquid and the second a determination by algebraic or graphic means from the location of two points on the expansion or compression curves of the indicator card based on an assumed law for the curves.

The first method of direct measurement is the only one that offers even a promise of accuracy, but even this is difficult to carry out because of the tendency of the measuring liquid to leak past piston or valves, which makes the result too large if the liquid be measured before the filling of the clearance space and too small if the liquid be measured after filling and drawing off. There is also a tendency in the latter case for some of the liquid to remain inside the space, besides the possibility in all cases of the failure to completely fill the space due to air pockets at high places.



FIG. 13.

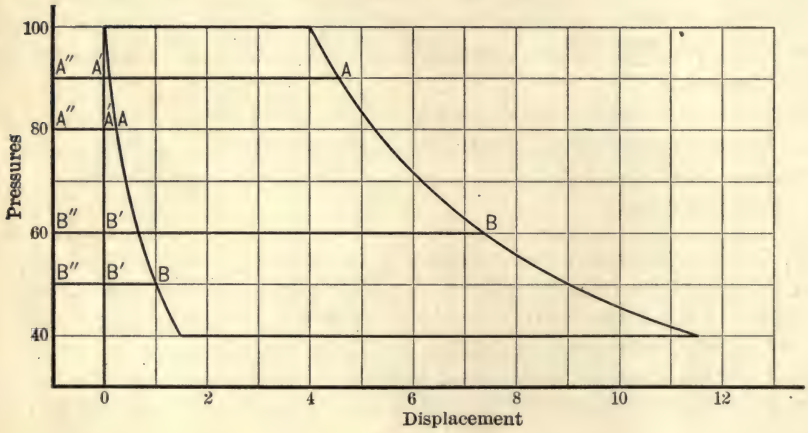


FIG. 14.

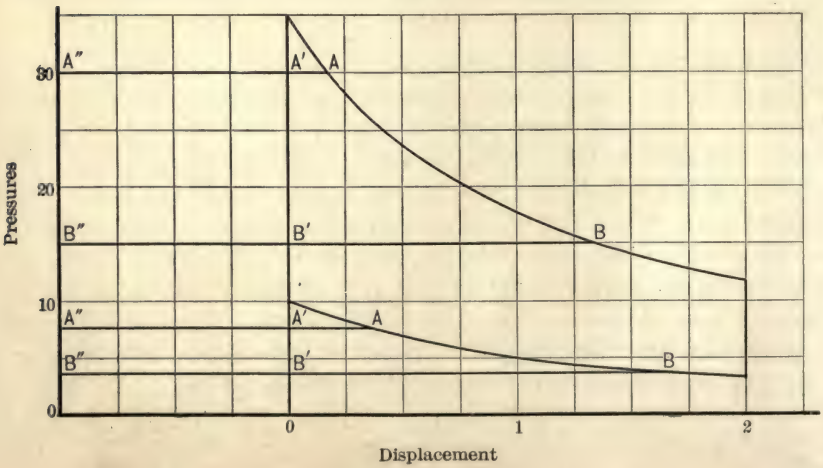


FIG. 15.

By the second general method any two points, A and B, on an expansion or mpression curve, Figs. 13, 14, 15, may be selected and horizontals drawn to e vertical line indicating the beginning of the stroke. The points A' and B' e distant from the unlocated axis an amount $A'A''=B'B''$, representing the earance.

Let the clearance volume $=Cl.$;
" the displacement up to $A=D_a$;
" the displacement up to $B=D_b$;
" the whole displacement $=D$;
" s be the exponent in $PV^s=$constant, which defines the law of the curve.

Then in general,

$$P_aV_a{}^s=P_bV_b{}^s.$$

ut

$$V_a=D_a+Cl,$$

id

$$V_b=D_b+Cl,$$

nce

$$P_a{}^{\frac{1}{s}}(D_a+Cl)=P_b{}^{\frac{1}{s}}(D_b+Cl),$$

$$Cl\left(P_a{}^{\frac{1}{s}}-P_b{}^{\frac{1}{s}}\right)=P_b{}^{\frac{1}{s}}D_b-P_a{}^{\frac{1}{s}}D_a,$$

hence the clearance in whatever units the displacement may be measured will be

$$Cl=\frac{P_b{}^{\frac{1}{s}}D_b-P_a{}^{\frac{1}{s}}D_a}{P_a{}^{\frac{1}{s}}-P_b{}^{\frac{1}{s}}},$$

$$Cl=\frac{P_b{}^{\frac{1}{s}}\left[D_b-\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}D_a\right]}{P_b{}^{\frac{1}{s}}\left[\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}-1\right]}=\frac{D_b-\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}D_a}{\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}-1},$$

id Cl, in per cent of the whole displacement will be

$$\text{Clearance as a fraction of displacement}=c=\frac{\frac{D_b}{D}-\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}\frac{D_a}{D}}{\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}-1}.$$

hen $s=1$ this takes the form

$$\text{Clearance in fraction of displacement}=c=\frac{\frac{D_b}{D}-\left(\frac{P_a}{P_b}\right)\frac{D_a}{D}}{\left(\frac{P_a}{P_b}\right)-1}. \quad . \quad . \quad (15)$$

To use such an expression it is only necessary to measure off the atmospher pressure below the atmospheric line, draw verticals at ends of the diagra and use the length of the horizontals and verticals to the points in the formu each horizontal representing one D and each vertical a P.

Graphic methods for the location of the axis of pressures, and hence t clearance, depend on the properties of the curves as derived from analyti For example, when $s=1$,

$$P_a V_a = P_b V_b,$$

which is the equation of the equilateral hyperbola, a fact that gives a comm name to the law, i.e., hyperbolic expansion or hyperbolic compression. T common characteristics of this curve may be used either separately or togeth the proof of which need not be given here, first that the diagonal of the rectan having two opposite corners on the curve when drawn through the other t corners will pass through the origin of coordinates, and second, that the oth diagonal drawn through two points of the curve and extended to intersect t axes of coordinates will have equal intercepts between each point and t nearest axis cut.

According to the first principle, lay off, Fig. 16, the vacuum line or a of volume XY and selecting any two points A and B, construct the rectan $ACBD$. Draw the diagonal CDE and erect at E the axis of pressures E then will EZ and EY be the axes of coordinates. According to the seco principle, proceed as before to locate the axes of volumes XY and select t points, A and B, Fig. 17. Draw a straight line through these points, whi represents the other diagonal of the rectangle $ACBD$, producing it to int sect XY at M and lay off $\overline{AN}=\overline{BM}$. Then will the vertical NE be the a of pressures. It should be noted that these two graphic methods apply o when $s=1$; other methods must be used when s is not equal to 1.

A method of finding the axis of zero volume is based upon the slope of t exponential curve,

$$PV^s = c.$$

Differentiation with respect to V gives

$$PsV^{s-1} + V^s \frac{dP}{dV} = 0,$$

or

$$\frac{Ps}{V} + \frac{dP}{dV} = 0,$$

whence

$$V = \frac{Ps}{\left(-\frac{dP}{dV}\right)} = sP\left(-\frac{dV}{dP}\right). \quad \ldots\ldots \quad (1)$$

In other words, the true volume at any given point on the known curve may be found by dividing the product of P and s by the tangent or the slope of the line at the given point, with the sign changed. This method gives results dependent for their accuracy upon the determination of the tangent to the curve, which is sometimes difficult.

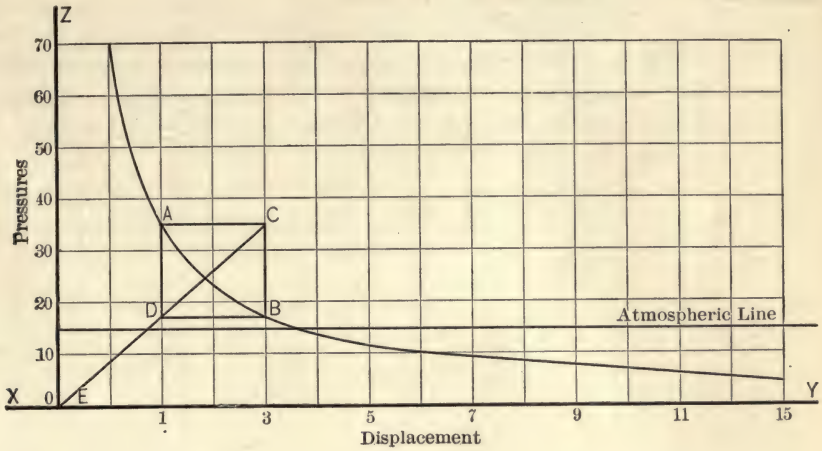


FIG. 16.

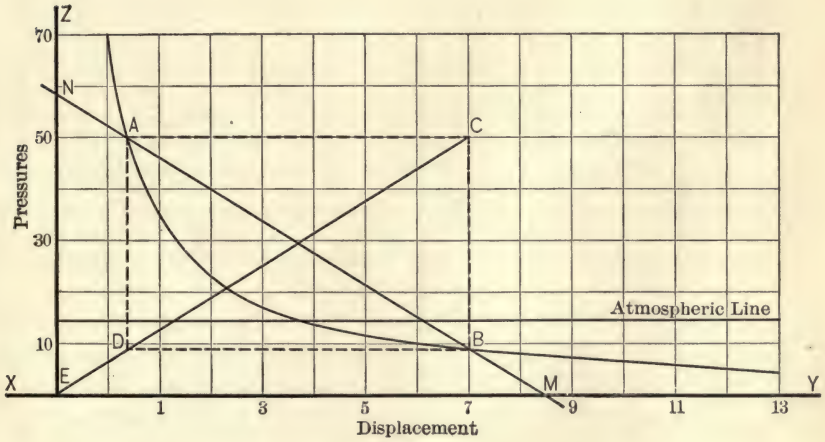


FIG. 17.

Graphic Methods of Locating the Clearance Line for Logarithmic or Hyperbolic Expansion and Compression Curves.

The following graphical solution is dependent upon the principle just given, and while not mathematically exact, gives results so near correct that the error is not easily measured. The curve ACB, Fig. 18, is first known experimentally or otherwise and therefore the value of s, and the axis FV from which pressures are measured is located. Assume that the axis of zero volume,

KP, is not known but must be found. Selecting any two convenient points, A and B, on the curve, complete the rectangle $AHBG$ with sides parallel and perpendicular to FV. The diagonal HG cuts the curve at C and the horizontal axis at E. From C drop the perpendicular CD. If now the distance DE be multiplied by the exponent s, and laid off DK, and the vertical KP erected, this may be taken as the zero volume axis.

It cannot be too strongly stated that methods for the finding of clearance or the location of the axes of pressures from the indicator card, much as they have been used in practice, are inaccurate and practically useless unless it is positively known beforehand just what value s has, since the assumed value

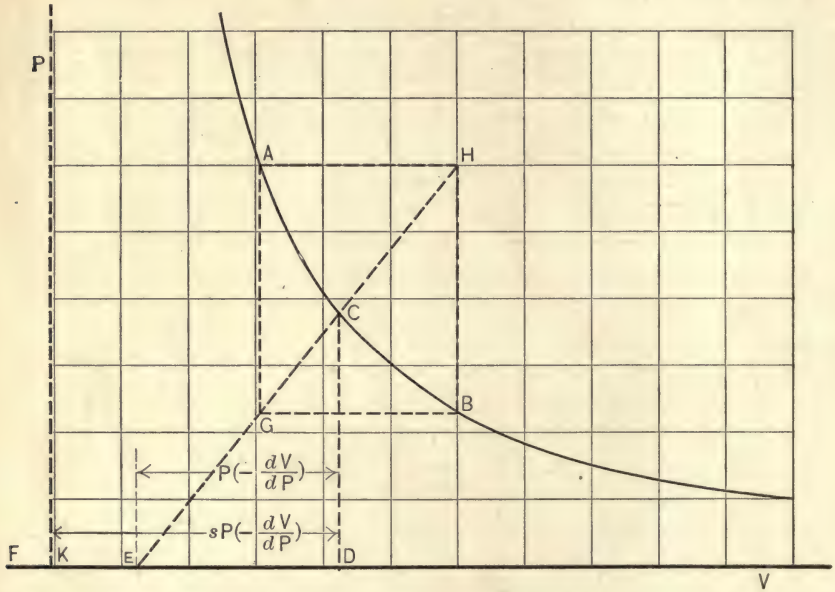


FIG. 18.—Graphic Method of Locating the Clearance Line for Exponential Expansion and Compression Curves.

of s enters into the work, and s for the actual diagram, as already explained, is affected by the substance, leakage, by moisture or wetness of vapor and by all heat interchange or exchange between the gas of vapor and its container.

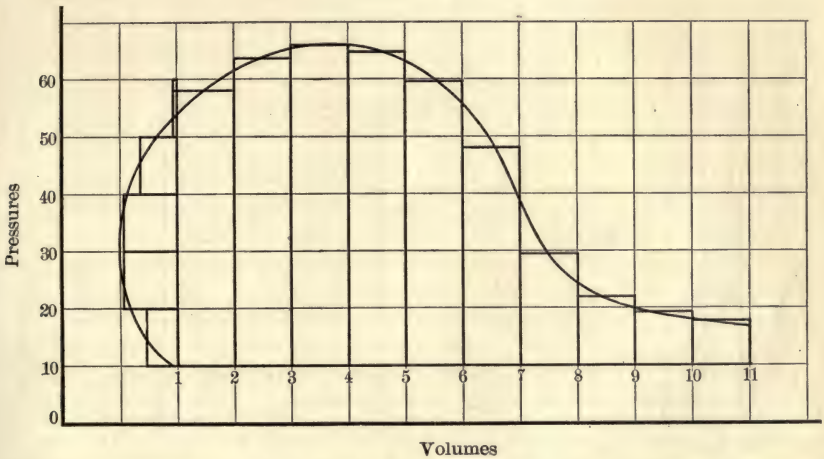
Prob. 1. If in card No. 6, Section 8, compression follows the law $PV^s = K$ where $s = 1.4$ and the barometer reads 29.9 ins. of mercury, locate the axes algebraically and graphically.

Prob. 2. If in card No. 3, Section 8, expansion follows the law $PV^s = K$ where $s = 1.37$ and the barometer reads 29.8 ins. of mercury, locate the axes algebraically and graphically.

Prob. 3. If in card No. 5, Section 8, expansion follows the law $PV^s = K$ where $s = 1$ and the barometer reads 27.5 ins. of mercury, locate axes algebraically and graphically.

13. Measurement of Areas of PV Diagrams and Indicator Cards. Areas pressure volume diagrams or indicator cards must be evaluated for the termination of work or mean effective pressure, except when calculation by rmula and hypothesis is possible. There are two general methods applicable both the indicator card and *PV* diagram, that of average heights, and the animeter measure, besides a third approximate but very useful method, pecially applicable to plotted curves on cross-section paper.

The third method assumes that the diagram may be divided into strips of ual width as in Fig. 19, which is very easily done if the diagram is plotted on oss-section paper. At the end of each strip, a line is drawn perpendicular to e axis of the strip, such that the area intercepted inside the figure is apparently ual to that outside the figure. If this line is correctly located, the area of the ctangular strip will equal the area of the strip bounded by the irregular lines.



IG. 19.—Approximate Method of Evaluating Areas and Mean Effective Pressures of Indicator Cards and P.V. Diagrams.

f the entire figure has irregular ends it may be necessary to subdivide one or oth ends into strips in the other direction, as is done at the left-hand side of ig. 19. The area of the entire figure will be equal to the summation of lengths f all such strips, multiplied by the common width. This total length may be btained by marking on the edge of a strip of paper the successive lengths in uch a way that the total length of the strip of paper when measured will be the otal length of the strips.

The mean height will be the total length of such strips divided by the num- er of strip-widths in the length of the diagram. By a little practice the proper ocation of the ends of the strips can be made with reasonable accuracy, and onsequently the results of this method will be very nearly correct if care is xercised.

By dividing the diagram into equal parts, usually ten, and finding the length f the middle of each strip, an approximation to the mean height of each strip

will be obtained; these added together and divided by the number will giv the mean height in inches from which the mean effective pressure may be foun by multiplying by the scale as above, or the area in square inches by multiply ing by the length in inches, which can be converted into work by multiplyin by the foot-pounds per square inch of area as fixed by the scales. As the pre sures usually vary most, near the ends of the diagram a closer approximatio can be made by subdividing the end strips, as is done in Fig. 20, which repr sents two steam engine indicator cards taken from opposite ends of the san cylinder and superimposed. The two diagrams are divided into ten equ spaces and then each end space is subdivided. The mean heights of the sul divisions are measured and averaged to get the mean height of the whole en division, or average pressure in this case for the division. The average heigh of divisions for diagram No. 1 are set down in a column on the left, while tho

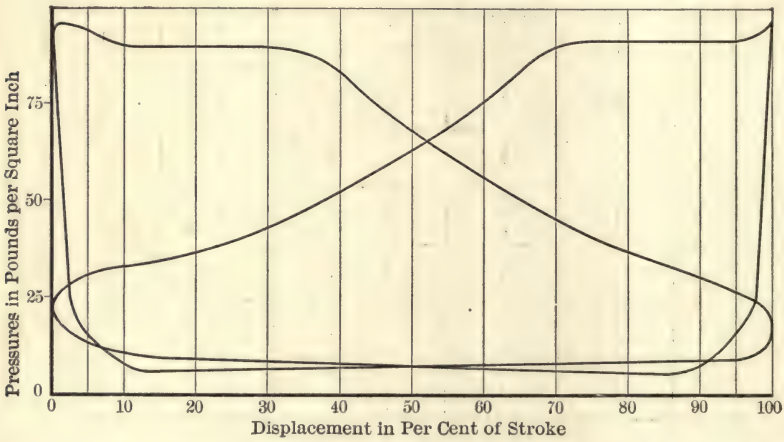


FIG. 20.—Simpson's Method for Finding Mean Effective Pressure of Indicator Cards.

for No. 2 are on the right; the sum of each column divided by ten and multipli by the spring scale gives the whole m.e.p. The heights of No. 1 in inch marked off continuously on a slip of paper measured a total of 11.16 ins. a for No. 2, 10.79 ins., these quantities divided by 10 (number of strips) and mul plied by the spring scale, 50, gives the m.e.p., as before. This method often designated as Simpson's rule.

The best and most commonly used method of area evaluation, whether f work or m.e.p. determination, is the planimeter, a well-known instrume specially designed for direct measurements of area.

14. Indicated Horse-power. Work done by the fluid in a cylinder, because is most often determined by indicator card measurements, measures the indicat horse-power, but the term is also applicable to work that would be done by t execution of a certain cycle of pressure volume changes carried out at a specifi rate. The mean effective pressure in pounds per square inch, whether of indicator card or PV cycle, when multiplied by piston area in square inch

ives the average force acting on the piston for one stroke, whether the cycle equired one, two or x strokes for its execution, and this mean force multiplied y the stroke in feet gives the foot-pounds of work done by the cycle. Therefore,

et m.e.p. = mean effective pressure in pounds per square inch for the cycle referred to one stroke;

" a = effective area of piston in square inches;

" L = length of stroke in feet;

" n = number of equal cycles completed per minute;

" N = number of revolutions per minute;

" S = mean piston speed = $2LN$ feet per minute;

" z = number of revolutions to complete one cycle = $\frac{N}{n}$. Then will the

ndicated horse-power be given by,

$$\left.\begin{aligned} \text{I.H.P.} &= \frac{(\text{m.e.p.})Lan}{33000} \quad \ldots\ldots\ldots (a) \\ &= \frac{(\text{m.e.p.})LaN}{33000z} \quad \ldots\ldots\ldots (b) \\ &= \frac{(\text{m.e.p.})aS}{33000 \times 2z} \quad \ldots\ldots\ldots (c) \end{aligned}\right\} (17)$$

When there are many working chambers, whether in opposite ends of the ame cylinder or in separate cylinders, the indicated horse-power of each should e found and the sum taken for that of the machine. This is important not nly because the effective areas are often unequal, as, for example, in opposite nds of a double-acting cylinder with a piston rod passing through one side nly or with two piston rods or one piston rod and one tail rod of unequal iameters, but also because unequal valve settings which are most common ill cause different pressure volume changes in the various chambers.

It is frequently useful to find the *horse-power per pound mean effective pressure*, hich may be symbolized by K_e, and its value given by

$$K_e = \frac{Lan}{33000} = \frac{LaN}{33000z}$$

sing this constant, which may be tabulated for various values of n, stroke nd bore, the indicated horse-power is given by two factors, one involving ylinder dimensions and cyclic speed or machine characteristics, and the other ne resultant PV characteristic, of the fluid, symbolically,

$$\text{I.H.P.} = K_e(\text{m.e.p.}).$$

hese tables of horse-power per pound m.e.p. are usually based on piston speed ather than rate of completion of cycles and are, therefore, directly applicable

when $z=\frac{1}{2}$ or $n=2N$, which means that the two cycles are completed in or revolution, in which case,

$$S=2LN=Ln,$$

and

$$K_e=\frac{aS}{33000},$$

whence

$$\text{I.H.P.}=K_e\,(\text{m.e.p.})=\frac{(\text{m.e.p.})aS}{33000}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18$$

Table XI at the end of this chapter gives values of (H.P. per lb. m.e.p.) (K_e for tabulated diameters of piston in inches and piston speeds in feet p minute. Tables are frequently given for what is called the engine constan which is variously defined as either

(*a*) $\frac{La}{33000}$, which must be multiplied by m.e.p.$\times n$ to obtain H.P., or

(*b*) $\frac{a}{33000}$, which must be multiplied by m.e.p.$\times L\times n$ to obtain H.P.

For an engine which completes two cycles per revolution, this is the same multiplying by m.e.p.$\times S$. Before using such a table of engine constants must be known whether it is computed as in (*a*) or in (*b*).

Example. A 9 in.$\times$12 in. double-acting steam engine runs at 250 R.P.M. and t mean effective pressure is 30 lbs. What is H.P. per pound m.e.p. and the I.H.P.?

$$K_e=\frac{Lan}{33000}=\frac{1\times63.6\times500}{33000};$$

$$\text{I.H.P.}=.9636\times30=28.908.$$

Prob. 1. A pump has a piston speed of 250 ft. per minute; piston diameter is 24 i What is the H.P. per pound mean effective pressure?

Prob. 2. A simple single-acting 2-cylinder engine has a piston 10 ins. in diameter w a 2-in. rod and a stroke of 15 ins. It runs with a mean effective pressure of 45 per square inch at a speed of 220 R.P.M. What is the H.P.?

Prob. 3. A gas engine has one working stroke in every four. If the speed is R.P.M. what must be the m.e.p. to give 6 H.P. when the cylinder has a diameter of 6 and a stroke of 12 ins.?

Prob. 4. An air compressor is found to have a mean effective pressure of 50 lbs. the cylinder is double acting and is 12 ins. diameter and 16 ins. stroke, what H.P. be needed to drive it at 80 R.P.M.?

Prob. 5. A gasoline engine has an engine constant (*a*) of .3. What must be m.e.p. to give 25 H.P.?

Prob. 6. A blowing engine has an m.e.p. of 10 lbs. Its horse-power is 500. W is the H.P. per pound m.e.p.?

Prob. 7. Two engines of the same size and speed are so run that one gives twice the power of the other. How will the engine constants and m.e.p. vary?

Prob. 8. From the diagrams following Section 9 what must have been the H.P. per pound m.e.p. to give 300 H.P. in each case?

Prob. 9. How will the H.P. per pound m.e.p. vary in two engines if the speed of one is twice that of the other, if the stroke is twice, if the diameter of piston is twice?

15. Effective Horse-power, Brake Horse-power, Friction Horse-power, Mechanical Efficiency, Efficiency of Transmission, Thermal Efficiency. Work is done and power developed primarily in the power cylinder of engines, and is transmitted through the mechanism with friction loss to some point at which it is utilized. There is frequently a whole train of transmission which may involve transformation of the energy into other forms, but always with some losses, including the mechanical friction. For example, a steam cylinder may drive the engine mechanism which in turn drives a dynamo, which transforms mechanical into electrical energy and this is transmitted to a distance over wires and used in motors to hoist a cage in a mine or to drive electric cars. There is mechanical work done at the end of the system and at a certain rate, so that there will be a certain *useful* or *effective* horse-power output for the system, which may be compared to the horse-power primarily developed in the power cylinders. A similar comparison may be made between the primary power or input and the power left after deducting losses to any intermediate point in the system. For example, the electrical energy per minute delivered to the motor, or *motor input*, is, of course, the output of the transmission line. Again, the electrical energy delivered to the line, or electrical transmission *input*, is the same as dynamo output, and mechanical energy delivered to the dynamo is identical with engine output. The comparison of these measurements of power usually takes one of two forms, and frequently both; first, a comparison by differences, and second, a comparison by ratios. The ratio of any horse-power measurement in the system to the I.H.P. of the power cylinder is the *efficiency* of the power system up to that point, the difference between the two is the horse-power loss up to that point. It should be noted that, as both the dynamo and motor transform energy from mechanical to electrical or vice versa, the engine mechanism transmits mechanical energy and the wires electrical energy, the system is made up of parts which have the function of (*a*) transmission without change of form, and (*b*) transformation of form. The ratio of output to input is always *an efficiency*, so that the *efficiency of the power system* is the *product* of all the *efficiencies of transformation* and of *transfer* or *transmission*, and the power loss of the system is the sum of transformation and transmission losses. Some of these efficiencies and losses have received names which are generally accepted and the meaning of which is generally understood by all, but it is equally important to note that others have no names, simply because there are not names enough to go around. In dealing with efficiencies and power losses that have accepted names these names may with reason be used, but in other cases where names are differently understood in different places or where there is no name, accurate description must be

relied on. As a matter of fact controversy should be avoided by definition of the quantity considered, whether descriptive names be used or not.

Effective horse-power is a general term applied to the output of a machine, or power system, determined by the form of energy output. Thus, for an engine it is the power that might be absorbed by a friction brake applied to the shaft, and in this case is universally called Brake Horse-power. The difference between brake and indicated horse-power of engines is the *friction horse-power* of the engine and the ratio of brake to indicated horse-power is the *mechanical efficiency* of the engine. For an engine, then, the effective horse-power or useful horse-power is the brake horse-power. When the power cylinders drive in one machine a pump or an air compressor, the friction horse-power of the machine is the difference between the indicated horse-power of the power cylinders and that for the pump or compressor cylinders, and the mechanical efficiency is the ratio of pump or air cylinder indicated porse-power to indicated horse-power of the power cylinders. Whether the indicated horse-power of the air or pump cylinders can be considered a measure of useful output or not is a matter of difference of opinion. From one point of view the machine may be as considered built for doing work on water or on air, in which case these horse-powers may properly be considered as useful output. On the other hand, the power pump is more often considered as a machine for moving water, in which case the useful work is the product of the weight of water moved into its head in feet, and includes all friction through ports, passages and perhaps even in pipes or conduits, which the indicated horse-power of the pump cylinder does not include, especially when leakage or other causes combine to make the pump cylinder displacement differ from the volume of water actually moved. With compressors the situation is still more complicated, as the air compressor may be considered useful only when its discharged compressed air has performed work in a rock drill, hoist or other form of an engine, in which case all sorts of measures of useful output of the compressor may be devised, even, for example, as the purely hypothetically possible work derivable from the subsequent admission and complete expansion of the compressed air in a separate air engine cylinder.

Too accurate a definition, then, of *output* and *input energy* in machines and power systems is not possible for avoidance of misunderstanding, which may affect questions both of power losses and efficiency of transmission and transformation whether in a power system or single machine. It is interesting to note here that not only is the *indicated work* of the power cylinder always considered the measure of power input for the system or machine, but, as in the other cases, it is itself *an output* or result of the action of heat on the vapor or gas and of the cycle of operations carried out. The ratio of the indicated power or cylinder work, to the heat energy both in foot-pound units, that was expended on the fluid is the *thermal efficiency* of the engine referred to indicated horse-power or the *efficiency* of *heat transformation* into work, the analysis of which forms the bulk of the subject matter of Chapter VI. Similarly, the ratio of any power measurement in the system to the equivalent of the heat supplied is the thermal efficiency of so much of the system as is included.

Example. It has been found that when the indicated horse-power of an engine is 50, a generator is giving out 700 amperes at 220 volts. At end of a transmis ion line is a otor using the output of the generator. This motor on test gave out 180 brake horse- ower. Assuming no losses in the transmission line, what was the efficiency of the otor, of the generator, of the engine, and of the system?

$$\text{Motor efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{180}{\frac{220 \times 700}{746}} = 87.2\%.$$

NOTE: Volts $\times$ amperes = watts, and, watts $\div$ 746 = H.P.

$$\text{Engine and generator efficiency} = \frac{\frac{220 \times 700}{746}}{250} = 82.4\%.$$

$$\text{Efficiency of system} = \frac{180}{250} = 72\% \quad \text{or} \quad 82.4 \times 87.2 = 72\%.$$

Prob. 1. An engine is belted to a pump; the I.H.P. of the engine is 50, of the pump 0, and the pump delivers 1200 gallons water per minute against 100-ft. head. What is he efficiency of each part and of the entire system.

Prob. 2. An engine is geared to air compressor. Upon test, the efficiency of the engine lone, gearing alone and compressor alone were each 80 per cent. When the com- ressor H.P. was 100 what was that of the engine?

Prob. 3. A water-wheel is run by the discharge from a pump. The B.H.P. of wheel s found to be 20 when the pump is delivering 45 gallons of water per minute at a ead of 1000 lbs. per square inch. The water I.H.P. of the pump is 30 and the team I.H.P. is 40. What are the efficiencies of each part of the system and the over-all fficiency?

Prob. 4. Perry gives a rule for the brake horse-power of steam engines as being qual to .95 I.H.P. $-$ 10. On this basis find the mechanical efficiency of a 500-H.P. ngine from 200 to 500 H.P. Show results by a curve with B.H.P. and per cent efficiency s coordinates.

Prob. 5. Perry gives a rule for the efficiency of an hydraulic line as $H = .7I - 25$ where I is the useful power of the pump and I is the indicated. Find I for values of H from 00 to 300 and plot a curve of results.

Prob. 6. An engine gives one I.H.P. for every 3 lbs. of coal per hour. One pound f coal contains 9,500,000 ft.-lbs. of energy. What is the thermal efficiency?

Prob. 7. A gas engine has a mechanical efficiency of 70 per cent when delivering ower to a generator which in turn has an efficiency of 90 per cent. If the engine uses 5 cu.ft. of gas per indicated horse-power hour and the gas contains 700,000 ft.-lbs. per ubic foot, what is the net thermal efficiency of the system?

16. Specific Displacement, Quantity of Fluid per Hour or per Minute per I.H.P. It has been shown that the work done in cylinders by pressure volume hanges of the vapor or gas depends on the mean effective pressure and on the lisplacement, or that there is a relation between I.H.P. and displacement. The quantity of fluid used also depends on the displacement and may be expressed

in cubic feet per minute at either the low pressure or high pressure condition when the work is done between two definite pressure limits, or in terms of pounds per minute or hour, which involves the application of fluid densities to volumes and which eliminates the double expression for the two conditions of pressure. The displacement per hour per horse-power, termed the *specific displacement*, is the basis of computations on the steam consumption of steam engines, the horse-power per cubic feet of free air per minute for air compressors, the horse-power per ton refrigeration for refrigerating machines and the consumption of fuel per hour per horse-power for gas and oil engines. It is therefore, a quantity of great importance in view of these applications. Applying the symbols already defined to displacement in one direction of one side of a piston

$$\text{Displacement in cu.ft. per stroke} = L \times \frac{a}{144};$$

$$\text{Displacement in cu.ft. per minute} = L \times \frac{a}{144} \times N;$$

$$\text{Displacement in cu.ft. per hour} = 60L \times \frac{a}{144} \times N.$$

$$\text{Indicated horse-power} = \frac{(\text{m.e.p.})Lan}{33000} = \frac{(\text{m.e.p.})LaN}{33000z}.$$

Whence expressing displacement per hour per I.H.P. or specific displacement in one direction for one side of a piston by D_s,

$$D_s = \frac{60L \times \frac{a}{144} \times N}{\frac{(\text{m.e.p.})LaN}{33000z}} = \frac{60 \times 33000z}{144(\text{m.e.p.})} = \frac{13750z}{(\text{m.e.p.})} \quad . \quad . \quad . \quad . \quad (19$$

From Eq. (19) *it appears that the specific displacement is equal to* $z \times 13{,}75($ *divided by the mean effective pressure in pounds per square inch.*

If two points, A and B, be so located on the indicator card, Fig. 21, as t have included between them a fluid transfer phase, either admission to, o expulsion from the cylinder, then calling δ_a = pounds per cubic foot or densit at point A, and δ_b = pounds per cubic foot or density at point B, the weigh of fluid present at A is,

$$(D_a + Cl)\delta_a \text{ lbs.},$$

and weight of fluid present at B is

$$(D_b + Cl)\delta_b \text{ lbs.},$$

whence the weight that has changed places or passed in and out per stroke i

$$(D_b + Cl)\delta_b - (D_a + Cl)\delta_a \text{ lbs. per stroke.}$$

both A and B lie on the same horizontal as A and B', $\delta_a = \delta_b = \delta$, the ensity of fluid at the pressure of measurement, whence the weight of fluid sed per stroke, will be

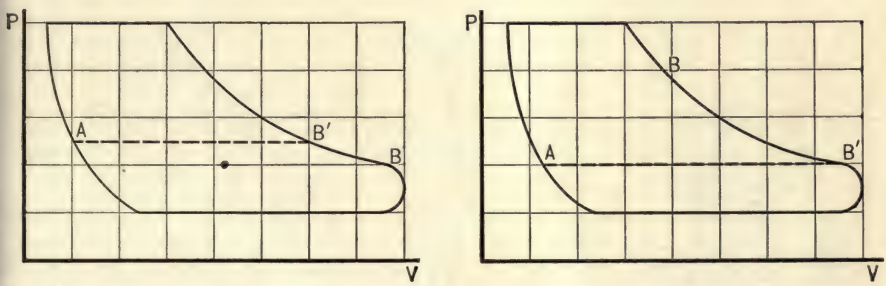
$$(D_b' - D_a)\delta,$$

nd the volume per stroke used at density δ is

$$D_b' - D_a \text{ cu.ft.},$$

hich compared to the displacement is

$$\frac{D_b' - D_a}{D},$$



IG. 21.—Determination of Consumption of Fluid per Hour per Indicated Horse-power from the Indicator Card.

This is the fraction of the displacement representing the volume of fluid passing through the machine at the selected pressure. Multiplying the specific displacement by this, there results,

$$\text{Cu.ft. of fluid per hr. at density } (\delta) \text{ per I.H.P.} = \frac{13750}{\text{(m.e.p.)}} \frac{D_b' - D_a}{D},$$

and

$$\text{Lbs. of fluid per hr. per I.H.P} = \frac{13750}{\text{(m.e.p.)}} \left(\frac{D_b - D_a}{D}\right)\delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

More generally, that is, when A and B are not taken at the same pressures

$$\text{Lbs. fluid per hr. per I.H.P.} = \frac{13750}{\text{(m.e.p.)}D}\Big[(D_b + Cl)\delta_b - (D_a + Cl)\delta_a.\Big] \quad . \quad (21)$$

The particular forms which this may take when applied to special cases will be examined in the succeeding chapters.

Example. An air compressor whose cylinder is 18×24 ins. (18 ins. in diameter and stroke 2 ft.) runs at 60 R.P.M. and is double acting. The mean effective air pressure is 50 lbs. per square inch. What is the specific displacement?

$$\text{Cu.ft. per hour} = \frac{60\,Lan}{144} = 60 \times 2 \times \frac{254.5}{144} \times 120 = 25{,}600.$$

$$\text{I.H.P.} = \frac{\text{m.e.p.}\,Lan}{33{,}000} = \frac{50 \times 2 \times 254.5 \times 120}{33{,}000} = 92.3;$$

$$\frac{\text{Cu.ft. per hour}}{\text{I.H.P.}} = \frac{25{,}600}{92.3} = 277,$$

or by the formula directly,

$$\frac{13{,}750}{\text{m.e.p.}} = \frac{13{,}750}{50} = 275.$$

Prob. 1. What will be the cubic feet of free air per hour per horse-power delivered by a 56×72-in. blowing engine with 4 per cent clearance and mean effective pressure of 10 lbs. per square inch?

Prob. 2. An 18×22-in. ammonia compressor works with a mean effective pressure of 45 lbs. per square inch. What is the weight of NH_3 per I.H.P. hour if the speed is 50 R.P.M. and compressor is double acting having a volumetric efficiency of 90 per cent? Use tabular NH_3 densities.

Prob. 3. A steam engine whose cylinder is 9×12 ins. runs at a speed of 300 R.P.M. and is double acting. If the m.e.p. is 60 lbs. and the density of steam at end of the stroke is .03, how many pounds of steam are used per hour per indicated horse-power?

17. Velocity Due to Free Expansion by PV Method. All the cases examined for the work done by PV cycles have been so far applied only to their execution in cylinders, but the work may be developed in nozzles accelerating the gas or vapor in free expansion, giving, as a consequence, a high velocity to the fluid. It was noted that for cylinders many combinations of phases might be found worthy of consideration as typical of possible actual conditions of practice, but this is not true of proper nozzle expansion, which has but one cycle, that of Fig. 22. That this is the cycle in question is seen from the following considerations. Consider a definite quantity of the gas or vapor approaching the nozzle from a source of supply which is capable of maintaining the pressure. It pushes forward that in front of it and work will be done, $ABCD$, equal to the admission of the same substance to a cylinder, so that its approach AB may be considered as a constant pressure, volume increasing phase for which the energy comes from the source of supply. This same substance expanding to the lower pressure will do the work $CBEF$; but there will be negative work equivalent to the pushing away or displacing of an equivalent quantity of fluid at the low pressure, or $FEGD$, making the work cycle $ABEG$, in which AG is the excess of

nitial over back pressure or the effective working pressure, remaining constant luring approach and lessening regularly during expansion to zero excess at E. The work done will be from Eq. (13),

$$W = P_b V_b + \frac{P_b V_b}{s-1}\left[1-\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}\right] - P_e V_e.$$

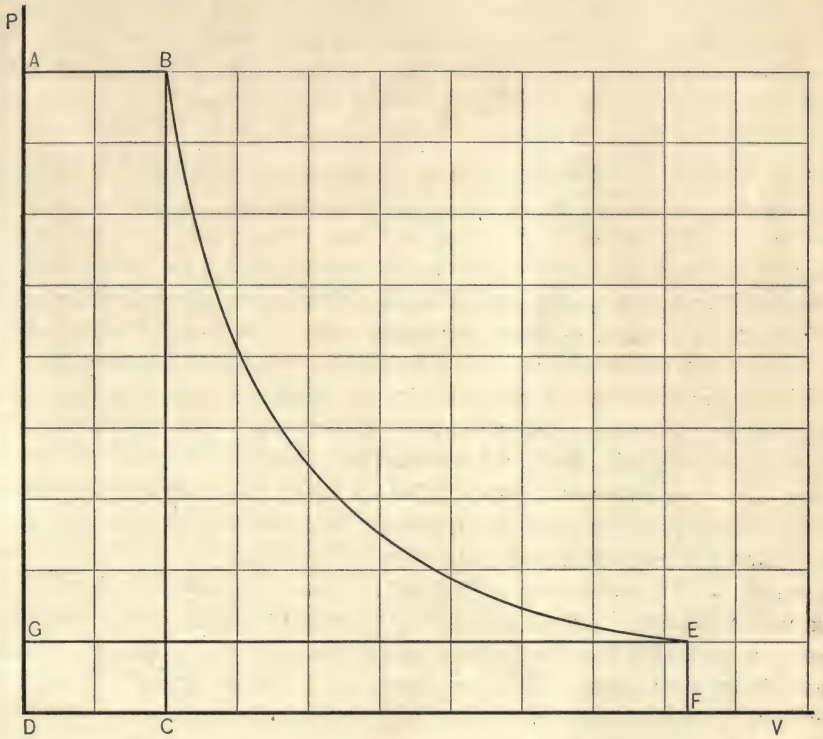


FIG. 22.—Pressure-Volume Diagram for Nozzle Expansion Measuring the Acceleration Velocity and Horse-power of Jets.

But

$$P_e V_e{}^s = P_e V_e V_e{}^{s-1} = P_b V_b{}^s = P_b V_b V_b{}^{s-1};$$

$$\therefore \quad P_e V_e = P_b V_b \left(\frac{V_b}{V_e}\right)^{s-1} = P_b V_b \left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}};$$

Whence

$$W = P_b V_b + \frac{P_b V_b}{s-1}\left[1-\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}\right] - P_b V_b \left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}$$

$$= \frac{s}{s-1} P_b V_b \left[1-\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Assuming the initial velocity to be zero, and the work of Eq. (22) to be don on 1 lb., the final or resultant velocity will be according to Eq. (6),

$$u = \sqrt{2gW}$$

$$= \sqrt{2g\frac{s}{s-1}P_bV_b\left[1-\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}\right]} \quad \ldots\ldots \quad (23$$

or

$$u = 8.02\sqrt{\frac{s}{s-1}P_bV_b\left[1-\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}\right]} \quad \ldots\ldots \quad (24$$

This velocity is in feet per second when pressures are in pounds per square foo and volumes in cubic feet of 1 lb. of substance, and is known as Zeuner' equation for the velocity of a gas or vapor expanding in a nozzle. It i generally assumed that such expansion, involving as it does very rapid motio of the fluid past the nozzle, is of the adiabatic sort, as there seems to be n time for heat exchange between fluid and walls. As already noted, the valu of s for adiabatic expansion of vapors is not constant, making the correc solution of problems on vapor flow through orifices practically impossible b this method of pressure volume analysis, but as will be seen later the therm method of solution is exact and comparatively easy.

NOTE. A comparison of Eqs. (22) and (13) and the figures correspondin will show that the area under the process curve, which is the same as the wor done during the compression or expansion, if multiplied by s will equal the are to the left of the process curve, which in turn represents, as in Fig. 21, fo engines, the algebraic sum of admission, complete expansion, and exhaust wor areas, or as in Fig. 9 for compressors, the algebraic sum of suction, compressio and delivery work areas. This statement must not be thought to refer to th work area of such a cycle as that of Fig. 8, where expansion is incomplete, nor case of over-expansion, Fig. 10.

Example. In Fig. 22 assume the initial pressure at 100 lbs. per square inch absolut back pressure at atmosphere, and expansion as being adiabatic. What will be t work per pound of steam and the velocity of the jet, if V_b is 4.36 cu.ft. and $s = 1.3$ f superheated steam?

$$W = \frac{s}{s-1}P_bV_b\left[1-\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}\right]$$

$$= \frac{1.3}{.3} \times 144 \times 100 \times 4.36\left[1-\left(\frac{14.7}{100}\right)^{\frac{.3}{1.3}}\right]$$

$$= 27{,}206 \times .608 = 16{,}541 \text{ ft.-lbs.};$$

$$u = \sqrt{2gW} = 8.02\sqrt{16{,}541};$$

$$= 1028 \text{ ft. per second.}$$

Prob. 1. Taking the same pressure range as above, find W and u for adiabatic expansion of air, also for isothermal expansion.

Prob. 2. How large must the effective opening of the suction valve be, in an air compressor 18×24 ins. to allow the cylinder to properly fill if the mean pressure-drop through the valve is 1 lb. per square inch and the compressor runs at 80 R.P.M.?

Prob. 3. What must be the diameter of the inlet valve in a gas engine to fill a cylinder of $\frac{1}{3}$ cu.ft. capacity if the lift of the valve is $\frac{3}{8}$ ins., allowing a pressure drop of 1 lb. per square inch? Engine makes 150 working strokes per minute.

Prob. 4. It has been found from experiment that the velocity of air issuing from a hole in plate orifice is 72 per cent of what would be expected from calculation as above when the absolute pressure ratio is 2 to 1, and 65 per cent when the absolute pressure ratio is $1\frac{1}{2}$ to 1. What will be the actual velocity for air flowing from a tank to atmosphere for these pressure ratios?

Prob. 5. CO_2 stored in a tank is allowed to escape through an orifice into the air. What will be the maximum velocity of the jet if the pressure on the tank be 100 lbs. per square inch gage?

NOTE: 1 lb. CO_2 at pressure of 100 lbs. per square inch gage occupies 1.15 cu.ft.

Prob. 6. If ammonia gas and hydrogen were allowed to expand from the same pressure, how would their maximum velocities compare? Vol. of 1 lb. of NH_3 at 50 lbs. per square inch gage is 4.5 cu.ft. Vol. of 1 lb. of H at same pressure is 77.5 cu.ft.

18. Weight of Flow through Nozzles. Applying an area factor to the velocity equation will give an expression for cubic flow per second which becomes weight per second by introducing the factor, density.

Let the area of an orifice at the point of maximum velocity, u, be A sq.ft., then will the cubic feet per second efflux be Au. Assume the point of maximum velocity, having area A, to be that part of the nozzle where the pressure has fallen to P_e, Fig. 22, and the gas or vapor to have the density δ_e pounds per cubic foot. Then will the nozzle flow in pounds per second be

$$w = uA\delta_e.$$

But the weight per cubic foot is the reciprocal of the cubic feet per pound, V_e, which it has already been assumed, is the final volume, of one pound of the fluid. Hence,

$$w = \frac{uA}{V_e}.$$

This may be put in terms of initial gas or vapor conditions for,

$$V_e = V_b\left(\frac{P_b}{P_e}\right)^{\frac{1}{s}}.$$

Whence

$$w = \frac{uA}{V_b\left(\frac{P_b}{P_e}\right)^{\frac{1}{s}}} = \frac{uA}{V_b}\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}$$

Substituting in this the value of u from Eq. (24),

$$w=\frac{A}{V_b}\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}\sqrt{2g\frac{s}{s-1}P_bV_b\left[1-\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}\right]}$$

$$=8.02\frac{A}{V_b}\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}\left\{\frac{s}{s-1}P_bV_b\left[1-\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}\right]\right\}^{\frac{1}{2}}\text{lbs. per sec. .} \quad (25)$$

This weight will be a maximum for a certain value of the pressure ratio, depending on the value of s only, and this value can be found by placing the first differential coefficient of w with respect to $\left(\frac{P_e}{P_b}\right)$ equal to zero.

$$\frac{dw}{d\left(\frac{P_e}{P_b}\right)}=0.$$

To accomplish this, rearrange Eq. (25) as follows:

$$w=A\left\{2g\frac{s}{s-1}\frac{P_b}{V_b}\left[\left(\frac{P_e}{P_b}\right)^{\frac{2}{s}}-\left(\frac{P_e}{P_b}\right)^{\frac{s+1}{s}}\right]\right\}^{\frac{1}{2}}.$$

But as the other factors do not enter to effect the result so long as P_b does not vary, w is a maximum when the bracket

$$\left[\left(\frac{P_e}{P_b}\right)^{\frac{2}{s}}-\left(\frac{P_e}{P_b}\right)^{\frac{s+1}{s}}\right],$$

is a maximum or when

$$\frac{2}{s}\left(\frac{P_e}{P_b}\right)^{\frac{2}{s}-1}-\left(\frac{s+1}{s}\right)\left(\frac{P_e}{P_b}\right)^{\frac{s+1}{s}-1}=0;$$

or

$$2\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}-1}-(s+1)\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}=0;$$

or

$$\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}\left[2\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}-1}-(s+1)\right]=0.$$

But as $\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}$ cannot be equal to zero in practice, then

$$2\left(\frac{P_e}{P_b}\right)^{\frac{1-s}{s}}-(s+1)=0,$$

which gives the condition that w is a maximum when

$$\left(\frac{P_e}{P_b}\right)^{\frac{1-s}{s}}=\frac{s+1}{2},$$

r maximum flow for given initial pressure occurs when

$$\left(\frac{P_e}{P_b}\right)=\left(\frac{2}{s+1}\right)^{\frac{s}{s-1}} \quad \ldots\ldots\ldots\ldots \quad (26)$$

For air expanding adiabatically $s=1.407$. Maximum flow occurs when $\frac{P_e}{P_b}=.528$ and for most common values of s it will be between .50 and .60

his result is quite remarkable and is verified by experiment reasonably closely. *t shows that, contrary to expectation, the weight of efflux from nozzles will not ontinuously and regularly increase with increasing differences in pressure, but or a given initial pressure the weight discharged per second will have reached s limit when the final pressure has been diminished to a certain fraction of the nitial, and any further decrease of the discharge pressure will not increase the low through an orifice of a given area.*

The subject of flow in nozzles will be treated more completely in hapter VI.

Prob. 1. For the following substances under adiabatic expansion determine the ressure ratio for maximum flow and find the rate of flow per square inch of orifice under his condition when flow is into a vacuum of 10 ins. of mercury with standard barometer:

(*a*) Carbon dioxide.
(*b*) Nitrogen.
(*c*) Hydrogen.
(*d*) Ammonia.
(*e*) Dry steam according to saturation law.

19. Horse-power of Nozzles and Jets. Although, strictly speaking, nozzles an have no horse-power, the term is applied to the nozzle containing the rifice through which flow occurs and in which a certain amount of work is one per minute in giving to a jet of gas or vapor initially at rest a certain nal velocity, and amount of kinetic energy. The foot-pounds of work per ound of fluid multiplied by the pounds flowing per second will give the foot-ounds of work developed per second within the nozzle, and this divided by 50 will give the horse-power developed by the jet, or the nozzle horse-power. ccordingly,

$$\left.\begin{aligned}
\text{H.P. of jet}&=\frac{W\times w}{550}=\frac{W}{550}\times\frac{A}{V_b}\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}\times u \quad \ldots\ldots\ldots \quad (a)\\
&=\frac{W}{550}\frac{A}{V_b}\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}\sqrt{2g\,W} \quad \ldots\ldots\ldots \quad (b)\\
&=.01458\frac{A}{V_b}\left(\frac{P_e}{P_b}\right)^{\frac{1}{s}}\left\{\frac{s}{s-1}P_bV_b\left[1-\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{s}}\right]\right\}^{\frac{3}{2}},(c)
\end{aligned}\right\} \quad (27)$$

where the expression in the bracket is the work done per pound of substance. The pressures are expressed in pounds per square foot, areas in square feet and volumes in cubic feet.

Example. A steam turbine operates on wet steam at 100 lbs. per square inch absolute pressure which is expanded adiabatically to atmospheric pressure. What must be the area of the nozzles if the turbine is to develop 50 H.P. ideally?

NOTE: 1 cu.ft. of steam at 100 lbs. = .23 lb.

By Eq. (26), maximum flow occurs when the pressure ratio is $\left(\frac{2}{s+1}\right)^{\frac{s}{s-1}}$, or, for this case when the pressure is $100 \div \left(\frac{2}{2.11}\right)^{\frac{1.11}{.11}} = 58$ lbs. per square inch absolute. As the back pressure is one atmosphere, the flow will not be greater than for the above critical pressure. Substituting it in Eq. (25) will give the flow weight w, and using the actual back pressure in Eq. (22) will give the work W.

$$\text{By Eq. (22), } W = \left(\frac{1.11}{.11}\right) \times \left(\frac{14400}{.23}\right)\left[1 - \left(\frac{2116}{14400}\right)^{\frac{.11}{1.11}}\right]$$

$$= 110000 \text{ ft.-lbs. per pound of steam.}$$

$$\text{By Eq. (25), } w = 8.02A \times .23 \times (.58)^{\frac{1}{1.11}} \left\{ \frac{1.11}{.11} \times \frac{14400}{.23}\left[1 - (.58)^{\frac{.11}{1.11}}\right] \right\}^{\frac{1}{2}}$$

$$= 198A \text{ lbs. of steam per second.}$$

By Eq. (27a),

$$\text{H.P.} = \frac{W \times w}{550} = 50 = \frac{110000 \times 198A}{550}$$

Whence

$$A = \frac{50}{39600} \text{ square feet.}$$

Prob. 1. What will be the horse-power per square inch of nozzle for a turbine using hot gases if expansion follows law $PVs = k$, when $s = 1.37$, the gases being at a pressure of 200 lbs. per square inch absolute and expanding to atmosphere.

Let the volume per pound at the high pressure be 2 cu.ft.

Prob. 2. What will be the horse-power per square inch of nozzle for the problem of Section 17?

Prob. 3. Suppose steam to expand according to law $PVs = k$, where $s = 1.111$, from atmosphere to a pressure of 2 lbs. per sq. inch absolute. How will the area of the orifice compare with that of the example to give the same horse-power?

NOTE: $V_b = 26.4$.

Prob. 4. Suppose steam to be superheated in the case of the example and of the last problem, how will this affect the area of nozzle?

NOTE: Let $V_b = 5$ and 32 respectively.

Prob. 5. How much work is done per inch of orifice if initial pressure is 100 lbs. absolute on one side and final 10 lbs. absolute on other side of a valve through which air is escaping?

GENERAL PROBLEMS ON CHAPTER I

1. An air compressor is required to compress 500 cu.ft. of free air per minute to a pressure of 100 lbs. per square inch gage; the compressor is direct connected to a steam engine. The mechanical efficiency of the machine is 80 per cent. What will be the steam horse-power if compression is (*a*) isothermal; (*b*) adiabatic?

2. A mine hoist weighing 10 tons is raised 4500 ft. in one minute. In the first 20 seconds it is accelerated from rest to a speed of 6000 ft. per minute; during the next 30 seconds speed is constant at this value, and during last 10 seconds it is brought to rest. What will be (*a*) work of acceleration for each period; (*b*) work of lift for each period; (*c*) total work supplied by engine; (*d*) horse-power during constant velocity period?

3. The engine driving the above hoist is driven by compressed air. If air is supplied at a pressure of 150 lbs. per square inch gage and is admitted for three-quarters of the stroke, then expanded adiabatically for the remainder of the stroke and exhausted to the atmosphere find (*a*) what must the piston displacement be to lift the hoist, the work of acceleration being neglected? (*b*) To what value could the air pressure be reduced if air were admitted full stroke?

4. It is proposed to substitute an electric motor for the above engine, installing a water-power electric plant at a considerable distance. The type of wheel chosen is one in which a jet of water issuing from a nozzle strikes against a series of revolving buckets. The available head at the nozzle is 1000 ft. Assuming the efficiency of the motor to be 85 per cent, transmission 80 per cent, generators 90 per cent, and water-wheels 60 per cent, what will be the cubic feet of water per minute?

5. A steam turbine consists of a series of moving vanes upon which steam jets issuing from nozzles impinge. It is assumed that for best results the speed of the vanes should be half that of the jets. The steam expands from 100 lbs. per square inch gage to 5 lbs. per square inch absolute. (*a*) What must be the best speed of vanes for wet steam where $s = 1.111$? (*b*) If 55 per cent of the work in steam is delivered by the wheel what must be the area of nozzle per 100 H.P., and weight of steam per hour 100 H.P?

NOTE: $V_b = 3.82$.

6. It has been found that a trolley car uses a current of 45 amperes at 550 volts when running 12 miles per hour. If the motor is 80 per cent efficient, what is tractive effort?

NOTE: Volts $\times$ amperes = watts, and watts $\div .746$ = H.P.

7. An engine whose cylinder is 18×24 ins. (diameter = 18 ins., stroke = 24 ins.) is double acting and runs at a speed of 125 R.P.M. Steam is admitted for one-quarter stroke at a pressure of 125 lbs. per square inch gage, allowed to expand for the rest of the stroke and then exhausted into a vacuum of 28.25 ins. of Hg. (*a*) Draw a PV diagram of the cycle and find the m.e.p. from the diagram and by calculation, and then find the horse-power. (*b*) Consider steam to be admitted one-half stroke without other change. How will the horse-power vary? (*c*) What will be the horse-power for one-quarter admission if the exhaust pressure is 15 lbs. per square inch absolute? (*d*) What will be the horse-power if the steam pressure be made 150 lbs. per square inch absolute, other conditions as in (*a*)? (*e*) Suppose all conditions as in (*a*) but the speed lowered to 75 R.P.M. What will be the horse-power?

8. Assuming that 50 per cent of the work in the jet is transformed to useful work, what must be the total area of the nozzles of a steam turbine to develop the same horse-

power as the engine in problem (7*a*), the pressure range being the same and s being 1.3? $V_b = 3.18$.

9. Water is being pumped from the bottom of a shaft 700 ft. deep at the rate of 1000 gallons per minute by an electrically driven pump. Efficiency of pump is 70 per cent, motor 90 per cent, transmission line 95 per cent, generator 85 per cent, and mechanical efficiency of engine 80 per cent. What will be the indicated horse-power of the engine? If the above installation were replaced with an air-driven pump of 65 per cent efficiency, efficiency of transmission being 100 per cent, and that of the compressor and engine 80 per cent, what would be the horse-power of this engine?

10. Show by a PV diagram, assuming any convenient scales, that the quantity of air discharged by a compressor and the horse-power, both decrease as the altitude increases, and that the horse-power per cubic foot of air delivered increases under the same condition.

11. A centrifugal pump is driven by a steam engine directly connected to it. The pump is forcing 1000 gallons of water per minute against a head of 250 ft. and runs at a speed of 450 R.P.M. The engine is double acting and its stroke equals the diameter of the cylinder. Steam of 100 lbs. per square inch gage is admitted for half stroke, allowed to expand the rest of the stroke so that $s = 1$, and is then exhausted to atmosphere. What must be the size of the engine if the pump efficiency is 65 per cent and the engine efficiency 75 per cent?

12. (*a*) What will be the pounds of steam used by this engine per hour per horse-power? (*b*) If the steam were admitted but one-quarter of the stroke and the initial pressure raised sufficiently to maintain the same horse-power, what would be the new initial pressure and the new value of the steam used per horse-power per hour?

NOTE: Weight of steam per cubic foot for (*a*) is .261; for (*b*) is .365.

13. If it were possible to procure a condenser for the above engine so that the exhaust pressure could be reduced to 2 lbs. per square inch absolute, (*a*) how much would the power be increased for each of the two initial pressures already given? (*b*) How would the steam consumption change?

14. A motor-fire engine requires a tractive force of 1300 lbs. to drive it 30 miles per hour, its rated speed. The efficiency of engine and transmission is 80 per cent. When the same engine is used to actuate the pumps 70 per cent of its power is expended on the water. What will be the rating of the engine in gallons per minute when pumping against a pressure of 200 lbs. per square inch?

15. A compressor when compressing air at sea level from atmosphere to 100 lbs. per square inch absolute, expends work on the air at the rate of 200 H.P., the air being compressed adiabatically. (*a*) How many cubic feet of free air are being taken into the compressor per minute and how many cubic feet of high pressure air discharged? Compressor is moved to altitude of 8000 ft. (*b*) What will be the horse-power if the same amount of air is taken in and how many cubic feet per minute will be discharged? (*c*) What will be the horse-power if the same number of cubic feet are discharged as in case (*a*) and what will be the number of cubic feet of low pressure air drawn in? (*d*) Should superheated ammonia be substituted for air at sea level, what would be the necessary horse-power?

16. A car weighing 40 short tons is at rest and is struck by a train of four cars, each of the same weight as the first. (*a*) Upon impact the single car is coupled to the train and all move off at a certain velocity. If the original velocity of the train was 3 miles per hour, what will it be after attachment of the extra car? (*b*) If instead of coupling, the extra car after impact moved away from the train at twice the speed the train was then moving, what would be the speed of train?

17. To drag a block of stone along the ground requires a pull of 1000 lbs. If it be placed on rollers the pull will be reduced to 300 lbs., while if it be placed on a wagon with well-made wheels, the pull will be but 200 lbs. Show by diagram how the work required to move it 1000 ft. will vary.

18. 100 cu. ft. of air, at atmospheric pressure, is compressed to six times its original pressure. (*a*) What will be the difference in horse-power to do this in 45 seconds isothermally and adiabatically at an elevation of 8000 ft. (*b*) What will be the final volumes? (*c*) What will be the difference in horse-power at sea level? (*d*) What will be the final volumes?

19. An engine operating a hoist is run by compressed air at 80 lbs. per square inch gage. The air is admitted half stroke, then expanded for the rest of the stroke so that $s = 1.3$ and then exhausted to atmosphere. The engine must be powerful enough to lift a ton cage and bring it up to a speed of 2000 ft. per minute in 15 seconds. What will be the necessary displacement per minute?

20. Construct PV diagrams for Probs. 1, 11, 13 and 15, showing by them that the work of admission, compression or expansion, and discharge or exhaust, is equal to that found algebraically.

21. The elongation of wrought iron under a force F is equal to the force times the length of the piece divided by 25,000,000 times the cross-section of metal in the piece. A $4\frac{1}{2}$ in. pipe has 3.75 sq. ins. of metal; a line of this pipe 1 mile long is running full of water with a velocity of 600 ft. per minute. This is stopped in 1 second by closing a valve. Assuming pipe did not burst, what would be the elongation?

22. Two steam turbines having nozzles of equal throat areas are operating on a steam pressure of 150 lbs. per square inch gage. One is allowing steam to expand to atmosphere the other to 2 lbs. per square inch absolute, both cases having an exponent for expansion of 1.11. Find the relation of the horse-power in the two cases.

23. The power from a hydro-electric plant is transmitted some distance and then used to drive motors of various sizes. At the time of greatest demand for current it has been found that 1000 horse-power is given out by the motors. Taking the average efficiency of the motors as 70 per cent, transmission efficiency as 85 per cent, generator efficiency as 85 per cent, and water-wheel efficiency as 70 per cent, how many cubic feet of water per second will the plant require if the fall is 80 ft.?

24. A small engine used for hoisting work is run by compressed air. Air is admitted for three-quarters of the stroke and then allowed to expand for the rest of the stroke in such a way that $s = 1.4$ and finally exhausted to atmosphere. For the first part of the hoisting, full pressure (80 lbs. per square inch gage) is applied, but after the load has been accelerated the pressure is reduced to 30 lbs. per square inch gage. If the engine has two cylinders each 9×12 in., is double acting and runs at 200 R.P.M., what will be the (*a*) horse-power in each case, (*b*) the specific displacement?

25. A steam engine has a cylinder 12×18 in., is double acting, and runs at 150 R.P.M. (*a*) What is the engine constant, and (*b*) horse-power per pound m.e.p.?

26. A water-power site has available at all times 3500 cu.ft. of water per minute at a 100-ft. fall. Turbines of 70 per cent efficiency are installed which take the place of two double-acting steam engines whose mechanical efficiencies were 85 per cent. The speed of the engines was 150 R.P.M., m.e.p. 100 lbs. per square inch, and stroke was twice the diameter. What was the size of each engine?

27. Assuming the frictional losses in a compressor to have been 15 per cent, how many cu.ft. of gas per minute could a compressor operated by the above engines compress from atmosphere to 80 lbs. per square inch gage if $s = 1.35$?

TABLE I

CONVERSION TABLE OF UNITS OF DISTANCE

Meters.[1]	Kilometers.	Inches.	Feet.	Statute Miles.	Nautical Miles.
1	0.001	39.37	3.28083	0.000621370	0.000539587
1000	1	39370.1	3280.83	0.62137	0.539587
0.0254	0.0000254	1	0.083333	0.0000157828	0.0000137055
0.304801	0.0003048	12	1	0.000189394	0.000164466
1609.35	1.60935	63360	5280	1.	0.868382
1853.27	1.85327	72963.2	6080.27	1.15157	1.

[1] In accordance with U. S. Standards (see Smithsonian Tables).

TABLE II

CONVERSION TABLE OF UNITS OF SURFACE

Sq. Meters.	Sq. Inches.	Sq. Feet.	Sq. Yards.	Acres.	Sq. Miles.
1	1550.00	10.76387	1.19599	.000247	
.000645	1	.00694			
.0929	144	1	.111		
.8361	1296	9	1	.000206	
4046.87	.....	43560	4840	1	.001562
2589999	.....	27878400	3097600	640	1

TABLE III

CONVERSION TABLE OF UNITS OF VOLUME

Cu. Meters.	Cu. Inches.	Cu. Feet.	Cu. Yards.	Litres (1000 Cu. Cm.)	Gallons (U.S.)
1	61023.4	35.3145	1.3079	1000	264.170
.....	1	.000578	.....	.016387	.00433
.028317	1728	1	.03704	28.317	7.4805
.76456	46656	27	1	.....	201.974
.001	61.023	.035314	.001308	1	.26417
.003785	231	.13368	.004951	3.7854	1

TABLE IV.

CONVERSION TABLE OF UNITS OF WEIGHT AND FORCE

Kilogrammes.	Metric Tons.	Pounds.	U. S. or Short Tons.	British or Long Tons.
1.	0.001	2.20462	0.00110231	0.000984205
1000.	1.	2204.62	1.10231	0.984205
0.453593	0.000453593	1.	0.0005	0.000446429
907.186	0.907186	2000.	1.	0.892957
1016.05	1.01605	2240.	1.12000	1.

TABLE V

CONVERSION TABLE OF UNITS OF PRESSURE

	Pounds per Square Foot.	Pounds per Square Inch.	Inches of Mercury at 32° F.	Atmospheres (Standard at Sea Level).
ne lb. per sq. ft.	1	0.006944	0.014139	0.0004724
ne lb. per sq. in.	144.	1.	2.03594	0.06802
ne ounce per sq. in.	9.	0.0625	0.127246	0.004252
ne atmosphere (standard at sea level)	2116.1	14.696	29.924	1.
ne kilogramme per square meter	20.4817	0.142234	0.289579	0.009678
ne gramme per square millimeter	204.817	1.42234	2.89579	0.09678
ne kilogramme per square centimeter	2048.17	14.2234	28.9579	0.9678
FLUID PRESSURES				
ne ft. of water at 39.1° F. (max. dens.)	62.425	0.43350	0.88225	0.029492
ne ft. of water at 62° F.	62.355	0.43302	0.88080	0.029460
ne in. of water at 62° F.	5.196	0.036085	0.07340	0.002455
ne in. of mercury at 32° F. (standard) [1]	70.7290	0.491174	1.	0.033416
ne centimeter of mercury at 0° C.	27.8461	0.193376	0.393701	0.013158
ne ft. of air at 32° F., one atmos. press.	0.08071	0.0005604	0.0011412	0.00003813
ne ft. of air, 62° F.	0.07607	0.0005282	0.0010755	0.00003594

[1] PRESSURES MEASURED BY THE MERCURY COLUMN. For temperatures other than 32° F., the density mercury, pounds per cubic inch, and hence the pressure, pounds per square inch, due to a column of mercury ch high, is given with sufficient accuracy by the following formula:

$$p = 0.4912 - (t - 32) \times 0.0001.$$

The mercurial barometer is commonly made with a brass scale which has its standard or correct length 62° F, and a linear coefficient of expansion of about 0.000001 for each degree Fahrenheit. Hence, to rect the standard of mercury at 32° F., the corrected reading will be

$$H_{32} = H_t - H_t \times \frac{t - 28.6}{11000},$$

ere H_t is the observed height at a temperature of t° F.

Table VI

CONVERSION TABLE OF UNITS OF WORK [1]

Kilogrammeters.	Foot-pounds.	Foot Tons (Short Tons).	Foot Tons (Long Tons).
1.	7.23300	0.00361650	0.00322902
0.138255	1.	0.000500	0.000446429
276.510	2000.	1.	0.892857
309.691	2240.	1.12000	1.

[1] See also more complete table of Units of Work and Energy in Chapter IV on Work and Heat.

Table VII

CONVERSION TABLE OF UNITS OF POWER

Foot-pounds per Second.	Foot-pounds per Minute.	Horse-power.	Cheval-Vapeur.	Kilogrammeters pe Minute.
1.	60.	0.00181818	0.00184340	8.29531
0.0166667	1.	0.000030303	0.0000307241	0.138252
550.000	33000.	1.	1.01387	4562.42
542.475	32548.5	0.986319	1.	4500.00
0.120550	7.23327	0.000219182	0.000222222	1.

Table VIII

UNITS OF VELOCITY

	Feet per Minute.	Feet per Secon
One foot per second	60.	1.
One foot per minute	1.	0.016667
One statute mile per hour	88.	1.4667
One nautical mile per hour = 1 knot	101.338	1.6890
One kilometer per hour	54.6806	0.911344
One meter per minute	3.28084	0.054581
One centimeter per second	2.00848	0.032808

TABLE IX

TABLE OF BAROMETRIC HEIGHTS, ALTITUDES, AND PRESSURES

(Adapted from Smithsonian Tables)

Barometric heights are given in inches and millimeters of mercury at its standard density (32° F.).

Altitudes are heights above mean sea level in feet, at which this barometric height is standard. (See Smithsonian Tables for corrections for latitude and temperature.)

Pressures given are the equivalent of the barometric height in lbs. per sq. in. and per sq. ft.

Standard Barometer.		Altitude, Feet above Sea Level.	Pressure, Pounds per	
Inches.	Centimeters.		Square Inch.	Square Foot.
17.0	43.18	15379	8.350	1202.3
17.2	43.69	15061	8.448	1216.6
17.4	44.20	14746	8.546	1230.7
17.6	44.70	14435	8.645	1244.8
17.8	45.21	14128	8.742	1259.0
18.0	45.72	13824	8.840	1273.2
18.2	46.23	13523	8.940	1287.3
18.4	46.73	13226	9.038	1301.4
18.6	47.24	12931	9.136	1315.6
18.8	47.75	12640	9.234	1329.7
19.0	48.26	12352	9.332	1343.8
19.2	48.77	12068	9.430	1357.9
19.4	49.28	11786	9.529	1372.1
19.6	49.78	11507	9.627	1386.3
19.8	50.29	11230	9.726	1400.4
20.0	50.80	10957	9.825	1414.6
20.2	51.31	10686	9.922	1428.7
20.4	51.82	10418	10.020	1442.9
20.6	52.32	10153	10.118	1457.0
20.8	52.83	9890	10.217	1471.2
21.0	53.34	9629	10.315	1485.3
21.2	53.85	9372	10.414	1499.4
21.4	54.36	9116	10.511	1513.6
21.6	54.87	8863	10.609	1527.7
21.8	55.37	8612	10.707	1541.8
22.0	55.88	8364	10.806	1556.0
22.2	56.39	8118	10.904	1570.1
22.4	56.90	7874	11.002	1584.3
22.6	57.40	7632	11.100	1598.4
22.8	57.91	7392	11.198	1612.6
23.0	58.42	7155	11.297	1626.7
23.2	58.92	6919	11.395	1640.8
23.4	59.44	6686	11.493	1655.0
23.6	59.95	6454	11.592	1669.3
23.8	60.45	6225	11.690	1683.3
24.0	60.96	5997	11.788	1697.4
24.2	61.47	5771	11.886	1711.6
24.4	61.98	5547	11.984	1725.7
24.6	62.48	5325	12.083	1739.9
24.8	62.99	5105	12.182	1754.0
25.0	63.50	4886	12.280	1768.2
25.2	64.01	4670	12.377	1782.3
25.4	64.52	4455	12.475	1796.5
25.6	65.02	4241	12.573	1810.7
25.8	65.53	4030	12.671	1824.8

TABLE IX—*Continued*

Standard Barometer.		Altitude, Feet above Sea Level.	Pressure, Pounds per	
Inches.	Centimeters.		Square Inch.	Square Foot.
26.0	65.04	3820	12.770	1838.9
26.1	66.30	3715	12.819	1846.0
26.2	66.55	3611	12.868	1853.1
26.3	66.80	3508	12.918	1860.2
26.4	67.06	3404	12.967	1867.3
26.5	67.31	3301	13.016	1874.3
26.6	67.57	3199	13.065	1881.4
26.7	67.82	3097	13.113	1888.5
26.8	68.08	2995	13 163	1895.5
26.9	68.33	2894	13.212	1902.6
27.0	68.58	2793	13.261	1909.7
27.1	68.84	2692	13.310	1916.7
27.2	69.09	2592	13.359	1923.8
27.3	69.34	2493	13.408	1930.9
27.4	69.60	2393	13.457	1938.0
27.5	69.85	2294	13.507	1945.1
27.6	70.10	2195	13.556	1952.1
27.7	70.35	2097	13.605	1959.2
27.8	70.61	1999	13.654	1966.3
27.9	70.87	1901	13.704	1973.3
28.0	71.12	1804	13.753	1980.4
28.1	71.38	1707	13.802	1987.5
28.2	71.63	1610	13.850	1994.5
28.3	71.88	1514	13.899	2001.6
28.4	72.14	1418	13.948	2008.7
28.5	72.39	1322	13.998	2015.7
28.6	72.64	1227	14.047	2022.8
28.7	72.90	1132	14.096	2030.0
28.8	73.15	1038	14.145	2037.0
28.9	73.40	943	14.194	2044.1
29.0	73.66	849	14.243	2051.2
29.1	73.92	756	14.293	2058.2
29.2	74.16	663	14.342	2065.3
29.3	74.42	570	14.392	2072.4
29.4	74.68	477	14.441	2079.4
29.5	74.94	384	14.490	2086.5
29.6	75.18	292	14.539	2093.6
29.7	75.44	261	14.588	2100.7
29.8	75.69	109	14.637	2107.7
29.9	75.95	+18	14.686	2114.7
29.92	76.00	0	14.696	2116.1
30.0	76.20	− 73	14.734	2121.7
30.1	76.46	−163	14.783	2128.8
30.2	76.71	−253	14.833	2135.9
30.3	76.96	−343	14.882	2143.0
30.4	77.22	−433	14.931	2150.1
30.5	77.47	−522	14.980	2157.2
30.6	77.72	−611	15.030	2164.2
30.7	77.98	−700	15.078	2171.3
30.8	78.23	−788	15.127	2178.4
30.9	78.48	−877	15.176	2185.5
31.0	78.74	−965	15.226	2192.6

TABLE X

VALUES OF s IN THE EQUATION PV^s = CONSTANT FOR VARIOUS SUBSTANCES AND CONDITIONS

Substance.		s	Remarks or Authority.
All gases	Isothermal	1	Accepted thermodynamic law
All gases and vapors	Constant pressure	0	
All saturated vapors	Isothermal	0	
All gases and vapors	Constant volume	∞	
Air	Adiabatic	1.4066	Smithsonian Tables
Air	Compressed in cylinder	1.4	Experience
Ammonia (NH_3)	Adiabatic, wet	1.1	Average
Ammonia (NH_3)	Adiabatic, superheated	1.3	Thermodynamics
Bromine	Adiabatic	1.293	Strecker
Carbon dioxide (CO_2)	Adiabatic	1.300	Röntgen, Wullner
Carbon monoxide (CO)	Adiabatic	1.403	Cazin, Wullner
Carbon disulphide (CS_2)	Adiabatic	1.200	Beyne
Chlorine (Cl)	Adiabatic	1.323	Strecker
Chloroform ($CCl_3CH(OH)_2$)	Adiabatic	1.106	Beyne, Wullner
Ether ($C_2H_5OC_2H_5$)	Adiabatic	1.029	Müller
Hydrogen (H_2)	Adiabatic	1.410	Cazin
Hydrogen sulph. (H_2S)	Adiabatic	1.276	Müller
Methane (CH_4)	Adiabatic	1.316	Müller
Nitrogen (N_2)	Adiabatic	1.410	Cazin
Nitrous oxide (NO_2)	Adiabatic	1.291	Wullner
Pintsch gas	Adiabatic	1.24	Pintsch Co.
Sulphide diox (SO_2)	Adiabatic	1.26	Cazin, Müller
Steam, superheated	Adiabatic	1.300	Smithsonian Tables
Steam, wet	Adiabatic	Variable	(From less than 1 to more than 1.2)
Steam, wet	Adiabatic	1.111	Rankine
Steam, wet	Adiabatic	$1+14\times\%$ moist.	Perry
Steam, wet	Adiabatic	$1.035+.1\times\%$ moist.	Gray
Steam, wet	Expanding in cylinder	1.	Average from practice
Steam, dry	Saturation law	1.0646	Regnault

TABLE XI

HORSE-POWER PER POUND MEAN EFFECTIVE PRESSURE.

VALUE OF $K_e = \frac{aS}{33000}$

Diameter of Cylinder, Inches.	Speed of Piston in Feet per Minute.								
	100	200	300	400	500	600	700	800	900
4	0.0381	0.0762	0.1142	0.1523	0.1904	0.2285	0.2666	0.3046	0.3427
4½	0.0482	0.0964	0.1446	0.1928	0.2410	0.2892	0.3374	0.3856	0.4338
5	0.0592	0.1190	0.1785	0.2380	0.2975	0.3570	0.4165	0.4760	0.5355
5½	0.0720	0.1440	0.2160	0.2880	0.3600	0.4320	0.5040	0.5760	0.6480
6	0.0857	0.1714	0.2570	0.3427	0.4284	0.5141	0.5998	0.6854	0.7711
6½	0.1006	0.2011	0.3017	0.4022	0.5028	0.6033	0.7039	0.8044	0.9050
7	0.1166	0.2332	0.3499	0.4665	0.5831	0.6997	0.8163	0.9330	1.0490
7½	0.1339	0.2678	0.4016	0.5355	0.6694	0.8033	0.9371	1.0710	1.2049
8	0.1523	0.3046	0.4570	0.6093	0.7616	0.9139	1.0662	1.2186	1.3709
8½	0.1720	0.2439	0.5159	0.6878	0.8598	1.0317	1.2037	1.3756	1.5476
9	0.1928	0.3856	0.5783	0.7711	0.9639	1.1567	1.3495	1.5422	1.7350
9½	0.2148	0.4296	0.6444	0.8592	1.0740	1.2888	1.5036	1.7184	1.9532
10	0.2380	0.4760	0.7140	0.9520	1.1900	1.4280	1.6660	1.9040	2.1420
11	0.2880	0.5760	0.8639	1.1519	1.4399	1.7279	2.0159	2.3038	2.5818
12	0.3427	0.6854	1.0282	1.3709	1.7136	2.0563	2.3990	2.7418	3.0845
13	0.4022	0.8044	1.2067	1.6089	2.0111	2.4133	2.8155	3.2178	3.6200
14	0.4665	0.9330	1.3994	1.8659	2.3324	2.7989	3.2654	3.7318	4.1983
15	0.5355	1.0710	1.6065	2.1420	2.6775	3.2130	3.7485	4.2840	4.8195
16	0.6093	1.2186	1.8278	2.4371	3.0464	3.6557	4.2650	4.8742	5.4835
17	0.6878	1.2756	1.9635	2.6513	3.3391	4.0269	4.6147	5.4026	6.1904
18	0.7711	1.5422	2.3134	3.0845	3.8556	4.6267	5.3987	6.1690	6.4901
19	0.8592	1.7184	2.5775	3.4367	4.2858	5.1551	6.0143	6.8734	7.7326
20	0.9520	1.9040	2.8560	3.8080	4.7600	5.7120	6.6640	7.6160	8.5680
21	1.0496	2.0992	3.1488	4.1983	5.2475	6.2975	7.3471	8.3966	9.4462
22	1.1519	2.3038	3.4558	4.6077	5.7596	6.9115	8.0643	9.2154	10.367
23	1.2590	2.5180	3.7771	5.0361	6.2951	7.5541	8.8131	10.072	11.331
24	1.3709	2.7418	4.1126	5.4835	6.8544	8.2253	9.5962	10.967	12.338
25	1.4875	2.9750	4.4625	5.9500	7.4375	8.9250	10.413	11.900	13.388
26	1.6089	3.2178	4.8266	6.4355	8.0444	9.6534	11.262	12.871	14.480
27	1.7350	3.4700	5.2051	6.9401	8.6751	10.410	12.145	13.880	15.615
28	1.8659	3.7318	5.5978	7.4637	9.3296	11.196	13.061	14.927	16.793
29	2.0016	4.0032	6.0047	8.0063	10.008	12.009	14.011	16.013	18.014
30	2.1420	4.2840	6.4260	8.5680	10.710	12.852	14.994	17.136	19.278
31	2.2872	4.5744	6.8615	9.1487	11.436	13.723	16.010	18.287	20.585
32	2.4371	4.8742	7.3114	9.7485	12.186	14.623	17.060	19.497	21.934
33	2.5918	5.1836	7.7755	10.367	12.959	15.551	18.143	20.735	23.326
34	2.7513	5.5026	8.2538	11.005	13.756	16.508	19.259	22.010	24.762
35	2.9155	5.8310	8.7465	11.662	14.578	17.493	20.409	23.224	26.240
36	3.0845	6.1690	9.2534	12.338	15.422	18.507	21.591	24.676	27.760
37	3.2582	6.5164	9.7747	13.033	16.291	19.549	22.808	26.066	29.324
38	3.4367	6.8734	10.310	13.747	17.184	20.620	24.057	27.494	30.930
39	3.6200	7.2400	10.860	14.480	18.100	21.720	25.340	28.960	32.580

TABLE XI—*Continued*

Diameter of Cylinder, Inches.	Speed of Piston in Feet per Minute.								
	100	200	300	400	500	600	700	800	900
40	3.8080	7.6160	11.424	15.232	19.040	22.848	26.656	30.464	34.272
41	4.0008	8.0016	12.002	16.003	20.004	24.005	28.005	32.006	36.007
42	4.1983	8.3866	12.585	16.783	20.982	25.180	29.378	33.577	37.775
43	4.4006	8.8012	13.202	17.602	22.003	26.404	30.804	35.205	39.606
44	4.6077	9.2154	13.823	18.431	23.038	27.646	32.254	36.861	41.469
45	4.8195	9.6390	14.459	19.278	24.098	28.917	33.737	38.556	43.376
46	5.0361	10.072	15.108	20.144	25.180	30.216	35.253	40.289	45.325
47	5.2574	10.515	15.772	21.030	26.287	31.545	36.802	42.059	47.317
48	5.3845	10.967	16.451	21.934	27.418	32.901	38.385	43.868	49.352
49	5.7144	11.429	17.143	22.858	28.572	34.286	40.001	45.715	51.429
50	5.9500	11.900	17.850	23.800	29.750	35.700	41.650	47.600	53.550
51	6.1904	12.381	18.571	24.762	30.952	37.142	43.333	49.523	55.713
52	6.4355	12.871	19.307	25.742	32.178	38.613	45.049	51.484	57.920
53	6.6854	13.371	20.056	26.742	33.427	40.113	46.798	53.483	60.169
54	6.9401	13.880	20.820	27.760	34.700	41.640	48.581	55.521	62.461
55	7.1995	14.399	21.599	28.798	35.998	43.197	50.397	57.596	64.796
56	7.4637	14.927	22.391	29.855	37.318	44.782	52.246	59.709	67.173
57	7.7326	15.465	23.198	30.930	38.663	46.396	54.128	61.861	69.594
58	8.0063	16.013	24.019	32.025	40.032	48.038	56.044	64.051	72.057
59	8.2849	16.570	24.854	33.139	41.424	49.709	57.993	66.278	74.563
60	8.5680	17.136	25.704	34.272	42.840	51.408	59.976	68.544	77.112

GENERAL FORMULA RELATING TO PRESSURE-VOLUME CALCULATIONS OF WORK AND POWER

$$\text{Work} = W = \text{Force} \times \text{Distance} = F \times L;$$

$$= \text{Pressure} \times \text{Area} \times \text{Distance} = P \times A \times L;$$

$$= \text{Pressure} \times \text{Volume change} = P \times (V_2 - V_1).$$

Force of acceleration = mass × acceleration.

$$F = \frac{w}{g} \times \frac{du}{d\tau}.$$

Work of acceleration $= \frac{1}{2}$ mass × difference of (velocity)^2,

$$= \frac{1}{2} \frac{w}{g} (u_2^2 - u_1^2) = \frac{w}{64.32} (u_2^2 - u_1^2).$$

Velocity due to work of acceleration = square root of the sum of (initial velocity)^2 plus $2g$ × work per pound of substance accelerated.

$$u_2 = \sqrt{u_1^2 + \frac{64.32W}{w}}$$

or if initial velocity is zero,

$$u_2 = \sqrt{64.32 \frac{W}{w}} = 8.02 \sqrt{\frac{W}{w}}.$$

Pressure Volume Relation for Expansion or Compression

$PV^s = P_1V_1^s = P_2V_2^s = K$, a constant. (See Table VIII for values of s.)

$$s = \frac{log \left(\frac{P_1}{P_2}\right)}{log \left(\frac{V_2}{V_1}\right)}.$$

(Note graphical method for finding s when variable, see text.)

Work done *during* a pressure volume change represented by the equation ($PV^s = K$) between points represented by 1 and 2 in figure = area under curve, W_1.

$$W_1 = \int_{V_1}^{V_2} P dV = K \int_{V_1}^{V_2} \frac{dV}{V^s}.$$

If $s = 1$,

$$\left.\begin{aligned} W_1 &= P_1 V_1 \log_e \frac{V_2}{V_1} \\ &= P_2 V_2 \log_e \frac{V_2}{V_1} \\ &= P_1 V_1 \log_e \frac{P_1}{P_2} \\ &= P_2 V_2 \log_e \frac{P_1}{P_2} \end{aligned}\right\} s = 1.$$

If s is not equal to 1,

$$\left.\begin{aligned} W_1 &= \frac{P_2 V_2}{s-1}\left[\left(\frac{V_2}{V_1}\right)^{s-1} - 1\right] \\ &= \frac{P_2 V_2}{s-1}\left[\left(\frac{P_1}{P_2}\right)^{\frac{s-1}{s}} - 1\right] \\ &= \frac{P_1 V_1}{s-1}\left[1 - \left(\frac{V_1}{V_2}\right)^{s-1}\right] \\ &= \frac{P_1 V_1}{s-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{s-1}{s}}\right] \end{aligned}\right\} s \neq 1.$$

Work of admission, complete expansion and exhaust for engines = area to left of curve (see figure) W_2. Same for admission, compression and expulsion for compressors. Both cases without clearance.

(When $s = 1$, W_2 is same as area under curve, W_1 (see above).

$$\text{When } s \neq 1 \quad \left.\begin{aligned} W_2 &= \frac{s}{s-1} P_2 V_2 \left[\left(\frac{V_2}{V_1}\right)^{s-1} - 1\right] \\ &= \frac{s}{s-1} P_2 V_2 \left[\left(\frac{P_1}{P_2}\right)^{\frac{s-1}{s}} - 1\right] \\ &= \frac{s}{s-1} P_1 V_1 \left[1 - \left(\frac{V_1}{V_2}\right)^{s-1}\right] \\ &= \frac{s}{s-1} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{s-1}{s}}\right] \end{aligned}\right\} = s \times W_1.$$

Clearance, expressed as a fraction of displacement = c, as a volume = Cl.

If $P_aV_a = P_bV_b^s$,

$$c = \frac{Cl}{D} = \frac{\frac{D_b}{D} - \frac{D_a}{D}\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}}{\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}} - 1}$$

Indicated horse-power = (mean effective pressure, pounds per square inch × effective area of piston, square inches × length of stroke, feet × number of working cycles performed per minute) divided by 33,000.

$$\text{I H.P.} = \frac{(\text{m.e.p.})Lan}{33{,}000}.$$

Specific displacement = displacement in one direction for one side of a piston, in cu.ft. per hour per H.P. $= D_s = 13{,}750\frac{z}{(\text{m.e.p.})}$,

where z is the number of strokes required to complete one cycle.

Velocity of a jet due to its own expansion = the square root of the product of $2g \times$ work done by admission, complete expansion and exhaust of 1 lb. of the substance.

$$u = \sqrt{2gW_2} = 8.02\left\{\frac{s}{s-1}P_1V_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{s-1}{s}}\right]\right\}^{\frac{1}{2}}.$$

Weight of flow through nozzle or orifices, pounds per second $= w =$ (velocity, feet per second × area, square feet, of orifice) ÷ (volume per pound of substance at section where area is measured).

$$w = \frac{uA}{V_2} = 8.02\frac{A}{V_1}\left(\frac{P_2}{P_1}\right)^{\frac{1}{s}}\left\{\frac{s}{s-1}P_1V_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{s-1}{s}}\right]\right\}^{\frac{1}{2}}$$

Maximum discharge w for a given initial pressure occurs when

$$\frac{P_1}{P_2} = \left(\frac{2}{s+1}\right)^{\frac{s}{s-1}}$$

CHAPTER II

WORK OF COMPRESSORS, HORSE-POWER AND CAPACITY OF AIR, GAS AND VAPOR COMPRESSORS, BLOWING ENGINES AND DRY VACUUM PUMPS.

1. General Description of Structure and Processes. There is quite a large class of machines designed to receive a cylinder full of some gas at one constant pressure and after the doing of work on the gas through decreasing volumes and rising pressures, to discharge the lesser volume of gas against a constant higher pressure. These machines are in practice grouped into sub-classes, each having some specific distinguishing characteristic. For example, blowing engines take in air at atmospheric pressure or as nearly so as the valve and port resistance will permit, and after compression deliver the air at a pressure of about three atmospheres absolute for use in blast furnaces. These blowing engines are usually very large, work at low but variable speeds, but always deliver against comparatively low pressures; they, therefore, have the characteristics of large but variable capacity and low pressures. A great variety of valves and driving gears are used, generally mechanically moved suction and automatic spring closed discharge valves, but all valves may be automatic. The compressor cylinder is often termed the blowing tub and the compressed or blast air frequently is spoken of as wind by furnace men. They are all direct-connected machines, an engine forming with the compressor one machine. The engine formerly was always of the steam type, but now a change is being affected to permit the direct internal combustion of the blast furnace waste gases in the cylinders of gas engines. These gas-driven blowing engines, showing approximately twice the economy of steam-driven machines, will in time probably entirely displace steam in steel plants, and this change will take place in proportion to the successful reduction of cost of repairs, increase of reliability and life of the gas-driven blowing engines to equal the steam-driven. Some low-pressure blowers are built on the rotary plan without reciprocating pistons, some form of rotating piston being substituted, and these, by reason of greater leakage possibilities, are adapted only to such low pressures as 5 lbs. per square inch above atmosphere or thereabouts. These blowers are coming into favor for blasting gas producers, in which air is forced through thick coal beds either by driving the air or by drawing on the gas produced beyond the bed. They are also used for forcing illuminating gas in cities through pipes otherwise too small, especially when the distances are long. In general very low pressures and large capacities are the characteristics of the service whether the work be that of blowing or exhausting or both. For still lower pressures, measurable by water or mercury columns, fans are used of the disk or propeller

or centrifugal type. These fans are most used for ventilation of buildings an mines, but a modification, based on the principles of the steam turbine reverse and termed turbo-compressors, is being rapidly adapted to such higher pre sures as have heretofore required piston compressors.

When high-pressure air is required for driving rock drills in mines and fo hoisting engines, for tools, as metal drills, riveters, chipping chisels, for ca air brakes, the compressors used to provide the air are termed simply ai compressors. These compressors usually take in atmospheric air and compre it to the desired pressure, the capacity required being usually adjustable they have valves of the automatic type throughout commonly, but in larg sizes frequently are fitted with mechanically operated suction valves t decrease the resistance to entrance of air and so increase economy, a com plication not warranted in small machines. When the pressures of deliver are quite high the compression is done in stages in successive cylinders, th discharge from the first or low-pressure cylinder being delivered through water cooler or *intercooler* to the second cylinder and occasionally to a thir in turn. This staging with intermediate or intercooling results in bett economy, as will be seen later in detail, and permits the attainment of th desired quantity of cool compressed air for subsequent use with the expenditur of less work, the extra complication and cost being warranted only whe machines are large and final pressures high.

In the operation of large steam condensers, non-condensible gases wi collect and spoil the vacuum, which can be maintained only by the continuou removal of these gases, consisting of air, carbon dioxide and gases of anim and vegetable decomposition originally present in the water. When these gas are separately removed the machine used is a special form of compressor terme a dry vacuum pump which, therefore, receives a charge at the absolute pre sure corresponding to the vacuum, or as nearly so as the entrance resistan permits, and after compression discharges into the atmosphere at a pressu in the cylinder above atmosphere equivalent to discharge resistance. Natura gas wells near exhaustion can sometimes be made to flow freely by the applic tion of a compressor capable of drawing a charge at a pressure below atmospher but whether the charge be received below atmospheric pressure or above in normal wells, the compressor will permit the delivery of the gas to dista cities or points of consumption even 250 miles away through smaller pip than would be otherwise possible. Natural-gas compressors, some steam- ar some gas-engine driven, are in use for both these purposes, compressing natur gas from whatever pressure may exist at the well to whatever is desired at t beginning of the pipe line.

In the preparation of liquid ammonia or carbonic acid gas for the marke as such, or in the operation of refrigerating machinery, wet or dry vapor is co pressed into a condenser to permit liquefaction by the combined effect of hi pressure and cooling. One form of refrigerating machine merely compress air, subsequently expanding it after preliminary cooling by water, so th after expansion is complete it will become extremely cold.

All these compressing machines have, as a primary purpose, either the removal of a quantity of low-pressure gases from a given place, or the delivery of a quantity of higher-pressure gas to another place or both, but all include compression as an intermediate step between constant-pressure admission and constant-pressure discharge as nearly as structure may permit. They will all involve the same sort of physical operations and can be analyzed by the same principles except the wet-vapor or wet-gas compressors, in which condensation or evaporation may complicate the process and introduce elements that can be treated only by thermal analysis later. Safe compressors cannot be built with zero cylinder clearance, hence at the end of delivery there will remain in the clearance space a volume of high-pressure gases equal to the volume of the clearance space. On the return stroke this clearance volume will expand until the pressure is low enough to permit suction, so that the new charge cannot enter the cylinder until some portion of the stroke has been covered to permit this *re-expansion* of clearance gases.

It is quite impossible to study here all the effects or influences of structure as indicated by the compressor indicator cards, but a quite satisfactory treatment can be given by the establishment of reference diagrams as standards of comparison and noting the nature of the differences between the actual cases and the standard reference diagram. These standard reference diagrams will really be pressure-volume diagrams, the phases of which correspond to certain hypotheses capable of mathematical expression, such as constant pressure, constant volume, expansion, and compression, according to some law, or with some definite value of s fixing either the heat-exchange character of the process or the substance, as already explained.

2. Standard Reference Diagrams or PV Cycles for Compressors and Methods of Analysis of Compressor Work and Capacity. All the standard reference diagrams will include constant-pressure lines corresponding to delivery and supply at pressures assumed equal to whatever exists outside the cylinder on either delivery or suction side, that is, assuming no loss of pressure on delivery or suction. The compression may be single or multi-stage with various amounts of cooling in the intercooler, but in multi-stage compression the standard reference diagram will be assumed to involve intermediate cooling of the gases to their original temperature, so that the gases entering all cylinders will be assumed to have the same temperature and to maintain it constant during admission. Another difference entering into the classification of standard reference diagrams is the laws of compression as defined by the exponent s. Integration of the differential work expression will take a logarithmic form for $s=1$, and an exponential form for all other values, thus giving two possible reference compression curves and two sets of work equations.

(*a*) The *isothermal* for which $s=1$, no matter what the gas, and which is the consequence of assuming that all the heat liberated by compression is continuously carried away as fast as set free, so that the temperature cannot rise at all.

(*b*) The *exponential* for which s has a value greater than one, generall different for every gas, vapor or gas-vapor mixture, but constant for any on gas, and also for dry vapors that remain dry for the whole process. Wet vapo having variable values of s cannot be treated by the simple pressure-volum analysis that suffices for the gases, but must be analyzed thermally. Th adiabatic value of s is a consequence of assuming no heat exchange at a between the gas and anything else and is a special case of the general exponen tial class.

Just why these two assumptions of thermal condition should result in th specified values of s will be taken up under the thermal analysis part of thi work, and is of no interest at this time.

As a consequence of these phase possibilities there may be established eigh standard reference diagrams or pressure-volume cycles defined by their phase as shown in Fig. 23, four for single-stage compression and two each for two an three stages. These might be extended by adding two more for four stage and so on, but as it seldom is desirable, all things being considered, to go beyon three, the analysis will stop with the eight cycles or reference diagrams show

Single-stage Compression Reference Cycles or PV Diagrams

Cycle 1. Single-stage Isothermal Compression without Clearance.
- Phase (*a*) Constant pressure supply.
- " (*b*) Isothermal compression.
- " (*c*) Constant pressure delivery.
- " (*d*) Constant zero-volume pressure drop.

Cycle 2. Single-stage Isothermal Compression with Clearance.
- Phase (*a*) Constant pressure supply.
- " (*b*) Isothermal compression.
- " (*c*) Constant pressure delivery.
- " (*d*) Isothermal re-expansion.

Cycle 3. Single-stage Exponential Compression without Clearance.
- Phase (*a*) Constant pressure supply.
- " (*b*) Exponential compression.
- " (*c*) Constant pressure delivery.
- " (*d*) Constant zero-volume pressure drop.

Cycle 4. Single-stage Exponential Compression with Clearance.
- Phase (*a*) Constant pressure supply.
- " (*b*) Exponential compression.
- " (*c*) Constant pressure delivery.
- " (*d*) Exponential re-expansion.

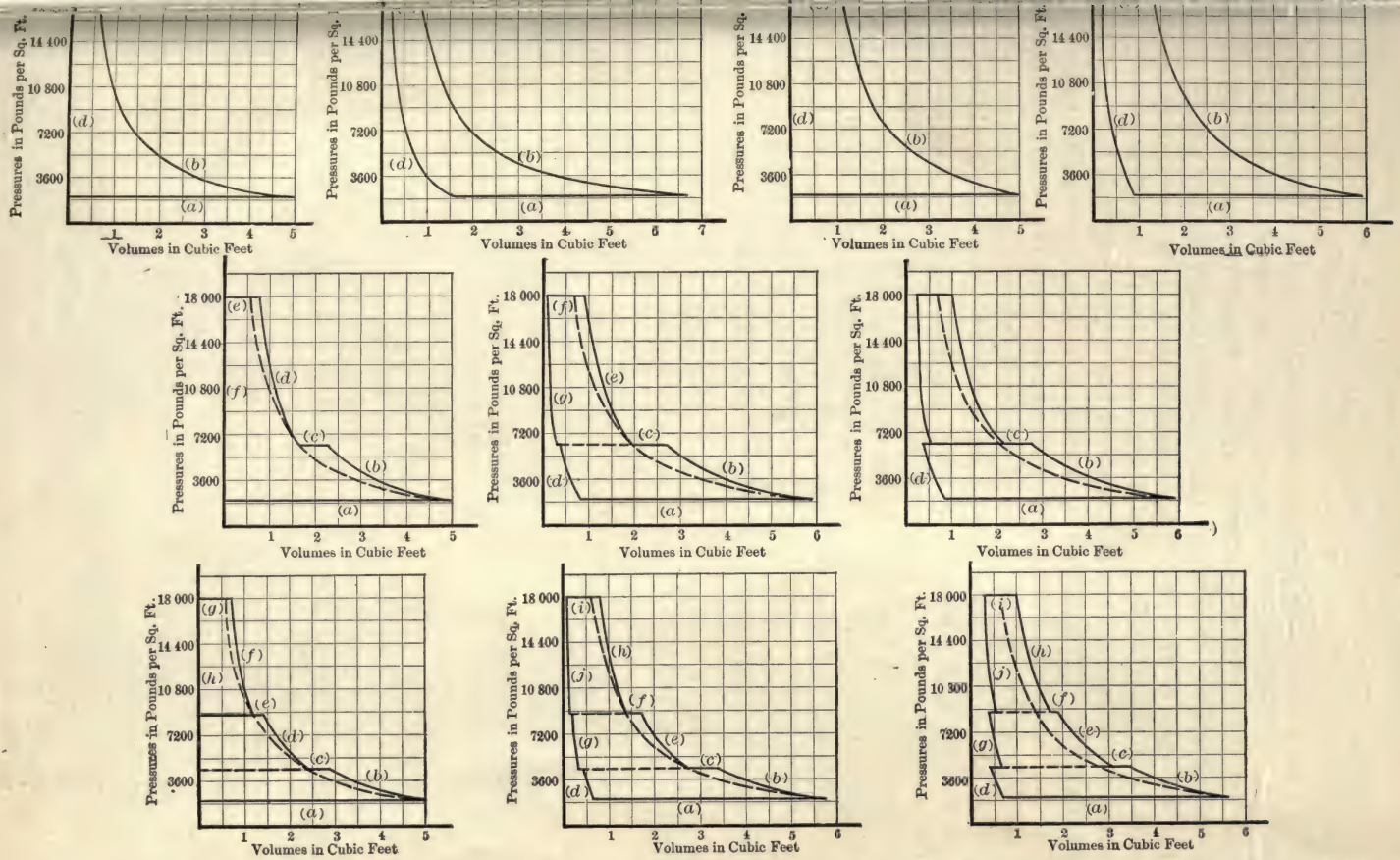


FIG. 23.—Standard Reference Cycles or P.V. Diagrams for Compressors; One-, Two-, and Three-Stage, with and without Clearance, Logarithmic and Exponential Laws.

Multi-stage Compression

The phases making up multi-stage compression cycles may be considered in two ways, first, as referred to each cylinder and intercooler separately, or second, as referred to the pressure volume changes of the gases themselves regardless of whether the changes take place in cylinders or intercoolers.

For example, if 10 cu.-ft. of hot compressed air be delivered from the first cylinder of 50 cu.-ft. displacement, the phase referred to this cylinder is a constant-pressure decreasing volume, delivery line whose length is $\frac{1}{5}$ of the whole diagram, exactly as in single-stage compression. If this 10 cu.-ft. of air delivered to an intercooler became 8 ft. at the same constant pressure as the first cylinder delivery, the phase would be indicated by a constant-pressure volume reduction line 2 cu.-ft long to scale, or referred to the original volume of air admitted to the first cylinder, a line $\frac{1}{25}$ of its length. Finally, admitting this 8 cu.-ft of cool air to the second cylinder and compressing it to $\frac{1}{5}$ of its volume would result in a final delivery line at constant pressure of a length of $\frac{1}{5}$ of the length of the second cylinder diagram, but as this represents only 8 cu.-ft the final delivery will represent only $\frac{1}{5} \times 8 = 1.6$ cu.-ft. This 1.6 cu.-ft. will when referred to the original 50 cu.-ft. admitted to the first cylinder, be represented by a constant-pressure line, $\frac{1.6}{50} = .032$, of the whole diagram length which in volume is equivalent to $\frac{1}{5}$ of the length of the second cylinder diagram. It should be noted also that three volume change operations take place at the intermediate pressure; first, first cylinder delivery; second volume decrease due to intercooling; third, second cylinder admission, the net effect of which referred to actual gas volumes, regardless of place where the changes happen, is represented by the volume decrease due to intercooling only. A diagram of volumes and pressures representing the resultant of all the gas processes is called in practice the *combined PV diagram* for the two cylinders, or when plotted from actual indicator cards with due regard for the different clearances of each cylinder the *combined indicator diagram*. It is proper in the study of the whole process of compression to consider the cycle consisting of phases referred to true gas volumes rather than phases referring to separate cylinder processes, which is equivalent to imagining the whole cycle carried out in one cylinder.

Intercooling effects measured by the amount of decrease of volume at constant pressure will, of course, depend on the amount of cooling or reduction of temperature, but in establishing a standard reference diagram some definite amount capable of algebraic description must be assumed as the intercooling hypothesis.

It has already been shown, Fig. 6, Chapter I, that from any original state of pressure and volume the exponential and isothermal could be drawn, diverging an amount depending on the difference between the defining exponent, n. If, after reaching a given state on the exponential curve, the gas be cooled

nstant pressure to its original temperature, the point indicating its condition ill lie by definition on the other curve or isothermal and the cooling process represented by a horizontal joining the two curves. Such intercooling as iis will be defined as *perfect intercooling*, for want of a better name, and its ressure-volume effects can be treated by the curve intersections. It is now ossible to set down the phases for the standard reference diagrams of multi-age compression, if in addition to the above it be admitted, as will be proved ter, that there is a best or most economical receiver pressure.

Two-stage Compression Reference Cycles or *PV* Diagrams

Cycle 5. Two-stage Exponential Compression without Clearance, Perfect Intercooling at Best Receiver Pressure.

Phase (*a*) Constant pressure supply.
" (*b*) Exponential compression to best receiver pressure.
" (*c*) Constant pressure perfect intercooling of delivered gas.
" (*d*) Exponential compression from best receiver pressure.
" (*e*) Constant pressure delivery.
" (*f*) Constant zero-volume pressure drop.

Cycle 6. Two-stage Exponential Compression with Clearance, Perfect Intercooling at Best Receiver Pressure.

Phase (*a*) Constant pressure supply.
" (*b*) Exponential compression to best receiver pressure.
" (*c*) Constant pressure perfect intercooling of delivered gas.
" (*d*) Exponential re-expansion of first stage clearance.
" (*e*) Exponential compression from best receiver pressure.
" (*f*) Constant pressure delivery.
" (*g*) Exponential re-expansion of second stage clearance.

Three-stage Compression Reference Cycles or *PV* Diagrams

Cycle 7. Three-stage Exponential Compression, without Clearance, Perfect Intercooling at Best Two Receiver Pressures.

Phase (*a*) Constant pressure supply.
" (*b*) Exponential compression to first receiver pressure.
" (*c*) Perfect intercooling at best first receiver presssure.
" (*d*) Exponential compression from best first to best second receiver pressure.
" (*e*) Perfect intercooling at best second receiver pressure.
" (*f*) Exponential compression from best second receiver pressure.
" (*g*) Constant pressure delivery.
" (*h*) Constant zero-volume pressure drop.

Cycle 8. Three-stage Adiabatic Compression with Clearance, Perfec
Intercooling at Best Two Receiver Pressures.

Phase (*a*) Constant pressure admission.
" (*b*) Exponential compression to best first receiver pressure.
" (*c*) Perfect cooling of delivered gas at best first receiver pressur
" (*d*) Exponential re-expansion of first stage clearance.
" (*e*) Exponential compression from best first to best secon
receiver pressure.
" (*f*) Perfect intercooling of delivered gas at best second receiv
pressure.
" (*g*) Exponential re-expansion of second stage clearance.
" (*h*) Exponential compression from best second receiver pressur
" (*i*) Constant pressure delivery.
" (*j*) Exponential re-expansion of third stage clearance.

It should be noted that cycles 6 and 8 may be sub-divided into any numb
of cases, of which some of the most characteristic are shown: (*a*) where th
clearance volume in each cylinder bears the same ratio to the displacemer
of that cylinder, and commonly called equal clearances; (*b*) where the clearanc
are such that the volume after re-expansion in the higher-pressure cylind
is equal to the volume of clearance in the next lower-pressure cylinder, causin
the combined diagram to have a continuous re-expansion line, a case whic
may be called proportionate clearance; and (*c*) the general case in which the
is no particular relation between clearances in the several cylinders.

By means of these definitions or their mathematical equivalents in symbo
it will be possible to calculate work as a function of pressures and volum
and by various transformations of a general expresssion for work of a referen
cycle to calculate the horse-power corresponding to the removal of a giv
volume of gas per minute from the low-pressure supply or to the delivery
another volume per minute to the high-pressure receiver or per unit weig
per minute. It will also be possible to calculate the necessary cylinder si
or displacement per unit of gas handled, and the horse-power necessary
drive the compressing piston at a specified rate and further to calculate th
work and horse-power of cylinders of given size and speed. In order that the
calculations of a numerical sort may be quickly made, which is quite necessa
if they are to be useful, the formulas must be definite and of proper form, th
form being considered proper when little or no algebraic transformation
necessary before numerical work is possible. While special expressions f
each case are necessary to facilitate numerical work, it is equally importan
if not more so, to make clear the broad general principles or methods of attac
because it is quite impossible to set down every case or even to conceive at th
time of writing of all different cases that must in future arise. The trea
ment, then, must be a combination of general and special, the general metho
being applied successively, to make them clear and as a matter of drill, n
to every possible case, but only to certain characteristic or type form

f cases, such as are here set down as standard reference diagrams. ndividual cases may be judged by comparison with these and certain factors f relation established which, being ratios, may be and are called efficiencies. hus, if a single-stage compressor should require two horse-power per cubic oot of free air compressed per minute, and Cycle I should for the same ressure limits require only one horse-power for its execution, then the efficiency f the real compression would be 50 per cent referred to Cycle I, and similar actors or efficiencies for other compressors similarly obtained; a comparison of the factors will yield information for a judgment of the two ompressors.

In what follows on the work and gas capacity of compressors two methods f attack will be used.

1. General pressure-volume analysis *in terms of gas pressures and volumes* esulting in the evaluation of work per cubic foot of low- or high- pressure aseous substance.

2. Transformation of results of (1) to yield volumetric efficiencies, mean ffective pressures, work, horse-power, and capacity *in terms of dimensions* f cylinders and clearances.

3. Single-stage Compressor, No Clearance, Isothermal Compression (Cycle 1), Work, Capacity, and Work per Cubic Foot in Terms of Pressures and Volumes. The standard reference diagram is represented by Fig. 24, on which the process (A to B) represents admission or supply at constant pressure; (B to C) compression at constant temperature; (C to D) delivery at constant pressure; and (D to A) zero-volume.

Let V_b = The number of cubic feet of low pressure gas in the cylinder after admission, represented to scale on the diagram by $\overline{AB}$ and equal to the volume at B;

" V_c = volume in cubic feet of the gas in cylinder when discharge begins, represented by $\overline{DC}$, which is the volume at C;

" P_b = absolute pressure in pounds per square foot, at which supply enters cylinder = (Sup.Pr.) = pressure at B;

" $p_b = P_b \div 144$ = absolute supply pressure in pounds per square inch = (sup.pr.);

" P_c = absolute pressure in pounds per square foot, at which delivery occurs = (Del. Pr.) = pressure at C;

" $p_c = P_c \div 144$ = absolute delivery pressure in pounds per square inch = (del.pr.);

" $R_p = \dfrac{P_c}{P_b}$ = ratio of delivery pressure to supply pressure;

" W = foot-pounds work done for the cycle;

" (H. P. Cap.) = volume of gas delivered in cubic feet per cycle, at temperature same as that of supply;

" (L. P. Cap.) = volume of gas drawn into cylinder, cubic feet per cycle. For this no clearance case (L. P. Cap.) = V_b.

Referring to Fig. 24, the work for the cycle is the sum of compression and delivery work, less admission work, or by areas

Net work $\overline{ABCD}$ = compression work $\overline{EBCG}$ + delivery work $\overline{GCDF}$ − admission work $\overline{EBAF}$

Algebraically this is equivalent to

$$W = P_b V_b \log_e \frac{P_c}{P_b} + P_c V_c - P_b V_b.$$

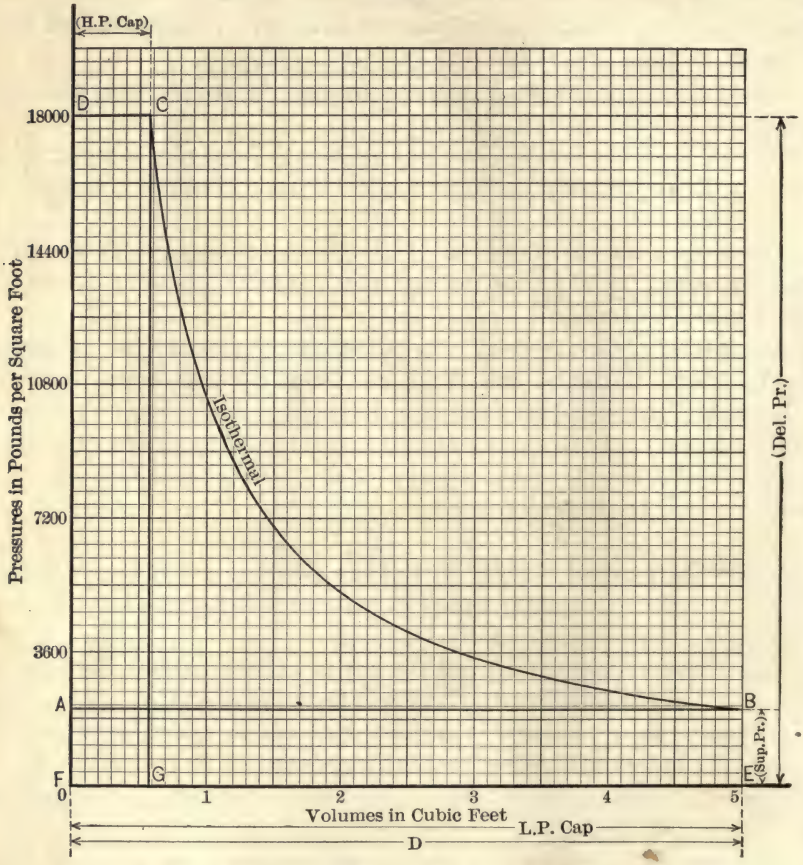


FIG. 24.—One-stage Compressor Cycle 1, No Clearance, Isothermal.

But since $P_c V_c = P_b V_b$ the expression becomes

$$W = P_b V_b \log_e \frac{P_c}{P_b}, \quad \ldots \quad (2$$

which is the work for the execution of the cycle when pressures and volum are in pounds per square foot, and cubic feet. The equivalent expressi for pounds per square inch and cubic feet is

$$W = 144\ p_b V_b \log_e \frac{p_c}{p_b}. \quad \ldots \quad (2$$

Since, when there is no clearance the volume taken into the cylinder for each cycle (L. P. Cap.) is equal to the volume at B, V_b, the expression Eq. (29) may be stated thus, symbolic form.

$$W = 144(\text{sup.pr.})(\text{L. P. Cap.}) \log_e R_p \quad . \quad . \quad . \quad . \quad . \quad (30)$$

The work per cubic foot of low pressure gas, foot-pounds, will be the above expression divided by (L. P. Cap.), or

$$\frac{W}{(\text{L. P. Cap.})} = 144\ (\text{sup.pr.}) \log_e R_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The work per cubic foot of high-pressure gas delivered will be

$$\frac{W}{(\text{H. P. Cap.})} = 144\ (\text{sup.pr.})\ R_p \log_e R_p, \quad . \quad . \quad . \quad . \quad (32)$$

since

$$P_b V_b = P_c V_c$$

or

$$V_b = V_c \frac{P_c}{P_b},$$

which expressed symbolically is

$$(\text{L.P. Cap.}) = (\text{H. P. Cap.}) \times R_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Expressions (31) and (32) for the isothermal compressor are especially useful as standards of comparison for the economy of the compressors using methods other than isothermal. It will be found that the work per cubic foot of either low pressure or cooled high-pressure gas is less by the isothermal process than by any other process discussed later, and that it is the limiting case for the economy of multi-stage compressors with a great number of stages. The fact that this process of isothermal compression is seldom if ever approached in practice does not make it any the less a suitable basis for comparison.

Example 1. Method of calculating Diagram Fig. 24.

Assumed Data.

$P_a = P_b = 2116$ lbs. per square foot. $\qquad V_a = V_d = 0.$

$P_c = P_d = 18{,}000$ lbs. per square foot. $\qquad$ Capacity $= 5$ cu.ft.

$$s = 1.$$

To obtain point C,

$$P_b V_b = P_c V_c \quad \text{or} \quad V_c = \frac{P_b}{P_c} V_b = \frac{5 \times 2116}{18{,}000} = .59.$$

$$\therefore \quad V_c = .59, \qquad P_c = 18{,}000.$$

Intermediate points B to C are obtained by assuming various pressures and findi corresponding volumes as for V_c.

Example 2. To compress and deliver 5 cu.ft. of air from atmospheric pressure (21 lbs. per square foot) to 8.5 atmospheres (18,000 lbs. per square foot) isothermall without clearance, how much work is necessary?

$$P_b = 2116 \qquad P_c = 18{,}000$$

$$V_b = 5$$

$$\frac{V_b}{V_c} = \frac{P_c}{P_b} = 8.5 \qquad \therefore \quad V_c = .588$$

Work of admission $= P_b V_b = 2116 \times 5 = 10{,}585$ ft.-lbs.

Work of compression $= P_b V_b \log_e \dfrac{P_c}{P_b} = 10{,}585 \times \log_e 8.5 = 22{,}600$ ft.lbs.

Work of delivery $= P_c V_c = 10{,}585$ ft.-lbs.

Total work $= 10{,}585 + 22{,}600 - 10{,}585 = 22{,}600$ ft.-lbs.

Or by the general formula,

$$W = (\text{sup.pr.})(\text{L.P.Cap.}) \log_e R_p = 2116 \times 5 \times \log_e 8.5 = 2116 \times 5 \times 2.14 = 22{,}652 \text{ ft.-lb}$$

Prob. 1. How many cubic feet of free air may be compressed and delivered p minute from 14 lbs. absolute to 80 lbs. per square inch absolute per horse-power in compressor with zero clearance if compression is isothermal?

Prob. 2. Gas is being forced through mains at the rate of 10,000 cu.ft. per minut under a pressure of 5 lbs. per square inch above atmosphere. The gas is taken into th compressor at atmospheric pressure and compression is isothermal. What horse power will be needed at sea level and at an elevation of 5000 feet?

Prob. 3. Natural gas is drawn from a well, compressed isothermally and forced throug a main at the rate of 200,000 cu.ft. per hour measured at the pressure on the suctio side. What steam horse-power will be required to operate the compressor if th mechanical efficiency be 80 per cent? Suction pressure is 8 lbs. per square inc absolute, delivery pressure 60 lbs. per square inch absolute.

Prob. 4. A vacuum cleaning pump is required to maintain a pressure of 14 lbs. p square inch absolute, move 500 cu.ft. of free air per minute and discharge it again an atmospheric pressure of 15 lbs. per square inch absolute. What horse-power wi be required (isothermal)?

Prob. 5. A blower furnishes 45 cu.ft. of air a minute at a pressure of 5 ins. of mercur above atmosphere. Assuming compression to be isothermal and supply pressure to b atmospheric, what horse-power will be needed?

Prob. 6. A compressor has a piston displacement of 3 cu.ft. At what speed can it b run if air be compressed isothermally from 1 to 10 atmospheres and the horse-powe supplied is 100?

Prob. 7. A tank of 1000 cu.ft. capacity contains air at atmospheric pressure. compressor taking air from atmosphere compresses it isothermally and discharges it int the tank until the pressure reaches 100 lbs. per square inch gage. What horse-powe will be required to fill tank at this pressure in ten minutes?

Prob. 8. A compressor receives air at atmosphere and compresses it isothermally t five atmospheres above atmosphere. It takes in 1000 cu.ft. of free air per minut How much would the capacity increase if the discharge pressure dropped to 3 atmo pheres and the horse-power remained the same?

Prob. 9. Suppose that the pressure in the above problem were raised to 8 atmospheres. How much would the capacity decrease if the horse-power remained the same and how much more power would be required to keep the capacity the same?

Prob. 10. By means of suitable apparatus, the water from the side of a waterfall is diverted to a vertical shaft, and in falling 126 ft. compresses air from atmospheric pressure to a value equal to 90 per cent of the head of the water. To deliver 1000 cu.ft. of compressed air per hour, how much water is required if the work of falling water is 80 per cent useful in compressing the air?

4. Single-stage Compressor with Clearance, Isothermal Compression, (Cycle 2). Work, Capacity, and Work per Cubic Foot in Terms of Pressures and Volumes.

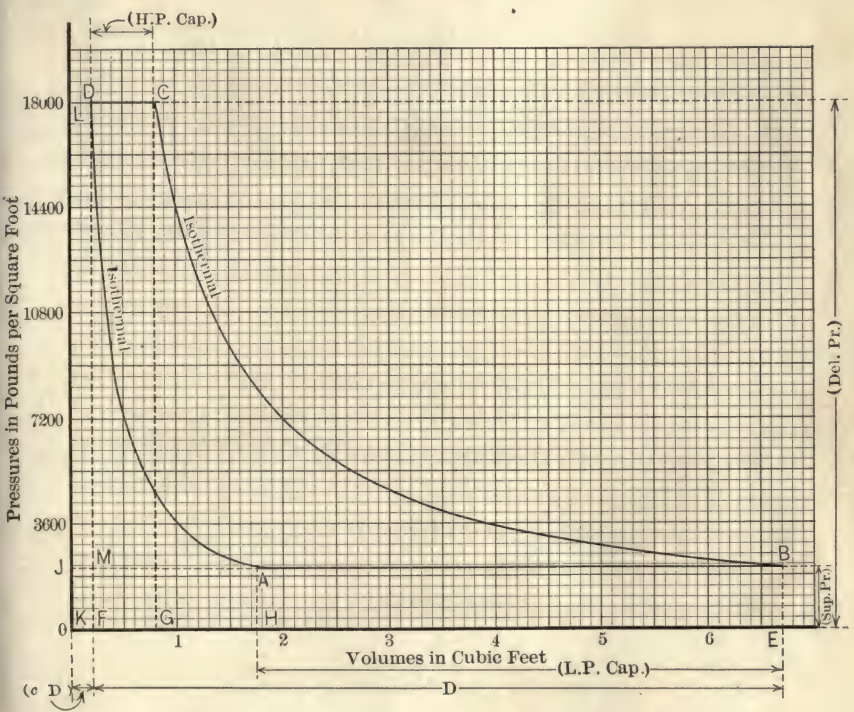


FIG. 25.—One-Stage Compressor Cycle 2, Clearance, Isothermal.

Referring to Fig. 25, the work of the cycle is, by areas.

$$\text{Net work area} = \overline{EBCG} + \overline{GCDF} - \overline{HADF} - \overline{EBAH} = \text{Area } \overline{ABCD}.$$

It is easily seen that this area is also equal to $(\overline{JBCL}) - (\overline{JADL})$, both of which are areas of the form evaluated in the preceding section. Accordingly

$$\text{Net work area} = \overline{JBCL} - \overline{JADL},$$

Algebraically,

$$W = P_b V_b \log_e \frac{P_c}{P_b} - P_a V_a \log_e \frac{P_d}{P_a}$$

$$= P_b(V_c - V_a) \log_e \frac{P_c}{P_b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3$$

which is the general expression for the work of the cycle in foot-pounds wh pressures are in pounds per square foot, and volumes in cubic feet. Substituti the symbolic equivalents and using pressures in pounds per square inch, the results, since $(V_b - V_a) = (\text{L. P. Cap.})$,

$$\text{Work} = 144\ (\text{sup.pr.})(\text{L. P. Cap.}) \log_e R_p, \quad . \quad . \quad . \quad . \quad (3$$

which is identical with Eq. (30), showing that *for a given low-pressure capac the work of isothermal compressors is independent of clearance.* The value the low-pressure capacity $(V_b - V_a)$ may not be known directly, but m be found if the volume before compression, V_b, the clearance volume befo re-expansion, V_d, and the ratio of delivery to supply pressure, R_p, are know thus

$$V_a = V_d R_p,$$

from which

$$(\text{L. P. Cap.}) = (V_b - V_d R_p). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3$$

From Eq. (35) the work per cubic foot of low-pressure gas is, in foot-pounds,

$$\frac{W}{(\text{L.P. Cap.})} = 144(\text{sup.pr.}) \log_e R_p \quad . \quad . \quad . \quad . \quad . \quad . \quad (3$$

and the work per cubic foot of high-pressure gas delivered, ft.-lbs.

$$\frac{W}{(\text{H.P. Cap.})} = 144(\text{sup.pr.}) R_p \log_e R_p \quad . \quad . \quad . \quad . \quad . \quad (3$$

By comparison, Eqs. (37) and (38) are found to be identical with (31) a (32) respectively, *since clearance has, as found above, no effect on the work d for a given volume of gas admitted, however much it may affect the work of cycle between given volume limits or work per unit of displacement.*

It is interesting to note that the work areas of Figs. 24 and 25 are eq when plotted on equal admission lines $\overline{AB}$ or delivery lines $\overline{CD}$ and a horizontal intercept xy will be equal in length on both if drawn at the sa pressure.

In what precedes, it has been assumed that $\overline{AB}$ represents admission volu and $\overline{CD}$ represents delivery volume which is true for these established cyc

reference, but it is well to repeat that for real compressors these are only apparent admission and delivery lines, as both neglect heating and cooling effects on the gas during its passage into and out of the cylinder. Also that in real compressors the pressure of the admission line cannot ever be as high as the pressure from which the charge is drawn and the delivery pressure must be necessarily higher than that which receives the discharge, in which cases the volume of gas admitted, as represented by $\overline{AB}$, even if the temperature did not change, would not equal the volume taken from the external supply, because it would exist in the cylinder at a lower pressure than it originally had, and a similar statement would be true for delivered gas.

Problems. Repeat all the problems of the last section, assuming any numerical value for the clearance up to 10 per cent of the displacement.

5. Single-stage Compressor Isothermal Compression. Capacity, Volumetric Efficiency, Work, Mean Effective Pressure, Horse-power and Horse-power per Cubic Foot of Substance, in Terms of Dimensions of Cylinder and Clearance.

Consider first the case where clearance is not zero. Then Fig. 25 is the reference diagram.

Let D = displacement = volume, in cubic feet, displaced by piston in one stroke = area of piston in sq.ft. × stroke in ft. = $(V_b - V_d)$.

" (H. P. Cap.) = high pressure capacity = vol. cu.ft. of gas delivered per cycle at temperature equal to that of supply = $(V_c - V_d)$;

" (L. P. Cap.) = low pressure capacity = vol. in cu.ft. of gas entering cylinder per cycle = $(V_b - V_a)$;

" E_v = volumetric efficiency = $\frac{\text{L.P. Cap.}}{D} = \frac{V_b - V_a}{V_b - V_d}$;

" Cl = volume of clearance, cubic feet = V_d

" c = clearance volume expressed as a fraction of the displacement;

" $= \frac{Cl}{D} = \frac{V_d}{V_b - V_a}$ whence $Cl = cD$;

" M.E.P. = mean effective pressure, lbs. per square foot = $\frac{W}{D}$;

" m.e.p. = mean effective pressure, lbs. per square inch = $\frac{W}{144D}$;

" N = number of revolutions per minute;

" n = number of cycles per minute;

" z = number of revolutions per cycle = $\frac{N}{n}$;

" I.H.P. = indicated horse-power of compressor;

The low-pressure capacity of the single-stage isothermal compressor wit clearance is,

$$(\text{L. P. Cap.}) = (V_b - V_a), \quad \text{but} \quad V_a = V_d \times \frac{P_d}{P_a}.$$

Whence $(\text{L. P. Cap.}) = \left(V_b - V_d \frac{P_d}{P_a}\right)$ for which may be substituted the symbo for displacement and clearance volumes, thus

$$(\text{L. P. Cap.}) = D + cD - cDR_p,$$

$$= D(1 + c - cR_p) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3$$

For convenience the term, Volumetric Efficiency, E_v is introduced. Sin this is defined as the ratio of the low-pressure capacity to the displacement,

$$E_v = \frac{(\text{L. P. Cap.})}{D} = 1 + c - cR_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4$$

Referring to Eq. (35) it is seen that the value of (L. P. Cap.) can substituted from Eq. (39) and the result is:

Work per cycle, foot-pounds, in terms of supply pressure, pound spersq ua inch, displacement cubic feet, clearance as a fraction of displacement, and rat of delivery to supply pressure is,

$$W = 144 \ (\text{sup.pr.}) D(1 + c - cR_p) \log_e R_p, \quad . \quad . \quad . \quad . \quad (4$$

or in the terms of the same quantities omitting clearance and introduci volumetric efficiency, E_v,

$$W = 144(\text{sup.pr.}) \ DE_v \log_e R_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4$$

To obtain the mean effective pressure for the cycle, the work done per cy is divided by displacement, D.

Mean effective pressure, pounds per square inch,

$$(\text{m.e.p.}) = \frac{W}{144D},$$

whence

$$(\text{m.e.p.}) = (\text{sup.pr.})(1 + c - cR_p) \log_e R_p \quad . \quad . \quad . \quad . \quad . \quad (4$$

or

$$(\text{m.e.p.}) = (\text{sup.pr.}) E_v \log_e R_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4$$

The indicated horse-power of the isothermal compressor is equal to the work per minute, in ft.-lbs. divided by 33,000. If n cycles are performed per minute, then

$$\text{I.H.P.} = \frac{Wn}{33000} = \frac{144n}{33000}\,(\text{sup.pr.})D(1+c-cR_p)\log_e R_p$$

$$= \frac{(\text{sup.pr.})n}{229.2} D(1+c-cR_p)\log_e R_p \quad \ldots\ldots\ldots \quad (45)$$

$$= \frac{(\text{sup.pr.})n}{229.2} DE_v \log_e R_p, \quad \ldots\ldots\ldots\ldots \quad (46)$$

Introducing the effective area of the piston, in square inches, a, and the piston speed S, feet per minute, then since

$$D = \frac{a}{144} Ln = \frac{a}{144}\,\frac{S}{2z} = \frac{aS}{288z},$$

$$\text{I.H.P.} = \frac{(\text{sup.pr.})aS}{66{,}000\,z} E_v \log_e R_p. \quad \ldots\ldots\ldots\ldots \quad (47)$$

The same expression for the indicated horse-power may be derived by the substitution of the value of (m.e.p.) Eq. (44) in the following general expression for indicated horse-power.

$$\text{I.H.P.} = \frac{(\text{m.e.p.})aS}{33000 \times 2z}.$$

Example 1. Method of calculating Diagram Fig. 25.

Assumed Data:

$P_a = P_b = 2116$ lbs. per square foot;
$P_d = P_c = 18{,}000$ lbs. per square foot;
$c = 3$ per cent. L.P. Cap. $= 5$ cu.ft. $s = 1$.

To obtain point D:

From formula Eq. (39), L.P. Cap. $= D(1+c-cR_p)$ or $5 = D(1+.03-.03\times 8.5)$,

or

$D = 6.5$ cu.ft. and $Cl. = .03 \times 6.5 = .195$ or approximately .2 cu.ft.

$\therefore\ V_d = .2$ cu.ft., $\qquad P_d = 18{,}000$ lbs. sq.ft.

To obtain point A:

$$P_a V_a = P_d V_d \quad \text{or} \quad V_a = \frac{P_d}{P_a} V_d = 8.5 \times .2 = 1.7,$$

$\therefore\ V_a = 1.7$ cu.ft., $\qquad P_a = 2116$ lbs. sq.ft.

Intermediate points D to A are obtained by assuming various pressures and finding the corresponding volumes as for V_a.

To obtain point B:

$$V_b = V_a + 5 = 6.7 \text{ cu.ft.} \qquad P_b = 2116 \text{ lbs. sq.ft.}$$

To obtain point C:

$$P_c V_c = P_b V_b, \quad \text{or} \quad V_c = \frac{P_b V_b}{P_c} = \frac{2116 \times 6.7}{18{,}000} = .79 \text{ cu.ft.}$$

$$\therefore\ V_c = .79 \text{ cu.ft.}, \qquad P_c = 18{,}000 \text{ lbs. sq.ft.}$$

Example 2. It is required to compress 1000 cu.ft. of air per minute from 1 to 8.5 atmospheres isothermally in a compressor having 4 per cent clearance. What must be the displacement work, per 100 cu.ft. of supplied and delivered air, and horse-power of machine? Speed is 150 R.P.M., compressor is double acting and stroke = 1.5 diameters. Neglect piston rods.

$$D = (\text{L. P. Cap.}) \div E_v \quad \text{and} \quad E_v = (1 + .04 - .04 \times 8.5) = .7,$$
$$\therefore\ D = 1000 \div .7 = 1428 \text{ cu.ft. per minute.}$$

Work per cu.ft. of supplied air = (sup.pr.) $144 \log_e R_p = 144 \times 14.7 \log_e R_p = 4530$ ft.-lbs.; Hence the work per 100 cu.ft. = 453,000 ft.-lbs.

Work per cu.ft. of delivered air $= 144(\text{sup.pr.}) R_p \log_e R_p = 144 \times 14.7 \times 8.5 \times 2.14 = 38{,}550$ ft.-lbs.

Hence the work per 100 cu.ft. = 3,855,000 ft.-lbs.

$$\text{I.H.P.} = \frac{4530000}{33000} = 137.2.$$

$$D = \frac{1428}{150 \times 2} = 4.76 \text{ cu.ft. per stroke.}$$

$$D = L \times A = 1.5d \times \frac{\pi}{4} d^2 = 1.18 d^3 = 4.76.$$

$$\text{Hence cylinder diameter} = \left(\frac{4.76}{1.18}\right)^{\frac{1}{3}} = 1.59 \text{ feet} = 19.1 \text{ inches.}$$

Prob. 1. How many cubic feet of free air per minute may be compressed isothermally to 100 lbs. per square inch absolute in a compressor having 6 per cent clearance if the horse-power supplied is 60?

Prob. 2. A compressor has a cylinder 18×24 ins., clearance 4 per cent, is double acting and runs at 150 R.P.M. If it compresses air from atmosphere to 100 lbs. per square inch gage, what will be its high- and low-pressure capacity, its horse-power, and

how will the horse-power and the capacity compare with these quantities in a hypothetical compressor of the same size but having zero clearance? How will the horse-power per cubic foot of delivered air compare?

Prob. 3. A manufacturer gives for a $10\frac{1}{4}\times12$ in. double-acting compressor running at 160 R.P.M. a capacity of 177 cu.ft. of free air per minute for pressures of 50–100 lbs. per square inch gage. What clearance does this assume for the lowest and highest pressure if the compression is isothermal? The horse-power is given as from 23 to 35. Check this.

Prob. 4. Air enters a compressor cylinder at 5 lbs. per square inch absolute and is compressed to atmosphere (barometer $=30\frac{1}{2}$ ins.). Another compressor receives air at atmosphere and compresses it to 3 atmospheres. If each has 6 per cent clearance what must be the size of each to compress 1000 cu.ft. of free air per minute, how will the total work compare in each machine, and how will the work per cubic foot of high and low pressure air compare in each? Assume compression to be isothermal.

Prob. 5. 1800 cu.ft. of free air per minute is to be compressed isothermally to a pressure of 100 lbs. per square inch gage. What must be the displacement and horse-power of a hypothetical zero clearance compressor, and how will they compare with those of a compressor with 6 per cent clearance?

Prob. 6. Consider a case of a compressor compressing air isothermally from atmosphere to 100 lbs. per square inch gage. Plot curves showing how displacement and horse-power will vary with clearance for a 1000 cu.ft. free air per minute capacity taking clearances from 1 per cent to 10 per cent.

Prob. 7. Two compressors of the same displacement, namely 1000 cu.ft. per minute, compress air isothermally from 50 lbs. per square inch gage to 150 lbs. per square inch gage. One has 5 per cent clearance, the other 10 per cent. How will their capacities and horse-power compare with each other and with a no clearance compressor?

Prob. 8. A 9×12 in. compressor is compressing air from atmosphere to 50 lbs. gage. How much free air will it draw in per stroke, and how much compressed air will it discharge per stroke for each per cent clearance?

Prob. 9. The volumetric efficiency of a compressor is .95 as found from the indicator card. It is double acting and has a cylinder 18×24 ins. What will be its capacity and required horse-power for 100 lbs. per square inch gage delivery pressure?

6. Single-stage Compressor, No Clearance Exponential Compression, (Cycle 3). Work, Capacity and Work per Cubic Foot, in Terms of Pressures and Volumes. The cycle of the single-stage exponential compressor without clearance is represented by Fig. 26. Referring to work areas on this diagram,

$$\begin{aligned}\text{Net work } \overline{ABCD} &= \text{compression work } \overline{EBCG}\\ &\quad +\text{delivery work } \overline{GCDF}\\ &\quad -\text{admission work } \overline{FABE}.\end{aligned}$$

Algebraically,

$$W=\frac{P_bV_b}{s-1}\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}}-1\right]+P_cV_c-P_bV_b.$$

But

$$P_c V_c V_c^{s-1} = P_b V_b V_b^{s-1},$$

or

$$P_c V_c = P_b V_b \left(\frac{V_b}{V_c}\right)^{s-1} = P_b V_b \left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}}$$

and

$$P_c V_c - P_b V_b = P_b V_b \left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right].$$

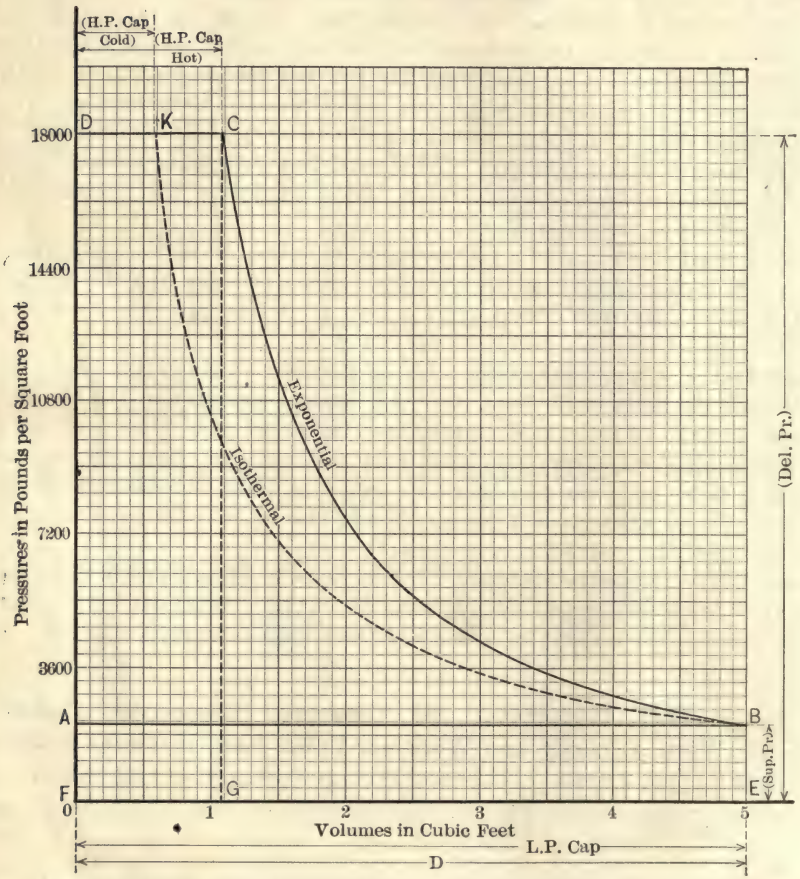


FIG. 26.—One-Stage Compressor Cycle 3, No Clearance, Exponential.

Substituting above

$$W = \frac{P_b V_b}{s-1}\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right] + P_b V_b \left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right]$$

$$= P_b V_b \left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right]\left(\frac{1}{s-1} + 1\right).$$

Whence

$$W=\frac{s}{s-1}P_bV_b\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}}-1\right], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

Eq. (48) gives the work in foot-pounds for the execution of the cycle when pressures are in pounds per square foot, and volumes in cubic feet.

The equivalent expression for pressures in pounds per square inch is

$$W=144\frac{s}{s-1}p_bV_b\left[\left(\frac{p_c}{p_b}\right)^{\frac{s-1}{s}}-1\right]. \quad . \quad . \quad . \quad . \quad . \quad (49)$$

When there is no clearance, as before, V_b represents the entire volume of displacement, which is also here equal to the volume admitted (L. P. Cap.), p_b is the supply pressure (sup.pr.) pounds per square inch absolute and $\frac{p_c}{p_b}$ is the ratio of delivery to supply pressure, R_p.

Accordingly, the work of an exponential, single-stage compressor with no clearance is

$$W=144\frac{s}{s-1}(\text{sup.pr.})(\text{L. P. Cap.})\left(R_p^{\frac{s-1}{s}}-1\right). \quad . \quad . \quad . \quad . \quad (50)$$

The work per cubic feet of low pressure gas, foot-pounds is

$$\frac{W}{(\text{L. P. Cap.})}=144\frac{s}{s-1}(\text{sup.pr.})\left(R_p^{\frac{s-1}{s}}-1\right). \quad . \quad . \quad . \quad . \quad . \quad (51)$$

Before obtaining the work per cubic foot of high-pressure gas, it is necessary to describe two conditions that may exist. Since the exponential compression is not isothermal, it may be concluded that a change in temperature will take place during compression. This change is a rise in temperature, and its law of variation will be presented in another chapter.

1. If the compressed air is to be used immediately, before cooling takes place, the high-pressure capacity or capacity of delivery will be equal to the volume at C, V_c and may be represented by (H. P. Cap. hot).

2. It more commonly occurs that the gas passes to a constant-pressure holder or reservoir, in which it stands long enough for it to cool approximately to the original temperature before compression, and the volume available after this cooling takes place is less than the actual volume discharged from the cylinder in the heated condition. Let this volume of discharge when reduced to the initial temperature be represented by (H. P. Cap. cold) which is represented by V_k, Fig. 26.

Since B and C in Fig. 26 lie on the exponential compression line, $P_b V_b^s = P_c V_c^s$,

$$V_b = V_c \left(\frac{P_c}{P_b}\right)^{\frac{1}{s}},$$

or

$$(\text{L. P. Cap.}) = (\text{H. P. Cap. hot})\ (R_p)^{\frac{1}{s}}. \quad . \quad . \quad . \quad . \quad . \quad (52$$

Hence, the work in foot-pounds per cubic foot of hot gas delivered from compressor is

$$\frac{W}{(\text{H. P. Cap. hot})} = 144 \frac{s}{s-1} (\text{sup.pr.}) R_p^{\frac{1}{s}} \left(R_p^{\frac{s-1}{s}} - 1\right). \quad . \quad . \quad . \quad . \quad (53$$

On the other hand, B and K lie on an isothermal and $P_b V_b = P_k V_k$, o since $P_k = P_c$,

$$V_b = V_k \frac{P_c}{P_b},$$

whence

$$(\text{L. P. Cap.}) = (\text{H. P. Cap. cold}) R_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (54$$

The work foot-pounds per cubic foot of gas cooled to its original temperature is, therefore,

$$\frac{W}{(\text{H. P. Cap. cold})} = 144 \frac{s}{s-1} (\text{sup. pr.}) R_p \left(R_p^{\frac{s-1}{s}} - 1\right), \quad . \quad . \quad . \quad . \quad (5$$

or

$$\frac{W}{(\text{H. P. Cap. cold})} = 144 \frac{s}{s-1} (\text{del. pr.}) \left(R_p^{\frac{s-1}{s}} - 1\right), \quad . \quad . \quad . \quad . \quad (56$$

This last equation is useful in determining the work required for the storin or supplying of a given amount of cool compressed air or gas, under conditio quite comparable with those of common practice.

Example 1. Method of calculating Diagram Fig. 26.
Assumed Data:

$P_a = P_b = 2116$ lbs. per square foot; $P_c = P_d = 18{,}000$ lbs. per square foot.
$Cl = 0$; $V_a = V_d = 0$; L. P. Capacity $= 5$ cu.ft.; $s = 1.4$ (adiabatic value of

To obtain point C:

$$P_c V_c^{1.4} = P_b V_b^{1.4} \quad \text{or} \quad V_c = V_b \div \left(\frac{P_c}{P_b}\right)^{\frac{1}{1.4}}$$

$P_c/P_b = 8.5$; $\log_e 8.5 = .929$, and $.71 \log_e 8.5 = .665$; $(P_c/P_b)^{\frac{1}{1.4}} = 4.6$,

ience $V_c = 5 \div 4.6 = 1.09$ cu. ft. $P_c = 18{,}000$ lbs. per sq.ft.

intermediate points B to C are obtained by assuming various pressures and finding he corresponding volumes as for V_c.

Example 2. To compress 5 cu.ft.of air from atmospheric pressure (2116 lbs. per quare foot) to 8.5 atmospheres (18,000 lbs. per square foot) adiabatically and with no learance requires how many foot-pounds of work?

$P_b = 2116$ lbs. sq.ft., $P_c = 18{,}000$ lbs. sq.ft.,
$V_b = 5$ cu.ft.

$$V_c = \frac{V_b}{\left(\frac{P_c}{P_b}\right)^{.71}} = 5 \div 4.57 = 1.092 \text{ cu.ft.}$$

Work of admission is

$$P_b V_b = 2116 \times 5 = 10{,}585 \text{ ft.-lbs.}$$

Work of compression, using γ to represent the adiabatic value of s is,

$$\frac{P_b V_b}{\gamma - 1}\left[\left(\frac{P_c}{P_b}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \frac{10{,}585}{.41}[(8.5)^{.29} - 1] = \frac{10{,}585}{.41} \times .860 = 22{,}350 \text{ ft.-lbs.}$$

Work of delivery is

$$P_c V_c = 18{,}000 \times 1.092 = 19{,}650 \text{ ft.-lbs.}$$

Total work $= 19{,}650 + 22{,}350 - 10{,}585 = 31{,}425$ ft.-lbs.,

r by the formula Eq. (50) directly

$$W = 144 \frac{\gamma}{\gamma - 1}(\text{sup. pr.})(\text{L. P. Cap.})\left(R_p^{\frac{\gamma-1}{\gamma}} - 1.\right)$$

$$= 144 + 3.46 \times 2116 \times 5 \times [(8.5)^{.29} - 1];$$

$$= 144 + 3.46 \times 2116 \times 5 \times .86 = 31{,}450 \text{ ft.-lbs.}$$

Prob. 1. A single-stage zero clearance compressor compresses air adiabatically from to 6 atmospheres. How many cubic feet of free air per minute can be handled if the ompressor is supplied with 25 H.P. net?

Prob. 2. The same compressor is used for superheated ammonia under the same ressure conditions. For the same horse-power will the capacity be greater or less nd how much?

Prob. 3. A dry-vacuum pump receives air at 28 ins. of mercury vacuum and delivers against atmospheric pressure. What will be the work per cubic foot of low-pressure ir and per cubic foot of high-pressure air hot? Barometer reads 29.9 ins.

Prob. 4. The manufacturer gives for a $10\frac{1}{4}\times12$ in. double acting compressor running at 160 R.P.M., a capacity of 177 cu.ft. of free air per minute and a horse-power o 25 to 35 when delivering against pressures from 50 to 100 lbs. Check these figures.

Prob. 5. A set of drills, hoists, etc., are operated on compressed air. For their opera tion 3000 cu.ft. of air at 70 lbs. gage pressure are required per minute. What must b the piston displacement and horse-power of a compressor plant to supply this air i compression is adiabatic and there is assumed to be no clearance?

Prob. 6. Air is compressed from atmosphere to 60 lbs. per square inch gage by a compressor having a 12×18 in. cylinder and running at 100 R.P.M. Find its capacity and horse-power at sea level and loss in capacity and horse-power if operated at an alti tude of 10,000 ft. for zero clearance.

Prob. 7. 10,000 cu.ft. of free air per minute at a pressure of 15 lbs. above atmos phere are compressed and delivered by a blowing engine. Find the horse-power required to do this and find how much free air could be delivered by same horse-power if th pressure were tripled.

Prob. 8. In a gas engine the mixture of air and gas is compressed in the cylinde before ignition. If the original pressure is 14 lbs. per square inch absolute, fina pressure 85 lbs. absolute and compression is adiabatic, what will be the work o compression only, per pound of mixture?

NOTE: Weight per cubic foot may be taken as .07 and γ as 1.38.

Prob. 9. A vacuum pump is maintaining a 25-in. vacuum and discharging the a removed against atmospheric pressure. Compare the work per cubic foot of low pres sure air with that of a compressor compressing from atmosphere to 110 lbs. above atmo phere.

7. Single-stage Compressor with Clearance, Exponential Compression (Cycle 4). Work, Capacity, and Work per Cubic Foot in Terms of Pressure and Volumes. When clearance exists in the cylinder, it is evident that volume equal to the clearance, V_d, will not be expelled during the deliver of compressed gas, and this volume will expand with fall in pressure as th piston returns, causing pressure-volume changes represented by the line D on the diagram, Fig. 27. Until the pressure has fallen to that of supply, th admission valve will not open, so that while the total volume in the cylinde at end of admission, is V_b, the volume V_a was already present by reaso of the clearance, and the volume taken in is $(V_b - V_a)$ which is the low-pressu capacity (L. P. Cap.).

The work area of the diagram is $ABCD$, which may be expressed as

$$\text{Work area} = \overline{JBCL} - \overline{JADL},$$

which areas are of the form evaluated in Section 6. Hence, the above expressio in algebraic terms is

$$W = \frac{s}{s-1}P_bV_b\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right] - \frac{s}{s-1}P_aV_a\left[\left(\frac{P_d}{P_a}\right)^{\frac{s-1}{s}} - 1\right]$$

$$= \frac{s}{s-1}P_b(V_b - V_a)\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right]. \quad \ldots\ldots\ldots \quad (5$$

This is the general expression for the work of the cycle, in foot-pounds, when the pressures are expressed in pounds per square foot, and volumes in cubic feet. Using symbolic equivalents

$$W = 144\frac{s}{s-1}(\text{sup.pr.})(\text{L.P.Cap.})\left[(R_p)^{\frac{s-1}{s}} - 1,\right] \quad . \quad . \quad . \quad . \quad (58)$$

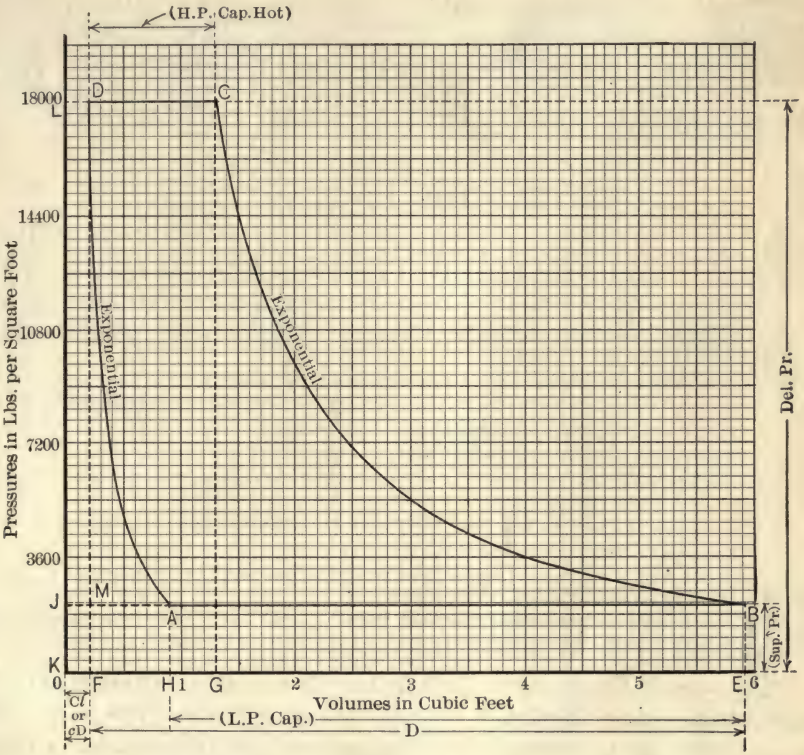


FIG. 27.—One-Stage Compressor Cycle 4, Clearance, Exponential.

Eq. (58) is identical with Eq. (50), showing *that for adiabatic as for isothermal compressors, the work done for a given low-pressure capacity is independent of clearance.* Due to this fact, the expressions derived for the exponential compressor without clearance will hold for that with clearance:

Work, in foot-pounds per cubic foot of low-pressure gas is,

$$\frac{W}{(\text{L. P. Cap.})} = 144\frac{s}{s-1}(\text{sup.pr.})(R_p^{\frac{s-1}{s}} - 1) \quad . \quad . \quad . \quad . \quad . \quad (59)$$

Work, in foot-pounds per cubic foot of hot gas delivered is,

$$\frac{W}{(\text{H. P. Cap. hot})} = 144\frac{s}{s-1}(\text{sup.pr.})R_p^{\frac{1}{s}}\left(R_p^{\frac{s-1}{s}} - 1\right). \quad . \quad . \quad . \quad (60)$$

Work, in foot-pounds per cubic foot of cooled gas to its original temperature is,

$$\frac{W}{(\text{H. P. Cap. cold})} = 144\frac{s}{s-1}(\text{del.pr.})\left(R_p^{\frac{s-1}{s}} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (61)$$

The relation of high-pressure capacity either hot or cold to the low-pressure capacity is also as given for the case of no clearance, as will be shown.

In Fig. 27, the high-pressure capacity, hot, is $\overline{DC} = V_c - V_d$. The low-pressure capacity is $\overline{AB} = V_b - V_a$, but $V_c P_c^{\frac{1}{s}} = V_b P_b^{\frac{1}{s}}$ and $V_d P_d^{\frac{1}{s}} = V_a P_a^{\frac{1}{s}}$, or

$$V_b = V_c R_p^{\frac{1}{s}} \quad \text{and} \quad V_a = V_d R_p^{\frac{1}{s}}.$$

Hence
$$(\text{L. P. Cap.}) = (\text{H. P. Cap. hot}) R_p^{\frac{1}{s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

If the delivered gas be cooled to its original temperature, then the volume after delivery and cooling will be

$$(\text{H. P. Cap. cold}) = \frac{(\text{L. P. Cap.})}{R_p},$$

or
$$(\text{L. P. Cap.}) = (\text{H. P. Cap. cold}) R_p \quad . \quad . \quad . \quad . \quad (63)$$

From the work relations given above, it is seen that in general, the work per unit of gas, or the horse-power per unit of gas per minute is independent of clearance.

8. Single-stage Compressor Exponential Compressor. Relation between Capacity, Volumetric Efficiency, Work, Mean Effective Pressure, Horse-power and H.P. per Cubic Foot of Substance and the Dimensions of Cylinder and Clearance. As indicated on Fig. 27, for the single-stage exponential compressor with clearance, the cylinder displacement D, is $(V_b - V_d)$. The low-pressure capacity per cycle is $(\text{L. P. Cap.}) = (V_b - V_a)$. The actual volume of gas or vapor delivered by the compressor is $(\text{H. P. Cap. hot}) = (V_c - V_d)$. This is, in the case of a gas at a higher temperature than during supply, but if cooled to the temperature which existed at B will become a less volume. This delivered volume after cooling is symbolized by (H. P. Cap. cold) and is equal to $(\text{L. P. Cap.}) \times \frac{(\text{sup.pr.})}{(\text{del.pr.})}$ or $\frac{(\text{L. P. Cap.})}{R_p}$ where R_p is the ratio of delivery pressure to supply pressure.

Volumetric efficiency, E_v, already defined as the ratio of low-pressure capacity to displacement is

$$E_v = \frac{V_b - V_a}{V_b - V_d} = \frac{(\text{L. P. Cap.})}{D}.$$

Clearance, c, expressed as a fraction of the displacement is the ratio of clearance volume, Cl, to displacement, D, and is,

$$c=\frac{Cl}{D}=\frac{V_d}{V_b-V_d}.$$

Mean effective pressure, pounds per square foot (M.E.P.), is the mean height of the diagram or the work area W, divided by displacement, D. If expressed in pounds per square inch the mean effective pressure will be indicated by

$$(\text{m.e.p.})=\frac{W}{144D}.$$

Let (I.H.P.) be indicated horse-power of the compressor;
" N the number of revolutions per minute;
" n the number of cycles per minute and
" z the number of revolutions per cycle, whence $n\times z=N$.

Then, the low-pressure capacity is

$$(\text{L. P. Cap.})=(V_b-V_a).$$

But

$$V_a=V_d\times\left(\frac{P_d}{P_a}\right)^{\frac{1}{s}},$$

since the re-expansion DA is exponential and similar to compression as to value of s, whence

$$(\text{L. P. Cap.})=(V_b-V_a)=V_b-V_dR_p^{\frac{1}{s}};$$

$$=D+Cl-V_dR_p^{\frac{1}{s}};$$

$$=D+cD-cDR_p^{\frac{1}{s}};$$

$$(\text{L. P. Cap.})=D\left(1+c-cR_p^{\frac{1}{s}}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

From this, by definition, the volumetric efficiency is

$$E_v=\frac{(\text{L.P.Cap.})}{D}=1+c-cR_p^{\frac{1}{s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

Referring to Eq.(57), in which may be substituted the value Eq. (64) for V_b-V_a), the work of the single-stage exponential compressor in terms of dis-

placement, clearance (as a fraction of displacement), and pressures of supp and delivery in pounds per square foot is,

$$W=\frac{s}{s-1}P_bD\left[1+c-c\left(\frac{P_c}{P_b}\right)^{\frac{1}{s}}\right]\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}}-1\right], \quad . \quad . \quad . \quad . \quad (6$$

or using pressures, pounds per square inch, and inserting the symbols, th may be stated in either of the following forms:

$$W=144\frac{s}{s-1}(\text{sup.pr.})D\left(1+c-cR_p^{\frac{1}{s}}\right)\left[R_p^{\frac{s-1}{s}}-1\right]. \quad . \quad . \quad . \quad (6$$

$$=144\frac{s}{s-1}(\text{sup.pr.})DE_v\left[R_p^{\frac{s-1}{s}}-1\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6$$

The mean effective pressure in pounds per square foot is this work divid by the displacement, in cubic feet, and may be converted to pounds per squa inch by dividing by 144, whence

Mean effective pressure, pounds per square inch,

$$(\text{m.e.p.})=\frac{s}{s-1}(\text{sup.pr.})(1+c-cR_p^{\frac{1}{s}})\left[R_p^{\frac{s-1}{s}}-1\right], \quad . \quad . \quad (6$$

$$=\frac{s}{s-1}(\text{sup.pr.})E_v\left[R_p^{\frac{s-1}{s}}-1\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7$$

The indicated horse-power of the single-stage exponential compressor fr (67) is,

$$\text{I.H.P.}=\frac{Wn}{33000}=\frac{s}{s-1}\frac{(\text{sup.pr.})nDE_v}{229.2}\left[R_p^{\frac{s-1}{s}}-1\right] \quad . \quad . \quad . \quad (7$$

Where n is the number of cycles per minute, or in terms of piston speed S a effective area of piston, square inches, and z the number of revolutions p cycle,

$$\text{I.H.P.}=\frac{s}{s-1}\frac{(\text{sup.pr.})aSE_v}{66000z}\left[R_p^{\frac{s-1}{s}}-1\right] \quad . \quad . \quad . \quad . \quad . \quad (7$$

Since it was found in Section 7, that the work per unit volume of gas is t same with clearance as without clearance, the horse-power per cubic foot p minute will also be independent of clearance. (See Eqs. (51), (53) and (56)).

Horse-power per cubic foot of gas supplied per minute

$$\frac{\text{I.H.P.}}{n(\text{L. P. Cap.})}=\frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2}\left[R_p^{\frac{s-1}{s}}-1\right] \quad . \quad . \quad . \quad (7$$

The horse-power per cubic foot of hot gas delivered per minute is

$$\frac{\text{I.H.P.}}{n(\text{H.P. Cap. hot})} = \frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2} R_p^{\frac{1}{s}} \left[R_p^{\frac{s-1}{s}} - 1 \right] \quad . \quad . \quad . \quad (74)$$

orse-power per cubic foot of gas delivered and cooled is

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap .cold})} = \frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2} R_p \left[R_p^{\frac{s-1}{s}} - 1 \right], \quad . \quad . \quad . \quad (75)$$

$$= \frac{s}{s-1}\frac{(\text{del.pr.})}{229.2} \left[R_p^{\frac{s-1}{s}} - 1 \right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

In the above formulæ (del.pr.) and (sup.pr.) indicate delivery pressure and pply pressure, in pounds per square inch.

Example 1. Method of calculating Diagram, Fig. 27.

ssumed data:

$$P_a = P_b = 2116 \text{ lbs. per square foot.}$$

$$P_c = P_d = 18{,}000 \text{ lbs. per square foot.}$$

$$Cl. = 3.5 \text{ per cent.} \quad \text{L. P. Capacity} = 5 \text{ cu.ft.} \quad s = 1.4.$$

o obtain point D:

$$\text{L. P. Cap.} = D\left(1 + c - cR_p^{\frac{1}{s}}\right) \quad \text{or} \quad 5 = D\left(1 + .035 - .035(8.5)\right); \quad .715$$

ence

$$D = 5 \div (1 + .035 - .035 \times 4.6) = 5.72 \text{ cu.ft. and } Cl = .035 \times 5.72 = .2 \text{ cu.ft.}$$

$$\therefore \quad V_d = .2 \text{ cu.ft.}; \quad P_d = 18{,}000 \text{ lbs. sq.ft.}$$

o obtain point A:

$$V_a = \left(\frac{P_d}{P_a}\right)^{\frac{1}{1.4}} V_d$$

$$= 4.6 \times .2 = .92;$$

$$\therefore \quad V_a = .92 \text{ cu.ft.}; \quad P_a = 2116 \text{ lbs. sq.ft.}$$

ıtermediate points D to A are obtained by assuming various pressures and finding the rresponding volumes as for V_a.

o obtain point B:

$$V_b = V_a + \text{L. P. Cap.} = .92 + 5 = 5.92.$$

$$\therefore \quad V_b = 5.92 \text{ cu.ft.}; \quad P_b = 2116 \text{ lbs. sq.ft.}$$

To obtain point C:

$$V_c = V_b \div \left(\frac{P_c}{P_b}\right)^{\frac{1}{1.4}}$$

$$= 5.92 \div 4.6 = 1.29 \text{ cu.ft.}$$

$$\therefore \quad V_c = 1.29 \text{ cu.ft.}; \qquad P_c = 18{,}000 \text{ lbs. sq.ft.}$$

Example 2. It is required to compress 1000 cu.ft. of air per minute from 1 to 8 atmospheres absolute so that $s = 1.4$, in a compressor having 4 per cent clearance. What must be the displacement of the compressor, work per 100 cu.ft. of supplied and delivered air, hot and cold, and horse-power of machine? Speed is 150 R.P.M., compressor double acting and stroke = 1.5 diameters.

$$D = \text{L. P. Cap.} \div E_v, \quad \text{and} \quad E_v = \left(1 + c - cR_p^{\frac{1}{s}}\right).$$

$$\therefore \quad E_v = \left(1 + .04 - .04 \times (8.5)^{.71}\right) = .86;$$

$$\therefore \quad D = 1000 \div .86 = 1162 \text{ cu.ft. per min.}$$

$$\text{Work per cubic foot of supplied air} = 144 \, \frac{s}{s-1} \, (\text{sup.pr})[R_p^{\frac{s-1}{s}} - 1],$$

$$= \quad 144 \times 3.46 \times 14.7 \times .86 = 6300 \text{ ft.-lbs.}$$

$$\therefore \quad \text{Work per 1000 cu.ft.} = 6{,}300{,}000 \text{ ft.-lbs.}$$

Work per cubic foot of delivered air cold is R_p times work per cubic foot of supplied; hence work per 100 cu.ft. of delivered cooled air is 5,350,000 ft.-lbs.

Work per cubic foot of delivered air hot is $R_p^{\frac{1}{s}}$ times work per cubic foot of supplied; hence work per 100 cu.ft. of hot delivered air is 2,800,000 ft.-lbs.

$$\text{I.H.P.} = \frac{(\text{m.e.p.})aS}{66{,}000z}; \quad z = \tfrac{1}{2}; \quad (\text{m.e.p.}) = \frac{s-1}{s} (\text{sup.pr.}) E_v \left[R_p^{\frac{s}{s-1}} - 1,\right]$$

or

$$(\text{m.e.p.}) = 3.46 \times 14.7 \times .86 \times .86 = 37.7 \text{ lbs. per square inch.}$$

$$a = \frac{\pi d^2}{4}; \quad S = 150 \times 2 \times \frac{1.5d}{12}; \quad \frac{aS}{288z} = D = 1162;$$

$$\therefore \quad d^3 = 5690 \quad \text{or} \quad d = 17.85.$$

$$a = 250 \text{ sq.inches.} \qquad S = 670 \text{ ft. per min.} \qquad \therefore \quad \text{I.H.P.} = 191.$$

Prob. 1. A dense-air ice machine requires that 4000 cu.ft. of air at 50 lbs. per square inch absolute be compressed each minute to 150 lbs. per square inch absolute. The compression being such that $s = 1.4$, clearance being 6 per cent, find the work required. What would be the work if clearance were double? Half?

Prob. 2. The compressor for an ammonia machine compresses from one atmosphere to 8 atmospheres absolute. With adiabatic compression and 4 per cent clearance, what will be work per cubic foot of vapor at the low pressure and at the high? Assume vapor to be superheated.

Prob. 3. On a locomotive an air-brake pump compresses air adiabatically from atmosphere to 80 lbs. per square inch gage. It is required to compress 50 cu.ft. of free air per minute and clearance is 5 per cent. What horse-power must be supplied to it?

Prob. 4. In a manufacturing process a tank must be maintained with a vacuum of 29 ins. when barometer reads 30 ins. To do this 100 cu.ft. of carbon dioxide must be removed from it per minute and returned under atmospheric pressure. Compression is adiabatic and clearance 7 per cent. How much power must be supplied to compressor and what should be its displacement?

Prob. 5. Two compressors each 12×18 in., double acting, with 8 per cent clearance, and running at 150 R.P.M. compress in the one case air, in the other carbon disulphide. The compression being adiabatic in each case, what (*a*), is the difference in power required, (*b*), in low-pressure capacities? Take pressures as 2 and 15 atmospheres of 26 inches mercury.

Prob. 6. A compressor is supplied with 40 horse-power. If it draws in air from atmosphere to what pressure can if 500 cu.ft. per minute be compressed, when $s = 1.38$ and clearance 10 per cent?

Prob. 7. For forcing gas through a main, a pressure of 50 lbs. per square inch gage is required. What is the work done per cubic foot of high-pressure gas, if a compressor having 6 per cent clearance is used, and s for the gas is 1.36? What should be its displacement?

Prob. 8. A gas compressor 20×22 ins. has a volumetric efficiency of 90 per cent, supply pressure = 4 lbs. per square inch and delivery 110 lbs. per square inch gage. What are its L. P., H. P., hot and cold capacities, and its I.H.P. if single acting at 70 R.P.M. when $s = 1.35$?

9. Two-Stage Compressor, no Clearance, Perfect Intercooling, Exponential Compression, Best Receiver Pressure, Equality of Stages (Cycle V). Work and Capacity in Terms of Pressures and Volumes. The common assumption in considering the multi-stage compressor is that in passing from one cylinder to the next, the gas is cooled to the temperature it had before entering the compressor, which has already (Section 2), been defined as "perfect intercooling."

This condition may be stated in other words by saying that the product of pressure and volume must be the same for gas entering each cylinder. If then the volume and pressure of gas entering the first stage be determined, fixing the volume entering the second stage will determine the pressure of the gas entering the second stage, or fixing the pressure of the gas entering the second stage will determine the volume that must be taken in.

Using subscripts referring to Fig. 28, for the no clearance case,

$$P_b V_b = P_d V_d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

The net work of the compressor, area $\overline{ABCDEF}$ = area $\overline{ABCH}$ first stage + area $\overline{HDEF}$ second stage. Using the general expression, Eq. (48) for these work areas with appropriate changes in subscripts

$$W = \frac{s}{s-1} P_b V_b \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} - 1 \right]. \quad \text{(first stage)}$$

$$+ \frac{s}{s-1} P_d V_d \left[\left(\frac{P_e}{P_d} \right)^{\frac{s-1}{s}} - 1 \right]. \quad \text{(second stage)}$$

But from the above, and since $P_c = P_d$,

$$W = \frac{s}{s-1} P_b V_b \left[\left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} + \left(\frac{P_e}{P_c} \right)^{\frac{s-1}{s}} - 2 \right], \quad \ldots \quad (78)$$

which is the general expression for work of a two-stage compressor without clearance, perfect intercooling, and may be restated with the usual symbols as follows:

$$W = 144 \frac{s}{s-1} \text{(sup.pr.)(L. P. Cap.)} \left[(R_{p1})^{\frac{s-1}{s}} + (R_{p2})^{\frac{s-1}{s}} - 2 \right], \quad (79)$$

in which (R_{p1}) and $R_{p2})$ are the ratios of delivery to supply pressures for the first stage and for the second stage respectively. From Eq. (79), work per cubic foot of gas supplied is,

$$\frac{W}{\text{(L. P. Cap.)}} = 144 \frac{s}{s-1} \text{(sup.pr.)} \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2 \right] \quad (80)$$

Work per cubic foot of gas discharge and cooled to its original temperature is

$$\frac{W}{\text{(H. P. Cap. cold)}} = 144 \frac{s}{s-1} \text{(sup.pr.)} R_p \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2 \right],$$

$$= 144 \frac{s}{s-1} \text{(del.pr.)} \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2 \right]. \quad \ldots \quad (81)$$

The low-pressure capacity stated in terms of high-pressure capacity hot, as actually discharged is

$$\text{(L. P. Cap.)} = \text{(H. P. Cap. hot)} R_{p2}^{\frac{1}{s}} R_{p1}, \quad \ldots \quad (82)$$

whence

Work per cubic foot hot gas discharged

$$\frac{W}{\text{(H. P. Cap. hot)}} = 144 \frac{s}{s-1} \text{(sup.pr.)} R_{p2}^{\frac{1}{s}} R_{p1} \left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2 \right] \quad (83)$$

Examination of Fig. 28 will show without analysis that there must be some *best-receiver pressure* at which least work will be required. For if the receiver pressure approached P_b then the compression would approach single stage and

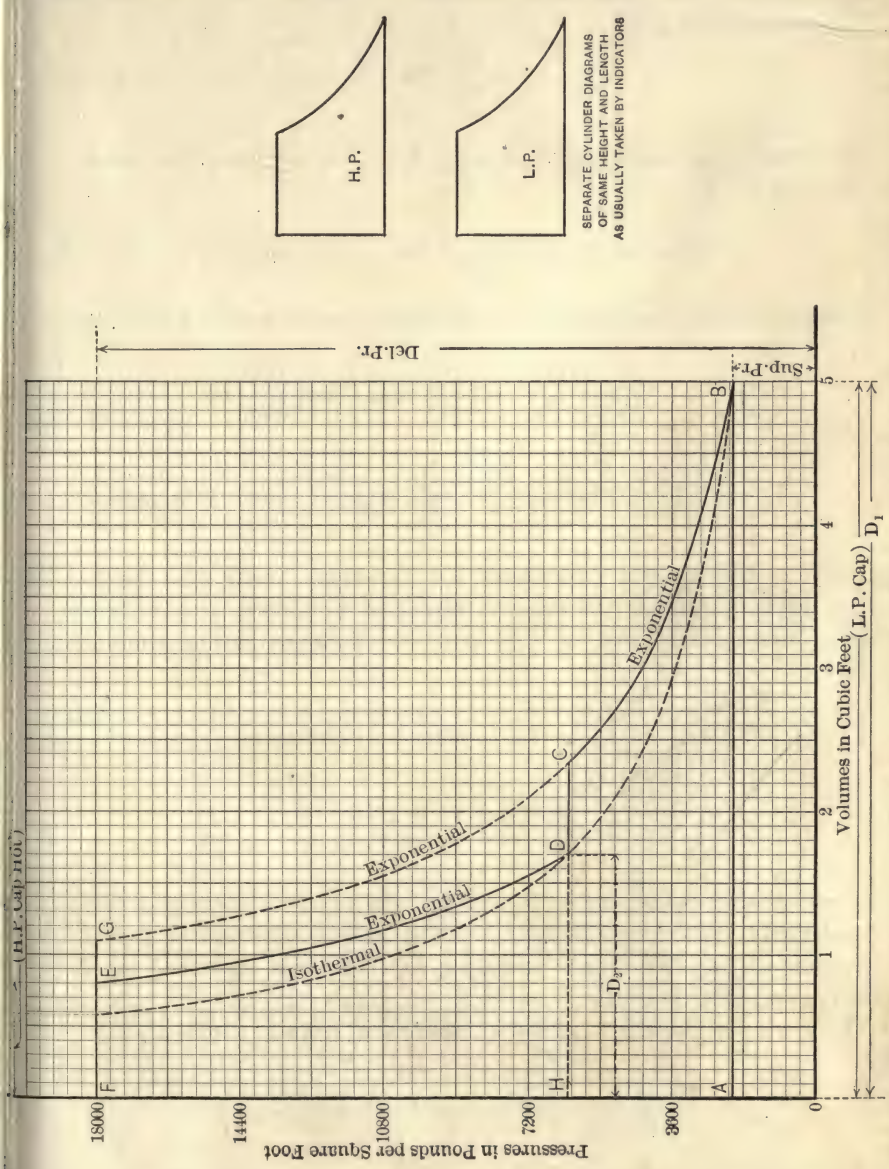


FIG. 28.—Two-Stage Compressor Cycle 5, No Clearance, Exponential. Perfect Intercooling, Best Receiver Pressure.

the compression line approach BCG. The same would be true as the receiver pressure approached $P_g = P_e$, whereas at any intermediate point C, intercooling causes the process to follow $BCDE$ with a saving of work over single-stage operation represented by the area $\overline{DCGE}$. This area being zero when C is at

either B or G, it must have a maximum value somewhere between, and t pressure at which this least-compressor work will be attained is the *best-recei pressure.*

By definition the best-receiver pressure is that for which W is a minimu or that corresponding to

$$\frac{dW}{dP_c}=0.$$

Performing this differentiation upon Eq. (78), equating the result to ze and solving for P_c,

$$(\text{Best rec.pr.})=(P_bP_e)^{\frac{1}{2}}=[(\text{sup. pr.})\ (\text{del. pr})]^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad (8)$$

Substituting this value in the general expression for work Eq. (78), noting th

$$\frac{P_c}{P_b}=\frac{(P_bP_e)^{\frac{1}{2}}}{P_b}=\left(\frac{P_e}{P_b}\right)^{\frac{1}{2}} \text{ and } \frac{P_e}{P_c}=\left(\frac{P_e}{P_b}\right)^{\frac{1}{2}}$$

$$W=2\frac{s}{s-1}P_bV_b\left[\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{2s}}-1\right], \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Eq. (85) is the general expression for two-stage work with perfect int cooling at best-receiver pressure in terms of pressures and volumes. S stituting the symbols for the pressures and volumes and noting that as Cycle 1,

$V_b=$(L. P. Cap.) and $V_e=$(H. P. Cap. hot) and using (R_p) for $\left(\frac{P_e}{P_b}\right)$,

$$W=288\frac{s}{s-1}(\text{sup.pr.})(\text{L. P. Cap.})\left(R_p^{\frac{s-1}{2s}}-1\right). \quad . \quad . \quad . \quad . \quad (9)$$

This equation gives the same value as Eq. (85), but in terms of different un

It should be noted here that the substitution of best-receiver pressure in expressions for the two stages preceding Eq. (78), *will show that the work d in the two cylinders is equal.*

The work per cubic foot of low-pressure gas, from Eq. (86) is,

$$\frac{W}{(\text{L. P. Cap.})}=288\frac{s}{s-1}(\text{sup.pr.})\left[R_p^{\frac{s-1}{2s}}-1\right]. \quad . \quad . \quad . \quad . \quad (9)$$

To transform Eq. (85) into a form involving delivery volumes, use the r tion from the diagram,

$$V_e=V_d\left(\frac{P_d}{P_e}\right)^{\frac{1}{s}}=V_b\left(\frac{P_b}{P_c}\right)\left(\frac{P_d}{P_e}\right)^{\frac{1}{s}}.$$

Whence

$$V_b = V_e\left(\frac{P_c}{P_b}\right)\left(\frac{P_e}{P_d}\right)^{\frac{1}{s}},$$

which for the best-receiver pressure becomes

$$V_b = V_e R_p^{\frac{s+1}{2s}}.$$

Substituting in Eq. (85),

$$W = 2\frac{s}{s-1}P_b V_e (R_p)^{\frac{s+1}{2s}}\left[R_p^{\frac{s-1}{2s}} - 1\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

Introducing the symbols,

$$W = 288\frac{s}{s-1}(\text{sup.pr.})(\text{H. P. Cap. hot})R_p^{\frac{s+1}{2s}}\left[R_p^{\frac{s-1}{2s}} - 1\right], \quad . \quad . \quad (89)$$

and

$$\frac{W}{(\text{H.P.Cap.hot})} = 288\frac{s}{s-1}(\text{sup.pr.})R_p^{\frac{s+1}{2s}}\left[R_p^{\frac{s-1}{2s}} - 1\right] \quad . \quad . \quad . \quad (90)$$

The volume of gas discharged at the higher pressure when reduced to its original temperature will become such that

$$\frac{(\text{L. P. Cap.})}{(\text{H. P. Cap. cold})} = \frac{P_e}{P_b} = R_p,$$

or

$$(\text{sup.pr.})\ (\text{L. P. Cap.}) = (\text{del.pr.})\ (\text{H. P. Cap. cold}), \quad . \quad . \quad . \quad (91)$$

which may be substituted in Eq. (86),

$$W = 288\frac{s}{s-1}(\text{del.pr.})(\text{H.P. Cap. cold})\left]R_p^{\frac{s-1}{2s}} - 1,\right]. \quad . \quad . \quad . \quad . \quad (92)$$

from which the work per cubic foot of gas delivered and cooled is,

$$\frac{W}{(\text{H.P. Cap. cold})} = 288\frac{s}{s-1}(\text{del.pr.})\left[R_p^{\frac{s-1}{2s}} - 1\right] \quad . \quad . \quad . \quad . \quad . \quad (93)$$

Example 1. Method of calculating diagram, Fig. 28.
Assumed data:

$$V_a = 0 \text{ cu.ft.} \quad P_a = 2116 \text{ lbs. per square foot.}$$

$$V_f = 0 \text{ cu.ft.} \quad P_c = P_d = \sqrt{P_a P_e} = 6172 \text{ lbs. sq.ft.}$$

$$V_b = 5 \text{ cu.ft.} \quad P_f = P_e = P_g = 18{,}000 \text{ lbs. sq.ft.}$$

$$V_d = V_b \frac{P_b}{P_d}.$$

$$s = 1.4.$$

To obtain point C:

$$V_c = V_e \div \left(\frac{P_c}{P_b}\right)^{\frac{1}{1.4}} = 2.36 \text{ cu.ft.;}$$

or

$$\therefore \quad V_c = 2.36 \text{ cu.ft.} \quad P_c = 6172 \text{ lbs. sq.ft}$$

To obtain point D:

$$V_d = V_b \times \frac{P_b}{P_c} = 5 \times \frac{2116}{6172} = 1.71 \text{ cu.ft.}$$

$$\therefore \quad V_d = 1.71 \text{ cu.ft.} \quad P_d = 6172 \text{ sq.ft.}$$

To obtain point E:

$$V_e = V_d \div \left(\frac{P_e}{P_d}\right)^{\frac{1}{1.4}},$$

but by definition

$$\left(\frac{P_e}{P_d}\right)^{\frac{1}{1.4}} = \left(\frac{P_d}{P_b}\right)^{\frac{1}{1.4}} = 2.14,$$

hence,

$$V_e = 1.71 \div 2.14 = .8 \text{ cu.ft.} \quad P_e = 18000 \text{ lbs. sq.ft.}$$

Example 2. To compress 5 cu.ft. of air from one atmosphere (2116 lbs. per square foot) to 8.5 atmospheres (18,050 lbs. per square foot) in two stages with best-receiver pressure and perfect intercooling requires how much work?

$$W = 288 \frac{s}{s-1} (\text{sup.pr.})(\text{L. P. Cap.})(R_p^{\frac{s-1}{2s}} - 1),$$

$$(\text{sup.pr.}) = 14.7. \qquad (\text{L. P. Cap.}) = 5. \qquad R_p = 8.5.$$

$$\therefore \quad W = 288 \times 3.463 \times 14.7 \times 5 \times \left(8.5^{\frac{s-1}{2s}} - 1\right) = 26{,}800 \text{ ft.-lbs.}$$

Prob. 1. Air at 14 lbs. per square inch absolute is compressed to 150 lbs. per square inch absolute by a two-stage compressor. What will be the work per cubic foot of air delivered? What will be the work per cubic foot if the air be allowed to cool to the original temperature, and how will this compare with the work per cubic foot of supplied air? Best receiver-pressure and perfect intercooling are assumed for the above compressor, $s = 1.4$.

Prob. 2. A compressor receives air at atmosphere and compresses it to half its volume, whereupon the air is discharged to the cooler and its temperature reduced to the original point. It then enters a second cylinder and is compressed to 80 lbs. absolute. What will be the work per cubic foot of supplied air in each cylinder and how will the work of compressing a cubic foot to the delivery pressure compare with the work done if compression were single stage, compression being adiabatic.

Prob. 3. Air is to be compressed from 15 lbs. per square inch absolute to 10 times this pressure. What would be the best-receiver pressure for a two-stage compressor? How many more cubic feet may be compressed per minute in two stage than one stage by the same horse-power?

Prob. 4. A manufacturer sells a compressor to run at best-receiver pressure when (sup.pr.) is 14 lbs. per square inch absolute and (del.pr.) 100 lbs. per square inch absolute. What will be the work per cubic foot of supply-pressure air done in each cylinder? Another compressor is so designed that the receiver pressure for same supply pressure and delivery pressure is 30 lbs. per square inch absolute, while a third is so designed that receiver pressure is 50 lbs. per square inch absolute. How will the work done in each cylinder of these machines compare with that of first machine?

Prob. 5. For an ice machine a compressor works between 50 and 150 lbs. per square inch absolute. It is single stage. Would the saving by making compression two stage at best-receiver pressure amount to a small or large per cent of the work in case of single stage, how much?

Prob. 6. A compressor has been designed to compress 1000 cu.ft. of carbon dioxide per minute from 15 to 150 lbs. per square inch absolute. What horse-power will be required at best-receiver pressure? Should delivery pressure change to 200 lbs., what power would be required? To 100 lbs. what power?

Prob. 7. A gas-compressing company operates a compressor which has to draw CO_2 gas from a spring and compress it to 150 lbs. per square inch gage. In the morning pressure on the spring is 10 lbs. gage, while by evening it has dropped to 5 lbs. absolute. If the compressor was designed for the first condition, how will the high-pressure capacity cold and horse-power per cubic foot of high-pressure gas at night compare with corresponding values in morning? Assume a barometric reading.

Prob. 8. On a mining operation a compressor is supplying a number of drills and hoists with air at 150 lbs. per square inch absolute, the supply pressure being 14 lbs. What will be the difference in horse-power per cubic foot of delivered air at compressor and per cubic foot received at drills if air is a long time in reaching drills?

Prob. 9. With a best-receiver pressure of 40 lbs. per square inch absolute and a supply pressure of 14 lbs. per square inch absolute, what horse-power will be required to compress and deliver 1000 cu.ft. of high-pressure air per minute at the delivery pressure for which compressor is designed and what is that delivery pressure?

10. Two-stage Compressor, with Clearance, Perfect Intercooling Exponential Compression, Best-receiver Pressure, Equality of Stages, (Cycle 6). Work and Capacity in Terms of Pressures and Volumes. The two-stage expo-

nential compressor with clearance and perfect intercooling is represented by the PV diagrams Figs. 29, 30, 31, which are clearly made up of two single-stage compression processes, each with clearance.

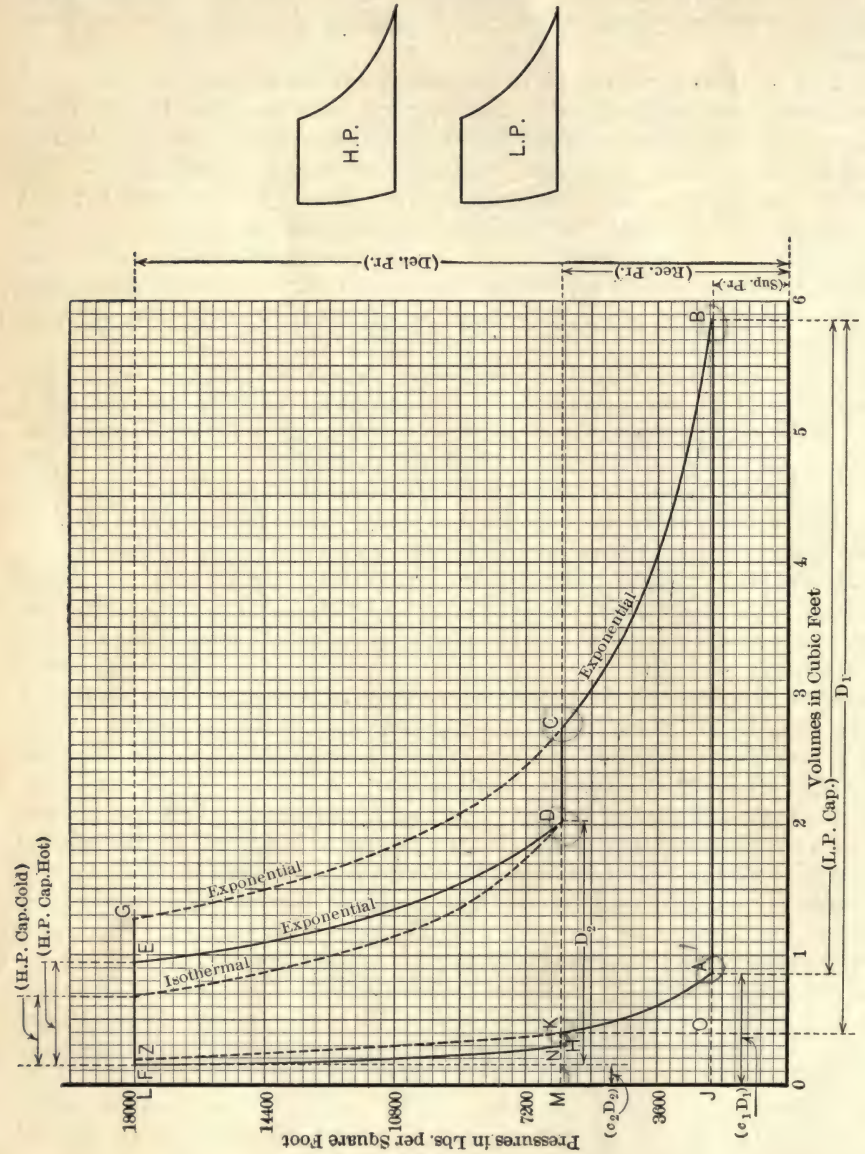


FIG. 29.—Two-stage Compressor, Cycle 6, Clearance, Exponential. Perfect Intercooling, Best-receiver Pressure. (H.P. Re-expansion Volume Less than L.P. Clearance.)

Applying Eq. (57) to the two stages and supplying proper subscripts, referring to Fig. 29,

$$W=\frac{s}{s-1}P_b(V_b-V_a)\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}}-1\right]. \quad . \quad \text{(first stage)}$$

$$+\frac{s}{s-1}P_d(V_d-V)_h\left[\left(\frac{P_e}{P_d}\right)^{\frac{s-1}{s}}-1\right]. \quad . \quad \text{(second stage)}$$

If the condition of perfect intercooling be imposed, it is plain that since h weight of gas entering the second stage must equal that entering the first t;e, and the temperature in each case is the same,

$$(V_d-V_h)P_d=(V_b-V_a)P_b,$$

noting also that

$$P_c=P_d,$$

$$W=\frac{s}{s-1}P_b(V_b-V_a)\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}}+\left(\frac{P_e}{P_c}\right)^{\frac{s-1}{s}}-2\right], \quad . \quad . \quad . \quad (94)$$

(94) is the general expression for the work of two-stage exponential compressor with perfect intercooling, P_c being the receiver pressure.

As in Section 9, let (R_{p1}) be the pressure ratio $\frac{P_c}{P_b}$ for the first stage and (R_{p2}) h pressure ratio $\frac{P_e}{P_c}$ for the second stage and using instead of P_b its equivalent d (sup.pr.) lbs. per square inch.

$$W=144\frac{s}{s-1}(\text{sup.pr.})(\text{L. P. Cap.})\left[(R_{p1})^{\frac{s-1}{s}}+(R_{p2},^{\frac{s-1}{s}}-2,\right] \quad . \quad . \quad (95)$$

ch is identical with (79), showing *that for two-stage compressors with perfect rcooling (as for single stage, Section 7), the work for a given low-pressure acity is independent of clearance.*

The work per cubic foot of gas supplied is given by Eq. (80); per cubic of cold gas delivered by Eq. (81) and per cubic foot hot gas delivered by (83).

The reasoning regarding best-receiver pressure followed out in Section 9, will l again in this case, and by putting $\frac{dW}{dP_c}=0$ in Eq. (94), and solving for P_c ill again be found that best-receiver pressure will be

$$(\text{best-rec. pr.})=(P_bP_e)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

Substitution of this value for P_c in Eq. (94), gives the following expression vork of the two-stage exponential compressor with best-receiver pressure,

$$W=2\frac{s}{s-1}P_b(V_b-V_a)\left[\left(\frac{P_e}{P_b}\right)^{\frac{s-1}{2s}}-1\right], \quad . \quad . \quad . \quad . \quad . \quad (97)$$

which may be expressed in terms of supply pressure, pounds per square in low-pressure capacity, cubic feet, and ratio of compression,

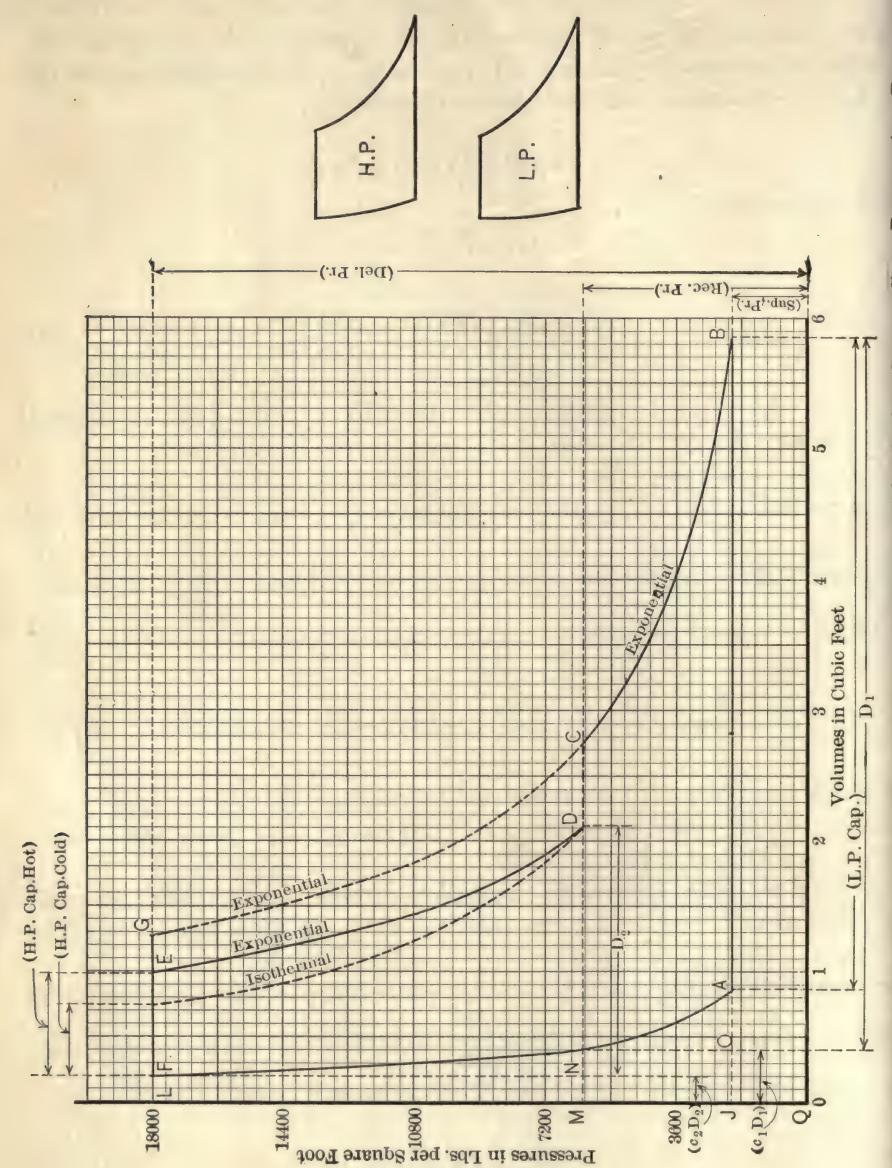


FIG. 30.—Two-stage Compressor, Cycle 6, Clearance, Exponential, Perfect Intercooling, Best-receiver Pressure.

$$W = 288\frac{s}{s-1}(\text{sup.pr.})(\text{L. P. Cap.})\left[R_p^{\frac{s-1}{2s}} - 1\right], \quad . \quad . \quad . \quad (\text{?})$$

which is the same as Eq. (86).

Substitution of the value of best-receiver pressure in the expression for

ork of the two stages separately will show the equality of work done in the spective stages for this case with clearance.

ork per cubic foot gas supplied to compressor is

$$\frac{W}{(\text{L. P. Cap.})} = 288\frac{s}{s-1}(\text{sup.pr.})\left[R_p^{\frac{s-1}{2s}} - 1\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

ork per cubic foot of high-pressure gas hot is

$$\frac{W}{(\text{H. P. Cap. hot})} = 288\frac{s}{s-1}(\text{sup.pr.})R_p^{\frac{s+1}{2s}}\left[R_p^{\frac{s-1}{2s}} - 1\right]. \quad . \quad . \quad (100)$$

The work per cubic foot of air delivered and cooled to its original temerature is,

$$\frac{W}{(\text{H. P. Cap. cold})} = 288\frac{s}{s-1}(\text{del.pr.})\left[R_p^{\frac{s-1}{2s}} - 1\right], \quad . \quad . \quad (101)$$

Due to the fact that clearance has no effect upon the work per cubic foot substance, as previously noted, Eqs. (99), (100) and (101) are identical with 37), (90) and (93).

11. Two-Stage Compressor, any Receiver Pressure, Exponential Compression. Capacity, Volumetric Efficiency, Work, Mean Effective Pressure and Horse-power, in Terms of Dimensions of Cylinders and Clearances. Referring to Fig. 29, let D_1 be the displacement of the first stage cylinder in cubic feet $= V_b - V_k$), D_2 the displacement of the cylinder of the second stage in cubic eet $= (V_a - V_f)$, c_1 the clearance of the first stage, stated as a fraction of the isplacement of that cylinder, so that the clearance of the first stage cubic eet $= c_1 D_1$, and that of the second stage $= c_2 D_2$.

The low-pressure capacity of the first stage (L. P. Cap.) in cubic feet is $V_b - V_a$), and, as for the single-stage compressor, is expressed in terms of displacement, clearance and ratio of compression of the first stage as follows, see Eq. (64):

$$(\text{L. P. Cap.}_1) = D_1\left(1 + c_1 - c_1 R_{p1}^{\frac{1}{s}}\right) = D_1 E_{v1}. \quad . \quad . \quad . \quad . \quad . \quad (102)$$

For the second stage, the low-pressure capacity (L. P. Cap.$_2$) is

$$(V_d - V_h),$$

nd is equal to

$$(\text{L. P. Cap.}_2) = D_2\left(1 + c_2 - c_2 R_{p2}^{\frac{1}{s}}\right) = D_2 E_{v2} \quad . \quad . \quad . \quad . \quad (103)$$

Volumetric efficiency of the first stage is given by

$$E_{v1} = 1 + c_1 - c_1 R_{p1}^{\frac{1}{s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

metric efficiency for second stage

$$E_{v2} = 1 + c_2 - c_2 R_{p2}^{\frac{1}{s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (105$$

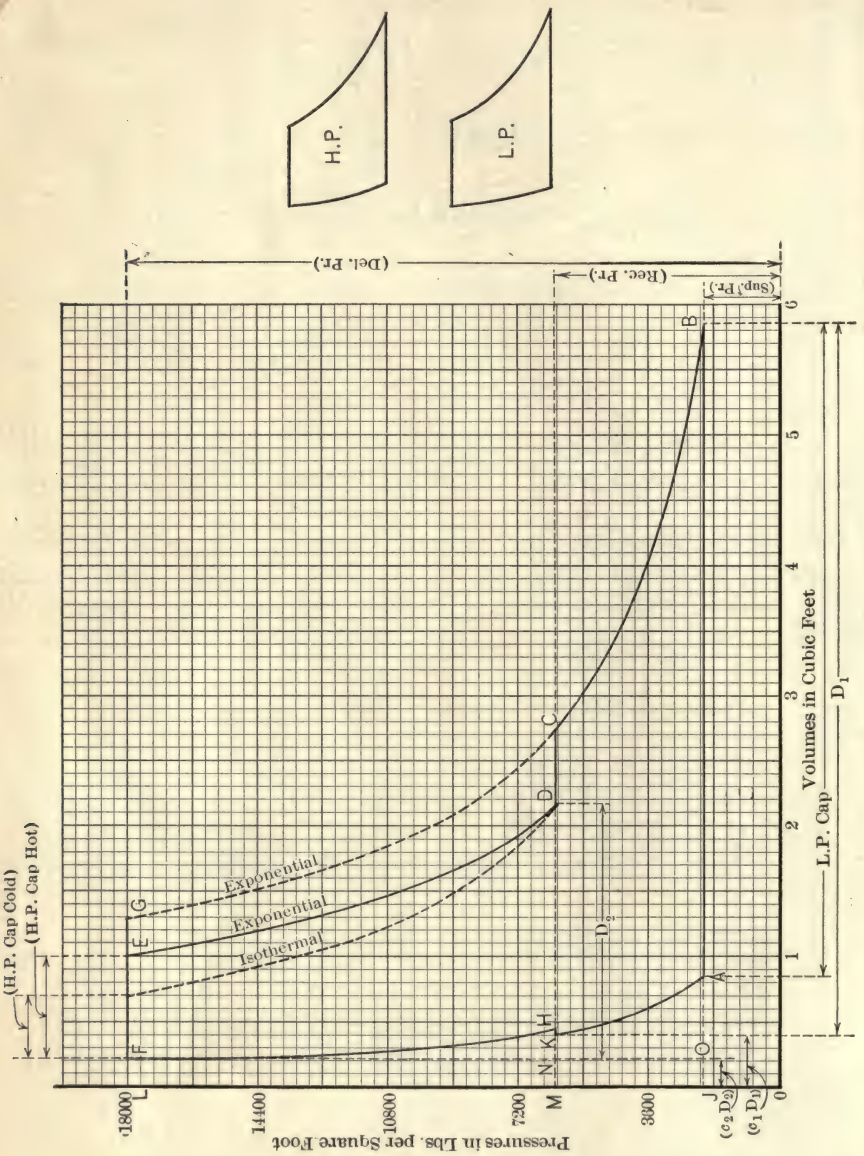


FIG. 31.—Two-stage Compressor, Cycle 6, Clearance, Exponential. Perfect Intercooling, Best-receiver Pressure.

It may be required *to find the receiver pressure* (incidental to the finding o work or horse-power) for a compressor with given cylinder sizes and deliver pressure. The condition assumed of perfect intercooling stipulates that

$$(\text{L. P. Cap.}_1)(\text{sup.pr.}) = (\text{L. P. Cap.}_2)\ (\text{rec.pr.}),$$

hence

$$(\text{rec.pr.}) = (\text{sup.pr.})\frac{(\text{L. P. Cap.}_1)}{(\text{L. P. Cap.}_2)}$$

$$= (\text{sup.pr.})\frac{D_1 E_{v1}}{D_2 E_{v2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

If the volumetric efficiencies are known or can be sufficiently well approximated this can be solved directly. If, however, E_{v1} and E_{v2} are not known, but the clearances are known, since these are both dependent upon the receiver pressure sought, the substitution of the values of these two quantities will give

$$(\text{rec.pr.}) = (\text{sup.pr.})\frac{D_1\left[1+c_1-c_1\left(\frac{\text{rec.pr.}}{\text{sup.pr.}}\right)^{\frac{1}{s}}\right]}{D_2\left[1+c_2-c_2\left(\frac{\text{del. pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}}\right]}, \quad . \quad . \quad . \quad (107)$$

an expression which contains the receiver pressure on both sides of the equation. This can be rearranged with respect to (rec. pr.), but results in a very complex expression which is difficult to solve and not of sufficient value ordinarily to warrant the expenditure of much labor in the solution. Therefore, the relations are left in the form (107). It may be solved by a series of approximations, the first of which is

$$(\text{rec.pr.}) = (\text{sup.pr.})\frac{D_1}{D_2} \text{ approx.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

With this value for the receiver pressure, substitution may be made in the second member of the Eq. (107), giving a result which will be very nearly correct. If desirable, a third approximation could be made.

To find the work of a two-stage exponential compressor in terms of displacement of cylinders, supply pressure, receiver pressure and delivery pressure, pounds per square inch, and volumetric efficiency of the first stage, E_{v1}, from (79) or (94),

$$W = 144\frac{s}{s-1}(\text{sup.pr.})D_1 E_{v1}\left[R_{p1}^{\frac{s-1}{s}} + R_{p2}^{\frac{s-1}{s}} - 2\right] \quad . \quad . \quad . \quad (109)$$

in which

$$R_{p1} = \frac{(\text{rec.pr.})}{(\text{sup.pr.})} \quad \text{and} \quad R_{p2} = \frac{(\text{del.pr.})}{(\text{rec.pr.})}.$$

To solve this the receiver pressure must be found as previously explained and the volumetric efficiency must be computed by Eq. (104) or otherwise be known.

It is impracticable to state work for this general case in terms of displacement and clearances directly, due to the difficulty of solving for (rec. pr.) in Eq.

(107). It may, however, be stated purely in terms of supply and delivery pressures, in pounds per square inch, displacement, in cubic feet, and volumetric efficiencies, as follows:

From Eq. (106),

$$R_{p1}=\frac{D_1E_{v1}}{D_2E_{v2}},$$

and

$$R_{p2}=\frac{\text{del.pr.}}{\text{sup.pr.}}\times\frac{D_2E_{v2}}{D_1E_{v1}}=R_p\frac{D_2E_{v2}}{D_1E_{v1}},$$

Hence

$$W=144\frac{s}{s-1}(\text{sup.pr.})D_1E_{v1}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_2E_{v2}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-2\right] \quad . \quad (110$$

The mean effective pressure of the two-stage compressor referred to the low pressure cylinder is found by dividing the work of the entire cycle Eq. (110), by the displacement of the first-stage cylinder, and by 144, to give pounds per square inch.

m.e.p. referred to first-stage cylinder, pounds per square inch is,

$$\frac{W}{144D_1}=\frac{s}{s-1}(\text{sup.pr.})E_{v1}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{1}}+\left(R_p\frac{D_2E_{v2}}{D_1E_{v2}}\right)^{\frac{s-1}{s}}-2\right]. \quad . \quad (111$$

It is well to note that this may also be found by multiplying (work done per cubic foot of gas supplied) by (volumetric efficiency of the first stage, E_{v1}) and dividing the product by 144.

In terms of the same quantities, an expression for indicated horse-power may be given as follows:

$$\text{I.H.P.}=\frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2}nD_1E_{v1}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_2E_{v2}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-2\right], \quad (112$$

where n is the number of cycles completed per minute by the compressor. For n may be substituted the number of revolutions per minute, divided by the revolutions per cycle,

$$n=\frac{N}{z},$$

The horse-power per cubic foot of gas supplied per minute is

$$\frac{\text{I.H.P}}{n(\text{L. P. Cap.})}=\frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_2E_{v2}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-2\right]. \quad . \quad (113$$

orse-power per cubic foot of gas delivered and cooled per minute.

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. cold})} = \frac{s}{s-1}\,\frac{(\text{del.pr.})}{229.2}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}} + \left(R_p\frac{D_2E_{v2}}{D_1E_{v1}}\right)^{\frac{s-1}{s}} - 2\right] \quad (114)$$

orse-power per cubic foot of hot gas delivered per minute

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. hot})} = \frac{s}{s-1}\,\frac{\text{sup.pr.}}{229.2}\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}} R_p^{\frac{1}{s}}$$

$$\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}} + \left(R_p\frac{D_2E_{v2}}{D_1E_{v1}}\right)^{\frac{s-1}{s}} - 2\right]. \quad . \quad (115)$$

or the case where clearance is zero or negligible, these expressions may be mplified by putting E_{v2} and E_{v1} equal to unity.

$$\text{I.H.P.} = \frac{s}{s-1}\,\frac{(\text{sup.pr.})}{229.2}nD_1\left[\left(\frac{D_1}{D_2}\right)^{\frac{s-1}{s}} + \left(R_p\frac{D_2}{D_1}\right)^{\frac{s-1}{s}} - 2\right]. \quad . \quad . \quad (116)$$

H.P. per cubic foot. gas supplied per minute

$$\frac{\text{I.H.P.}}{n(\text{L. P. Cap.})} = \frac{s}{s-1}\,\frac{(\text{sup.pr.})}{229.2}\left[\left(\frac{D_1}{D_2}\right)^{\frac{s-1}{s}} + \left(R_p\frac{D_2}{D_1}\right)^{\frac{s-1}{s}} - 2\right]. \quad . \quad (117)$$

H.P. per cubic foot gas delivered and cooled per minute

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. cold})} = \frac{s}{s-1}\,\frac{(\text{del.pr.})}{229.2}\left[\left(\frac{D_1}{D_2}\right)^{\frac{s-1}{s}} + \left(R_p\frac{D_2}{D_1}\right)^{\frac{s-1}{s}} - 2\right]. \quad (118)$$

H.P. per cubic foot hot gas delivered per minute

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. hot})} = \frac{s}{s-1}\,\frac{\text{sup.pr.}}{229.2}\left(\frac{D_1}{D_2}\right)^{\frac{s-1}{s}} R_p^{\frac{1}{s}}\left[\left(\frac{D_1}{D_2}\right)^{\frac{s-1}{s}} + \left(R_p\frac{D_2}{D_1}\right)^{\frac{s-1}{s}} - 2\right] \quad (119)$$

Example 1. Method of calculating diagram, Figs. 29, 30, 31.
ssumed data.

$P_a = P_b = 2116$ lbs. per square foot;

$P_c = P_d = P_h = P_k = 6172$ lbs. per square foot.

$P_g = P_e = P_f = 18{,}000$ lbs. per square foot.

l(H. P.) = 7.5 per cent; Cl(L. P.) 7.5 per cent; $s = 1.4$; L. P. Capacity = 5 cu.ft.

To obtain point K.

From formula Eq. (64),

$$\text{L. P. Cap.} = D_1(1 + c_1 - c_1 R_{p1s}^{\frac{1}{s}})$$

$5 = D_1(1 + .075 - .075 \times 2.14)$, hence $D_1 = 5.45$ cu.ft.

$$Cl_1 = V_k = 5.45 \times .075 = .41 \text{ cu.ft.}$$

$$\therefore \quad V_k = .4 \text{ cu.ft.}; \qquad P_k = 6172 \text{ lbs. sq.ft.}$$

To obtain point A:

$$V_a = V_k \left(\frac{P_k}{P_a}\right)^{\frac{1}{1.4}} = .4 \times 2.14 = .856 \text{ cu.ft.}$$

$$\therefore \quad V_a = .85 \text{ cu.ft.}; \qquad P_a = 2116 \text{ lbs. sq.ft.}$$

To obtain point B:

$$V_b = V_a + 5 = .85 + 5 = 5.85 \text{ cu.ft.}; \qquad P_b = 2116. \text{ lbs. sq.ft.}$$

To obtain point C:

$$V_c = V_b \div \left(\frac{P_c}{P_b}\right)^{\frac{1}{1.4}} = 5.85 \div 2.14 = 2.73,$$

$$\therefore \quad V_c = 2.73 \text{ cu.ft.}; \qquad P_c = 6172 \text{ lbs. sq.ft.}$$

To obtain point D:

Volume at D is the displacement plus clearance of H. P. cylinder. This cannot b found until the capacity is known. The capacity is the amount gas which must b taken in each stroke and which is also the amount actually delivered by L. P. cylind cooled to original temperature. The amount of cool gas taken in by the second cylind is

$$\text{(L. P. Cap.}_2) = \left(\frac{\text{sup.pr.}}{\text{rec.pr.}}\right)\text{(L. P. Cap.}_1) = \frac{2116}{6172} \times 5 = 1.7 \text{ cu.ft.}$$

But

$$D_2 = \frac{\text{(L. P. Cap.}_2)}{\left(1 + c_2 - c_2 R_{p2}^{\frac{1}{s}}\right)} = \frac{1.7}{1 + .075 - .075 \times 2.14} = 1.88 \text{ cu.ft.}$$

$$Cl_2 = V_f = c_2 D_2 = .075 \times 1.88 = .14 \text{ cu.ft.}$$

$$V_d = Cl_2 + D_2 = 1.88 + .14 = 2.02 \text{ cu.ft.}$$

$$P_d = 6172 \text{ lbs. sq.ft.}$$

Other points are easily determined by relations too obvious to warrant setting dow

Example 2. What will be the capacity, volumetric efficiency and horse-power p 1000 cu.ft. of free air and per 1000 cu.ft. of hot compressed air per minute for the follo ing compressor: Two-stage, double-acting cylinders, $22\frac{1}{4}$ and $34\frac{1}{4} \times 24$ in., runnir at 100 R.P.M. High-pressure clearance 6 per cent, low-pressure 4 per cent. Supp

essure 14 lbs. per square inch absolute. Delivery pressure 115 lbs. per square inch solute.

The capacity will be the cylinder displacement times the volumetric efficiency.

D_1 = displacement of a $34\frac{1}{4}\times24$ in. cylinder, or 12.8 cu.ft. and D_2 = displacement of $22\frac{1}{4}+24''$ cylinder or 5.4 cu.ft. To obtain the volumetric efficiency, make use of pproximation of formula Eq. (108),

$$(\text{rec.pr.}) = (\text{sup.pr.})\frac{D_1}{D_2} = 14\times\frac{12.8}{5.4} = 33.2 \text{ lbs. sq.in,}$$

nd then by Eq. (107) checking,

$$(\text{rec.pr.}) = 14\times\frac{12.8\left[1+.04-.04\left(\frac{33.2}{14}\right)^{.714}\right]}{5.4\left[1+.06-.06\left(\frac{115}{33.2}\right)^{.714}\right]} = 35.1 \text{ lbs. sq.in.}$$

$$E_{v1} = 1+c_1-c_1(R_{p1})^{\frac{1}{s}} \quad \text{from Eq. (104),}$$

$$= 1+.04-.04\times(2.5)^{\frac{1}{s}} = 96.8 \text{ per cent}$$

herefore the capacity will be,

$$200\times12.8\times.968 = 2480 \text{ cu.ft. per minute;}$$

$$E_{v2} = 1+c_2-c_2(R_{p2})^{\frac{1}{s}} \quad \text{from Eq. (105),}$$

$$= 1+.06-.06\times(3.28)^{.714} = 92 \text{ per cent.}$$

rom Eq. (113), I H.P. per cu.ft. (sup.pr.) air per minute is,

$$= \frac{s}{s-1}\,\frac{\text{sup.pr.}}{229.2}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}} + \left(R_p\frac{D_2E_{v2}}{D_1E_{1v}}\right)^{\frac{s-1}{s}} - 2\right],$$

$$= \frac{1.4}{.4}\times\frac{14}{229.2}\left[\left(\frac{12.8\times.968}{5.4\times.92}\right)^{.286} + \left(8.22\frac{5.4\times.92}{12.8\times.968}\right)^{.286} - 2\right] = .15.$$

Whence horse-power per 1000 cu.ft. of free air per minute is, = 150.

From Eq. (115) horse-power per cubic foot (del.pr.) air, hot = that of (sup.pr.) ir $\times R_p^{\frac{1}{s}}\left(\frac{D_1E_{v1}}{D_2E_2}\right)^{\frac{s-1}{1}}$ or 5.85 times that of (sup.pr.) air.

$\therefore$ Horse-power per 1000 cu.ft. of hot (del.pr. air) = $150\times5.85 = 877$.

Prob. 1. A two-stage double-acting compressor has volumetric efficiencies as shown y cards of 98 per cent and 90 per cent for the high- and low-pressure cylinders respect- vely. It is running at 80 R.P.M. and compressing from atmosphere to 80 lbs. per square nch gage. If the cylinders are $15\frac{1}{4}\times25\frac{1}{4}\times18$ ins., and speed is 120 R.P.M., what

horse-power is being used and how many cubic feet of free and compressed air (hot and cold) are being delivered per minute, when s equals 1.41?

Prob. 2. What horse-power will be needed to drive a two-stage compressor $10\frac{1}{4}$ ins. and $16\frac{1}{4}\times12$ ins., double acting, with 5 per cent clearance in each cylinder at 160 R.P.M. when the supply pressure is atmosphere, delivery pressure 100 lbs. per square inch gage, when s equals 1.35?

Prob. 3. A thousand cubic feet of free air per minute must be compressed by a two-stage compressor to 80 lbs. per square inch gage from a supply pressure of 10 lbs. per square inch absolute. The volumetric efficiencies for the high- and low-pressure cylinders are 85 per cent and 95 per cent respectively, and the receiver pressure is 25 lbs. per square inch absolute. What will be the displacement of each cylinder and the horse-power per cubic foot of (sup.pr.) air?

Prob. 4. How many cubic feet of free air can be compressed in two-stage compressor $18\frac{1}{4}\times30\frac{1}{4}\times24$ ins. with 5 per cent clearance in high-pressure cylinder and 3 per cent in low if (sup.pr.) is atmosphere and (del.pr.) 80 lbs. per square inch gage? How would the answer be affected if clearance were taken as zero? Take $s=1.41$.

Prob. 5. The volumetric efficiency of the low-pressure cylinder is known to be 95 per cent, and of the high-pressure cylinder 85 per cent. The cylinder sizes are $15\frac{1}{4}\times25\frac{1}{4}\times1$ ins. and speed is 120 R.P.M. What horse-power must be supplied to the machine if the mechanical efficiency is 80 per cent and the pressure ratio 10 with a (sup.pr.) of one atmosphere?

Prob. 6. A compressor runs at 120 R.P.M. and is double acting. It is compressing air from 14 lbs. per square inch absolute to pressures ranging from 70 lbs. per square inch gage to 100 lbs. per square inch gage. The cylinders are $20\frac{1}{4}\times32\frac{1}{4}\times24$ ins., and clearances 8 per cent and 4 per cent. Find the approximate receiver pressure, capacity and horse-power for the range of discharge pressure, for $s=1.3$.

Prob. 7. The volumetric efficiency of the low-pressure cylinder of a two-stage compressor is known to be 95 per cent, the receiver pressure as shown by gage is 40 lbs., delivery pressure 100 lbs., and supply pressure one atmosphere. What will be the horse-power if the machine runs at 120 R.P.M. and the low-pressure cylinder is 18×12 in. $s=1.4$.

Prob. 8. An air compressor appears to require more power to run it than should be necessary. It is a double-acting $18\times30\times24$ in. machine running at 100 R.P.M. The volumetric efficiencies are 85 per cent and 90 per cent respectively and supply and delivery pressures 14 lbs. and 110 lbs. per square inch, both absolute. What would be the minimum work per cubic foot of (sup.pr.) air, per cubic foot of (del.pr.) air, hot and cold, for adiabatic compression?

Prob. 9. The efficiency of the driving gear on an electric-driven compressor is 75 per cent. Power is being supplied at the rate of 150 H.P. How much air should be compressed per minute from 4 lbs. per square inch absolute to 100 lbs. per square inch gage, if the receiver pressure is 35 lbs. per square inch absolute and the low-pressure volumetric efficiency is 90 per cent, s being 1.4?

12. Two-stage Compressor with Best Receiver Pressure Exponential Compression. Capacity, Volumetric Efficiency, Work, Mean Effective Pressures and Horse-power in Terms of Dimensions of Cylinders and Clearances. For the two-stage exponential compressor with or without clearance, and perfect intercooling, the best-receiver pressure was found to be (Eq. 84),

$$(\text{best-rec.pr.})=[(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (120$$

This expression Eq. (120) for best-receiver pressure makes it possible to aluate R_{p1} and R_{p2} as follows:

$$_{p1}\text{for(best-rec.pr.)} = \frac{\text{best-rec.pr.}}{\text{sup.pr.}} = \frac{[(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{2}}}{\text{sup.pr.}} = \left[\frac{\text{del.pr.}}{\text{sup.pr.}}\right]^{\frac{1}{2}} = R_p^{\frac{1}{2}}, \quad (121)$$

nd

$$_{p2}\text{ for (best-rec. pr.)} = \frac{(\text{del.pr.})}{(\text{best-rec.pr.})} = \frac{(\text{del.pr.})}{[(\text{sup.pr.})(\text{del.pr.})]^{\frac{1}{2}}}$$

$$= \left[\left(\frac{\text{del. pr.}}{\text{sup.pr.}}\right)\right]^{\frac{1}{2}} = R_p^{\frac{1}{2}}, \quad . \quad (122)$$

The use of these values for R_{p1} and R_{p2} in the expressions previously given r volumetric efficiency for the general case, Eqs. (104) and (105) results in

olumetric efficiency, first stage

$$E_{v1} = (1 + c_1 - c_1 R_p^{\frac{1}{2s}}), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (123)$$

nd volumetric efficiency, second stage

$$E_{v2} = (1 + c_2 - c_2 R_p^{\frac{1}{2s}}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (124)$$

The work was found to be represented by Eq. (98), which may be stated n terms of displacement and volumetric efficiency of the first stage, as follows:

$$W = 288\frac{s}{s-1}(\text{sup.pr.})D_1E_{v1}\left[R_p^{\frac{s-1}{2s}} - 1\right], \quad . \quad . \quad . \quad . \quad (125)$$

here $R_p = \dfrac{(\text{del.pr.})}{(\text{sup.pr.})}$ and where (sup.pr.) is in pounds per square inch.

If the clearance is known for the first stage this becomes by the use of Eq. (104),

$$W = 88\,\frac{s}{s-1}(\text{sup.pr.})D_1(1 + c_1 - c_1 R_p^{\frac{1}{2s}})\left[R_p^{\frac{s-1}{2s}} - 1\right], \quad . \quad . \quad . \quad . \quad (126)$$

which is a direct statement of the work of a two-stage adiabatic compressor vith perfect intercooling in terms of supply pressure and delivery pressure, ounds per square inch, displacement, cubic feet and clearance as a fraction

of displacement, *provided the cylinder sizes and clearances are known to be such as to give best-receiver pressures.*

The *mean effective pressure* reduced to first-stage displacement, in pounds per square inch, may be derived from either Eq. (125) or (126) by dividing the work by the displacement of the first-stage cylinder, and again dividing by 144

$$\left.\begin{aligned}\text{m.e.p.} &= \frac{W}{144D_1} = \frac{2s}{s-1}\,(\text{sup.pr.})E_{v1}\left[R_p^{\frac{s-1}{2s}} - 1\right] \\ &= \frac{2s}{s-1}\,(\text{sup.pr.})\left(1 + c_1 - c_1 R_p^{\frac{1}{2s}}\right)\left[R_p^{\frac{s-1}{2s}} - 1\right]\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (127)$$

Since the work done is equally divided between the two cylinders when best receiver pressure is maintained, the mean effective pressure, in pounds per square foot, for each cylinder will be, one-half the total work divided by the displacement of the cylinder in question,

$$\text{m.e.p., first stage} = \frac{W}{288D_1} = \frac{s}{s-1}(\text{sup.pr.})E_{v1}\left[R_p^{\frac{s-1}{2s}} - 1\right]. \quad . \quad . \quad . \quad . \quad . \quad (128)$$

Note that this is one-half as great as the m.e.p. of the compressor reduced to first stage, (127),

$$\text{m.e.p., second stage} = \frac{W}{288D_2} = \frac{s}{s-1}(\text{sup.pr.})\frac{D_1}{D_2}E_{v1}\left[R_p^{\frac{s-1}{2s}} - 1\right], \quad . \quad . \quad . \quad (129)$$

But

$$(\text{sup.pr.})\frac{D_1 E_{v1}}{D_2 E_{v2}} = (\text{rec.pr.}) = \left[(\text{sup.pr.})(\text{del.pr.})\right]^{\frac{1}{2}},$$

whence,

$$\text{m.e.p., second stage} = \frac{s-1}{s}\left[(\text{sup.pr.})(\text{del.pr.})\right]^{\frac{1}{2}}E_{v2}\left[R_p^{\frac{s-1}{2s}} - \right]1. \quad . \quad . \quad . \quad (130)$$

It is next necessary to investigate what conditions must be fulfilled to obtain the best-receiver pressure, the value of which is stated, Eq. (120). The condition of perfect intercooling provides that the temperature of the gas entering the second stage is the same as that entering the first stage, and hence that the product (volume entering second stage) × (pressure when entering second stage) must be equal to the product (volume entering first stage) × (pressure of supply to first stage), or

$$(\text{L. P. Cap.}_2)(\text{rec. pr.}) = (\text{L. P. Cap.}_1)(\text{sup. pr.}). \quad . \quad . \quad . \quad (13$$

ombining with Eq. (120)

$$\frac{\text{(L. P. Cap.}_1\text{)}}{\text{(L. P. Cap.}_2\text{)}} = \frac{[\text{(sup.pr.)(del.pr.)}]^{\frac{1}{2}}}{\text{(sup.pr.)}} = \left[\frac{\text{(del.pr.)}}{\text{(sup.pr.)}}\right]^{\frac{1}{2}} = R_p^{\frac{1}{2}},$$

$$R_p^{\frac{1}{2}} = \overset{(1)}{\frac{\text{(L. P. Cap.}_1\text{)}}{\text{(L. P. Cap.}_2\text{)}}} = \overset{(2)}{\frac{D_1E_{v1}}{D_2E_{v2}}} = \frac{D_1\left[1+\overset{(3)}{c_1}-c_1R_p^{\frac{1}{2s}}\right]}{D_2\left[1+c_2-c_2R_p^{\frac{1}{2s}}\right]}. \quad . \quad . \quad . \quad (132)$$

From this three-part equation proper values may be found to fulfill requirements of best-receiver pressure for:

1. The ratio of capacities for a given ratio of pressures, or conversely, the ratio of pressures when capacities are known;
2. The ratio of cylinder displacements for known volumetric efficiencies;
3. The ratio of cylinder displacements when the clearances and ratio of compression are known, or conversely, with known displacements and clearances he ratio of pressures which will cause best-receiver pressure to exist. This st case in general is subject to solution most easily by a series of approximations.

There is, however, a special case which is more or less likely to occur in practice, and which lends itself to solution, that of *equal clearance percentages*. If $c_1 = c_2$ the parenthesis in the numerator of part (3) of Eq. (132) becomes equal o the parenthesis in the denominator, and evidently the volumetric efficiency f the two cylinders are equal, hence for equal clearance percentages in the two ages,

$$\frac{D_1}{D_2} = R_p^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

A case which leads to the same expression, Eq. (133), is that of zero clearance, condition that is often assumed in machines where the clearance is quite mall.

The work per cycle, Eq. (126), when multiplied by the number of cycles erformed per minute, n, and divided by 33,000, gives

$$\text{I.H.P.} = \frac{s}{s-1}\frac{\text{(sup.pr.)}}{114.6}nD_1(1+c_1-c_1R_p^{\frac{1}{2s}})(R_p^{\frac{s-1}{2s}}-1), \quad . \quad (134)$$

rom which are obtained the following:

I.H.P. per cubic foot supplied per minute

$$\frac{\text{I.H.P.}}{n\text{(L. P. Cap.)}} = \frac{s}{s-1}\frac{\text{(sup.pr.)}}{114.6}(R_p^{\frac{s-1}{2s}}-1), \quad . \quad . \quad . \quad . \quad (135)$$

I.H.P. per cubic foot delivered and cooled per minute

$$\frac{\text{I.H.P.}}{n\text{(H. P. Cap. cold)}} = \frac{s}{s-1}\frac{\text{(del.pr.)}}{114.6}(R_p^{\frac{s-1}{2s}}-1), \quad . \quad . \quad . \quad (136)$$

and I.H.P. per cubic foot delivered hot per minute

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. hot})} = \frac{s}{s-1}\,\frac{(\text{sup.pr.})}{114.6} R_p^{\frac{s+1}{2s}} (R_p^{\frac{s-1}{2s}} - 1). \quad (137)$$

These expressions, Eqs. (165), (166) and (167) are all independent of clearance.

Example. What will be the capacity, volumetric efficiency and horse-power per 1000 cu.ft. of free air and per 1000 cu.ft. of hot compressed air per minute for the following compressor for $s = 1.4$? Two-stage, double-acting, cylinders $22\frac{1}{4} \times 34\frac{1}{4} \times 24$ ins. running at 100 R.P.M. Low-pressure clearance 5 per cent, high-pressure clearance such as to give best-receiver pressure. Supply pressure 15 lbs. per square inch absolute, delivery pressure 105 lbs. per square inch absolute.

Capacity will be cylinder displacement times low pressure volumetric efficiency, or $200 D_1 \times E_{v1}$.

$$D_1 = 17.5 \text{ cu.ft.}$$

$$E_{v1} \text{ from Eq. (123)} = (1 + c_1 - c_1 R_p^{\frac{1}{2s}})$$
$$= 1 + .05 - .05 \times 7^{.357} = 95 \text{ per cent.}$$

Therefore low pressure capacity $= 200 \times 12.8 \times .95 = 2430$ cu.ft. per minute.

Horse-power per cubic foot of (sup.pr.) air per minute is from Eq. (135)

$$\frac{s}{s-1}\,\frac{\text{sup.pr.}}{114.6}(R_p^{\frac{s-1}{2s}} - 1),$$
$$= \frac{1.4}{.4} \times \frac{15}{105}\,(7^{\frac{.4}{2 \cdot 8}} - 1) = .16.$$

Therefore, horse-power per 1000 cu.ft. of sup.pr. air $= 160$.

Horse-power per cubic foot of (del.pr.) air, hot, is from Eq. (137)

$$R_p^{\frac{1+s}{2s}} \text{ times power per cu.ft. of (sup.pr.) air,}$$

hence,

$160 \times 5.3 = 850 =$ horse-power per 1000 cu.ft. of (del.pr.) air, hot, per minute.

Problem Note. In the following problems, cylinders are assumed to be proportioned with reference to pressures so as to give best-receiver pressure. Where data conflict the conflict must be found and eliminated.

Prob. 1. Air is compressed adiabatically from 14 lbs. per square inch absolute to 80 lbs. per square inch gage, in a $20\frac{1}{4} \times 32\frac{1}{4} \times 24$ in. compressor, running at 100 R.P.M. the low-pressure cylinder has 3 per cent clearance. What will be horse-power required, to run compres or and what will be the capacity in cubic feet of low pressure and in cubic feet of (del.pr.) air?

Prob. 2. What must be the cylinder displacement of a two-stage compressor with 4 per cent clearance in each cylinder to compress 500 cu.ft. of free air per minute from 14 lbs. per square inch absolute to 85 lbs. per square inch gage, so that s equals 1.4. What will be the horse-power per cubic foot of (del.pr.) air hot and cold?

Prob. 3. A two-stage compressor is compressing gas with a value of $s = 1.2$ from 10 lbs. per square inch gage to 100 lbs. per square inch gage. The cylinders are $18\frac{1}{4} \times 30\frac{1}{4} \times 24$ ins., and speed is 100 R.P.M. If the clearance is 5 per cent in the low

ɔssure cylinder and 10 per cent in the high, what will be the cubic foot of (sup.pr.) gas ndled per minute and what will be the horse-power at best receiver pressure?

Prob. 4. A manufacturer states that his $20\frac{1}{4}\times32\frac{1}{2}\times24$ in. double-acting compres- r when running at 100 R.P.M. at sea level will have a capacity of 2390 cu.ft. of free air r minute, pressure range being from atmosphere to 80 lbs. per square inch gage. At st-receiver pressure what clearance must the compressor have, compression being iabatic?

Prob. 5. The cylinder sizes of a two-stage compressor are given as $10\frac{1}{4}\times16\frac{1}{4}\times12$ s., and clearance in each is 5 per cent. What will be the best-receiver pressures when erating between atmosphere and following discharge pressures, 60, 70, 80, 90, 100 d 110 lbs. per square inch gage, for s equal 1.4?

Prob. 6. 1500 cu.ft. of air at 150 lbs. per square inch gage pressure are needed per nute for drills, hoists, etc. The air is supplied from 3 compressors of the same size d speed, 120 R.P.M. Each has 4 per cent clearance in each cylinder. What will sizes of cylinders and the horse-power of the plant for best-receiver pressure, when =1.41?

Prob. 7. The cards taken from a compressor show volumetric efficiencies of 95 per nt and 80 per cent in low- and high-pressure cylinder respectively. What will be el.pr.) for best-receiver pressure if compressor is $15\frac{1}{4}\times25\frac{1}{4}\times18$ ins., and (sup.pr.) 15 s. per square inch absolute to 10 lbs. absolute, and what will be the work in each case, being 1.35?

Prob. 8. A manufacturer gives a range of working pressure of his $10\frac{1}{4}\times16\frac{1}{4}\times12$ in. mpressor from 80–100 lbs. per square inch gage. If clearances are, low 4 per cent, gh 8 per cent, and (sup.pr.) is atmosphere, find by trial which end of the range comes arest to giving best-receiver pressure? If clearances were equal which would give st-receiver pressure?

Prob. 9 A $16\frac{1}{4}\times25\frac{1}{4}\times16$ in. compressor is rated at 1205 cu.ft. free air per minute at 5 R.P.M. at sea level. What would be the clearance if compressor were compressing r from atmosphere to 100 lbs. gage at sea level? With same clearance what would be e size of a low-pressure cylinder to give the same capacity at altitude of 10,000 ft. with e same clearance and the same (del.pr.), best-receiver pressure always being maintained?

13. Three-Stage Compressor, no Clearance, Perfect Intercooling Exponential Compression (Cycle 7), Best Two Receiver Pressures, Equality of Stages. Work and Capacity, in Terms of Pressures and Volumes. The three-stage exponential compressor cycle with no clearance, perfect intercooling Cycle is shown in Fig. 32. The net work area, $\overline{ABCDEFGHJKA}$, is made up of three areas which may be computed individually by the formulæ for single stage Eq. (48), provided the requisite pressures and volumes are known, as follows:

$$\left.\begin{aligned} W = &\frac{s}{s-1}P_bV_b\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}}-1\right] \quad \text{(first stage)}\\ &+\frac{s}{s-1}P_dV_d\left[\left(\frac{P_e}{P_d}\right)^{\frac{s-1}{s}}-1\right] \quad \text{(second stage)}\\ &+\frac{s}{s-1}P_fV_f\left[\left(\frac{P_g}{P_f}\right)^{\frac{s-1}{s}}-1\right] \quad \text{(third stage)} \end{aligned}\right\} \quad . \quad . \quad (138)$$

But the condition of perfect intercooling provides that for no clearance,

$$P_b V_b = P_d V_d = P_f V_f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

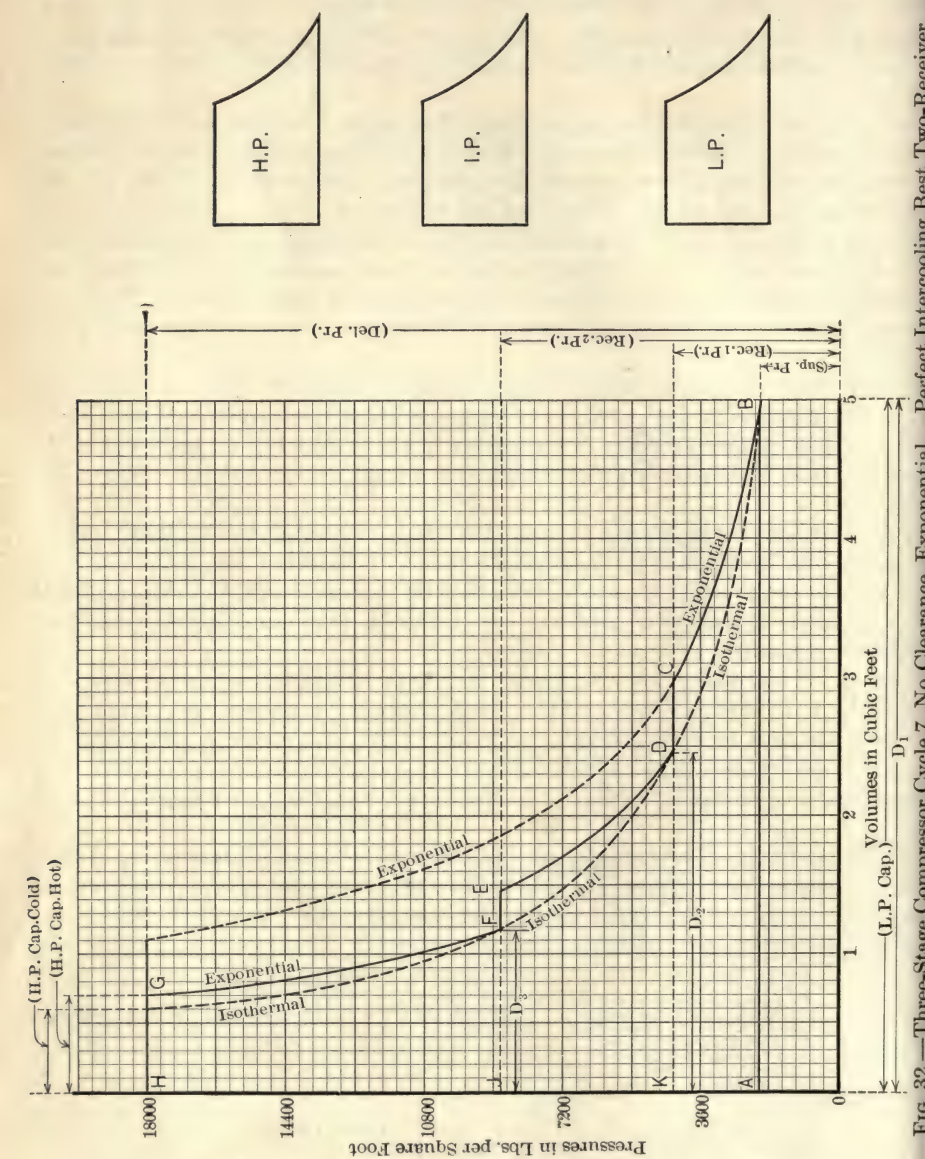


FIG. 32.—Three-Stage Compressor Cycle 7, No Clearance, Exponential. Perfect Intercooling Best Two-Receiver

and it may be noted that $P_d = P_c$, and $P_f = P_e$. Accordingly,

$$W = \frac{s}{s-1} P_b V_b \left[\left(\frac{P_c}{P_a}\right)^{\frac{s-1}{s}} + \left(\frac{P_e}{P_c}\right)^{\frac{s-1}{s}} + \left(\frac{P_g}{P_e}\right)^{\frac{s-1}{s}} - 3 \right] \quad . \quad . \quad (14$$

Pressures in this expression are in pounds per square foot.

Changing the equation to read in terms of supply pressure pounds per square ich, low-pressure capacity cubic feet, and ratios of pressures, first stage R_{p1}), second stage (R_{p2}) and third stage (R_{p3}), it becomes

Work done by three-stage compressor, perfect intercooling

$$W = 144\frac{s}{s-1}(\text{sup.pr.})(\text{L. P. Cap.})\left[(R_{p1})^{\frac{s-1}{s}} + (R_{p2})^{\frac{s-1}{s}} + (R_{p3})^{\frac{s-1}{s}} - 3\right], \quad (141$$

rom this the following expressions are derived:

Work per cubic foot supplied

$$\frac{W}{\text{L.P.Cap.}} = 144\frac{s}{s-1}(\text{sup.pr.})\left[(R_{p1})^{\frac{s-1}{s}} + (R_{p2})^{\frac{s-1}{s}} + (R_{p3})^{\frac{s-1}{s}} - 3\right]. \quad . \quad . \quad (142)$$

Work per cubic foot gas delivered and cooled

$$\frac{W}{(\text{H.P. Cap. cold})} = 144\frac{s}{s-1}(\text{del.pr.})\left[(R_{p1})^{\frac{s-1}{s}} + (R_{p2})^{\frac{s-1}{s}} + (R_{p3})^{\frac{s-1}{s}} - 3\right]. \quad (143)$$

Work per cubic foot gas, as delivered hot

$$\frac{W}{(\text{H. P. Cap. hot})} = 144\frac{s}{s-1}(\text{sup.pr.})(R_{p1})(R_{p2})(R_{p3})^{\frac{1}{s}}\left[(R_{p1})^{\frac{s-1}{s}}\right.$$

$$\left. + (R_{p2})^{\frac{s-1}{s}} + (R_{p3})^{\frac{s-1}{s}} - 3\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (144)$$

Best Two Receiver Pressures. Referring to Fig. 32, P_c is the pressure in the st receiver (1 rec.pr.) and P_e is the pressure in the second receiver, (2 rec.pr.). is evident that if either receiver pressure be fixed and the other is varied, e work necessary to compress a given initial volume of gas will be varied, d will have a minimum value for some particular value of the varying receiver essure. By a variation of both receiver pressures a minumim may be found r the work when both receiver pressures have some specific relation to supply d delivery pressures. For instance, assume that P_c is fixed. Then a change P_e can change only the work of the second and third stages, and the three-age compressor may be regarded as consisting of

One single-stage compressor, compressing form P_b to P_c.
One two-stage compressor, compressing from P_c to P_g.

In this two-stage compressor, best-receiver pressure is to exist, accord g to Eq. (84),

$$P_e = (\text{best 2 rec.pr.}) = (P_c P_g)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (145)$$

Similar reasoning, assuming P_e fixed and making P_c variable, would sho that

$$P_c = (\text{best 1 rec.pr.}) = (P_e P_b)^{\frac{1}{2}}. \quad \ldots \quad (146$$

Eliminate P_c from Eq. (145) and the expression becomes,

$$P_e = (\text{best 2 rec.pr.}) = (P_b P_g^2) = \left[(\text{sup.pr.})(\text{del.pr.})^2\right]^{\frac{1}{3}} \quad (147$$

Similarly, from Eq. (146)

$$P_c = (\text{best 1 rec.pr.}) = (P_b^2 P_g) = \left[(\text{sup.pr.})^2(\text{del.pr.})\right]^{\frac{1}{3}} \quad (14$$

From these expressions may be obtained,

$$\left.\begin{aligned} &\frac{P_c}{P_b} = \frac{P_e}{P_c} = \frac{P_g}{P_e} = \left(\frac{P_g}{P_b}\right)^{\frac{1}{3}} \\ \text{or} \quad &R_{p1} = R_{p2} = R_{p3} = R_p^{\frac{1}{3}} \end{aligned}\right\} \quad \ldots \quad (14$$

Substitution in Eq. (140) gives,

Work, three-stage, best-receiver pressure no clearance

$$W = 3\frac{s}{s-1} P_b V_b \left[\left(\frac{P_g}{P_b}\right)^{\frac{s-1}{3s}} - 1\right]. \quad \ldots \quad (15$$

Arranging this equation to read in terms of supply pressure, pounds per squa inch, low-pressure capacity, cubic feet, and ratio of pressures

Work, three-stage best-receiver pressure

$$W = 432\frac{s}{s-1}(\text{sup.pr.})(\text{L. P. Cap.})(R_p^{\frac{s-1}{3s}} - 1), \quad \ldots \quad (1$$

The work of the compressor is equally divided between the three sta when best-receiver pressures are maintained, which may be proven by substi tion of Eq. (149) in the three parts of Eq. (138), and

Work of any one stage of three-stage compressor with best-receiver pressure

$$W_1 = W_2 = W_3 = 144\frac{s}{s-1}(\text{sup.pr.})(\text{L. P. Cap.})(R_p^{\frac{s-1}{3s}} - 1). \quad \ldots \quad (1$$

From Eq.(151), may be derived the expressions for work per unit of capacity.

Work per cubic foot low-pressure gas is,

$$\frac{W}{(\text{L. P. Cap.})} = 432\frac{s}{s-1}(\text{sup.pr.})\left[R_p^{\frac{s-1}{3s}} - 1\right]. \quad . \quad . \quad . \quad . \quad . \quad (153)$$

Since

$$(\text{L. P. Cap.}) = (\text{H. P. Cap. cold})R_p.$$

Work per cubic foot cooled gas delivered is,

$$\frac{W}{(\text{H. P. Cap. cold})} = 432\frac{s}{s-1}(\text{sup.pr.})R_p(R_p^{\frac{s-1}{3s}} - 1). \quad . \quad . \quad . \quad (154)$$

Again, from Fig. 32,

$$V_g\left(\frac{P_g}{P_f}\right)^{\frac{1}{s}} = V_f = V_b\left(\frac{P_b}{P_f}\right), \text{ or } V_b = V_g\left(\frac{P_f}{P_b}\right)\left(\frac{P_g}{P_f}\right)^{\frac{1}{s}},$$

which is to say that, when best-receiver pressures are maintained,

$$(\text{L. P. Cap.}) = (\text{H. P. Cap. hot})R_p^{\frac{2}{3}}R_p^{\frac{1}{3s}},$$

$$= (\text{H. P. Cap. hot})R_p^{\frac{2s+1}{3s}}, \quad . \quad . \quad . \quad . \quad (155)$$

Hence

Work per cubic foot hot gas delivered

$$\frac{W}{(\text{H. P. Cap. hot})} = 432\frac{s}{s-1}(\text{sup.pr.})R_p^{\frac{2s+1}{3s}}(R_p^{\frac{s-1}{3s}} - 1). \quad . \quad . \quad (156)$$

Example 1. Method of calculating Diagram, Fig. 32.
Assumed data.

$P_a = P_b = 2116$ lbs. per sq.ft.

$P_c = P_d =$ best first-receiver pressure $= P_a^{\frac{2}{3}}P_g^{\frac{1}{3}} = 4330.$

$P_e = P_f =$ best second-receiver pressure $= P_b^{\frac{1}{3}}P_g^{\frac{2}{3}} = 8830.$

$P_g = P_h = 18{,}000$ lbs. per sq.ft.

$V_a = V_h = 0$ cu.ft. $\quad V_b = 5$ cu.ft. $\quad s = 1.4.$

To obtain point C:

$$V_c = V_b \div \left(\frac{P_c}{P_b}\right)^{\frac{1}{1.4}} = 5 \div 1.67 = 3 \text{ cu.ft.}$$

$$\therefore \quad V_c = 3 \text{ cu.ft.} \qquad P_c = 4330 \text{ lbs. sq.ft.}$$

Intermediate points B to C may be found by assuming various pressures and finding the corresponding volumes as for V_c.

To obtain point D:

$$V_d = V_b \times \frac{P_b}{P_d} = 5 \times \frac{2116}{4330} = 2.44 \text{ cu.ft.}$$

$$\therefore \quad V_d = 2.44 \text{ cu.ft.}, \qquad P_d = 4330 \text{ lbs. sq.ft.}$$

To obtain point E:

$$V_e = V_d \div \left(\frac{P_e}{P_d}\right)^{\frac{1}{1.4}}, \text{ but } \quad \frac{P_e}{P_d} = \frac{P_d}{P_b},$$

by assumption of best-receiver pressure.

Hence $V_e = 2.44 \div 1.67 = 1.46$ cu.ft., an $P_e = 8830$ lbs. sq.ft.

Intermediate points D to E may be found by assuming various pressures and finding corresponding volumes as for V_e, and succeeding points are found by similar method to these already used.

$$V_g = .72, \qquad P_g = 18{,}000,$$

Example 2. What will be the horse-power required to compress 100 cu.ft. of free air per minute from 15 lbs. per square inch absolute to 90 lbs. per square inch gage in a no-clearance, three-stage compressor if compression be adiabatic? What will be the work per cubic foot of (del.pr.) air hot or cold?

From Eq. (153) work per cubic foot of (sup.pr.) air is,

$$432 \frac{s}{s-1} (\text{sup.pr.})(R_p^{\frac{s-1}{3s}} - 1),$$

$$= 432 \times \frac{1.4}{.4} \times 15 \times (7^{.0952} - 1) = 4500 \text{ ft.-lbs.},$$

or

$$\text{H.P. for 100 cu.ft. per minute} = \frac{4500 \times 100}{33{,}000} = 13.6.$$

From Eq. (154) work per cubic foot of (del.pr.) air cold is R_p times that per cubic foot of (sup.pr.) air, or in this case is 31,500 ft.-lbs.

From Eq. (156) work per cubic foot of (del.pr.) air hot is $R_p^{\frac{2s+1}{3s}}$ times that per cubic foot of (sup.pr.) air, or in this case $5.8 \times 45{,}000 = 46{,}200$ ft.-lbs.

Prob. 1. What work will be required to supply 2000 cu.ft. of air at 200 lbs. per inch ge pressure if compressing is done adiabatically by three-stage compressors, taking r at atmosphere, neglecting the clearances?

Prob. 2. A motor is available for running a compressor for compressing gas, for nich s equals 1.3. If 60 per cent of the input of the motor can be expended on the r, to what delivery pressure can a cubic foot of air at atmospheric pressure be com- essed in a zero clearance three-stage machine? How many cubic feet per minute uld be compressed to a pressure of 100 lbs. gage per H.P. input to motor?

Prob. 3. Two compressors are of the same size and speed. One is compressing r so that exponent is 1.4, the other a gas so that exponent is 1.1. Each is three stage. hich will require the greater power to drive, and the greater power per cubic foot (sup.pr.) gas, and per cubic foot of (del.pr.) gas, hot, and how much more, neglect- g clearance?

Prob. 4. How will the work per cubic foot of (sup.pr.) air and per cubic foot of (del. .) air differ for a three-stage compressor compressing from atmosphere to 150 lbs. per uare inch gage from a single- and a two-stage, neglecting clearance?

Prob. 5. A table in "Power" gives the steam used per hour in compressing air to rious pressures single stage. A value for air compressed to 100 lbs. is 9.9 lbs. steam r hour per 100 cu.ft. of free air. Using the same ratio of work to steam, find the value r the steam if compression had been three-stage, zero clearances to be assumed.

Prob. 6. A 5 in. drill requires 200 cu.ft. of free air per minute at 100 lbs. per square ich gage pressure. What work will be required to compress air for 20 such drills if ree-stage compressors are used, compared to single-stage for no clearance?

Prob. 7. What would be the steam horse-power of a compressor delivering 150 .ft. of air per minute at 500 lbs. per square inch pressure if compression is three-stage, iabatic, clearance zero, and mechanical efficiency of compressor 80 per cent?

14. Three-stage Compressor with Clearance, Perfect Intercooling Expo- ential Compression (Cycle 8), Best-receiver Pressures, Equality of Stages. ork and Capacity in Terms of Pressures and Volumes. The pressure- olume diagrams of the three-stage compression is shown in Figs. 33, 34 and 5, on which the clearance volume and displacements, low-pressure capacity id high-pressure or delivery capacity for hot gas are indicated.

If perfect intercooling exists, as is here assumed,

$$\left.\begin{aligned} &(V_b - V_a)P_b = (V_d - V_l)P_d = (V_f - V_j)P_f \\ \text{id also}\quad &(\text{L. P. Cap.})P_b = (\text{H.P. Cap. cold})P_g. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (157)$$

Apply Eq. (57) to the three stages and the entire work done is,

$$\left.\begin{aligned} W = &\frac{s}{s-1}P_b(V_b - V_a)\left[\left(\frac{P_c}{P_b}\right)^{\frac{s-1}{s}} - 1\right] \quad \text{(first stage)} \\ +&\frac{s}{s-1}P_d(V_d - V_l)\left[\left(\frac{P_e}{P_d}\right)^{\frac{s-1}{s}} - 1\right] \text{(second stage)} \\ +&\frac{s}{s-1}P_f(V_f - V_j)\left[\left(\frac{P_g}{P_f}\right)^{\frac{s-1}{s}} - 1\right] \quad \text{(third stage)} \end{aligned}\right\} \quad . \quad . \quad . \quad (158)$$

By use of the above conditions of perfect intercooling Eq. (157) this expression becomes,

$$W=\frac{s}{s-1}P_b(V_b-V_a)\left[R_{p1}^{\frac{s-1}{s}}+R_{p2}^{\frac{s-1}{s}}+R_{p3}^{\frac{s-1}{s}}-3\right], \quad . \quad . \quad (159)$$

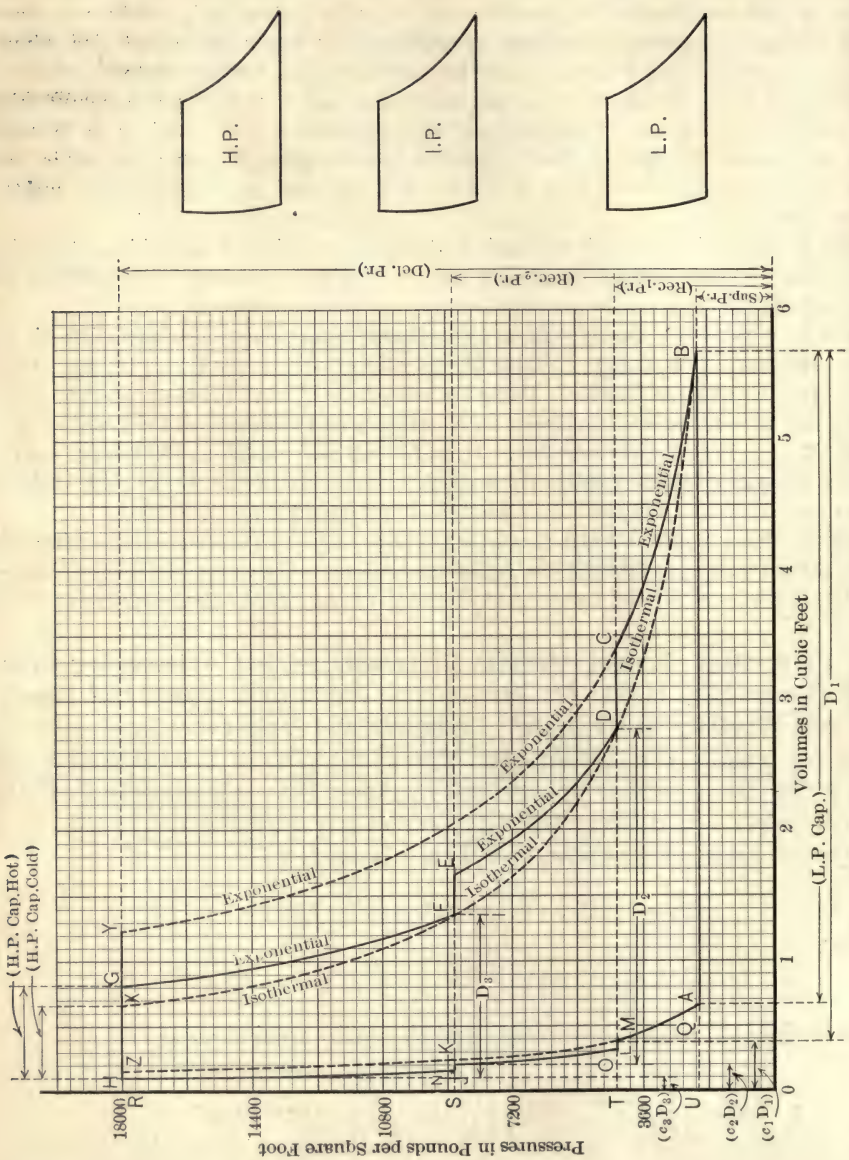


Fig. 33.—Three-stage Compressor, Cycle 7, Clearance, Exponential. Perfect Intercooling. Best Two-receiver

in which

$$R_{p1}=\frac{P_c}{P_b}, \quad R_{p2}=\frac{P_e}{P_d}, \quad R_{p3}=\frac{P_g}{P_f}.$$

In terms of supply pressure, pounds per square inch, low-pressure capacity, cubic foot and ratios of pressures as above, the work of a three-stage compressor with perfect intercooling and with clearance is

$$W = 144\frac{s}{s-1}(\text{sup.pr.})(\text{L. P. Cap.})\left[(R_{p1})^{\frac{s-1}{s}} + (R_{p2})^{\frac{s-1}{s}} + (R_{p3})^{\frac{s-1}{s}}_{l} - 3\right], \quad (160)$$

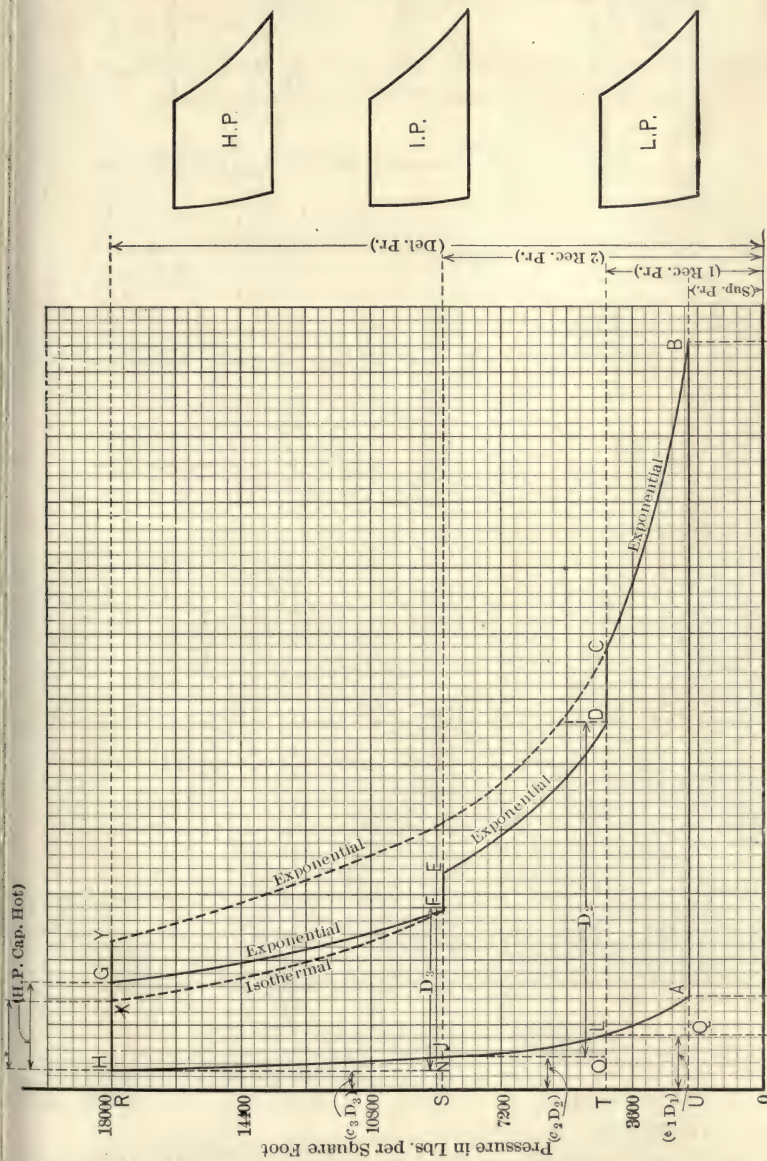


FIG. 34.—Three-stage Compressor, Cycle 7, Clearance, Exponential. Perfect Intercooling, Best Two-receiver Pressures, Re-expansion Volumes Equal to Lower Pressure Cylinder Clearances.

which is identical with Eq. (141), *showing that clearance has no effect upon the work for a given capacity.*

It readily follows that the work per unit of gas is independent of clearan and hence Eqs. (142), (143) and (144), will give a correct value for the w

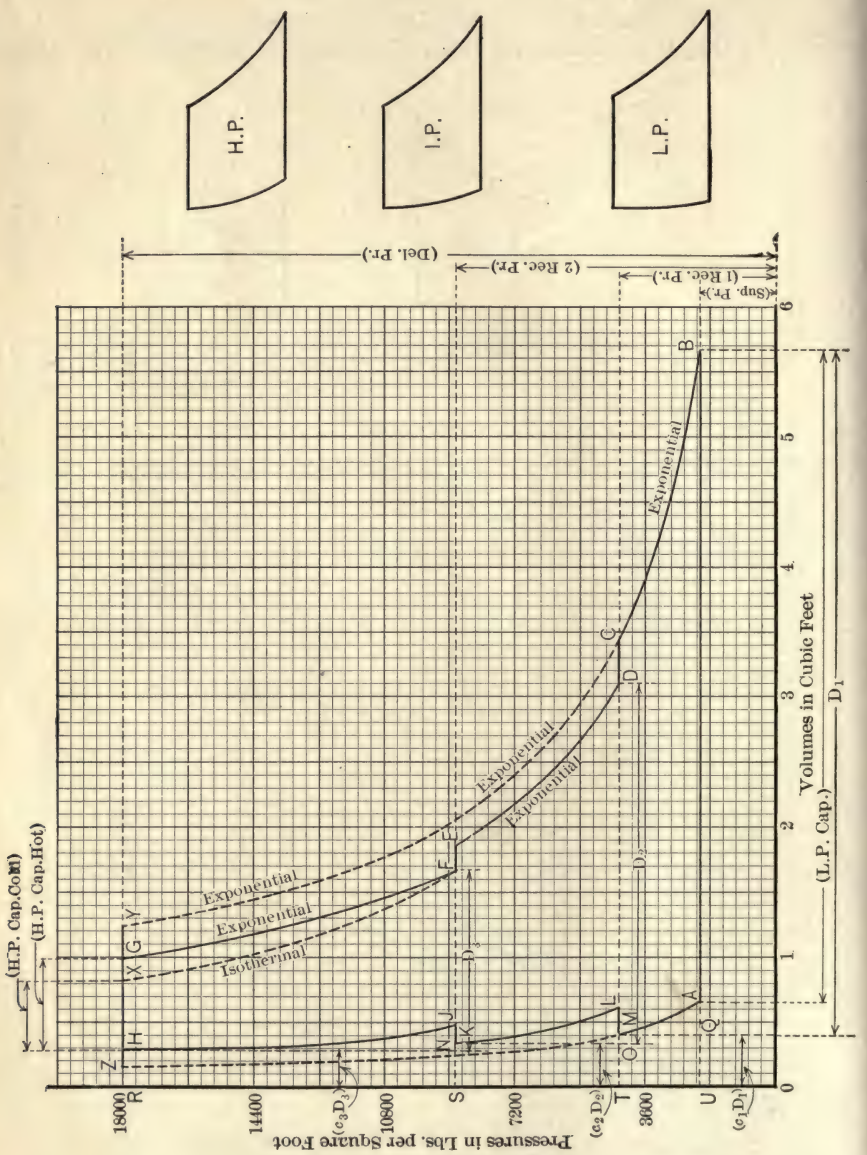


FIG. 35.—Three-stage Compressor Cycle. Clearance Exponential. Perfect Intercooling. Best Two-receiver

per cubic foot of gas supplied, per cubic feet delivered and cooled, and cubic foot as delivered hot, respectively.

Since in two-stage compressors the reasoning leading to the determinat of best-receiver pressure applies equally well with and without clearance, since the value of best-receiver pressures for three-stage are found by c

sidering the three-stage a combination of one- and two stage-compressors, the same expressions for best-receiver pressures will hold with clearance as without; see Eqs. (147) and (148).

$$P_e = (\text{best 2 rec.pr.}) = [(\text{sup.pr.})(\text{del.pr.})^2]^{\frac{1}{3}}.$$

$$P_c = (\text{best 1 rec.pr.}) = [(\text{sup.pr.})^2(\text{del.pr.})]^{\frac{1}{3}}.$$

The use of these expressions for best-receiver pressures leads to the same result as for no clearance Eq. (150), except for the volumes,

Work, three-stage best-receiver's pressure with clearance

$$W = 3\frac{s}{s-1}P_b(V_b - V_a)\left[\left(\frac{P_g}{P_b}\right)^{\frac{s-1}{3s}} - 1\right] \quad . \quad . \quad . \quad . \quad . \quad (161)$$

which is stated below in terms of supply pressure, pounds per square inch low-pressure capacity, cubic foot, and ratio of compression R_p,

Work, three-stage best-receiver pressure.

$$W = 432\frac{s}{s-1}(\text{sup.pr.})(\text{L. P. Cap.})(R_p^{\frac{s-1}{3s}} - 1) \quad . \quad . \quad . \quad (162)$$

which is identical with Eq. (151).

From this may be obtained expressions for the work per cubic foot of low-pressure gas supplied to compressor per cubic foot of gas delivered and cooled, and per cubic foot of gas as delivered hot from the compressor, when the receiver-pressures are best, and these will be respectively identical with Eqs. (153), (154), and (156), in the foregoing section.

15. Three-stage Compressor, any Receiver-pressure Exponential Compression. Capacity, Volumetric Efficiency, Work, Mean Effective Pressure, and Horse-power in Terms of Dimensions of Cylinders and Clearances.

D_1 = displacement of the first-stage cylinder, in cubic feet = $(V_b - V_m)$;

D_2 = displacement of the second-stage cylinder, in cubic feet = $(V_d - V_k)$;

D_3 = displacement of the third-stage cylinder, in cubic feet = $(V_f - V_h)$.

c_1, c_2, c_3 are the clearances of the first, second and third stages respectively, stated as fractions of the displacement, so that,

Clearance volume, 1st stage, in cubic feet = $V_m = c_1 D_1$;

Clearance volume, 2d stage, in cubic feet = $V_k = c_2 D_2$;

Clearance volume, 3d stage, in cubic feet = $V_h = c_3 D_3$.

The low-pressure capacity of the first stage, and hence for the compress is $(V_b - V_a)$, and in terms of clearance, c_1, and displacement D_1 of the fir stage is, according to Eq. (64),

$$(\text{L. P. Cap.}_1) = D_1(1 + c_1 - c_1 R_{p1}^{\frac{1}{s}}) = D_1 E_{v1}. \quad . \quad . \quad . \quad . \quad (16$$

For the second stage, the low-pressure capacity is $(V_d - V_l)$ and is equal t

$$(\text{L. P. Cap.}_2) = D_2(1 + c_2 - c_2 R_{p2}^{\frac{1}{s}}) = D_2 E_{v2}, \quad . \quad . \quad . \quad . \quad (16$$

and for the third stage $(V_f - V_j)$ or,

$$(\text{L. P. Cap.}_3) = D_3(1 + c_3 - c_3 R_{p3}^{\frac{1}{s}}) = D_3 E_{v3}. \quad . \quad . \quad . \quad . \quad (16$$

The volumetric efficiency of 1st stage is

$$E_{v1} = (1 + c_1 - c_1 R_{p1}^{\frac{1}{s}}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16$$

Volumetric efficiency of second stage is

$$E_{v2} = (1 + c_2 - c_2 R_{p2}^{\frac{1}{s}}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16$$

Volumetric efficiency of third stage is

$$E_{v3} = (1 + c_3 - c_3 R_{p3}^{\frac{1}{s}}), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16$$

The work of the three-stage compressor with the assistance of Eq. (163) ma be stated in terms of supply pressure, pounds per square inch, displacemen of first-stage cylinder, in cubic feet and volumetric efficiency of first stage, an also ratios of compression existing in the first, second, and third stages,

$$W = 144 \frac{s}{s-1} (\text{sup.pr.}) D_1 E_{v1} \left[(R_{p1})^{\frac{s-1}{s}} + (R_{p2})^{\frac{s-1}{s}} + (R_{p3})^{\frac{s-1}{s}} - 3 \right] . \quad (16$$

To make use of this formula for the work of the compressor the tw receiver pressures must be known, and it is, therefore, important to derive relation between receiver pressures, displacements and clearances or volumetr efficiencies.

The assumption of perfect intercooling which has already been made us of in obtaining Eq. (169), regardless of the receiver-pressure, requires that- see Eq. (157):

$$(\text{L.P. Cap.}_1)\ (\text{sup.pr.}) = (\text{L.P. Cap.}_2)\ (1\ \text{rec.pr.})$$
$$= (\text{L.P. Cap.}_3)(2\ \text{rec.pr.}). \quad . \quad (170$$

Using values of capacities in Eqs. (163), (164), and (165) and solving for irst- and second-receiver pressures.

$$(1 \text{ rec.pr.}) = (\text{sup.pr.})\frac{(\text{L. P. Cap.}_1)}{(\text{L. P. Cap.}_2)} = (\text{sup.pr.})\frac{D_1E_{v1}}{D_2E_{v2}}, \quad . \quad . \quad (171)$$

nd

$$(2 \text{ rec.pr.}) = (\text{sup.pr.})\frac{(\text{L. P. Cap.}_1)}{(\text{L. P. Cap.}_3)} = (\text{sup.pr.})\frac{D_1E_{v1}}{D_3E_{v3}}. \quad . \quad . \quad (172)$$

Then

$$R_{p1} = \frac{(1 \text{ rec.pr.})}{(\text{sup.pr.})} = \frac{D_1E_{v1}}{D_2E_{v2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (173)$$

$$R_{p2} = \frac{(2 \text{ rec.pr.})}{(1 \text{ rec.pr.})} = \frac{D_2E_{v2}}{D_3E_{v3}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (174)$$

By definition, $(\text{del.pr.}) = R_p\ (\text{sup.pr.})$,

$$R_{p3} = \frac{(\text{del.pr.}}{(2 \text{ rec.pr.})} = R_p\frac{(\text{sup.pr.})}{(2 \text{ rec.pr.})} = R_p\frac{D_3E_{v3}}{D_1E_{v1}}. \quad . \quad . \quad . \quad . \quad . \quad (175)$$

The work of the three-stage compressor may then be stated in terms of upply pressure, pounds per square inch, displacements, cubic feet, volumetric fficiencies, and overall ratio of compression, R_p, as follows:

$$W = 144\frac{s}{s-1}(\text{sup.pr.})D_1E_{v1}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}} + \left(\frac{D_2E_{v2}}{D_3E_{v3}}\right)^{\frac{s-1}{s}} + \left(R_p\frac{D_3E_{v3}}{D_1E_{v1}}\right)^{\frac{s-1}{s}} - 3\right]. \quad . \quad (176)$$

In Terms of Pressures, Displacements, and Clearances, an expression can be written by substitution of values of E_{v1}, E_{v2} and E_{v3} from Eqs. (166), (167) and (168), but it becomes a long expression, further complicated by the fact that R_{p1}, R_{p2} and R_{p3} remain in it. This may be solved by the approximation based first upon the assumption that all volumetric efficiencies are equal to each other or to unity when

$$\left.\begin{aligned} R_{p1} &= \frac{D_1}{D_2} \\ R_{p2} &= \frac{D_2}{D_3} \\ R_{p3} &= \frac{D_3}{D_1} \end{aligned}\right\} \text{(If volumetric efficiencies are each equal to each other or to unity)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (177)$$

This process amounts to the same thing as evaluating E_{v1}, E_{v2}, and E_{v3} from Eqs. (166), (167) and (168), making use of the approximation Eq. (177) and substituting the values found in Eq. (176).

Since the above can be done with any expression which is in terms of volumetric efficiencies, the following formulæ will be derived from Eq. (176), as it stands.

The *mean effective pressure* of the three-stage compressor reduced to the first stage cylinder is found by dividing the work of the entire cycle, Eq. (176) by displacement of the first stage, and by 144 to reduce to pounds per square inch (m.e.p.) reduced to first stage cylinder,

$$\frac{W}{144D_1}=\frac{s}{s-1}(\text{sup.pr.})E_{v1}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+\left(\frac{D_2E_{v2}}{D_3E_{v3}}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3E_{v3}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-3\right]. \quad (178)$$

Note here that this may also be obtained by multiplying (work per cubic foot supplied) by (volumetric efficiency of first stage) and dividing the product by 144.

The *indicated horse-power* of a compressor performing n cycles per minute will be equal to the work per cycle multiplied by n and divided by 33,000, or for the three-stage compressor with general receiver pressures,

$$\text{I.H.P.}=\frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2}nD_1E_{v1}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+\left(\frac{D_2E_{v2}}{D_3E_{v3}}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3E_{v3}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-3\right]. \quad (179)$$

For n may be substituted the number of revolutions per minute, N, divided by the revolutions required to complete one cycle

$$n=\frac{N}{z}.$$

The horse-power per cubic foot of gas supplied per minute is

$$\frac{\text{I.H.P.}}{n(\text{L. P. Cap.})}=\frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+\left(\frac{D_2E_{v2}}{D_3E_{v3}}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3E_{v3}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-3\right]. \quad (180)$$

Horse-power per cubic foot gas delivered and cooled per minute is

$$\frac{\text{I.H.P.}}{n(\text{H.P.Cap.cold})}=\frac{s}{s-1}\frac{(\text{del.pr.})}{229.2}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+\left(\frac{D_2E_{v2}}{D_3E_{v3}}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3E_{v3}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-3\right]. \quad (181)$$

Horse power per cubic foot hot gas delivered per minute is

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. hot})}=\frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2}\left(\frac{D_1E_{v1}}{D_3E_{v3}}\right)^{\frac{s-1}{s}}R_p^{\frac{1}{s}}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+\left(\frac{D_2E_{v2}}{D_3E_{v3}}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3E_{v3}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-3\right]. \quad (182)$$

The last equation is obtained by means of the relation

$$\begin{aligned}(\text{L. P. Cap.})&=(\text{H. P. Cap. hot})\times\left(\frac{\text{del.pr.}}{2\text{ rec.pr.}}\right)^{\frac{1}{s}}\times\left(\frac{2\text{ rec.pr.}}{\text{sup.pr.}}\right)\\&=(\text{H. P. Cap. hot})\times\left(R_p\frac{D_3E_{v3}}{D_1E_{v1}}\right)^{\frac{1}{s}}\left(\frac{D_1E_{v1}}{D_3E_{v3}}\right)\\&=(\text{H. P. Cap. hot})\times R_p^{\frac{1}{s}}\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}. \quad (183)\end{aligned}$$

If clearance is zero or negligible, these expressions may be rewritten, putting E_v, E_{v2} and E_{v3} each equal to unity.

$$\text{I.H.P.}=\frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2}nD_1\left[\left(\frac{D_1}{D_2}\right)^{\frac{s-1}{s}}+\left(\frac{D_2}{D_3}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3}{D_1}\right)^{\frac{s-1}{s}}-3\right]. \quad (184)$$

H.P. per cubic foot of gas supplied per minute is

$$\frac{\text{I.H.P.}}{n(\text{L.P. Cap.})}=\frac{(\text{sup.pr.})}{229.2}\left[\left(\frac{D_1}{D_2}\right)^{\frac{s-1}{s}}+\left(\frac{D_2}{D_3}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3}{D_1}\right)^{\frac{s-1}{s}}-3\right] \quad (185)$$

H.P. per cubic foot delivered and cooled per minute is

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. cold})}=\frac{(\text{del. pr.})}{229.2}\left[\left(\frac{D_1}{D_2}\right)^{\frac{s-1}{s}}+\left(\frac{D_2}{D_3}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3}{D_1}\right)^{\frac{s-1}{s}}-3\right]. \quad (186)$$

H.P. per cubic foot hot gas delivered per minute is

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. hot})}=\frac{s}{s-1}\frac{(\text{sup.pr.})}{229.2}\left(\frac{D_1}{D_3}\right)^{\frac{s-1}{s}}R_p^{\frac{1}{s}}\left[\left(\frac{D_1}{D_2}\right)^{\frac{s-1}{s}}+\left(\frac{D_2}{D_3}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3}{D_1}\right)^{\frac{s-1}{s}}-3\right]. \quad (187)$$

Example 1. Method of calculating Diagram, Fig. 35.
Assumed data:

$$P_a = P_b = 2116 \text{ lbs. per square foot.}$$

$$P_c = P_d = P_l = P_m = 4330 \text{ lbs. per square foot.}$$

$$P_e = P_f = P_j = P_k = 8830 \text{ lbs. per square foot.}$$

$$P_g = P_h = 18{,}000 \text{ lbs. per square foot.}$$

$c_1 = 7.5$ per cent for all cylinders; $s = 1.4$.

L.P. capacity 5 cu.ft.

To obtain point M:

From formula Eq. (163) L. P. $\text{Cap}_1. = D_1(1 + c_1 - c_1 R_{p1}^{\frac{1}{s}})$

or

$$5 = D_1(1 + .075 - .075 \times 1.67) \quad \text{or} \quad D_1 = 5.3 \text{ cu.ft. and clearance volume}$$

$$V_m = 5.3 \times .075 = .387 \text{ cu.ft.}$$

Therefore,

$$V_m = .39 \text{ cu.ft.}; \quad P_m = 4330 \text{ lbs. sq.ft.};$$

To obtain Point A:

$$V_a = V_m \times \left(\frac{P_m}{P_a}\right)^{\frac{1}{1.4}}. = .39 \times 1.67 = .67 \text{ cu.ft.}$$

Additional points M to A may be found by assuming pressures and finding corresponding volumes as for V_a.

To obtain point B:

$$V_b = V_A + (\text{L. P. } \text{Cap}_1.) = .67 + 5 = 5.67 \text{ cu.ft.}$$

Therefore,

$$V_b = 5.67 \text{ cu.ft.}; \quad P_b = 2116 \text{ lbs. sq. ft.};$$

To obtain point C:

$$V_c = V_b \div \left(\frac{P_c}{P_B}\right)^{\frac{1}{1.4}} = 5.67 \div 1.67 = 3.45 \text{ cu.ft.}$$

Therefore,

$$V_c = 3.45 \text{ cu.ft.}; \quad P_c = 4330 \text{ lbs. sq.ft.}$$

Intermediate points B to C may be found by assuming various pressures and finding corresponding volumes as for V_b.

To obtain point D:

Volume at D is the displacement plus clearance of the intermediate cylinder. This cannot be found until the capacity is known. Applying the same sort of relations as were used in calculating the diagram for the two-stage case with clearance,

$$D_2(1+C_2-C_2R_{p2})^{\frac{1}{s}}=2.44 \text{ or } D_2=2.57,$$

and clearance volume.

$$V_k=.075\times2.57=.192 \text{ cu.ft.},$$

hence,

$$V_d=2.57+.19=2.76 \text{ cu.ft.}$$

Therefore,

$$V_d=2.76 \text{ cu.ft}; \qquad P_d=4330 \text{ lbs. sq.ft.};$$

The rest of the points are determined by methods that require no further explanation and as pressures were fixed only volumes are to be found. These have the following values, which should be checked:

$V=1.65$; $V_f=1.32$; $V_g=.79$; $V_h=.09$; $V_j=.15$; $V_l=.32$; $V_x=.65$; $V_y=1.23$; $V_z=.14$.

Example 2. A three-stage compressor is compressing air from atmosphere to 140 lbs. per square inch absolute. The low-pressure cylinder is 32×24 ins. and is known to have a clearance of 5 per cent. From gages on the machine it is noted that the first-receiver pressure is 15 lbs. per square inch gage and the second-receiver pressure is 70 lbs. per square inch gage. What horse-power is being developed if the speed is 200 R.P.M. and $s=1.4$? From the formula Eq. (169),

$$W=144\frac{s}{s-1}(\text{sup.pr.})D_1E_{v_1}\left[R_{p_1}^{\frac{s-1}{s}}+R_{p2}^{\frac{s-1}{s}}+R_{p3}^{\frac{s-1}{s}}-3\right].$$

From gage readings

$$R_{p_1}=\frac{30}{15}=2. \qquad R_{p2}=\frac{70}{30}=2.33; \qquad R_{p3}=\frac{140}{70}=2.$$

$$E_{v_1}=(1+c_1-c_1R_{p1}^{\frac{1}{s}}) \text{ from Eq. (166)},$$

$$E_{v1}=(1+.05-.05\times1.65)=67.5 \text{ per cent.}$$

hence,

$$W=144\times\frac{1.4}{.4}\times15\times11.2\times.675(1.22+1.28+1.22-3);$$

$$=59{,}200 \text{ ft.lbs. per stroke or } 200\times59{,}200 \text{ ft.} = \text{lbs. per minute};$$

$$=358 \text{ I.H.P.}$$

Example 3. Another compressor has cylinders $12\times20\times32\times24$ in. and it is know that the volumetric efficiencies of the high, intermediate and low-pressure cylinders ar respectively 70 per cent, 85 per cent and 98 per cent. The (del.pr.) is 150 lbs. per squar inch absolute. What is the horse-power in this case if the speed is 100 R.P.M.?

From the formula Eq. (176),

$$W=144\frac{s}{s-1}(\text{sup.pr.})D_1E_{v1}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+\left(\frac{D_2E_{v2}}{D_3E_{v3}}\right)^{\frac{s-1}{s}}+\left(R_p\frac{D_3E_{v3}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-3\right]$$

$$=144\times\frac{14}{4}\times15\times11.2\times.98\left[\left(\frac{11.2\times.98}{4.36\times.85}\right)^{.286}+\left(\frac{4.36\times.85}{1.59\times.70}\right)^{.286}\right.$$

$$\left.+\left(10\frac{1.59\times.7}{11.2\times.98}\right)^{.286}-3\right]$$

$$=(1.309+1.495+1-3)=66{,}400 \text{ ft.-lbs. per stroke,}$$

Whence $$\text{I.H.P.}=\frac{200\times66{,}400}{33{,}000}=402.$$

Prob. 1. What will be the horse-power required to drive a $12\times22\times34\times30$ in. three stage compressor with volumetric efficiencies of 75, 85, and 95 per cent in the high intermediate and low-pressure cylinders, at 100 R.P.M. when compressing natural ga from 25 lbs. per square inch gage to 300 lbs. per square inch gage, adiabatically?

Prob. 2. A three-stage compressor for supplying air for a compressed-air locomo tive receives air at atmosphere and delivers it at 800 lbs. per square inch gage. Shoul the receiver pressures be 50 lbs. and 220 lbs. respectively in the first and second and th volumetric efficiency of the first stage 90 per cent, what would be its displacemen and horse-power when compressing 125 cu.ft. of free air per minute, adiabatically What are the cylinder displacements?

Prob. 3. Find the work done on a gas, the value for s of which is 1.3, in compressin it from atmosphere to 7 atmospheres absolute, adiabatically in three stages, the com pressor having a low-pressure cylinder displacement of 60 cu.ft. per minute and a volu metric efficiency of 95 per cent, first receiver pressure being 2 atmospheres absolut and second-receiver pressure 4 atmospheres absolute. If air were being compresse instead of the above gas, how would the work vary?

Prob. 4. The cylinders of a compressor are $8\times12\times18\times24$ ins. and clearance suc as to give volumetric efficiencies of 80, 90 and 98 per cent in the different cylinders in th order given. Compressor is double acting, running at 120 R.P.M. and compressing ai adiabatically from 14 lbs. per square inch absolute to 150 lbs. per square inch gage What is the capacity in cubic feet per minute, work done per cubic feet of (sup.pr.) ai (del.pr.) air hot and cold and the horse-power of the compressor? .What would b the effect on these quantities if the clearances were neglected?

Prob. 5. If the cylinders of a compressor are $10\times14\times20\times18$ ins., and clearanc are 8, 5 and 3 per cent, what work is being done in adiabatically compressing air fro 10 lbs. per square inch absolute to 100 bs. per square inch gage?

NOTE: Solve by approximate method.

Prob. 6. For special reasons it is planned to keep the first-receiver pressure of three-stage compressor at 30 lbs. per square inch absolute, the second-receiver pressure 60 lbs. per square inch absolute, and the line pressure at 120 lbs. per square inch absolu

le (sup.pr.) is 14 lbs. per square inch absolute. If the clearances are 4 per cent in t: low and 8 per cent in the intermediate and high-pressure cylinders, what must be t: cylinder sizes to handle 500 cu.ft. of free air per minute at 120 R.P.M. and what ɹwer must be supplied to the compressor on a basis of 80 per cent mechanical effi- ɔncy, for a value of s equal to 1.39?

Prob. 7. Should the above pressures (Prob. 6) be gage pressures instead of absolute, ɦw would the quantities to be found be affected?

Prob. 8. The receiver pressures on a CO_2 gas compressor are 50 lbs. per square inch ɜolute, and 200 lbs per square inch absolute, the (del.pr.) being 1000 lbs. per square i h absolute. The mach ne has a low-pressure cylinder 8×10 ins. with 3 per cent ɔarance. What horse-power will be required to run it at 100 R.P.M. and what would l the resultant horse-power and capacity if each pressure were halved? (Sup.pr.) = 14.7 l. per square inch.

16. Three-stage Compressor with Best-receiver Pressures Exponential (mpression. Capacity, Volumetric Efficiency, Work, Mean Effective Pressure ad Horse-power in Terms of Dimensions of Cylinders and Clearances. It vs found that for the three-stage adiabatic compressor with perfect inter- ɔling, the work was a minimum if the first and second receivers had pressures ɗfined as follows, see Eqs. (147) and (148):

$$(\text{best 1 rec.pr.}) = [(\text{sup.pr.})^2(\text{del.pr.})]^{\frac{1}{3}}. \quad . \quad . \quad . \quad . \quad . \quad (188)$$

$$(\text{best 2 rec.pr.}) = [(\text{sup.pr.})(\text{del.pr.})^2]^{\frac{1}{3}}. \quad . \quad . \quad . \quad . \quad . \quad (189)$$

$$R_{p1} = \frac{(\text{best 1 rec.pr.})}{(\text{sup.pr.})} = \left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)^{\frac{1}{3}} = R_p^{\frac{1}{3}}, \quad . \quad . \quad . \quad . \quad (190)$$

$$R_{p2} = \frac{(\text{best 2 rec.pr.})}{(\text{best 1 rec.pr.})} = \left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)^{\frac{1}{3}} = R_p^{\frac{1}{3}}, \quad . \quad . \quad . \quad . \quad (191)$$

$$R_{p3} = \frac{(\text{del.pr.})}{(\text{best 2 rec.pr.})} = \left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)^{\frac{1}{3}} = R_p^{\frac{1}{3}}. \quad . \quad . \quad . \quad . \quad (192)$$

The use of these values in connection with expressions previously given f volumetric efficiency, Eqs. (166), (167) and (168), gives,

Vlumetric efficiency of first stage $= E_{v1} = (1 + c_1 - c_1 R_p^{\frac{1}{3s}})$ . . . . (193)

Vlumetric efficiency of second stage $= E_{v2} = (1 + c_2 - c_2 R_p^{\frac{1}{3s}})$ . . . . (149)

Vlumetric efficiency of third stage $= E_{v3} = (1 + c_3 - c_3 R_p^{\frac{1}{3s}})$ . . . . (195)

The work of the three-stage compressor with best-receiver pressures, Eq. (162), when expressed in terms of displacement and volumetric efficiency becomes

$$W = 432\frac{s}{s-1}(\text{sup.pr.})D_1E_{v1}(R_p^{\frac{s-1}{3s}}-1) \quad . \quad . \quad . \quad . \quad . \quad (19$$

where

$$R_p = \frac{(\text{del.pr.})}{(\text{sup.pr.})}$$

If clearance is known, the value of E_{v1} may be ascertained by Eq. (19 and inserted in Eq. (196). Since this may be so readily done the substitutio will not here be made.

The *mean effective pressure* of the compressor referred to the first stage obtained by dividing the work Eq. (196) by 144 D_1:

(m.e.p.) referred to first-stage cylinder

$$\frac{W}{144D_1} = 3\frac{s}{s-1}(\text{sup.pr.})E_{v1}(R_p^{\frac{s-1}{3s}}-1). \quad . \quad . \quad . \quad . \quad (19$$

The mean effective pressures of the respective stages, due to the equali of work done in the three stages will be as follows:

For first stage

$$(\text{m.e.p.}) = \frac{s}{s-1}(\text{sup.pr.})E_{v1}(R_p^{\frac{s-1}{3s}}-1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (19$$

For second stage

$$(\text{m.e.p.}) = \frac{s}{s-1}(\text{sup.pr.})\frac{D_1}{D_2}E_{v1}(R_p^{\frac{s-1}{3s}}-1). \quad . \quad . \quad . \quad . \quad . \quad (19$$

For third stage

$$(\text{m.e.p.}) = \frac{s}{s-1}(\text{sup.pr.})\frac{D_1}{D_3}E_{v1}(R_p^{\frac{s-1}{3s}}-1). \quad . \quad . \quad . \quad . \quad . \quad (20$$

But

$$(\text{sup.pr.})\frac{D_1E_{v1}}{D_2E_{v2}} = (1 \text{ rec.pr.}) = [(\text{sup.pr.})^2(\text{del.pr.})]^{\frac{1}{3}},$$

and also

$$(\text{sup.pr.})\frac{D_1E_{v1}}{D_3E_{v3}} = (2 \text{ rec.pr.}) = [(\text{sup.pr.})(\text{del.pr.})^2]^{\frac{1}{3}}.$$

Hence

For second stage

$$(\text{m.e.p.}) = \frac{s}{s-1}[(\text{sup.pr.})^2(\text{del.pr.})]^{\frac{1}{3}} E_{v2}(R_p^{\frac{s-1}{3s}} - 1). \quad . \quad (201)$$

For third stage

$$(\text{m.e.p.}) = \frac{s}{s-1}[(\text{sup.pr.})(\text{del.pr.})^2]^{\frac{1}{3}} E_{v3}(R_p^{\frac{s-1}{3s}} - 1). \quad . \quad . \quad (202)$$

Conditions to Give Best-receiver Pressures. All the foregoing discussion of best-receiver pressures for the three-stage compressor can apply only to cases in which all the conditions are fulfilled necessary to the existence of best-receiver pressures. These conditions are expressed by equations (173), (174), (175), (190), (191), and (192), which may be combined as follows:

$$\left.\begin{aligned} R_p^{\frac{1}{3}} &= \frac{\overset{(1)}{(\text{L. P. Cap.}_1)}}{(\text{L. P. Cap.}_2)} = \frac{\overset{(2)}{(\text{L. P. Cap.}_2)}}{(\text{L. P. Cap.}_3)} = \frac{\overset{(3)}{D_1 E_{v1}}}{D_2 E_{v2}} = \frac{\overset{(4)}{D_2 E_{v2}}}{D_3 E_{v3}} \\ &= \frac{\overset{(5)}{D_1(1+c_1-c_1 R_p^{\frac{1}{3s}})}}{D_2\left(1+c_2-c_2 R_p^{\frac{1}{3s}}\right)} = \frac{\overset{(6)}{D_2(1+c_2-c_2 R_p^{\frac{1}{3s}})}}{D_3\left(1+c_3-c_3 R_p^{\frac{1}{3s}}\right)} \end{aligned}\right\} \quad . \quad . \quad (203)$$

Parts (1) and (2) of this equation state the requirements in terms of capacities; (3) and (4) in terms of displacements and volumetric efficiencies; (5) and (6) in terms of displacements and clearances. In order, then, that best-receiver pressure may be obtained, there must be a certain relation between the given ratio of compression and dimensions of cylinders and clearances. Since, after the compressor is once built these dimensions are fixed, a given multi-stage compressor can be made to give best-receiver pressures only when compressing through a given range, i.e., when R_p has a definite value. If R_p has any other value the receiver pressures are not best, and the methods of the previous Section (15) must be applied.

When *clearance percentages* are *equal* in all three cylinders, $c_1 = c_2 = c_3$, and the volumetric efficiencies are all equal then, when best-receiver pressures exist, Eq. (203) becomes,

$$R_p^{\frac{1}{3}} = \frac{D_1}{D_2} = \frac{D_2}{D_3} = \text{for equal clearance per cent.} \quad . \quad . \quad (204)$$

Evidently this same expression holds if *clearances* are all *zero* or negligible. What constitutes negligible clearance is a question requiring careful thought and is dependent upon the ratio of compression and the percentage of error allowable.

Indicated horse-power of the compressor is found by multiplying the wor per cycle, Eq. (196) by the number of cycles per minute, n, and dividing th product by 33,000.

$$\text{I.H.P.} = \frac{s}{s-1}\frac{(\text{sup.pr.})}{76.4} n D_1 E_{v1} (R_p^{\frac{s-1}{3s}} - 1) \quad . \quad . \quad . \quad . \quad (205$$

From this are obtained the following:

H.P. per cubic foot supplied per minute

$$\frac{\text{I.H.P.}}{n(\text{L. P. Cap.})} = \frac{s}{s-1}\frac{(\text{sup.pr.})}{76.4}(R_p^{\frac{s-1}{3s}} - 1). \quad . \quad . \quad . \quad . \quad . \quad (206$$

H.P. per cubic foot delivered and cooled per minute

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. cold})} = \frac{s}{s-1}\frac{(\text{del.pr.})}{76.4}(R_p^{\frac{s-1}{3s}} - 1). \quad . \quad . \quad . \quad . \quad (207$$

H.P. per cubic foot delivered hot per minute

$$\frac{\text{I.H.P.}}{n(\text{H. P. Cap. hot})} = \frac{s}{s-1}\frac{(\text{sup.pr.})}{76.4} R_p^{\frac{2s+1}{3s}} (R_p^{\frac{s-1}{3s}} - 1). \quad . \quad . \quad (208$$

(See Eq. (156)).

It is useful to note that these expressions are all independent of clearanc which is to be expected, since the multi-stage compressor may be regarded as series of single-stage compressors, and in single stage such an independenc was found for work and horse-power per unit of capacity.

Example. If the following three-stage compressor be run at best-receiver pressure what will be the horse-power and the best-receiver pressures? Compressor has lo pressure cylinder 32×24 ins. with 5 per cent clearance, is compressing air from atmo phere to 140 lbs. per square inch absolute, so that s equals 1.4 and it runs at 100 R.P.M

From the formula Eq. (196)

$$W = \frac{432s}{s-1}(\text{sup.pr.}) D_1 E_{v1} \left(R_p^{\frac{s-1}{3s}} - 1\right)$$

From the formula Eq. (188)

$$\begin{aligned}(\text{best 1 rec.pr.}) &= [(\text{sup.pr.})^2(\text{del.pr.})]^{\frac{1}{3}} \\ &= (15)^2 \times 140]^{\frac{1}{3}} = 31.6.\end{aligned}$$

From Eq. (189)

$$(\text{best 2 rec.pr.}) = [(\text{sup.pr.})(\text{del.pr.})^2]^{\frac{1}{3}}$$

$$= [15 \times (140)^2]^{\frac{1}{3}} = 66.5.$$

From Eq. (193)

$$E_{v1} = \left(1 + c_1 - c_1 R_p^{\frac{1}{3s}} \right)$$

$$= 1 + .05 - .05 \times \left(\frac{140}{15}\right)^{3s} = 96.5;$$

ence,

$$W = 432 \times \frac{1.4}{.4} \times 15 \times 11.2 \times 96.5 \times (9.35^{.95} - 1) = 59{,}000 \text{ ft.-lbs.},$$

;

$$\text{I.H.P.} = \frac{59{,}000 \times 200}{33{,}000} = 358$$

Prob. 1. There is available for running a compressor 175 H.P. How many cu.ft. free air per minute can be compressed from atmosphere to 150 lbs. per square inch ıge by a three-stage adiabatic compressor with best-receiver pressures?

Prob. 2. The low-pressure cylinder of a three-stage compressor has a capacity of cu.ft. per stroke. If the stroke of all three cylinders is 18 ins., what must be the ameters of the intermediate and high to insure best-receiver pressures, if clearance e neglected, and (sup.pr.) be 1 lb. per square inch absolute and (del.pr.) 15 lbs. per uare inch absolute, s being 1.4.

Prob. 3. The above compressor is used as a dry-vacuum pump for use with a sur-ce condenser. If 800 cu.ft. of (sup.pr.) gas must be handled per minute what horse-ower will be needed to run it? What will be the horse-power per cubic foot of atmos-neric air?

Prob. 4. Will a $15 \times 22 \times 34 \times 24$ in. compressor with clearances of 3, 5 and 8 per cent low, intermediate and high-pressure cylinders respectively be working at best-receiver essures when (sup.pr.) is 15 lbs. per square inch absolute and (del.pr.) 150 lbs. per uare inch absolute? If not, find by trial, the approximate (del.pr.), for which this achine is best, with s equal to 1.4?

Prob. 5. For the best (del.pr.) as found above find the horse-power to run the achine at 100 R.P.M. and also the horse-power per cu.ft. of (del. pr.) air cold?

Prob. 6. Should this compressor be used for compressing ammonia would the st (del. pr.) change, and if so what would be its value? Also what power would be eded for this case?

Prob. 7. Compare the work necessary to compress adiabatically in three stages from) lbs. per square inch absolute to 200 lbs. per square inch absolute, the following gases:

Air; Oxygen; Gas-engine mixtures, for which $s = 1.36$.

Prob. 8. For 5 per cent clearance in all the cylinders what must be the cylinder ratio r best-receiver pressure and a pressure ratio of 10?

Prob. 9. A compressor, the low-pressure cylinder of which is 30×20 ins. with 5 p cent clearance is compressing air adiabatically from atmosphere to 150 lbs. per squa inch gage, at best-receiver pressure. Due to a sudden demand for air the (del. pr drops to 100 lbs. per square inch gage. Assuming that the (1 rec. pr.) dropped to 5 lb per square inch gage and (2 rec.pr.) dropped to 40 lbs. per square inch gage, how muc would the speed rise if the power supplied to machine was not changed?

17. Comparative Economy or Efficiency of Compressors. As the prin duty of compressors of all sorts is to move gas or vapor from a region of lo to a region of high pressure, and as this process always requires the expenditu of work, the compressor process which is most economical is the one th accomplishes the desired transference with the least work. In this sense, the economy of compression means something different than efficiency, as ord narily considered. Ordinarily, efficiency is the ratio of the energy at one poi in a train of transmission or transformation, to the energy at another poin whereas with compressors, economy of compression is understood to mean th ratio of the work required to compress and deliver a unit of gas, moving from a low- to a high-pressure place, to the work that would have been require by some other process or hypothesis, referred to as a standard. This econom of compression must not be confused with efficiencyof compressors as machine as it is merely a comparison of the work in the compressor cylinder for an actu case or hypothesis to that for some other hypothesis taken as a standard. Th standard of comparison may be any one of several possible, and unfortunatel there is no accepted practice with regard to this standard. It will, therefor be necessary to specify the standard of reference whenever economy of compre sion is under consideration. The following standards have been used wit some propriety and each is as useful, as it supplies the sort of information reall desired.

First Standard. The work per cubic foot of supply gas necessary to co press *isothermally* (Cycle 1), from the supply pressure to the delivery pressu of the existing compressor and to deliver at the high pressure is less than that any commercial process of compression, and may be taken as a standard f comparison. Since, however, actual compressors never depart greatly fro the adiabatic law, their economy compared with the isothermal standard w always be low, making their performance seem poor, whereas they may be nearly perfect as is possible, so that it may appear that some other standa would be a better indication of their excellence.

Second Standard. The work per cubic foot of gas supplied when compress *adiabatically in a single stage* (Cycle 3), if taken as a standard, will indicat high economy, near unity for single-stage compressors, and an economy abo unity for most multi-stage compressors. For the purpose of comparison will be equally as good as the first standard,and the excess of the economy ov unity will be a measure of the saving over single-stage adiabatic compressio Since, however, single-stage adiabatic compression is not the most economi obtainable in practice for many cases, this standard may give an incorr idea of the perfection of the compressor.

Third Standard. Due to the facts noted above, it may be a better indication of the degree of perfection of the compressor to compare the work per cubic foot of gas supplied with that computed for the standard *adiabatic cycle most nearly approaching that* of the compressor. This standard is, however, open to the objection that a multi-stage compressor is not referred to the same cycle as a single-stage compressor, and a multi-stage compressor with other than best-receiver pressure is not referred to the same cycle as another operating with best-receiver pressure. This is, therefore, not a desirable standard for comparing compressors of different types with one another, although it does show to what extent the compressor approaches the hypothetical best condition for its own type and size.

Other standards might be chosen for special reasons, each having a value in proportion as it supplies the information that is sought.

It is seen from the discussion of the second standard that its only advantage over the first is in that it affords a measure of the saving or loss as compared with the single-stage adiabatic compressor cycle.

If the first standard, that of the isothermal compressor cycle, be adopted for the purpose of comparison, it at once gives a measure of comparison with the isothermal, which is more and more nearly approached as the number of stages is increased, though never quite reached, or as the gas is more effectively cooled during compression. It may be regarded as the limiting case of multi-stage compression with perfect intercooling, or the limiting case of continuous cooling.

In order to ascertain how nearly the actual compressor approaches the adiabatic cycle most nearly representing its working conditions, the economy of each of the various reference cycles heretofore discussed may be tabulated or charted, and the economy of the cycle as compared with that of the actual performance of the compressor will give the required information. The process of computation by which this information is obtained will depend upon the nature of information sought. The economy of actual compressor compared with the isothermal may be stated in any of the following ways:

$$\left.\begin{array}{l} \dfrac{\text{Computed work per cubic foot supplied, isothermal}}{\text{Indicated work per cu.ft. actual gas supplied to compressor}} \quad (a) \\[2ex] \dfrac{\text{I.H.P. per cubic foot per minute supplied, isothermal}}{\text{I.H.P. per cubic foot per minute actual supplied}} \quad (b) \\[2ex] \text{Single stage} \\[2ex] \dfrac{\text{(m.e.p.) isothermal, pounds per square inch, no clearance}}{\text{(m.e.p.) actual} \div \text{true volumetric efficiency}} \quad (c) \\[2ex] \text{Multi-stage} \\[2ex] \dfrac{\text{(m.e.p.) isothermal, no clearance}}{\text{(m.e.p.) reduced to first stage} \div \text{first stage vol. eff.}} \quad (d) \end{array}\right\} \quad (209)$$

In this connection it is useful to note that for the case of the no-clearan cycles, the work per cubic foot of supply is equal to the mean effective pressu (M.E.P.) in pounds per square foot, and when divided by 144 gives (m.e.p in pounds per square inch. Also, that in cases with clearance, or even actu compressors with negligible clearance, but in which, due to leakage and oth causes, the true volumetric efficiency is not equal to unity,

$$\text{Work per cubic foot gas supplied} \times E_v = 144(\text{m.e.p.}). \quad . \quad . \quad (21$$

The information that is ordinarily available to determine the econon of the compressor will be in the form of indicator cards from which the (m.e.p for the individual cylinders may be obtained with ordinary accuracy. T volumetric efficiency may be approximated from the indicator cards also, b with certain errors due to leakage and heating, that will be discuss later. If by this or other more accurate means the true volumetr efficiency is found, the information required for the use of Eq. (209) (or (*d*) is available. Evaluation of the numerator may be had by Eq. (3 which is repeated below, or by reference to the curve sheets found at the e of this chapter. (Fig. 50.)

Mean effective pressure, in pounds per square inch for the isothermal co pressor without clearance is given by

$$(\text{m.e.p.})\ \text{isothermal} = (\text{sup.pr.})\ \log_e R_p. \quad . \quad . \quad . \quad . \quad . \quad (21$$

The curve sheet mentioned above also gives the economy of adiaba cycles of single stage, also two and three stages with best-receiver pressur The value of s will depend upon the substance compressed and its conditio The curve sheet is arranged to give the choice of the proper value of s applyi to the specific problem.

If it is required to find the economy of an actual compressor referred the third standard, i.e., that hypothetical adiabatic cycle which most near approaches the actual, then

Economy by third standard is

$$\frac{\text{Econ. actual referred to isothermal}}{\text{Econ. hypothetical referred to isothermal}} \quad . \quad . \quad . \quad . \quad (21$$

It is important to notice that for a vapor an isothermal process is not o following the law $P \times V = \text{constant}$. What has, in this section, been called isothermal is correctly so called only so long as the substance is a gas. Sin however, the pressure-volume analysis is not adequate for the treatment vapors, and as they will be discussed under the subject of Heat and Wo Chapter VI, it is best to regard this section as referring only to the treatm of gases, or superheated vapors which act very nearly as gases. Howev it must be understood that whenever the curve follows the law $P \times V = \text{consta}$ the isothermal equations for work apply, even if the substance be a va and the process is not isothermal.

18. Conditions of Maximum Work of Compressors. Certain types of compressors are intended to operate with a delivery pressure approximately constant, but may have a varying supply pressure. Such a case is found in pumps or compressors intended to create or maintain a vacuum and in pumping natural gas from wells to pipe lines. The former deliver to atmosphere, thus having a substantially constant delivery pressure. The supply pressure, however, is variable, depending upon the vacuum maintained. In order that such a compressor may have supplied to it a sufficient amount of power to keep it running under all conditions, it is desirable to learn in what way this power required will vary, and if it reaches a maximum what is its value, and under what conditions.

Examine first the expression for work of a single-stage adiabatic compressor with clearance. The work per cycle will vary directly as the mean effective pressure. Eq. (69.)

$$(\text{m.e.p.}) = \frac{s}{s-1}(\text{sup.pr.})\left[1+c-c\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)^{\frac{1}{s}}\right]\left[\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)^{\frac{s-1}{s}}-1\right]. \quad (213)$$

This will have a maximum value when

$$\frac{d(\text{m.e.p.})}{d(\text{sup.pr.})} = 0,$$

or when

$$\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)^{\frac{s-1}{s}} - \frac{s}{1+c}\left[1+c-c\frac{s-1}{s}\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)^{\frac{1}{s}}\right] = 0. \quad . \quad . \quad . \quad (214)$$

Solving this for the value of supply pressure will give that supply pressure at which the work will be a maximum, in terms of a given delivery pressure, clearance and the exponent s.

The assumption most commonly used is that clearance is zero. If this is true or the assumption permissible, the above equation becomes simplified,

$$\frac{(\text{del.pr.})}{(\text{sup.pr.})} = s^{\frac{s}{s-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (215)$$

The value of s for air, for instance, is 1.406, and hence the ratio of compression for maximum work for the hypothetical air compressor is

$$(1.406)^{3.46} = 3.26. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (216)$$

It may be noted that when $s=1$ in the above expression, the value of the ratio of compression become indeterminate. To find the supply pressure for maximum work in this case, take the expression for mean effective pressure for the isothermal compressor ($s=1$), Eq. (43),

$$(\text{m.e.p.}) = (\text{sup.pr.})\left[1+c-c\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)\right]\log_e\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right). \quad . \quad . \quad . \quad (217)$$

Differentiate with respect to (sup.pr.) and place the differential coefficient equa to zero. This process results in the expression

$$\log_e\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)+\frac{c}{1+c}\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)-1=0. \quad . \quad . \quad . \quad . \quad . \quad (218$$

When $c=0$, this becomes,

$$\log_e\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)=1 \quad \text{or} \quad \left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)=2.72. \quad . \quad . \quad . \quad . \quad (219$$

The expressions Eqs. (215) and (219) given are easily solved, but Eqs. (214 and (218) are not, and to facilitate computations requiring their solutio the results of the computation are given graphically on the chart, Fig. 48, a the end of this chapter.

The mean effective pressure for a compressor operating under maximu work conditions may be found by substituting the proper ratio of compressio found as above, in Eq. (213) or (217). In Fig. 48 are found also the result of this computation in the form of curves. Note in these curves that the mea effective pressure is expressed as a decimal fraction of the delivery pressur

The discussion so far applies to only single-stage compressors. The proble of *maximum work for multi-stage compressors* is somewhat different, and it solution is not so frequently required. Moreover, if the assumption of perfec intercooling is made, the results are not of great value, as a still greater amoun of power might be required, due to the failure for a period of time of the suppl of cooling water. Consider this case first.

If *intercooling be discontinued* in a multi-stage compressor, the volum entering the second stage will equal that delivered from the first, and similarl for the third and second stages. The entire work done in all stages will be th same as if it had all been done in a single stage. It might be questioned a to whether this would hold, when the ratio of compression is much less tha designed. The first stage will compress until the volume has become as sma as the low-pressure capacity of the second stage. If the delivery pressure i reached before this volume is reached, there is no work left to be done in th second or any subsequent stages, and, due to the pressure of the gas, the valve if automatic, will be lifted in the second and higher stages, and the gas wi be blown through, with only friction work. It appears then that under th condition of no intercooling the multi-stage compressor acts the *same as single stage*, and the conditions of maximum work will be the same.

If *intercooling is maintained perfect* there will still be a range of pressure on which all the work of compression is done in the first stage, merely blowin the discharge through receivers, valves and cylinders in the upper stages. I this range is such that this continues beyond a ratio of pressures, which gives maximum (m.e.p.) for the single stage, then the maximum will have been reache while the compressor is operating single stage, and *the single-stage* formul and curves may be *applied to this case also.*

That this condition frequently exists with multi-stage compressors of dinary design is shown by the fact that the ratio of compression in each stage i seldom less than 3, and more frequently 3.5, 4 or even more. The ratio c compression giving maximum work for single stage, has values from 2.5 t 3.26, dependent on clearance and the value of s for the gas compressed, and i therefore, less for the majority of cases.

If a curve be drawn, Fig. 36, with ratio of compression as abscissas and (i.e.p. ÷ del.pr.) as ordinates, so long as the action is single stage, a smooth curve ll result, but when the ratio of compression is reached above which the second linder begins to act, the curve changes direction suddenly, falling as the ratio

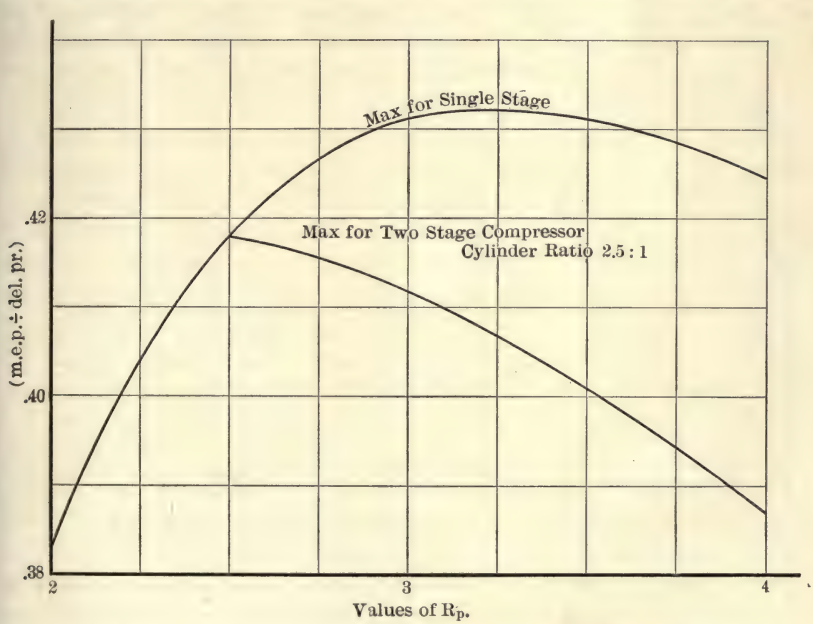


Fig. 36.—Curve of Relation between Mean Effective Pressure per Pound of Delivery Pressure in Terms of Pressure Ratio for Air, showing Maximum Value.

compression increases. Hence, if the ratio of cylinders is such that the single-ge *maximum is not reached before the second stage begins to operate*, the highest nt of the curve, or *maximum work for a given delivery pressure will occur* en the ratio of supply and delivery pressures is such as to make *first-receiver ssure equal to delivery pressure.*

19. Actual Compressor Characteristics. Air or gas compressors are very mmonly made double acting, so that for a single cylinder, two cycles will b performed during one revolution, one in each end of the cylinder. If a rod ends through one of these spaces and not through the other, the displace-nt of that end of the cylinder will be less than the other by a volume equal t the area of rod multiplied by the stroke. To avoid mechanical shock a the end of either stroke, it is necessary to leave some space between the

piston and cylinder head. Passages must also be provided, communicat with inlet and discharge valves. The total volume remaining in this sp and in the passages when the piston is at the nearer end of its stroke constit the clearance. The amount of this clearance volume varies from .5 or .(one per cent in some very large compressors to as much as 4 or 5 per cen the volume of displacement in good small cylinders.

In order to study the performance of an actual compressor and to comp it with the hypothetical cycle, it is necessary to obtain an indicator card, knowing the clearance and barometric pressure to convert the indicator c into a pressure volume diagram, by methods explained in Chapter I. Fig. 7 is such a diagram for a single-stage compressor. In the pipe leading to

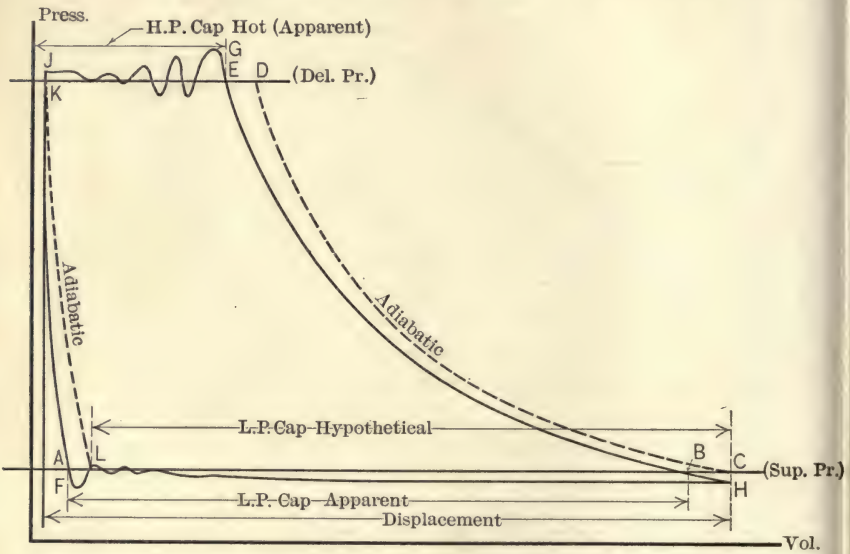


FIG. 37.—Compressor Indicator Card Illustrating Departure from Reference Cycle.

intake valve the pressure is determined and a horizontal line $\overline{AB}$ is dr on the diagram at a height to represent the supply pressure. Simila discharge pressure is determined and drawn on the diagram, $\overline{KE}$. Cons the four phases of this diagram in succession.

1. *Intake Line.* At a point somewhat below A the intake valve op say at the point F. This remains open till a point H is reached at or near end of the stroke. The line connecting these two points indicates variat of pressures and volumes throughout the supply stroke. In general this will lie below the supply-pressure line AB due to first, the pressure neces to lift the inlet valve from its seat against its spring and inertia, if autom and support it, and second, friction in the passages leading to the cylinder the point where the supply pressure was measured. While the former is n constant the latter varies, depending upon the velocity of gases in the pass The piston attains its highest velocity near the middle of the stroke,

using the intake line to drop below the supply pressure more at this part stroke. These considerations do not, however, account completely for the rm of the intake line. Frequently the first portion of the line lies lower than te last portion, even at points where the piston velocities are equal. This more prominent on a compressor having a long supply pipe, and is due to te forces required to accelerate the air in the supply pipe while piston velocity increasing, and to retard it while piston velocity is decreasing. In com-essors where the inlet valve is mechanically operated and the supply pipe ng, it is possible to obtain a pressure at the end of the intake, line H, even excess of the supply pressure. The effect of this upon volumetric efficiency ll be noted later.

The apparent fluctuations in pressure during the first part of the intake e may be attributed, first, to inertia vibrations of the indicator arm, in which se the fluctations may not indicate real variations of pressure; second, the licator card may show true variations of pressure due to inertia of the gases the supply pipe, since a moment before the valve opened at F the gases were tionary in the supply pipe. When F is reached the piston is already in motion d a very considerable velocity is demanded in the supply pipe to supply te demand. This sudden acceleration can be caused only by a difference in essure, which is seen to exist below and to the right of F on the diagram. he suddenness of this acceleration may start a surging action which will cause re and fall of pressure to a decreasing extent immediately after. A third cause possible, that is, a vibration of the inlet valve due to its sudden opening hen it is of the common form, mechanical valves change the conditions. It closed by weight or a spring and opened by the pressure difference. Between tese forces the valve disk may vibrate, so affecting the pressure.

2. *Compression Line.* From the time the inlet valve closes at the point until the discharge valve opens at the point G, the gases within the cylinder e being compressed. The compression is very nearly adiabatic in ordinary ractice, but due to the exchange of heat between the cylinder walls, at first m walls and later from gas to walls, which are cooled by water jacket to event the metal from overheating, there is a slight departure from the adia-tic law almost too small to measure.

A second factor which influences the form of this curve to a greater extent leakage. This may occur around the piston, permitting gas to escape from e end of the cylinder to the other. During the compression process there is st an excess of pressure in the other end of the cylinder due to reexpansion, nding to increase pressures on the first part of compression. Later, the essure rises and the pressure on the other side of the piston falls to supply essure. During this period leakage past piston tends to decrease successive essures or lowers the compression line. Leakage also occurs through either charge or inlet valves. The former will raise the compression line, while cessive leakage of the inlet valve will lower it.

It is then evident that unless the nature of the leakage is known, it is possible to predict the way in which it will change this line. It is, however,

more frequently the case that the piston and inlet leakage are large as compared with the discharge valves, in which case the actual compression line has a tendency to fall lower than the adiabatic as the volumes are decreased. Compression lines lowered by leakage are often mistaken for proofs of effective cooling, and cases have been known where isothermal compression of air was claimed on what proved to be evidence only of bad leakage.

3. *Delivery Line.* After the delivery pressure of the compressor has been exceeded sufficiently, the discharge valve is opened and the gas is delivered to the discharge pipe or receiver till the end of the stroke is reached and at the point J the valve closes. The same group of factors influence the form of this line as act upon the intake line; spring resistance of discharge valve; friction in discharge passages varying with piston velocity; inertia of gases in delivery pipe; sudden acceleration of gases in delivery pipe when discharge valve opens, and inertia of indicator arm, but in addition a strong tendency for the valve to chatter or jump open and shut alternately. Leakage also occurs through intake valve and past piston during this process, with the result that less gas passes through the discharge valve than is shown on the indicator card.

4. *Reexpansion Line.* From the time the discharge valve closes, at J, till the intake valve opens, at F, the gas which remained in the clearance space after delivery expands, due to the advancing of the piston, till the pressure has fallen to such an amount that the intake valve will open. The same factors influence this line as the compression line. Heat is exchanged with the jacketed cylinder walls, at first cooling and later heating the gas as the pressure falls. This, for any given volume, changes the pressure. Leakage occurs inward through the discharge valve and outward through intake valve and past piston. If these last two are in excess, the pressures will fall more rapidly than if the expansion were that of a constant quantity of gas.

Work due to gas friction and inertia, it should be noted, is fully represented on the indicator card, and may be regarded as being equal to that extra area below the supply-pressure line and above the delivery-pressure line. In the combined card of a two-stage compressor there would be an overlapping of the diagrams due to this frictional loss.

Low-pressure Capacity. Referring to the adiabatic compression and expansion lines, CD and KL, Fig. 37, it is seen that the *low-pressure capacity of the hypothetical* cycle is the volume, LC.

The *apparent low-pressure capacity* of *the actual* compressor, measured at the supply pressure is AB. This is not, however, the true volume of gas at supply pressure and temperature that is taken in, compressed, and finally delivered per cycle. First, the valves, passages and walls are not at the same temperature as the entering gas, due to the heat left from the compression of the previous charge. This causes the temperature of the gas within the cylinder to be something higher than the supply gas outside. This causes it to be less dense, and hence an equivalent weight of gas at supply temperature and pressure would occupy a volume somewhat less than AB. Second, the gas which occupies the volume AB has not all entered the cylinder through the intake

valve. After reexpansion is completed the intake valve opens and gas enters the end of the cylinder under consideration. At the same time compression is taking place in the other end, and later delivery. During these processes whatever gas leaks past the piston tends to fill the end of the cylinder in which intake is going on. Leakage past the discharge valve also tends to fill the cylinder with leakage gas. Both of these tendencies decrease the quantity of gas entering through this intake valve, and its true amount when reduced to external supply pressure and temperature is, therefore, less than the volume AB.

The *true low-pressure capacity* of the compressor is the true volume of gas under external supply conditions that enters the cylinder for each cycle. This cannot be determined from the indicator card except by making certain assumptions which involve some error at best. It can, however, be ascertained by means of additional apparatus, such as meters or calibrated nozzles or receivers, by means of which the true amount of gas compressed per unit of time is made known. This reduced to the volume per cycle under supply pressure and temperature will give the true low-pressure capacity.

Volumetric efficiency is defined as being the ratio of low-pressure capacity to displacement. On the diagram, Fig. 37, the displacement is represented to the volume scale by the horizontal distance between verticals through the extreme ends of the diagram, K and H. Since there are three ways in which the low-pressure capacity may be approximated or determined, there is a corresponding number of expressions for volumetric efficiency.

1. The *volumetric efficiency* of the *hypothetical* cycle is

$$E_v(\text{hypothetical}) = \frac{(\text{hypothetical L. P. Cap.})}{(\text{displacement})}, \quad . \quad . \quad . \quad . \quad . \quad (220)$$

and this is evaluated and used in computations in the foregoing sections of this chapter.

2. The *apparent volumetric efficiency* is

$$E_v(\text{apparent}) = \frac{(\text{apparent L. P. Cap.})}{(\text{displacement})}, \quad . \quad . \quad . \quad . \quad . \quad (221)$$

and would be very nearly equal to the true volumetric efficiency were it not for leakage valve resistance and heating during suction, but due to this may be very different from it.

3. The *true volumetric efficiency* is

$$E_v(\text{true}) = \frac{(\text{true L. P. Cap.})}{(\text{displacement})}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (222)$$

In *problems of design or prediction* it is necessary either to find dimensions, speeds and power necessary to give certain actual results, or with given dimensions and speeds to ascertain the probable power and capacity or other

characteristics of actual performance. Since it is impossible to obtain actual performance identical with the hypothetical, and since the former cannot be computed, the most satisfactory method of estimate is to perform the computations on the hypothetical cycle, as is explained in previous sections of this chapter, and then to apply to these results factors which have been found by comparing actual with hypothetical performance on existing machines as nearly like that under discussion as can be obtained. This necessitates access to data on tests performed on compressors in which not only indicator cards are taken and speed recorded, but also some reliable measurement of gas compressed.

The following factors or ratios will be found of much use, and should be evaluated whenever such data is to be had:

$$e_1 = \frac{E_v(\text{true})}{E_v\,(\text{hypothetical})} = \frac{(\text{true L. P. Cap.})}{(\text{hypothetical L. P. Cap.})}. \quad . \quad . \quad (223)$$

$$e_2 = \frac{E_v(\text{true})}{E_v(\text{apparent})} = \frac{(\text{true L. P. Cap.})}{(\text{apparent L. P. Cap.})}. \quad . \quad . \quad . \quad . \quad . \quad (224)$$

$$e_3 = \frac{\text{true I.H.P.}}{\text{hypothetical I.H.P.}} = \frac{\text{true m.e.p.}}{\text{hypothetical m.e.p.}}. \quad . \quad . \quad . \quad . \quad (225)$$

Then

$$\left.\begin{aligned} &\frac{\text{true work per cu.ft. gas, supplied}}{\text{hypothetical work per cu.ft. gas, supplied}} \\ &= \frac{\text{true I.H.P. per cu.ft. gas supplied}}{\text{hypothetical I.H.P. per cu.ft. supplied}} = \frac{\text{true } \dfrac{\text{I.H.P.}}{(\text{L. P. Cap.})}}{\text{hypothetical } \dfrac{\text{I.H.P.}}{(\text{L. P. Cap.})}} \\ &= \frac{\text{true m.e.p.}}{\text{hypothetical m.e.p.}} \div \frac{\text{true L. P. Cap.}}{\text{hypothetical L. P. Cap.}} = \frac{e_3}{e_1} \end{aligned}\right\} . \quad (226)$$

This ratio can be used to convert from hypothetical work per cubic foo gas supplied to probable true work per cubic foot.

Multi-stage Compressors are subject in each stage to all of the characteristic described for single stage to a greater or less extent. Valve resistance, friction and inertia affect the intake and discharge lines; heat transfer and leakag influence the form of compression and reexpansion lines, and the true capacit of the cylinder is made different from the apparent due to leakage, pressur and temperatures changes.

In addition to these points it is useful to note one special way in which th multi-stage compressor differs from the single stage. The discharge of the firs stage is not delivered to a reservoir in which the pressure is constant, but receiver of limited capacity. The *average* rate at which gas is delivered t the receiver must equal the average rate at which it passes to the next cylinde The momentary rate of supply and removal is not constantly the same, howeve

This causes a rise or fall of pressure. It is evident that this pressure fluctuation is greatest for a small receiver. Very small receivers are not, however, used in gas compressors due to the necessity of cooling the gas as it passes from one stage to the next. To accomplish this a large amount of cooling surface must be exposed, requiring a large chamber in which it can be done. Thus, it is seen that the hypothetical cycles assumed for multi-stage compressors do not truly represent the actual cycle, but the difference can never be very great, due to the large size of receiver which must always be used.

Another way in which the performance of this multi-stage compressor commonly differs from assumptions made in the foregoing discussions is in

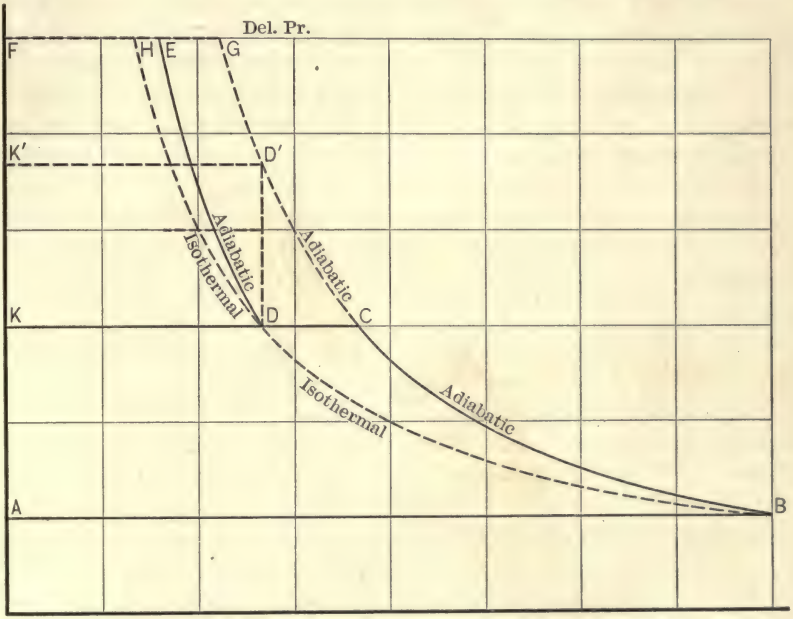


FIG. 38.—Effect of Loss of Intercooling in Two-stage Compressors on Receiver-pressure and Work Distribution in the Two Cylinders.

regard to intercooling. It seldom occurs that the gases enter all stages at the same temperature. In the several stages the temperature of the gases will depend on the amount of compression, on the cooling surface and on the amount and temperature of cooling water. The effect of variations in temperature upon the work and receiver pressures will be taken up later. It may be noted now, however, that if all cooling water is shut off, the gas passes from one cylinder to the next *without cooling*, there is no decrease in volume in the receiver. For simplicity take the case of zero clearance, two-stage (Fig. 38). Let $\overline{ABCDEF}$ be the cycle for perfect intercooling. AB and KD are the low-pressure capacities of the first and second stages respectively. If now, intercooling ceases, the gas will no longer change volume in the receiver. The receiver gas, in order to be made sufficiently dense to occupy the same

volume (KD) as it did before, must be subjected to a greater pressure in th first stage. The new receiver line will be $K'D'$. The work of the firs stage will therefore be $\overline{ABD'K'}$; of the second stage $\overline{K'D'GF}$, and the tota work in the new condition is greater than when intercoooling was perfect by a amount represented by the area $\overline{DCGE}$.

In the case where clearance is considered, the effect is the same, except tha the increasing receiver pressure, increasing the ratio of compression of the firs stage, causes the volumetric efficiency of the first stage to become less, an hence lessens the capacity of the compressor. The effect on work per unit o capacity is the same as without clearance.

The question as to *how many stages* should be used for a given compresso is dependent upon the ratio of compression largely, and so is due, first, to con siderations of economy, which can be understood from the foregoing sections second, for mechanical reasons, to avoid high pressures in large cylinders; third for thermal reasons, to avoid such high temperatures that the lubrication o the cylinders would be made difficult, or other dangers, such as explosions involved.

Practice varies very widely as to the limiting pressures for single, two three or four-stage compressors. Air compressors of a single stage are com monly used for ratios of compression as high as 6 or 7 (75 to 90 lbs. gage). Fo ratios greater than these, two-stage compressors are used, especially for large sizes, up to ratios of 34 to 51 (500 to 750 lbs. gage). Some three-stage con pressors are used for ratios as low as 11 or 14 (150 or 200 lbs. gage), althoug installations of this nature are rare, and are warranted only when power is costl and the installation permanent and continuously used to warrant the hig investment cost. As a minimum ratio for three stages, 11 (150 lbs. gag is used for large units, while a few small units compress as high as 135 or eve 170 atmospheres (2000 or 2500 lbs. gage). A notable use for the four-sta compressor is for charging the air flask of automobile torpedoes used by th various navies, which use pressures from 1600 to 3000 lbs. per square inc (110 to 200 atmospheres). These require special design of valves, cylinde and packings to withstand the extremely high pressures, small clearances, a special precautions against leakage, due to the great loss of volumetric efficien and economy that would otherwise result.

20. Work at Partial Capacity in Compressors of Variable Capacity. is seldom that a gas compressor is run continuously at its full capacity. the duty of the compressor is to charge storage tanks, it may be made to r at its full capacity until the process is completed and then may be stopp entirely, by hand. Even where the compressed gas is being used continuou it is common practice to have a storage reservoir into which the compressor m deliver. This enables the compressor to deliver a little faster or slower th the demand for a short period without a great fluctuation pressure in e reservoir. For many purposes hand regulation is not sufficient or is o expensive, hence the demand for automatic systems of capacity regulati These systems may be classified in a general way in accordance with the met

of driving. Some methods of power application permit of speed variations while others require constant speed. The former provides in itself a means of regulating capacity within certain limits, while, if the compressor must run at constant speed, some additional means of gas capacity control must be provided.

Compressors driven by an independent steam engine, or steam cylinders constituting part of the same machine may be made to run at any speed required within a very wide range and still kept low enough for safety. If driven by gear, belt, rope, chain or direct drive from a source of power whose speed is constant, the speed of the compressor cannot be varied. Electric motor, gas-engine, oil-engine or water-power drives are subject to only limited speed alteration and may, therefore, be placed in the constant speed class.

Regulation of Capacity by Means of Speed Change. If the speed of the compressor is decreased below normal:

1. Displacement of piston is decreased in proportion to the speed.

2. Mean effective pressure, as to hypothetical considerations, is the same, but due to the decrease of velocities in gas passages, the frictional fall of pressure during inlet and delivery is not so great, and hence the mean effective pressure is not quite so great. If the compressor is multi-stage, since a smaller quantity of gas is passing through the intercooler, it is probable that the intercooling is more nearly perfect, thus decreasing the mean effective pressures in the succeeding stages.

3. The volumetric efficiency is changed, due first to the fact that leakage is about the same in total amount per minute as at full speed, but the total quantity of gas being less, leakage is a larger percentage of the total; second, the inertia of gases in the supply pipe, as well as their friction, has been decreased. The former tends to decrease vulmetric efficiency, while the latter may tend to increase or decrease it. It may be expected that the true volumetric efficiency will be somewhat greater at fractional speed than at full speed.

For any compressor there is a speed of maximum economy above and below which the economy is less, though it may be that this most economical speed is greater than any speed of actual operation.

It is not desirable at this point to discuss the effect of speed variation upon the economy of the engine or other motor supplying the power. The reasoning above applies to the term economy as applied to the compression effect obtained per unit of power applied in the compression cylinder. It might be noted here, however, that the decrease of speed has little effect upon the mechanical efficiency of the compressor as a machine, since frictional resistance between solid parts remains nearly constant, and, therefore, power expended in friction will vary as the speed, as does approximately also the power to drive the compressor. The ratio of frictional power to total may then be expected to remain nearly constant.

Regulation of Capacity at Constant Speed may be accomplished in a number of ways:

1. Intermittent running;

2. Throttling the supply to compressor;

3. Periodically holding open or shut the intake valve;
4. Closing intake valve before end of intake stroke, or holding intake valve open until compression stroke has been partially completed;
5. Large clearance;
6. Variable clearance,

The *first* necessitates some means for stopping and starting the compressor, which is simple with electric drive, and may be accomplished in other cases by means of a detaching clutch or other mechanical device. The pressure in the reservoir is made to control this stopping and starting device by means of a regulator. This arrangement is made to keep the pressure in the reservoir between certain fixed limits, but does not maintain a constant pressure. The economy of compression in this case is evidently the same as at full speed continuous running, provided there is no loss in the driving system due to starting and stopping, which may not be the case. This method of regulation is used mainly for small compressors in which inertia is not great, such as supply the air brakes on trolley cars. The sudden change of load on the driving machinery would be too great if large compressors were arranged in this way.

If the compressor whose capacity is regulated by intermittent running is *multiti-stage*, the constant supply of water to the intercoolers while the compressor is stopped will lower the temperature of the cooling surface, causing more nearly perfect intercooling when the compressor is started. Leakage, on the other hand, will permit the loss of pressure to a greater or less extent in the receivers while the compressor is stationary, which must be replaced after starting before effective delivery is obtained.

Throttling the gas supply to the compressor has certain effects that may be studied by referring to Fig. 39, which represents the hypothetical cycles most nearly approaching this case. In order to reduce from the full-load low-pressure capacity, AB, to a smaller capacity, AE, the supply pressure is decreased by throttling to the pressure of B', such that B' and E lie on the same adiabatic. The work area $A'B'EA$ is entirely used up in overcoming the throttle resistance and is useless friction, so that economy is seriously reduced by this method of regulation. Such compressors may use almost as much power at partial as at full capacity.

It is easily seen that this method of regulation would be undesirable, its only advantage being simplicity.

The effect of throttling upon a *multi-stage* compressor may be illustrated as in Fig. 40, by considering the two-stage compressor cycle without clearance $ABCDEF$. The ratio of compression of the first cylinder is determined with perfect intercooling by the ratio of displacements $P_c = P_b\left(\frac{D_1}{D_2}\right)^s$. When the supply pressure is throttled down to P_b', the new receiver pressure will be $P_c' = P_b'\left(\frac{D_1}{D_2}\right)^s$, a pressure much lower than P_c. Hence the receiver pressure is decreased, less work done in the first stage, and far more than half the work

of compression done in the second stage. If best-receiver pressure existed at normal capacity, it does not exist in the throttled condition.

The intake valve may be held wide open or completely closed during one or more revolutions, thereby avoiding the delivery of any gas during that period. If the intake valve is held wide open, the indicator card would be as shown in Fig. 41*A* in full lines, *ABCD*, the dotted lines showing the cycle performed when

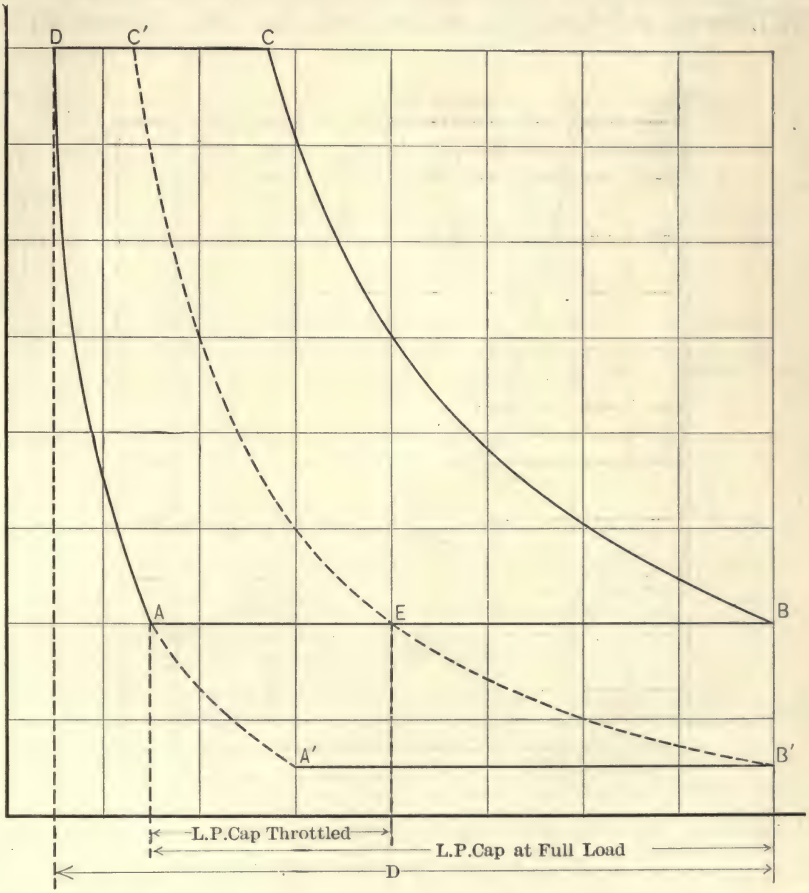


FIG. 39.—Effect of Throttling the Suction of One-stage Compressors, on Capacity and Economy.

normal operation is permitted. With the inlet valve open in this way there is a loss of power due to friction of the gas in passage during both strokes, measured by the area within the loop.

Closing the inlet valve and holding it shut will give an indicator card of the form *EFG*, Fig. 41*B*, which will be a single line retraced in both directions except for probable leakage effects. If leakage is small, there will be but little

area enclosed between the lines. At a high speed this might be expected to incur less lost power than the former plan.

Certain types of compressors are made with an intake valve controlled by a drop cut-off, much like the steam valve of the Corliss engine. The effect of this is to cut off the supply of gas before the end of the stroke, after which time the gas must expand hypothetically according to the adiabatic law. The return stroke causes it to compress along the same line continued up to the delivery pressure, as indicated by the line *FEG*, Fig. 41*C*. There is little work

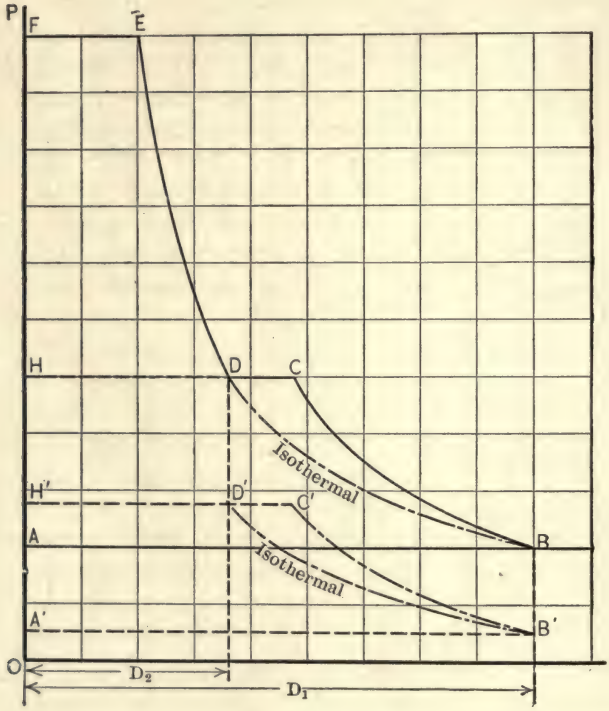


FIG. 40.—Effect of Throttling Multi-stage Compressors on Receiver-pressure, Work Distribution, Capacity and Economy.

lost in the process, none, if the line is superimposed as in the figure, and hence the process is the same as if only the cycle *AEGD* were performed.

The same quantity of gas might have been entrapped in the cylinder by holding the intake valve open until the end of the stroke and on the return till the point *E*, Fig. 41*D*, was reached, then closing it. The same compression line *EG* will be produced. The line *AB* will not coincide with *BE*, due to friction of the gas in passages, and hence will enclose between them a small area representing lost work, which may be no larger than that lost in the process *EFE*.

If such an automatic cut-off were applied only to the first stage of a *multi-stage* compressor, the effect would be to lower receiver pressures as in the throttling process. To avoid this, the best practice is to have a similar cut-off to act on the supply to all of the stages. If this is properly adjusted, the receiver pressures can be maintained the same as at full load. An additional advantage of this system is that even if the compressor is to be used for a delivery pressure for which it was not originally designed, the relative cut-offs may be so adjusted as to give and maintain best-receiver pressure.

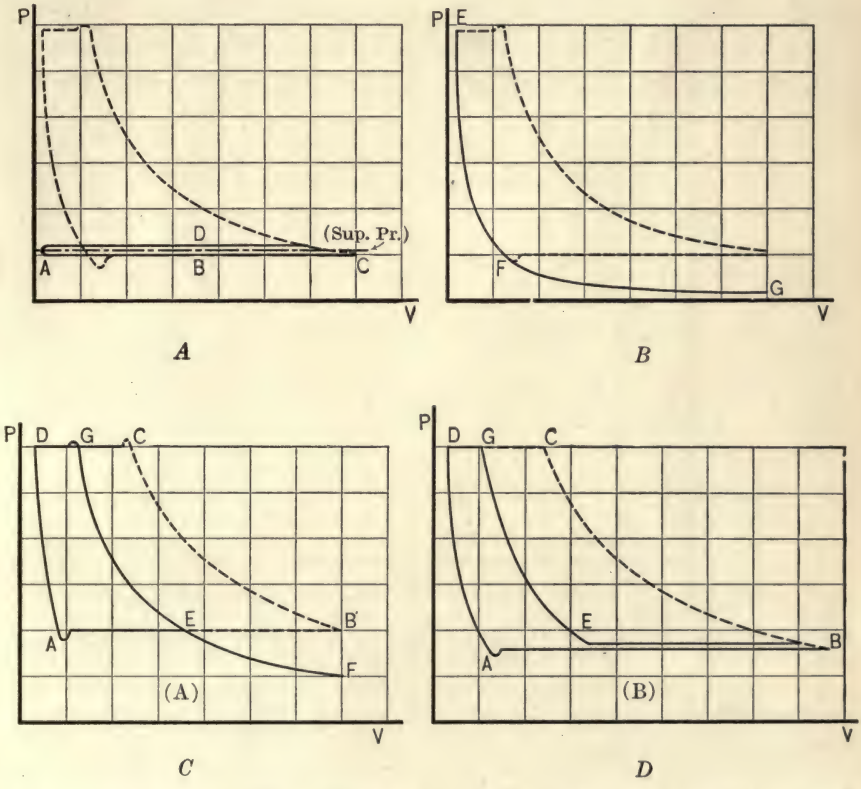


FIG. 41.—Control of Compressor Capacity by *A*. Open Inlet Valve; *B*. Closed Inlet Valve; *C*. Suction Cut-off; *D*. Delayed Suction Closure.

Since the low-pressure capacity per cycle of a compressor involves clearance and ratio of compression as two of its variables, it is possible to change capacity by changing either the clearance or the ratio of compression.

$$(\text{L. P. Cap.}) = DE_v = D(1+c-cR_p^{\frac{1}{s}}) \quad . \quad . \quad . \quad . \quad . \quad (227)$$

Assuming that clearance is a fixed amount and not zero, it is evident that an increase in the ratio of compression decreases the capacity, and when it has

reached a certain quantity will make the capacity zero. If the clearance is large, making the coefficient of $R_p^{\frac{1}{s}}$ large in the equation the effect of a change in that factor is increased. Fig. 42 indicates the hypothetical performance of a compressor with large clearance. When the pressure of delivery is low (say P_e) the capacity is large, AB. The cycle is then $ABCD$. An increase of the delivery pressure to P_e' changes the cycle to $A'BC'D'$ and the low pressure capacity is $A'B$. If the compressor is delivering to a receiver from which no gas is being drawn, the delivery pressure will continue to rise and the capacity

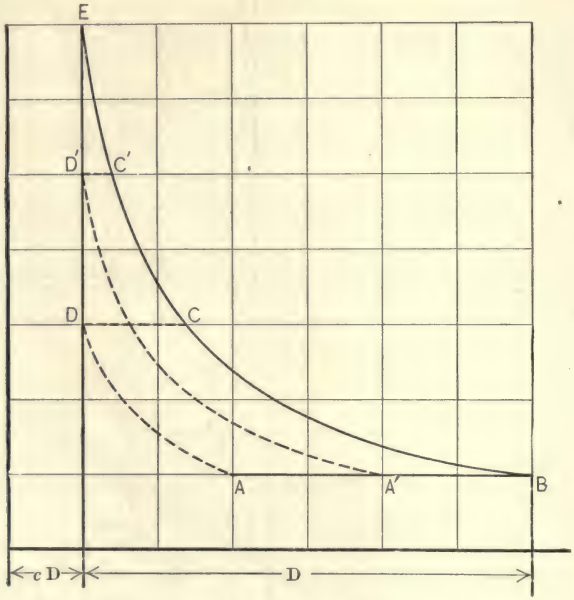


FIG. 42.—Variation of Compressor Capacity with Rise of Delivery Pressure, Fixed Clearance, Pressure for Zero Delivery.

to decrease till the capacity approaches zero as the delivery pressure approaches the pressure P_e as a limit.

$$(\text{limiting del.pr.}) = (\text{sup.pr.})\left(\frac{1+c}{c}\right)^s \quad . \quad . \quad . \quad . \quad . \quad (228)$$

When the limiting condition has been reached and the capacity has become zero, the compression and reexpansion lines coincide and enclose zero area between them; hence, the mean effective pressure and the indicated horsepower are zero, for the hypothetical case. Leakage will prevent a perfect coincidence of the lines and cause some power to be required in addition to that of friction.

Such a simple method of regulation as this is used for some small compressors driven constantly from some source of power used primarily for other purposes. When it is not necessary to have a constant delivery pressure,

but only to keep it between certain limits, this may be made use of, especially if the limits of pressure are quite wide.

The expression for low-pressure capacity Eq. (227), suggests the possibility of decreasing capacity by the increase of clearance. The effect of this is shown in Fig. 43. The original compression cycle (full capacity) is shown by $ABCD$, with a clearance volume of cD, so that the axis of zero volume is OP. Increasing the clearance to $c'D$ causes a smaller volume $C'D$ to be delivered and due to the more sloping re-expansion DA', a smaller volume of gas is taken in, $A'B$.

It has been shown in previous sections that clearance has no effect upon the economy of a compressor so far as hypothetical considerations are regarded. In practice it is found that a slight loss of economy is suffered at light load,

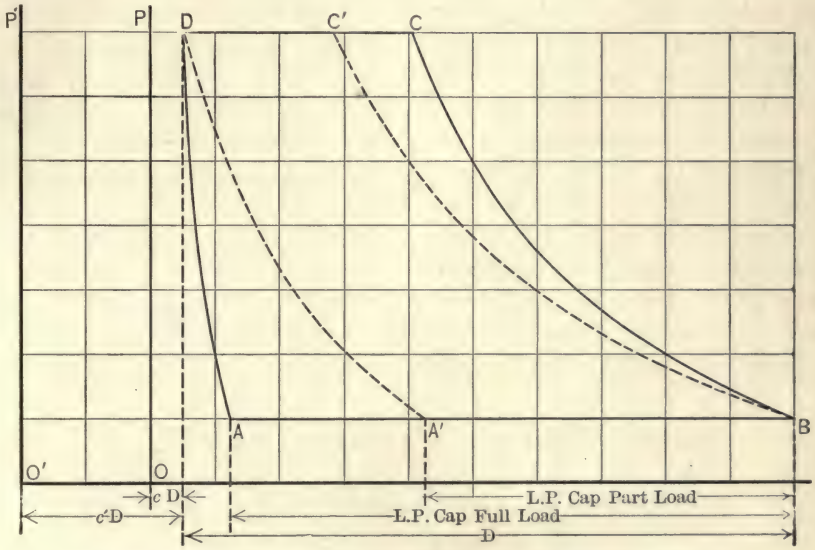


FIG. 43.—Variation of Compressor Capacity by Changing Clearance.

as might be expected, due to greater leakage per unit of capacity. The additional clearance is provided in the form of two or more chambers connected to the clearance space of the compressor by a passage in which is a valve automatically controlled by the receiver pressure.

In the *multi-stage* compressor, decreasing the capacity of the first stage by an increase of its clearance would evidently permit a decrease of receiver pressures unless the capacity of each of the various stages is decreased in the same proportion. Eq. (132) gives the condition which must be fulfilled to give best receiver pressure for a two-stage compressor.

$$R_p^{\frac{1}{2}} = \frac{D_1\left[1+c_1-c_1R_p^{\frac{1}{2s}}\right]}{D_2\left[1+c_2-c_2R_p^{\frac{1}{2s}}\right]}.$$

Since D_1, D_2, and R_p remain fixed, for any chosen value of clearance of the first stage, c_1, the clearance of the second stage, c_2, to give best-receiver pressure can be found,

$$c_2 = \frac{[1 - c_1(R_p^{\frac{1}{2s}} - 1)]D_1}{(R_p^{\frac{1}{2s}} - 1)R_p^{\frac{1}{2s}}D_2}. \quad \ldots\ldots\ldots \quad (229)$$

For every value of first-stage clearance there is a corresponding clearance of second stage that will give best-receiver pressure, found by this equation. Similar reasoning can be applied to three- or four-stage compressors.

21. Graphic Solution of Compressor Problems. In order to obviate the necessity of working out the formulas given in this chapter each time a problem is to be solved, several of them have been worked out for one or more cases and results arranged to give a series of answers graphically. By the use of the charts made up of these curves many problems may be solved directly and in many others certain steps may be shortened. A description of each chart, its derivation and use is given in subsequent paragraphs.

Chart, Fig. 44. This chart gives the work required to compress and deliver a cubic foot of (sup.pr.) air or the horse-power to compress and deliver 1000 cu.ft. of (sup.pr.) air per minute if the ratio of pressure (del.pr.) ÷ (sup.pr.), the value of s and the (sup.pr.) are known, and compression occurs *in one stage.* The work or H.P. for any number of cubic feet is directly proportional to number of feet. The curves are dependent upon the formulas, Eq. (31), for the case when $s=1$, and Eq. (51) for the case when s is not equal to 1. These formulas are:

Eq. (31), W per cu.ft. $= 144$ (sup.pr) $\log_e R_p$;

" (51), W per cu.ft. $= 144\frac{s}{s-1}$ (sup.pr.)$\left(R_p^{\frac{s-1}{s}} - 1\right)$.

These equations are difficult to solve if an attempt is made to get a relation between the work and ratio of pressures. This relation may, however, be worked out for a number of values of pressure ratios and results plotted to form a curve by which the relation may be had for any other ratio within limits. This has been done in this figure in the following manner:

On a horizontal base various values of R_p are laid off, starting with the value 2 at the origin. The values for work were then found for a number of values of R_p with a constant value of (sup.pr.) and s. A vertical work scale was then laid off from origin of R_p and a curve drawn through the points found by the intersection of horizontal lines through values of work, with vertical lines through corresponding values of R_p. The process was then repeated for other values of s and curves similar to the first, drawn for the other values of s. From the construction so far completed it is possible to find the work per cubic foot for any pressure ratio and any value of s for *one* (sup.pr.) by pro-

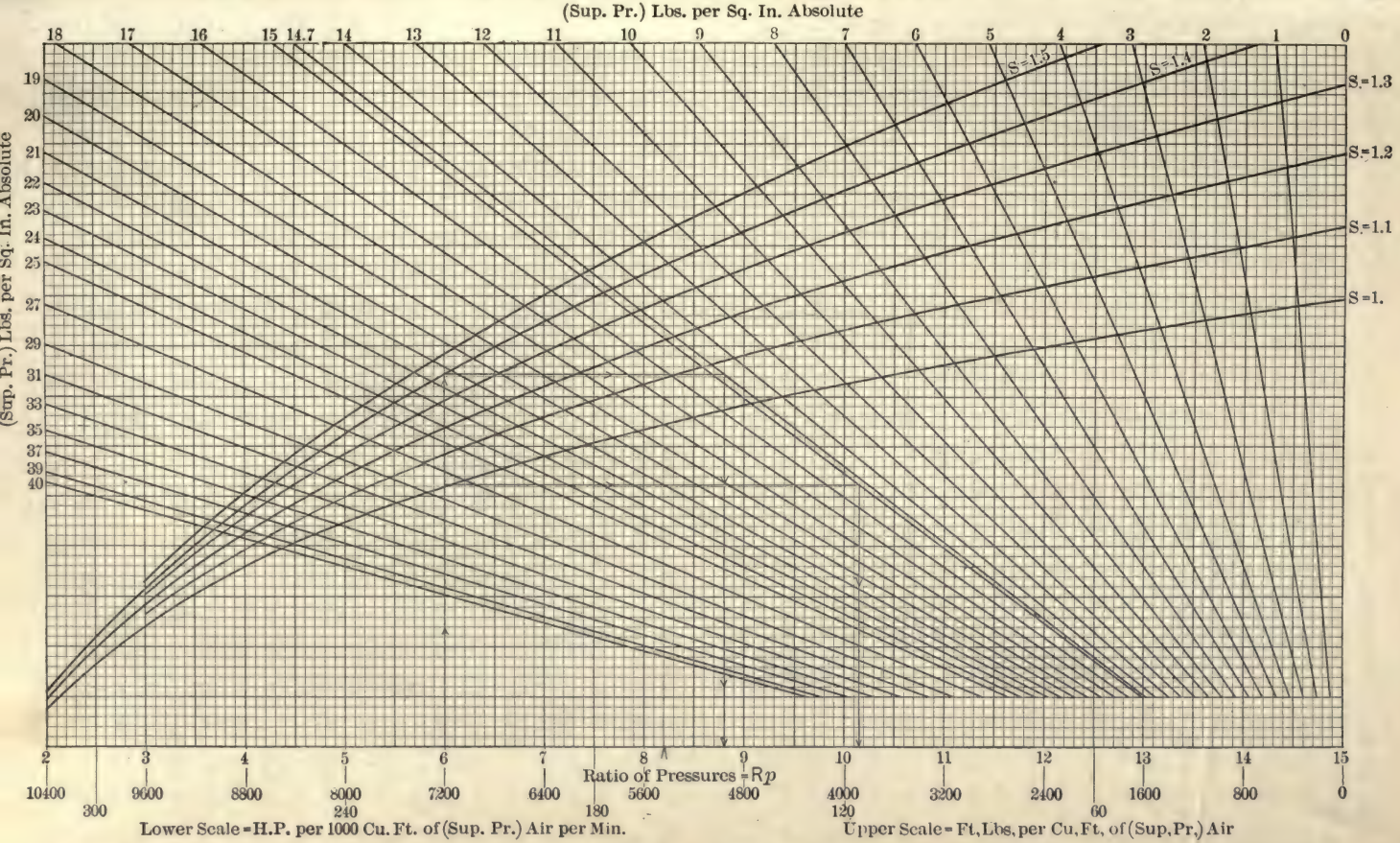


FIG. 44.—Work per Cubic Foot and Horse-power per 1000 Cubic Feet per Minute of Supply Pressure Gas for *Single-stage Compressors.*

jecting up from the proper value of R_p to the curve of value of s and then horizontally to the scale of work. It will be noted from these formulas, however, that the work may be laid off on the horizontal base and a group of lines drawn so that the slope of the line equals ratio of work for any supply pressure to that for the (sup.pr.) originally used. For convenience, in order that the group of s curves and the latter group may be as distinct as possible, the origin of the latter group is taken at the opposite end of the base line. If from the point for work originally found, a projection is made horizontally to the proper (sup.pr.) curve, the value for work with this (sup.pr.) will be found directly below. It will be noted that from point of intersection of the vertical from the R_p value with the s curve, it is only necessary to project horizontally far enough to intersect the desired (sup.pr.) curve, and since no information of value will be found by continuing to the work scale for the original (sup.pr.) this is omitted from the diagram.

In brief, then, the use of this chart consists in projecting upward from the proper value of R_p to the proper s curve, then passing horizontally to the value of (sup.pr.) and finally downward to the work scale. As an example of use of the curve Ex. 2 of Section (8) may be solved directly. This is to find the work to compress 1000 cu.ft. of free air from 1 to $8\frac{1}{2}$ atmospheres adiabatically. On the curve project upward from $R_p = 8.5$ to curve of $s = 1.406$, then over to 14.7 (sup.pr.) curve and down to read work = 6,300,000 as found, for example, by use of formulas in Section (8).

Chart, Fig. 45. This gives the work required to compress and deliver a cubic foot of (sup.pr.) air or the horse-power to compress and deliver 1000 cu.ft. of (sup.pr.) air per minute if the ratio of pressures, the value of s and (sup.pr.) are known and if compression occurs *in two stages* with best-receiver pressure and perfect intercooling. The work or H.P. for any other number of cubic feet may be found by multiplying work per foot by the number of feet. The method of arriving at this chart was exactly the same as that for one stage.

As an example of the use of the chart, Example 2 of Section (9) may be solved directly. This problem calls for the work to compress 5 cu.ft. of free air from 1 to $8\frac{1}{2}$ atmospheres adiabatically in two stages. Project upward from $R_p = 8.5$ to curve $s = 1.406$, then over to 14.7 curve and down to read 5320 ft.-lbs. per cubic foot, which is same as found from the formula in Section (9).

Chart, Fig. 46. This chart gives the work necessary to compress and deliver a cubic foot of (sup.pr.) air or horse-power to compress and deliver 1000 cu.ft. of (sup.pr.) air per minute, if the ratio of pressures, the value of s, and the (sup.pr.) are known and if the compression occurs *in three stages* with best receiver pressures and perfect intercooling. The work or horse-power for any other number of cubic feet may be found by multiplying the work for one foot by the number of feet.

As an example of use of this chart, Example 2 of Section (13) may be solved directly byit. This calls for the horse-power to compress 100 cu.ft. free air per minute adiabatically in three stages from 15 lbs. per square inch abs. to 90 lbs. per square inch gage. From $R_p = 7$, project to curve of $s = 1.4$ the

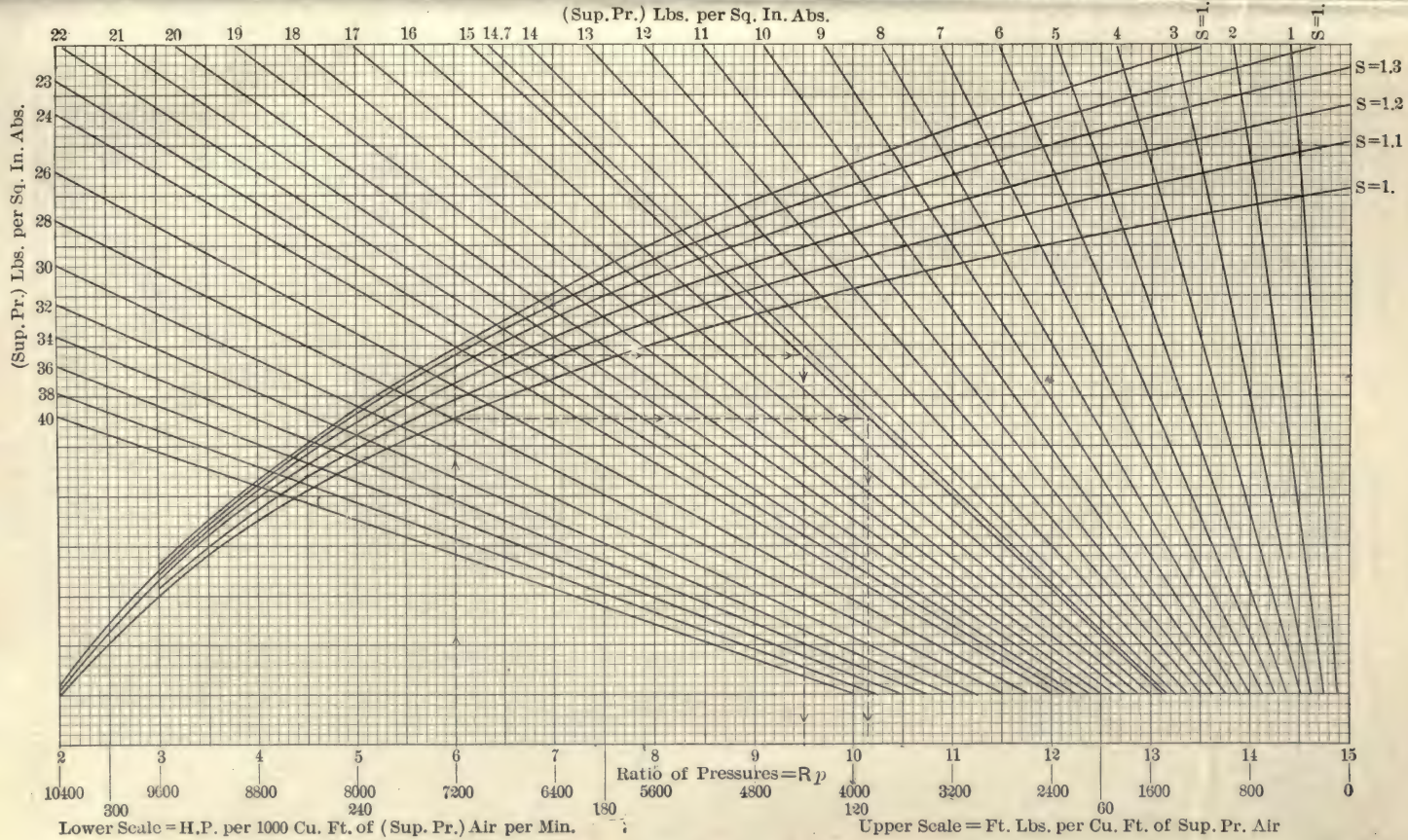


FIG. 45.—Work per Cubic Foot and Horse-power per 1000 Cubic Feet per Minute of Supply Pressure Gas for *Two-stage Compressors*, Best Receiver-pressure, Perfect Intercooling.

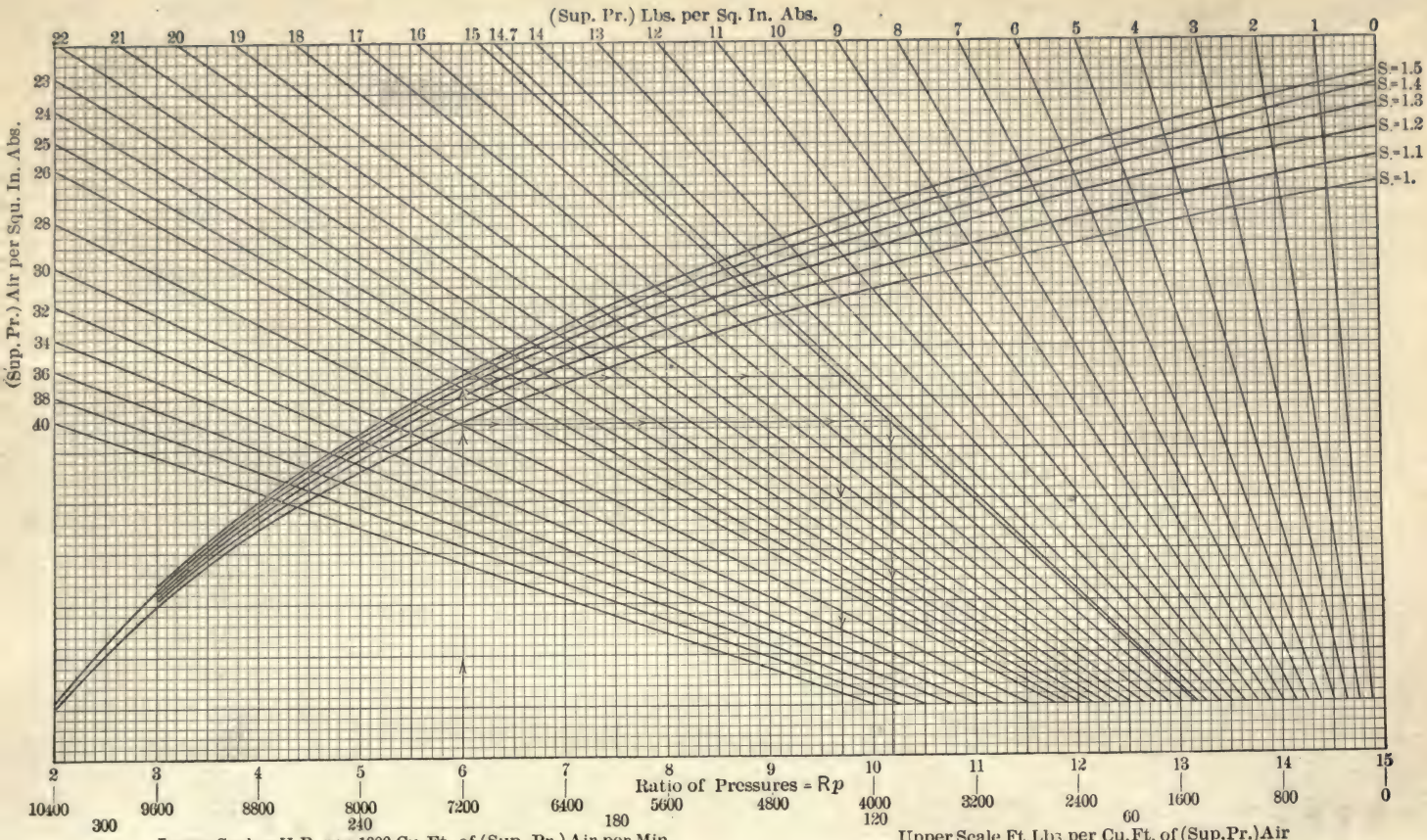


FIG. 46.—Work per Cubic Foot and Horse-power per 1000 Cubic Feet per Minute of Supply Pressure Gas for *Three-stage Compressors*,

er to (sup.pr.) = 15 and down, and the horse-power will be found to be 13.6 before by use of formulas.

Chart, Fig. 47. This chart is for finding the (m.e.p.) of compressors. In the se of multi-stage compressors with best-receiver pressure and perfect inter-oling, the (m.e.p.) of each cylinder may be found by considering each cylinder a single-stage compressor, or the (m.e.p.) of the compressor referred to the P. cylinder may be found.

The chart depends on the fact that the work per cubic foot of (sup.pr.) gas equal to the (m.e.p.) for the no clearance case and that the (m.e.p.) with arance is equal to the (m.e.p.) for no clearance, times the volumetric iciency. Diagrams 1, 2 and 4 are reproductions of Figs. 45, 46 and 47 to a aller scale and hence need no explanation as to derivation. Their use may briefly given. From the proper ratio of pressures project upward to the per curve, then horizontally to the (sup.pr.) and downward to read work per bic feet of (sup.pr.) gas.

The volumetric efficiency diagram was drawn in the following manner: om Eq. (65) vol. eff. $= (1+c-cR_p^{\frac{1}{s}})$, showing that it depends upon three riables, R_p, c and s. A horizontal scale of values of R_p was laid off. Values $R_p^{\frac{1}{s}}$ were found and a vertical scale of this quantity laid off from the same gin as the R_p values. Through the intersection of the verticals from various lues of R_p with the horizontals drawn through the corresponding values of $_p)^{\frac{1}{s}}$ for a known value of s, a curve of this value of s was drawn. In a similar y curves of other values of s were drawn. From the construction so far mpleted it is possible to find the value of $(R_p)^{\frac{1}{s}}$ by projecting upward from any lue of R_p to the curve of s and then horizontally to the scale of $(R_p)^{\frac{1}{s}}$. Values volumetric efficiencies found for various clearances and the values of $(R_p)^{\frac{1}{s}}$ laid off on a horizontal base, with the origin at the opposite end of scale m that of R_p values, in order that clearance curves and s curves might be distant as possible. These clearance curves were drawn through the inter-tion of horizontals through the $(R_p)^{\frac{1}{s}}$ values and of verticals through the lmetric efficiency values corresponding to them for the particular clearance question.

To find volumetric efficiency then it is merely necessary to project from value R_p to the correct s curve, then across to the proper clearance and finally wn to volumetric efficiency. As the value of $(R_p)^{\frac{1}{s}}$ is not desired, the hori-ntal projection is carried only to the intersection with the clearance curve d not to the edge of the diagram. To find the (m.e.p.) for single stage, the rk per cubic foot is found from the diagram and then the volumetric efficiency, th as described above. *The product is* (*m.e.p.*)

For multi-stage compressors with perfect intercooling and best-receiver essure, as stated above, the (m.e.p.) of each cylinder may be found, considering

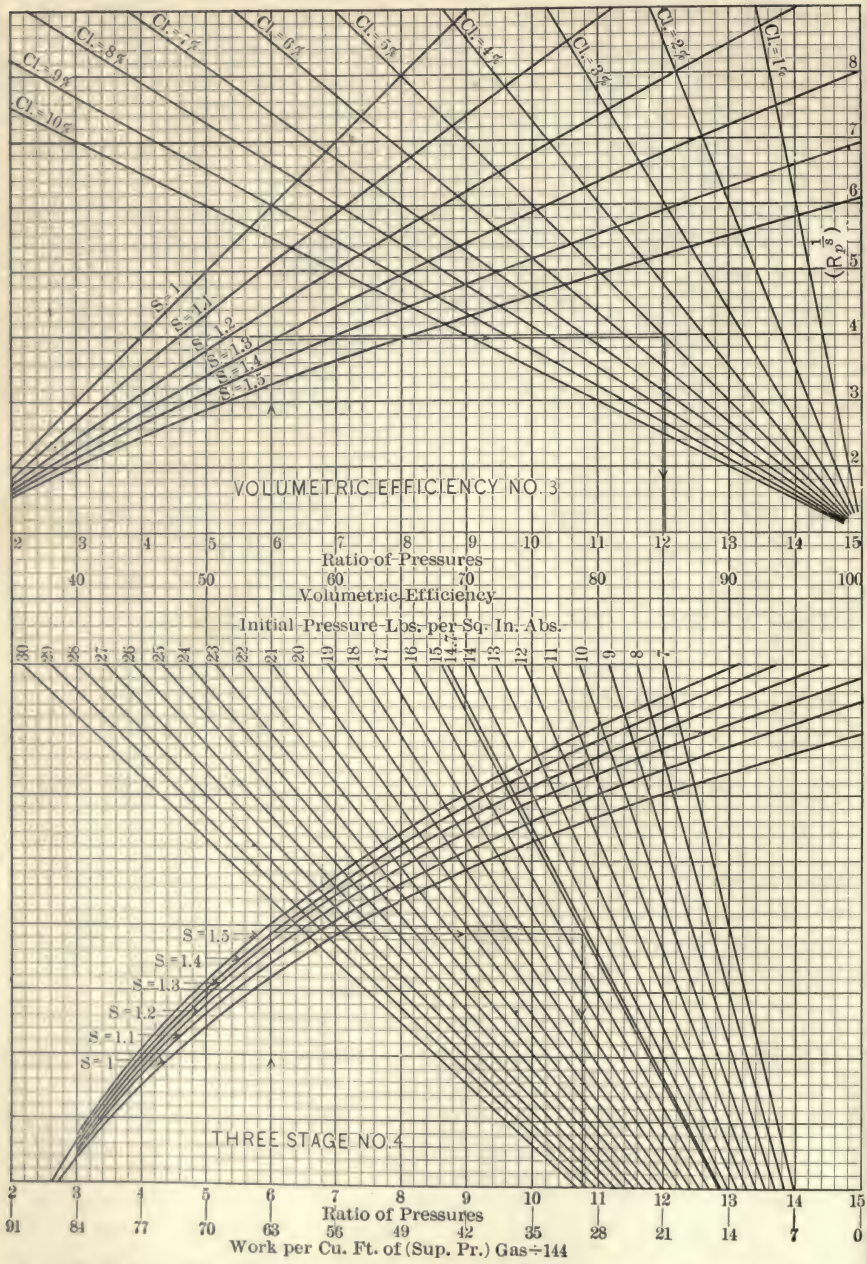


FIG. 47.—Mean Effective Pressure of Compressors, One-, Two-, and Three-stages.

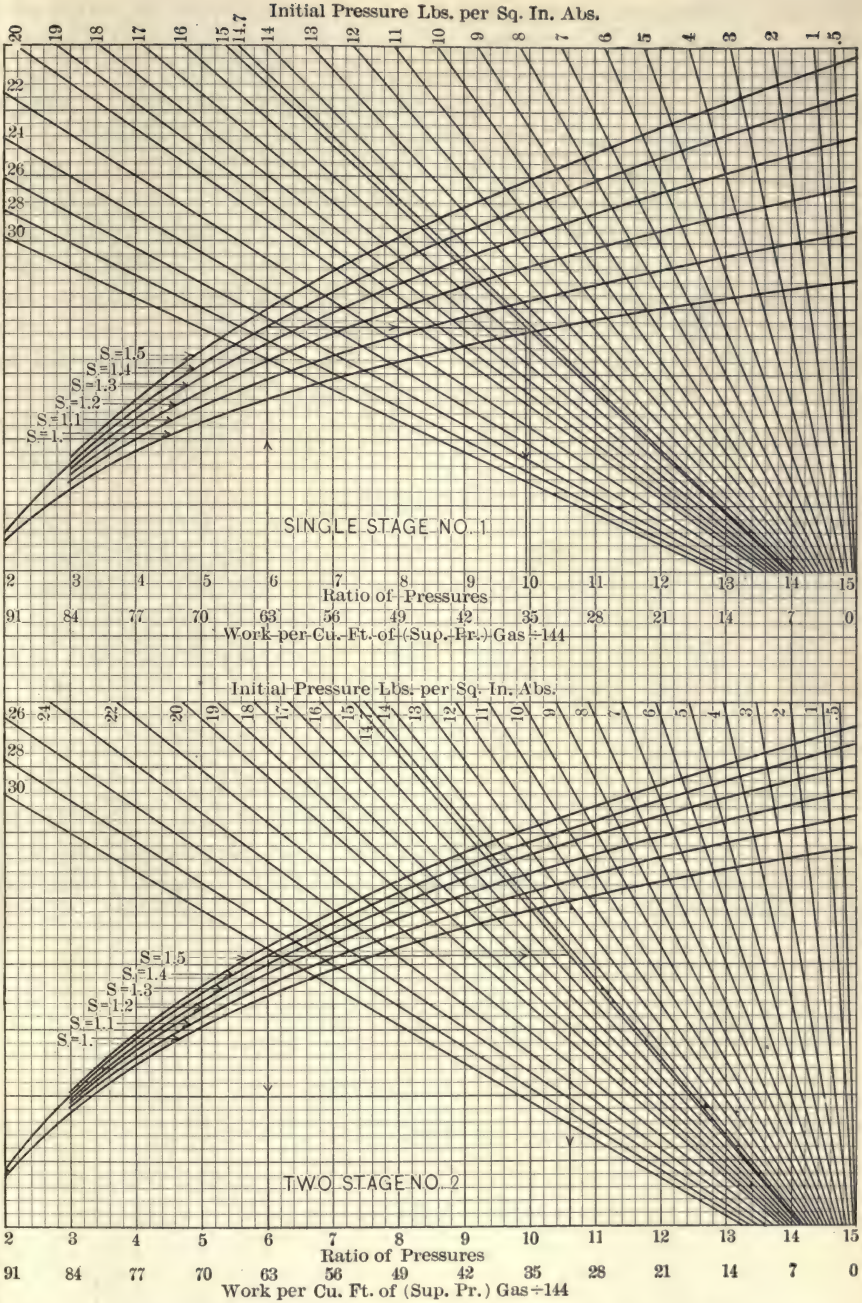


FIG. 47.—Mean Effective Pressure of Compressors, One-, Two-, and Three-stages.

each to be a single-stage compressor and remembering that (1 rec.pr.) becomes (sup.pr.) for second stage, and (del.pr.) for first stage and that (2 rec.pr.) becomes (sup.pr.) for third stage, (del.pr.) for second stage. The (m.e.p.) reduced to low-pressure cylinder is found by taking work per cubic feet of (sup.pr.) gas and multiplying by volumetric efficiency of low-pressure cylinder.

To illustrate the use of this curve the example of Section (16) may be solved. Projecting upward from the pressure ratio of 9.35 to the line of $s=1.4$ and then over to (sup.pr.) = 15 in diagram 4, since compression is three stage and from 15 lbs. per square inch to 140 lbs. per square inch, work per cubic foot or (m.e.p.), is found for no clearance to be 37.8 abs. per square inch. Since best-receiver pressure assumed is 31.6, which gives a ratio of 2.1 for the low-pressure cylinder. From diagram 3, by projecting upward from $R_p=2.1$ and over to the 5 per cent clearance line volumetric efficiency is 96.5. The product gives (m.e.p.)reduced to low-pressure cylinder and is 36.5. From the $\frac{(\text{m.e.p.})LaN}{33{,}000}$ formula, horse-power is found to be 358 as before.

Chart, Fig. 48. As mentioned in Section 18, there is one (sup.pr.), which for a given (del.pr.) will give the maximum work of compression. The chart, Fig. 48, originated by Mr. T. M. Gunn, gives a graphical means of finding this value of (sup.pr.) when the (del.pr.), clearance and value of s are known. It also gives on the right-hand of the chart a means for finding the (m.e.p.) for this condition. The figure was drawn by means of Eqs. (214) and (218). For the value of $s=1$ the ratio of (del.pr.) to (sup.pr.) was found for cases of clearances from 0 per cent to 15 by means of Eq. (218) by trying values of this ratio which would fulfill the condition of the equation, $\log_e\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)=1-\frac{c}{1+c}\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)$. For values of s not 1, Eq. (214) was used, and a set of values of R_p found for the values of $s=1.4$ and 1.2 by trial, the correct value of R_p being that which satisfied the equation,

$$\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)^{\frac{s-1}{s}}=\frac{s}{1+c}\left[1+c-c\frac{s-1}{s}\left(\frac{\text{del.pr.}}{\text{sup.pr.}}\right)^{\frac{1}{s}}.\right]$$

As an example the work for the case where $s=1.2$ and $c=10$ per cent is given.

$$\text{Try } R_p=2.6, \text{ then, } R_p^{\frac{.2}{1.2}}=\frac{1.2}{1+.1}\left[1+.1-.1\times\frac{.2}{1.2}\times 2.6^{.833}\right]$$

$$=1.091(1.1-.01667\times 2.218),$$

$$=1.161$$

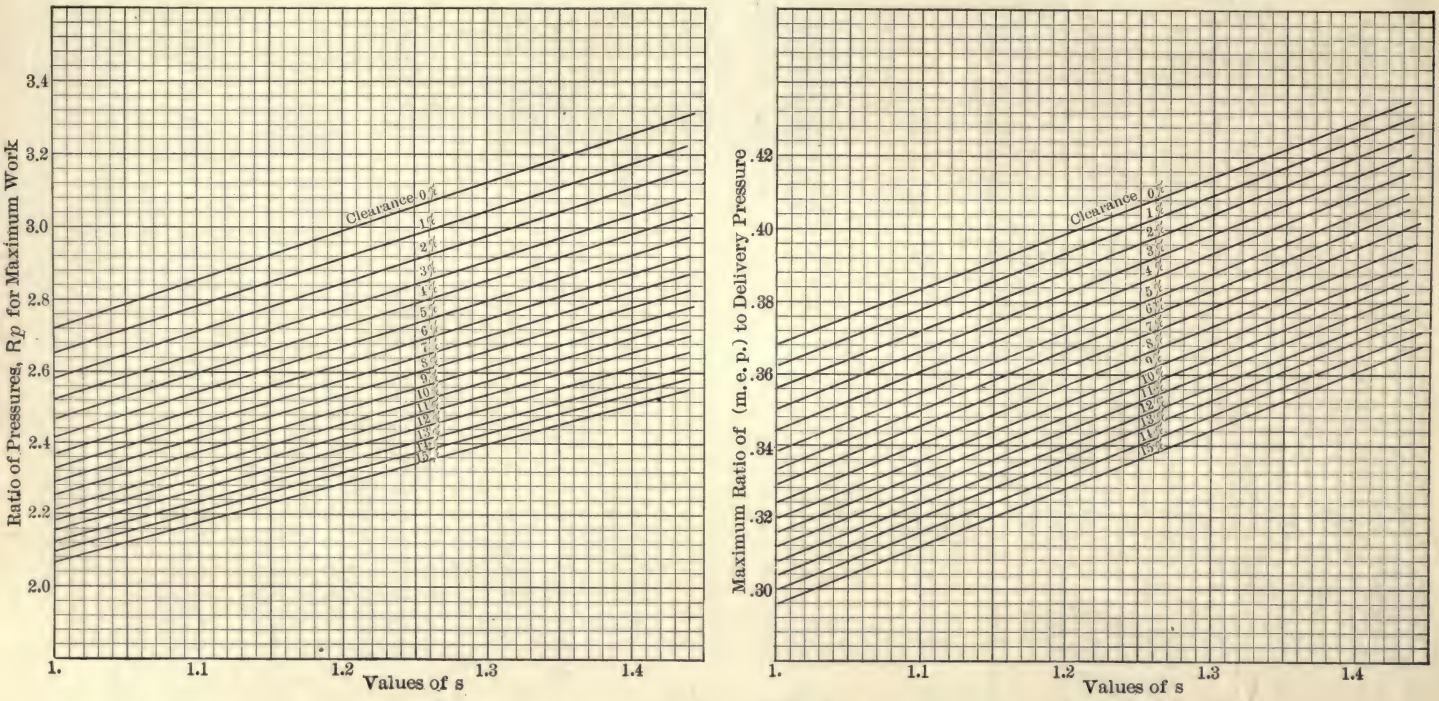


FIG. 48.—Value of Supply Pressure that Results in Maximum Work and Corresponding Mean Effective Pressure.

$R_p = (1.161)^6 = 2.45$, which shows the value of 2.6 to be incorrect. For a second trial take 2.45, and then,

$$R_p^{\frac{1}{6}} = 1.091(1.1 - .01667 \times 2.45^{.833}),$$
$$= 1.1627$$
$$= (1.1617)^6 = 2.458,$$

which is sufficiently close. Therefore the value of R_p for $s = 1.2$ and clearance = 10 per cent, is 2.45; that is, maximum work will occur for a given (del.pr.) when (sup.pr.) is $\frac{1}{2.45}$ times the (del.pr.)

When the values for R_p had been obtained a horizontal axis of values of s and a vertical one of R_p values, were laid off and the points for clearance curves laid off to their proper values referred to these axes. Through points as plotted the clearance lines were drawn. The right-hand diagram was plotted in a similar manner from Eqs. (213) and (217), for s not equal to 1 and equal to 1 respectively.

The latter formula was rearranged in the form

$$\left(\frac{\text{m.e.p.}}{\text{del.pr.}}\right) = \frac{\log_e R_p}{R_p} E_v,$$

the last term being found from curve of Fig. 45. The value of R_p for each value of the clearance was taken from the left-hand diagram, and substituted in the above expression to obtain $\left(\frac{\text{m.e.p.}}{\text{del.pr.}}\right)$ for the case of $s = 1$. Eq. (213) was put in form

$$\left(\frac{\text{m.e.p.}}{\text{del.pr.}}\right) = \frac{s}{(s-1)R_p}(R_p^{\frac{s-1}{s}} - 1)E_v,$$

and values of R_p for each value of the clearance found in the left-hand diagram were substituted, together with E_v values from Fig. 45 and the value of $\left(\frac{\text{m.e.p.}}{\text{del.pr.}}\right)$ found for each case of clearance when $s = 1.4$. When the points for $s = 1$ and $s = 1.4$ had been found, a horizontal axis of values of s and a vertical one of values of R_p were laid off, and points for the clearance curves plotted as for the left-hand diagram and the curves drawn in.

To find the (sup.pr.) to give maximum work for any (del.pr.) it is only necessary to project from the proper value of s to the proper clearance curve, and then horizontally to read the value of R_p. The (del.pr.) divided by this gives the (sup.pr.) desired. To obtain the (m.e.p.) project upward from the value of s to the clearance curve, then horizontally to read the ratio $\left(\frac{\text{m.e.p.}}{\text{del.pr.}}\right)$ *The (del.pr.) times this quantity gives the m.e.p.*

As an example of the use of this chart let it be required to find the (sup.pr.) for the case of maximum work for 9×12 in. double-acting compressor running 200 R.P.M., having 5 per cent clearance and delivering against 45 lbs. per square inch gage. Also the horse-power. Compression such that $s = 1.3$.

Projecting from the value 1.3 for s on the left-hand diagram to the line of 5 per cent clearance find R_p to be 2.8, hence (sup.pr.) $=\frac{60}{2.8}=21.4$ lbs. per square inch absolute $=6.4$ lbs. per square inch gage. Again, projecting from value 1.3 for s on right-hand diagram to line of 5 per cent clearance find that $\frac{(\text{m.e.p.})}{(\text{del.pr.})}=.383$, hence (m.e.p.) $=23$ and

$$\text{I.H.P.}=\frac{23\times1\times64\times400}{33{,}000}=17.8.$$

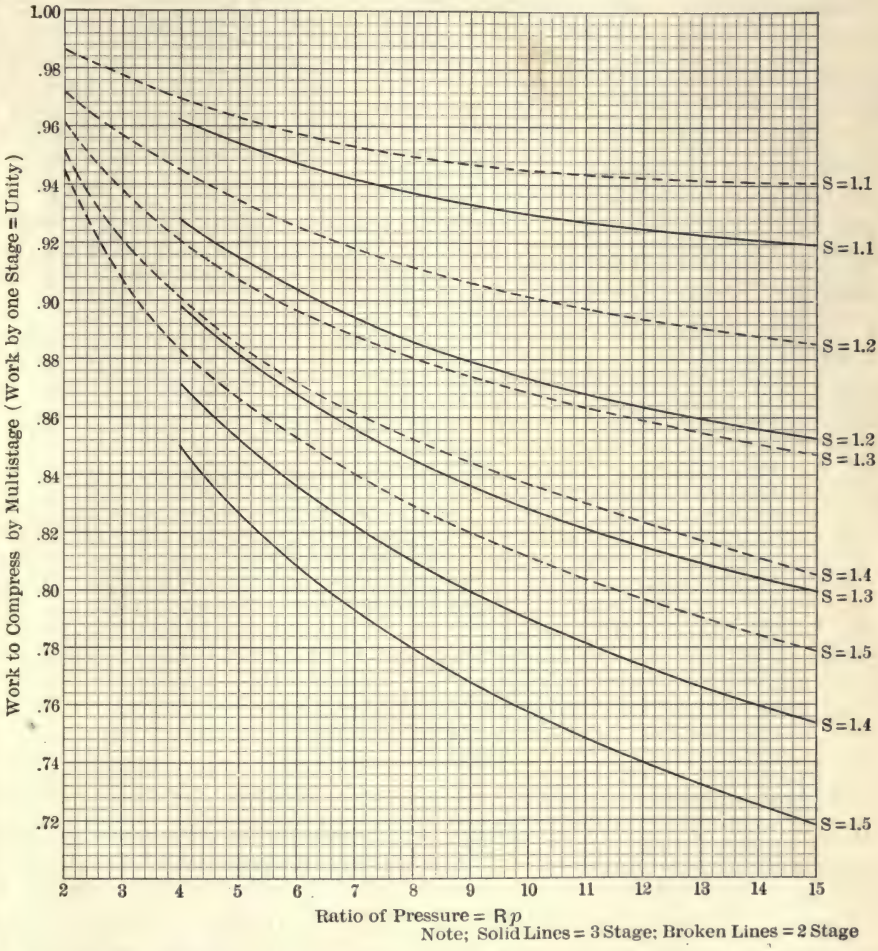


FIG. 49.—Relative Work of Two- and Three-stage Compressors Compared to Single Stage.

Chart, Fig. 49. This chart is designed to show the saving in work done in compressing and delivering gases by two-stage or three-stage compression with best-receiver pressure and perfect intercooling over that required for compressing and delivering the same gas between the same pressures in one stage.

The chart was made by laying off on a horizontal base a scale of pressure ratios. From the same origin a scale of work for two or three stage divided by the work of one stage was drawn vertically. For a number of values of R_p the work to compress a cubic foot of gas was found for one, two and three stage for each value of s. The values found by dividing the work of two or three stage by the work of single stage were plotted above the proper R_p values and opposite the proper ratio values and curves drawn through all points for one value of s. To find the saving by compressing in two or three stages project from the proper R_p value to the chosen s curve for the,desired number of stages, then horizontally to read the ratio of multi-stage to one-stage work. This value gives per cent power needed for one stage that will be required to compress the same gas multi-stage. Saving by multi-stage as a percentage of single stage is one minus the value read.

To illustrate the use of this chart, find the per cent of work needed to compress a cubic foot of air adiabatically from 1 to $8\frac{1}{2}$ atmospheres in two stages compared to doing it in one stage. From examples under chart Nos. 44 and 46 it was found that work was 6300 ft.-lbs. and 5320 ft.-lbs. respectively, for one- and two-stage compression, or that two stage was 84.5 per cent of one stage. From R_p, $8\frac{1}{2}$ project up on Fig. 49 to $s=1.406$ for two stage and over to read 84.6 per cent, which is nearly the same.

Chart, Fig. 50. This chart, designed by Mr. T. M. Gunn, shows the economy compared to isothermal compression.

The chart was drawn on the basis of the following equation:

$$\text{Economy (isothermal)} = \frac{\text{m.e.p. isothermal (no clearance)}}{\text{m.e.p. actual} \div E_v \text{ actual}}$$

$$= \frac{(\text{sup.pr.}) \log_e R_p}{\frac{s}{s-1}(\text{sup.pr.})(R_p^{\frac{s-1}{s}} - 1)}.$$

$$= \frac{\log_e R_p}{\frac{s}{s-1}(R_p^{\frac{s-1}{s}} - 1)}$$

Values of this expression were worked out for each exponent, for assumed values of R_p. A scale of values of R_p was laid off horizontally and from the same origin a vertical scale of values of the ratio of isothermal to adiabatic. The results found were then plotted, each point above its proper R_p and opposite its ratio value. Curves were then drawn through all the points found for the same value of s. In a similar way a set of curves for two stage and a set for three stage were drawn.

This chart is also useful in obtaining the (m.e.p.) of the cycle if the (sup.pr.) and the volumetric efficiency of the cylinder be known. A second horizontal scale laid off above the R_p scale shows the (m.e.p.) per pound of (sup.pr. for) the isothermal no-clearance cycle. This is found to be equal to $\log_e R_p$, since

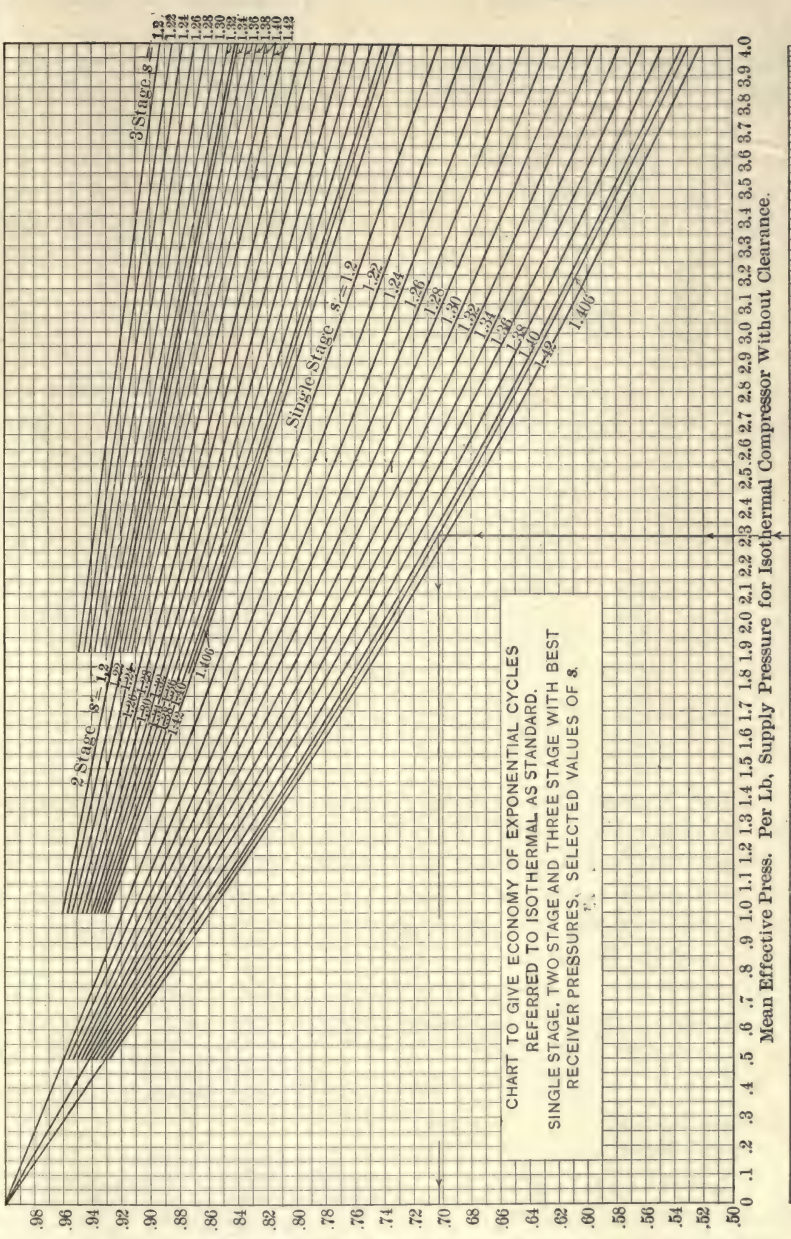

CHART TO GIVE ECONOMY OF EXPONENTIAL CYCLES
REFERRED TO ISOTHERMAL AS STANDARD.
SINGLE STAGE, TWO STAGE AND THREE STAGE WITH BEST
RECEIVER PRESSURES, SELECTED VALUES OF s.
Single Stage s = 1.2
1.22
1.24
1.26
1.28
1.30
1.32
1.34
1.36
1.40
1.406
2 Stage s = 1.2
3 Stage s = 1.3
Economy Compared with Isothermal Standard.
Mean Effective Press. Per Lb. Supply Pressure for Isothermal Compressor Without Clearance.
Ratio of Delivery Press. to Supply Press.

FIG. 50.

the (m.e.p.) for no clearance is equal to the work per cubic foot of (sup.pr.) gas, which, in turn, for the isothermal case is (sup.pr.) $\log_e R_p$ or $\log_e R_p$ when (sup.pr.) = 1.

Knowing the ratio of pressures, economy compared to isothermal can be found as explained above. Also knowing R_p the (m.e.p.) per pound initial is found from the upper scale.

Since the latter quantity is assumed to be known, by multiplying it by factor just found there is obtained (m.e.p.) isothermal. Since volumetric efficiency is assumed known, all the factors are known for the first equation given above which, rearranged, reads

$$\text{(m.e.p.) actual} = \frac{\text{m.e.p. isothermal (no clearance)}}{\text{(economy isothermal)} \div E_v},$$

Chart, Fig. 51. This chart is drawn to give the cylinder displacement for a desired capacity, with various values of R_p, s and clearance. From the formula Eq. (64)

$$\text{(L. P. Cap.)} = D(1+c-cR_p^{\frac{1}{s}}).$$

The right-hand portion of the diagram is for the purpose of finding values of $(R_p)^{\frac{1}{s}}$ for various values of R_p and s, and is constructed as was the similar curve in Fig. 45. The values of the lower scale on the left-hand diagram give values of $D = \text{(L. P. Cap.)} \div (1+c-cR_p^{\frac{1}{s}})$, where capacity is taken at 100 cu.ft., this scale was laid out and the clearance curves points found by solving the above equation for various values of $(R_p)^{\frac{1}{s}}$ for each value of c. To obtain the displacement necessary for a certain capacity with a given value of R_p, c and s, project upward from R_p to the proper s curve across to the c curve and down to read displacement per hundred cubic feet. Also on the left-hand diagram are drawn lines of piston speed, and on left-hand edge a scale of cylinder areas and diameters to give displacements found on horizontal scale. To obtain cylinder areas or *approximate* diameters in inches project from displacement to piston speed line and across to read cylinder area or diameter. Figures given are for 100 cu.ft. per minute. For any other volume the displacement and area of cylinder will be as desired volume to 100 and diameters will be as $\sqrt{\text{desired volume to 100}}$.

As an example, let it be required to find the low-pressure cylinder size for a compressor to handle 1500 cu.ft. of free air per minute. Receiver pressure to be 45 lbs. per square inch gage and (sup.pr.) to be atmosphere. Piston speed limited to 500 ft. per minute. Compression to be so that $s = 1.4$ and clearance = 4 per cent. Projecting upward from $R_p = 4$ to $s = 1.4$, across to $c = 4\%$, and down to piston speed = 500, find the diameter of a cylinder for 100 cu.ft. per minute is 6.3. For 1500 cu.ft. diameter will be as $\sqrt{15} \times 6.3 = 3.9 \times 6.3 = 24$ ins.

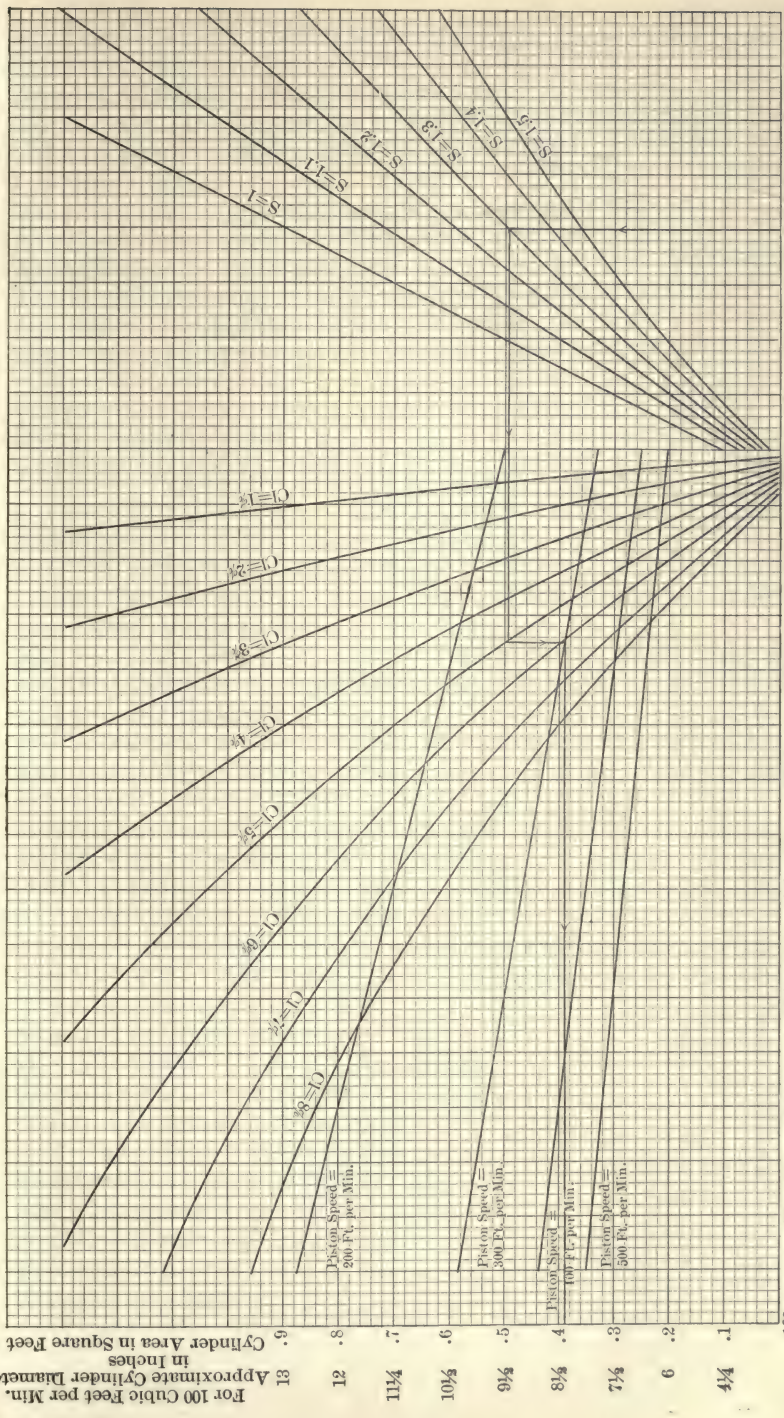


FIG. 51.—Compressor Cylinder Displacement for Given Capacity.

GENERAL PROBLEMS ON CHAPTER II.

Prob. 1. One hundred cubic feet of H_2S are compressed from 15 lbs. per square inch absolute to 160 lbs. per square inch absolute.

(*a*) Find work done if compression occurs isothermally in a no-clearance one-stage compressor;

(*b*) Adiabatically in a two-stage, no-clearance compressor;

(*c*) Adiabatically in two-stage compressor each cylinder having 5 per cent clearance;

Prob. 2. Air is being compressed in three plants. One is single-stage, the second is two-stage, and the third is three-stage Considering the compressors to have no clearance and 1000 cu.ft. of free air compressed adiabatically per minute from atmosphere to 150 lbs. per square inch gage, what will be the horse-power required and cylinder sizes in each case?

Prob. 3. A two-stage compressor with 5 per cent clearance in the high and 3 per cent in the low-pressure cylinder is compressing air from 14 lbs. per square inch absolute to 125 lbs. per square inch gage. What is the best-receiver pressure and what must be the size of the cylinders to handle 500 cu.ft. of free air per minute?

Prob. 4. A manufacturer gives a capacity of 970 cu.ft. for a $22\frac{1}{4}\times24$ in. single-stage compressor running at 142 R.P.M. when working pressures are 50 to 100 lbs. per square inch gage. What would be the clearance for each of these pressures assuming $s=1.4$?

Prob. 5. The card taken from a single-stage compressor cylinder showed an apparent volmetric efficiency of a 95 per cent and a mean value for s of 1.3. What is the clearance and what would be the (m.e.p.) for the ratio of pressures of 6?

Prob. 6. A compressor with double-acting cylinder 12×14 ins., having 6 per cent clearance, is forcing air into a tank. Taking the volumetric efficiency as the mean of that at the start and the end, how long will it take to build up 100 lbs. per square inch gage pressure in a tank of 1000 cu.ft. capacity if the speed is 100 M.R.P. and compression is isothermal, and how long will it take to do it if the compression is adiabatic and the air in the tank does not cool during filling? What is the maximum attainable pressure?

Prob. 7. It is desired to supply 1000 cu.ft. of air per minute at a pressure of 80 lbs. per square inch gage. A two-stage compressor is to be used, the clearance in low pressure of which is 3 per cent. What must be the displacement of the low-pressure cylinder and what will be the horse-power of the compressor?

Prob. 8. The low-pressure cylinder of a compressor is 18×24 ins. and has a clearance of 4 per cent. The receiver pressure is 60 lbs. per square inch absolute. The high-pressure cylinder has a 5 per cent clearance. What must be its diameter, stroke being same as low, so that compressor will operate at its designed receiver pressure?

Prob. 9. The discharge pressure of a two-stage compressor is 120 lbs. per square inch absolute and the supply pressure is 15 lbs. per square inch absolute The compressor is $10\times16\times12$ ins. The clearance in the low-pressure cylinder is 3 per cent. What must be the clearance in the high-pressure cylinder for the machine to operate at best-receiver pressure? Compression is adiabatic.

Prob. 10. If the clearance in the high-pressure cylinder of Prob. 9 were reduced to 3 per cent, would the receiver pressure increase or decrease, how much and why?

Prob. 11. If the discharge pressure in Prob. 9 fell to 100 lbs. per square inch absolute, what would be the new best-receiver pressure and why? Would the original clearance allow the new best-receiver pressure to be maintained?

Prob. 12. The discharge pressure for which a $20\frac{1}{4}\times32\frac{1}{4}\times24$ in. compressor is designed, is 100 lbs. per square inch gage, supply pressure being 14 lbs. per square inch absolute. The discharge pressure is raised to 125 lbs. per square inch gage. The clearance on the high-pressure cylinder can be adjusted. To what value must it be changed to enable the compressor to carry the best-receiver pressure for the new discharge pressure? Low-pressure clearance is 5 per cent at all times and compression being adiabatic.

Prob. 13. A manufacturer builds his $15\frac{1}{4}\times25\frac{1}{4}\times18$ in. compressors with low-pressure cylinders of larger diameter for high altitude work. What would be the diameter of a special cylinder for this compressor to work at an altitude of 10,000 ft. and what would be the horse-power per cubic foot of low-pressure air in each case?

Prob. 14. A three-stage compressor has 4 per cent clearance in all the cylinders. The low-pressure cylinder is 34×36 ins., delivery pressure 200 lbs. per square inch gage, supply pressure 14 lbs. per square inch absolute. What must be the size of the other cylinders for the machine to operate at best-receiver pressure.

Prob. 15. The cylinders of a two-stage compressor are given as $10\frac{1}{4}$ and $16\frac{1}{4}$ ins., the stroke being 12 ins. The machine has a capacity of 440 cu.ft. per minute at 160 R.P.M., the supply pressure is 14 lbs. per square inch absolute and the delivery pressure 100 lbs. per square inch gage. What is the clearance of each cylinder?

Prob. 16. To perform the required useful refrigerating effect in an ice plant, it is necessary to compress 250 cu.ft. of ammonia vapor per minute from 30 lbs. per square inch gage to 150 lbs. per square inch gage. What must be the size of the compressor to handle this at 75 R.P.M. if it be double acting and the clearance be 5 per cent?

Prob. 17. The maximum delivery pressure of a type of compressor is controlled by making the clearance large so that the volumetric efficiency will decrease as the pressure rises and become zero at the desired pressure. What must be the clearance for a single-stage compressor where the supply pressure is 14 lbs. per square inch absolute and the maximum delivery pressure 140 lbs. per square inch absolute? What will be the volumetric efficiency of the same machine at a delivery pressure of $\frac{1}{2}$ the maximum? At $\frac{1}{4}$?

Prob. 18. A three-stage compressor has a clearance of 5 per cent in each cylinder. What must be the cylinder ratios for the best-receiver pressures when the machine is compressing to 170 lbs. per square inch gage from atmosphere?

Prob. 19. Show why it was very essential to keep the clearance low in cylinders of three-stage compressor used for compressing air for air-driven cars, where the delivery pressure carried was 2500 lbs. per square inch, by assuming numerical data and calculating numerical proof.

Prob. 20. With water falling 150 ft. and used to compress air directly, how many cubic feet of air could be compressed per cubic foot of water?

Prob. 21. Air is compressed from atmosphere to 150 lbs. per square inch absolute, isothermally in one stage. How much more work would be required per cubic foot if compression were adiabatic? How much of this excess would be saved by compressing two stage? Three stage?

Prob. 22. 150 I.H.P. is delivered to the air cylinders of a $14\frac{1}{4}\times22\frac{1}{4}\times18$ in. compressor, running at 120 R.P.M. The supply pressure is 15 lbs. per square inch absolute. The volumetric efficiency as found from the indicator card is 95 per cent. What was the discharge pressure?

Prob. 23. The clearance in the high-pressure cylinder of a compressor is 5 per cent, which allows the compressor to run with the best-receiver pressure for a discharge of 00 lbs. per square inch absolute when the compressor is at sea-level. What would the

clearance be if the discharge pressure were kept the same and the altitude were 10,000 ft. to keep the best-receiver pressure?

Prob. 24. How many cubic feet of supply-pressure air may be compressed per minute from 1 to 8 atmospheres absolute by 100 horse-power if the compression in all cases is adiabatic?

(*b*) Three stage, no clearance;

(*c*) Two stage with 5 per cent clearance;

(*d*) Single stage with 5 per cent clearance;

(*a*) Two stage, no clearance

Prob. 25. The capacity of a $14\frac{1}{4}\times22\frac{1}{4}\times14$ in. compressor when running at 140 R.P.M. is said to be 940 cu.ft. for working pressures of 80 to 100 lbs. per square inch gage and atmospheric supply at sea level. Check these figures.

Prob. 26. What horse-power would be required by an $18\frac{1}{4}\times30\frac{1}{4}\times24$ in. compressor operating at 100 P.P.M. and on a working pressure of 100 lbs. per square inch gage if the clearance in low-pressure cylinder is 4 per cent? What would be the capacity?

Prob. 27. By means of water jackets on a compressor cylinder the va ue for s of compression curve in single-stage machine is lowered to 1.3. Compare the work to compress 1000 cu.ft. of air from 1 to 8 atmospheres absolute with this condition with that required for isothermal and adiabatic compression.

Prob. 28. What must be the size of cylinders in a three-stage compressor for compressing gas from 50 lbs. per square inch absolute to 600 lbs. per square inch absolute when s equals 1.3 and each cylinder has 3 per cent clearance. Compressor to run at 100 R.P.M. to be double acting and handle 1000 cu.ft. of gas per hour?

Prob. 29. How many cu.ft. of free air could be compressed per minute with an available horse-power of 1000 H.P. from atmosphere to 150 lbs. per square inch gage; (*a*) if compression is isothermal; (*b*) if compression be single-stage adiabatic; (*c*) if compression be three-stage adiabatic?

Prob. 30. A single-stage compressor is compressing air adiabatically at an altitude of 6000 ft. to a pressure of 80 lbs. per square inch gage. The cylinder has 2 per cent clearance. What must be the size of the cylinder to compress 2000 cu.ft. of free air per minute if the piston speed is limited to 600 ft. per minute? What, if the clearance be zero?

Prob. 31. What would be the size of the two-stage compressor for same data as in Prob. 30?

CHAPTER III

WORK OF PISTON ENGINES. HORSE POWER AND CONSUMPTION OF PISTON ENGINES USING STEAM, COMPRESSED AIR, OR ANY OTHER GAS OR VAPOR UNDER PRESSURE.

1. Action of Fluid in Single Cylinders. General Description of Structure and Processes. The most commonly used class of engines is that in which the operation is dependent on the pushing action of high-pressure fluids on pistons in cylinders, and this includes all piston steam engines of the reciprocating or straight-line piston path group as well as the less common rotary group, having pistons moving in curved and generally circular paths. In these same engines there may be used compressed air as well as steam, and equally as well the vapors of other substances or any other gases, without change of structure, except perhaps as to proportions, providing only that the substance to be used be drawn from a source of supply under high pressure, be admitted to the cylinder, there used and from it discharged or exhausted to a place of lower pressure. This place of lower pressure may be the open air or a closed chamber; the used fluid may be thrown away and wasted or used again for various purposes without in any way affecting the essential process of obtaining work at the expense of high-pressure gases or vapors. It is evident that, regarding a piston as a movable wall of a cylinder, whenever a fluid acts on one side with greater pressure than on the other, the piston will move toward the lower pressure end of the cylinder, and in so moving can exert a definite force or overcome a definite resistance, measured by the difference in pressure on the two sides and the areas exposed to the pressure. It is not so evident, but just as true, that the piston may be made to move from one end of the cylinder to the other when the average pressure on one side is greater than the average pressure on the other, and also do work even if the excess of pressure should reverse in direction during the stroke and provided only some energy storage device is added. In the common steam or compressed-air engine this is a flywheel with the usual connecting rod and crank mechanism, uniting the reciprocating piston movement with the continuous rotary movement of the flywheel mass. In certain forms of pumps the energy is stored in extra cylinders at times of excess and given out at times of deficiency in the path of the piston, so that its motion from end to end of cylinder may not be interrupted even if the pressure on the driving side should fall below that on the resisting side, assuming, of course, the average pressure for the whole stroke to be greater on the driving side than on the resisting side.

It appears, therefore, that piston movement in engines of the common form and structure, and the doing of work by that movement is not a question of maintaining a continuously greater pressure on one side than on the other. On the contrary, the process is to be studied by examination of the average pressure on the driving side and that on the resisting side, or by comparing the whole work done on one side with the whole work done on the other side by the fluid. The work done by the fluid on one side of a piston may be positive or negative, positive when the pressures are assisting motion, negative when they are resisting it. It is most convenient to study the action of fluids in cylinders by considering the whole action on one side from the beginning of movement at one end to the end of movement at the same point, after the completion of one complete forward and one complete return stroke. All the work done by the pressure of the fluid on the forward stroke on the side of the piston that is apparently moving away from the fluid is positive work, all the work done by the pressure of the fluid on the same side of the piston during the return stroke is negative, and for this stroke the side of the piston under consideration is apparently moving toward the fluid or pushing it.

For the complete cycle of piston movement covering the two strokes the work done on one side is the algebraic sum of the forward stroke work, considered positive, and the back stroke work, considered negative. During the same time some pressures are acting on the other side of the piston, and for them also there will be a net work done equal to the corresponding algebraic sum. The work available for use during the complete two strokes, or one revolution, will be the sum of the net work done by the fluid on the two sides of the piston during that time, or the algebraic sum of two positive and two negative quantities of work. Methods of *analysis of the work* of compressed fluids in cylinders are consequently *based* on the action in *one end of a cylinder*, treated as if the other end did not exist.

Just how the high-pressure fluid from a source of supply such as a boiler or an air compressor is introduced into one end of a cylinder, how it is treated after it gets there, and how expelled, will determine the nature of the variation in pressure in that end that acts on that side of the piston, and these are subjects to be studied. To determine the work done in the cylinder end by the fluid, it is necessary to determine laws of pressure change with stroke, and these are fixed first by valve action controlling the distribution of the fluid with respect to the piston and second by the physical properties of the fluid in question.

It is necessary that the cylinder be fitted with a valve for getting fluid into a cylinder, isolating the charge from the source of supply and getting it out again, and it may be that one valve will do, or that two or even more are desirable but this is a structural matter, knowledge of which is assumed here and not concerned with the effects under investigation. The first step in the process is, of course, admission of fluid from the source of supply to the cylinder at one end, which may continue for the whole, or be limited to a part

of the stroke. When admission ceases or supply is cut off before the end of the stroke there will be in the cylinder an isolated mass of fluid which will, of course, expand as the piston proceeds to the end. Thus the forward stroke, considering one side of the piston only, always consists of full pressure admission followed by expansion, the amount of which may vary from zero to a very large amount; in fact the final volume of the fluid after expansion may be hundreds of times as great as at its beginning, when supply was cut off.

At the end of this forward stroke an exhaust valve is opened, which permits communication of the cylinder with the atmosphere in non-condensing steam and most compressed-air engines, or with another cylinder, or with a storage chamber, or with a condenser in the case of a steam engine in which the pressure approximates a perfect vacuum. If at the moment of exhaust opening the cylinder pressure is greater or less than the back pressure, there will be a more or less quick equalization either up or down before the piston begins to return, after which the return or exhaust stroke will proceed with some back-pressure resistance acting on the piston, which is generally though not always constant. This may last for the whole back stroke or for only a part, as determined by the closure of the exhaust valve. When the exhaust valve closes before the end of the return stroke the unexpelled steam will be trapped and compressed to a pressure depending partly on the point of the stroke when closure begins and the pressure at the time, and partly on the clearance volume of the cylinder into which the trapped steam is compressed. Of course, at any time near the end of the stroke the admission valve may be opened again, and this may occur, 1st, before compression is complete, which will result in a sudden pressure rise in the cylinder before the end of the stroke to equalize it with the source of supply; 2d, just at the end of the stroke, which may result in a rise or a fall to equalize, or no change at all, depending on whether compression has raised the cylinder pressure not quite to supply pressure, or to something greater than it, or to a value just equal to it; 3d, after the end of the stroke, which will result in a reexpansion of the steam previously under compression, and then a sudden rise. It may be said in general that in cylinders there are carried out with more or less variation the following processes:

Forward stroke, constant-pressure admission followed by expansion.

Back stroke, constant-pressure exhaust followed by compression, while at both ends of the stroke there may or may not be a vertical line on a pressure-volume diagram representing a constant-volume change of pressure.

These processes will result in a cycle of pressure-volume changes which will be a closed curve made of more or less accurately defined phases, and the work of the cycle will be the area enclosed by the cyclic curve. Of course, there are causes of disturbance which make the phases take on peculiar characteristics. For example, the valve openings and closures may not take place as desired or as presupposed with respect to piston positions; leakage may occur, steam may condense during the operations in the cylinder, and water of condensation

may evaporate; the resistance through valves will always make the cylinder pressure during admission less than in the supply chamber and greater during exhaust than the atmosphere or than in exhaust receiver or condenser and may through the valve movements make what might have been a constant-pressure straight line become a curve. There will, by reason of these influences, encountered in real engines, be an almost infinite variety of indicator cards or pressure-volume cycles for such engines.

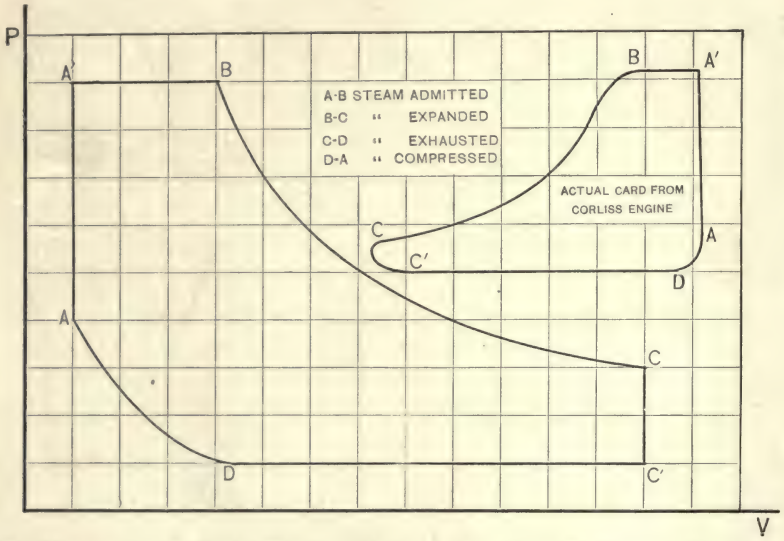


FIG. 52.—Diagram to Indicate Position of Admission, Cut-off, Release, Compression on Engine Indicator Card.

The various points of the stroke at which important events occur, important in their pressure-volume significance, have names, as do also the lines between the points, and these names are more or less commonly accepted and generally understood as follows: letters referring to the diagram Fig. 52.

Point Names: Events of Cycle.

A. *Admission* is that point of the stroke where the supply valve is opened.

B. *Cut-off* is that point of the stroke where the supply valve is closed.

C. *Release* is that point of the stroke where the exhaust valve is opened.

D. *Compression* is that point of the stroke where the exhaust valve is closed.

Names of Lines, or Periods:

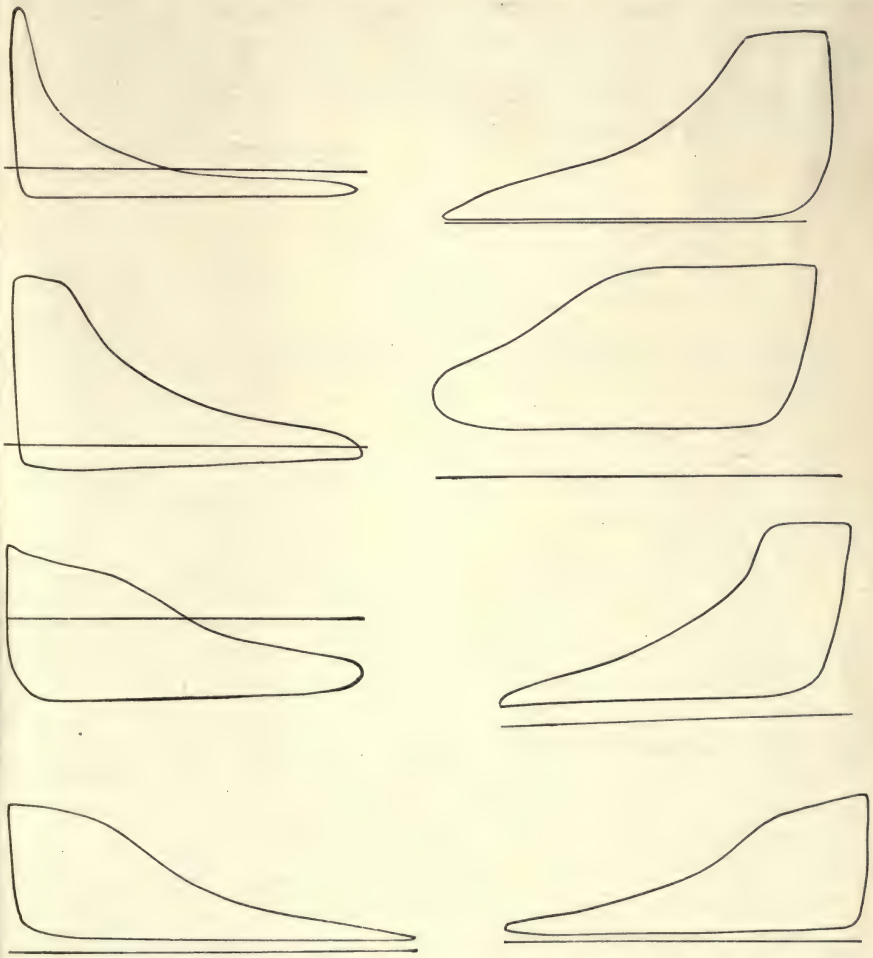
A–B. *Admission* or *steam line* joins the points of admission and cut-off.

B–C. *Expansion line* joins the points of cut-off and release.

C–D. *Exhaust line* joins the points of release and compression if there is any, or admission if there is not.

D–A. *Compression line* joins the points of compression and admission

By reason of the interferences discussed, these points on actual indicator ards may be difficult to locate, one line merging into the next in so gradual manner as to make it impossible to tell where the characteristic point lies, s will be apparent from Fig. 53, in which is reproduced a number of actual ndicator cards. In such cases equivalent points must be located for study



IG. 53.—Actual Steam Engine Indicator Cards Showing Distortion of Lines and Uncertain Location of Characteristic Points.

These same terms, which it appears sometimes refer to points and sometimes to lines, are also used in other senses, for example, *cut-off* is commonly used to mean the fraction of stroke completed up to the point of ut-off, and *compression* that fraction of stroke remaining incomplete at the oint of compression, while compression is also sometimes used to express he pressure attained at the end of the compression line. In general, there

is nothing in the use of these words to indicate just which of the various meanings is intended except the text, and experience will soon eliminate most of the possible chances of confusion.

Prob. 1. Draw a card in which admission and exhaust are late. Draw a card in which there is no compression, in which compression is very early, so that compression pressure is equal to admission pressure. Draw a card with 0 per cent cut-off, and cut-off = 100 per cent. Draw cards with same cut-off but with varying initial pressures.

Prob. 2. The following diagrams, Fig. 54, are reproductions of indicator cards actually taken from engines. Explain what features are peculiar to each and if possible give an explanation of the cause.

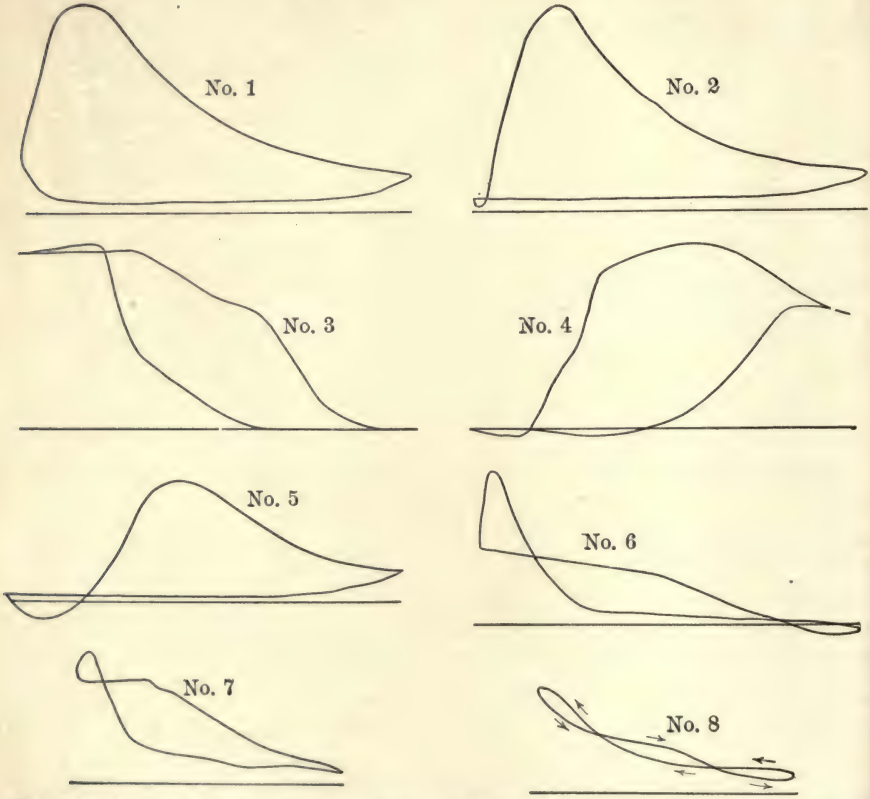


FIG. 54.—Indicator Diagram from Steam Engines with Improperly Set Valve Gear.

For example, in No. 1 a line of pressure equalization between the end of the compression line and the beginning of the admission line inclines to the right instead of being perpendicular, as in a perfect diagram. This is due to the fact that admission does not occur until after the piston has begun to move outward, so that pressure rise does not occur at constant volume, but during a period of increasing volume.

2. Standard Reference Cycles or PV Diagrams for the Work of Expansive Fluids in a Single Cylinder. Simple Engines. To permit of the derivation of a formula for the work of steam, compressed air, or any other fluid in a

cylinder, the various pressure volume, changes must be defined algebraically. The first step is, therefore, the determination of the cycle or pressure-volume diagram representative of the whole series of processes and consisting of a number of well-known phases or single processes. These phases, ignoring all sorts of interferences due to leakage or improper valve action, will consist of constant-pressure and constant-volume lines representing fluid movement into or from the cylinder, combined with expansion and compression lines representing changes of condition of the fluid isolated in the cylinder. These *expansion and compression lines* represent strictly thermal phases, laws for which will be assumed here, but derived rigidly later in the part treating of the thermal analysis; however, all cases can be represented by the *general expression*

$$PV^s = C,$$

in which the character of the case is fixed by fixing the value of s. For all gases and for vapors that do not contain liquid or do not form or evaporate any during expansion or compression, i.e., continually superheated, the exponent s may have one of two *characteristic values*. The first is *isothermal* expansion and compression, and for this process s is the same for all substances and equal to unity. The second is for *exponential* expansion or compression and for this process s will have values peculiar to the gas or superheated vapor under discussion, but it is possible that more than one substance may have the same value, as may be noted by reference to Section 8, Chapter I, from which the value $s = 1.406$ for air and $s = 1.3$ for superheated steam or ammonia adiabatically expanding are selected for illustration.

When steam or any other vapor not so highly superheated as to remain free from moisture during treatment is expanded or compressed in cylinders different values of s must be used to truly represent the process and, of course, there can be no isothermal value, since *there can be no change of pressure of wet vapors without a change of temperature*. For steam expanding adiabatically the value of s is not a constant, as will be proved later by thermal analysis, so that the exact solution of problems of adiabatic expansion of steam under ordinary conditions becomes impossible by pressure-volume analysis and can be handled only by thermal analysis. However, it is sometimes convenient or desirable to find a solution that is approximately correct, and for this a sort of average value for s may be taken. Rankine's average value is $s = 1.111 = \frac{1}{9}^0$ for adiabatic expansion of ordinarily wet steam, and while other values have been suggested from time to time this is as close as any and more handy than most. The value $s = 1.035 + .14 \times$ (the original dryness fraction), is given by Perry to take account of the variation in original moisture.

Steam during expansion adiabatically, tends to make itself wet, the condensation being due to the lesser heat content by reason of the work done; but if during expansion heat be added to steam originally just dry, to keep it so continuously, as the expansion proceeds, it may be said to follow the *saturation law* of steam, for which $s = 1.0646$. This is a strictly experimental value found

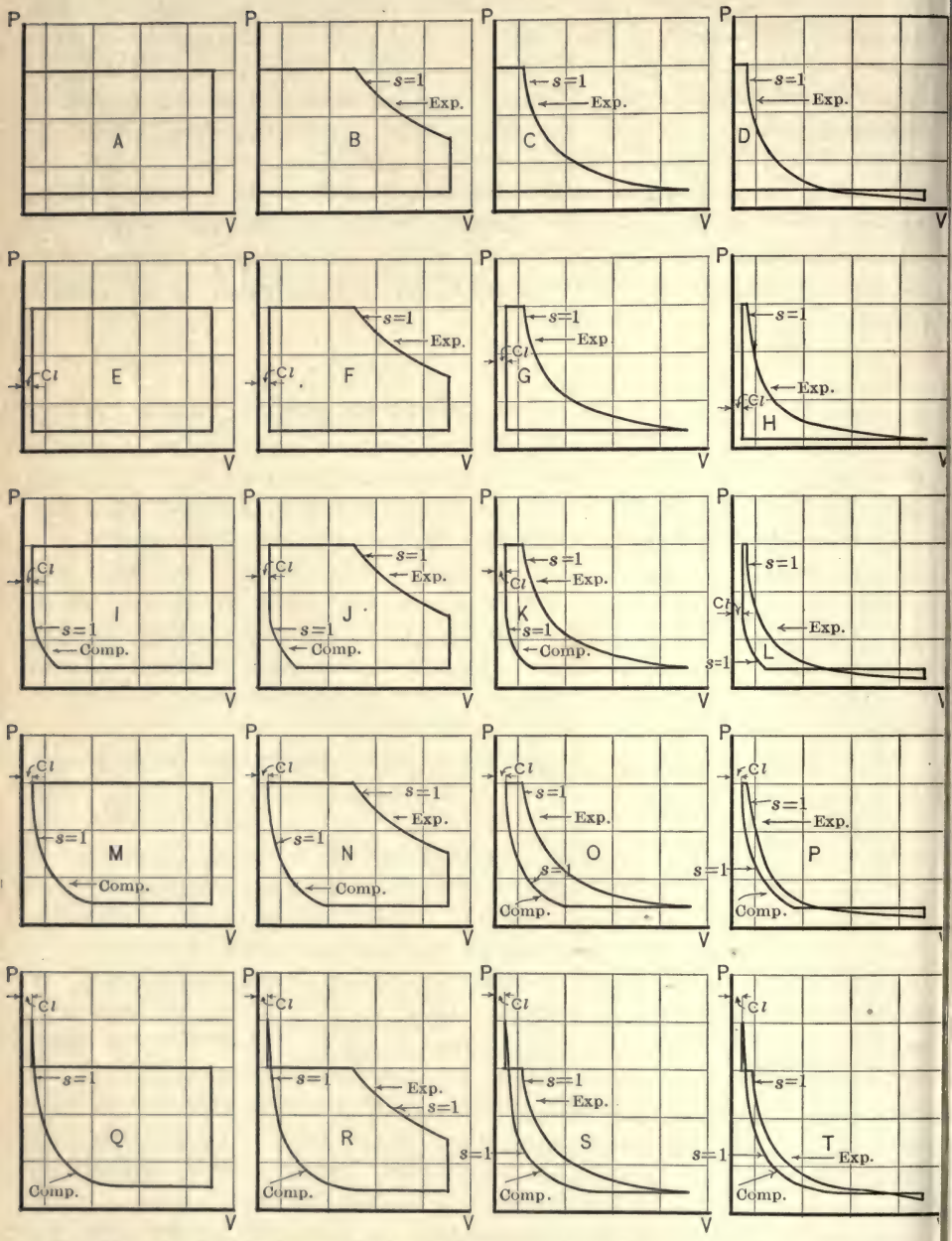


FIG. 55.—Standard Reference Cycles or Pressure-volume Diagrams for Expansive Fluid in Simple Engines.

by studying the volume occupied by a pound of just dry steam at various pressures, quite independent of engines.

Direct observation of steam engine indicator cards has revealed the fact that while, in general, the pressure falls faster at the beginning of expansion and slower at the end than would be the case if $s=1$, yet the total work is about the same as if s had this value all along the curve. This law of expansion and compression, which may be conveniently designated as the *logarithmic law*, is almost universally accepted as representing about what happens in actual steam engine cylinders. Later, the thermal analysis will show a variation of wetness corresponding to $s=1$, which is based on no thermal hypothesis whatever, but is the result of years of experience with exact cards. Curiously

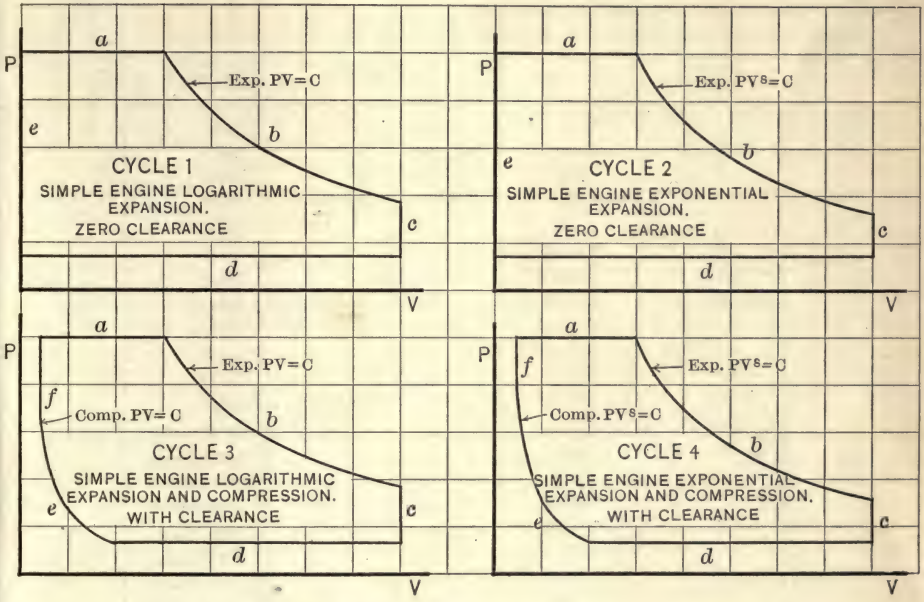


FIG. 56.—Simple Engine Reference, Cycles or *PV* Diagrams.

enough, this value of s is the same as results from the thermal analysis of constant temperature or isothermal expansion for gases, but it fails entirely to represent the case of isothermal expansion for steam. That $s=1$ for isothermal gas expansion and actual steam cylinder expansion is a mere coincidence, a fact not understood by the authors of many books often considered standard, as in them it is spoken of as the isothermal curve for steam, which it most positively is not. This discussion of the expansion or compression laws indicates that analysis falls into two classes, first, that for which $s=1$, which yields a logarithmic expression for work, and second, that for which s is greater or less than one, which yields an exponential expression for work, and the former will be designated as the *logarithmic* and the latter as the *exponential* laws, for convenience.

The phases to be considered then may be summed up as far as this analysis is concerned as:

1. Admission or exhaust, pressure constant, $P = \text{const.}$
2. Admission or exhaust, volume constant, $V = \text{const.}$
3. Expansion, $PV = \text{const.}$, when $s = 1$.
4. Expansion, $PV^s = \text{const.}$, when s is greater or less than 1.
5. Compression, $PV = \text{const.}$, when $s = 1$.
6. Compression, $PV^s = \text{const.}$, when s is greater or less than 1.

Considering all the possible variations of phases, there may result any or the cycles represented by Fig. 55. These cycles have the characteristics indicated by the following table, noting the variation in the law of the expansion or compression that may also be possible.

Cycle.	Clearance.	Expansion.	Compression.
A	Zero	Zero	Zero
B	Zero	Little	Zero
C	Zero	Complete	Zero
D	Zero	Over-expansion	Zero
E	Little	Zero	Zero
F	Little	Little	Zero
G	Little	Complete	Zero
H	Little	Over-expansion	Zero
I	Little	Zero	Little
J	Little	Little	Little
K	Little	Complete	Little
L	Little	Over-expansion	Little
M	Little	Zero	Complete
N	Little	Little	Complete
O	Little	Complete	Complete
P	Little	Over-expansion	Complete
Q	Little	Zero	Too much
R	Little	Little	Too much
S	Little	Complete	Too much
T	Little	Over-expansion	Too much

It is not necessary, however, to derive algebraic expressions for all these cases, since a few general expressions may be found involving all the variables in which some of them may be given a zero value and the resulting expression will apply to those cycles in which that variable does not appear. The resulting cycles, Fig. 56, that is is convenient to treat are as follows:

SIMPLE ENGINE REFERENCE CYCLE OR *PV* DIAGRAMS

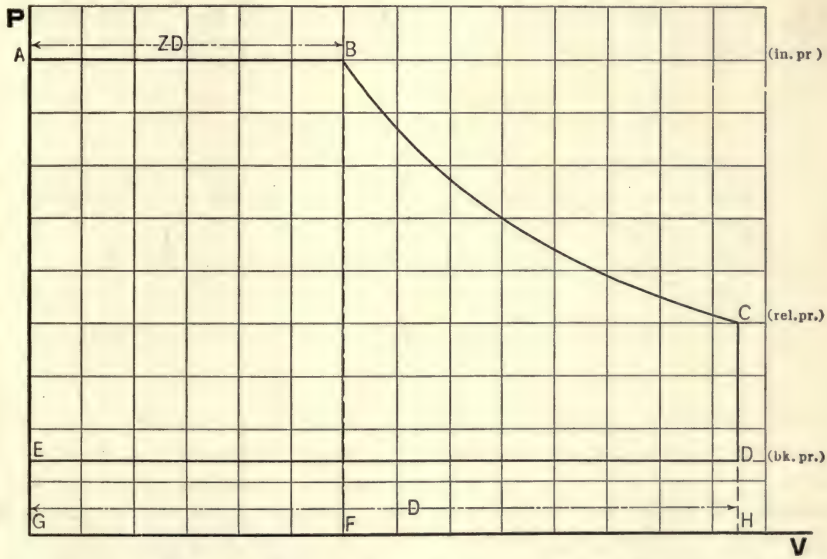
CYCLE 1. Simple Engine, Logarithmic Expansion without Clearance.

Phase (*a*) Constant pressure admission.
" (*b*) Expansion $PV = \text{const.}$ (may be absent).
" (*c*) Constant volume equalization of pressure with exhaust (may be absent).
" (*d*) Constant pressure exhaust.
" (*e*) Constant (zero) volume admission equalization of pressure with supply.

YCLE II. Simple Engine, Exponential Expansion without Clearance.
Phase (*a*) Constant pressure admission.
" (*b*) Expansion PV^s = const. (may be absent).
" (*c*) Constant-volume equalization of pressure with exhasut (may be absent).
" (*d*) Constant-pressure exhaust.
" (*e*) Constant (zero) volume admission equalization of pressure with supply.

YCLE III. Simple Engine, Logarithmic Expansion and Compression with Clearance.
Phase (*a*) Constant pressure admission.
" (*b*) Expansion PV = const. (may be absent).
" (*c*) Constant volume equalization of pressure with exhaust (may be absent).
" (*d*) Constant pressure exhaust.
" (*e*) Compression PV = const. (may be absent).
" (*f*) Constant volume admission, equalization of pressure with supply (may be absent).

YCLE IV. Simple Engine, Exponential Expansion and Compression with Clearance.
Phase (*a*) Constant pressure admission.
" (*b*) Expansion PV^s = const. (may be absent).
" (*c*) Constant volume equalization of pressure with exhaust (may be absent).
" (*d*) Constant pressure exhaust.
" (*e*) Compression PV^s = const. (may be absent).
" (*f*) Constant admission, equalization of pressure with supply (may be absent).



IG. 57.—Work of Expansive Fluid in Single Cylinder with No Clearance. Logarithmic Expansion for Cycle I. Exponential for Cycle II.

3. Work of Expansive Fluid in Single Cylinder without Clearance. Logarithmic Expansion, Cycle I. Mean Effective Pressure, Horse-power and Consumption of Simple Engine. Referring to the diagram, Fig. 57, the *net work*, whether expansion be incomplete, perfect, or excessive, is the sum of

admission and expansion work less the back-pressure work, or by areas, net work area, $\overline{ABCDE}$ = admission work area $\overline{ABFG}$
+expansion work area $\overline{FBCH}$
−back-pressure work area $\overline{GEDH}$

or algebraically,

$$W = P_b V_b + P_b V_b \log_e \frac{P_b}{P_c} - P_d V_d, \quad \ldots \ldots \quad (230)$$

Work of cycle in foot-pounds is

$$\left.\begin{aligned} W &= P_b V_b \left(1+\log_e \frac{P_b}{P_c}\right) - P_d V_d \quad (a) \\ &= P_c V_c \left(1+\log_e \frac{P_b}{P_c}\right) - P_d V_d \quad (b) \\ &= P_b V_b \left(1+\log_e \frac{V_c}{V_b}\right) - P_d V_d \quad (c) \\ &= P_c V_c \left(1+\log_e \frac{V_c}{V_b}\right) - P_d V_d \quad (d) \end{aligned}\right\} \quad \ldots \ldots \quad (231)$$

As V_d or V_c represent the whole displacement, the *mean effective pressure* will be obtained by dividing Eqs. (231), by V_d or V_c, giving,

$$\left.\begin{aligned} \text{M.E.P.} &= P_c \left(1+\log_e \frac{P_b}{P_c}\right) - P_d \quad (a) \\ &= P_c \left(1+\log_e \frac{V_c}{V_b}\right) - \mathrm{P}_d \quad (b) \\ \text{m.e.p.} &= p_c \left(1+\log_e \frac{p_b}{p_c}\right) - p_d \quad (c) \\ &= p_c \left(1+\log_e \frac{V_c}{V_b}\right) - p_d \quad (d) \\ &= p_b \left(1+\log_e \frac{V_c}{V_b}\right)\left(\frac{V_b}{V_c}\right) - p_d \quad (e) \end{aligned}\right\} \quad \ldots \ldots \quad (232)$$

Similarly, dividing the Eq. (232) by the volume of fluid admitted, V_b, will give the *work per cubic foot*, which is a good measure of economy, *greatest economy being defined by maximum work per cubic foot, which, it may be noted, is the inverse of the compressor standard.*

$$\left.\begin{aligned} \text{Work per cu.ft. supplied} &= P_b \left(1+\log_e \frac{P_b}{P_c}\right) - P_d \frac{V_d}{V_b} \quad (a) \\ &= P_b \left(1+\log_e \frac{V_c}{V_b}\right) - P_d \frac{V_d}{V_b} \quad (b) \end{aligned}\right\} \quad \ldots \ldots \quad (233)$$

According to Eq. (19), Chapter I, the piston displacement in cubic feet per hour per I.H.P. is for $z=1$, $\frac{13{,}750}{\text{(m.e.p.}}$, and this multiplied by the fraction of whole displacement occupied in charging the cylinder or representing admission, which is

$$\frac{V_b}{V_d} \quad \text{or} \quad \frac{V_b}{V_c},$$

will give the *cubic feet of high pressure fluid supplied per hour per I.H.P.*, hence

$$\left.\begin{aligned}\text{Cu.ft. supplied per hr. per I.H.P.} &= \frac{13{,}750}{\text{(m.e.p.)}} \times \frac{V_b}{V_c} \ (a) \\ &= \frac{13{,}750}{\text{(m.e.p.)}} \times \frac{P_c}{P_b} \ (b)\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (234)$$

Introducing a density factor, this can be transformed to *weight of fluid.* If then δ_1 is the density of the fluid as supplied in pounds per cubic foot,

$$\left.\begin{aligned}\text{Lbs. fluid supplied per hr. per I.H.P.} &= \frac{13{,}750}{\text{(m.e.p.)}} \times \frac{V_b}{V_c} \times \delta_1 \ (a) \\ &= \frac{13{,}750}{\text{(m.e.p.)}} \times \frac{P_c}{P_b} \times \delta_1 \ (b)\end{aligned}\right\} \quad \ldots \quad (235)$$

All these expressions, Eqs. (230) to (235), for the work of the cycle, the mean effective pressure, work per cubic feet of fluid supplied, cubic feet and pounds of fluid supplied per hour per I.H.P., are in terms of diagram point conditions and must be transformed so as to read in terms of more generally defined quantities for convenience in solving problems. The first step is to introduce quantities representing supply and back pressures and the amount of expansion, accordingly:

Let (in.pr.) represent the initial or supply pressure p_b expressed in pounds per square inch;

" (rel.pr.) represent the release pressure p_c, in pounds per square inch;

" (bk.pr.) represent the back pressure p_d, in pounds per square inch;

" R_V represent the ratio of expansion defined as the ratio of largest to smallest volume on the expansion line $\left(\frac{V_c}{V_b}\right)$, or $\left(\frac{V_d}{V_b}\right)$ which is, of course, equal to the ratio of supply to release pressure $\left(\frac{p_b}{p_c}\right)$, when the logarithmic law is assumed;

" D represent the displacement in cubic feet which is V_d or V_c when no clearance is assumed;

Let Z represent the fraction of stroke or displacement completed up to cut-off so that ZD represents the volume V_b admitted to the cylinder.

In this case when clearance is zero, $Z = \frac{1}{R_V}$.

Work of the cycle in foot-pounds

$$\left.\begin{aligned} W &= 144\left[(\text{in.pr.})\frac{1+\log_e R_V}{R_V} - (\text{bk.pr.})\right]D \quad (a) \\ &= 144[(\text{rel.pr.})(1+\log_e R_V) - (\text{bk.pr.})]D \quad (b) \end{aligned}\right\} \quad . \quad . \quad (236)$$

$$\left.\begin{aligned} \text{m.e.p.} &= (\text{rel.pr.})(1+\log_e R_V) - (\text{bk.pr.}) \quad (a) \\ &= (\text{in.pr.})\left(\frac{1+\log_e R_V}{R_V}\right) - (\text{bk.pr.}) \quad (b) \\ &= (\text{in.pr.})Z\left(1+\log_e \frac{1}{Z}\right) - (\text{bk.pr.}) \quad (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (237)$$

$$\left.\begin{aligned} \text{Work per cu.ft. supplied} &= 144[(\text{in.pr.})(1+\log_e R_V) - (\text{bk.pr.})R_V] \quad (a) \\ &= 144\left[(\text{in.pr.})\left(1+\log_e \frac{1}{Z}\right) - \frac{(\text{bk.pr.})}{Z}\right] \quad (b) \end{aligned}\right\} \quad . \quad (238)$$

$$\left.\begin{aligned} \text{Cu.ft. supplied per hr. per I.H.P.} &= \frac{13{,}750}{\text{m.e.p.}}\frac{1}{R_V} \quad (a) \\ &= \frac{13{,}750}{(\text{m.e.p.})}Z \quad (b) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (239)$$

Lbs. supplied per hr. per I.H.P.

$$\left.\begin{aligned} &= \frac{13{,}750}{(\text{m.e.p.})}\frac{S_1}{R_V} \quad (a) \\ &= \frac{13{,}750}{(\text{m.e.p.})}ZS_1 \quad (b) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (240)$$

The indicated horse-power may be found by multiplying the work of the cycle, Eq. (236), by the number of cycles performed per minute n and dividing the product by 33,000.

$$\text{I.H.P.} = \frac{Dn}{229.2}\left[(\text{in.pr.})\left(\frac{1+\log_e R_V}{R_V}\right) - (\text{bk.pr.})\right], \quad . \quad . \quad . \quad (241)$$

or

$$\text{I.H.P.} = \frac{Dn(\text{m.e.p.})}{229.2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (242)$$

In any of these expressions where R_V is the ratio of greatest to smallest volume during expansion, either R_P, ratio of greater to smaller pressures, or $\frac{1}{Z}$, the

reciprocal of the cut-off, may be substituted, since the expressions apply only to the logarithmic law, and clearance is assumed equal to zero. When clearance is not zero; it is shown later that the cut-off as a fraction of stroke is not the reciprocal of R_P or R_V.

These expressions are perfectly general, but convenience in calculation will be served by deriving expressions for certain special cases. The first of these is the case of no expansion at all, the second that of complete expansion without over-expansion. This latter gives the most economical operation from the hypothetical standpoint, because no work of expansion has been left unaccomplished and at the same time no negative work has been introduced by over-expansion.

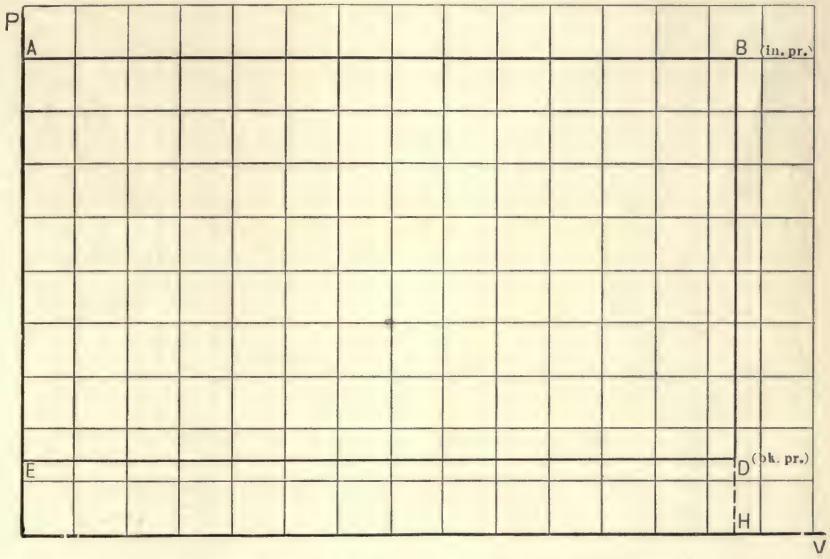


FIG. 58.—First Special Case of Cycles I and II. Expansion = zero. Full Stroke Admission.

First Special Case. If there is no *expansion*, together with the above assumption of no clearance, the diagram takes the form (Fig. 58), and

$$W = 144D[(\text{in.pr.}) - (\text{bk.pr.})] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (243)$$

$$\text{m.e.p.} = (\text{in.pr.}) - (\text{bk.pr.}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (244)$$

$$\text{Work per cu.ft. supplied} = 144[(\text{in.pr.}) - (\text{bk.pr.})]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (245)$$

$$\text{Cu.ft. supplied per hr. per I.H.P.} = \frac{13{,}750}{(\text{in.pr.}) - (\text{bk.pr.})}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (246)$$

$$\text{Lbs. supplied per hr. per I.H.P.} = \frac{13{,}750\delta_1}{(\text{in.pr.}) - (\text{bk.pr.})}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

Second Special Case. When the *expansion* is *complete* without over-expansion, no clearance, the points C and D, Fig. 59, coincide, and (rel.pr.) = (bk.pr.), hence $R_V = R_P = \frac{\text{(in.pr.)}}{\text{(bk.pr.)}} = \frac{1}{Z}$. *This value of cut-off, Z, is known as best cut-off, as it is that which uses all the available energy of the fluid by expansion.*

$$W = 144D(\text{in.pr.})\frac{\log_e R_P}{R_P} = 144D(\text{bk.pr.})\log_e R_P. \quad . \quad . \quad . \quad (248)$$

$$(\text{m.e.p.}) = (\text{in.pr.})\frac{\log_e R_P}{R_P}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (249)$$

$$\text{Work per cu.ft. supplied} = 144(\text{in.pr.})\log_e R_P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (250)$$

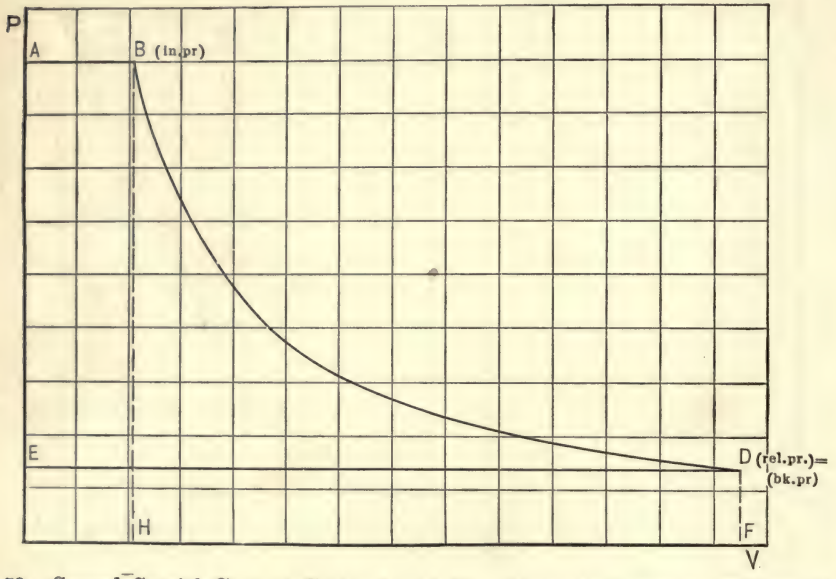


FIG. 59.—Second Special Case of Cycles I and II. Complete Expansion Without Over Expansion Case of Best Cut-off.

$$\text{Cu.ft. supplied per hr. per I.H.P.} = \frac{13{,}750}{(\text{in.pr.})\log_e R_P}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (251)$$

$$\text{Lbs. supplied per hr. per I.H.P.} = \frac{13{,}750\delta_1}{(\text{in.pr.})\log_e R_P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (252)$$

Example 1. Method of calculating diagrams. Fig. 57 and Fig. 59.

Assumed data for Fig. 57.

$P_a = P_c = 90$ lbs. per sq.in. abs. $V_a = V_e = 0$ cu.ft.
$P_d = P_e = 14$ lbs. per sq.in. abs. $V_c = V_d = 13.5$ cu.ft.
$V_b = 6$ cu.ft.

To obtain point C:

$$P_c = P_b \times \frac{V_b}{V_c} = 90 \times \frac{6}{13.5} = 40 \text{ lbs. per sq.in. abs.}$$

Assumed data for Fig. 59.

$$P_a = P_b = 90 \text{ lbs. per sq.in.} \quad V_a = V_e = 0 \text{ cu.ft.}$$
$$P_d = P_e = 14 \text{ lbs. per sq.in.} \quad V_d = 13.5 \text{ cu.ft.}$$

To obtain point B:

$$V_b = V_d \times \frac{P_d}{P_b} = 13.5 \times \frac{14}{90} = 2.1 \text{ cu.ft.}$$

Example 2. A simple double-acting engine admits steam at 100 per square inch absolute for $\frac{1}{4}$ stroke, allows it to expand to the end of the stroke and then exhausts it against a back pressure of 5 lbs. per square inch absolute. If the engine has no clearance, a 7×9-in. cylinder and runs at 300 R.P.M., what is the horse-power and steam consumption when steam is expanding according to the logarithmic law? Note: 1 cu.ft. steam at 100 lbs. per square inch absolute weighs .2258 lb.

From Eq. (237_b),

$$\text{m.e.p.} = (\text{in.pr.})\left(\frac{1+\log_e R_V}{R_v}\right) - (\text{bk.pr.})$$

$$= 100\frac{(1+\log_e 4)}{4} - 5 = 100\frac{(1+1.386)}{4} - 5 = 54.7:$$

$$\text{I.H.P.} = \frac{(\text{m.e.p.})Lan}{33{,}000} = \frac{54.7 \times .75 \times 38.5 \times 600}{33{,}000} = 28:$$

or directly from Eq. (242)

$$\text{I.H.P.} = \frac{Dn(\text{m.e.p.}),}{229.2}$$

$$= \frac{.2 \times 600 \times 54.7}{229.2} = 28,$$

$$\text{Lbs. steam per I.H.P.} = \frac{13{,}750}{\text{m.e.p.}} \times \frac{\delta_1}{R_V},$$

$$= \frac{13750}{54.7} \times \frac{.2258}{4} = 14.13.$$

Therefore, steam per hour used by engine $= 14.15 \times 28 = 396$ lbs.

Prob. 1. A steam engine has a cylinder 12×18 ins. with no clearance. It runs at 200 R.P.M. and is double-acting. If the steam pressure be fixed at 100 lbs. per square inch absolute, and the back pressure at 10 lbs. per square inch abs., show how the horse-power and steam consumption will vary as cut-off increases. Take cut-off from $\frac{1}{8}$ to $\frac{5}{8}$ by eighths. Plot.

Prob. 2. Two engines of the same size and design as above are running on a steam pressure of 100 lbs. per square inch absolute, but one exhausts through a long pipe to the atmosphere, the total back pressure being 20 lbs. per square inch absolute, while the other exhausts into a condenser in which the pressure is but 3 lbs. per square inch absolute. If the cut-off is in each case $\frac{3}{8}$, how will the I.H.P. and steam used in the two cases vary?

Prob. 3. By finding the water rate and the horse-power in the two following cases, show the saving in steam and loss in power due to using steam expansively. A pump having a cylinder 9×12 ins. admits steam full stroke, while an engine of same size admits it but $\frac{1}{4}$ of the stroke; both run at the same speed and have the same back pressure.

Prob. 4. Steam from a 12×24 in. cylinder is exhausted at atmospheric pressure (15 lbs. per square inch absolute) into a tank, from which a second engine takes steam. Neither engine has clearance. The first engine receives steam at 100 lbs. per square inch absolute and the cut-off is such as to give complete expansion. The second engine exhausts into a 24 in. vacuum and its cut-off is such that complete expansion occurs in its cylinder. Also the cylinder volume up to cut-off equals that of the first cylinder at exhaust. If the stroke is the same in both engines and the speed of each is 200 R.P.M., what is the diameter of the larger cylinder, the total horse-power developed, the total steam used, and the work per cubic foot of steam admitted to the first cylinder, the water rate of each engine and the total horse-power derived from each pound of steam?

Prob. 5. The steam pressure for a given engine is changed from 80 lbs. per square inch gage to 120 lbs. per square inch gage. If the engine is 12×16 ins., running 250 R.P.M. with a fixed cut-off of 25 per cent and no clearance, the back pressure being 15 lbs. per square inch absolute, what will be the horse-power and the water rate in each case?

NOTE: 1 cu.ft. of steam at 80 and 120 lbs. weighs .215 and .3 lb. respectively.

Prob. 6. By trial, find how much the cut-off should have been shortened to keep the H.P. constant when the pressure was increased and what effect this would have had on the water rate.

Prob. 7. A certain type of automobile engine uses steam at 600 lbs. per square inch absolute pressure. The exhaust is to atmosphere. For a cut-off of $\frac{1}{8}$ and no clearance, what would be the water rate?

NOTE: for 600 lbs. $\delta_1 = 1.32$.

Prob. 8. Engines are governed by throttling the initial pressure or shortening the cut-off. The following cases show the effect of light load on economy. Both engines, 12×18 ins., running at 200 R.P.M., with 125 lbs. per square inch absolute. Initial pressure and back pressure of 10 lbs. per square inch absolute. The load is sufficient to require full steam pressure at $\frac{1}{2}$ cut-off for each engine. Load drops to a point where the throttle engine requires but 50 lbs. per square inch absolute initial pressure with the cut-off still fixed at $\frac{1}{2}$. What is the original load and water rate, and new load and water-rate for each engine?

NOTE: δ for 125 lbs. absolute $= .279$ and for 50 lbs. $= .117$ lb.

Prob. 9. The guarantee for a simple engine 18×24 ins., running at 200 R.P.M., states that the I.W.R. when cut-off is $\frac{1}{4}$ will not exceed 15 lbs. if the initial pressure be 100 lbs. per square inch gage, and back pressure 5 lbs. per square inch absolute. If engine has no clearance, see if this would be possible.

4. Work of Expansive Fluid in Single Cylinder without Clearance. Exponential Expansion Cycle II. Mean Effective Pressure, Horse-power and

Consumption of Simple Engines. Referring to the diagram, Fig. 57, the work is given by the same areas as for Cycle I, but its algebraic expression is different because s is greater than 1 and an exponential expansion results on integration instead of a logarithmic one.

In general, from Eq. (13a), Section 7, Chapter I,

$$W = P_b V_b + \frac{P_c V_c}{s-1}\left[\left(\frac{V_c}{V_b}\right)^{s-1} - 1\right] - P_d V_d \quad . \quad . \quad . \quad . \quad . \quad (253)$$

Putting this in terms of initial conditions by the relations

$$\left(\frac{P_b}{P_c}\right) = \left(\frac{V_c}{V_b}\right)^s = R_V^s \text{ and } V_c = V_d = V_b R_V.$$

Also

$$P_c V_c = P_b V_b \left(\frac{V_b}{V_c}\right)^{s-1} = \frac{P_b V_b}{R_V{}^{s-1}};$$

there results

$$W = P_b V_b + \frac{P_b V_b}{(s-1)R_V{}^{s-1}}(R_V{}^{s-1} - 1) - P_d V_b R_V$$

$$= P_b V_b\left[1 + \frac{1}{(s-1)R_V{}^{s-1}}(R_V{}^{s-1} - 1)\right] - P_d V_b R_V.$$

$$\left.\begin{aligned} W &= P_b V_b\left(\frac{s}{s-1} - \frac{1}{(s-1)R_V{}^{s-1}}\right) - P_d V_b R_V && (a)\\ &= 144D\left[\frac{\text{(in.pr.)}}{R_V}\left(\frac{s}{s-1} - \frac{1}{(s-1)R_V{}^{s-1}}\right) - \text{(bk.pr.)}\right] && (b)\\ &= 144D\left[Z\ \text{(in.pr.)}\,\frac{s - Z^{s-1}}{s-1} - \text{(bk.pr.)}\right] && (c) \end{aligned}\right\} \quad . \quad . \quad (254)$$

which is the general equation for work of this cycle.

Dividing by V_b, the volume of fluid supplied,

$$\left.\begin{aligned} \text{Work per cu.ft. supplied} &= P_b\left(\frac{s}{s-1} - \frac{1}{(s-1)R_V{}^{s-1}}\right) - P_d R_V && (a)\\ &= 144\,\text{(in.pr.)}\left(\frac{s}{s-1} - \frac{1}{(s-1)R_V{}^{s-1}}\right) - \text{(bk.pr.)}R_V && (b) \end{aligned}\right\} \quad . \quad . \quad . \quad (255)$$

Similarly, the *mean effective pressure* results from dividing the work by the displacement, $V_d = V_b R_V$.

$$\left.\begin{aligned} \text{M.E.P.} &= \frac{P_b}{R_V}\left(\frac{s}{s-1} - \frac{1}{(s-1)R_V{}^{s-1}}\right) - P_d && (a)\\ \text{or}\quad (\text{m.e.p.}) &= \frac{\text{(in.pr.)}}{R_V}\left(\frac{s}{s-1} - \frac{1}{(s-1)R_V{}^{s-1}}\right) - \text{(bk.pr.)} && (b)\\ &= Z\,\text{(in.pr.)}\left(\frac{s - Z^{s-1}}{s-1}\right) - \text{(bk.pr.)} && (c) \end{aligned}\right\} \quad . \quad (256)$$

First special case of no expansion, when $R_V = 1$, results the same diagram as in the previous section, Fig. 58, and exactly the same set of formulas.

Second special case when the expansion is *complete* without over-expansion is again represented by Fig. 59 and for it

$$P_d = P_b\left(\frac{V_b}{V_d}\right)^s = \frac{P_b}{R_V{}^s}.$$

Whence

$$W = P_b V_b\left(\frac{s}{s-1} - \frac{1}{(s-1)R_V{}^{s-1}} - \frac{1}{R_V{}^{s-1}}\right),$$

or

Work for complete expansion is

$$\left.\begin{aligned} W &= P_b V_b \frac{s}{s-1}\left(1 - \frac{1}{R_V{}^{s-1}}\right) && (a)\\ &= 144\frac{(\text{in.pr.})D}{R_V}\ \frac{s}{s-1}\left(1 - \frac{1}{R_V{}^{s-1}}\right) && (b)\end{aligned}\right\}, \quad \ldots\ (257)$$

which is the general equation for the work of V_b or $\left(\frac{D}{R_V}\right)$ cubic feet of fluid when the economy is best or for best cut-off.

The work per cubic foot of fluid supplied for this case of *complete expansion* gives the maximum value for Eq. (255) and is obtained by dividing Eq. (257) by V_b or $\left(\frac{D}{R_V}\right)$.

$$\left.\begin{aligned} \text{Max. work per cu.ft. supplied} &= P_b \frac{s}{s-1}\left(1 - \frac{1}{R_V{}^{s-1}}\right) && (a)\\ &= 144(\text{in.pr.})\frac{s}{s-1}\left(1 - \frac{1}{R_V{}^{s-1}}\right) && (b)\end{aligned}\right\}, \quad \ldots\ (258)$$

which is the general equation for maximum work per cubic foot of fluid supplied.

The expression for *mean effective pressure becomes for this case of best cut-off,*

or,

$$\left.\begin{aligned} \text{M.E.P.} &= \frac{P_b}{R_V}\ \frac{s}{s-1}\left(1 - \frac{1}{R_V{}^{s-1}}\right) && (a)\\ (\text{m.e.p.}) &= \frac{(\text{in.pr.})}{R_V}\ \frac{s}{s-1}\left(1 - \frac{1}{R_V{}^{s-1}}\right) && (b)\end{aligned}\right\}. \quad \ldots\ (259)$$

It is convenient to note that in using Eqs. (257), (258) and (259) it may be desirable to evaluate them without first finding R_V. Since

$$R_V = \frac{V_c}{V_b} = \left(\frac{P_b}{P_c}\right)^{\frac{1}{s}} = R_P{}^{\frac{1}{s}},$$

this substitution may be made, and

$$R_V^{s-1} = R_P^{\frac{s-1}{s}}$$

Example. Compare the horse-power and the steam consumption of a 9×12 in. simple double-acting engine with no clearance and running at 250 R.P.M. when initial pressure is 100 lbs. per square inch absolute and cut-off is $\frac{1}{4}$, if

(a) steam remains dry and saturated throughout expansion,

(b) remains superheated throughout expansion, and

(c) if originally dry and suffers adiabatic expansion.

Back pressure is 10 lbs. per square inch absolute.

$$(\text{m.e.p.}) = \frac{(\text{in.pr.})}{R_V}\left(\frac{s}{s-1} - \frac{1}{(s-1)R_V^{s-1}}\right) - (\text{bk.pr.}).$$

For case (a) $s = 1.0646$ and $(\text{m.e.p.}) = \frac{100}{4}\left(\frac{1.0646}{.0646} - \frac{1}{.0646 \times 4^{\cdot 0646}}\right) - 10 = 48.6;$

" (b) $s = 1.3$ and $(\text{m.e.p.}) = \frac{100}{4}\left(\frac{13.}{.3} - \frac{1}{.3 \times 4^{\cdot 3}}\right) - 10 = 43.5;$

" (c) $s = 1.111$ and $(\text{m.e.p.}) = \frac{100}{4}\left(\frac{1.111}{.111} - \frac{1}{.111 \times 4^{\cdot 111}}\right) - 10 = 47.5.$

$$\text{I.H.P.} = (\text{m.e.p.})Lan = \frac{(\text{m.e.p.})1.0 \times 63.6 \times 500}{33{,}000} = .965 \text{ m.e.p.}$$

$\therefore$ I.H.P. for case (a) = 46.9,
" " (b) = 42.0,
" " (c) = 45.8.

From Eq. (240), lbs. steam per hour per $\text{I.H.P.} = \frac{13{,}750}{\text{m.e.p.}} \times \frac{\delta_1}{R_v}$

$\therefore$ For case (a) steam per hr. $= 46.9 \times \frac{13{,}750}{48.6} \times \frac{\delta_1}{4}$;

" (b) steam per hr. $= 42 \times \frac{13{,}750}{43.5} \times \frac{\delta_1}{4}$;

" (c) steam per hr. $= 45.8 \times \frac{13{,}750}{47.5} \times \frac{\delta_1}{4}$.

Prob. 1. On starting a locomotive steam is admitted full stroke, while in running the valve gear is arranged for $\frac{5}{8}$ cut-off. If the engine were 18×30 ins., initial pressure 150 lbs. per square inch absolute, back pressure 15 lbs. per square inch absolute, what would be the difference in horse-power with the gear in normal running position and in the starting position for a speed of 20 miles per hour with 6-ft. driving wheels? Consider the steam to be originally dry and expanding adiabatically. What would be the difference in steam per horse-power hour for the two cases and the difference in total steam? Clearance neglected.

Prob. 2. Consider a boiler horse-power to be 30 lbs. of steam per hour; what must be the horse-power of a boiler to supply the following engine? Steam is supplied in a super-

heated state and remains so throughout expansion. Initial density of steam = .21 lbs. per cubic foot. Engine is 12×20 ins., double-acting, 200 R.P.M., no clearance, initial pressure 125 lbs. per square inch absolute, back pressure a vacuum of 26 ins. of mercury. Cut-off at maximum load $\frac{3}{4}$, no load, $\frac{1}{16}$. What per cent of rating of boiler will be required by the engine at no load?

Prob. 3. While an engine driving a generator is running, a short circuit occurs putting full load on engine, requiring a $\frac{3}{4}$ cut-off. A moment later the circuit-breaker opens and only the friction load remains, requiring a cut-off of only $\frac{3}{32}$. The engine being two-cylinder, double-acting, simple, 12×18 ins., running at 300 R.P.M., and having no clearance, what will be the rate at which it uses steam just before and just after circuit-breaker opens if the steam supplied is at 125 lbs. per square inch absolute and is just dry, becoming wet on expanding, and back pressure is 3 lbs. per square inch absolute?

Prob. 4. A pumping engine has two double-acting steam cylinders each 9×12 ins. and a fixed cut-off of $\frac{1}{2}$. It runs at 60 R.P.M. on 80 lbs. per square inch absolute steam pressure and atmospheric exhaust. Cylinder is jacketed so that steam stays dry throughout its expansion. How much steam will it use per hour? Neglect clearance.

Prob. 5. If an engine 10×14 ins. and running 250 R.P.M. has such a cut-off that complete expansion occurs for 90 lbs. per square inch absolute initial pressure, and at atmospheric (15 lbs. absolute) exhaust, what will be the horse-power and steam used per hour, steam being superheated at all times, and what would be the value for the horse-power and steam used if full stroke admission occurred?

Prob. 6. The steam consumption of an engine working under constant load is better than that of a similar one working under variable load. For a 16×24 ins. engine running at 250 R.P.M. on wet steam of 125 lbs. per square inch absolute and atmospheric exhaust, find the horse-power and steam used per horse-power per hour for best condition and by taking two lighter and three heavier loads, show by a curve how steam used per horse-power per hour will vary.

Prob. 7. For driving a shop a two cylinder single-acting engine, 6×6 ins., running at 430 R.P.M., is used. The cut-off is fixed at $\frac{1}{2}$ and intitial pressure varied to control speed. Plot a curve between horse-power and weight of steam per hour per horse-power for 20, 40, 60, 80, 100, 120 lbs. per square inch absolute initial pressure and atmospheric exhaust. Steam constantly dry. Clearance zero.

NOTE: δ_1 for above pressures equals .05, .095, .139, .183, .226, and .268 lbs. per cubic foot, respectively.

Prob. 8. Taking the loads found in Prob. 7, find what cut-off would be required to cause the engine to run at rated speed for each load if the initial steam pressure were 100 lbs. per square inch absolute, and the back pressure atmosphere, and a plot curve between horse-power and steam used per horse-power hour for this case.

Prob. 9. For working a mine hoist a two-cylinder, double-acting engine is used in which compressed air is admitted $\frac{3}{8}$ stroke at 125 lbs. per square inch absolute and then allowed to expand adiabatically and exhaust to atmosphere. If the cylinders are 18×24 ins. and speed is 150 R.P.M., find the horse-power and cubic feet of high pressure air needed per minute.

5. Work of Expansive Fluid in Single Cylinder with Clearance. Logarithmic Expansion and Compression; Cycle III. Mean Effective Pressure, Horse-power, and Consumption of Simple Engines. As in previous cycles, the net work of the cycle is equal to the algebraic sum of the positive work

one on the forward stroke and the negative work on the return stroke. By reas, Fig. 60, this is

$$\text{Work area} = \overline{JABN} + \overline{NBCW} - \overline{WDEO} - \overline{OEFJ}.$$

Expressed in terms of diagram points this becomes

$$W = \left\{ \begin{array}{l} P_b(V_b - V_a) + P_b V_b \log_e \dfrac{V_c}{V_b} \\ \\ -P_d(V_d - V_e) - P_e V_e \log_e \dfrac{V_e}{V_f} \end{array} \right\} \quad \ldots\ldots \quad (260)$$

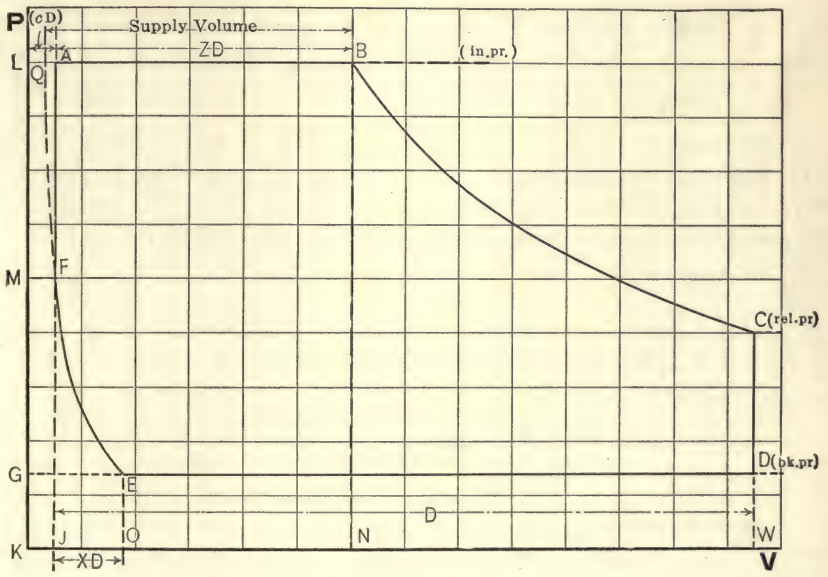


FIG. 60.—Work of Expansive Fluid in Single Cylinder with Clearance. Logarithmic Expansion for Cycle III. Exponential for Cycle IV.

Expressing this in terms of displacement, in cubic feet D; clearance as a fraction of displacement, c; cut-off as a fraction of displacement, Z; compression as a fraction of displacement, X; initial pressure, in pounds per square inch (in.pr.), and exhaust or back pressure, in pounds per square inch (bk.pr):

$$P_b = 144(\text{in.pr.}); \qquad (V_d - V_e) = (1-X)D;$$

$$P_d = 144\ (\text{bk.pr.}); \qquad V_b = D(Z+c);$$

$$(V_b - V_a) = ZD; \qquad V_e = D(X+c).$$

$$\frac{V_c}{V_b} = \frac{D(1+c)}{D(Z+c)} = \frac{1+c}{Z+c}, \qquad \frac{V_e}{V_f} = \frac{D(X+c)}{Dc} = \frac{X+c}{c}.$$

Whence

Work in ft.-lbs. per cycle is

$$W = \left\{ \begin{array}{l} 144D \left\{ \text{(in.pr.)} \left[Z + (Z+c) \log_e \frac{1+c}{Z+c} \right] \right. \\ \left. - \text{(bk.pr.)} \left[(1-X) + (X+c) \log_e \frac{X+c}{c} \right] \right\} \end{array} \right\}, \quad . \quad . \quad . \quad (261)$$

From Eq. (261), the *mean effective pressure*, pounds per square inch, follows by dividing by $144D$:

$$\text{(m.e.p.)} = \left\{ \begin{array}{ll} \text{(in.pr.)} \left[Z + (Z+c) \log_e \frac{1+c}{Z+c} \right]. \quad . \quad . & \text{(mean forward press.)} \\ - \text{(bk.pr.)} \left[(1-X) + (X+c) \log_e \frac{X+c}{c}. \quad . \right] & \text{(mean bk.pr.)} \end{array} \right\} \quad (262)$$

This is a general expression of very broad use in computing probable mean effective pressure for the steam engine with clearance and compression, or for other cases where it is practicable to assume the logarithmic law to hold. Fig. 117, at the end of this chapter, will be found of assistance in evaluating this expression.

Indicated horse-power, according to expressions already given, may be found by either of the following equations:

$$\text{I.H.P.} = \frac{\text{(m.e.p.)}Lan}{33{,}000} = \frac{144\text{(m.e.p.)}Dn}{33{,}000} = \frac{\text{(m.e.p.)}Dn}{229.2},$$

where L is stroke in feet, a is effective area of piston, square inches, n is the number of cycles performed per minute and D the displacement, cubic feet.

It might seem that the work per cubic foot of fluid supplied could be found by dividing Eq. (261) by the admission volume,

$$(V_b - V_a,)$$

but this would be true only when no steam is needed to build up the pressure from F to A. This is the case only when the clearance is zero or when compression begins soon enough to carry the point F up to point A, i.e., when by compression the pressure of the clearance fluid is raised to the initial pressure.

It is evident that the fluid supplied may perform the two duties: first, building up the clearance pressure at constant or nearly constant volume, and second, filling the cylinder up to cut-off at constant pressure. To measure the steam supplied in terms of diagram quantities requires the fixing of the volume of live steam necessary to build up the pressure from F to A and adding it to the apparent admission volume $(V_b - V_a)$. This can be done by producing

the compression line EF to the initial pressure Q, then LQ is the volume that the clearance steam would have at the initial pressure and QA the volume of live steam necessary to build up the pressure. The whole volume of steam admitted then is represented by $\overline{QB}$ instead of $\overline{AB}$ or by $(V_b - V_q)$ instead of by $(V_b - V_a)$, and calling this the supply volume,

$$(\text{Sup.Vol.}) = (V_b - V_q.)$$

But

$$V_q = V_e \frac{P_e}{P_q} = V_e \frac{(\text{bk.pr.})}{(\text{in.pr.})} = (X+c)D \frac{(\text{bk.pr.})}{(\text{in.pr.})}.$$

Hence

$$(\text{Sup.Vol.}) = D\left[(Z+c) - (X+c)\frac{(\text{bk.pr.})}{(\text{in.pr.})}\right], \quad . \quad . \quad . \quad . \quad (263)$$

which is the cubic feet of fluid admitted at the initial pressure for the displacement of D cubic feet by the piston. Dividing by D there results

$$\frac{(\text{Sup.Vol.})}{D} = (Z+c) - (X+c)\frac{(\text{bk.pr.})}{(\text{in.pr.})}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (264)$$

which is the ratio of admission volume to displacement or cubic feet of live steam admitted per cubic foot of displacement.

Dividing the work done by the cubic feet of steam supplied gives the economy of the simple engine in terms of volumes, or

$$\text{Work per cu.ft. of fluid supplied} = \frac{W}{(\text{Sup.Vol.})}$$

$$= 144\frac{(\text{in.pr.})\left[Z+(Z+c)\log_e \frac{1+c}{Z+c}\right] - (\text{bk.pr.})\left[(1-X)+(X+c)\log_e \frac{X+c}{c}\right]}{(Z+c)-(X+c)\frac{(\text{bk.pr.})}{(\text{in.pr.})}}; \quad (265)$$

It is more common to express economy of the engine in terms of the weight of steam used per hour per horse-power or the "water rate," which in more general terms may be called the consumption per hour per I.H.P.

Let δ_1 be the density or weight per cubic foot of fluid supplied, then the weight per cycle is (Sup.Vol.) δ_1, and this weight is capable of performing W foot-pounds of work or (Sup.Vol.) δ_1 lbs. per minute will permit of $\frac{W}{33{,}000}$ horse-power. But (Sup.Vol.) δ_1 lbs. per mintue corresponds to 60 (Sup.Vol.) δ_1 lbs. per hour, whence the number of pounds per hour per horse-power is

$$\frac{60\,(\text{Sup.Vol.})\delta_1}{W/33{,}000},$$

which is the pounds consumption per hour per I.H.P., whence

$$\text{Consumption in lbs. per hr. per I.H.P.} = \frac{60 \times 33{,}000(\text{Sup.Vol.})\delta_1}{W}, \quad . \quad . \quad . \quad (266)$$

which is the general expression for consumption in terms of the cubic feet of fluid admitted per cycle, δ_1 initial density, and the work per cycle.

As work is the product of mean effective pressure in pounds per square foot, (M.E.P.,) and the displacement in cu.ft. or $W = (\text{M.E.P.})D$, or in terms of mean effective pressure pounds per square inch $W = 144\ (\text{m.e.p.})D$, the consumption may also be written

$$\text{Consumption in lbs.,per hr. per I.H.P.} = \frac{60 \times 33{,}000\,(\text{Sup.Vol.})\delta_1}{144\,(\text{m.e.p.})D}$$

$$= \frac{13{,}750}{(\text{m.e.p.})} \frac{(\text{Sup.Vol.})\delta_1}{D}$$

$$= \frac{13{,}750}{(\text{m.e.p.})}\left[(Z+c) - (X+c)\frac{(\text{bk.pr.})}{(\text{in.pr.})}\right]\delta_1, \quad . \quad . \quad . \quad . \quad . \quad (267)$$

which gives the water rate in terms of the mean effective pressure, cut-off, clearance, compression, initial and back pressures and initial steam density. It is sometimes more convenient to introduce the density of fluid at the back pressure δ_2, which can be done by the relation (referring to the diagram),

$$P_q V_q = P_e V_e \text{ or } \frac{(\text{in.pr.})}{(\text{bk.pr.})} = \frac{V_e}{V_q} = \frac{\delta_1}{\delta_2},$$

whence

$$\delta_2 = \delta_1 \frac{(\text{bk.pr.})}{(\text{in.pr.})}.$$

This on substitution gives

$$\text{Consumption in lbs., per hr. per I.H.P.}$$

$$= \frac{13.750}{(\text{m.e.p.})}\left[(Z+c)\delta_1 - (X+c)\delta_2.\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (268)$$

Since the step taken above of introducing δ_2 has removed all pressure or volume ratios from the expression, Eq. (268) is general, and not dependent upon the logarithmic law. It gives the consumption in terms of mean effective pressure, cut-off, clearance, compression and the density of steam at initial and back pressure, which is of very common use.

It cannot be too strongly kept in mind that all the preceding is true only when no steam forms from moisture water during expansion or compression or no steam condenses, which assumption is known to be untrue. These formulæ are, therefore, to be considered as merely convenient approximations, although they

are almost universally used in daily practice. (See the end of this chapter for diagrams by which the solution of this expression is facilitated.)

Special Cases. First, *no expansion and no compression* would result in Fig. 61. For it

$$W = 144D[(\text{in.pr.}) - (\text{bk.pr.})]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (269)$$

$$(\text{m.e.p.}) = (\text{in.pr.}) - (\text{bk.pr.}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (270)$$

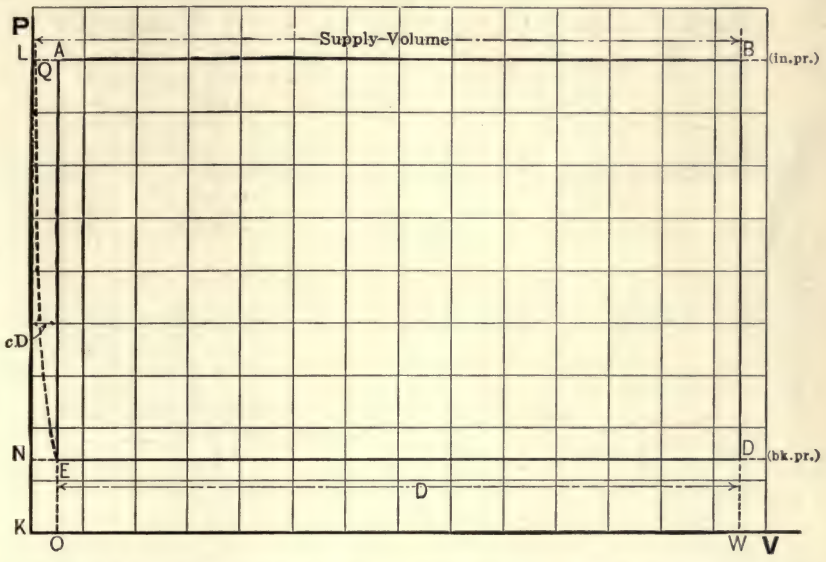


FIG. 61.—First Special Case of Cycles III and IV. Expansion and Compression both Zero, but Clearance Finite.

The volume of fluid supplied per cycle is QB, or from Eq. (263) it is

$$(\text{Sup.Vol.}) = D\left[1 + c - c\frac{(\text{bk.pr.})}{(\text{in.pr.})}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (271)$$

$$\text{Consumption in lbs. per hr. per I.H.P.} = \frac{13{,}750}{(\text{in.pr.}) - (\text{bk.pr.})}\left[1 + c - c\frac{(\text{bk.pr.})}{(\text{in.pr.})}\right]\delta_1, \quad (272)$$

or in terms of initial and final densities,

$$\text{Consumption in lbs. per hr. per I.H.P.} = \frac{13{,}750}{(\text{in.pr.}) - (\text{bk.pr.})}[(1+c)\delta_1 - c\delta_2] \quad (273)$$

The second special case is that of complete expansion and compression, as indicated in Fig. 62. Complete expansion provides that the pressure at the

end of expansion be equal to the back pressure, and complete compression that the final compression pressure be equal to the initial pressure.

Here

$$\frac{V_c}{V_b}=\frac{V_e}{V_a}=\frac{1+c}{Z+c}=\frac{X+c}{c}=\frac{(\text{in.pr.})}{(\text{bk.pr.})},$$

and hence

$$\frac{V_c-V}{V_b-V_a}=\frac{\text{in.pr.}}{\text{bk.pr.}}, \quad \text{but} \quad (V_c-V_e)=D\left[1+c-c\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)\right]$$

and

$$(V_b-V_a)=ZD.$$

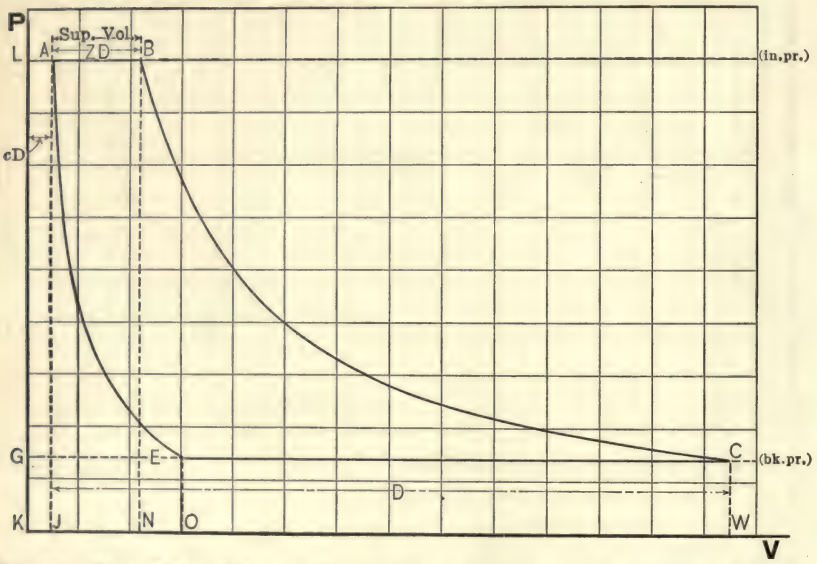


FIG. 62.—Second Special Case of Cycles III and IV. Perfect Expansion and Perfect Compression with Clearance.

Hence by substitution

$$\frac{D\left[1+c-c\dfrac{(\text{in.pr.})}{(\text{bk.pr.})}\right]}{ZD}=\frac{(\text{in.pr.})}{(\text{bk.pr.})},$$

from which

$$Z=(1+c)\frac{(\text{bk.pr.})}{(\text{in. pr.})}-c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (274)$$

Again,

$$X=\frac{V_e-V_a}{V_c-V_a}=\frac{cD\left[\left(\dfrac{\text{in.pr.}}{\text{bk.pr.}}\right)-1\right]}{D}=c\left[\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)-1\right] \quad . \quad . \quad . \quad (275)$$

Eq. (274) gives the cut-off as a fraction of the displacement necessary to give complete expansion, while Eq. (275) gives the compression as a fraction of displacement to give complete compression, both in terms of clearance, initial pressure and back pressure, provided the logarithmic law applies to expansion and compression.

Substitution of the values given above in Eq. (261) gives, after simplification,

$$W = 144D[(1+c)(\text{bk.pr.}) - c(\text{in.pr.})] \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}. \quad . \quad . \quad (276)$$

$$(\text{m.e.p.}) = [(1+c)(\text{bk.pr.}) - c(\text{in.pr.})] \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}. \quad . \quad . \quad . \quad . \quad (277)$$

In this case the volume supplied is exactly equal to that represented by the admission line AB, and is equal to

$$(\text{Sup.Vol.}) = ZD. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (278)$$

Hence, the consumption, in pounds fluid per hour per I.H.P. in terms of initial density, is

$$\text{Consumption in lbs. per hr. per I.H.P.} = \frac{13{,}750}{(\text{m.e.p.})} Z\delta_1,$$

but

$$\frac{Z}{\text{m.e.p.}} = \frac{(1+c)\frac{(\text{bk.pr.})}{(\text{in.pr.})} - c}{(\text{in.pr.})\left[(1+c)\frac{(\text{bk.pr.})}{(\text{in.pr.})} - c\right] \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}} = \frac{1}{(\text{in.pr.}) \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}},$$

hence

$$\text{Consumption in lbs. fluid per hr. per I.H.P.} = \frac{13{,}750\delta_1}{(\text{in.pr.}) \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}}. \quad . \quad . \quad (279)$$

This last equation is interesting in that it shows the consumption (or water rate, if it is a steam engine) is independent of clearance, and dependent only upon initial density, and on the initial and final pressures.

An expression may also be easily derived for the consumption in terms of initial and final density, but due to its limited use, will not be introduced here.

Example 1. Method of calculating Diagrams Fig. 60 and Fig. 62. Assumed data for Fig. 60:

$P_a = P_a = P_b = 90$ lbs. per square inch abs. $V_a = V_f = .5$ cu.ft.
$P_g = P_e = P_d = 14$ lbs. per square inch abs. $V_d = V_c = 135$ cu.ft.
$P_f = 50$ lbs. per square inch abs. $V_b = 6$ cu.ft.

To obtain point C:

$$P_c = P_b \times \frac{V_b}{V_c} = 90 \times \frac{6}{13.5} = 40 \text{ lbs. per sq.in. abs.}$$

To obtain point E:

$$V_e = V_f \times \frac{P_f}{P_e} = .5 \times \frac{50}{14} = 1.78 \text{ cu.ft.}$$

To obtain point Q:

$$V_q = V_f \times \frac{P_f}{P_q} = .5 \times \frac{50}{90} = .278 \text{ cu.ft.}$$

Intermediate points from B to C and E to Q are found by assuming volumes and computing the corresponding pressures by relation $P_x V_x = P_b V_b$ or $P_x V_x = P_e V_e$.

$$\text{Clearance is } \frac{V_a}{V_d - V_a} = \frac{.5}{13} = 3.8 \text{ per cent,}$$

$$\text{Cut-off is } \frac{V_b - V_a}{V_d - V_a} = \frac{5.5}{13} = 42.3 \text{ per cent,}$$

$$\text{Compression is } \frac{V_e - V_a}{V_d - V_a} = \frac{1.28}{13} = 9.9 \text{ per cent.}$$

Assumed data for Fig. 62.

$P_a = P_b = 90$ lbs. per square inch absolute. $V_a = .5$ cu.ft.
$P_e = P_c = 14$ lbs. per square inch absolute. $V_d = 13.5$ cu.ft.

To obtain point B:

$$V_b = V_c \frac{P_c}{P_b} = 13.5 \times \frac{14}{90} = 2.11 \text{ cu.ft.}$$

To obtain point E:

$$V_e = V_a \times \frac{P_a}{P_e} = .5 \times \frac{90}{14} = 3.2 \text{ cu.ft.}$$

Intermediate points from B to C and from A to E are to be found by assuming various volumes and finding the corresponding pressures from relation $P_x V_x = P_a V_a$ or $P_x V_x = P_b V_b$.

Example. 2. What will be the horse-power of, and steam used per hour by the following engine:

(*a*) cut-off 50 per cent, compression 30 per cent,

(*b*) complete expansion and compression,

(*c*) no expansion or compression.

Cylinder, 12×18-in. double-acting, 200 R.P.M., 7 per cent clearance, initial pressure 85 lbs. per square inch gage, back pressure 15 lbs. per square inch absolute, and logarithmic expansion and compression.

NOTE: δ for 85 lbs. gage = .23, δ_1 for 15 lbs. absolute = .038 cu.ft.

(*a*) From Eq. (262)

$$(\text{m.e.p.}) = (\text{in.pr.})\left[Z+(Z+c)\log_e\frac{1+c}{Z+c}\right]-(\text{bk.pr.})\left[(1-X)+(X+c)\log_e\frac{X+c}{c}\right],$$

$$=100\left[.5+(.5+.07)\log_e\frac{1+.07}{.5+.07}\right]-15\left[(1-.3)+(.3+.07)\log_e\frac{.3+.07}{.07}\right]$$

$$=86-20=66 \text{ lbs. sq.in.}$$

$$\text{I.H.P.}=\frac{(\text{m.e.p.})\,Lan}{33{,}000}=\frac{66\times1.5\times113.1\times400}{33{,}000}=135.$$

From Eq. (267) steam per hour per I.H.P. in pounds is

$$\frac{13{,}750}{(\text{m.e.p.})}\left[(Z+c)-(X+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)\right]\delta_1,$$

$$=\frac{13{,}750}{66}\left[(.5+.07)-(.3+.07)\times\frac{15}{100}\right]\times.23=25 \text{ lbs.}$$

Hence total steam per hour $=25\times135=3380$ lbs.

(*b*) From Eq. (277)

$$(\text{m.e.p.})=[(1+c)(\text{bk. pr.})-c(\text{in. pr.})]\log_e\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right),$$

$$=[(1+.07)\times15-.07\times100]\log_e 6.67=17.2 \text{ lbs. sq.in.}$$

$$\text{I.H.P.}=\frac{17.2\times1.5\times113.1\times400}{33{,}000}=35.4.$$

From Eq. (279)

$$\text{Steam per I.H.P. per hour}=\frac{13{,}750\delta_1}{(\text{in.pr.})\log_e\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)}=\frac{13{,}750\times.23}{100\times1.9}=16.6,$$

Total steam per hour $=16.6\times35.4=588$ lbs.

(*c*) From Eq. (270) (m.e.p.) = (in.pr.) − (bk.pr.) $=100-15=85$ lbs. sq.in.

$$\therefore\ \text{I.H.P.}=\frac{85\times1.5\times13.1\times400}{33{,}000}=174.5.$$

From Eq. (273)

$$\text{Steam per I.H.P. per hour}=\frac{13{,}750}{(\text{m.e.p.})}[(1+c)\delta_1-c\delta_2]-\frac{13{,}750}{85}[1.07\times.23-.07\times.038]=35.4.$$

Total steam per hour $=174.5\times35.4=6100$ lbs.

Prob. 1. What will be the horse-power and water rate of a 9×12 in. simple engine having 5 per cent clearance, running at 250 R.P.M. on 100 lbs. per square inch abso-

lute initial pressure and 5 lbs. per square inch absolute back pressure when the cut-off is $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$, expansion follows the logarithmic law, and there is no compression?

NOTE: δ for 100 lbs. absolute = .23, δ for 5 lbs. absolute = .014.

Prob. 2. Will a pump with a cylinder 10×15 ins. and 10 per cent clearance give the same horse-power and have the same water rate as a pump with cylinders of the same size but with 20 per cent clearance, both taking steam full stroke? Solve for a case of 125 lbs. per square inch absolute initial pressure, atmospheric exhaust and a speed of 50 double strokes. No compression.

NOTE: δ for 125 lbs. absolute = .283, δ for 15 lbs. absolute = .039.

Prob. 3. Solve the above problem for an engine of the same size, using steam expansively when the cut-off is $\frac{1}{2}$ and R.P.M. 200, steam and exhaust pressure as in Prob. 2 and compression zero.

Prob. 4. Two engines, each 9×15 ins., are running on an initial pressure of 90 lbs. per square inch absolute, and a back pressure of one atmosphere. One has no clearance, the other 8 per cent. Each is cutting off so that complete expansion occurs. The speed of each is 200 and neither has any compression. What will be the horse-power and water rate?

NOTE: δ for 90 lbs. = .24, δ for 15 lbs. = .039.

Prob. 5. By finding the horse-power and water rate of a 12×18 in. double-acting engine with 8 per cent clearance, running at 150 R.P.M. on an initial steam pressure of 90 lbs. per square inch absolute and atmosphere exhaust for a fixed cut-off of $\frac{1}{2}$ and variable compression from 0 to the point where the pressure at the end of compression is equal to 125 per cent of the initial pressure, plot the curves between compression and horse-power, and compression and water rate to show the effect of compression on the other two.

NOTE: δ for 90 lbs. = .21, δ for 15 lbs. = .039.

Prob. 6. A steam engine is running at such a load that the cut-off has to be $\frac{3}{8}$ at a speed of 150 R.P.M. The engine is 14×20 ins. and has no clearance. Initial pressure 100 lbs. per square inch absolute and back pressure 5 lbs. per square inch absolute. What would be the cut-off of an engine of the same dimensions but with 10 per cent clearance under similar conditions?

Prob. 7. The steam pressure is 100 lbs. per square inch gage and the back pressure is 26 ins. of mercury vacuum when the barometer is 30 ins. For a 16×22 in. engine with 6 per cent clearance running at 125 R.P.M., cut-off at $\frac{1}{2}$ and 30 per cent compression, what will be the horse-power and the water rate? Should the steam pressure be doubled what would be the horse-power and the water rate? If it should be halved?

NOTE: δ for 100 lbs. gage = .2017, δ for 26 ins. Hg. = .0058.

Prob. 8. While an 18×24 in. simple engine with 4 per cent clearance at speed of 150 R.P.M. is running with a $\frac{1}{4}$ cut-off and a compression of $\frac{1}{5}$ on a steam pressure of 125 lbs. per square inch gage, and a vacuum of 28 ins. of mercury, the condenser fails and the back pressure rises to 17 lbs. per square inch absolute. What will be the change in the horse-power and water rate if all other factors stay constant? What would the new cut-off have to be to keep the engine running at the same horse-power and what would be the water rate with this cut-off?

NOTE: δ for 125 lbs. gage = .315, δ for 28 in. Hg. = .0029, δ for 17 lbs. absolute = .0435.

Prob. 9. Under normal load an engine has a cut-off of $\frac{3}{8}$, while under light load the cut-off is but $\frac{1}{16}$. What per cent of the steam used at normal load will be used

at light load for the following data? Cylinder 10×18 ins.; 7 per cent clearance; 200 R.P.M.; initial pressure 120 lbs. per square inch gage; back pressure 2 lbs. per square inch absolute; compression at normal load 5 per cent; at light load 25 per cent.

NOTE: δ for 120 lbs. gage = .304, δ for 2 lbs. absolute = .0058.

6. Work of Expansive Fluid in Single Cylinder with Clearance; Exponential Expansion and Compression, Cycle IV. Mean Effective Pressure, Horsepower and Consumption of Simple Engines. As pointed out in several places, the logarithmic expansion of steam only approximates the truth in real engines and is the result of no particular logical or physically definable hypothesis as to the condition of the fluid, moreover its equations are of little or no value for compressed air or other gases used in engine cylinders. All expansions that can be defined by conditions of physical state or condition of heat, including the adiabatic, are expressible approximately or exactly by a definite value of s, not unity, in the expression $PV^s = \text{const.}$ All these cases can then be treated in a group and expressions for work and mean effective pressure found for a general value of s, for which particular values belonging to, or following from any physical hypothesis can be substituted. The area under such expansion curves is given by Eq. (13) Chapter I, which applied to the work diagram, Fig. 60, in the same manner as was done for logarithmic expansion, gives the *net work:*

$$\left.\begin{aligned} W = P_b(V_b - V_a) + \frac{P_b V_b}{s-1}\left[1 - \left(\frac{V_b}{V_c}\right)^{s-1}\right] \qquad & \text{(area } JABCWJ\text{)} \\ -P_d(V_d - V_e) - \frac{P_d V_e}{s-1}\left[\left(\frac{V_e}{V_f}\right)^{s-1} - 1\right] \qquad & \text{(area } WDEFJW\text{)} \end{aligned}\right\} . \quad (280)$$

Introducing the symbols,

$$P_b = 144(\text{in.pr.}), \qquad V_b = D(Z+c).$$

$$P_d = 144(\text{bk.pr.}), \qquad V_e = D(X+c).$$

$$(V_b - V_a) = ZD, \qquad \left(\frac{V_b}{V_c}\right) = \frac{Z+c}{1+c}.$$

$$(V_d - V_e) = D(1-X), \qquad \left(\frac{V_e}{V_f}\right) = \frac{X+c}{c}.$$

$$W = 144D\left\{(\text{in.pr.})\left[Z + \frac{Z+c}{s-1}\left[1 - \left(\frac{Z+c}{1+c}\right)^{s-1}\right]\right]\right.$$

$$\left. -(\text{bk.pr.})\left[(1-X) + \frac{X+c}{s-1}\left[\left(\frac{X+c}{c}\right)^{s-1} - 1\right]\right]\right\} . . \quad (281)$$

Eq. (281) gives the work in foot-pounds for D cubic feet of displacement in a cylinder having any clearance c, cut-off Z, and compression X, between two

pressures, and when the law of expansion is $PV^s = \text{const.}$ and s anything except unity, but constant.

The mean effective pressure, pounds per square inch, is obtained by dividing the expression for work by $144D$, giving

$$\left.\begin{aligned}(\text{m.e.p.}) = (\text{in.pr.})\left\{Z+\frac{Z+c}{s-1}\left[1-\left(\frac{Z+c}{1+c}\right)^{s-1}\right]\right\} \quad &(\text{mean for'd pr.})\\ -(\text{bk.pr.})\left\{(1-X)+\frac{X+c}{s-1}\left[\left(\frac{X+c}{c}\right)^{s-1}-1\right]\right\} \quad &(\text{mean bk.pr.})\end{aligned}\right\}, \qquad (282)$$

which is the general expression for mean effective pressure for this cycle.

It was pointed out in Section (5) that the cubic feet of fluid admitted at the initial pressure was not represented by $\overline{AB}$, Fig. 60, but by $\overline{QB}$, and the same is true for this case, so that the

$$(\text{Sup.Vol.}) = V_b - V_q.$$

But when the expansion and compression laws have the form $PV^s = c$

$$V_q = V_e\left(\frac{P_e}{P_q}\right)^{\frac{1}{s}} = D(X+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}.$$

Whence

$$(\text{Sup. Vol.}) = D\left[(Z+c)-(X+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}\right] \quad . \quad . \quad . \quad . \quad . \quad (283)$$

Eq. (283) gives for these cases what was given by Eq. (263) for the logarithmic law, the cubic feet of fluid supplied at the initial pressure for the displacement of D cubic feet in terms of cut-off, clearance, compression and the pressures. From this, by division there is found the volume of high pressure fluid per cu.ft. of displacement

$$\frac{(\text{Sup.Vol.})}{D} = (Z+c)-(X+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}} \quad . \quad . \quad . \quad . \quad . \quad (284)$$

The consumption is given by the general expression already derived, Eq. (34), from which is obtained,

Consumption lbs. per hr. per I.H.P.

$$= \frac{13{,}750}{(\text{m.e.p.})}\left[(Z+c)-(X+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}\right]\delta_1. \quad . \quad . \quad . \quad . \quad . \quad (285)$$

Eq. (285) gives the water rate or gas consumption in terms of mean effective pressure, initial and back pressure, cut-off, clearance, compression and initial fluid density. Introducing the density at the back pressure by the relation,

$$P_q V_q^s = P_e V_e^s;$$

$$\frac{V_e}{V_q} = \frac{\delta_1}{\delta_2} = \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{s}};$$

$$\delta_1 = \delta_2 \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{s}},$$

there results

$$\text{Consumption lbs. per hr. per I.H.P.} = \frac{13{,}750}{(\text{m.e.p.})}\left[(Z+c)\delta_1 - (X+c)\delta_2\right], \quad . \quad (286)$$

which is identical with Eq. (268) and is, as previously observed, a general expression, no matter what the laws of expansion and compression, in terms of mean effective pressure, cut-off, clearance, compression and the initial and final steam density.

The *first special case of full admission, no compression* might at first thought appear to be the same as in the preceding section, where the logarithmic law was assumed to hold, and so it is as regards work and mean effective pressure, Eq. (269) and (270), but referring to Fig. 61 it will be seen that since the exponential law is now assumed instead of the logarithmic, the point Q will be differently located (nearer to A than it was previously if s is greater than 1), and hence the supply volume $\overline{QB}$ is changed, and its new value is

$$(\text{Sup.Vol.}) = D\left[1+c-c\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}\right]. \quad . \quad . \quad . \quad . \quad . \quad (287)$$

Hence, consumption stated in terms of initial density of the fluid δ_1, is

$$\text{Consumption lbs. per hr. per I.H.P.}$$

$$= \frac{13{,}750}{(\text{in.pr.}) - (\text{bk.pr.})}\left[1+c-c\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}\right]\delta_1. \quad . \quad . \quad . \quad (288)$$

Stated in terms of initial and final densities, the expression is as before, Eq. (273).

The *second special case, complete expansion and compression* is again represented by Fig. 62. From the law of expansion it is evident that

$$V_b = V_c\left(\frac{P_c}{P_b}\right)^{\frac{1}{s}},$$

or stated in symbolic form,

$$D(Z+c)=D(1+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}},$$

whence

$$Z=(1+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}-c. \quad \ldots\ldots\ldots \quad (289)$$

Again referring to Fig. 62,

$$X=\frac{V_e-V_a}{V_c-V_a}=\frac{cD\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{s}}-cD}{D}.$$

Whence

$$X=c\left[\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{s}}-1\right]. \quad \ldots\ldots\ldots \quad (290)$$

Eq. (289) gives the cut-off as a fraction of displacement necessary to give complete expansion, and (290), the compression fraction to give complete compression, both in terms of clearance, initial and back pressures, and the exponent s, in the equation of the expansion or compression line, $PV^s=\text{const.}$

The work of the cycle becomes for this special case, by substitution in Eq. (281),

$$W=144D\ (\text{in.pr.})\frac{s}{s-1}\left[(1+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}-c\right]\left[1-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right], \quad (291)$$

and the mean effective pressure, lbs. per sq.in., is

$$(\text{m.e.p.})=(\text{in.pr.})\frac{s}{s-1}\left[(1+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}-c\right]\left[1-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right]. \quad \ldots \quad (292)$$

The volume of fluid supplied is,

$$(\text{Sup.Vol.})=ZD, \quad \ldots\ldots\ldots \quad (293)$$

hence

$$\text{Consumption, lbs. per hr. per I.H.P.}=\frac{13{,}750}{\text{m.e.p.}}Z\delta_1,$$

but

$$\frac{Z}{(\text{m.e.p.})}=\frac{\left[(1+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}-c\right]}{(\text{in.pr.})\frac{s}{s-1}\left[(1+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}-c\right]\left[1-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right]}$$

$$=\frac{1}{(\text{in.pr.})\frac{s}{s-1}\left[1-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right]},$$

whence

Consumption lbs. fluid per hr. per I.H.P. is,

$$\frac{13{,}750\times\delta_1}{(\text{in.pr.})\frac{s}{s-1}\left[1-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right]}, \quad \ldots\ldots \quad (294)$$

the expression for smallest consumption (or water rate if steam) of fluid for the most economical hypothetical cycle, which may it be noticed, is again independent of clearance.

The expressions for work and mean effective pressure are not, however, independent of clearance, and hence, according to the hypothetical cycles here considered, it is proved that large clearance decreases the work capacity of a a cylinder of given size, but does not affect the economy, provided complete expansion and compression are attained, a conclusion similar to that in regard to clearance effect on compressor capacity and economy. Whether the actual performance of gas or steam engines agrees with this conclusion based only on hypothetical reasoning, will be discussed later.

Example 1. What will be the horse-power of and steam used per hour by the following engine: 12×18-in. double-acting, 200 R.P.M., 7 per cent clearance, initial pressure 85 lbs. per square inch gage, back pressure 15 lbs. per square inch absolute, and expansion such that $s=1.3$.

(*a*) cut-off = 50 per cent; compression = 30 per cent;
(*b*) complete expansion and compression;
(*c*) no expansion or compression;

NOTE: δ for 85 lbs. gage = .18; for 15 lbs. absolute. = .03.

(*a*) From Eq. (282)

$$(\text{m.e.p.}) = (\text{in. pr.})\left\{Z+\frac{Z+c}{s-1}\left[1-\left(\frac{Z+c}{1+c}\right)^{s-1}\right]\right\} - (\text{bk. pr.})\left\{(1-X)+\frac{X+c}{s-1}\left[\left(\frac{X+c}{c}\right)^{s-1}-1\right]\right\}$$

$$=100\left\{.5+\frac{.57}{.3}\left[1-\left(\frac{.57}{1.07}\right)^{.3}\right]\right\}-15\left\{.7+\frac{.37}{.3}\left[\left(\frac{.37}{.07}\right)^{.3}-1\right]\right\}=59.8 \text{ lbs. sq.in.}$$

$$\text{I.H.P.}=\frac{59.8\times1.5\times113.1\times400}{33{,}000}=122.$$

From Eq. (286)

Steam per hour per I.H.P.

$$=\frac{13{,}750}{(\text{m.e.p.})}\left[(Z+c)\delta_1-(X+c)\delta_2\right]=\frac{13{,}750}{59.8}[(.57)\times.18-(.37)\times.03]=20.9 \text{ lbs.}$$

∴ Steam per hour = 122 × 20.9 = 2560 lbs.

(*b*) From Eq. (292)

$$(\text{m.e.p.}) = (\text{in. pr.})\frac{s}{s-1}\left[(1+c)\left(\frac{\text{bk. pr.}}{\text{in. pr.}}\right)^{\frac{1}{s}} - c\right]\left[1-\left(\frac{\text{bk. pr.}}{\text{in. pr.}}\right)^{\frac{s-1}{s}}\right],$$

$$= \frac{1.3}{.3}\times 100\left[1.07\times\left(\frac{15}{100}\right)^{\frac{1}{1.3}} - .07\right]\left[1-\left(\frac{15}{100}\right)^{\frac{.3}{1.3}}\right] = 26.2 \text{ lbs. sq.in.},$$

$$\text{I.H.P.} = \frac{26.2\times 1.5\times 113.1\times 200}{3000} = 64$$

From Eq. (294) steam used I.H.P. per hour is,

$$\frac{13{,}750\delta_1}{(\text{in.pr.})\frac{s}{s-1}\left[1-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right]\frac{1.3}{.3}\times 150\left[1-\left(\frac{15}{100}\right)^{\frac{.3}{1.3}}\right]} = \frac{13{,}750\times .18}{} = 16.5 \text{ lbs.},$$

hence total steam per hour $= 16.5\times 64 = 1060$ lbs.

(*c*) From Eq. (270) which holds for any value of s, m.e.p. $= 100 - 15 = 85$ lbs. sq.in.

and $$\text{I.H.P.} = \frac{85\times 1.5\times 113.1\times 400}{33{,}000} = 174.5,$$

From Eq. (288) steam per I.H.P. hour

$$= \frac{13{,}750\delta_1}{(\text{m.e.p.})}\left[1+c-c\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}\right] = \frac{13{,}750\times .18}{85}\left[1+.07-.07\times\left(\frac{15}{100}\right)^{\frac{1}{1.3}}\right] = 24.5 \text{ lbs.},$$

and total steam per hour $= 174.5\times 24.5 = 2475$ lbs.

Prob. 1. Two simple engines, each 12×18 ins., with 6 per cent clearance are operated at $\frac{1}{2}$ cut-off and with 20 per cent compression. One is supplied with air at 80 lbs. per square inch gage, and exhausts it to atmosphere; the other with initially dry steam which becomes wet on expansion and which also is exhausted to atmosphere. For a speed of 200 R.P.M. what is the horse-power of each engine and the cubic feet of stuff supplied per horse-power hour ?

Prob. 2. A crank-and-flywheel two-cylinder, double-acting, pumping engine is supplied with dry steam and the expansion is such that it remains dry until exhaust. The cylinder size is 24×36 ins., cut-off to give perfect expansion, clearance 5 per cent, compression to give perfect compression, initial pressure 50 lbs. per square inch absolute. back pressure 5 lbs. per square inch absolute. What is the horse-power and water rate? What would be the horse-power and water rate of a full-stroke pump of the same size and clearance but having no compression, running on the same pressure range and quality of steam.

NOTE: δ for 50 lbs. absolute $= .12$, δ for 15 lbs. absolute $= .0387$.

Prob. 3. Should the cylinder of the following engine be so provided that the steam was always kept dry, would there be any change in the horse-power developed a

compared with steam expanded adiabatically, and how much? Cylinder 20×24 ins., initial pressure 125 lbs. per sq. in. gage, back pressure 26 ins. vacuum, standard barometer, clearance 6 per cent, cut-off $\frac{3}{5}$, compression 10 per cent, and speed 125 R.P.M.

Prob. 4. What will be the total steam used per hour by a 20×28-in. double-acting engine running at 150 R.P.M. if the initial pressure be 125 lbs. per square inch absolute, back pressure one atmosphere, clearance 8 per cent, compression zero, for cut-off $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, and $\frac{1}{2}$, if steam expands adiabatically and is originally dry and saturated?

NOTE: δ for 125 lbs. absolute = .283, δ for 15 lbs. absolute = .0387.

Prob. 5. An engine which is supplied with superheated steam is said to have an indicated water rate of 15 lbs. at $\frac{1}{4}$ cut-off and one of 25 lbs. at $\frac{1}{8}$ cut-off. See if this is reasonable for the following conditions: engine is 15×22 ins., 7 per cent clearance, no compression, initial pressure 100 lbs. per square inch gage., back pressure 28-in. vacuum, barometer 30 ins. and speed 180 R.P.M.

NOTE: δ for 100 lbs. gage = .262, δ for 28 in. Hg = .0029.

Prob. 6. The tank capacity of a locomotive is 4500 gals. The cylinders are 18×36 ins., initial pressure 200 lbs. per square inch gage, exhaust atmospheric, cut-off $\frac{1}{2}$, clearance 7 per cent, speed 200 R.P.M., no compression. The stream is dry at start and expansion adiabatic, how long will the water last if 40% condenses during admission?

NOTE: δ for 200 lbs. gage = .471, δ for 15 lbs. absolute = .0387.

Prob. 7. To drive a hoist, an air engine is used, the air being supplied for $\frac{1}{2}$ stroke at 80 lbs. per square inch gage expanded adiabatically and exhausted to atmosphere. If the clearance is 8 per cent and there is no compression how many cubic feet of air per hour per horse-power will be needed? What, with complete compression?

Prob. 8. A manufacturer rates his 44×42-in. double-acting engine with a speed of 100 R.P.M. at 1000 H.P. when running non-condensing, initial pressure 70 lbs. per square inch gage and cut-off $\frac{1}{4}$. No clearance is mentioned and nothing said about manner of expansion. Assuming $s = 1.0646$, find on what clearance basis this rating is made.

Prob. 9. The water supply of a town is supplied by a direct-acting non-condensing pump with two cylinders, each 24×42 ins., with 10 per cent clearance, and no compression, initial pressure being 100 lbs. per square inch gage. What must be the size of the steam cylinder of a crank-and-flywheel pump with 6 per cent clearance to give the same horse-power on the same steam and exhaust pressures with a cut-off of $\frac{1}{2}$? Speed in each case to be 50 R.P.M.

NOTE: δ for 100 lbs. gage = .262, δ for 15 lbs. abs. = .0387.

7. Action of Fluid in Multiple-expansion Cylinders. General Description of Structure and Processes. When steam, compressed air, or any other high pressure working fluid is caused to pass through more than one cylinder in series, so that the exhaust from the one is the supply for the next, the engine is, in general, a multiple-expansion engine, or more specifically, a compound when the operations are in two expansion stages, triple for three, and quadruple for four stages. It must be understood that while a compound engine is one in which the whole pressure-volume change from initial to back pressure takes place in two stages, it does not necessarily follow that the machine is a two-cylinder one, for the second stage of expansion may take place in two cylinders, in each of which, half of the steam is put through identical operations; this

would make a three-cylinder compound. Similarly, triple-expansion engines, while they cannot have less than three may have four or five or six cylinders Multiple expansions engine, most of which are compound, are of two classes with respect to the treatment and pressure-volume changes of the steam, first *without receiver*, and second, *with receiver*. A receiver is primarily a chamber large in proportion to cylinder volumes, placed between the high- and low-pressure cylinders of compounds or between any pair of cylinders in triple or quadruple engines, and its purpose is to provide a reservoir of fluid so that the exhaust from the higher into it, or the admission to the lower from it, will be accomplished without a material change of pressure, and this will be accomplished as its volume is large in proportion to the charge of steam received by it or delivered from it. With a receiver of infinite size the exhaust line of a high-pressure cylinder discharging into it will be a constant-pressure line, as will also the admission line of the low-pressure cylinder. When, however, the receiver is of finite size high-pressure exhaust is equivalent to increasing the quantity of fluid in the receiver of fixed volume and must be accompanied by a rise of pressure except when a low-pressure cylinder may happen to be taking out fluid at the same rate and at the same time, which in practice never happens. As the receiver becomes smaller in proportion to the cylinders, the pressure in it will rise and fall more for each high-pressure exhaust and low-pressure admission with, of course, a constant average value. The greatest possible change of pressure during high-pressure exhaust and low-pressure admission would occur when the receiver is of zero size, that is when there is none at all, in which case, of course, the high-and low-pressure pistons must have synchronous movement, both starting and stopping at the same time, but moving either in the same or opposite directions. When the pistons of the no-receiver compound engines move in the same direction at the same time, one end of the high-pressure cylinder must exhaust into the opposite end of the low; but with oppositely moving pistons, the exhaust from high will enter the same end of the low. It is plain that a real receiver of zero volume is impossible, as the connecting ports must have some volume and likewise that an infinite receiver is equally impracticable, so that any multiple-expansion real engine will have receivers of finite volume with corresponding pressure changes during the period when a receiver is in communication with a cylinder. The amount of these pressure changes will depend partly on the size of the receiver with respect to the cylinders, but also as well, on the relation between the periods of flow into receiver, by high-pressure exhaust and out of it, by low-pressure admission, which latter factor will be fixed largely by crank angles, and partly by the settings of the two valves, relations which are often extremely complicated.

For the purpose of analysis it is desirable to treat the two limiting cases of no receiver and infinite receiver, because they yield formulas simple enough to be useful, while an exact simple solution of the general case is impossible. These simple expressions for hypothetical cases which are very valuable for estimates and approximations are generally close to truth for an actual engine especially if intelligently selected and used.

Receivers of steam engines may be simple tanks or temporary storage chambers or be fitted with coils or tubes to which live or high-pressure steam is supplied and which may heat up the lower pressure, partly expanded steam passing from cylinder to cylinder through the receiver. Such receivers are *reheating receivers*, and as noted, may heat up the engine steam or may evaporate any moisture it might contain. As a matter of fact there can be no heating of the steam before all moisture is first evaporated, from which it appears that the action of such reheating receivers may be, and is quite complicated thermally, and a study of these conditions must be postponed till a thermal method of analysis is established. This will introduce no serious difficulty, as such reheating receivers assist the thermal economy of the whole system but little and have little effect on engine power, likewise are now little used. *Reheating of* air or other gases, as well as *preheating them before admission* to the high-pressure cylinder is a necessary practice, when the supply pressure is high, to prevent freezing of moisture by the gases, which get very cold in expansion if it be carried far. This is likewise, however, a thermal problem, not to be taken up till later.

Multiple-expansion engines are built for greater economy than is possible in simple engines and the reasons are divisible into two classes, first mechanical, and second thermal. It has already been shown that by expansion, work is obtained in greater amounts as the expansion is greater, provided, of course, expansion below the back pressure is avoided, and as high initial and low back pressures permit essentially of most expansion, engines must be built capable of utilizing all that the steam or compressed gas may yield. If steam followed the logarithmic law of expansion, pressure falling inversely with volume increase, then steam of 150 lbs. per square inch absolute expanding to 1 lb. per square inch absolute would require enough ultimate cylinder space to allow whatever volume of steam was admitted up to cut-off to increase 150 times. This would involve a *valve gear* and cylinder structure capable of admitting $\frac{1}{150} = .0067$ of the cylinder volume. It is practically impossible to construct a valve that will accurately open and close in this necessarily short equivalent portion of the stroke. This, however, is not the worst handicap even mechanically, because actual cylinders cannot be made without some *clearance*, usually more than 2 per cent of the displacement and in order that any steam might be admitted at all, the clearance in the example would have to be less than .67 per cent of the total volume, which is quite impossible. These two mechanical or structural limitations, that of *admission valve gear* and that of *clearance limits*, supply the first argument for multiple-expansion engines, the structure of which is capable of utilizing any amount of expansion that high boiler pressure and good condenser vacuum make available. For, if neglecting clearance, the low-pressure cylinder had ten times the volume of the high, then the full stroke admission of steam to the high followed by expansion in the low would give ten expansions, while admission to the high for $\frac{1}{15}$ of its stroke would give 15 expansions in it, after which this final volume would increase in the low ten times, that is, to 150 times the original

volume, and cylinder admission of $\frac{1}{15}$ or 6.7 per cent is possible with ordinary valve gears, as is also an initial volume of 6.7 per cent of a total cylinder volume, even with clearance which in reasonably large engines may be not over 2 per cent of the whole cylinder volume.

It is evident that the higher the initial and the lower the back pressures the greater the expansion ratio will be for complete expansion, and as in steam practice boiler pressures of 225 lbs. per square inch gage or approximately 240 lbs. per square inch absolute with vacuum back pressures as low as one or even half a pound per square inch are in use, it should be possible whether desirable or not, to expand to a final volume from 250 to 500 times the initial in round numbers. This is, of course, quite impossible in simple engine cylinders, and as it is easy with multiple expansion there is supplied another mechanical argument for *staging*. Sufficient expansion for practical purposes in locomotives and land engines under the usually variable load of industrial service is available for even these high pressures by compounding, but when the loads are about constant as in waterworks pumping engines, and marine engines for ship propulsion triple expansion is used for pressures in excess of about 180 lbs. gage.

Use of very high initial and very low back pressures will result in simple engines, in a possibility of great unbalanced forces on a piston, its rods, pins and crank, when acting on opposite sides, and a considerable fluctuation in tangential turning force at the crank pin. Compounding will always reduce the unbalanced force on a piston, and when carried out in cylinders each of which has a separate crank, permits of a very considerable improvement of turning effort. So that, not only does multiple expansion make it possible to utilize to the fullest extent the whole range of high initial and low back pressures but it may result in a better force distribution in the engine, avoiding shocks making unnecessary, excessively strong pistons, and rods and equalizing turning effort so that the maximum and minimum tangential force do not depart too much from the mean.

The second or thermal reason for bothering with multiple-expansion complications in the interest of steam economy is concerned with the prevention of steam loss by *condensation and leakage*. It does not need any elaborate analysis to show that low-pressure steam will be cooler than high-pressure steam and that expanding steam in a cylinder has a tendency to cool the cylinder and piston walls, certainly the inner skin at least, so that after expansion and exhaust they will be cooler than after admission; but as admission follows exhaust hot live steam will come into contact with cool walls and some will necessarily condense, the amount being smaller the less the original expansion; hence in any one cylinder of a multiple-expansion engine the condensation may be less than a simple engine with the same range of steam pressures and temperatures. Whether all the steam condensation during admission added together will equal that of the simple engine or not is another question. There is no doubt, however, that as the multiple expansion engine can expand usefully to greater degree than a simple engine, and so cause a lower temperature by expansion, that it has a greater chance to

eevaporate some of the water of initial condensation and so get some work out of the extra steam so evaporated, which in the simple engine might have emained as water, incapable of working until exhaust opened and lowered he pressure, when, of course, it could do no good. It is also clear that steam or compressed-air leakage in a simple engine is a direct loss, whereas in a compound high-pressure cylinder leakage has at least a chance to do some work in the low-pressure cylinder. The exact analysis of the thermal reasons or greater economy is complicated and is largely concerned with a study of steam condensation and reevaporation, but the fact is, that multiple-expansion engines are capable of greater economy than simple. The thermal analysis must also consider the influence of the reheating receiver, the steam-jacketed working cylinder, and the use of superheated steam, their effects on the possible work per pound of steam and the corresponding quantity of heat expended to secure it, and for air and compressed gas the parallel treatment of preheating and reheating.

To illustrate the action of steam in multiple-expansion engines some indicator cards are given for a few typical cases in Figs. 63 to 66, together with the combined diagrams of pressure-volume changes of the fluid in all cylinders to the same scale of pressures and volumes, which, of course, makes the diagram look quite different, as indicator cards are usually taken to the same base length, fixed by the reducing motion, and to different pressure scales, to get as large a height of diagram as the paper will permit. Fig. 63 shows four sets of cards taken from an engine of the compound no-receiver type, namely, a Vauclain compound locomotive. In this machine there are two cylinders, one high pressure and one low, on each side, the steam from the high pressure exhausting directly into the low-pressure cylinder so that the only receiver space is made up of the clearance and connecting passages between the cylinders. Starting with set **A**, the cards show a decreasing high pressure cut-off of 76 per cent in the case of set **A** to 54 per cent in the case of set **D**. The letters *A*, *B*, *C* and *D* refer in each case to admission, cut-off, release and compression, the use of primed letters denoting the low-pressure cylinder.

In set **A** the high-pressure admission line *AFB* may be considered as made up of two parts, the part *AF* representing pressure rise at constant volume, which is the admission of steam to the clearance space at dead center to raise the pressure from that at the end of compression to that of boiler pressure. From *F* to *B* admission occurred at constant pressure, steam filling the cylinder volume as the piston moved outward. At *B* cut-off or closure of the steam valve occurred and the steam in the cylinder expanded. At *C*, release or opening of the exhaust valve of the high-pressure cylinder occurred and the admission valve of the low-pressure cylinder opened, the steam dropping in pressure until the pressure in both high- and low-pressure clearance became equal, and then expanding in both cylinders, as the exhaust from the high and admission to the low occurred, the exhaust line *CD* of the high pressure and the admission line *F'B'* of the low pressure being identical except for the slight pressure drop

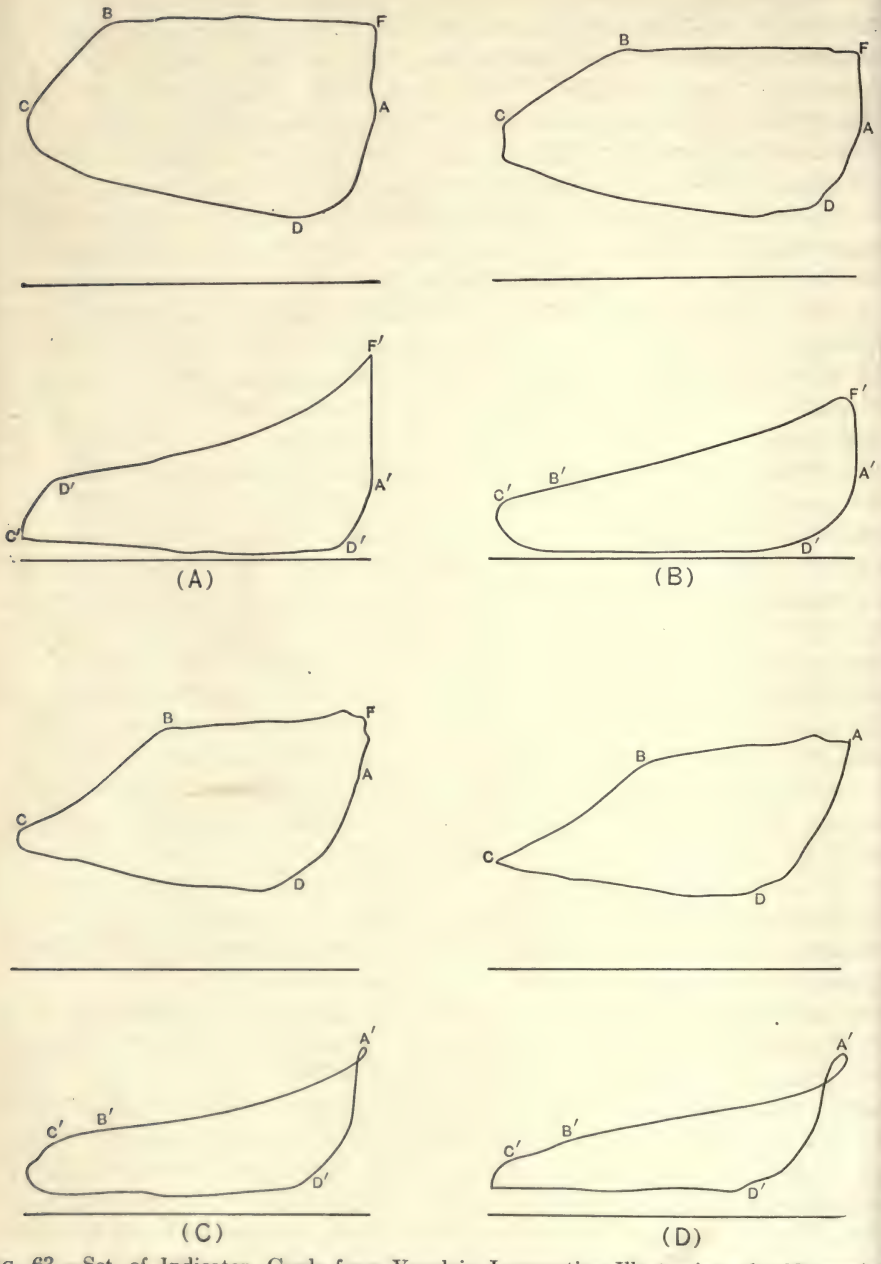


FIG. 63.—Set of Indicator Cards from Vauclain Locomotive Illustrating the No-receiver Compound Steam Engine.

in the passages between the high- and the low-pressure cylinders. At D the high-pressure exhaust valve closed and compression of the steam trapped in the high-pressure cylinder occurred to point A, thus closing the cycle. From point D' in the low-pressure cylinder, which corresponds to D in the high pressure, no more steam was admitted to the low-pressure cylinder. What steam there was in the low expanded to the point C' when the exhaust valve opened and the pressure dropped to the back pressure and the steam was exhausted at nearly constant back pressure to D', when the exhaust valve closed and the steam trapped in the cylinder was compressed to A', at which point steam was again admitted and the cycle repeated.

In set **B** the cycle of operation is exactly the same as in set **A**. In set **C** the cycle is the same as in **A**, but there are one or two points to be especially noted, as they are not present in set **A**. The admission line of the high-pressure cylinder is not a constant pressure, but rather a falling pressure one, due to throttling of the steam, or *"wire drawing,"* as it is called, through the throttle valve or steam ports, due to the higher speed at which this card was taken. It will also be noticed that the compression pressure is higher in this case, due to earlier closing of the exhaust valve, which becomes necessary with the type of valve gear used, as the cut-off is made earlier. In the low-pressure card it will be seen that the compression pressure is greater than the admission pressure and hence there is a pressure drop instead of rise on admission. In set **D** the peculiarities of **C** are still more apparent, the compression in high-pressure cylinder being equal to admission pressure and above it in the low-pressure cylinder. The wire drawing is also more marked, as the speed was still higher when this set of cards was taken.

In Fig. 64, one set of the cards of Fig. 63 is redrawn on cross-section paper and then combined. Cards taken from the different cylinders of a multiple-expansion engine will in nearly all cases have the same length, the greatest that can be conveniently handled by the indicator, and will be to two different pressures scales, in as much as that indicator spring will be chosen for each cylinder which will give the greatest height of card consistent with safety to the instrument. To properly compare the cards they must be reduced to the same pressure scale, and also to the same volume scale. As the lengths represent volumes, the ratio of the two volume scales will be as that of the cylinder volumes or diameters squared. Hence, the length of the high-pressure card must be decreased in this ratio or the low increased. As a rule it is found more convenient to employ the former method. When the cards have been reduced to a proper scale of pressures and volumes the clearance must be added to each in order that the true volume of the fluid may be shown. The cards may now be placed with these atmospheric lines and zero volume lines coinciding and will then appear in their true relation. In this case the cylinder ratio was 1.65, the indicator springs 100 lbs. and 70 lbs. respectively and clearance 5 per cent in each cylinder.

The steps in combining the cards were as follows: The zero volume lines were first drawn perpendicular to the atmospheric line and at a distance from the end of

the card equal to the length of the card times the clearance. PV axes were laid off and a line drawn parallel to the zero-pressure line at a distance above it equal to 14.7 lbs. to scale of combined diagram. This scale was taken to be that of the high-pressure diagram. A number of points $A'B'C'$, etc., were then chosen on the low-pressure card, and the corresponding points $a'b'c'$, etc., plotted by making the distances of a',b', etc., from the zero-volume line equal to those of A',B', etc., and the distances of the new points above the atmosphere .7 the distances of the original. By joining the points as plotted, the new diagram for the low-pressure card was formed. The high-pressure card was then redrawn

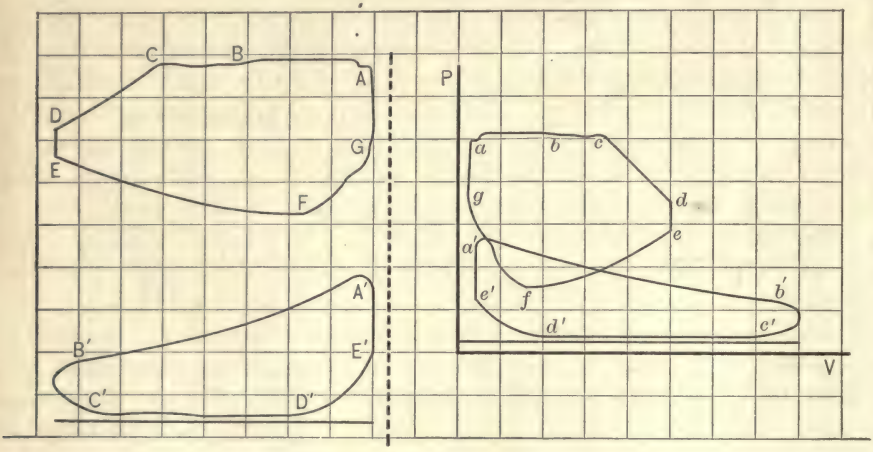


FIG. 64.—Diagram to Show Method of Combining the High- and Low-pressure Cylinder Indicator Cards of the No-receiver Compound Engine.

by taking a number of points A, B, C, etc., and plotting new points a, b, c, etc., so that the distances of a, b, c, etc., from the zero-volume line were $\frac{1}{1.65}$ the distances of A, B, C, etc., while the distances of new points above the atmospheric line were the same as for the original points.

In Fig. 65 are shown two cards from a compound steam engine with receiver. Diagram **A** shows the cards as taken, but transferred to cross-section paper for ease in combining, and with the zero-volume axis added. On the high-pressure card admission occurred practically at constant volume, piston being at rest at dead center, at A, bringing the pressures in the cylinder up to the initial pressure at B. Admission continued from B to C at nearly constant pressure, the piston moving slowly with correspondingly small demand for steam and consequently little wire drawing. From C to D the piston is moving more rapidly and there is in consequence more wire drawing, admission being no longer at constant pressure. At D the steam valve closes and expansion occurs, to E, where release occurs, the pressure falling to that in the receiver. From F to G exhaust occurs with increase of pressure due to the steam being forced into the receiver, (receiver+decreasing H.P. cyl.vol.) while from G to H the

pressure falls, due to the low-pressure cylinder taking steam from the receiver and consequently volume of receiver, (receiver+increasing L.P. cyl.vol.+ decreasing H.P. cyl.vol.) increasing. At H exhaust closes, a very slight compression occurring from H to A.

On the low-pressure card, admission occurred at A' and continued to B' at constant volume, the piston being on dead center as from A to B in high-pressure cylinder. From B' to C' admission occurred with falling pressure due to increase in receiver volume, (receiver+increasing L.P. cyl.vol.), and from C' to D' admission still took place, but with less rapidly falling pressure, as high-pressure cylinder is now exhausting and receiver volume, (receiver+increasing L.P. cyl.vol.+decreasing H.P. cyl.vol.) was receiving some steam as well as

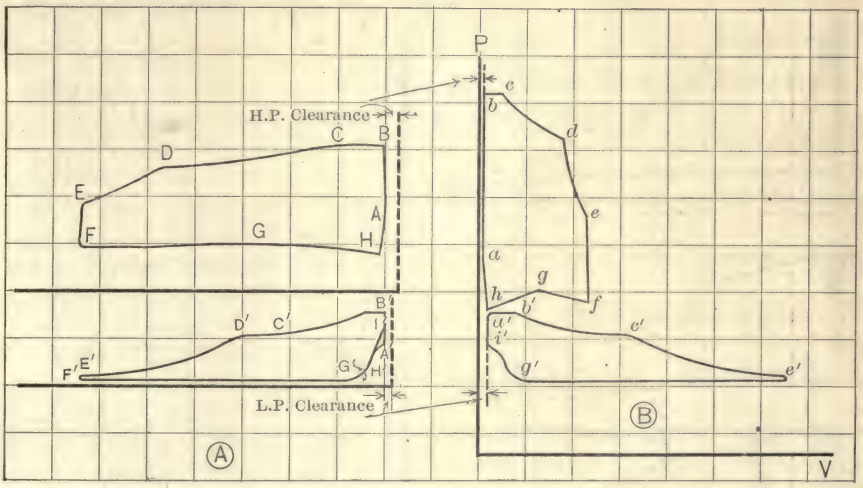


FIG. 65.—Indicator Cards from a Compound Engine with a Receiver, as Taken and as Combined.

delivering. At D' admission ceased and expansion took place to E' where release occurred, the pressure falling to the back pressure and continuing from F' to G', where the exhaust valve closed and compression took place to A', thus completing the cycle. At H' leakage past the exhaust valve was so great as to cause the curve to fall off considerably from H' to A', instead of continuing to be a true compression curve, ending at I, as it should have done. The combined diagrams are shown in **B**.

In Fig. 66 are shown a set of three cards from a triple-expansion pumping engine with large receivers and cranks at 120°. In diagram **A** the cards are shown with the same length and with different pressure scales as taken, but with the zero volume line added and transferred to cross-section paper. On the high-pressure card admission occurred at A, causing a constant volume pressure rise to B, the piston being at rest with the crank at dead center. From B to C admission occurred at nearly constant pressure to C, where steam was

cut off and expansion took place to D. At this point release occurred, the pressure dropping at constant volume to E with the piston at rest. From E exhaust took place with slightly increasing pressure, since the intermediate cylinder was taking no steam, the intermediate piston being beyond the point of cut-off. The pressure rise is slight, however, due to the size of the receiver, which is large compared to the cylinder. At two-thirds of the exhaust stroke, point F, the back pressure became constant and then decreased, for at this point the speed of the intermediate piston increased and the receiver pressure fell. At G exhaust closed and a slight pressure rise occurred to A, due to the restricted passage of the closing exhaust valve. On the intermediate card

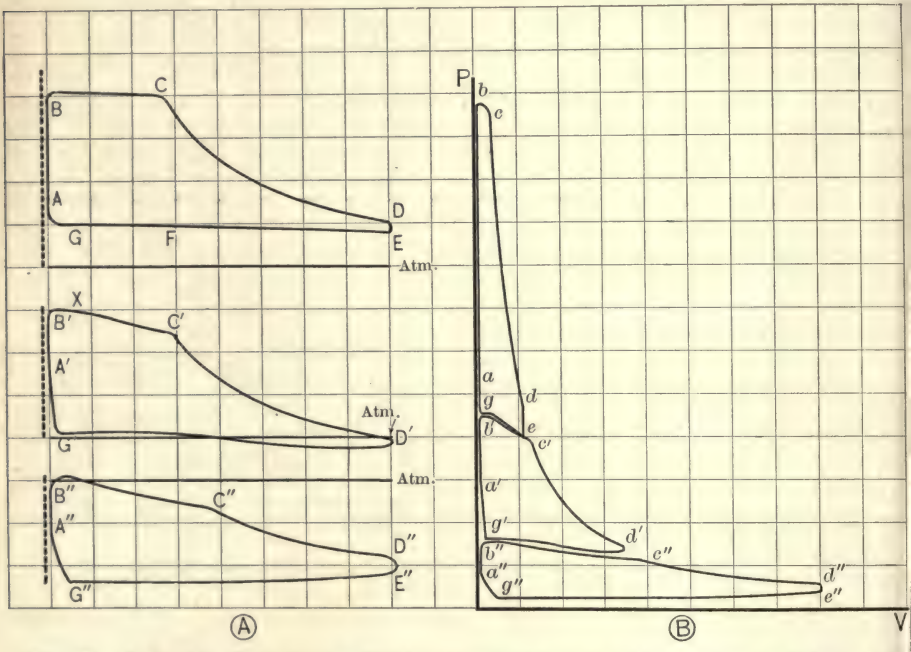


FIG. 66.—Indicator Cards from a Triple-expansion Engine with Receiver as Taken and Combined.

admission occurred at A', the pressure rising to B'. From B' the admissio was at nearly constant pressure to X while the piston speed was low and the at a falling pressure to C'. Pressure was falling, since the steam was supplie from a finite receiver into which no steam was flowing during intermediat admission. At C' cut-off occurred and steam expanded to D', where relea took place, and the steam was exhausted. As in the case of the high-pressu cylinder the back pressure was rising for two-thirds of the stroke, since t steam was being compressed into the receiver or rather into a volume ma up of receiver and intermediate cylinder volume, which is, of course, a decrea ing one, since the cylinder volume is decreasing. At two-thirds of the stro the low-pressure cylinder begins to take steam and the receiver volume

now increased, inasmuch as it was made up of the receiver portion of the intermediate cylinder and a portion of the low-pressure cylinder, and the low-pressure cylinder volume increased faster than intermediate decreased for the same amount of piston travel. At G' exhaust closed and a slight compression occurred to A', thus completing the cycle.

On the low-pressure card admission occurred at A'' and the pressure rose at constant volume to B'', and then admission continued first at constant pressure and then falling, as in the intermediate cylinder, to the point of cut-off at C''. From here expansion took place to D''. At this point the exhaust valve opened, the pressure fell nearly to back pressure at E'', and the steam was exhausted at practically constant back pressure to G'', where the exhaust valve closed and there was compression to A'', thus completing the cycle. The combined diagram is shown in **B.**

Prob. 1. In Fig. 67 are shown six sets of indicator cards from compound engines. The cylinder sizes and clearances are given below. Explain the cylinder events and the shape of lines for each card and form a combined diagram for each set.

No. 1. From a four-valve Corliss engine, 26×48 ins., with 3 per cent clearance in each cylinder.

No. 2. From a single-valve engine, $12 \times 20 \times 12$ ins., with 33 per cent clearance in high-pressure cylinder and 9 per cent in low.

No. 3. From a four-valve Corliss engine $22 \times 44 \times 60$ ins., with 2 per cent clearance in the high-pressure cylinder and 6 per cent in low.

No. 4. From a single-valve engine $18 \times 30 \times 16$ ins., with 30 per cent clearance in the high-pressure cylinder and 8 per cent in the low.

No. 5. From a single-valve engine $11\frac{1}{2} \times 18\frac{1}{2} \times 13$ ins., with 7 per cent clearance in the high and 10 per cent in the low.

No. 6. From a double-valve engine $14 \times 28 \times 24$ ins., with 3.5 per cent clearance in the high-pressure cylinder and 6.5 per cent in the low.

Prob. 2. In Fig. 68 are shown four sets of indicator cards from triple-expansion marine engines. The cylinder sizes and clearances are given below. Explain the cylinder events and the shape of the lines of each card and form a combined diagram of each set.

No. 1. From the engine of a steam-ship, cylinders $21.9 \times 34 \times 57$ ins. $\times 39$ ins. with 6 per cent clearance in each and fitted with simple slide valves.

No. 2. From an engine $20 \times 30 \times 50 \times 48$ ins.

No. 3. From the engine of a steam-ship with cylinders $22 \times 35 \times 58 \times 42$ ins., assume clearance 7 per cent in each cylinder.

No. 4. From the engine of the steamer "Aberdeen," with cylinders $30 \times 45 \times 70 \times 54$ ins., and with 4 per cent clearance in the high, 7 per cent in the intermediate, and 8 per cent in the low.

Prob. 3. In Fig. 69 are shown some combined cards from compound engines. Explain the cylinder events and the shape of the lines and reproduce the indicator cards.

Prob. 4. In Fig. 70 are shown some combined cards from triple-expansion engines. Draw the individual cards and explain the cylinder events and shape of lines.

8. Standard Reference Cycles or PV Diagrams for the Work of Expansive Fluids in Two-cylinder Compound Engines. The possible combinations of admission with all degrees of expansion for forward strokes and of exhaust with

all degrees of compression for back strokes, with and without clearance, in each of the two cylinders of the compound engine, that may have any volume relation one to the other and any size of receiver between, and finally, any sort of periodicity of receiver receipt and discharge of fluid, all make possible a

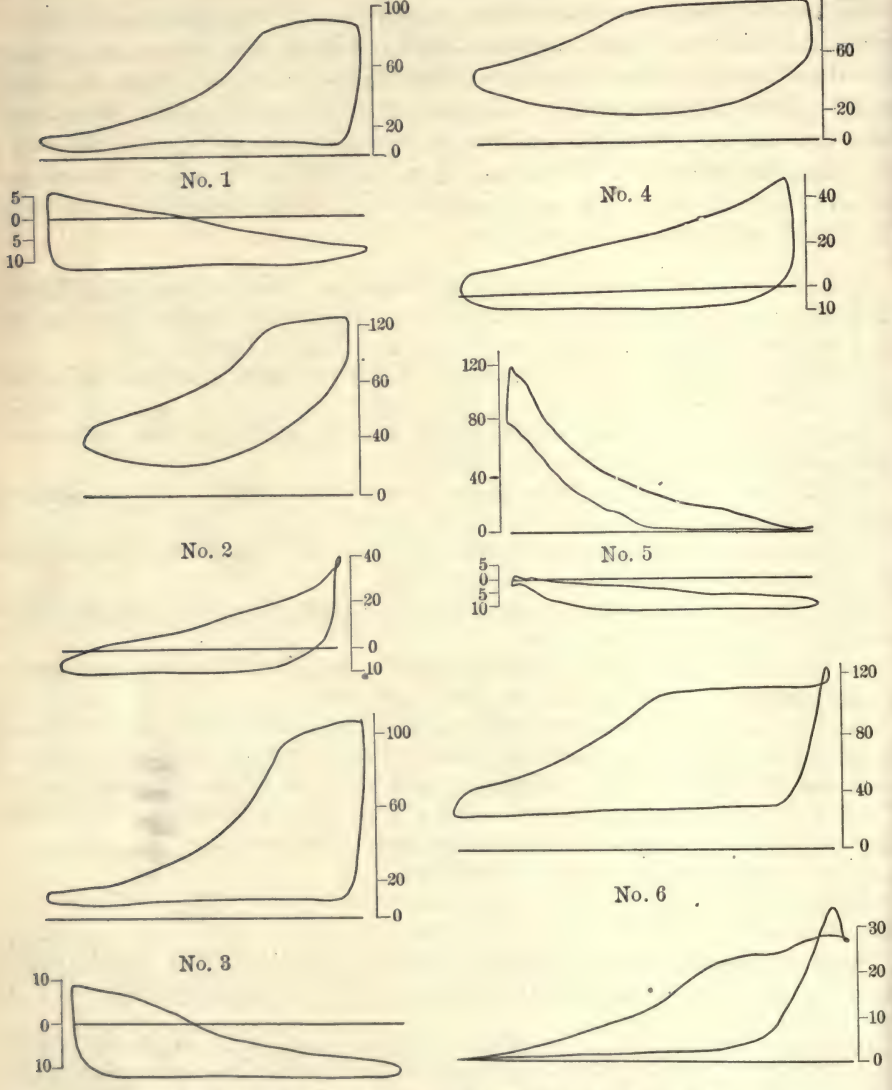


Fig. 67.—Six Sets of Compound Engine Indicator Cards.

very large number of cycles. In order that analysis of these conditions of working may be kept within reasonable space, it is necessary to proceed as was done with compressors and simple engines, concentrating attention on such type forms as yield readily to analytical treatment and for which the formulas

are simple even if only approximate with respect to actual engines, but, of course, keeping in mind the possible value of the formulas, as those that teach

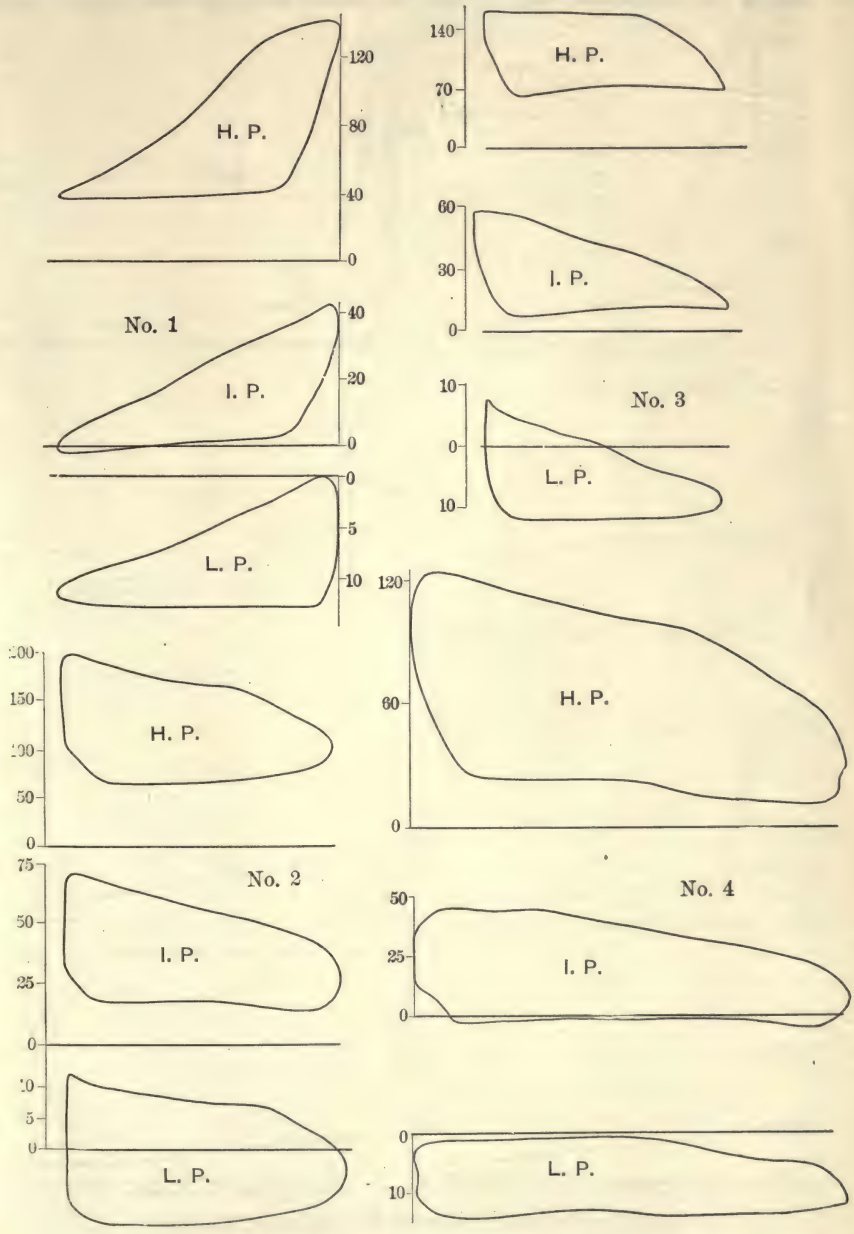


FIG. 68.—Four Sets of Triple-expansion Engine Indicator Cards.

no principles or fail to assist in solving problems must be discarded as useless. The work that fluids under pressure can do by losing that pressure is no

different in compound than in simple engines, *if the fluid has a chance to do what it can.* Provided the structure is such as will not interfere with the completeness of the expansion, and no fluid is wasted in filling dead spaces without

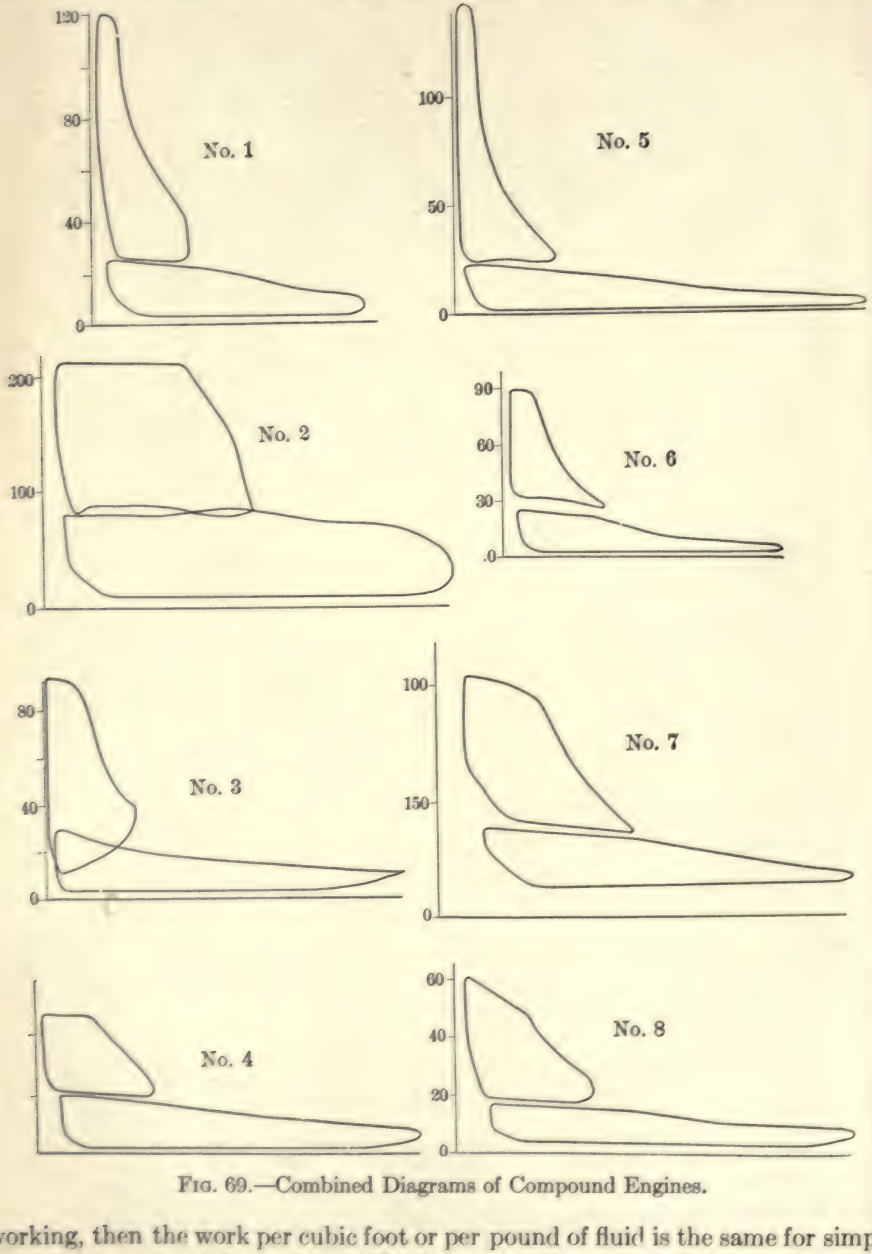


FIG. 69.—Combined Diagrams of Compound Engines.

working, then the work per cubic foot or per pound of fluid is the same for simple, compound and triple engines. Furthermore, there is a horse-power equivalence between the simple and compound, if, in the latter case the steam

dmitted up to cut-off may be assumed to be acting only in the low-pressure ylinder, that is, ignoring the high-pressure cylinder except as it serves as a

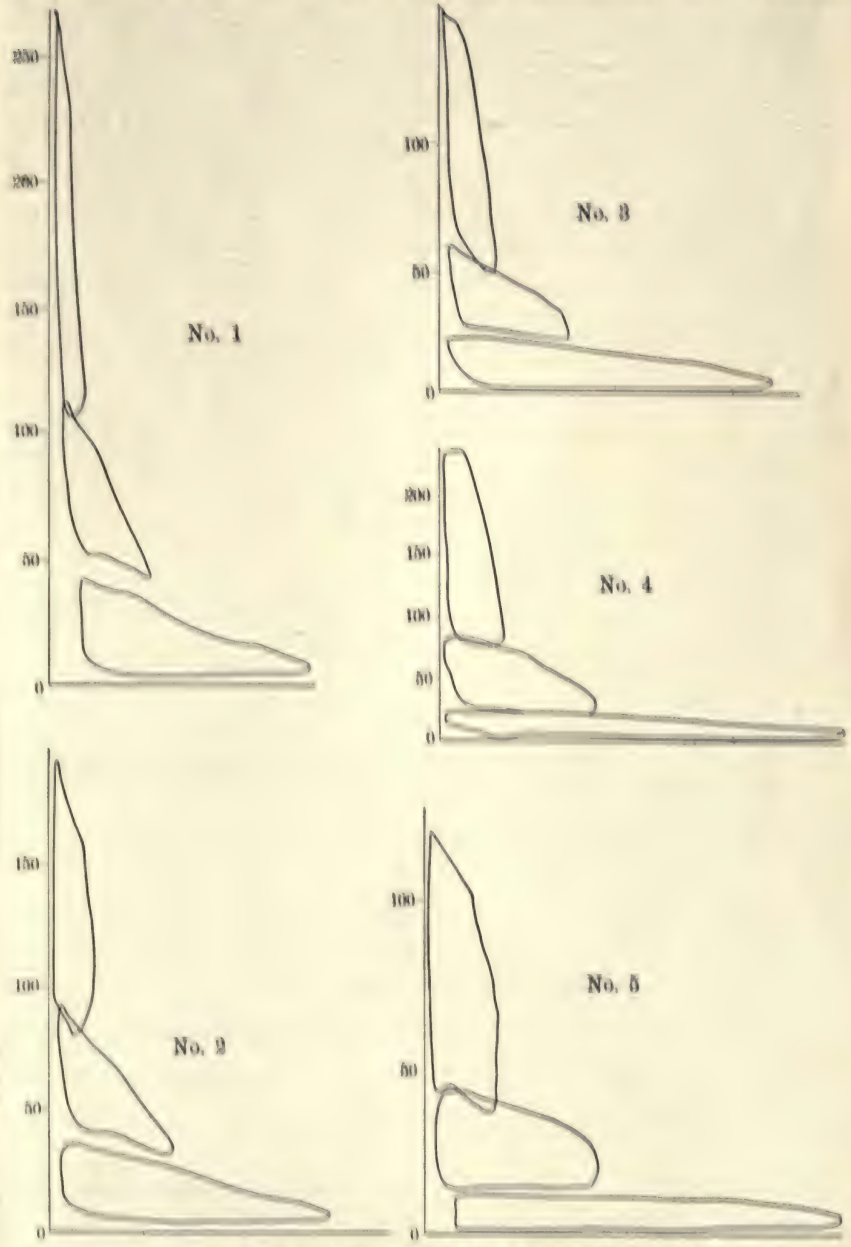


Fig. 70.—Combined Diagrams of Triple-expansion Engines.

ut-off means or meter. This should be clear from a comparison of Figs. 71, A and B. In Fig. 71A, representing the case of the simple engine without

clearance and with complete expansion, the volume admitted, $\overline{AB}$, expand to the back pressure on reaching the full cylinder volume $\overline{DC}$, and exhaust at constant back pressure, the work represented by the area $\overline{ABCD}$. It shoul be clear that no difference will result in the work done if a line be drawn acros the work area as in Fig. 71*B*, all work done above the line $\overline{HG}$ to be develope in the high-pressure cylinder and that below in the low. This is merely equiv alent to saying that a volume of steam $\overline{AB}$ is admitted to the high-pressur cylinder expands completely to the pressure at G on reaching the full high pressure cylinder volume, after which it exhausts at constant pressure (int a receiver of infinite capacity), this same amount being subsequently admitte without change of pressure to the low-pressure cylinder, when it again expand completely. Thus, it appears that the working of steam or compressed a

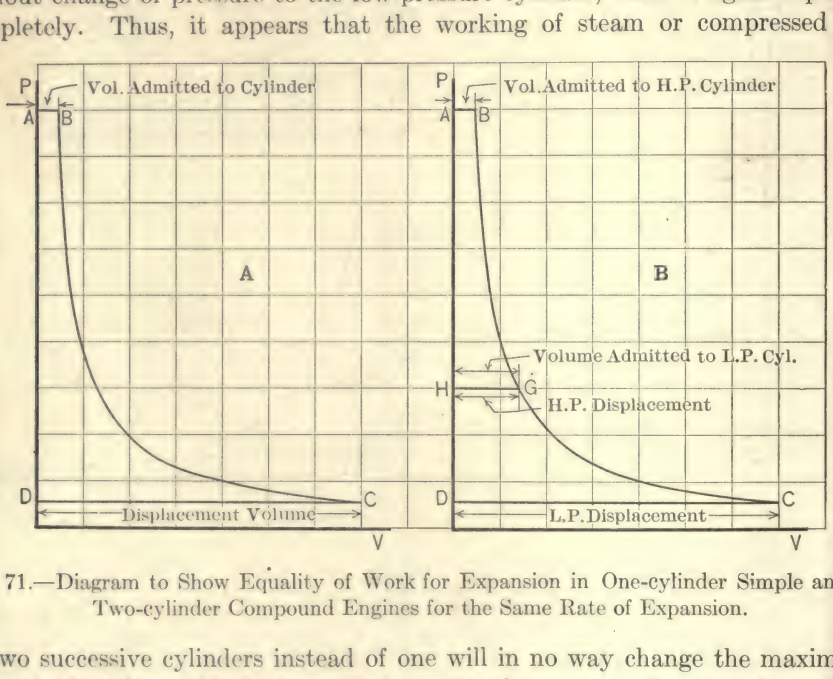


FIG. 71.—Diagram to Show Equality of Work for Expansion in One-cylinder Simple and Two-cylinder Compound Engines for the Same Rate of Expansion.

in two successive cylinders instead of one will in no way change the maximu amount of work a cubic foot supplied can do, the compounding merely maki it easier to get this maximum. In simple engine cases, Fig. 71*A*, the cut-off in p cent of stroke is $100 \times \frac{AB}{DC}$, which is a very small value, leaving but little time open and close the admission valve, whereas in the compound case the per ce cut-off in the high-pressure cylinder is $100 \times \frac{AB}{HG}$, and in the low-pressure cylind $100 \times \frac{HG}{DC}$, which are quite large enough ratios to be easily managed with ordina valve gears.

Compounding does, however, introduce possibilities of loss not present the single-stage expansion, if the dimensions or adjustments are not rig which may be classed somewhat improperly as *receiver losses*, and these

of two kinds, one due to incomplete expansion in the high and the other to over-expansion there. Thus, in Fig. 72, if $ABCEFGDA$ represent a combined compound diagram for the case of complete expansion in the high-pressure cylinder continued in the low without interruption but incomplete there, DC represents the *volume in the low-pressure cylinder at cut-off, and at the same time the total high-pressure cylinder volume.*

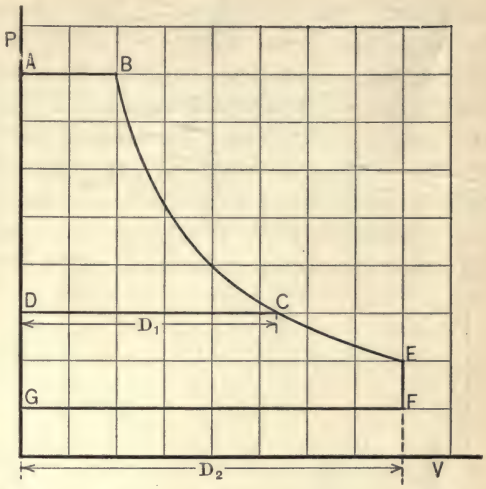


FIG. 72.—Diagram to Show Correct Low-pressure Cut-off for No Receiver Loss.

If now, the low-pressure cut-off be made to occur later, Fig. 73, then the volume that the steam would occupy when expansion began in the low-pressure cylinder is represented by $\overline{D'C'}$. This adjustment could not, of course, change the high-pressure total volume $\overline{DC}$, so that at release in the high-pressure cylinder the pressure would drop abruptly to such a value in the receiver as corresponds to filling the low pressure up to its cut-off, and work be lost equal to area $\overline{CC'C''}$.

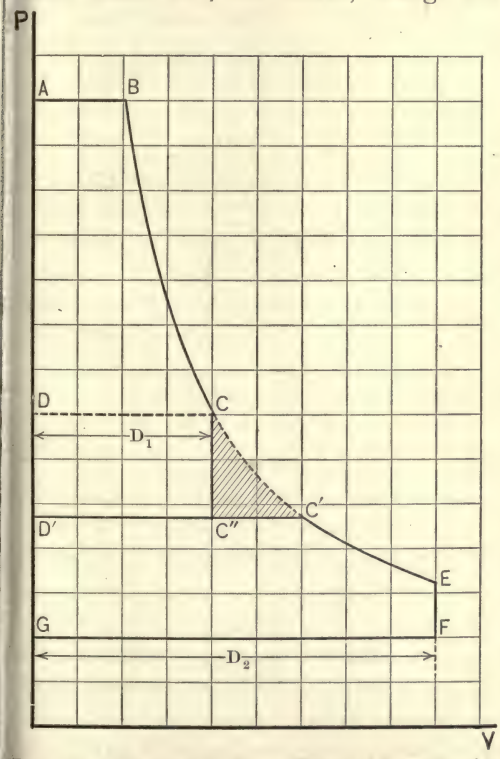


FIG. 73.—Diagram to Show Effect of Lengthening L.P. Cut-off Introducing a Receiver Loss Due to Incomplete High-pressure Cylinder Expansion.

A shortening of the low-pressure cut-off will have an equally bad effect by introducing negative work as indicated in Fig. 74, in which the L.P. cut-off volume is reduced from $\overline{DC}$ to $\overline{D'C'}$. Expansion in the high pressure proceeds as before till the end of the stroke, at which time it has a volume $\overline{DC}$ greater than the low pressure can receive $\overline{D'C'}$, hence the receiver pressure must rise to such a value as will reduce the volume the required amount, introducing the negative work $\overline{CC'C''}$. Changes of low-pressure cut-off may, therefore, introduce negative work, change the receiver pressure and, of course, modifiy the distribution of work between high and low, but

these same effects might also have resulted from changes of high-pressure cut-off or of cylinder ratio.

For such conditions as have been assumed it seems that compounding does not increase the work capacity of fluids, but may make it easier to realize this capacity, *introducing at the same time certain rather rigid relations between cut-offs and cylinder volumes as necessary conditions to its attainment.* It can also be shown that the same proposition is true when there are clearance and compression, that is, in real cylinders and when the receiver is real or not infinite in size, or when the exhaust of high and admission of low, are not constant-pressure lines. The former needs no direct proof, as inspection of previous diagrams makes it clear, but the latter requires some discussion.

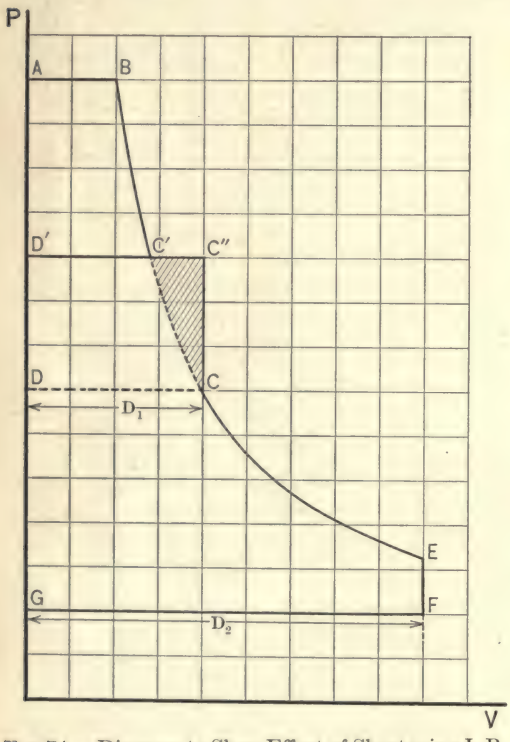


FIG. 74.—Diagram to Show Effect of Shortening L.P. Cut-off, Introducing a Receiver Loss Due to Over-expansion in the High-pressure Cylinder.

A real receiver of finite size is at times in communication with the high-pressure cylinder during its exhaust, and at other times with the low-pressure cylinder during admission, and these two events may take place at entirely independent times, be simultaneous as to time, or overlap in all sorts of ways. Suppose the *periods* to be *independent* and there be *no cylinder clearance*, then at the beginning of high-pressure exhaust two separate volumes of fluid come together, the contents of both the high-pressure cylinder and the receiver, and this double volume is compressed by the H.P. piston into the receiver, in which case the high-pressure exhaust would take place with rising pressure. Following this will come low-pressure admission, during which the volume of fluid in the receiver expands into the low-pressure cylinder up to its cut-off, and if the same volume is thus taken out of the receiver as entered it previously, low-pressure admission will take place with falling pressure, the line representing it exactly coinciding with that for the high-pressure exhaust. Independence of H.P. cylinder exhaust and L.P. cylinder admission, as to time, may result in a cycle such as is represented in Fig. 75 for the case of no cylinder clearance. On this diagram the receiver line is DC, an expansion or compression line referred to a second axis of volumes KJ, placed away from the axis of purely cylinder volumes

by the distance $\overline{LD}$, equal to the receiver volume to scale. All diagram points are referred to the axis AI except those on the line DC.

This same case of time *independence of H.P. exhaust and L.P. admission* yields quite a different diagram when the cylinder *clearance* is considered. Such a case is represented by the diagram, Fig. 76, which also serves to illustrate the effect of incomplete expansion and compression as to equalization of receiver with cylinder pressures. At high-pressue release the volume of fluid in the H.P. cylinder is $\overline{ML}$ and its pressure is $\overline{LR}$. This is about to come into communication with the receiver volume $\overline{ON}$ from which the low-pressure cylinder finished taking fluid and which is, therefore, at the same pressure

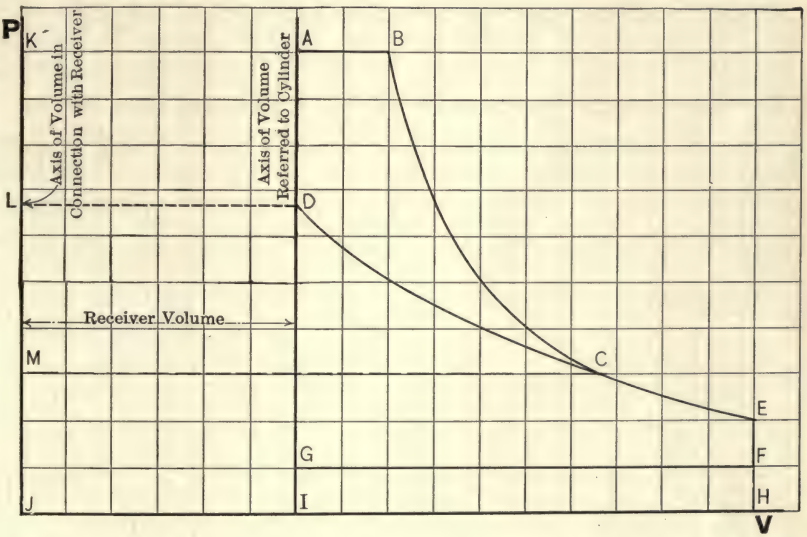


FIG. 75.—Diagram to Illustrate Variable Receiver Pressure for the Case of Independent High-pressure Exhaust and Low-pressure Admission with Zero Clearance.

as the L.P. cylinder cut-off $\overline{KS}$. The question, therefore, is—what will be the pressure at P in both H.P. cylinder and receiver when $\overline{LM}$ cu.ft. of fluid at $\overline{LR}$ pressure combines with $\overline{ON}$ cu.ft. at pressure $\overline{KS}$, and together occupy a volume $\overline{ON}+\overline{LM}$. By hypothesis the pressure after mixture is

$$\frac{(\text{first volume} \times \text{its pressure}) + (\text{second volume} \times \text{its pressure})}{\text{sum of volumes}}.$$

From this or the graphic construction following, the point P is located. If the high-pressure expansion had continued to bring LQ to the receiver pressure $\overline{KS}$, it would reach it at X. At this hypothetic point there would be a volume $\overline{NX}$ in the H.P. cylinder to add to the volume $\overline{ON}$ in the receiver at the same pressure, resulting in $\overline{OX}$ cu.ft. This fluid would have a higher pressure at the lesser volume of receiver and H.P. cylinder and the value is found by a compres-

sion line through X, $XPAT$ referred to the receiver axis. This same line is also the exhaust of the H.P. cylinder from P to A. A similar situation exists at admission to the L.P. cylinder as to pressure equalization and location of admission line. At the end of the L.P. compression there is in the L.P. cylinder $\overline{FE}$ cu.ft. at pressure $\overline{EH}$, to come into communication with the receiver volume $\overline{CB}$ cu.ft. at pressure $\overline{BG}$, that at which H.P. exhaust ended. Producing the L.P. compression line to I, the volume $\overline{BI}$ is found, which, added to receiver, results in no pressure change. An expansion line, referred to the receiver axis through I, fixes the equalized pressure J and locates the L.P. admission line JK, which, it must be observed, does not coincide with the H.P. exhaust.

So far only *complete independence* of the time of H.P. exhaust and L.P. admission have been considered, and it is now desirable to consider the diagram-

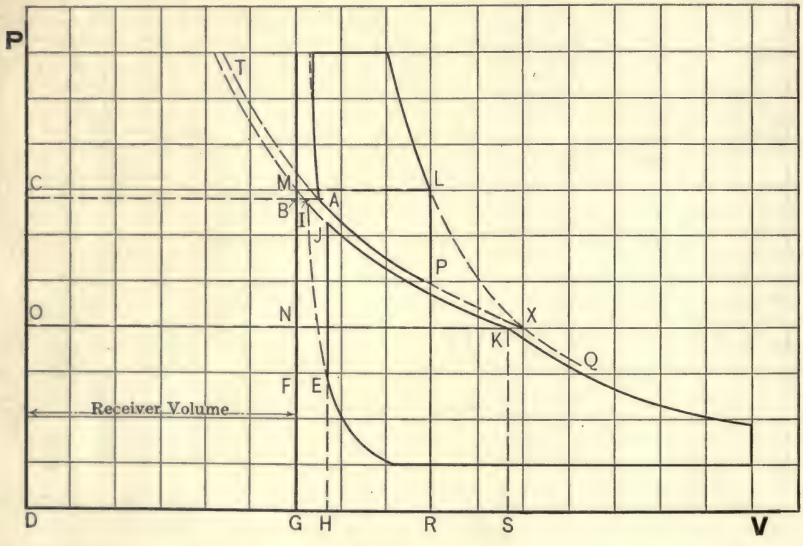


FIG. 76.—Diagram to Illustrate Variable Receiver Pressure for the Case of Independent High-pressure Exhaust and Low-pressure Admission with Clearance.

matic representation of the results of *complete coincidence*. Such cases occur in practice with the ordinary tandem compound stationary steam engine and twin-cylinder single-crosshead Vauclain compound steam locomotive. In the latter structure both pistons move together, a single valve controlling both cylinders, exhaust from high taking place directly into low, and for exactly equal coincident time periods. The diameter of the low-pressure cylinder being greater than the high, the steam at the moment of release suffers a drop in pressure in filling the low-pressure clearance, unless, as rarely happens, the pressure in the low is raised by compression to be just equal to that at H.P. release. After pressure equalization takes place, high-pressure exhaust and low-pressure admission events are really together a continuation of expansion, the volume occupied by the steam at any time being equal to the difference between

the two cylinder displacements and clearances up to that point of the stroke. Before this period of communication, that is, between high-pressure cut-off and release, the volume of the expanding fluid is that of the high-pressure displacement up to that point of the stroke, together with the high-pressure clearance. After the period of communication the volume of the expanding fluid is that of the low pressure cylinder up to that point of the stroke, together with the low pressure clearance, plus the high-pressure displacement not yet swept out, and the high-pressure clearance.

These fluid processes cannot be clearly indicated by a single diagram, because a diagram drawn to indicate volumes of fluid will not show the volumes in the cylinders without distortion. If there be *no clearance*, Fig. 77 will assist in showing the way in which two forms of diagram for this purpose are derived. Referring to Fig. 77*A*, the volume $\overline{AB}$ admitted to the high pressure cylinder expands in it until it occupies the whole H.P. cylinder volume $\overline{DC}$. At this point expansion proceeds in low and high together, with decreasing volumes in high and increasing in low until the low-pressure cylinder volume is attained at E. The line BCE then indicates the pressures and volumes of the fluid expanding, but *does not clearly show the volume in either cylinder between C and E, with the corresponding pressure.* It is certain, however, that when the volume in the H.P. cylinder becomes zero the pressure must have fallen to a value the same as that in the low when the fluid completely fills it, or $P_f = P_e$.

As the high-pressure piston returns, on the exhaust stroke, the low-pressure piston advances an equal distance, on its admission stroke, sweeping through a greater volume than the high pressure, in the ratio of low-pressure to high-pressure displacements. If at any point of the stroke the volume remaining in the high-pressure cylinder be x, and the high- and low-pressure displacements be respectively D_1, and D_2, then $(D_1 - x)$ is the volume swept *out* by high-pressure piston, x the volume remaining in it, and $\frac{D_2}{D_1}(D_1 - x)$ the volume swept *in* by the low-pressure piston. Then the total volume still in the two cylinders is, for a point between C and E,

$$V = x + \frac{D_2}{D_1}(D_1 - x) = D_2 - x\left(\frac{D_2}{D_1} - 1\right).$$

Since the equation of the curve CE is, $PV = P_c V_c = P_c D_1$, the value of V may be substituted, giving $P\left[D_2 - x\left(\frac{D_2}{D_1} - 1\right)\right] = P_c D_1, = \text{constant}$. Dividing by $\left(\frac{D_2}{D_1} - 1\right)$ this becomes $P\left[\frac{D_2}{\left(\frac{D_2}{D_1} - 1\right)} - x\right] = P\left[\frac{D_2 D_1}{D_2 - D_1} - x\right] = \text{a new constant, so}$

that if a new axis LM be laid off on **B**, $\overline{GV} = \left(\frac{D_2 - D_1}{D_2 D_1}\right)$ from the axis GP any point on FC will be distant from the new axis LM an amount $\left[\left(\frac{D_2 D_1}{D_2 - D_1} - \right)x\right]$ as the product of this distance and the pressure P, is constant, the curve FC is an equilateral hyperbola referred to the axis LM. Therefore Fig. 77*B* is

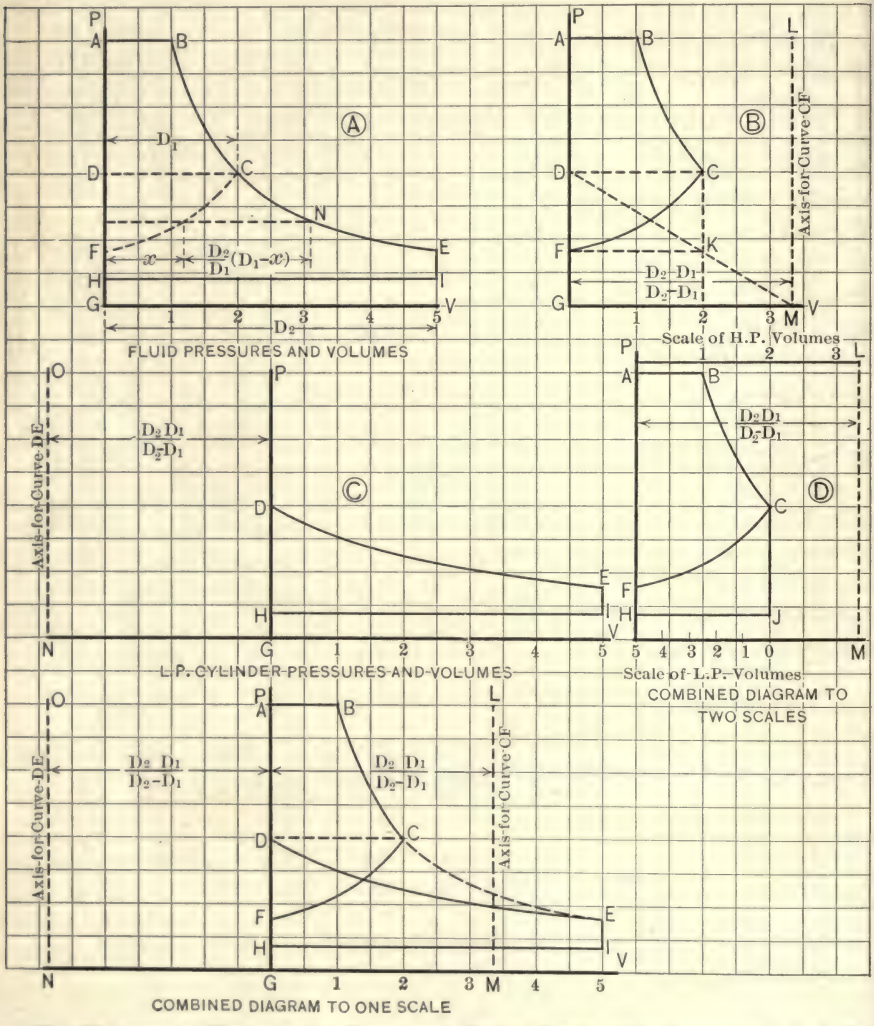


FIG. 77.—Diagram to Illustrate the Compound Engine Cycle with No-receiver, and Exact Coincidence of H.P. Exhaust and L.P. Admission Periods. No Clearance.

the pressure-volume representation of the entire cycle of the high-pressure cylinder.

In Fig. 77*C* is shown the corresponding pressure-volume diagram for the low-pressure cylinder, for which it may similarly be shown that the curve DE may be

plotted to an axis NO at a distance to the left of the axis GP equal to the same quantity,

$$GN = \frac{D_2 D_1}{D_2 - D_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (295)$$

These diagrams, 77A, B and C may be superposed, as in Fig. 77E, giving one form of combined diagram used for this purpose, and the one most nearly comparable with those already discussed. In this diagram, the area $\overline{ABCFA}$ represents the work of the high-pressure cylinder, and $\overline{DEIHD}$, the work of the low-pressure cylinder. Together, they equal the work of the enclosing figure $\overline{ABEIHA}$, and hence the work of the low-pressure cylinder must also be represented by the area $\overline{FCEIHF}$.

It is not difficult to show that if a vertical, CJ, be dropped from the point C to the exhaust line HJ, the figure $\overline{CFHJ}$, in Fig. 77D is similar to $\overline{DEIH}$, in Fig. 77E, reversed, but drawn to a different horizontal scale. Here the length of the low-pressure diagram is made equal to the length of the high-pressure diagram. The two scales of volumes are shown above and below the figure. While this appears to be a very convenient diagram, it will be found to be less so when clearance and compression are considered.

It may be noted that since it has been shown that the curves CF and DE are of the same mathematical form (hyperbolic) as the expansion line CE, they may be plotted in the same way after having in any way found the axis. The location of this axis may be computed as given above, or may be found graphically by the method given in connections with the subject of clearance, Chapter I, and shown in Fig. 77B. Knowing two points that lie on the curve, C and F, the rectangle $CDFK$ is completed. Its diagonal, DK, extended, cuts the horizontal axis GV in the point M, which is the base of the desired axis ML.

If now the axis NO and the figure $DEIH$, part of which is referred to this axis, be reversed about the axis GP, Fig. 77C, NO will coincide with ML, Fig. 77D, and Fig. 78 results. Note that the axis here may be found graphically, in a very simple way. Draw the vertical CK from C to the axis GV and the horizontal DJ to the vertical IE extended. $\overline{DC}$ is then the high-pressure displacement and $\overline{DJ}$ the low-pressure displacement. Draw the two diagonals DK and JG, extending them to their intersection X. By similar triangles it may be shown that a horizontal line, UW, will have an intercept between these two lines, JG and DK, equal to the volume of fluid present in the two cylinders combined. The intersection X is the point at which this volume would become zero if the mechanical process could be continued unchanged to that point, and is, therefore, on the desired axis ML extended. T being the intersection of WU with the axis GP, when the volume UT is present in the high-pressure cylinder, TW gives the volume in the low-pressure cylinder.

When *clearance* and *compression* are considered, this diagram is changed in many respects, and is shown in Fig. 79. The axes OP, OV and OV' are laid out, with OZ equal to the clearance and ZK, the displacement, of the high-pressure

cylinder, and OQ and QY, clearance and displacement of the low-pressure cylinder. It is necessary to know high-pressure cut-off, $\frac{\overline{AB}}{\overline{ZK}}$; high-pressure compression, $\frac{\overline{ZE''}}{\overline{ZK}}$; and low-pressure compression, $\frac{\overline{I'Q}}{\overline{YQ}}$, in addition to the initial and back pressures, in order to lay out the diagram. The drop in pressure CD at release is due to the coming together of (volume V_c at pressure P_c), with (volume V_j at pressure P_j). If the volume V_j (measured from axis OP) were decreased sufficiently to raise the pressure in the low-pressure clearance to the pressure

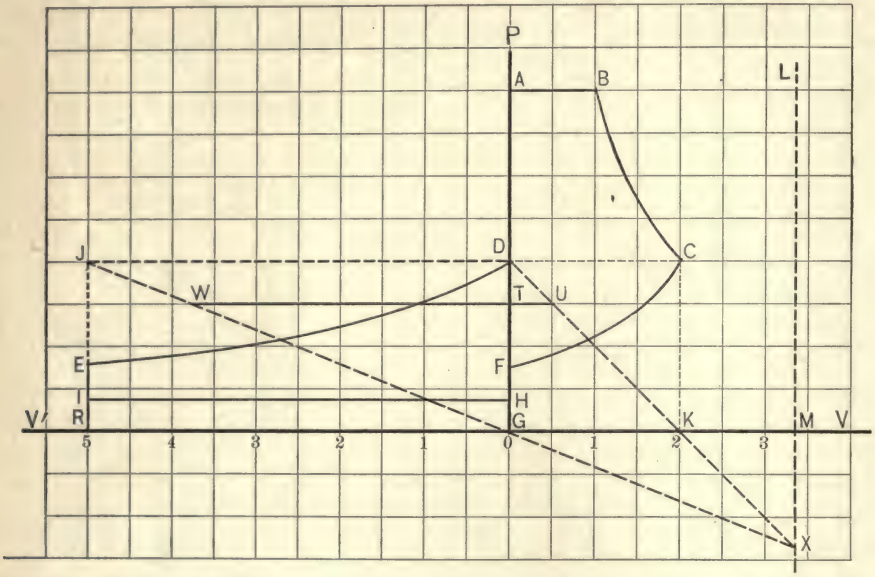


FIG. 78.—Diagram to Show Volume of Steam in the Cylinders of the No-receiver Compound Engine at any Point of the Stroke for the Case of No-clearance and Coincidence of H.P. Exhaust with L.P. Admission.

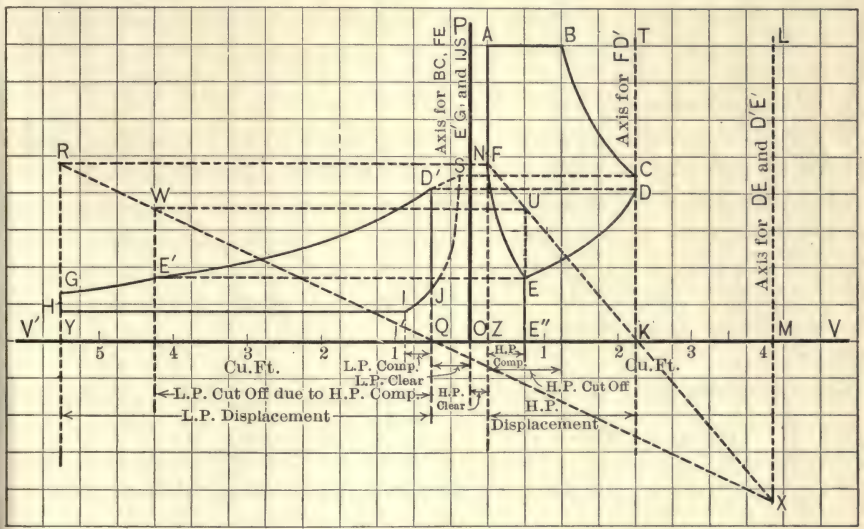
at C, the volume would become V_s, as indicated at the point S, and the volumes now combined in the hypothetical condition would occupy the volume $\overline{SC}$.

Increasing this volume to $\overline{D'D}$, that actually occupied after communication, the pressure would fall along the curve SD', which is constructed on KV' and KT as axes. The pressure of D is then laid out equal to the pressure at D'. To find the axis, ML, for the curves DE and $D'E'$, from any convenient point N on ZA, draw the line NK extended to X. Extend HG to R, at a height equal to that of N, and draw RQX, and through the intersection draw the desired axis XML. The fraction of stroke completed at E' in the low pressure at cut-off must be equal to that completed at E in the high pressure at compression, and may be laid out graphically by projecting up from E to the point U on the line NK and horizontally from U to W on the line RQ. Projecting down from W to the curve at E' locates the point of effective cut-off in the low-pressure cylinder.

fter the supply from the high-pressure cylinder has been cut off at E', the xpansion is that of the volume in the low-pressure cylinder and its clearance, and ence the curve $E'G$ is constructed on OP as an axis.

While in this last case a zero receiver volume has been assumed, there is othing to prevent a receiver volume being interposed between H.P. and L.P. o that common expansion takes place with a volume greater than assumed y so much as this volume, the effect of which is to decrease the slope of DE nd $D'E'$. Such receivers usually consist of the spaces included in a valve ody and connecting passages and may be treated generally as increased L.P. learances.

The most common of all relations between H.P. exhaust and L.P. admis- ion is, of course, that of *partial coincidence of periods*, as it is thus with all cross-



IG. 79.—Diagram to Show Volume of Steam in the Cylinders of the No-receiver Compound Engine at any Point of the Stroke for the Case of Clearance and Coincidence of H.P. Exhaust with L.P. Admission.

ompound and triple-expansion engines having separate cranks for the ndividual cylinders. For these there is no simple fixed relation between the eriods, for, while crank angles are generally fixed in some comparatively simple elation, such as 90°, 180° and 270° for compounds and 120° for triples, they re sometimes set at all sorts of odd angles for better balance or for better rning effort. Even if the angles were known the receiver line would have be calculated point by point. When the H.P. cylinder begins to discharge into receiver for, say, a cross compound with cranks at 90°, steam is compressed nto the receiver, and so far the action is the same as already considered for ndependence of periods, but at near mid-stroke the low-pressure admission pens while high-pressure exhaust continues. This will cause the receiver ressure to stop rising and probably to fall until the low pressure cuts off, which

may occur before the H.P. exhaust into the receiver ceases. If it does, the receive pressure will again rise. Exact determination of such complex receiver lines i not often wanted, and when needed is best obtained graphically point by point Its value lies principally in fixing exactly the *work distribution between cylinders*, which is not of great importance except for engines that are to work a constant load nearly all the time, such as is the case with city water work pumping, and marine engines. While equations could be derived for thes cases, they are not worth the trouble of derivation, because they are too cumbersome, and graphic methods are to be substituted or an approximation to be made

Four kinds of approximation are available, as follows, all of which ignor partial coincidence of periods:

1. Receiver pressure constant at some mean value and clearance ignored
2. Receiver pressure constant at some constant value and clearance con sidered with compression zero or complete.
3. Receiver pressure fluctuates between fixed limits as determined by a assumed size, clearance ignored.
4. Receiver pressure fluctuates between fixed limits as determined by a assumed size, clearance considered, with compression zero or complete.

These are not all of equal difficulty in solution, and the one to be used i that nearest the truth as to representation of conditions, which is usually th most difficult, provided time permits or the information is worth the trouble Quickest work is accomplished with assumption (1) and as this is most ofte used in practical work it indicates that its results are near enough for mo purposes.

This discussion leads, therefore, to the analytical study of the following cycle

INFINITE RECEIVER, ZERO CYLINDER CLEARANCE.

CYCLES V, AND VI (Fig. 80).

H.P. CYLINDER EVENTS

(*a*) Admission at constant supply pressure to H.P. cylinder.
(*b*) Expansion in H.P. cyl. (may be zero) by law $PV=c$ for (V $PV^s=c$ for (VI).
(*c*) Equalization of H.P. cyl. pressure with receiver pressu at constant volume (may be zero).
(*d*) Exhaust into infinite receiver at constant pressure fro H.P. cylinder.
(*e*) Equalization of H.P. cylinder pressure with supply pressu at constant zero volume.

L.P. CYLINDER EVENTS

(*f*) Admission from receiver at constant receiver pressure L.P. cylinder.
(*g*) Expansion in L.P cylinder (may be zero) by law $PV=c$ f (V); $PV^s=c$ for (VI).
(*h*) Equalization of L.P. cylinder pressure with back pressu at constant volume (may be zero).
(*i*) Exhaust at constant back pressure for L.P. cylinder.
(*j*) Equalization of L.P. cylinder pressure with receiver pressu at constant zero volume.

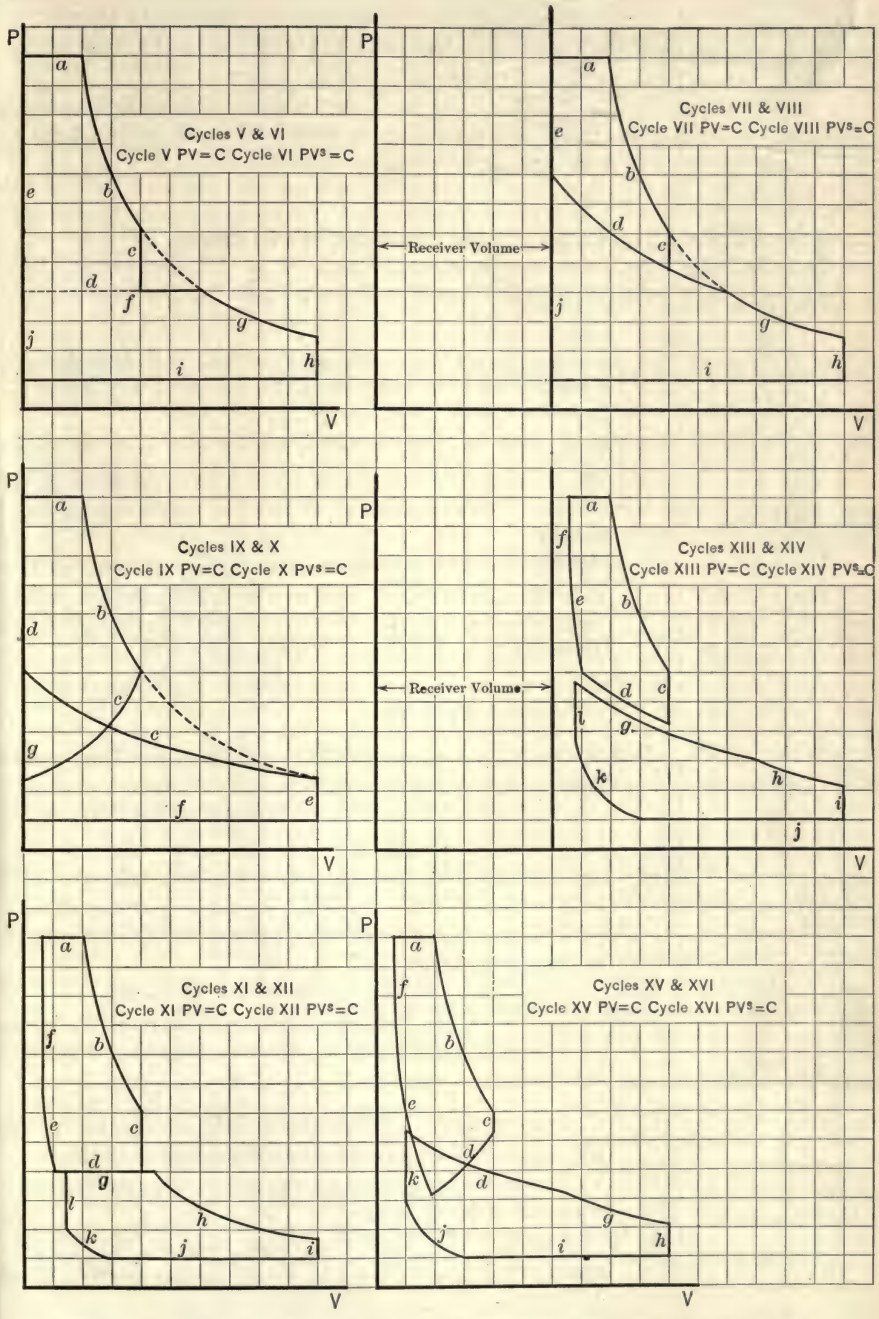


FIG. 80.—Compound Engine Standard Reference Cycles or *PV* Diagrams.

Relations between H.P. and L.P. Cylinder Events

(1) H.P. exhaust and L.P. admission independent as to time, coincident as to representation (except as to length).
(2) H.P. expansion line produced coincides as to representation with L.P. expansion line.
(3) The length of the constant pressure receiver line up to the H.P. expansion line produced is equal to the length of the L.P. admission line.

Finite Receiver, Zero Cylinder Clearance.

CYCLES VII, AND VIII, (Fig. 80).

H.P. Cylinder Events

(*a*) Admission at constant supply pressure to H.P. cylinder.
(*b*) Expansion in H.P. cylinder (may be zero) by law $PV=c$ for (VII); $PV^s=c$ for (VIII).
(*c*) Equalization of H.P. cylinder pressure with receiver pressure at constant volume (may be zero) with a change of receiver pressure toward that at H.P. cylinder release (may be zero).
(*d*) Exhaust into finite receiver from H.P. cylinder at rising pressure equivalent to compression of fluid in H.P. cylinder and receiver into receiver by law $PV=c$ for (VII) and $PV^s=c$ for (VIII).
(*e*) Equalization of H.P. cylinder pressure with supply pressure at constant zero volume.

L.P. Cylinder Events

(*f*) Admission from receiver to L.P. cylinder at falling pressure equivalent to expansion of fluid in receiver into receiver and L.P. cylinder together by law $PV=c$ for (VII), $PV^s=c$ for (VIII).
(*g*) Expansion in L.P. cylinder (may be zero) by law $PV=c$ for (VII); $PV^s=c$ for (VIII).
(*h*) Equalization of L.P. cylinder pressure with back pressure at constant volume (may be zero).
(*i*) Exhaust at constant back pressure for L.P. cylinder.
(*j*) Equalization of L.P. cylinder pressure with receiver pressure at constant zero volume to value resulting from H.P. exhaust.

Relation between H.P. and L.P. Cylinder Events

(1) H.P. exhaust and L.P. admission independent as to time, coincident as to representation, except as to length.
(2) H.P. expansion line produced coincides as to representation with L.P. expansion line.
(3) The length of the receiver pressure line up to the H.P. expansion line produced is equal to the length of the L.P. admission line.

No Receiver, Zero Cylinder Clearance.

CYCLES IX, AND X, (Fig. 80).

H.P. Cylinder Events

(*a*) Admission at constant supply pressure to H.P. Cylinder.
(*b*) Expansion in H.P. cylinder (may be zero) by law $PV=c$ for (IX); $PV^s=c$ for (X).

Both H.P. and L.P. Simultaneously

(*c*) Transference of fluid from H.P.to L.P. cylinder with simultaneous continuation of expansion until all fluid is in L.P. cylinder and expanded to its full volume by law $PV=c$ for (IX); $PV^s=c$ for (X).

H.P. Cylinder Events

(*d*) Equalization of H.P. cylinder pressure to the pressure of supply.

L.P. Cylinder Events

(*e*) Equalization of L.P. cylinder pressure with back pressure at constant volume (may be zero).
(*f*) Exhaust at constant back pressure for L.P. cylinder.
(*g*) Equalization of L.P. cylinder pressure to the pressure in H.P. cylinder at the end of its expansion.

Infinite Receiver, with Cylinder Clearance.

CYCLES XI, AND XII, (Fig. 80).

H.P. Cylinder Events

(*a*) Admission at constant supply pressure to H.P. cylinder.
(*b*) Expansion in H.P. cylinder (may be zero) by law $PV=c$ for (XI); $PV^s=c$ for (XII).
(*c*) Equalization of H.P. cylinder pressure with receiver pressure at constant volume (may be zero) pressure.
(*d*) Exhaust into infinite receiver at constant pressure from H.P. cylinder.
(*e*) Compression in H.P. cylinder to clearance volume (may be zero) by law $PV=c$ for (XI); $PV^s=c$ for (XII).
(*f*) Equalization of H.P. cylinder pressure with supply pressure at constant clearance volume, may be zero.

L.P. Cylinder Events

(*g*) Admission from receiver at constant-receiver pressure to L.P. cylinder.
(*h*) Expansion in L.P. cylinder (may be zero) by law $PV=c$ for (XI); $PV^s=c$ for (XII).
(*i*) Equalization of L.P. cylinder pressure with back pressure at constant volume (may be zero).
(*j*) Exhaust at constant back pressure for L.P. cylinder.
(*k*) Compression in L.P. cylinder to clearance volume by law $PV=c$ for (XI); $PV^s=c$ for (XII) (may be zero).
(*l*) Equalization of L.P. cylinder pressure with receiver pressure at constant clearance volume without change of receiver pressure (may be zero).

Relations between H.P. and L.P. Cylinder Events

(1) H.P. exhaust and L.P. admission independent as to time, coincident as to representation except as to length.

(2) L.P. expansion line does not coincide as to representation with H.P. expansion line produced by reason of clearance influence except in one special and unusual case.

(3) The length of the constant-receiver pressure line intercepted between H.P. compression line and H.P. expansion line produced is equal to the same intercept between L.P. expansion line and L.P. compression line produced. This is equivalent to the condition that the volume taken in by low is equal to expelled by the high reduced to the same pressure.

Finite Receiver, with Cylinder Clearance.

CYCLES XIII, AND XIV, (Fig. 80).

H.P. Cylinder Events

(a) Admission at constant supply pressure to H.P. cylinder.

(b) Expansion in H.P. cylinder (may be zero) by law $PV=c$ for (XIII); $PV^s=c$ for (XIV).

(c) Equalization of H.P. cylinder pressure with receiver pressure at constant volume (may be zero) toward that at H.P. cylinder release (may be zero).

(d) Exhaust into finite receiver from H.P. cylinder at rising pressure equivalent to compression of fluid in H.P. cylinder and receiver into receiver by law $PV=c$ for (XIII); $PV^s=c$ for (XIV).

(e) Compression in H.P. cylinder to clearance volume (may be zero) by law $PV=c$ for (XIII); $PV^s=c$ for (XIV).

(f) Equalization of H.P. cylinder pressure with supply pressure at constant clearance volume.

L.P. Cylinder Events

(g) Admission from receiver to L.P. cylinder at falling pressure equivalent to expansion of fluid in receiver into receiver and L.P. cylinder together by $PV=c$ for (XIII); $PV^s=c$ for (XIV).

(h) Expansion in L.P. cylinder (may be zero) by law $PV=c$ for (V); $PV=c$ for (VI).

(i) Equalization of L.P. cylinder pressure with back pressure at constant volume (may be zero).

(j) Exhaust at constant back pressure for L.P. cylinder.

(k) Compression in L.P. cylinder to clearance volume by law $PV=c$ for (XI); $PV^s=c$ for (XII) (may be zero).

(l) Equalization of L.P. cylinder pressure with receiver pressure at constant clearance volume with change of receiver pressure in direction of L.P. compression pressure (may be zero).

Relations between H.P. and L.P. Cylinder Events

(1) H.P. exhaust and L.P. admission independent as to time, representation and length.
(2) L.P. expansion line does not coincide as to representation with H.P. expansion line produced by reason of clearance influence except in one special and unusual case.
(3) The high-pressure exhaust and low-pressure admission lines do not coincide as to representation by reason of clearance influences.
(4) There is a relation between the lengths of the L.P. admission and H.P. exhaust lines, but not a simple one.

No Receiver, with Cylinder Clearance.

Cycles XV, and XVI, (Fig. 80).

H.P. Cylinder Events

(*a*) Admission at constant-supply pressure to H.P. cylinder.
(*b*) Expansion in H.P. cylinder (may be zero) according to law $PV=c$ for (XV); $PV^s=c$ for (XVI).
(*c*) Equalization of pressures in H.P. cylinder after expansion with that in L.P. after compression at constant volume (may be zero).
Also L.P. Event
(*d*) Transference of fluid from H.P. to L.P. cylinder until all fluid is in L.P. cylinder and expanded to its full volume by same law as (*b*).
(*e*) Compression in H.P. cylinder to clearance volume (may be zero) by law $PV=c$ for (XV); $PV^s=c$ for (XVI).
(*f*) Equalization of pressure in H.P. cylinder with supply at constant-clearance volume (may be zero).

L.P. Cylinder Events

(*g*) Expansion in L.P. cylinder may be zero by law $PV=c$ for (XV); $PV^s=c$ for (XVI).
(*h*) Equalization of pressure in L.P. cylinder with back pressure, at constant volume (may be zero).
(*i*) Exhaust at constant pressure for L.P. cylinder.
(*j*) Compression in L.P. cylinder to clearance, may be zero by law $PV=c$ for (XV); $PV^s=c$ for (XVI).
(*k*) Equalization of L.P. cylinder pressure with H.P. cylinder pressure.

Cycle XVII, Fig. 81, for the triple expansion is defined in the same way as the corresponding case of compounds Cycle V, with appropriate alterations in wording to account for a third or intermediate cylinder between high- and low-pressure cylinders and an additional receiver. Thus, high- pressure cylinder exhausts into first, and intermediate cylinder into second receiver: intermediate cylinder receives its supply from first, and low-pressure cylinder from second receiver. This being the case, it is unnecessary to write out the cylinder events, noting their relation to the corresponding compound case.

9. Compound Engine with Infinite Receiver, Logarithmic Law. No Clearance, Cycle V. General Relations between Pressures, Dimensions, and Work. It must be understood that the diagrams representing this cycle, Fig. 82, indicating (*A*) *incomplete expansion* and (*B*) *over-expansion* in both cylinders,

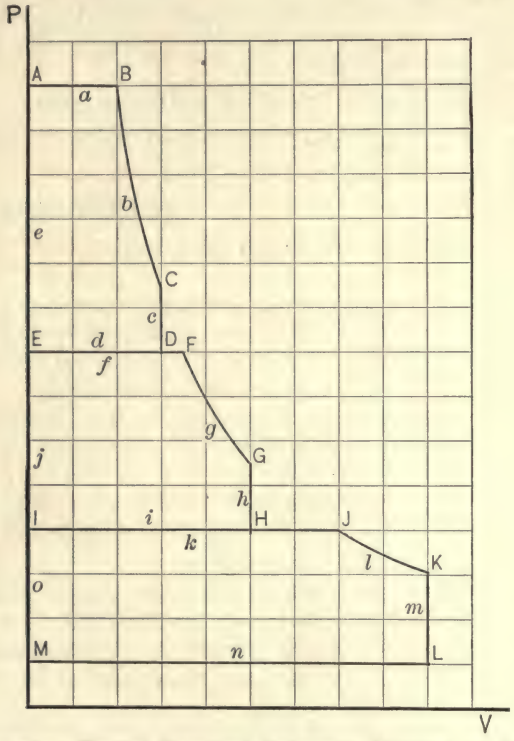


FIG. 81.—Triple-expansion Engine Standard Reference Diagram or *PV* Cycle for Infinite Receiver.

may just as well stand for over, complete or incomplete expansion in all possible combinations in the two cylinders. Applying the principles already derived for calculating the work areas,

High-pressure cylinder work

$$W_H = P_b V_b \left(1 + \log_e \frac{V_c}{V_b}\right) - P_d V_d, \quad \ldots \ldots \quad (296$$

Low-pressure cylinder work

$$W_L = P_e V_e \left(1 + \log_e \frac{V_f}{V_e}\right) - P_g V_g, \quad \ldots \ldots \quad (297$$

Total work

$$W = P_b V_b\left(1+\log_e \frac{V_c}{V_b}\right)+P_e V_e\left(1+\log_e \frac{V_f}{V_e}\right)-P_a V_a - P_g V_g, \quad . \quad . \quad (298)$$

pressure being in pounds per square foot, and volumes in cubic feet.

In theses expressions the receiver pressure $P_e = P_a$ is unknown, but determinate as it is a function of initial pressure and certain volumes, giving it the value,

$$P_e = P_a = P_b \frac{V_b}{V_e},$$

is merely satisfying the condition that the point E at which expansion begins in the low-pressure cylinder must lie in the expansion line of the high. Substituting this value there results

$$W = P_b V_b\left(1+\log_e \frac{V_c}{V_b}\right)+P_b V_b\left(1+\log_e \frac{V_f}{V_e}\right)-P_b V_b \frac{V_d}{V_e}-P_g V_g$$

$$= P_b V_b\left[2+\log_e \frac{V_c}{V_b}+\log_e \frac{V_f}{V_e}-\frac{V_d}{V_e}\right]-P_g V_g. \quad . \quad (299)$$

This is a perfectly general expression for the *work* of the fluid expanding to any degree in two cylinders in succession when the clearance is zero and receiver volume infinite, in terms of initial and back pressures, pounds per square foot, the volumes occupied by the fluid in both cylinders at cut-off, and at full stroke in cubic feet. Dividing this by the volume of the low-pressure cylinder V_g gives the *mean effective pressure* referred to the low-pressure cylinder, from which the horse-power may be determined without considering the high-pressure cylinder at all. Hence, in the same units as are used for P_b and P_g,

$$(\text{M.E.P. referred to L.P.}) = P_b \frac{V_b}{V_g}\left[2+\log_e \frac{V_c}{V_b}+\log_e \frac{V_f}{V_e}-\frac{V_d}{V_e}\right]-P_g. \quad (300)$$

Proceeding as was done for simple engines, the *work per cubic feet of fluid supplied* is found by dividing Eq. (299) by the volume admitted to the high-pressure cylinder V_b, whence,

$$\text{Work per cu.ft. supplied} = P_b\left[2+\log_e \frac{V_c}{V_b}+\log_e \frac{V_f}{V_e}-\frac{V_d}{V_e}\right]-P_g \frac{V_g}{V_b}. \quad . \quad . \quad . \quad (301)$$

Also applying the *consumption* law with respect to horse-power,

$$\text{Cu.ft. supplied per hour per I.H.P.} = \frac{13{,}750}{(\text{m.e.p. ref. to low})} \frac{V_b}{V_g}. \quad . \quad . \quad . \quad . \quad (302)$$

$$\text{Lbs. supplied per hr. per I.H.P.} = \frac{13{,}750}{(\text{m.e.p. ref. to low})} \frac{V_b}{V_g}\delta_1. \quad . \quad . \quad . \quad . \quad (303)$$

These last five equations, (299), (300), (301), (302), (303), while characteristic, are not convenient for general use in their present form, but are rendered so by substituting general symbols for initial and back pressures displacement, cut-off, and amount of expansion for each cylinder.

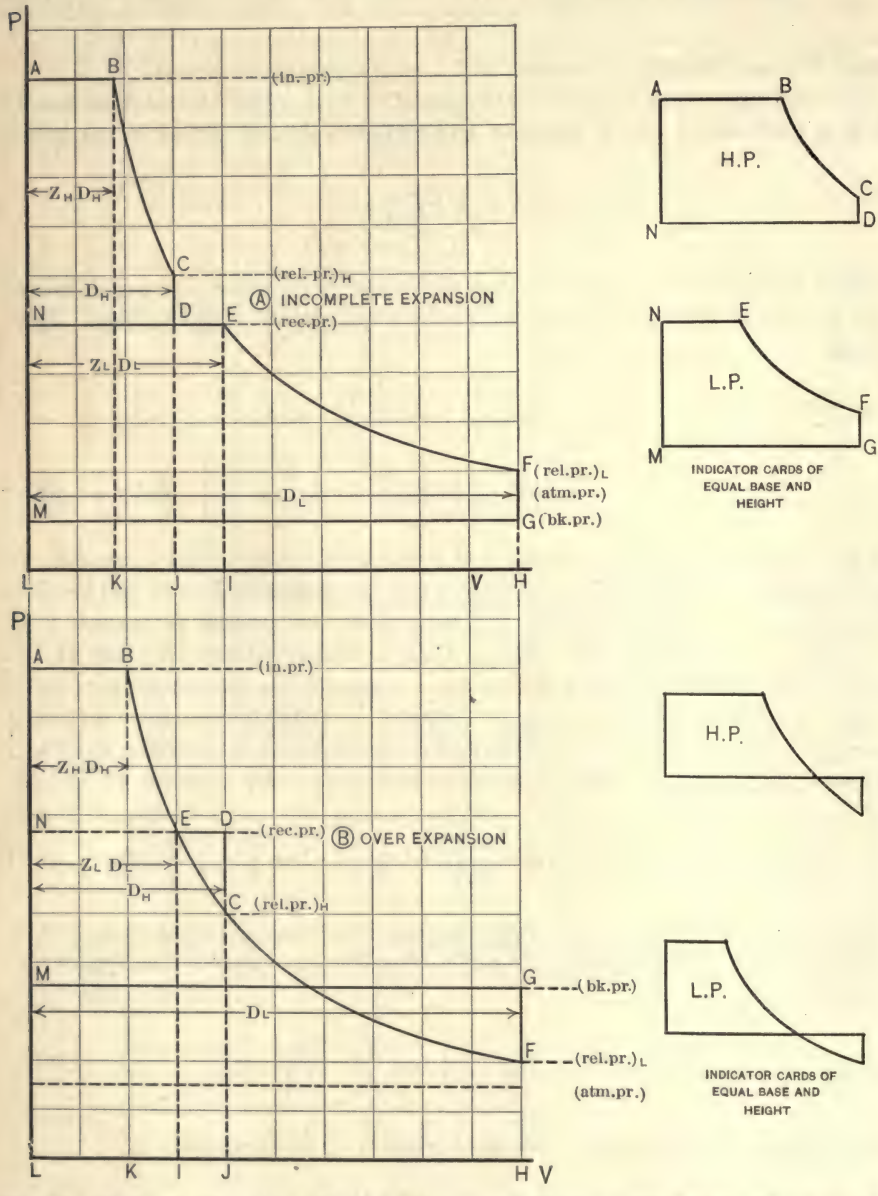


FIG. 82.—Work of Expansive Fluid in Compound Engine with Infinite Receiver, Zero Clearance. Cycle V, Logarithmic Expansion and Cycle VI Exponential Expansion.

Let (in.pr.) = initial or supply pressure, pounds per square inch $= \frac{P_b}{144}$;

" $(\text{rel.pr.})_H$ = release pressure, in H.P. cylinder pounds per square inch $= \frac{P_c}{144}$;

" $(\text{rel.pr.})_L$ = release pressure in L.P. cylinder, pounds per square inch $= \frac{P_f}{144}$;

" (rec.pr.) = receiver pressure, pounds per square inch $= \frac{P_d}{144} = \frac{P_e}{144}$;

" (bk.pr.) = back pressure, pounds per square inch $= \frac{P_g}{144}$;

" R_H = ratio of expansion in high-pressure cylinder $= \frac{V_c}{V_b}$;

" R_L = ratio of expansion in low-pressure cylinder $= \frac{V_f}{V_e}$;

" R_V = ratio of expansion for whole expansion $= \frac{V_f}{V_b}$;

" D_H = displacement of high-pressure cylinder $= V_d = V_c$;

" D_L = displacement of low-pressure cylinder $= V_f = V_g$;

" R_C = cylinder ratio $= \frac{D_L}{D_H} = \frac{V_g}{V_d}$;

" Z_H = fraction of displacement completed up to cut-off in high-pressure

" cylinder, so that $Z_H D_H = V_b = \frac{D_H}{R_H}$;

" Z_L = fraction of displacement completed up to cut-off in low-pressure cylinder, so that $Z_L D_L = V_e = \frac{D_L}{R_L}$.

Substitution of these general symbols in Eqs. (299), (300), (301), (302), and (303) gives another set of five equations in useful form for direct substitution of ordinary data as follows:

Work of cycle

$$\left.\begin{aligned} &= 144 D_H (\text{in.pr.}) \left[\frac{1}{R_H}(2 + \log_e R_H + \log_e R_L) - \frac{R_L}{R_C R_H}\right] - 144 (\text{bk.pr.}) D_L \quad (a) \\ &= 144 D_L \left\{ (\text{in.pr.}) \frac{Z_H}{R_C}\left(2 + \log_e \frac{1}{Z_H} + \log_e \frac{1}{Z_L} - \frac{1}{R_C Z_L}\right) - (\text{bk.pr.}) \right\} \quad (b) \\ &= 144 D_L \left\{ (\text{in.pr.}) \frac{1}{R_H R_C}\left(2 + \log_e R_H + \log_e R_L - \frac{R_L}{R_C}\right) - (\text{bk.pr.}) \right\} \quad (c) \end{aligned}\right\} . \quad (304)$$

(m.e.p.) lbs. per sq.in. referred to L.P. cyl.

$$\left.\begin{aligned} &= (\text{in.pr.}) \frac{1}{R_H R_C}\left(2 + \log_e R_H + \log_e R_L - \frac{R_L}{R_C}\right) - (\text{bk.pr.}) \quad (a) \\ &= (\text{in.pr.}) \frac{Z_H}{R_C}\left(2 + \log_e \frac{1}{Z_H} + \log_e \frac{1}{Z_L} - \frac{1}{Z_L R_C}\right) - (\text{bk.pr.}) \quad (b) \end{aligned}\right\} . \quad . \quad . \quad (305)$$

Work per cu.ft. supplied

$$\left.\begin{aligned} &= 144\left[(\text{in.pr.})\left(2+\log_e R_H+\log_e R_L-\frac{R_L}{R_C}\right)-(\text{bk.pr.})R_C R_H\right] \quad (a)\\ &= 144\left[(\text{in.pr.})\left(2+\log_e \frac{1}{Z_H}+\log_e \frac{1}{Z_L}-\frac{1}{Z_L R_C}\right)-(\text{bk.pr.})\frac{R_C}{Z_H}\right] \quad (b)\end{aligned}\right\}. \quad (306)$$

Cu.ft. supplied per hr. per I.H.P.

$$\left.\begin{aligned} &= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\,\frac{1}{R_H R_C} \quad (a)\\ &= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\,\frac{Z_H}{R_C} \quad (b)\end{aligned}\right\}. \quad \ldots \quad (307)$$

From this, of course, the weight in pounds supplied per I.H.P. results directly from multiplication by the density of the fluid.

To these characteristic equations for evaluating work, mean pressure, economy and consumption in terms of the initial and final pressures and cylinder dimensions there may be added a series defining certain *other general relations* of value in fixing the cycle for given dimensions and initial and final pressures, and in predicting dimensions for specified total work to be done and its division between high- and low-pressure cylinders.

Returning to the use of diagram points and translating into the general symbols as each expression is derived, there results,

Receiver pressure $= P_d = P_e = P_b\frac{V_b}{V_e}$.

$$\therefore \quad \left.\begin{aligned} (\text{rec.pr.}) &= (\text{in.pr.})\frac{Z_H D_H}{Z_L D_L} = (\text{in.pr.})\frac{Z_H}{R_C Z_L} \quad (a)\\ &= (\text{in.pr.})\frac{R_L}{R_C R_H} \quad (b)\end{aligned}\right\}. \quad \ldots \quad (308)$$

High-pressure cylinder release pressure $= P_c = P_b\frac{V_b}{V_c}$,

$$\therefore \quad \left.\begin{aligned} (\text{rel.pr.})_H &= (\text{in.pr.})\frac{1}{R_H} \quad (a)\\ &= (\text{in.pr.})Z_H \quad (b)\end{aligned}\right\}. \quad \ldots \quad (309)$$

Low-pressure cylinder release pressure $= P_f = P_e\frac{V_e}{V_f}$.

$$\therefore \quad \left.\begin{aligned} (\text{rel.pr.})_L &= (\text{in.pr.})\frac{1}{R_C R_H} \quad (a)\\ &= (\text{in.pr.})\frac{Z_H}{R_C} \quad (b)\\ &= \frac{(\text{in.pr.})}{R_V} \quad (c)\\ &= \frac{(\text{rel.pr.})_H}{R_C} \quad (d)\end{aligned}\right\}. \quad \ldots \quad (310)$$

Division of work between cylinders may be made anything for a given load by suitably proportioning cylinders, and equations giving the necessary relations to be fulfilled can be set down. It is quite common for designers to fix on equal division of work for the most commonly recurring or average load or that corresponding to some high pressure cut-off or low-pressure terminal pressure, generally the latter. Therefore, a general expression for dimensional relations to be fulfilled for equal division of work is useful. On the other hand, for an engine the dimensions of which are determined, it is often necessary to find the work division for the imposed conditions, so that the following equations are of value.

From Eqs. (296) and (297), noting that $P_d = P_e = P_b \frac{V_b}{V_e}$,

$$\frac{\text{High-pressure cylinder work}}{\text{Low-pressure cylinder work}} = \frac{P_b V_b \left[\left(1 + \log_e \frac{V_c}{V_b} \right) - \frac{V_d}{V_e} \right]}{P_b V_b \left(1 + \log_e \frac{V_f}{V_e} \right) - P_g V_g}$$

$$= \frac{\left(1 + \log_e \frac{V_c}{V_b} - \frac{V_d}{V_e} \right)}{\left(1 + \log_e \frac{V_f}{V_e} \right) - \frac{P_g V_g}{P_b V_b}}.$$

$$\left. \begin{aligned} &= \frac{1 + \log_e R_H - \frac{R_L}{R_C}}{1 + \log_e R_L - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) R_C R_H} \qquad (a) \\ &= \frac{1 + \log_e \frac{1}{Z_H} - \frac{1}{R_C Z_L}}{1 + \log_e \frac{1}{Z_L} - \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) \frac{R_C}{Z_H}} \qquad (b) \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad (311)$$

This is a general expression for work division between the cylinders in terms of (*a*) ratio of expansion in each cylinder, initial and back-pressure ratio and cylinder ratio, or, in terms of (*b*) cut-off in each, associated with cylinder and pressure ratios.

This expression Eq. (311) is less frequently used in its general form as above, than in special forms in which the work of the two cylinders in made equal, or the expression made equal to unity. The conditions thus found for *equal division of work* between cylinders may be expressed either (*a*) in terms of initial and back pressures, release pressure of low-pressure cylinder and ratio of L.P. admission volume to H.P. displacement, and cylinder ratio, or (*b*) cut-off in high- and low-pressure cylinders, initial and back pressures and cylinder ratio. Still more special conditions giving equality of work may be found (*c*) when the

cylinder ratio is made such that equality of work is obtained at all loads by equalizing high and low cut-offs.

(*a*) To find the first set of conditions, equate Eqs. (296) and (297) from the first part of this section, and by simplification there results,

$$\log_e \frac{V_c}{V_b} - \frac{V_c}{V_e} = \log_e \frac{V_f}{V_e} - \frac{P_g}{P_f},$$

or

$$\log_e \left(\frac{V_c}{V_b} \frac{V_e}{V_f} \right) = \frac{V_c}{V_e} - \frac{P_g}{P_f};$$

$$\therefore \frac{V_c}{V_b} \frac{V_e}{V_f} = e^{\left(\frac{V_c}{V_e} - \frac{P_g}{P_f} \right)}.$$

Introducing the usual symbols and putting in addition

$$\frac{\text{Low-pressure admission volume}}{\text{High-pressure displacement volume}} = \frac{V_e}{V_c} = x = Z_L R_C = \frac{(\text{rel.pr.})_H}{(\text{rec.pr.})};$$

Therefore,

$$R_C = \left[e^{\left[\frac{(\text{bk.pr.})}{(\text{rel.pr.})_L} - \frac{1}{x} \right]} \frac{(\text{in.pr.})}{(\text{rel.pr.})_L} x \right]^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad (312)$$

This is of value when a given release pressure is to be reached in the low pressure cylinder and with a particular value of low-pressure cut-off volume as fixed by x in terms of high-pressure cylinder displacement.

(*b*) Again for equal division of work, make Eq. (311) equal to unity, whence,

$$1 + \log_e \frac{1}{Z_H} - \frac{1}{R_C Z_L} = 1 + \log_e \frac{1}{Z_L} - \frac{(\text{bk.pr.})}{(\text{in.pr.})} \frac{R_C}{Z_H},$$

or

$$\frac{R_C^2}{Z_H} \frac{(\text{bk.pr.})}{(\text{in.pr.})} + R_C \log_e \frac{Z_L}{Z_H} - \frac{1}{Z_L} = 0,$$

which may be reduced to the following, solving for R_C,

$$R_C = \frac{\left[\left(\log_e \frac{Z_L}{Z_H} \right)^2 + 4 \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right) \frac{1}{Z_H Z_L} \right]^{\frac{1}{2}} - \log_e \frac{Z_L}{Z_H}}{\frac{2}{Z_H} \frac{(\text{bk.pr.})}{(\text{in.pr.})}}. \quad . \quad . \quad . \quad (313)$$

Equal division of work for given initial and back pressure is to be obtained by satisfying these complex relations Eq. (313) between the two cut-offs, or their equivalent ratios of expansion in connection with a given cylinder ratio, or the relation between pressures and volumes in Eq. (312) equally complex.

(*c*) An assumption of *equal cut-off* in both cylinders gives results which are of interest and practical value, although it is a special case. Eq. (313) then becomes, when $Z_H = Z_L$ or $R_H = R_L$,

$$R_C = \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (314)$$

As would be expected, this may also be derived from Eq. (312), since $\frac{1}{x} = \frac{\text{bk.pr.}}{(\text{rel.pr.})_L}$ and $x = \frac{(\text{rel.pr.})_L}{(\text{bk.pr.})}$ under these conditions.

The receiver pressure under these conditions is constant, and is, from Eq. (308)

$$(\text{rec.pr.}) = \frac{(\text{in.pr.})}{R_C} = \frac{(\text{in.pr.})}{\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}}} = \Big[(\text{in.pr.})(\text{bk.pr.})\Big]^{\frac{1}{2}} \quad . \quad . \quad . \quad (315)$$

The high-pressure *release pressure* is not affected by any change in the low-pressure cut-off, and hence Eq. (309) gives the value of high-pressure release, pressure for the case. Low-pressure release pressure Eq. (310) may be expressed for the case of equal cut-off,

$$\left.\begin{aligned}(\text{rel.pr.})_L = \frac{(\text{in.pr.})Z_H}{\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}}} &= Z_H\Big[(\text{in.pr.})(\text{bk.pr.})\Big]^{\frac{1}{2}} \quad (a) \quad . \quad . \quad . \\ &= \frac{1}{R_H}\Big[(\text{in.pr.})(\text{bk.pr.})\Big]^{\frac{1}{2}} \quad (b) \quad . \quad . \quad . \quad .\end{aligned}\right\} \quad (316)$$

The foregoing equations up to and including Eq. (311), are perfectly general, and take special forms for special conditions, the most important of which is that of *complete expansion in both cylinders*, the equations of condition for which are, referring to Fig. 82.

$$P_c = P_d;$$
$$V_d = V_e;$$
$$P_f = P_g,$$

which, when fulfilled, yield the diagram, Fig. 83. These equations of condition are equivalent to fixing a relation between the cut-off in both the high- and low-pressure cylinders, and the volume of high-pressure cylinder with respect to the low-pressure volume, so that

$$V_b = V_f \frac{P_f}{P_b}, \text{ or symbolically,}$$

$$\left.\begin{aligned}Z_H = \frac{D_L}{D_H}\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right) = R_C\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right) \quad (a) \\ R_H = \frac{1}{R_C}\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right) \quad (b)\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (317)$$

Similarly the low-pressure cylinder cut-off volume must equal the high-pressure displacement volume or $D_H = Z_L D_L$,

$$\left.\begin{aligned} Z_L &= \frac{D_H}{D_L} = \frac{1}{R_C} \quad (a) \\ R_L &= R_C \quad (b) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (318)$$

indicating that low-pressure cut-off is the reciprocal of the cylinder ratio. Making the necessary substitution there result the following equations for this cycle which, it must be noted, is that for most economical use of fluid in compound

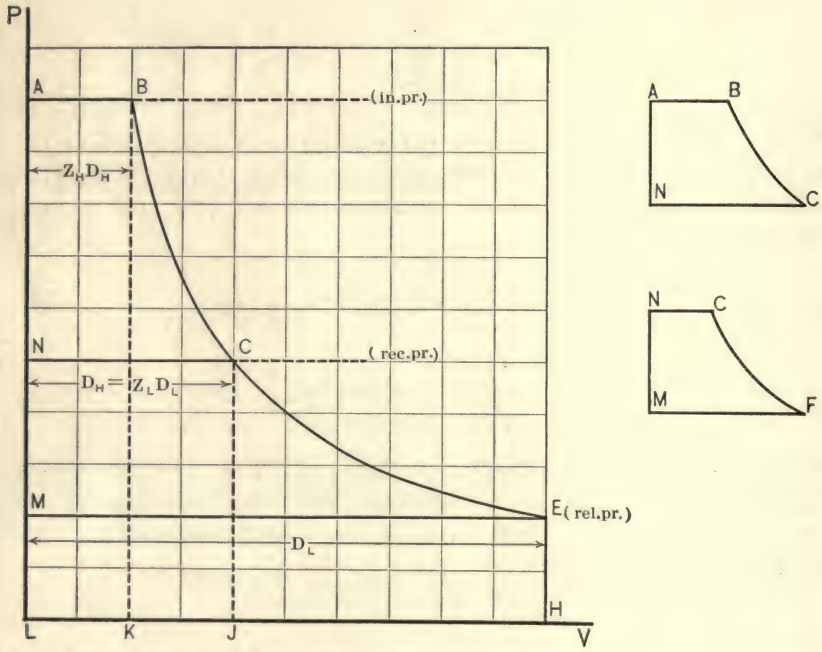


FIG. 83.—Special Case of Cycles V and VI, Complete Expansion in both Cylinders of Compound Engine with Infinite Receiver and Zero Clearance.

cylinders without clearance and with infinite receiver, and in which the same work is done as in Cycle I, for simple engines at best cut-off.

From Eq. (308)

$$(\text{rec.pr.}) = (\text{in.pr.})\frac{R_L}{R_C R_H} = \frac{(\text{in.pr.})}{R_C}\frac{R_C}{\frac{1}{R_C}\frac{(\text{in.pr.})}{(\text{bk.pr.})}} = R_C(\text{bk.pr.}) \quad . \quad . \quad . \quad (319)$$

From Eq. (309),

$$(\text{rel.pr.})_H = \frac{(\text{in.pr.})}{R_H} = (\text{in.pr.})\frac{R_C(\text{bk.pr.})}{(\text{in.pr.})} = R_C(\text{bk.pr.}) = (\text{in.pr.})_L \quad . \quad (320)$$

From Eq. (310),

$$(\text{rel.pr.})_L = \frac{(\text{in.pr.})}{R_C R_H} = \frac{(\text{in.pr.})}{R_C \frac{1}{R_C} \frac{(\text{in.pr.})}{(\text{bk.pr.})}} = (\text{bk.pr.}). \quad . \quad . \quad . \quad . \quad . \quad . \quad (321)$$

From Eq. (311),

$$\left.\begin{aligned} \frac{\text{High pressure cylinder work}}{\text{Low pressure cylinder work}} &= \frac{1+\log_e R_H - \frac{R_L}{R_C}}{1+\log_e R_L - \frac{(\text{bk.pr.})}{(\text{in.pr.})} R_C R_H} = \frac{\log_e R_H}{\log_e R_L} \quad (a) \\ &= \frac{\log_e \frac{1}{Z_H}}{\log_e \frac{1}{Z_L}} \quad (b) \end{aligned}\right\} . \quad (322)$$

For the case of most economical operation, that of complete and perfect expansion in both cylinders, there may be set down the four characteristic Eqs. (304), (305), (306), (307) with suitable modifications to meet the case. These become

$$\text{Work of cycle} = 144(\text{in.pr.}) D_L \frac{\log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)}{\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)} = 144(\text{in.pr.}) D_L \frac{\log_e R_V}{R_V}. \quad . \quad . \quad (323)$$

$$(\text{m.e.p.}) \ (\text{ref. to L.P.}) = \frac{W}{144 D_L} = (\text{in.pr.}) \frac{\log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)}{\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)} = (\text{in.pr.}) \frac{\log_e R_V}{R_V}. \quad . \quad . \quad (324)$$

$$\text{Work per cu.ft. supplied} = 144(\text{in.pr.}) \log_e \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right) = 144(\text{in.pr.}) \log_e R_V. \quad (325)$$

$$\left.\begin{aligned} \text{Cu.ft. per hr. per I.H.P.} &= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})} \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right) \quad (a) \\ &= \frac{13{,}750}{(\text{m.e.p.ref. to L.P.})} \frac{1}{R_V} \quad (b) \end{aligned}\right\} . \quad . \quad . \quad . \quad (226)$$

For equal division of work with complete expansion in both cylinders, the ratios of Eqs. (317) and (318) becomes

$$\left.\begin{aligned} R_H = R_L &= \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}} \quad (a) \\ Z_H = Z_L &= \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{2}} \quad (b) \end{aligned}\right\} , \quad . \quad . \quad . \quad . \quad . \quad . \quad (327)$$

and this is evidently a case to which Eqs. (314) and (315) apply without change.

Example. 1. Method of calculating diagram, Fig. 82.
Assumed data for Case A:

$$P_a = P_b = 100 \text{ lbs. per sq.in. abs.} \qquad V_a = V_n = V_m = 0 \text{ cu.ft.}$$
$$P_n = P_d = P_e = 50 \text{ lbs. per sq.in. abs.} \qquad V_c = V_d = .6 \text{ cu.ft.}$$
$$P_m = P_g = 10 \text{ lbs. per sq.in. abs.} \qquad V_f = V_g = 2 \text{ cu.ft.}$$
$$P_c = 60 \text{ lbs. per sq.in. abs.} \qquad V_e = .8 \text{ cu.ft.}$$

To obtain point B:

$$V_b = V_c \times \frac{P_c}{P_b} = .6 \times \frac{60}{100} = .36 \text{ cu.ft.}$$

To obtain point F:

$$P_f = P_e \times \frac{V_e}{V_f} = 50 \times \frac{.8}{2} = 20 \text{ lbs. per sq.in.}$$

To construct the indicator cards:

Lay off ND of the PV diagrams to equal the length of the card, and NA perpendicular to it at N to equal the height of the card. Cut off equals $AB \div ND$. From A on card lay off this ratio times the length of the card. From D on the card lay off a perpendicular equal to CD of the PV diagram reduced by the same proportion as AN of the card is to AN of the diagram. Join the points B and C by a curve through points located from intermediate points on the PV diagram. The low-pressure card is constructed in same manner.

Example. 2. A 12- and 18×24-in. steam engine without clearance runs on 150 lbs. per square inch absolute initial pressure, 10 lbs. per square inch absolute back pressure, and has a speed of 125 R.P.M. What will be (*a*) the horse-power for ½ cut-off in H.P. cylinder, (*b*) pounds of steam per I.H.P. hour, (*c*) terminal pressures, (*d*) L.P. cut-off for continuous expansion, (*e*) work done in each cylinder.

NOTE: δ for 150 lbs. = .332.

(*a*) From Eq. (305)

(m.e.p.) referred to L.P. cylinder is

$$(\text{in.pr.}) \frac{1}{R_H R_C} \left(2 + \log_e R_H + \log_e R_L - \frac{R_L}{R_C} \right) - (\text{bk.pr.}).$$

In this case

$$R_H = 2, \quad R_C = \left(\frac{18}{12} \right)^2 = 2.25, \quad R_L = 2.25,$$

since vol. of L.P. cyl. at cut-off must be equal to the entire volume of the high for continuous expansion, hence

$$(\text{m.e.p.}) = 150 \times \frac{1}{2 \times 2.25} \times (2 + .69 + .81 - 1) - 10 = 73.3 \text{ lb. sq. inch,}$$

and

$$\text{I.H.P.} = \frac{(\text{m.e.p.})\ Lan}{33{,}000} = 282.$$

(*b*) From Eq. (307)

$$\text{Cu.ft. per hour per I.H.P.} = \frac{13{,}750}{(\text{m.e.p.})} \frac{Z_H}{R_C} = \frac{13{,}750}{73.3} \times \frac{.5}{2.25} = 41.7,$$

(*c*) From Eq. (309)

$$(\text{rel.pr.})_H = (\text{in.pr.})Z_H,$$

$$= 150 \times \tfrac{1}{2} = 75 \text{ lbs. sq.in.}$$

and from Eq. (310) we have

$$(\text{rel.pr.})_L = \frac{(\text{rel.pr.})_H}{R_C},$$

$$= \frac{75}{2.25} = 33.3 \text{ lbs. sq.in.}$$

$$Z_L = \frac{1}{2.25} = .444.$$

(*e*) From Eq. (311)

$$\frac{\text{H.P. work}}{\text{L.P. work}} = \frac{1 + \log_e \frac{1}{Z_H} - \frac{1}{R_C Z_L}}{1 + \log_e \frac{1}{Z_L} - \frac{(\text{bk.pr.})}{(\text{in.pr.})} \frac{R_C}{Z_H}},$$

$$= \frac{1 + .69 - \frac{1}{2.25 \times .44}}{1 + .81 - \frac{10}{150} \times \frac{2.25}{.5}} = \frac{.69}{1.51} = .456,$$

or

$$\text{H.P.work} = .456 \times \text{L.P. work},$$

also

$$\text{H.P. work} + \text{L.P. work} = 282 \text{ I.H.P.}$$

Hence

$$\text{H.P. work} = 88 \text{ I.H.P.}$$

and

$$\text{L.P. work} = 194 \text{ I.H.P.}$$

Prob. 1. What must be the cylinder diameters of a cross compound engine to run on 100 lbs. per square inch absolute steam pressure, 18 ins. of mercury vacuum and to develop 150 H.P. at a speed of 200 R.P.M. with $\frac{1}{2}$ cut-off in each cylinder, if cylinder ratio is 3 and stroke is 18 ins.? Engine is double-acting and assumed to have no clearance.

Prob. 2. What will be the release pressure in each cylinder and the receiver pressure of the engine of Prob. 1? If cut-off were reduced to $\frac{1}{8}$ in H.P. cylinder, how would these pressures be affected and to what extent? How would the horse-power change?

Prob. 3. A 15- and 22×30-in. infinite receiver engine has no clearance, a speed of 150 R.P.M., initial pressure 125 lbs. per square inch gage. What will be the horse-power and steam consumption for a H.P. cut-off of $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{2}$, and that value which will give complete expansion in high-pressure cylinder? Low-pressure cut-off to be fixed at $\frac{1}{2}$.

NOTE: δ for 150 lbs. gage = .363.

Prob. 4. What will be the release and receiver pressures, and the work done in each cylinder for Prob. 3?

Prob. 5. An 18 and 24×30-in. infinite receiver engine is to be operated so as to give complete expansion in both cylinders. What will be the cut-off to accomplish this and what horse-power will result if the initial pressure is 100 lbs. and back pressure 10 lbs. per square inch absolute?

Prob. 6. Draw the PV diagram for following cases. Cylinder ratio 1 to 2.5, (in.pr.), 100 lbs. per square inch absolute, (bk.pr.), 20 lbs. per square inch absolute, H.P. cut-off $(a)=\frac{1}{5}$, $(b)=\frac{1}{2}$, $(c)=\frac{3}{5}$. L.P. cut-off $(a)=\frac{1}{5}$, $(b)=\frac{4}{10}$, $(c)=\frac{3}{4}$.

Prob. 7. For the following conditions find the horse-power, steam used per hour, receiver pressure and release pressures. Engine, 10- and 15×24-in. 150 R.P.M., 125 lbs. per square inch gage initial pressure, 2 lbs. per square inch absolute, back pressure, $\frac{1}{2}$ cut-off in high-pressure cylinder, $\frac{4}{10}$ cut-in low-pressure cylinder with infinite receiver.

NOTE: δ for 125 lbs. = .311.

Prob. 8. An infinite receiver engine is to develop 150 H.P. at 200 R.P.M. when initial pressure is 150 lbs. per square inch absolute. Cylinder ratio is 1 to 3 and back pressure is one atmosphere. What must be its size if the stroke is equal to the low-pressure cylinder diameter for $\frac{1}{4}$ cut-off in the high-pressure cylinder and $\frac{1}{2}$ cut-off in the low-pressure cylinder?

Prob. 9. Find by trial the cut-offs at which work division will be equal for an infinite receiver engine with a cylinder ratio of 2.5, an initial pressure of 100 lbs. per square inch absolute and a back pressure of 5 lbs. per square inch absolute?

10. Compound Engine with Infinite Receiver, Exponential Law. No Clearance, Cycle VI. General Relations between Pressures, Dimensions and Work. Again referring to Fig. 82, which may be used to represent this cycle also, the work of each cylinder may be expressed as follows, by the assistance of Eq. (254) derived in Section 4.

$$W_H = 144 D_H \left[Z_H(\text{in.pr.}) \left(\frac{s - Z_H^{s-1}}{s-1} \right) - (\text{rec.pr.}) \right] \quad . \quad . \quad . \quad (328$$

$$W_L = 144 D_L \left[Z_L(\text{rec.pr.}) \left(\frac{s - Z_L^{s-1}}{s-1} \right) - (\text{bk.pr.}) \right], \quad . \quad . \quad . \quad (329$$

where Z_H is the cut-off in the high pressure, $= \frac{V_b}{V_d}$ and Z_L, low-pressure cut-o $= \frac{V_e}{V_g}$. In combining these into a single equation for the total work, the ter for receiver pressure (rec.pr.) should be eliminated. Referring to Fig. 82,

$$(\text{rec.pr.}) = P_d = P_e = P_b \left(\frac{V_b}{V_e} \right)^s = (\text{in.pr.}) \left(\frac{Z_H}{R_C Z_L} \right)^s, \quad . \quad . \quad . \quad (330$$

hence

$$W = 144\, D_L \left\{ (\text{in.pr.}) \left[\left(\frac{Z_H}{R_C} \right) \left(\frac{s - Z^{s-1}{}_H}{s-1} \right) - \frac{1}{R_C} \left(\frac{Z_H}{R_C Z_L} \right)^s + Z_L \left(\frac{Z_H}{R_C Z_L} \right)^s \left(\frac{s - Z_L^{s-1}}{s-1} \right) \right] - (\text{bk.pr.}) \right\}, \quad . \quad (33$$

a rather complex expression which permits of little simplification, but offers no particular difficulty in solution.

Mean effective pressure referred to the low-pressure cylinder is

$$(\text{m.e.p. ref. to L.P.}) = (\text{in.pr.})\left[\left(\frac{Z_H}{R_C}\right)\left(\frac{s - Z_H{}^{s-1}}{s-1}\right) - \frac{1}{R_C}\left(\frac{Z_H}{R_C Z_L}\right)^s + Z_L\left(\frac{Z_H}{R_C Z_L}\right)^s \left(\frac{s - Z_L{}^{s-1}}{s-1}\right)\right] - (\text{bk.pr.}). \quad (332)$$

Work per cubic feet fluid supplied may be found by dividing Eq. (331) by the *supply volume*, which in terms of low-pressure displacement is

$$(\text{Sup.Vol.}) = D_L \frac{Z_H}{R_C}. \quad (333)$$

The consumption of fluid, cubic feet per hour per indicated horse-power is

$$\text{Consumption cu.ft. per hr. per I.H.P.} = \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})} \frac{Z_H}{R_C}, \quad (334)$$

which is the same expression as for the logarithmic law. Multiplying this by δ_1, the initial density of the fluid, pounds per cubic feet, gives *consumption, pounds* per fluid hour per I.H.P.

The receiver pressure has already been determined in Eq. (330).

Release pressure of the high-pressure cylinder is

$$(\text{rel.pr.})_H = (\text{in.pr.}) Z_H{}^s, \quad (335)$$

and for the low-pressure cylinder,

$$\left.\begin{aligned} (\text{rel.pr.})_L &= (\text{in.pr.})\left(\frac{Z_H}{R_C}\right)^s \quad (a) \\ &= \frac{(\text{in.pr.})}{R_V{}^s} \quad (b) \end{aligned}\right\} \quad (336)$$

where R_V is the ratio of maximum volume in the low pressure, to, volume at cut-off in the high, and equals $\frac{R_C}{Z_H}$.

The distrubition of work between the high- and low-pressure cylinders may be found as follows, by means of Eqs. (328) and (329), eliminating (rec.pr.) by means of Eq. (330)

$$\frac{W_H}{W_L} = \frac{Z_H\left(\frac{s - Z_H{}^{s-1}}{s-1}\right) - \left(\frac{Z_H}{R_C Z_L}\right)^s}{R_C\left[Z_L\left(\frac{Z_H}{R_C Z_L}\right)^s \left(\frac{s - Z_L{}^{s-1}}{s-1}\right) - \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)\right]}. \quad (337)$$

Equality of work in the two cylinders will be obtained if this expression is equal to unity, giving a complex relation between high- and low-pressure cut-offs, cylinder ratio and ratio of initial and back pressures, to be satisfied. It is found at once that the simple conditions for equality in the case of logarithmic law will not give equality of work for the exponential law. There is, however, a case under this law which yields itself to analysis, that of complete expansion in both cylinders, without over-expansion. The conditions for equality of work for this case will be treated after deriving work and mean effective pressure for it.

Complete expansion, without over-expansion, in both cylinders may be represented by Fig. 83.

$$Z_H = \frac{\overline{AB}}{\overline{NC}}, \quad Z_L = \frac{\overline{NC}}{\overline{ME}},$$

and since

$$\overline{NC} = D_H \text{ and } \overline{ME} = D_L,$$

$$R_C = \frac{D_L}{D_H} = \frac{\overline{ME}}{\overline{NC}} = \frac{1}{Z_L}.$$

The true ratio of expansion $= R_V = \frac{\overline{ME}}{\overline{AB}} = \frac{1}{Z_H Z_L} = \frac{R_C}{Z_H}$, but this is also equal to $\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{s}}$ due to the law of the curve, $P_b V_b^s = P_e V_e^s$.

By means of Eq. (257) in Section (4) the work of the two cylinders may be evaluated,

$$\left.\begin{aligned} W_H &= 144(\text{in.pr.}) D_H Z_H \frac{s}{s-1}\left(1 - Z_H{}^{s-1}\right) \quad (a) \\ &= 144(\text{in.pr.}) D_L \frac{Z_H}{R_C} \frac{s}{s-1}\left(1 - Z_H{}^{s-1}\right) \quad (b) \end{aligned}\right\} \quad . \quad . \quad (338)$$

$$W_L = 144(\text{bk.pr.}) D_L \frac{s}{s-1}\left(\frac{1}{Z_L{}^{s-1}} - 1\right),$$

but since

$$Z_L = \frac{1}{R_C},$$

$$\left.\begin{aligned} W_L &= 144(\text{bk.pr.}) D_L \frac{s}{s-1}(R_C{}^{s-1} - 1) \quad (a) \\ &= 144(\text{in.pr.}) D_L \frac{s}{s-1}\left(\frac{Z_H}{R_C}\right)^s (R_C{}^{s-1} - 1) \quad (b) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (339$$

The total work is evidently the same as that of a cylinder equal in size to the low-pressure cylinder with a cut-off equal to $\frac{Z_H}{R_C}$, working between the given (in.pr.) and (bk.pr.) and may be stated by reference to Eq. (257), Section 4, or by taking the sum of W_H and W_L given above,

$$W = 144(\text{in.pr.})D_L\frac{Z_H}{R_C}\frac{s}{s-1}\left\{(1-Z_H{}^{s-1})+\left(\frac{Z_H}{R_C}\right)^{s-1}(R_C{}^{s-1}-1),\right\},$$

which reduces to

$$W = 144(\text{in.pr.})D_L\frac{Z_H}{R_C}\frac{s}{s-1}\left\{1-\left(\frac{Z_H}{R_C}\right)^{s-1}\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (340)$$

For this case of complete expansion in both cylinders, the ratio of high- to low-pressure work is given by division and cancellation,

$$\frac{W_H}{W_L} = \frac{1-Z_H{}^{s-1}}{\left(\frac{Z_H}{R_C}\right)^{s-1}(R_C{}^{s-1}-1)} = \frac{\left(\frac{R_c}{Z_H}\right)^{s-1}-R_C{}^{s-1}}{R_C{}^{s-1}-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (341)$$

Equality of work, obtained by placing this expression equal to unity, provides the condition that

$$2R_C{}^{s-1}-1 = \left(\frac{R_C}{Z_H}\right)^{s-1} = \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{s-1}{s}},$$

or

$$R_C = \left\{\frac{\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{s-1}{s}}+1}{2}\right\}^{\frac{1}{s-1}}, \quad . \quad . \quad . \quad . \quad . \quad (342)$$

for equal work and complete expansion, and

$$Z_H = \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}; \quad R_C = \left\{\frac{\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}+1}{2}\right\}^{\frac{1}{s-1}} \quad . \quad . \quad . \quad . \quad . \quad (343)$$

Since $Z_L = \frac{1}{R_C}$ for complete expansion, and (in Fig. 83) $P_cV_c{}^s = P_fV_f{}^s$, the receiver pressure, P_c, is

$$(\text{rec.pr.}) = (\text{bk.pr.})\left(\frac{V_e}{V_c}\right)^s = (\text{bk.pr.})R_C{}^s, \quad . \quad . \quad . \quad . \quad . \quad (344)$$

in which R_C will have the value given above if work is equally distributed.

Example 1. What will be (*a*) the horse-power, (*b*) consumption, (*c*) work ratio, (*d*) receiver and release pressures for the following conditions? Engine 12 and 18×24 ins., running at 125 R.P.M. on initial air pressure of 150 lbs. per square inch absolute, and back pressure of 10 lbs. per square inch absolute, with ½ cut-off in high-pressure cylinder and continuous expansion in low-pressure cylinder. Exponent of expansion curve = 1.4 for compressed air, infinite receiver.

(*a*) From Eq. (332)

$$(\text{m.e.p.}) = (\text{in.pr.})\left[\left(\frac{Z_H}{R_C}\right)\left(\frac{s - Z_H{}^{s-1}}{s-1}\right) - \frac{1}{R_C}\left(\frac{Z_H}{R_C Z_L}\right)^s + Z_L\left(\frac{Z_H}{R_C Z_L}\right)^s\left(\frac{s - Z_L{}^{s-1}}{s-1}\right)\right] - (\text{bk.pr.})$$

which, on substituting values from above, gives for (m.e.p.) 63 lbs. per sq. inch. Hence, the indicated horse-power = 242.

(*b*) From Eq. (334)

$$\text{Compressed air per hour per I.H.P.} = \frac{13{,}750}{\text{m.e.p.}}\,\frac{Z_H}{R_C}\ \text{cu.ft.},$$

which, on substitution, gives

$$\frac{13{,}750}{63} \times \frac{.5}{2.25} = 48.3\ \text{cu.ft.}$$

(*c*) From Eq. (337)

$$\frac{W_H}{W_L} = \frac{Z_H\left(\frac{s - Z_H{}^{s-1}}{s-1}\right) - \left(\frac{Z_H}{R_C Z_L}\right)^s}{R_C\left[Z_L\left(\frac{Z_H}{R_C Z_L}\right)^s\left(\frac{s - Z_L{}^{s-1}}{s-1}\right) - \left(\frac{\text{bk.pr.}}{\text{in. pr.}}\right)\right]}$$

which gives

$$\frac{W_H}{W_L} = \frac{.5\left(\frac{1.4 - .5^{.4}}{.4}\right) - \left(\frac{.5}{2.25 \times \frac{1}{2.25}}\right)^{1.4}}{2.25\left[\frac{1}{2.25}\left(\frac{.5}{2.25 \times \frac{1}{2.25}}\right)^{1.4}\left(\frac{1.4 - \left(\frac{1}{2.25}\right)^{.4}}{.4}\right) - \frac{10}{150}\right]} = .294.$$

and

$$W_H + W_L = 242\ \text{I.H.P.}$$

Hence

$$W_H = 56\ \text{I.H.P.}$$

and

$$W_L = 184\ \text{I.H.P.}$$

(*d*) From Eq. (330)

$$(\text{rec.pr.}) = (\text{in.pr.})\left(\frac{Z_H}{R_C Z_L}\right)^s = 150\left(\frac{.5}{2.25 \times \frac{1}{2.25}}\right)^{1.4} = 57\ \text{lbs. per sq.in.}$$

From Eq. (335)

$$(\text{rel.pr.})_H = (\text{in.pr.}) Z_H{}^s,$$

$$= 150 \times (.5)^{1.4} = 57\ \text{lbs. per sq.in.}$$

From Eq. (336)

$$(\text{rel.pr.})_L = (\text{in.pr.}) \div R^s{}_V,$$

$$= 150 \div 21.85 = 6.85 \text{ lbs. per sq.in.}$$

These values may be compared with those of Ex. 1, Section 9, which were for the same data with logarithmic expansion.

Prob. 1. What will be the horse-power and steam used per hour by the following engine under the conditions given? Cylinders 18 and 30×48 ins., speed 100 R.P.M., initial pressure 150 lbs. per square inch absolute, back-pressure 10 lbs. per square inch absolute, steam continually dry. Cut-off at first $\frac{1}{3}$ in high-pressure and $\frac{1}{2}$ in low, and then $\frac{1}{2}$ in each infinite receiver.

Prob. 2. The very large receiver of a compound pumping engine is fitted with safety valve which is to be set to blow at 25 per cent above ordinary pressure. The cylinder ratio is 1 to 3.5, and cut-offs are $\frac{3}{8}$ in high and $\frac{1}{2}$ in low. If initial pressure is 125 lbs. per square inch gage, for what must valve be set? What vacuum must be carried in the condenser to have complete expansion in low-pressure cylinder? Superheated steam.

Prob. 3. A compound engine is to be designed to work on superheated steam of 125 lbs. per square inch absolute, initial pressure and on an 18-inch vacuum. The load which it is to carry is 150 horse-power and piston speed is to be 500 ft. per minute at 200 R.P.M. Load is to be equally divided between cylinders and there is to be complete expansion in both cylinders. What must be cylinder sizes, and what cut-offs will be used for an infinite receiver?

Prob. 4. How will the economy of the two following engines compare? Each is 14 and 20×24 ins., runs at 200 R.P.M., on compressed air of 100 lbs. per square inch gage pressure, and 15 lbs. per square inch absolute exhaust pressure. Low-pressure cut-off of each is $\frac{1}{2}$ and high pressure of one is $\frac{1}{4}$, the other, $\frac{3}{8}$. Infinite receivers.

Prob. 5. A compound engine 12 and 18×24 ins. is running at 200 R.P.M. on superheated steam of 100 lbs. per square inch absolute pressure and exhausting to a condenser in which pressure is 10 lbs. per square inch absolute. The cut-off is $\frac{1}{3}$ in high-pressure cylinder and $\frac{1}{2}$ in low-pressure cylinder. Compare the power and steam consumption under this condition with corresponding values for wet steam under same conditions of pressure and cut-off and infinite receivers.

Prob. 6. The initial pressure of an engine is 150 lbs. per square inch absolute, the back pressure one atmosphere, the cylinder ratio 3. As operated, both cut-offs are at $\frac{1}{2}$. What will be the receiver pressure, high-pressure release pressure, and low-pressure release pressure? What will be the new values of each if (*a*) high-pressure cut-off is made $\frac{1}{4}$, (*b*) $\frac{3}{4}$, without change of anything else, (*c*) low pressure cut-off is made $\frac{1}{4}$, (*d*) $\frac{3}{4}$, without change of anything else? Infinite receiver, $s = 1.3$.

Prob. 7. In the above problem for $\frac{1}{2}$ cut-off in each cylinder how will the release and receiver pressures change if (*a*) initial pressure is raised 25 per cent, (*b*) lowered 25 per cent, (*c*) back pressure raised 25 per cent, (*d*) lowered 25 per cent?

Prob. 8. How many pounds of initially dry steam per hour will be required to supply an 18-in. and 24×30-in. engine running at $\frac{1}{2}$ cut-off in each cylinder if speed

be 200 R.P.M., initial pressure 100 lbs. per square inch gage and back pressure 5 lbs. per square inch absolute? Expansion to be adiabatic and receiver infinite.

NOTE: δ for 100 lbs. = .26.

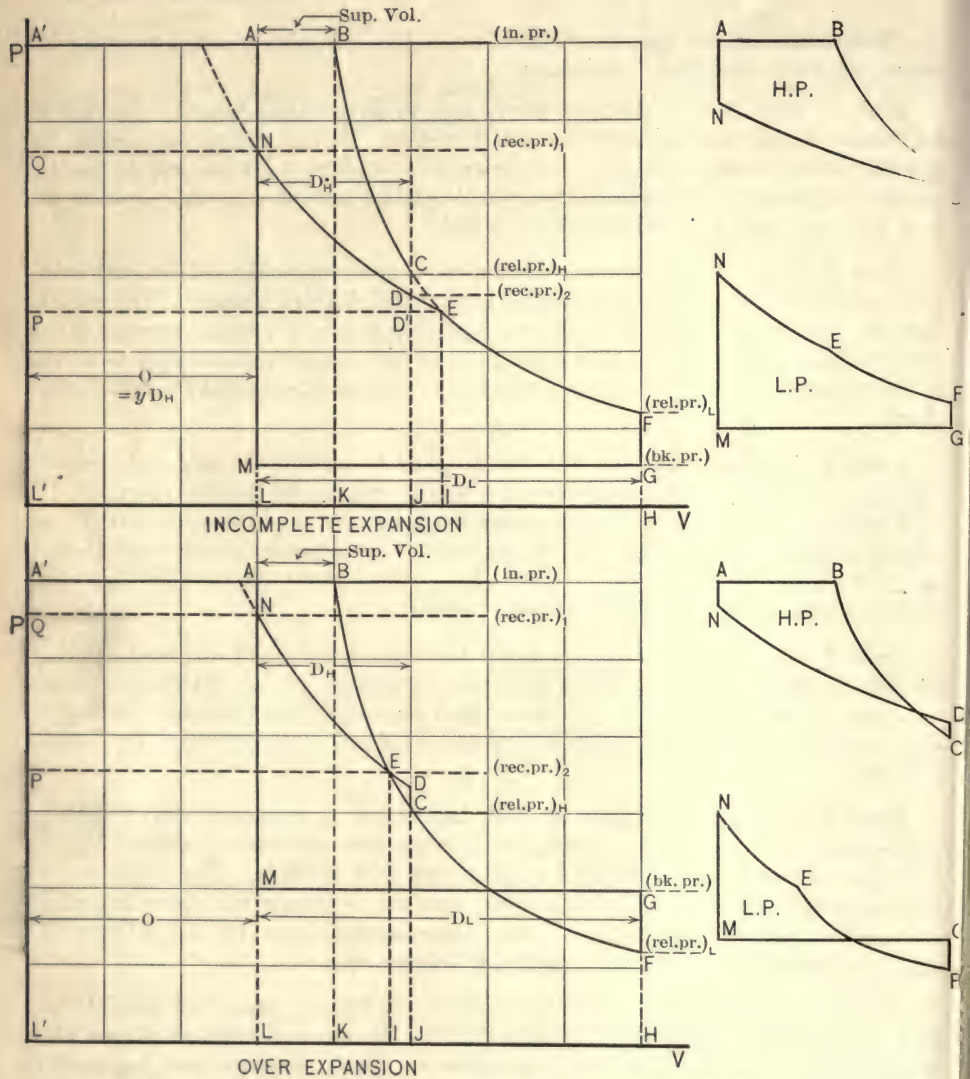


FIG. 84.—Work of Expansive Fluid in Compound Engines with Finite Receiver, Zero Clearance. Cycle VII, Logarithmic Expansion; Cycle VIII, Exponential Expansion.

11. Compound Engine with Finite Receiver. Logarithmic Law. No Clearance, Cycle VII. General Relations between Dimensions and Work when H.P. Exhaust and L.P. Admission are not Coincident. The diagrams, Fig. 84, while showing only two degrees of expansion, that of over and under in both cylinders, suffice for the derivation of equations applicable to all degrees in either cylinder. Volumes measured from the axis AL are those

occupied by the fluid in either cylinder alone, while fluid volumes entirely in the receiver, or partly in receiver, and in either cylinder at the same time are measured from the axis $A'L'$. No confusion will result if all volumes represented by points be designated by the (V) with a subscript, and to these a constant O representing the receiver volume be added when part of the fluid is in the receiver. Then,

High-pressure cylinder work is

$$W_H = P_b V_b \left(1 + \log_e \frac{V_c}{V_b}\right) - P_n O \log_e \left(\frac{V_c + O}{O}\right). \quad . \quad . \quad . \quad . \quad (345)$$

Low-pressure cylinder work is

$$W_L = P_n O \log_e \left(\frac{V_e + O}{O}\right) + P_e V_e \log_e \frac{V_f}{V_e} - P_g V_g. \quad . \quad . \quad . \quad (346)$$

Total work

$$W = P_b V_b \left(1 + \log_e \frac{V_c}{V_b}\right) + P_n O \log_e \left(\frac{V_e + O}{O}\right)$$
$$+ P_e V_e \log_e \frac{V_f}{V_e} - P_n O \log_e \left(\frac{V_c + O}{O}\right) - P_g V_g. \quad . \quad (347)$$

These expressions include some terms not known as initial data and may be reduced by the following relations,

$$P_b V_b = P_c V_c = P_e V_e = P_f V_f,$$

and

$$P_n O = P_e (V_e + O) = P_b V_b \left(\frac{V_e + O}{V_e}.\right)$$

Hence

$$W = P_b V_b \left[1 + \log_e \frac{V_c}{V_b} + \left(\frac{V_e + O}{V_e}\right) \log_e \left(\frac{V_e + O}{O}\right)\right.$$
$$\left. + \log_e \frac{V_f}{V_e} - \left(\frac{V_e + O}{V_e}\right) \log_e \left(\frac{V_c + O}{O}\right)\right] - P_g V_g. \quad (348)$$

Dividing by the low-pressure cylinder displacement, V_g, the result will be the mean effective pressure referred to the low-pressure cylinder,

(M.E.P. ref. to L.P.)

$$= P_b \frac{V_b}{V_g} \left[1 + \log_e \frac{V_c}{V_b} + \left(\frac{V_e + O}{V_e}\right) \log_e \left(\frac{V_e + O}{O}\right) + \log_e \frac{V_f}{V_e}\right.$$
$$\left. - \left(\frac{V_e + O}{V_e}\right) \log_e \left(\frac{V_c + O}{O}\right)\right] - P_g. \quad . \quad . \quad . \quad . \quad . \quad (349)$$

A similar division but with the volume supplied, V_b as the divisor, gives Work per cu.ft. supplied

$$=P_b\left[1+\log_e\frac{V_c}{V_b}+\left(\frac{V_e+O}{V_e}\right)\log_e\left(\frac{V_e+O}{O}\right)+\log_e\frac{V_f}{V_e}\right.$$
$$\left.-\left(\frac{V_e+O}{V_e}\right)\log_e\left(\frac{V_c+O}{O}\right)\right]-P_g\frac{V_g}{V_b}. \quad . \quad . \quad . \quad . \quad (350)$$

Also as in previous cases

Cu.ft. supplied per hr. per I.H.P.

$$=\frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\times\frac{V_b}{V_g}. \quad . \quad . \quad . \quad . \quad (351)$$

Of course, the weight per hour per I.H.P. follows from Eq. (351) by introducing the density as a multiplier.

While the last four equations can be used for the solution of problems, it is much better to transform them by introducing dimensional relations as in the previous cases developed.

Let $(\text{rec.pr.})_1$ = maximum receiver pressure P_n, which is also the initial admission pressure for the low-pressure cylinder;

" $(\text{rec.pr.})_2$ = minimum receiver pressure P_e, which is the terminal admission pressure for the low-pressure cylinder and that at which expansion begins there;

" $y=\dfrac{\text{receiver volume}}{\text{high-pressure cyl. displ.}}=\dfrac{O}{V_c}=\dfrac{O}{D_H}$, and, $\dfrac{O}{D_L}=\dfrac{D_H y}{D_L}=\dfrac{y}{R_C}$.

Other symbols necessary are unchanged from the meaning imposed in Section (9).

Substitution in Eqs. (348), (349), (350), and (351) gives the following set in a form for direct substitution of ordinary data:

Work of cycle

$$\left.\begin{aligned}
&=144(\text{in.pr.})Z_H D_H\left\{1+\log_e\frac{1}{Z_H}+\log_e\frac{1}{Z_L}\right.\\
&\left.+\left(1+\frac{y}{Z_L R_C}\right)\left[\log_e\left(1+\frac{Z_L R_C}{y}\right)-\log_e\left(1+\frac{1}{y}\right)\right]\right\}-144(\text{bk.pr.})D_L \quad (a)\\
&=144(\text{in.pr.})\frac{D_H}{R_H}\left\{1+\log_e R_H+\log_e R_L+\left(1+\frac{R_L y}{R_C}\right)\left[\log_e\left(1+\frac{R_C}{R_L y}\right)\right.\right.\\
&\left.\left.-\log_e\left(1+\frac{1}{y}\right)\right]\right\}-144(\text{bk.pr.})D_L \quad (b)
\end{aligned}\right\}. \quad (35)$$

(m.e.p. ref. to L.P.)

$$\left.\begin{aligned}
&= (\text{in.pr.})\frac{Z_H}{R_C}\left\{1+\log_e\frac{1}{Z_H}+\log_e\frac{1}{Z_L}+\right.\\
&\qquad\left.\left(1+\frac{y}{Z_L R_C}\right)\left[\log_e\left(1+\frac{Z_L R_C}{y}\right)-\log_e\left(1+\frac{1}{y}\right)\right]\right\}-(\text{bk.pr.}) \quad (a)\\
&= (\text{in.pr.})\frac{1}{R_H R_C}\left\{1+\log_e R_H+\log_e R_L+\left(1+\frac{R_L y}{R_C}\right)\left[\log_e\left(1+\frac{R_C}{R_L y}\right)\right.\right.\\
&\qquad\left.\left.-\log_e\left(1+\frac{1}{y}\right)\right]\right\}-(\text{bk.pr.}) \quad (b)
\end{aligned}\right\}. \quad (353)$$

Work per cu.ft. supplied

$$\left.\begin{aligned}
&= 144(\text{in.pr.})\left\{1+\log_e\frac{1}{Z_H}+\log_e\frac{1}{Z_L}\right.\\
&+\left(1+\frac{y}{Z_L R_C}\right)\left[\log_e\left(1+\frac{Z_L R_C}{y}\right)-\log_e\left(1+\frac{1}{y}\right)\right]\Big\}-144(\text{bk.pr.})\frac{R_C}{Z_H} \quad (a)\\
&= 144(\text{in.pr.})\left\{1+\log_e R_H+\log_e R_L+\left(1+\frac{R_L y}{R_C}\right)\left[\log_e\left(1+\frac{R_C}{R_L y}\right)\right.\right.\\
&\qquad\left.\left.-\log_e\left(1+\frac{1}{y}\right)\right]\right\}-144(\text{bk.pr.})R_C R_H \quad (b)
\end{aligned}\right\}. \quad (354)$$

Cu.ft. supplied per hr. per I.H.P.

$$\left.\begin{aligned}
&= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\times\frac{Z_H}{R_C} \quad (a)\\
&= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\times\frac{1}{R_H R_C} \quad (b)
\end{aligned}\right\}. \quad . \quad . \quad . \quad . \quad . \quad (355)$$

It is desirable at this point to introduce a series of expressions fixing the relations between the dimensions, the cycle that may follow, and the fluctuations in the receiver pressure, and for the selection of cylinder and receiver dimensions for a required output of work and division of it between cylinders.

In doing this it will be convenient to start with diagram points and finally substitute general symbols in each case. There will first be established the *maximum and minimum receiver pressures and the fluctuations.*

Maximum receiver pressure

$$P_n = P_e\frac{(V_e+O)}{O} = \left(\frac{P_b V_b}{V_e}\right)\left(\frac{V_e+O}{O}\right) = P_b\left(\frac{V_b}{O}+\frac{V_b}{V_e}\right).$$

$$\left.\begin{aligned}
\therefore \quad (\text{rec.pr.})_1 &= (\text{in.pr.})\left(\frac{Z_H D_H}{O}+\frac{Z_H D_H}{Z_L D_L}\right) = (\text{in.pr.})\left(\frac{Z_H}{y}+\frac{Z_H}{Z_L R_C}\right) \quad (a)\\
&= (\text{in.pr.})\left(\frac{1}{R_H y}+\frac{R_L}{R_H R_C}\right) \quad (b)
\end{aligned}\right\}. \quad (356)$$

Minimum receiver pressure

$$P_e = P_b \frac{V_b}{V_e};$$

$$\left.\begin{aligned} \therefore \quad (\text{rec.pr.})_2 &= (\text{in.pr.})\frac{Z_H D_H}{Z_L D_L} = (\text{in.pr.})\frac{Z_H}{Z_L R_C} \quad (a) \\ &= (\text{in.pr.})\frac{R_L}{R_H R_C} \quad (b) \end{aligned}\right\} \quad . \quad . \quad . \quad (357)$$

Fluctuation in receiver pressure $= (P_n - P_e) = P_b \frac{V_b}{O}$.

$$\left.\begin{aligned} \therefore \quad (\text{rec.pr.})_1 - (\text{rec.pr.})_2 &= (\text{in.pr.})\frac{Z_H D_H}{O} = (\text{in.pr.})\frac{Z_H}{y} \quad (a) \\ &= (\text{in.pr.})\frac{1}{R_H y} \quad (b) \end{aligned}\right\} \quad . \quad (358)$$

It is interesting to note that the *minimum receiver pressure* is exactly the *same as* the value of the constant-receiver pressure *for infinite receiver*, so that limiting the size of receiver does not affect the point E, but only raises point N higher, tending to throw more work on the L.P. cylinder for the same valve setting.

The *two release pressures* P_c and P_f can be evaluated as in the case of the infinite receiver, as both these points lie on the common expansion line, which is not at all affected by the receiver-pressure changes, and the values are the same as for the infinite receiver, and are here reproduced from Eqs. (309) and (310) with new numbers to make the set of equations complete:

$$\left.\begin{aligned} (\text{rel. pr.})_H &= (\text{in.pr.})\frac{1}{R_H} \quad (a) \\ &= (\text{in.pr.})Z_H \quad (b) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (359)$$

$$\left.\begin{aligned} (\text{rel.pr.})_L &= (\text{in.pr.})\frac{1}{R_C R_H} \quad (a) \\ &= (\text{in.pr.})\frac{Z_H}{R_C} \quad (b) \\ &= \frac{(\text{in.pr.})}{R_V} \quad (c) \\ &= \frac{(\text{rel.pr.})_H}{R_C} \quad (d) \end{aligned}\right\} \quad , \quad . \quad . \quad . \quad . \quad . \quad (360)$$

where R_V is the ratio of maximum volume in the low- to the volume at cut-off in the high-pressure cylinder.

Division of work between the cylinders cannot, as pointed out, be the same as for the infinite receiver, the tendency being to throw more work on the low as the receiver becomes smaller, assuming the cut-off to remain the same. As, therefore, equal division was obtainable in the case of infinite receiver with

equal cut-offs when the cylinder ratio was equal to the square root of initial over back pressure, it is evident that a finite receiver will require unequal cut-offs. As increase of low-pressure admission period or cut-off fraction lowers the receiver pressure and reduces the low-pressure work, it follows that with the finite receiver the low-pressure cut-off must be greater than the high for equal work division, and it is interesting to examine by analysis the ratio between them to determine if it should be constant or variable.

For *equal work division* Eqs. (345) and (346) should be equal, hence by diagram points

$$P_b V_b\left(1+\log_e \frac{V_c}{V_b}\right)-P_n O \log_e\left(\frac{V_c+O}{O}\right)=P_n O \log_e\left(\frac{V_e+O}{O}\right)+P_e V_e \log_e \frac{V_f}{V_e}-P_g V_g.$$

$$P_b V_b\left(1+\log_e \frac{V_c}{V_b}\right)-P_b V_b\left(\frac{V_e+O}{V_e}\right)\log_e\left(\frac{V_c+O}{O}\right)$$
$$=P_b V_b\left(\frac{V_e+O}{V_e}\right)\log_e\left(\frac{V_e+O}{O}\right)+P_b V_b \log_e \frac{V_f}{V_e}-P_g V_g.$$

$$1+\log_e R_H-\left(1+\frac{R_L y}{R_C}\right)\log_e\left(1+\frac{1}{y}\right)=\left(1+\frac{R_L y}{R_C}\right)\log_e\left(1+\frac{RC}{R_L y}\right)+\log R_L$$
$$-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)R_H R_C,$$

hence for equal division of work, the following relations must be satisfied:

$$(\log_e R_H-\log_e R_L)=\log_e \frac{R_H}{R_L}=\left(1+\frac{R_L y}{R_C}\right)\log_e\left[\left(1+\frac{R_C}{R_L y}\right)\left(1+\frac{1}{y}\right)\right]$$
$$-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)R_H R_C-1. \quad . \quad (361)$$

It will be shown later that when expansion is complete in both cylinders and work equal that the high-pressure cylinder cut-off or the equivalent ratio of expansion bears a constant relation to that of the low, according to

$$\frac{R_H}{R_L}=a^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (362)$$

in which a is a constant depending only on the size of the receiver. It will also be shown that the cylinder ratio is a constant function of the initial and back pressures and the receiver volume for equal division of work according to

$$R_C=\frac{1}{a}\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (363)$$

in which (a) is the same constant as in Eq. (362). It is impo tant to know if these same values will also give equal division for this general case. Substituting them in Eq. (361)

$$2\log_e a=\left(1+\frac{R_L y a}{\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}}}\right)\log_e\left[\left(1+\frac{\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}}}{R_L y a}\right)\left(1+\frac{1}{y}\right)\right]-\frac{a R_L}{\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}}}-1.$$

Here there is only one variable, R_L, the evaluation of which can be made by inspection, for if

$$R_L = \frac{1}{a}\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}},$$

the equation will become

$$2 \log_e a = (1+y) \log_e \left(1+\frac{1}{y}\right)^2 - 2 = 2(1+y) \log_e \left(1+\frac{1}{y}\right) - 2,$$

or

$$a = e^{[(1+y) \log_e \left(1+\frac{1}{y}\right)-1]}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (364)$$

which is a constant.

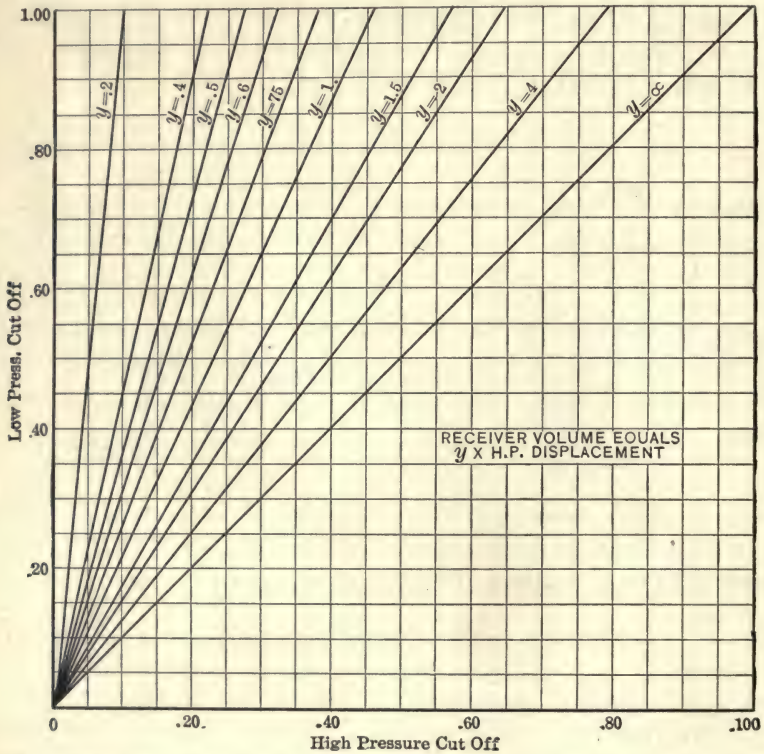


FIG. 85.—Diagram to Show Relation of High- and Low-Pressure Cut-offs for Equal Work in the Two Cylinders of a Finite-receiver Compound Engine with Zero Clearance and Logarithmic Law.

As only one constant value of low-pressure ratio of expansion or cut-off satisfies the equation for equal division of work when there is a fixed ratio between the values for high and low, that necessary for equal division with complete expansion in both, it is evident that equal division of work between the two cylinders cannot be maintained at all values of cut-off by fixing the ratio between them. As the relation between these cut-offs is a matter of some interest and as it cannot be derived by a solution of the general equation it is given by the curve, Fig. 85, to scale, the points of which were calculated.

A *special case* of this cycle of sufficient importance to warrant derivation of equations because of the simplicity of their form and consequent value in estimating when exact solutions of a particular problem are impossible, *is the case of complete and perfect expansion in both cylinders.* For it the following equations of condition hold, referring to Fig. 84,

$$P_c = P_d = P_e;$$
$$V_d = V_c = V_e;$$
$$P_f = P_g;$$

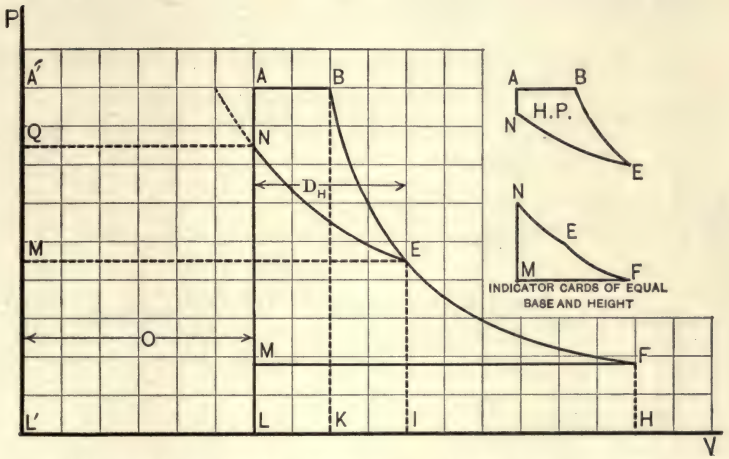


FIG. 86.—Special Case of Cycles VII and VIII Complete Expansion, in both Cylinders of the Finite Receiver Compound Engine. Zero Clearance.

which when fulfilled yield the diagram, Fig. 86. These equations of conditions are equivalent to fixing the cut-off in both high- and low-pressure cylinders, and the volume of the high- with respect to the low-pressure volume. Accordingly,

$$V_b = V_e \frac{P_e}{P_b},$$

$$\left.\begin{aligned} Z_H &= R_C \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right) = \frac{R_C}{R_V} \quad (a) \\ R_H &= \frac{1}{R_C}\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right) = \frac{R_V}{R_C} \quad (b) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (365)$$

Also for the low-pressure cylinder the cut-off volume must equal the whole high-pressure volume, or $D_H = Z_L D_L$. Therefore,

$$\left.\begin{aligned} Z_L &= \frac{1}{R_C} \quad (a) \\ R_L &= R_C \quad (b) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (366)$$

Substituting these equations of condition in the characteristic set Eqs. (352), (353), (354), and (355), there results the following for most economical operation:

Work of cycle

$$W = 144(\text{in.pr.})R_C\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)D_H\left\{1+\log_e\left[\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)\frac{1}{R_C}\right]+\log_e R_C\right.$$

$$\left.+(1+y)\ \log_e\left(1+\frac{1}{y}\right)-\log_e\left(1+\frac{1}{y}\right)\right\}-144(\text{bk.pr.})D_L$$

$$=144(\text{bk.pr.})D_L\log_e\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right) \qquad (a)$$

$$=144(\text{in.pr.})D_L\frac{\log_e\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)}{\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)}=144(\text{in.pr.})D_L\frac{\log_e R_V}{R_V} \qquad (b) \qquad (367)$$

$$(\text{m.e.p. ref. to L.P.})=144\frac{W}{144D_L}$$

$$=(\text{bk.pr.})\log_e\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)=(\text{in.pr.})\frac{\log_e\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)}{\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)}=(\text{in.pr.})\frac{\log_e R_V}{R_V}. \qquad (368)$$

$$\text{Work per cu.ft. supplied}=\frac{W}{Z_H D_H}$$

$$=144\frac{(\text{bk.pr.})D_L}{R_C\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)D_H}\log_e\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)=144(\text{in.pr.})\log_e\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)$$

$$=144(\text{in.pr.})\log_e R_V. \qquad (369)$$

Cu.ft. supplied per hr. per I.H.P.

$$=\frac{13{,}750}{(\text{m.e.p. ref. to L. P.})}\times\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)=\frac{13{,}750}{(\text{m.e.p. ref. to L. P.})}\times\frac{1}{R_V}. \qquad (370)$$

For this special case of best economy the receiver and release pressures, of course, have special values obtained by substituting the equations of condition Eqs. (365) and (366), in Eqs. (356), (357), (358), (359), (360).

$$(\text{rec.pr.})_1=(\text{in.pr.})\left(\frac{1}{R_H y}+\frac{R_L}{R_H R_C}\right)=(\text{in.pr.})\left(\frac{R_C}{R_V y}+\frac{R_C}{R_V}\right)$$

$$=(\text{in.pr.})\frac{R_C}{R_V}\left(\frac{1}{y}+1\right). \qquad (371)$$

$$(\text{rec.pr.})_2=(\text{in.pr.})\frac{R_L}{R_H R_C}=(\text{in.pr.})\frac{R_C}{R_V}=(\text{bk.pr.})R_C. \qquad (372)$$

Therefore

$$(\text{rec.pr.})_1-(\text{rec.pr.})_2=(\text{in.pr.})\frac{R_C}{R_V y}; \quad \ldots\ldots\ldots \quad (373)$$

$$(\text{rel.pr.})_H=(\text{in.pr.})\frac{1}{R_H}=(\text{bk.pr.})R_C=(\text{rec.pr.})_2; \quad \ldots\ldots \quad (374)$$

$$(\text{rel.pr.})_L=(\text{in.pr.})\frac{1}{R_C R_H}=(\text{bk.pr.}). \quad \ldots\ldots\ldots \quad (375)$$

These last two expressions might have been set down at once, but are orked out as checks on the previous equations.

For equal division of work in this special case the general Eq. (361) becomes

$$\log_e R_H-\log_e R_L=\log_e\frac{R_H}{R_L}=\left(1+\frac{R_C y}{R_C}\right)\log_e\left[\left(1+\frac{R_C}{R_C y}\right)\left(1+\frac{1}{y}\right)\right]$$
$$-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)-1=(1+y)\log_e\left(1+\frac{1}{y}\right)^2-2,$$

$$\log_e\frac{R_H}{R_L}=2\left[(1+y)\log_e\left(1+\frac{1}{y}\right)-1\right].$$

Therefore

$$\frac{R_H}{R_L}=e^{2\left[(1+y)\log_e\left(1+\frac{1}{y}\right)-1\right]}=a^2. \quad \ldots\ldots\ldots \quad (376)$$

This term, a, has already been used in previous discussions of equality of ork, while the derivation of its value has not been made up to this point.

This indicates that ratio of cut-offs or individual ratios of expansion is a nction of the receiver size for equal division of work.

From Eq. (376) the cylinder ratio can be found in terms of a, and the ratio expansion. Referring to Fig. 86,

$$\frac{R_H}{R_L}=\frac{\dfrac{V_e}{V_b}}{\dfrac{V_f}{V_e}}=\frac{V_e^2}{V_b V_f}=a^2,$$

$$V_e=a\sqrt{V_b V_f},$$

$$R_C=\frac{V_f}{V_e}=\frac{V_f}{a\sqrt{V_b V_f}}=\frac{1}{a}\sqrt{\frac{V_f}{V_b}}=\frac{1}{a}\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}}=\frac{1}{a}\sqrt{R_V}, \quad \ldots \quad (377)$$

hence the cylinder ratio is equal to a constant depending on the receiver size, ultiplied by the value for the infinite receiver, i.e., the square root of the initial ivided by back pressure.

The high-pressure cylinder ratio of expansion is

$$R_H = \frac{V_e}{V_b} = a\sqrt{\frac{V_b V_f}{V_b}} = a\sqrt{\frac{V_f}{V_b}} = a\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}} = a\sqrt{R_V}, \quad . \quad . \quad . \quad (378)$$

and the corresponding value for the low-pressure cylinder is

$$R_L = \frac{V_f}{V_e} = \frac{V_f}{a\sqrt{V_b V_f}} = \frac{1}{a}\sqrt{\frac{V_f}{V_b}} = \frac{1}{a}\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}} = \frac{1}{a}\sqrt{R_V}. \quad . \quad . \quad . \quad (379)$$

For convenience in calculation Table, XII of values of a and a^2 is added for various size of receivers.

TABLE XII.

$\frac{\text{Receiver Vol.}}{\text{H.P. Cyl. Disp.}} = y$	$a = e^{\left[(1+y)\log_e\left(1+\frac{1}{y}\right)-1\right]}$	a^2
.5	1.915	2.64
.75	1.624	3.67
1.0	1.474	2.17
1.5	1.322	1.75
2.0	1.243	1.55
2.5	1.198	1.437
3.0	1.164	1.359
4.0	1.1223	1.262
5.0	1.0973	1.204
7.0	1.0690	1.143
10.0	1.0478	1.098
14.0	1.0366	1.068
20.0	1.0228	1.046
Infinite	1.0	1.0

At the end of this chapter there is presented a chart which gives the relation between cylinder and receiver volumes, cylinder ratio, and high- and low-pressure cut-offs graphically.

The corresponding values of maximum and minimum receiver pressure for equal division of work for this case of best economy are

$$(\text{rec.pr.})_1 = (\text{bk.pr.})\frac{1}{a}\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}}\left(\frac{1}{y}+1\right) = \frac{\sqrt{(\text{in.pr.})(\text{bk.pr.})}}{a}\left(\frac{1}{y}+1\right) \quad . \quad (380$$

$$(\text{rec.pr.})_2 = (\text{bk.pr.})\frac{1}{a}\sqrt{\frac{\text{in.pr.}}{\text{bk.pr.}}} = \frac{\sqrt{(\text{in.pr.})(\text{bk.pr.})}}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (381$$

$$(\text{rec.pr.})_1 - (\text{rec.pr.})_2 = \frac{\sqrt{(\text{in.pr.})(\text{bk.pr.})}}{ay} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (382$$

Example 1. Method of calculating Diagram, Fig. 84.
Assumed data for case A:

$P_a = P_b = 120$ lbs. per sq.in. abs. $\quad V_a = V_n = V_m = 0$ cu.ft.
$P_m = P_g = 10$ lbs. per sq.in. abs. $\quad V_b = .4$ cu.ft.
$V_e = 1$ cu.ft.
$O = 1.2$ cu.ft.
$V_c = .8$ cu.ft.

To find point C:

$$P_c = P_b \frac{V_b}{V_c} = \frac{120 \times .4}{.8} = 60 \text{ lbs. per sq. inch.}$$

To obtain point E:

$$P_e = P_b \frac{V_b}{V_e} = \frac{120 \times .4}{1} = 48 \text{ lbs. per sq. inch.}$$

To obtain point D:

$$P_e(V_e + O) = P_d(V_c + O) \quad \text{or} \quad P_d = \frac{48 \times 2.2}{2} = 53 \text{ lbs. per sq. inch.}$$

To obtain point N:

$$P_n(O + V_n) = P_e(V_e + O) \quad \text{or} \quad P_n = \frac{48 \times 2.2}{1.2} = 88 \text{ lbs. per sq. inch.}$$

To obtain point F:

$$P_f = \frac{P_e V_e}{V_f} = \frac{48 \times 1}{2} = 24 \text{ lbs. per sq. inch.}$$

Example 2. Find (a) the horse-power, (b) steam used per hour, (c) the release and receiver pressures for a 12- and 18×24-in. engine with receiver twice as large as the low-pressure cylinder when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., and cut-offs ½ in high-pressure and such a value in the low pressure as to give complete expansion, and clearances zero.

(a) From Eq. (353)

$$\text{(m.e.p.)} = \text{(in.pr.)} \frac{1}{R_H R_C} \left\{ 1 + \log_e R_H + \log_e R_L + \left(\frac{1 + R_L y}{R_C} \right) \left[\log_e \left(1 + \frac{R_C}{R_L y} \right) - \log_e \left(1 + \frac{1}{y} \right) \right] \right\} - \text{bk.pr.}$$

which on substituting the above values gives

$$\text{(m.e.p.)} = \frac{150}{2 \times 2.25} \left\{ 1 + .69 + .81 + \frac{(1 + 2.25 \times 4.5)}{2.25} \left(.8 - .8 \right) \right\} - 10 = 73.3 \text{ lbs.}$$

hence I.H.P. = 282.

(b) From Eq. (355) we have

Cu.ft. steam per hour per horse-power $= \frac{13{,}750}{\text{(m.e.p.)}} \times \frac{1}{R_H R_C} = \frac{13{,}750}{73.3} \times \frac{1}{2 \times 2.25} = 41.7,$

(*c*) From Eqs. (356) and (357) for maximum and minimum receiver pressures respectively:

$$(\text{in.pr.})\left(\frac{1}{R_{HV}}+\frac{R_L}{R_H R_C}\right) \quad \text{and} \quad (\text{in.pr.})\frac{R_L}{R_H R_C},$$

$$\text{maximum receiver pressure} = 150\left(\frac{1}{2\times 4.5}+\frac{2.25}{2\times 2.25}\right) = 91.5 \text{ lbs. per sq. inch.}$$

$$\text{minimum receiver pressure} = 150\times\frac{2.25}{2\times 2.25} = 75 \text{ lbs. per sq. inch.}$$

From Eqs. (359) and (360) for release pressures

$$(\text{in.pr.})Z_H \quad \text{and} \quad \frac{(\text{in.pr.})}{RV},$$

$$\text{high pressure cylinder release pressure} = 150\times .5 = 75 \text{ lbs., per sq. inch.}$$

$$\text{low pressure cylinder release pressure} = \frac{150}{.444} = 33.9 \text{ lbs. per sq. inch.}$$

These results may be compared with those of Example 1 of Sections 9 and 10, which are derived for same engine, with data to fit the special cycle described in the particular section.

NOTE: In all the following problems clearance is to be neglected.

Prob. 1. A 12- and 18×24-in. engine has a receiver equal to 5 times the volume of the high-pressure cylinder. It is running on an initial pressure of 150 lbs. per square inch gage and exhausts to the atmosphere. It has a speed of 150 R.P.M. and the cut-offs are $\frac{3}{10}$ and $\frac{1}{4}$ in high- and low-pressure cylinders respectively. What is the horse-power and the steam used in cubic feet per hour?

Prob. 2. What will be the release pressures, and variation of receiver pressure for an engine in which the cylinder ratio is 3, cut-offs $\frac{2}{3}$ and $\frac{1}{2}$, in high and low, initial pressure is 100 lbs. per square inch absolute, and receiver 2 times low-pressure cylinder volume?

Prob. 3. Show whether or not the following engine will develope equal cylinder work for the conditions given. Cylinder diameters, 15 and 22 in., initial pressure 135 lbs. per square inch gage, back pressure 10 lbs. per square inch absolute, cut-offs $\frac{1}{4}$ and $\frac{2}{5}$, receiver volume 4 times high-pressure cylinder, strokes equal.

Prob. 4. For the same conditions as above, what low-pressure cut-off would give equal work?

Prob. 5. What will be the most economical load for a 16- and 24×30-in. engine running at 125 R.P.M. on 150 lbs. per square inch absolute initial pressure and atmospheric backpressure? What will be the economy at this load?

Prob. 6. What will be the release and receiver pressures for the above engine if the receiver has a volume of 15 cu.ft.?

Prob. 7. Find the cut-offs and cylinder ratio for equal work division and complete expansion when initial pressure is 150 lbs. per square inch absolute and back pressure is 10 lbs. per square inch absolute, receiver four H.P. volumes.

Prob. 8. Will a 14- and 20×20-in. engine, with a receiver volume equal to 5 times the H.P. cylinder and running on $\frac{2}{5}$ cut-off on the high-pressure cylinder and $\frac{3}{5}$ cut-off on the low, with steam pressure of 100 lbs. per square inch gage and back pressure of 5 lbs. per square inch absolute, have complete expansion and equal work distribution? If not, what changes must be made in the cut-off or initial pressure?

Prob. 9. What must be the size of an engine to give 200 I.H.P. at 150 R.P.M. on an initial steam pressure of 150 lbs. per square inch absolute, and 10 lbs. per square inch absolute back pressure, if the piston speed is limited to 450 ft. per minute and complete expansion and equal work distribution is required? Receiver is to be 6 times the volume of high-pressure cylinder and H.P. stroke equal to diameter.

12. Compound Engine with Finite Receiver. Exponential Law, No Clearance. Cycle VIII. General Relations between Pressures, Dimensions, and Work, when High Pressure Exhaust and Low-pressure Admission are Independent. The diagram Fig. 84 may be used to represent this cycle, as well as cycle VII, by conceiving a slight change in the slope of the expansion and receiver lines. Using the same symbols as those of the preceding section, and the expression for work as found in Section 7, Chapter I,

$$W_H = P_b V_b + \frac{P_b V_b}{s-1}\left[1-\left(\frac{V_b}{V_e}\right)^{s-1}\right] - \frac{P_n O}{s-1}\left[1-\left(\frac{O}{V_c+O}\right)^{s-1}\right]$$

$$= 144 D_H \left\{ (\text{in.pr.}) Z_H + \frac{(\text{in.pr.}) Z_H}{s-1}\left(1-Z_H{}^{s-1}\right) - \frac{(\text{rec.pr.})_1 y}{s-1}\left[1-\left(\frac{y}{y+1}\right)^{s-1}\right]\right\},$$

but

$$(\text{rec.pr.})_1 = P_n = P_b\left(\frac{V_b}{V_e}\right)^s\left(\frac{V_e+O}{O}\right)^s,$$

or

$$(\text{rec.pr.})_1 = (\text{in.pr.})\left(\frac{Z_H}{R_C Z_L}\right)^s\left(\frac{y+Z_L R_C}{y}\right)^s$$

$$= (\text{in.pr.})\left(\frac{Z_H}{y}\right)^s\left(\frac{y}{R_C Z_L}+1\right)^s,$$

and the last term in the equation for W_H within the bracket may therefore be written

$$\frac{(\text{in.pr.}) Z_H}{s-1}\left(\frac{y}{Z_H}\right)\left(\frac{Z_H}{y}\right)^s\left(\frac{y}{R_C Z_L}+1\right)^s\left[1-\left(\frac{y}{y+1}\right)^{s-1}\right],$$

or

$$\frac{(\text{in.pr.})\, Z_H}{s-1}\left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_C Z_L}+1\right)^s\left[1-\left(\frac{y}{y+1}\right)^{s-1}\right],$$

and hence by simplifying the first two terms also,

$$W_H = 144(\text{in.pr.})\frac{Z_H D_H}{s-1}\left\{s - Z_H{}^{s-1} - \left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_C Z_L}+1\right)^s\left[1-\left(\frac{y}{y+1}\right)^{s-1}\right]\right\}. \quad (383)$$

Work of the low-pressure cylinder may be expressed in terms of pressure and volumes at N, E, and G, but it is convenient to use instead of the pressure at N or at E, its equivalent in terms of the point B. The pressure at N is

$$(\text{rec.pr.})_1 = (\text{in.pr.})\left(\frac{Z_H}{y}\right)^s\left(\frac{y}{R_C Z_L}+1\right)^s$$

and when multiplied by the receiver volume yD_H, it becomes

$$(\text{rec.pr.})_1 y D_H = (\text{in.pr.}) D_H Z_H \left(\frac{y}{Z_H}\right)\left(\frac{Z_H}{y}\right)^s\left(\frac{y}{R_C Z_L}+1\right)^s$$
$$= (\text{in.pr.}) D_H Z_H \left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_C Z_L}+1\right)^s$$

At E the product of pressure and volume is

$$(\text{rec.pr.})_2 \times Z_L D_L = (\text{in.pr.}) Z_H D_H \left(\frac{Z_H}{R_C Z_L}\right)^{s-1}.$$

Using these quantities, the following equation gives the work of the low-pressure cylinder:

$$W_L = 144(\text{in.pr.})\frac{Z_H D_H}{s-1}\left\{\left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_C Z_L}+1\right)^s\left[1-\left(\frac{y}{y+R_C Z_L}\right)^{s-1}\right]\right.$$
$$\left. + \left(\frac{Z_H}{Z_L R_C}\right)^{s-1}\left[1 - Z_L{}^{s-1}\right]\right\} - 144(\text{bk.pr.})D_L, \quad . \quad (384)$$

and the total work is, by adding (W_H) and (W_L),

$$W = 144(\text{in.pr.})\frac{Z_H D_H}{s-1}\left\{s - Z_H{}^{s-1} + \left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_C Z_L}+1\right)^s\left[\left(\frac{y}{y+1}\right)^{s-1}\right.\right.$$
$$\left.\left. - \left(\frac{y}{y+R_C Z_L}\right)^{s-1}\right] + \left(\frac{Z_H}{Z_L R_C}\right)^{s-1}\left(1 - Z_L{}^{s-1}\right)\right\} - 144(\text{bk.pr.})D_L, \quad . \quad . \quad (385)$$

This Eq. (385) is the general expression for work of the zero clearance compound engine with exponential expansion, no clearance, and finite receiver. From this the following expressions are derived:

(m.e.p. ref. to L.P.)

$$= \frac{(\text{in.pr.})}{s-1}\frac{Z_H}{R_C}\left\{s - Z_H{}^{s-1} + \left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_C Z_L}+1\right)^s\left[\left(\frac{y}{y+1}\right)^{s-1}\right.\right.$$
$$\left.\left. - \left(\frac{y}{y+R_C Z_L}\right)^{s-1}\right] + \left(\frac{Z_H}{Z_L R_C}\right)^{s-1}\left(1 - Z_L{}^{s-1}\right)\right\} - (\text{bk.pr.}). \quad . \quad (386)$$

Work per cu.ft. supplied is

$$=144\,\frac{(\text{in.pr.})}{s-1}\left\{s-Z_H{}^{s-1}+\left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_CZ_L}+1\right)^s\left[\left(\frac{y}{y+1}\right)^{s-1}-\left(\frac{y}{y+R_CZ_L}\right)^{s-1}\right]+\left(\frac{Z_H}{Z_LR_C}\right)^{s-1}(1-Z_L{}^{s-1})\right\}-144(\text{bk.pr.})\frac{R_C}{Z_H}. \quad (387)$$

Cu.ft. supplied per hr. per I.H.P.

$$=\frac{13{,}750}{(\text{m.e.p. ref. to L. P.})}\frac{Z_H}{R_C}; \quad (388)$$

$$(\text{rec.pr.})_2=(\text{in.pr.})\left(\frac{Z_H}{Z_LR_C}\right)^s; \quad (389)$$

$$(\text{rec.pr.})_1=(\text{in.pr.})\left(\frac{Z_H}{Z_LR_C}\right)^s\left(1+\frac{R_CZ_L}{y}\right)^s; \quad (390)$$

$$(\text{rel.pr.})_H=(\text{in.pr.})Z_H{}^s; \quad (391)$$

$$(\text{rel.pr.})_L=(\text{rec.pr.})_2Z_L{}^s=(\text{in.pr.})\left(\frac{Z_H}{R_C}\right)^s. \quad (392)$$

If work is equally divided between the cylinders, W_H, Eq. (383), and W_L, Eq. (384), will become equal, hence

$$s-Z_H{}^{s-1}-\left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_CZ_L}+1\right)^s\left[1-\left(\frac{y}{y+1}\right)^{s-1}\right]=\left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_CZ_L}+1\right)^s\left[1-\left(\frac{y}{y+R_CZ_L}\right)^{s-1}\right]\left(\frac{Z_H}{R_CZ_L}\right)^{s-1}(1-Z_L{}^{s-1})-\frac{(\text{bk.pr.})}{(\text{in.pr.})}\frac{R_C}{Z_H}(s-1). \quad (393)$$

This equation shows conditions to be fulfilled in order that an equal division of work may be obtained. It does not yield directly to a general solution.

When *expansion* is *complete* in *both cylinders*,

$$Z_L=\frac{1}{R_C} \quad \text{and} \quad \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)=\left(\frac{Z_H}{R_C}\right)^s.$$

Introducing these values in the general expression Eq. (385) for work of this cycle, it may be reduced to the following:

$$W=144(\text{in.pr.})Z_HD_H\frac{s}{s-1}\left[1-\left(\frac{Z_H}{R_C}\right)^{s-1}\right]. \quad (394)$$

From which are obtained

$$(\text{m.e.p. ref. to L. P.})=(\text{in.pr.})\frac{Z_H}{R_C}\frac{s}{s-1}\left[1-\left(\frac{Z_H}{R_C}\right)^{s-1}\right]. \quad (395)$$

$$\text{Work per cu.ft. supplied}=144(\text{in.pr.}\ \frac{s}{s-1}\left[1-\left(\frac{Z_H}{R_C}\right)^{s-1}\right] \quad (396)$$

$$\text{Cu.ft. supplied per hr. per I.H.P.} = \frac{13{,}750}{(\text{m.e.p. ref. to L. P.})}\left(\frac{Z_H}{R_C}\right)$$

$$= \frac{13{,}750}{(\text{m.e.p. ref. to L. P.})}\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}. \quad (397)$$

If work is equally divided and complete expansion is maintained in both cylinders Eq. (381) becomes

$$s - {Z_H}^{s-1} - {Z_H}^{s-1}\frac{(y+1)^s}{y^{s-1}}\left[1-\left(\frac{y}{y+1}\right)^{s-1}\right]$$

$$-{Z_H}^{s-1}\frac{(y+1)^s}{y^{s-1}}\left[1-\left(\frac{y}{y+1}\right)^{s-1}\right] - {Z_H}^{s-1} + \frac{1}{{R_V}^{s-1}} + \frac{s-1}{{R_V}^{s-1}} = 0$$

which may be simplified to the form,

$$2{Z_H}^{s-1}\left\{1+\frac{(y+1)^s}{y^{s-1}}\left[1-\left(\frac{y}{y+1}\right)^{s-1}\right]\right\} = \frac{s}{{R_V}^s} - s. \quad . \quad . \quad . \quad (398)$$

where R_V is the ratio of maximum low-pressure volume, to the high-pressure volume at cut-off,

$$R_V = \frac{R_C}{Z_H},$$

hence

$$Z_H = \frac{R_C}{R_V},$$

and the value of R_V may be found from original data,

$$R_V = \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{s}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (399)$$

Eq. (398) may easily be solved for Z_H, from which the required cylinder ratio may be found by,

$$R_C = Z_H R_V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (400)$$

This is the cylinder ratio which gives equal work in the two cylinders and complete expansion in both, when used with the value found for the high pressure cut-off Z_H, the assumed initial and back pressures, and the assumed ratio, y, of receiver volume to high-pressure displacement.

Example. Find (*a*) the horse-power, (*b*) steam used per hour, (*c*) the release nd receiver pressures of a 12- and 18×24-in. engine, with a receiver twice as large as ne low-pressure cylinder when the initial pressure is 150 lbs. per square inch absolute, ack pressure 10 lbs. per square inch absolute, speed 125 R.P.M., and cut-offs $\frac{1}{2}$ in the igh and such a value in the low as to give complete expansion. Exponent for ex-ansion curve = 1.4.

(*a*) From Eq. (386)

$$\text{m.e.p.}) = \frac{(\text{in.pr.})}{s-1}\frac{Z_H}{R_C}\left\{ s - {Z_H}^{s-1} + \left(\frac{Z_H}{y}\right)^{s-1}\left(\frac{y}{R_C Z_L}+1\right)^{s}\left[\left(\frac{y}{y+1}\right)^{s-1} - \left(\frac{y}{y+R_C Z_L}\right)^{s-1}\right] + \left(\frac{Z_H}{Z_L R_C}\right)^{s-1}\left([1-{Z_L}^{s-1}]\right)\right\} - \text{bk.pr}$$

vhich, on substituting above values, gives

$$\frac{.50}{.4}\times\frac{.5}{2.25}\left\{1.4-.5^{.4}+\left(\frac{.5}{4.5}\right)^{.4}\left(\frac{4.5}{2.25\times\frac{1}{2.25}}+1\right)^{1.4}\left[\left(\frac{4.5}{5.5}\right)^{.4}-\left(\frac{4.5}{4.5+2.25\times\frac{1}{2.25}}\right)^{.4}\right] + \left(\frac{.5}{\frac{1}{2.25}\times 2.25}\right)^{.4}\left[1-\left(\frac{1}{2.25}\right)^{.4}\right]\right\} - 10$$

or

$$(\text{m.e.p.}) = 57.5 \text{ lbs. per sq.in.},$$

ience

$$\text{I.H.P.} = 221.$$

(*b*) From Eq. (388)

$$\text{Cubic feet of steam per hour per horse-power} = \frac{13{,}750}{\text{m.e.p.}}\frac{Z_H}{R_C},$$

$$= \frac{13{,}750}{57.5}\times\frac{.5}{2.25} = 53.2 \text{ cu.ft.},$$

ience total pounds per hour will be

$$53.2\times 221\times .332 = 3910.$$

From Eqs. (389) to (392):

$$(\text{rec.pr.})_1 = (\text{in.pr.})\left(\frac{Z_H}{Z_L R_C}\right)^{s}\left(1+\frac{R_C Z_L}{y}\right)^{s},$$

$$(\text{rec.pr.})_2 = (\text{in.pr.})\left(\frac{Z_H}{Z_L R_C}\right)^{s},$$

$$(\text{rel.pr.})_H = (\text{in.pr.}){Z_H}^{s},$$

$$(\text{rel.pr.})_L = (\text{rec.pr.})_2 {Z_L}^{s},$$

These, on substitution of the proper numerical values, become:

$$(\text{rec.pr.})_1 = 150 \times \left(\frac{.5}{1}\right)^{1.4} \left(\times 1 + \frac{1}{4.5}\right)^{1.4} = 75 \text{ lbs. per sq. inch,}$$

$$(\text{rec.pr.})_2 = 150 \times (.5)^{1.4} = 57 \text{ lbs., "}$$

$$(\text{rel.pr.})_H = 150 \times (.5)^{1.4} = 57 \text{ lbs, "}$$

$$(\text{rel.pr.})_L = 57 \times \left(\frac{1}{2.25}\right)^{1.4} = 32.1 \text{ lbs. "}$$

NOTE: In all the following problems clearance is assumed to be zero.

Prob. 1. A $12 \times 18 \times 24$ in. engine is running on superheated steam of 150 lbs. per square inch absolute pressure, and exhausts to the atmosphere. If the speed is 100 R.P.M., high-pressure cut-off $\frac{1}{3}$, low pressure cut-off $\frac{1}{2}$, and receiver volume 10 cu.ft., what horse-power will be developed and what steam used per hour?

Prob. 2. What would be the effect on the power and the economy of (*a*) changing to wet steam in the above? (*b*) to compressed air?

Prob. 3. What would be the receiver and the release pressures for each case?

Prob. 4. Will there be equal work distribution between the two cylinders?

Prob. 5. It is desired to obtain complete expansion in a $14 \times 22 \times 36$-in. engine running on fluid which gives a value for s of 1.2. Initial pressure is 100 lbs. per square inch gage, and back pressure 5 lbs. per square inch absolute. What must be the cut-offs and what power will be developed at 500 ft. piston speed? Receiver = 3 $\times$H.P. volume.

Prob. 6. How large must the receiver be for the above engine in order that the pressure in it shall not fluctuate more than 5 lbs. per sq. inch?

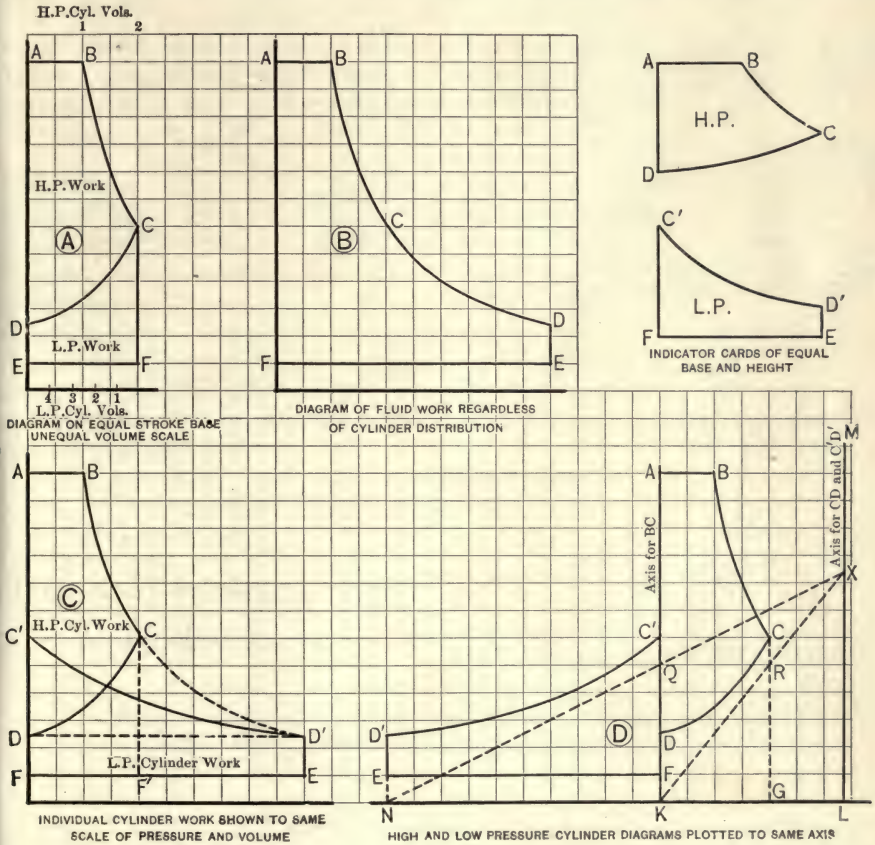
Prob. 7. An engine is to run on steam which will give a value of $s = 1.1$, and to develope 500 horse-power at 100 R.P.M. Piston speed is not to exceed 500 ft. per minute. Steam pressure, 150 lbs. per square inch absolute, back pressure, 5 lbs. per square inch absolute. Complete expansion and equal work distribution, for this load are to be accomplished. What will be the cylinder sizes and the high-pressure cut-off if the receiver is to be 3 times the high-pressure cylinder volume?

Prob. 8. What will be the steam used per hour by the engine of Prob. 7, and what will be the variation in the receiver pressure?

Prob. 9. If the high-pressure cut-off were halved, how would the power and economy be affected?

13. Compound Engine without Receiver, Logarithmic Law. No Clearance, Cycle IX. General Relations between Dimensions and Work when High-Pressure Exhaust and Low-Pressure Admission are Coincident. Such a peculiar case as this admits of but little modification of the cycle compared with the receiver cases, because the low-pressure expansion is necessarily a direct continuation of the high pressure without any possible break. There can be no over-expansion in the high nor can expansion there be incomplete, as there is, properly speaking, no back pressure with which to compare the high-pressure cylinder terminal pressure. There may, however, be over and incomplete expansion in the low-pressure cylinder. It might appear that the high-pressure cylinder negative work was equal to the low-pressure admission work, as each is represented by the area below *DC*, Fig. 87*A*, but this is not the case, since

ne diagram is drawn to two different scales of volumes, showing the pressure-troke relation between high and low. This is apparent from the diagram, Fig. 7C showing fluid volumes in each cylinder to a single scale on which $ABCD$ the work done in the high-pressure cylinder, $ABD'EF$ the whole work, whence $CD'EF$ is the part done in the low-pressure cylinder. There is, of course, o low-pressure cut-off or even admission as ordinarily considered. *The cycle, o far as the work to be done is concerned, is the same as for a simple engine,*



IG. 87.—Work of Expansion in the No-receiver Compound Engine, Zero Clearance, Cycle IX, Logarithmic Expansion, Cycle X, Exponential Coincident Piston Movement.

nd the only reason for introducing formulas for overall work, work per cubic eet supplied, (m.e.p. referred to low), and fluid consumption, is to put them into orm for immediate substitution of dimensional relations. Because of the absence f cut-off in the low, the distribution of work between high and low will epend *solely on the cylinder ratio and high-pressure cut-off*, for, the earlier the igh-pressure cut-off, and the larger the high-pressure cylinder, the greater he fraction of the total work that will be done there, as there is only a fixed mount available, and the less there will be left to be done in the low

The diagrams of the two cylinders are plotted to combined axes in Fig. 87*D*. The points Q and R at equal heights. KN is the L. P. displacement, and KG that of the H.P. It has been shown in Section 8, that the expansion lines CD and $C'D'$ may be plotted to the axes LN and LXM, the point X being the intersection of NQ and KR extended, and that the distance

$$\overline{KL}=\frac{D_L D_H}{D_L-D_H}=D_H\frac{R_C}{R_C-1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (401)$$

and

$$\overline{GL}=\frac{D_L D_H}{D_L-D_H}-D_H=\frac{D_H{}^2}{D_L-D_H}=D_H\frac{1}{R_C-1}; \quad . \quad . \quad . \quad (402)$$

also

$$\overline{NL}=\frac{D_L D_H}{D_L-D_H}+D_L=\frac{D_L{}^2}{D_L-D_H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (403)$$

Hence the work area under CD is

$$W_{cd}=\overline{GL}\times P_c \log_e\frac{\overline{KL}}{\overline{GL}}=144(\text{rel.pr.})_H\frac{D_H{}^2}{D_L-D_H}\log_e\frac{D_L}{D_H}$$

but

$$(\text{rel.pr.})_H=(\text{in.pr.})Z_H,$$

hence

$$W_H=144(\text{in.pr.})Z_H D_H\left\{1+\log_e\frac{1}{Z_H}-\frac{1}{R_C-1}\log_e R_C\right\}. \quad . \quad (404)$$

Again the work area under $C'D'$ is

$$W_{c'd'}=\overline{KL}\times P_c \log_e\frac{\overline{NL}}{\overline{KL}}=144(\text{rel.pr.})_H\frac{D_L D_H}{D_L-D_H}\log_e\frac{D_L}{D_H},$$

hence

$$W_L=144(\text{in.pr.})Z_H D_H\left(\frac{R_C}{R_C-1}\right)\log_e R_c-144(\text{bk.pr.})D_L, \quad . \quad (405)$$

and the total work,

$$W=144(\text{in.pr.})Z_H D_H\left\{1+\log_e\frac{1}{Z_H}-\frac{1}{R_C-1}\log_e R_C+\frac{R_C}{R_C-1}\log_e R_C\right\}-144(\text{bk.pr.})D_L$$

$$=144(\text{in.pr.})Z_H D_H\left\{1+\log_e\frac{1}{Z_H}+\left(\frac{R_C}{R_C-1}-\frac{1}{R_C-1}\right)\log_e R_C\right\}-144(\text{bk.pr.})D_L.$$

But

$$\left(\frac{R_C}{R_C-1}-\frac{1}{R_C-1}\right)=1,$$

and

$$\log_e\frac{1}{Z_H}+\log_e R_C=\log_e\frac{R_C}{Z_H}=\log_e R_V,$$

so that

$$W=144(\text{in.pr.})Z_HD_H\left(1+\log_e\frac{R_C}{Z_H}\right)-144(\text{bk.pr.})D_L, \quad . \; . \quad (406)$$

which shows by its similarity to the work of the simple engine that, as before stated, the total work is the same for this cycle as if the entire expansion were made to take place in a single cylinder.

This same result could have been attained in another way sufficiently interesting to warrant setting it down. Since the low-pressure work is represented truly to scale by $\overline{C'D'EF}$, Fig. 87*C*, the mean effective pressure of the low-pressure cylinder is given by the area divided by V_e. By contracting all volumes proportionately, $C'D'$ takes the position CD' and $C'F$ the position CF', hence

$$\frac{\text{area } CD'EF}{V_e-V_{f'}}-P_e$$

represents the mean effective pressure in the low-pressure cylinder just as truly. Therefore,

$$\text{L.P. cylinder work}=\left(\frac{\text{area } CD'EF}{V_e-V_{f'}}-P_e\right)V_e$$

$$=P_cV_c\left(\frac{V_e}{V_e-V_{f'}}\right)\log\frac{V_e}{V_c}-P_eV_c.$$

As the high-pressure work is (total − low),

$$\text{H.P. cylinder work}=P_bV_b\left(1+\log_e\frac{V_e}{V_b}\right)-P_eV_e-P_cV_c\left(\frac{V_e}{V_e-V_{f'}}\right)\log_e\frac{V_e}{V_c}+P_eV_e$$

$$=P_bV_b\left[1+\log_e\frac{V_e}{V_b}-\left(\frac{\frac{V_e}{V_c}}{\frac{V_e}{V_c}-1}\right)\log_e\frac{V_e}{V_c}\right].$$

Introducing symbols

$$\text{L.P. cylinder work}=144(\text{in.pr.})Z_HD_H\left(\frac{R_C}{R_C-1}\right)\log_eR_C-144(\text{bk.pr.})D_L. \quad (407)$$

$$\text{H.P. cylinder work}=144(\text{in.pr.})Z_HD_H\left[1+\log_e\frac{R_C}{Z_H}-\frac{R_C}{R_C-1}\log_eR_C\right]$$

$$=144(\text{in.pr.})Z_HD_H\left[1+\log_e\frac{1}{Z_H}-\frac{1}{R_C-1}\log_eR_C\right]. \; . \quad (408)$$

which check with Eqs. (404) and (405).

Dividing the total work by the low-pressure cylinder volume and the high-pressure admission volume in turn,

$$\left.\begin{aligned}(\text{m.e.p. ref. to L.P.}) &= (\text{in.pr.})\frac{Z_H}{R_C}\left(1+\log_e\frac{R_C}{Z_H}\right)-(\text{bk.pr.}) \quad (a)\\ &= (\text{in.pr.})\frac{1}{R_H R_C}[1+\log_e(R_H R_C)]-(\text{bk.pr.}) \quad (b)\end{aligned}\right\} \quad (409)$$

$$\left.\begin{aligned}\text{Work per cu.ft. supplied} &= 144(\text{in.pr.})Z_H\left(1+\log_e\frac{R_C}{Z_H}\right)-(\text{bk.pr.})R_C \quad (a)\\ &= 144(\text{in.pr.})\frac{1}{R_H}[1+\log_e(R_H R_C)]-(\text{bk.pr.})R_C \quad (b)\end{aligned}\right\} \quad (410)$$

$$\left.\begin{aligned}\text{Cu.ft. supplied per hr. per I.H.P.} &= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\times\frac{Z_H}{R_C} \quad (a)\\ &= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\frac{1}{R_H R_C} \quad (b)\end{aligned}\right\} \quad (411)$$

For equal division of work there can obviously be only one setting of the high-pressure cut-off for a given cylinder ratio and any change of load to be met by a change of initial pressure or of high-pressure cut-off will necessarily unbalance the work. Equating the high-pressure and low-pressure work expressions, Eqs. (404) and (405),

$$1+\log_e\frac{1}{Z_H}-\frac{1}{R_C-1}\log_e R_C=\frac{R_C}{R_C-1}\log_e R_C-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)\frac{R_C}{Z_H},$$

or

$$1+\log_e\frac{1}{Z_H}+\frac{(\text{bk.pr.})}{(\text{in.pr.})}\frac{R_C}{Z_H}-\frac{R_C+1}{R_C-1}\log_e R_C=0.$$

Another relation exists between Z_H and R_C, namely, that

$$\frac{1}{Z_H}=\frac{R_V}{R_C},$$

where R_V is the ratio of volumetric expansion. Then

$$1+\log_e\frac{R_V}{R_C}-\frac{R_C+1}{R_C-1}\log_e R_C+\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)R_V=0,$$

but

$$\log_e\frac{R_V}{R_C}-\frac{R_C+1}{R_C-1}\log_e R_C=\log_e\left[\left(\frac{R_V}{R_C}\right)\left(\frac{1}{R_C^{\frac{R_C+1}{R_C-1}}}\right)\right]=\log_e\frac{R_V}{R_C^{\frac{2R_C}{R_C-1}}},$$

hence

$$\log_e\frac{R_C^{\frac{2R_C}{R_C-1}}}{R_V}=1+\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)R_V. \quad (412)$$

With this formula it is possible to find the necessary ratio of cylinder displacements for given initial and back pressures and for given ratio of expansion R_V.

For convenience in solving this, a curve is given in Fig. 88 to find value of R_C when $R_C^{\frac{2R_C}{R_C-1}}$ has been found.

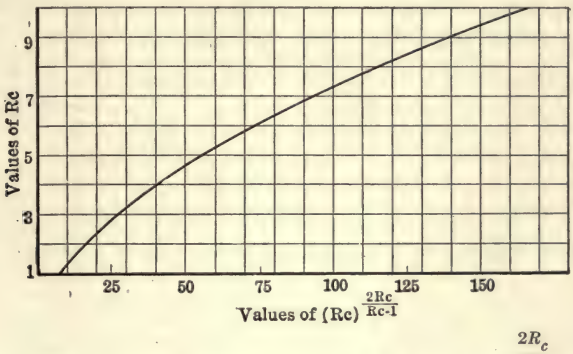


FIG. 88.—Curve to Show Relation between Values of R_E and $(R_E)^{\frac{2R_c}{R_c-1}}$ for Use in Solving Eq. (412), Giving Cylinder Ratio in Terms of Ratio of Expansion for the No-receiver Compound Engine without Clearance.

The complete expansion case of this cycle results from the condition

$$P_d = P_e \quad \text{or} \quad (\text{rel.pr.})_L = (\text{bk.pr.}) \quad \text{or} \quad R_V = \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right),$$

which when applied to Fig. 87, transforms the diagrams to the form Fig. 89.

It also follows that

$$(\text{bk.pr.})D_L = (\text{in.pr.})Z_H D_H$$

and

$$\frac{R_C}{Z_H} = \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)$$

These conditions will, of course, reduce the total work Eq. (406) to the common value for all cycles with logarithmic expansion and likewise those for mean effective pressure, work per cubic foot supplied, and consumption. For the equal division of work under this condition, Eq. (412), becomes

$$R_C^{\frac{2R_C}{R_C-1}} = 7.39R, \quad \ldots \ldots \ldots \quad (413)$$

since $\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)R_V = 1$ and R may represent ratio of expansion or ratio of

initial to back pressures, these being equal. Fig. 90 gives a curve showing the relation between cylinder ratio and ratio of expansion established by the above condition.

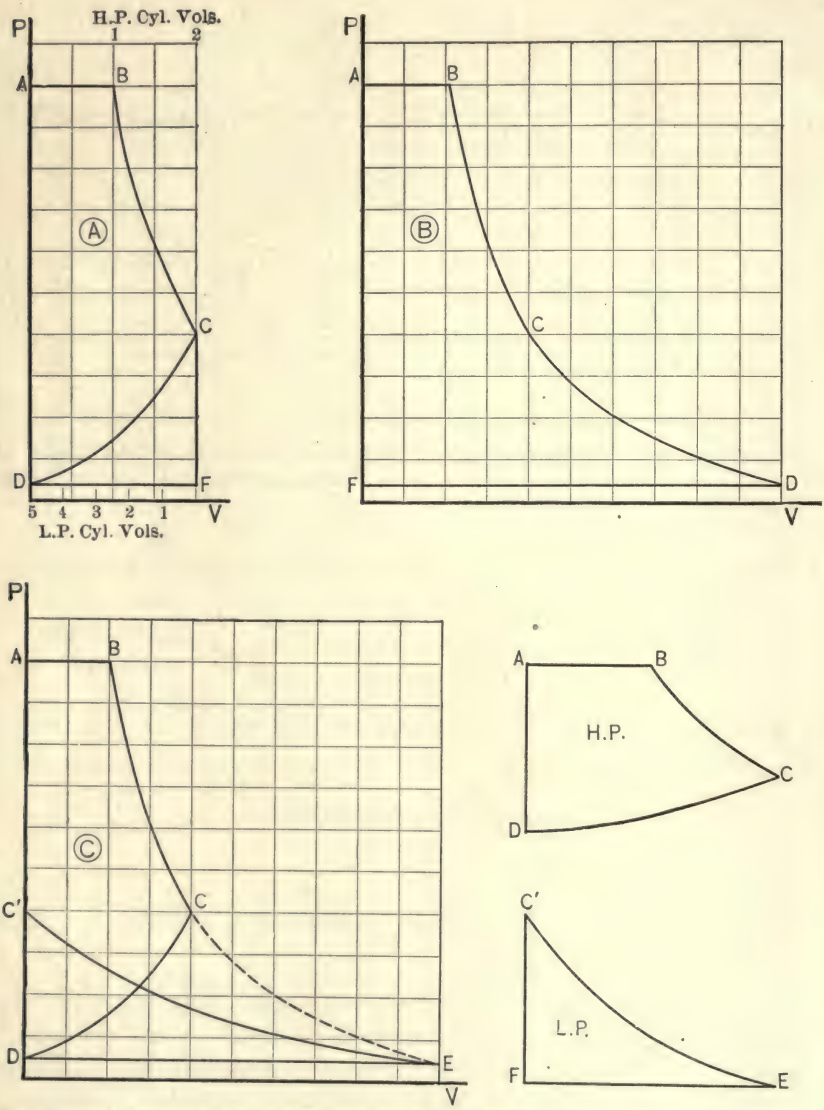


Fig. 89.—Special Case of Cycles IX and X. Complete Expansion in both Cylinders of the No-Receiver Compound Engine, Zero Clearnace.

Example 1. Method of calculating Diagram, Fig. 87.

A. As described in the text this diagram is drawn to two-volume scales, so that there may be two volumes for one point.

Assumed data:

$P_a = P_b = 120$ lbs. per sq.in. abs. $\quad V_a = V_c = V_d = V_g = V_f = 0$ cu.ft.
$P_e = P_f = 10$ lbs. per sq.in. abs. $\quad V_b = 1$ cu.ft.
$V_c = 2$ cu.ft.
$V_d = V_e = 5$ cu.ft.

To locate point C:

$$P_c = \frac{P_b V_b}{V_c} = \frac{120 \times 1}{2} = 60 \text{ lbs. per sq.in.}$$

To locate point D:

$$P_d = \frac{P_c V_c}{V_d} = \frac{60 \times 2}{5} = 24 \text{ lbs. per sq.in.}$$

To locate intermediate points from C to D. The volume at any intermediate point is (the volume of low-pressure cylinder up to that point) + (volume of high-

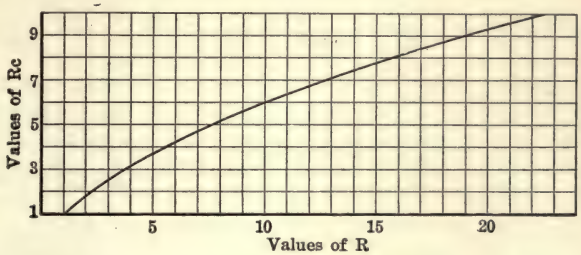


FIG. 90.—Curve to Show Relation between Values of R_C, the Cylinder Ratio, and R the Ratio of Initial to Back Pressure for Complete Expansion in the No-receiver Compound Engine without Clearance (Eq. (413).)

pressure cylinder from that point to end of stroke), e.g., at $\frac{3}{4}$ stroke the volume in low, is $.75 \times 5$, and the volume in the high is $.25 \times 2$, or total 4.25, and the pressure at that point is found by the PV relation as above.

B. Assumed data:

$P_a = P_b = 120$ lbs. per sq.in. abs. $\quad V_a = V_f = 0$ cu.ft.
$P_e = P_f = 10$ lbs. per sq.in. abs. $\quad V_d = V_e = 5$ cu.ft.
$V_b = 1$ cu.ft.

To locate point D:

$$P_d = \frac{P_b V_b}{V_d} = \frac{120 \times 1}{5} = 24 \text{ lbs. per sq.in.}$$

Intermediate points from B to D found by assuming volumes and computing pressures from the PV relation as above.

C. Figure $ABCD$ constructed as in **A**. Figure $C'D'EF$ is figure $CDEF$ of **A** to the same pressure scale but to a volume scale 2.5 times as large.

D. Figures constructed as in **C**.

To draw indicator cards. The volume and pressure scales are chosen and from diagram A, a distance $\overline{AB}$ is laid off to the volume scale, $\overline{AD}$ is then laid off equal to $\overline{AD}$ of diagram A to the pressure scale. Point C is located to these scales and joined to B and D by drawing curves through the intermediate points plotted from the PV diagram to the scales of the card. For the low-pressure card EF is laid off to the volume scale, and FC' and ED' to pressure scale. C' and D' are then joined in same manner as C and D for high-pressure card.

Example 2. Find (*a*) the horse-power, and (*b*) steam used per hour for a $12\times18\times24$ in. engine with no clearance when initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., and cut-off in the high-pressure cylinder is $\frac{1}{2}$, there being no receiver.

(*a*) From Eq. (409) we have

$$(\text{m.e.p.})=(\text{in.pr.})\frac{1}{R_H R_C}[1+\log_e(R_H R_C)]-(\text{bk.pr.}),$$

$$=150\times\frac{1}{2\times2.25}\times(1+.8)-10=50 \text{ lbs. sq.in.}$$

hence

$$\text{I.H.P.}=192.$$

(*b*) From Eq. (411) we have

$$\text{Cubic feet of steam per I.H.P. per hour}=\frac{13{,}750}{(\text{m.e.p.})}\times\frac{1}{R_H R_C},$$

$$=\frac{13{,}750}{50}\times\frac{1}{2\times2.25}=61.2,$$

hence the weight of steam used per hour will be

$$61.2\times.332\times192=3890 \text{ pounds.}$$

Example 3. What will be the cylinder ratio and the high-pressure cut-off to give equal work distribution for a ratio of expansion of 6, an initial pressure of 150 lbs. per square inch absolute and back pressure of 10 lbs. per square inch absolute?

Ratio of back to initial pressures is .067 and

$$R_V=6,$$

hence from Eq. (412)

$$\log_e\frac{R_C^{\frac{2R_C}{R_C-1}}}{6}=1.40,$$

or

$$R_C^{\frac{2R_C}{R_C-1}}=24.36,$$

and from Fig. 88

$$R_C=2.8.$$

From the relation $Z_H=\frac{R_C}{R_V}=$ high-pressure cut off $=\frac{2.8}{6}=.446.$

Prob. 1. A compound locomotive has no receiver and runs on an initial pressure of 175 lbs. per square inch gage and atmospheric exhaust. The cylinders are 18 and 30×42 in. The steam pressure may be varied, as may also the cut-off to a limited degree. For a speed of 200 R.P.M. and a constant cut-off of $\frac{3}{4}$, find how the power will vary with initial pressure and for constant initial pressure equal to boiler pressure show how the power at the same speed will vary from $\frac{1}{2}$ cut-off to full stroke.

Prob. 2. Show how the steam used per horse-power hour will vary in above problem.

NOTE: δ for 175 lbs. = .416.

Prob. 3. With the cut-off at $\frac{3}{4}$, what should the initial pressure be to give equal work distribution?

Prob. 4. With full boiler pressure and $\frac{3}{4}$ cut-off what would be terminal pressure in the low-pressure cylinder?

Prob. 5. What must be size of cylinders for a tandem compound engine with negligable receiver volume to run at 125 R.P.M. with complete expansion and equal work distribution on an initial pressure of 125 lbs. per square inch gage and a back pressure of 5 lbs. per square inch absolute, when carrying a load of 500 horse-power, the piston speed to be less than 500 ft. per minute?

Prob. 6. What will be the steam used by the above engine in pounds per hour?

NOTE: δ for 125 lbs. = .311.

Prob. 7. A builder gives following data for a tandem compound steam engine. Check the horse-power and see if the work is equally divided at the rated load. Cylinders 10 ins. and 17½×15 ins., initial pressure 125 lbs., speed 250 R.P.M., horse-power 155. Neglect the receiver volume.

Prob. 8. Another manufacturer gives for his engine the following, check this: Cylinders 20 and 32×18 ins., initial pressure 100 lbs., atmospheric exhaust, speed 200 R.P.M., horse-power 400. Neglect the receiver volume.

14. Compound Engine without Receiver, Exponential Law, Cycle X. General Relations between Dimensions and Work when High-pressure Exhaust and Low-pressure Admission are Coincident. Referring to Fig. 87*D* it is desirable first to evaluate the work areas *CDKG* and *C'D'NK*. As before,

$$\overline{GL} = D_H \frac{1}{R_C - 1} \quad \text{and} \quad \overline{KL} = D_H \frac{R_C}{R_C - 1},$$

hence

$$W_{cd} = \frac{P_c \times \overline{GL}}{s-1}\left[1 - \left(\frac{\overline{GL}}{\overline{KL}}\right)^{s-1}\right] = 144 \frac{(\text{rel.pr.})_H}{s-1} \frac{D_H}{R_C - 1}\left[1 - \left(\frac{1}{R_C}\right)^{s-1}\right],$$

but

$$(\text{rel.pr.})_H = (\text{in.pr.}) Z_H{}^s,$$

so that

$$W_H = 144(\text{in.pr.}) \frac{D_H Z_H}{s-1}\left[s - Z_H{}^{s-1} - Z_H{}^{s-1}\left(\frac{1 - \left(\frac{1}{R_C}\right)^{s-1}}{R_C - 1}\right)\right], \quad . \quad . \quad . \quad (414)$$

$$W_{c'd'} = \frac{P_c \times \overline{KL}}{s-1}\left[1 - \left(\frac{\overline{KL}}{\overline{NL}}\right)^{s-1}\right] = 144 \frac{(\text{rel.pr.})_H}{s-1} \frac{D_H R_C}{R_C - 1}\left[1 - \left(\frac{1}{R_C}\right)^{s-1}\right]$$

whence

$$W_L = 144(\text{in.pr.})\frac{D_H Z_H^s}{s-1} R_C \left[\frac{1-\left(\frac{1}{R_C}\right)^{s-1}}{R_C-1}\right] - 144(\text{bk.pr.})D_L. \quad . \quad . \quad (415)$$

It is to be expected that the sum of high- and low-pressure work will be of a form similar to that which would be obtained if all work were performed in a single cylinder of a displacement equal to that of the low pressure, adding,

$$W = W_H + W_L = 144(\text{in.pr.})\frac{D_H Z_H}{s-1}\left[s - Z_H{}^{s-1} - Z_H{}^{s-1}\left(\frac{1-\left(\frac{1}{R_C}\right)^{s-1}}{R_C-1}\right)\right.$$

$$\left. + Z_H{}^{s-1} R_C \left(\frac{1-\left(\frac{1}{R_C}\right)^{s-1}}{R_C-1}\right)\right] - 144(\text{bk.pr.})D_L$$

$$= 144(\text{in.pr.})\frac{D_H Z_H}{s-1}\left\{s - Z_H{}^{s-1} + Z_H{}^{s-1}\left[1-\left(\frac{1}{R_C}\right)^{s-1}\right]\right\} - 144(\text{bk.pr.})D_L.$$

whence, substituting $\frac{Z_H}{R_C} = \frac{1}{R_V}$

$$W = 144(\text{in.pr.})\frac{D_H Z_H}{s-1}\left[s - \left(\frac{1}{R_V}\right)^{s-1}\right] - 144(\text{bk.pr.})D_L; \quad . \quad . \quad . \quad . \quad . \quad (416)$$

$$(\text{m.e.p. ref. to L.P.}) = \frac{(\text{in.pr.})}{s-1}\frac{Z_H}{R_C}\left[s - \left(\frac{1}{R_V}\right)^{s-1}\right] - (\text{bk.pr.}) \quad . \quad . \quad . \quad . \quad . \quad (417)$$

$$\text{Work per cu.ft. supplied} = 144\frac{(\text{in.pr.})}{s-1}\left[s - \left(\frac{1}{R_V}\right)^{s-1}\right] - 144(\text{bk.pr.})\frac{R_C}{Z_H} \quad . \quad (418)$$

$$\text{Cu.ft. supplied per hr. per I.H.P.} = \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\frac{Z_H}{R_C}$$

$$= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\frac{1}{R_V}. \quad . \quad (419)$$

Conditions for *equal division of work* between high- and low-pressure cylinders may be obtained by equating Eqs. (414) and (415),

$$s - Z_H{}^{s-1} - Z_H{}^{s-1}\left[\frac{1-\left(\frac{1}{R_C}\right)^{s-1}}{R_C-1}\right] = Z_H{}^{s-1} R_C \left[\frac{1-\left(\frac{1}{R_C}\right)^{s-1}}{R_C-1}\right] - \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)\frac{R_C}{Z_H}(s-1).$$

Rearranging

$$\frac{(R_C+1)}{(R_C-1)}Z_H{}^{s-1}\left[1-\left(\frac{1}{R_C}\right)^{s-1}\right]+Z_H{}^{s-1}-s-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)\frac{R_C}{Z_H}(s-1)=0.$$

The last term in the first member of this equation may be expressed as

$$\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)R_V(s-1)$$

and the relation

$$Z_H=\frac{R_C}{R_V}$$

exists between Z_H and R_C, hence, making these substitutions,

$$\frac{R_C+1}{R_C-1}[R_C{}^{s-1}-1]+R_C{}^{s-1}=R_V{}^{s-1}\left[s+\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)R_V(s-1)\right]. \quad . \quad (420)$$

which is not a simple relation, but can be solved by trial.

The assumption of *complete expansion* in the low-pressure cylinder (it is always complete in high, for this cycle), leads to this following relations:

$$\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)=R_V{}^s,$$

hence

$$144(\text{bk.pr.})D_L=144(\text{in.pr.})\frac{R_C}{R_V{}^s}D_H,$$

and from Eq. (414),

$$W_H=144(\text{in.pr.})\frac{D_HZ_H}{s-1}\left[s-\left(\frac{1}{R_V}\right)^{s-1}-\frac{R_C}{Z_H}\frac{s-1}{R_V{}^s},\right]$$

but

$$\frac{R_C}{Z_H}=R_V, \quad \text{and} \quad \frac{R_V}{Z_H}\frac{s-1}{R_C{}^s}=\frac{s-1}{R_V{}^{s-1}}, \quad \text{and,} \quad \frac{1}{R_V}=\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}$$

$$W=144(\text{in.pr.})D_HZ_H\frac{s}{s-1}\left[1-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right]. \quad . \quad . \quad . \quad (421)$$

$$(\text{m.e.p. ref. to L.P.})=\text{in.pr.}\frac{Z_H}{R_C}\frac{s}{s-1}\left[1-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right]. \quad . \quad . \quad (422)$$

The expression for *equality of work* Eq. (420) becomes, for this case of complete expansion,

$$\frac{R_C+1}{R_C-1}(R_C{}^{s-1}-1)+R_C{}^{s-1}=sR_V{}^{s-1}+(s-1), \quad . \ . \ . \quad (423)$$

by which it is not difficult to find the ratio of expansion R_V, which gives equality of work for given values of s, and R_C, the cylinder ratio. Values for R_V for various values of R_C and s are given by the curves of Fig. 91.

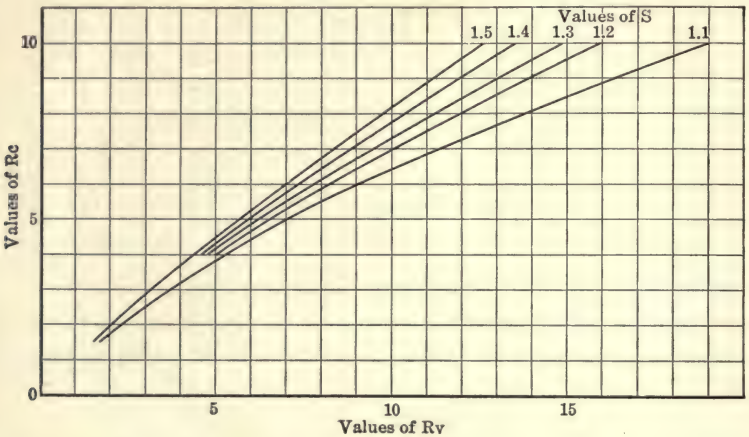


FIG. 91.—Curves to Show Relation between R_C the Cylinder Ratio, and R_V the Ratio of Expansion, for Various Values of (s), Applied to the No-receiver Compound Engine without Clearance, when the Expansion is not Logarithmic.

Example 1. Find (*a*) the horse-power, and (*b*) the steam used per hour for a 12- and 18×24-in. engine with no receiver when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., cut-off in high-pressure cylinder is $\frac{1}{2}$, there being no receiver and steam having expansion, such that $s=1.3$.

From Eq. (417)

$$(\text{m.e.p.})=\frac{(\text{in.pr.})}{s-1}\frac{Z_H}{R_C}\left[s-\left(\frac{1}{R_V}\right)^{s-1}\right]-(\text{bk.pr.}),$$

which, on substituting the above values, becomes

$$\frac{150}{.3}\times\frac{.5}{2.25}\left[1.3-\left(\frac{.1}{4.5}\right)\right]-10=63.3 \text{ lbs. per sq.in.}$$

hence the indicated horse-power = 243.

(*b*) From Eq. (419) the steam used per hour in cu.ft. per horse-power is

$$\frac{13{,}750}{\text{m.e.p.}}\frac{Z_H}{R_C'}$$

which, for the data given above, becomes

$$\frac{13{,}750}{63.3}\times\frac{.5}{2.25}=48.2 \text{ cu.ft.}$$

or pounds per hour total, is, $48.2\times243\times.332=3880$.

Example 2. What will be the high-pressure cut-off and cylinder ratio to give equal work distribution and complete expansion for an initial pressure of 150 lbs. per square inch absolute, and back pressure of 10 lbs. per square inch absolute?

From relation $R_V{}^s=\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)$, $R_V=6.9$ and from this, by the curve of Fig. 91, $R_C=5.4$.

For complete expansion

$$Z_H=\frac{R_C}{R_V}=\frac{5.4}{6.9}=.8$$

Prob. 1. A tandem compound engine without receiver has cylinders 18- and 30×42-ins. and runs at 200 R.P.M. What will be the horse-power developed at this speed if the initial pressure is 175 lbs. per square inch gage, back pressure atmosphere, high-pressure cut-off $\frac{1}{2}$, and s has a value of (*a*) 1.1, (*b*) 1.3? Compare the results with Prob. 1 of Sec. 13.

Prob. 2. What will be the weight of steam used per horse-power per hour for the two cases of the above problem? Compare these results with those of Prob. 2, Sec. 13.

NOTE: $\delta=.416$.

Prob. 3. What must be the cut-off in a 10- and 15×20-in. compressed air engine running on 100 lbs. per square inch gage initial pressure and atmospheric back pressure, to give complete expansion, and what will be the horse-power per 100 ft. per minute piston speed, s being 1.4?

Prob. 4. It is desired to run the following engine at its most economical load. What will this load be and how much steam will be needed per hour?

Cylinders 20- and 32×18 ins., speed 150 R.P.M., steam pressure 100 per square inch gage, atmospheric exhaust, dry saturated steam.

Prob. 5. Should the load increase 50 per cent in Prob. 4, how would the cut-off change and what would be th effect on the amount of steam used?

Prob. 6. What would be gain in power and the economy of the engine of Prob. 4 were superheated steam used, for which $s=1.3$?

Prob. 7. In a 14- and 20×24-in. engine will the work be equally divided between the cylinders for the following conditions? If not, what per cent will be done in each? Steam pressure 100 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, $s=1.2$, cut-off $=\frac{1}{2}$.

Prob. 8. What would be the work and steam used by the above engine if there were complete expansion and equal distribution?

15. Compound Engine with Infinite Receiver. Logarithmic Law. With Clearance and Compression, Cycle XI. General Relations between Pressures, Dimensions and Work. In terms of point pressures and volumes, Fig. 92, the

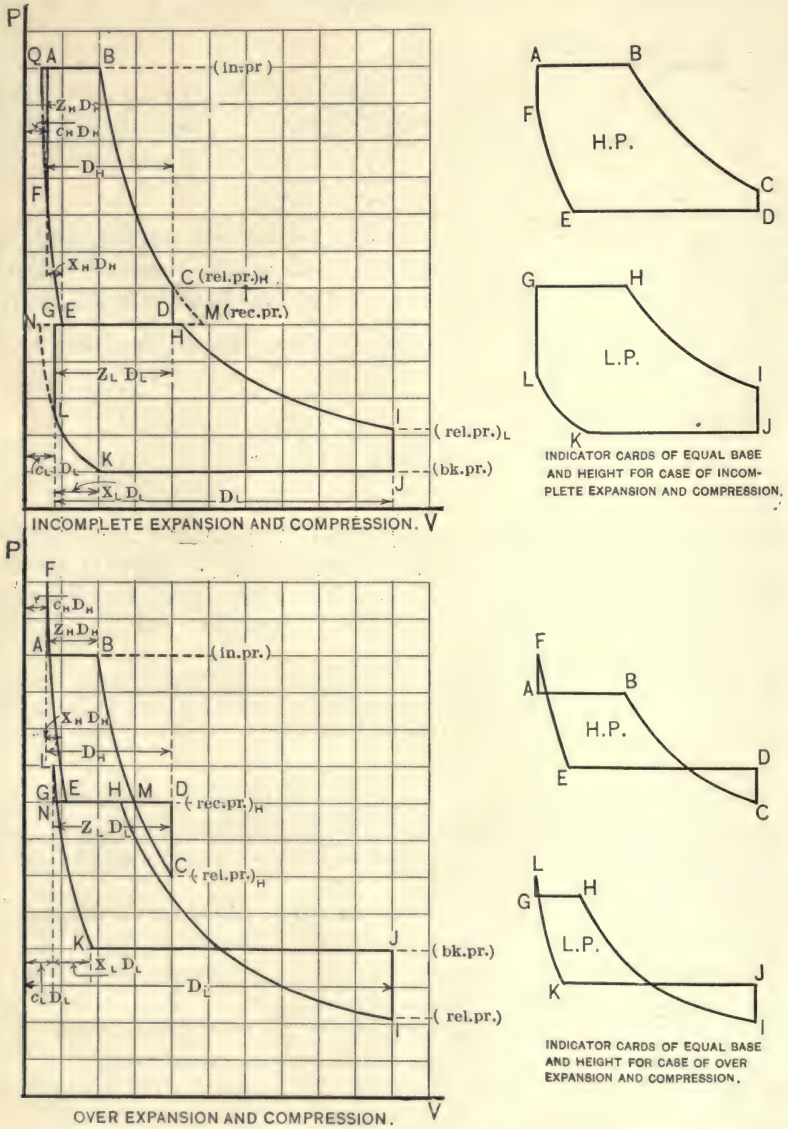


FIG. 92.—Work of Expansive Fluid in Compound Engines with Infinite Receiver, with Clearance, Cycle X Logarithmic, and Cycle IX Exponential Expansion, and Compression.

work of the two cylinders may be written down at once as if each were independent of the other, the connection between them being fixed first by making the back pressure of the high equal to the initial pressure of the low, or to

he receiver pressure, and second by making the volume admitted to the low qual to that discharged from the high reduced to the same pressure. This ast condition may be introduced in either of two ways,

(*a*) $\overline{EM}=\overline{NH}$,
(*b*) [(PV) on H.P. expansion line$-(PV)$ on H.P. comp. line],
$=$[(PV) on L.P. expansion line$-(PV)$ on L.P. comp. line].

Without introducing the last relation

$$W_H=P_bV_b\left(1+\log_e\frac{V_c}{V_b}\right)-P_fV_f\left(1+\log_e\frac{V_e}{V_f}\right)-(P_a-P_f)V_a-P_d(V_d-V_e); \quad (424)$$

$$W_L=P_hV_h\left(1+\log_e\frac{V_i}{V_h}\right)-P_lV_l\left(1+\log_e\frac{V_k}{V_l}\right)-(P_g-P_l)V_g-P_j(V_j-V_k) \quad (425)$$

$$W=W_H+W_L;$$

$$W=P_bV_b\log_e\frac{V_c}{V_b}+P_hV_h\log_e\frac{V_i}{V_h}-P_eV_e\log_e\frac{V_e}{V_f}-P_kV_k\log_e\frac{V_k}{V_l}$$
$$+P_bV_b-P_fV_f-P_aV_a+P_fV_f-P_dV_d+P_eV_e$$
$$+P_hV_h-P_lV_l-P_gV_g+P_lV_l-P_jV_j+P_kV_k.$$

The second condition is

$$P_bV_b-P_eV_e=P_hV_h-P_kV_k; \quad \text{or} \quad P_bV_b+P_kV_k=P_eV_e+P_hV_h,$$

or

$$P_bV_b+P_kV_k+P_eV_e+P_hV_h=2(P_bV_b+P_kV_k). \quad (426)$$

Substituting

$$\left.\begin{aligned}W=P_bV_b\log_e\frac{V_c}{V_b}+P_hV_h\log_e\frac{V_i}{V_h}-P_eV_e\log_e\frac{V_e}{V_f}-P_kV_k\log_e\frac{V_k}{V_l}\\ +2(P_bV_b+P_kV_k)-P_aV_a-P_gV_g-P_dV_d-P_jV_j\end{aligned}\right\}. \quad (427)$$

This expression, Eq. (427) contains, however, the receiver pressure which is elated to the release pressure by

$$(\text{rec.pr.})=P_g=P_e=P_d=P_h=P_i\frac{V_i}{V_h}=(\text{rel.pr.})_L\frac{\text{L.P. max.vol.}}{\text{L.P. cut-off vol.}}.$$

Introducing this

$$\left.\begin{aligned}W=P_bV_b\log_e\frac{V_c}{V_b}+P_iV_i\log_e\frac{V_i}{V_h}-P_i\frac{V_i}{V_h}V_e\log_e\frac{V_e}{V_f}-P_kV_k\log_e\frac{V_k}{V_l}\\ +2(P_bV_b+P_kV_k)-P_aV_a-P_i\frac{V_i}{V_h}V_g-P_i\frac{V_i}{V_h}V_d-P_jV_j\end{aligned}\right\}. \quad (428)$$

Introducing the usual symbols in Eqs. (427) and (428) and in additio the following:

Z = cut-off as fraction of stroke, so that $Z_H D_H$ is the displacement volum up to cut-off.

c = clearance volume divided by displacement, so that $c_H D_H$ is the clearan volume and $(Z_H+c_H)D_H$ is the volume in the high-pressure cylind at cut-off.

X = that fraction of the stroke during which compression is taking place that $(X_H+c_H)D_H$ is the volume in the high-pressure cylinder whe compression begins.

Applying the general symbols to Eq. (427),

$$W = 144 \left\{ \begin{array}{l} (\text{in.pr.})(Z_H+c_H)D_H \log_e \left(\dfrac{1+c_H}{Z_H+c_H}\right) \\ \quad +(\text{rec.pr.})(Z_L+c_L)D_L \log_e \left(\dfrac{1+c_L}{Z_L+c_L}\right) \\ \quad -(\text{rec.pr.})(X_H+c_H)D_H \log_e \left(\dfrac{X_H+c_H}{c_H}\right) \\ \quad -(\text{bk.pr.})(X_L+c_L)D_L \log_e \left(\dfrac{X_L+c_L}{c_L}\right) \\ \quad +2(\text{in.pr.})(Z_H+c_H)D_H - (\text{in.pr.})c_H D_H - (\text{rec.pr.})(1+c_H)D_H \\ \quad +2(\text{bk.pr.})(X_L+c_L)D_L - (\text{rec.pr.})c_L D_L - (\text{bk.pr.})(1+c_L)D_L \end{array} \right\} . \quad (42$$

This expression gives the work in terms of initial, receiver and back pressur the valve periods, cut-off and compression, the clearances and cylinder d placements.

Substitution of the symbols in Eq. (428) will give another equivalent expr sion in terms of the same quantities except that low-pressure cylinder relea pressure will take the place of receiver pressure. This is

$$W = 144 \left\{ \begin{array}{l} (\text{in.pr.})(Z_H+c_H)D_H \log_e \left(\dfrac{1+c_H}{Z_H+c_H}\right) + (\text{rel.pr.})_L(1+c_L)D_L \log_e \left(\dfrac{1+c_L}{Z_L+c_L}\right) \\ \quad -(\text{rel.pr.})_L \left(\dfrac{1+c_L}{Z_L+c_L}\right)(X_H+c_H)D_H \log_e \left(\dfrac{X_H+D_H}{c_H}\right) \\ \quad -(\text{bk.pr.})(X_L+c_L)D_L \log_e \left(\dfrac{X_L+c_L}{c_L}\right) \\ \quad +2(\text{in.pr.})(Z_H+c_H)D_H - (\text{in.pr.})c_H D_H - (\text{rel.pr.})_L \left(\dfrac{1+c_L}{Z_L+c_L}\right)(1+c_H)D_H \\ \quad +2(\text{bk.pr.})(X_L+c_L)D_L - (\text{rel.pr.})_L \left(\dfrac{1+c_L}{Z_L+c_L}\right)c_L D_L - (\text{bk.pr.})(1+c_L)D_L \end{array} \right\} (4$$

It is sometimes more convenient to involve the cylinder ratio and low-essure displacement than the two displacements as involved in Eq. (430) d the ratios of expansion instead of cut-offs. This may be done by

$$R_C = \frac{D_L}{D_H},$$

d

$$R_H = \frac{V_c}{V_b} = \frac{1+c_H}{Z_H+c_H}; \qquad R_L = \frac{V_l}{V_h} = \frac{1+c_L}{Z_L+c_L}, \quad . \quad . \quad . \quad . \quad (431)$$

d it should be noted here that the *ratio of expansion* in each cylinder *is no ger the reciprocal of its cut-off*, as was the case when clearance was zero, nor the whole ratio of expansion equal to the product of the two separate ones cause the low-pressure cylinder expansion line is not a continuation of that the high. Making these substitutions for cylinder and expansion ratios, l. (430) becomes,

$$= \frac{}{44D_L}\left\{\begin{array}{l} (\text{in.pr.})(1+c_H)\dfrac{1}{R_H R_C}\log_e R_H + (\text{rel.pr.})_L(1+c_L)\log_e R_L \\ -(\text{rel.pr.})_L(X_H+c_H)\dfrac{R_L}{R_C}\log_e\left(\dfrac{X_H+c_H}{c_H}\right) - (\text{bk.pr.})(X_L+c_L)\log_e\left(\dfrac{X_L+c_L}{c_L}\right) \\ +2(\text{in.pr.})(1+c_H)\dfrac{1}{R_H R_C} - (\text{in.pr.})\dfrac{c_H}{R_C} - (\text{rel.pr.})_L\dfrac{R_L}{R_C}(1+c_H) \\ +2(\text{bk.pr.})(X_L+c_L) - (\text{rel.pr.})_L R_L c_L - (\text{bk.pr.})(1+c_L) \end{array}\right\} \quad (432)$$

It is interesting to note that this reduces to Eq. 304 of Section 9, by making earance and compression zero.

From any of the expressions for work, but more particularly (430) and 32), the usual expressions for (m.e.p.) referred to low-pressure cylinder, work er cubic foot supplied, and consumption per hour per I.H.P. can be found, it as these are long they are not set down, but merely indicated as follows:

$$(\text{m.e.p. ref. to L.P.}) = \frac{W}{144D_L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (433)$$

$$\text{ork per cu.ft. supplied} = \frac{W}{D_H\left[(Z_H+c_H)-(X_H+c_H)\left(\dfrac{\text{rec.pr.}}{\text{in.pr.}}\right)\right]} \quad . \quad . \quad . \quad (434)$$

u.ft. sup. per hr. per I.H.P.

$$= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\left[(Z_H+c_H)-(X_H+c_H)\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)\right]\frac{1}{R_C}$$

$$= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\left[(Z_L+c_L)-(X_L+c_L)\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)\right]\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right). \quad (435)$$

As the receiver pressure is related to the initial and back pressures and t the relation between the amount taken out of the receiver to that put in, whic is a function of the compression as well as the cut-off and cylinder ratio, it i expressed only by a complicated function which may be derived from th equivalence of volumes in the high and low, reduced to equal pressure.

$$P_bV_b-P_eV_e=P_hV_h-P_kV_k, \text{ or } P_hV_h+P_eV_e=P_bV_b+P_kV_k=P_h(V_h+V_e).$$

Therefore,

$$P_h=P_b\frac{V_b}{V_h+V_e}+P_k\frac{V_k}{V_h+V_e}.$$

Introducing symbols

$$(\text{rec.pr.})=(\text{in.pr.})\frac{(Z_H+c_H)D_H}{(Z_L+c_L)D_L+(X_H+c_H)D_H}+(\text{bk.pr.})\frac{(X_L+c_L)D_L}{(Z_L+c_L)D_L+(X_H+c_H)D_H}$$

Hence

$$(\text{rec.pr.})=(\text{in.pr.})\frac{(Z_H+c_H)}{(Z_L+c_L)R_C+(X_H+c_H)}+(\text{bk.pr.})\frac{(X_L+c_L)R_C}{(Z_L+c_L)R_C+(X_H+c_H)}. \quad (436$$

This Eq. (436) gives the receiver pressure in terms of initial and back pressures the two clearances and compressions, the cylinder ratio and the cut-off in eac cylinder.

Proceeding in a similar way, the release pressures can be found in terms o initial data,

$$P_c=P_b\frac{V_c}{V_b},$$

or

$$\left.\begin{aligned}(\text{rel.pr.})_H&=(\text{in.pr.})\left(\frac{Z_H+c_H}{1+c_H}\right) \quad (a)\\ &=(\text{in.pr.})\frac{1}{R_H}. \quad (b)\end{aligned}\right\} \quad (437$$

And

$$P_i=P_h\frac{V_h}{V_i}=P_b\left(\frac{\frac{V_b}{V_i}}{1+\frac{V_e}{V_h}}\right)+P_k\left(\frac{\frac{V_k}{V_i}}{1+\frac{V_e}{V_h}}\right)$$

or

$$\left.\begin{aligned}(\text{rel.pr.})_L&=(\text{in.pr.})\left[\frac{\left(\frac{(Z_H+c_H)}{(1+c_L)R_C}\right)}{1+\frac{(X_H+c_H)}{(Z_L+c_L)R_C}}\right]+(\text{bk.pr.})\left[\frac{\frac{(X_L+c_L)}{(1+c_L)}}{1+\frac{(X_H+c_H)}{(Z_L+c_L)R_C}}\right] \quad (a)\\ &=(\text{in.pr.})\left[\frac{\left(\frac{1+c_H}{1+c_L}\right)\frac{1}{R_HR_C}}{1+\frac{(X_H+c_H)R_L}{(1+c_L)R_C}}\right]+(\text{bk.pr.})\left[\frac{\frac{(X_L+c_L)}{(1+c_L)}}{1+\frac{(X_H+c_H)R_L}{(1+c_L)R_C}}\right] \quad (b)\end{aligned}\right\} \quad (43$$

These three pressures all reduce to those of Eqs. (308), (309), (310), Section 9, when clearance and compression are zero.

Equal work in both cylinders is, of course, possible, but it may be secured by an almost infinite variety of combinations of clearance, compression and cut-off in the two cylinders for various ratios of expansion; it is, therefore, not worth while setting down the equation of condition to be satisfied, but reference may be had to Eqs. (424) and (425), which must be made equal to each other, the result of which must be combined with the equation of cylinder relations.

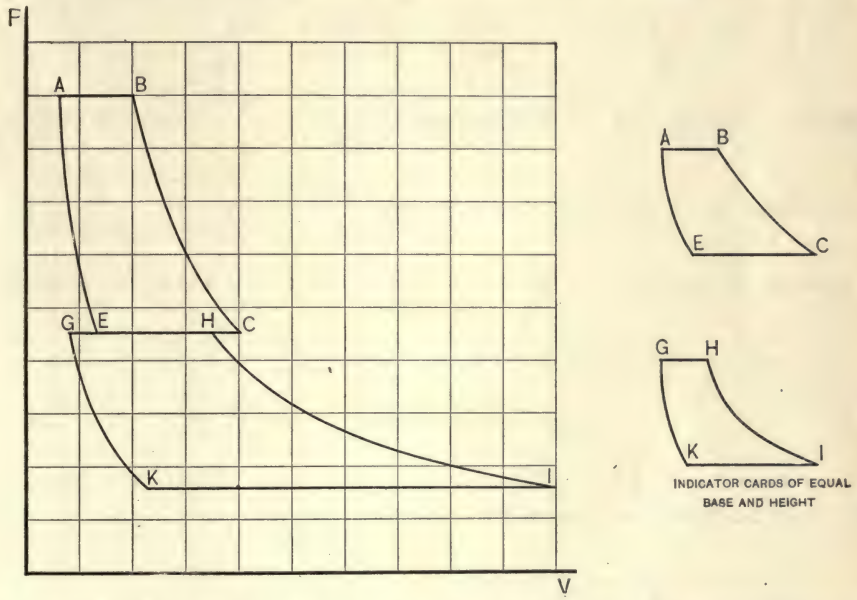


FIG. 93.—Special Case of Cycles XI and XII Complete Expansion and Compression in both Cylinders, of Compound Engine with Clearance and Infinite Receiver.

There are certain special cases of this cycle for which equations expressing important relations are simpler, and they are for that reason worth investigating. Those that will be examined are

(*a*) Complete expansion and compression in both cylinders, Fig. 93.

(*b*) Complete expansion in both cylinders with no compression, any clearance, Fig. 94.

(*c*) Any amount of expansion and compression but equal in both cylinders, equal clearance percentages and a cylinder ratio equal to the square root of the ratio of initial to back pressures, Fig. 95.

Case (*a*) When *both expansion and compression* are *complete* in both cylinders, Fig. 93,

$$W_H = 144(\text{in.pr.})Z_H D_H \log. \left(\frac{(\text{in.pr.})}{(\text{rec.pr.})}\right), \quad . \quad . \quad . \quad . \quad (439)$$

$$W_L = 144(\text{rec.pr.})Z_L D_L \log_e \left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})}\right), \quad . \quad . \quad . \quad . \quad (440)$$

but

$$Z_H D_H(\text{in.pr}) = Z_L D_L(\text{rec.pr.})$$

and

$$\log_e\left(\frac{(\text{in.pr.})}{(\text{rec.pr.})}\right)+\log_e\left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})}\right)=\log_e\left(\frac{(\text{in.pr.})}{(\text{rec.pr.})}\,\frac{(\text{rec.pr.})}{(\text{bk.pr.})}\right)=\log_e\frac{(\text{in.pr.})}{(\text{bk.pr.})},$$

hence

$$W=144(\text{in.pr.})Z_H D_H \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})} \quad . \quad . \quad . \quad . \quad . \quad . \quad (441)$$

$$(\text{m.e.p. ref. to L.P.})=(\text{in.pr.})\frac{Z_H}{R_C}\log_e\left(\frac{(\text{in.pr.})}{(\text{bk.pr.})}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (442)$$

$$\text{Work per cu.ft. supplied} = 144(\text{in.pr.})\log_e\left(\frac{(\text{in.pr.})}{(\text{bk.pr.})}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (443)$$

$$\text{Consumption, cu.ft. per hr. per I.H.P.} = \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\,\frac{Z_H}{R_C} \quad . \quad . \quad . \quad . \quad (444)$$

Equality of work in high- and low-pressure cylinders is obtained by making

$$\left(\frac{(\text{in.pr.})}{(\text{rec.pr.})}\right)=\left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})}\right)=\left(\frac{(\text{in.pr.})}{(\text{bk.pr.})}\right)^{\frac{1}{2}},$$

or

$$(\text{rec.pr.})=[(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (445)$$

It is desirable to know what clearances and displacements will permit of equal work and complete expansion and compression.

$$(\text{rec.pr.})=(\text{in.pr.})\frac{V_b}{V_c}=(\text{in.pr.})\left(\frac{Z_H+c_H}{1+c_H}\right)$$

$$=(\text{bk.pr.})\frac{V_t}{V_h}=(\text{bk.pr.})\left(\frac{1+c_L}{Z_L+c_L}\right),$$

hence

$$\left(\frac{(\text{in.pr.})}{(\text{rec.pr.})}\right)=\frac{1+c_H}{Z_H+c_H}=\left(\frac{(\text{in.pr.})^{\frac{1}{2}}}{(\text{bk.pr.})}\right),$$

$$\left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})}\right)=\frac{1+c_L}{Z_L+c_L}=\left(\frac{(\text{in.pr.})}{(\text{bk.pr.})}\right)^{\frac{1}{2}},$$

or calling

$$\left(\frac{(\text{in.pr.})}{(\text{bk.pr.})}\right)=R_P,$$

$$Z_H=\frac{1+c_H-c_H R_P^{\frac{1}{2}}}{R_P^{\frac{1}{2}}}, \quad \text{and,} \quad Z_L=\frac{1+c_L-c_L R_P^{\frac{1}{2}}}{R_P^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad (446)$$

Equating discharge of high and intake of low-pressure cylinders,

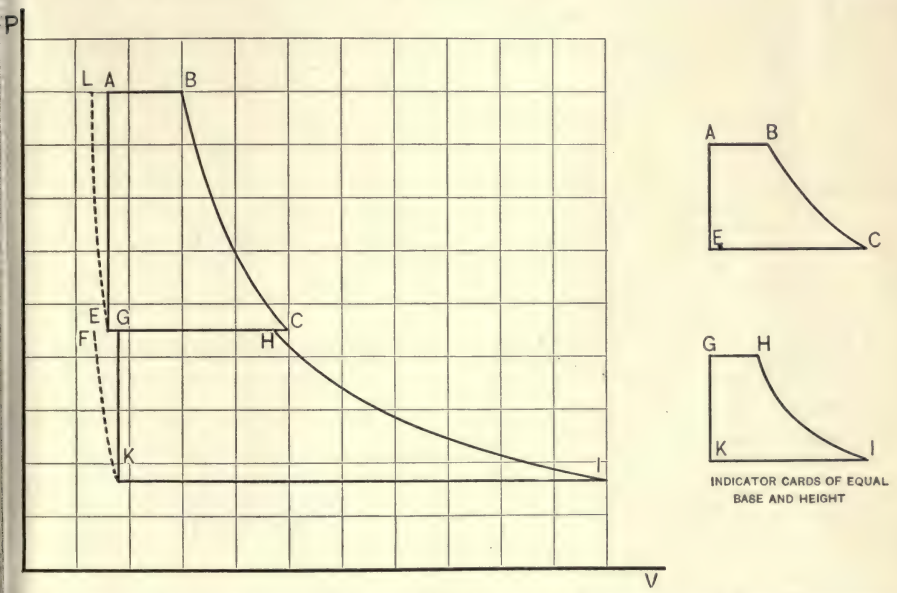
$$Z_H D_H R_P^{\frac{1}{2}}=Z_L D_L \quad \text{or} \quad \frac{D_L}{D_H}=R_C=\frac{Z_H}{Z_L}R_P^{\frac{1}{2}}.$$

Inserting in this the values just found for Z_H and Z_L,

$$R_C = \frac{1+c_H-c_H R_P^{\frac{1}{2}}}{1+c_L-c_L R_P^{\frac{1}{2}}} R_P^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (447)$$

vhich is the required relation between cylinder sizes, clearances and ratio of ressures, which, together with cut-offs given in Eq. (446), will give equal work nd complete expansion and compression. The compression in the high-ressure cylinder is such that

$$\left.\begin{aligned} c_H + X_H &= c_H R_P^{\frac{1}{2}} \\ X_H &= c_H(R_P^{\frac{1}{2}}-1) \\ \text{nd for L.P. cylinder.} \quad X_L &= c_L(R_P^{\frac{1}{2}}-1). \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (448)$$



G. 94.—Special Case of Cycles XI and XII. Complete Expansion and Zero Compression in both Cylinders of Compounds Engine with Clearance and Infinite Receiver.

Case (*b*) With *complete expansion and no compression*, both cylinders, any earance, Fig. 94,

$$W_H = 144 D_H \left[(\text{in.pr.})(Z_H+c_H)\log_e \left(\frac{(\text{in.pr.})}{(\text{rec.pr.})}\right) - c_H[(\text{in.pr.})-(\text{rec.pr.})] \right] \quad (449)$$

$$W_L = 144 D_L \left[(\text{rec.pr.})(Z_L+c_L)\log_e \left(\frac{(\text{rec.pr.})}{(\text{bk.pr.})}\right) - c_L[(\text{rec.pr.})-(\text{bk.pr.})] \right] \quad (450)$$

ith the added requirement that the high-pressure discharge volume, $\overline{EC}$ = low essure admission volume $\overline{FH}$, or

$$D_H = D_L \left[Z_L + c_L - c_L \left(\frac{(\text{bk.pr.})}{(\text{rec.pr.})}\right) \right], \quad . \quad . \quad . \quad . \quad . \quad . \quad (451)$$

and

$$(\text{rec.pr.}) = (\text{bk.pr.})\frac{V_4}{V_h} = (\text{bk.pr.})\frac{1+c_L}{Z_L+c_L}, \quad \ldots\ldots \quad (452)$$

hence

$$\left[\frac{(\text{bk.pr.})}{(\text{rec.pr.})}\right] = \frac{Z_L+c_L}{1+c_L},$$

which substituted in Eq. (451) and rearranging gives

$$R_C = \frac{1}{Z_L+c_L-c_L\left(\dfrac{Z_L+c_L}{1+c_L}\right)}, \quad \ldots\ldots \quad (453)$$

Eq. (453) indicates that for this special case of complete expansion and no compression the cylinder ratio required to give this case, is determined entirely by the L.P. cut-off and clearance. If the cylinder ratio and clearance are fixed, the required cut-off in the L.P. cylinder can be found by solving Eq. (453) for Z_L

$$Z_L = \frac{1+c_L}{R_C} - c_L, \quad \ldots\ldots \quad (454)$$

and from Eq. (452),

$$(\text{rec.pr.}) = (\text{bk.pr.})\left[\frac{1+c_L}{\dfrac{1+c_L}{R_C}-c_L+c_L}\right] = (\text{bk.pr.})R_C. \quad \ldots \quad (455)$$

Cut-off in the high-pressure cylinder is determined by clearance, initial pressure and receiver pressure, which in turn depends on low-pressure cut-off and clearance Eq.(452), or may be reduced to cylinder ratio and low-pressure clearance by Eq.(454), as follows

$$\frac{V_c}{V_b} = \frac{1+c_H}{Z_H+c_H} = \left(\frac{(\text{in.pr.})}{(\text{rec.pr.})}\right) = \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})}\right)\frac{Z_L+c_L}{1+c_L},$$

hence

$$Z_H = \frac{(1+c_H)(1+c_L)}{R_P(Z_L+c_L)} - c_H.$$

Eliminate Z_L by Eq. (454),

$$Z_H = \frac{(1+c_H)(1+c_L)}{\left(R_P\ \dfrac{1+c_L}{R_C}-c_L+c_L\right)} - c_H = (1+c_H)\frac{R_C}{R_P} - c_H. \quad \ldots \quad (456)$$

Since the high- and low-pressure cut-offs are functions of cylinder and clearance dimensions, and of R_P, the rato of initial and back pressures, the work of high and low-pressure cylinders may be expressed entirely in terms of these quantities

$$W_H = 144D_H(\text{in.pr.})\left\{(1+c_H)\frac{R_C}{R_P}\log_e\left(\frac{R_P}{R_C}\right) - c_H\left(1-\frac{R_C}{R_P}\right)\right\}. \quad \ldots \quad (457)$$

$$W_L = 144D_H R_C(\text{bk.pr.})\left[(1+c_L)\log_e R_C - c_L(R_C-1)\right]. \quad \ldots \quad (458)$$

Hence, total work by addition is

$$W = 144 D_H (\text{in.pr.}) \frac{R_C}{R_P} \left\{ (1+c_H) \log_e \left(\frac{R_P}{R_C} \right) - c_H \left(\frac{R_P}{R_C} - 1 \right) \right.$$
$$\left. + (1+c_L) \log_e R_C - c_L (R_C - 1) \right\} \quad . \quad . \quad . \quad (459)$$

Expressions might be easily written for mean effective pressure referred to the L.P. cylinder, work per cubic foot fluid supplied, and consumption, but will be omitted for brevity. It is important to note, however, the volume of fluid used per cycle is not $\overline{AB}$, but is $\overline{LB}$, Fig. 94, and is,

$$(\text{Sup.Vol.}) = D_H \left[(Z_H + c_H) - c_H \frac{(\text{rec.pr.})}{(\text{in.pr.})} \right] = D_H \left[(Z_H + c_H) - c_H \frac{R_C}{R_P} \right] . \quad (460)$$

$$(\text{m.e.p. ref. to L.P.}) = \frac{W}{144 D_H R_C} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (461)$$

$$(\text{Work per cu.ft. supplied}) = \frac{W}{(\text{Sup.Vol.})} . \quad . \quad . \quad . \quad . \quad (462)$$

$$\text{Consumption cu.ft per hr. per I.H.P.} = \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})} \left[(Z_H + c_H) \right.$$
$$\left. - c_H \frac{R_C}{R_P} \right] \frac{1}{R_C} . \quad . \quad (463)$$

Equality of work, secured by equating Eqs. (457) and (458) gives

$$(1+c_H) \log_e \left(\frac{R_P}{R_C} \right) - c_H \left(\frac{R_P}{R_C} - 1 \right) = (1+c_L) \log_e R_C - c_L (R_C - 1) . \quad . \quad (464)$$

This equation may be satisfied in an infinite number of ways. One case worth noting is that of equal clearances, when it is evident that if

$$c_H = c_L, \quad \text{and} \quad \frac{R_P}{R_C} = Rc, \quad \text{or} \quad R_C = \sqrt{R_P}$$

the Eq. (464) is satisfied. This last condition is the same as that which satisfied Case (*a*) with complete compression.

Case (*c*), Fig. 95, assumes that

$$c_H = c_L = c, \quad Z_H = Z_L = Z, \quad X_H = X_L = X,$$

and

$$R_C = \left(\frac{\text{in.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{2}} = \sqrt{R_P}$$

and corresponds to the first special case considered in Section 9, which lead in the no-clearance case to equality of high- and low-pressure work.

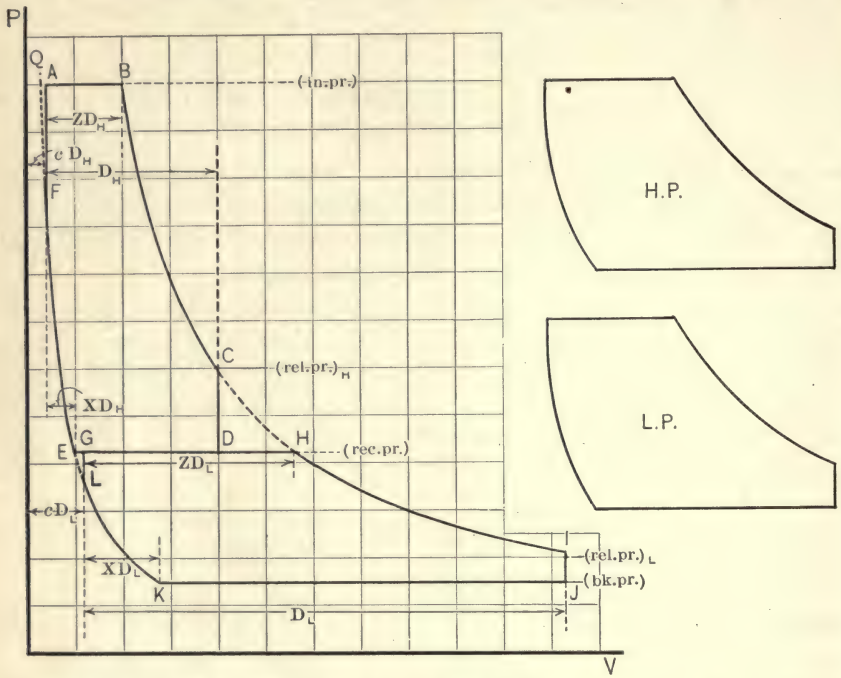


FIG. 95.—Special Case of Cycles XI and XII, Equal Per Cent Clearance in Each Cylinder of Compound Engine with Infinite Receiver and Cylinder Ratio Equal to the Square Root of Initial Divided by Back Pressure.

The assumptions already made are sufficient to determine the receiver pressure. By Eq. (436)

$$(\text{rec.pr.}) = (\text{in.pr.})\frac{Z+c}{(Z+c)\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}}+X+c}+(\text{bk.pr.})\frac{(X+c)\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}}}{(Z+c)\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}}+X+c}$$

$$=\frac{(Z+c)\left(\frac{\text{in.pr.}}{\text{bk.pr.}}^{\frac{1}{2}}\right)+X+c}{(Z+c)\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}}+X+c}[(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}}$$

$$=[(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (465)$$

The work of the high-pressure cylinder may now be evaluated.

$$W_H = 144D_H(\text{in.pr.})\left\{Z\left[1+\log_e\left(\frac{1+c}{Z+c}\right)\right]-c\right\}$$

$$-144D_H[(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}}\left\{(X+c)\log_e\left(\frac{X+c}{c}\right)+1-X\right\}. \quad . \quad (466)$$

The low-pressure cylinder work may be similarly stated,

$$W_L = 144 D_L[(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}} \left\{ Z\left[1+\log_e\left(\frac{1+c}{Z+c}\right)\right]-c \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (467)$$

$$-144 D_L(\text{bk.pr.}) \left\{ (X+c)\log_e\left(\frac{X+c}{c}\right)+1-X \right\},$$

but

$$D_L[(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}} = D_H R_C[(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}}$$

$$= D_H\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{2}}[(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}}$$

$$= D_H(\text{in.pr.}),$$

and similarly,

$$D_L(\text{bk.pr.}) = D_H[(\text{in.pr.})(\text{bk.pr.})]^{\frac{1}{2}}$$

With these substitutions the value of *low-pressure work*, W_L, Eq. (467), becomes *equal to high pressure work*, Eq. (466), hence the total work

$$W = 2W_H = 2W_L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (468)$$

Example 1. Method of calculating Diagrams, Fig. 92.
Assumed data:

$P_q = P_a = P_b = 120$ lbs. per square inch abs.
$P_n = P_g = P_e = P_d = P_h = 50$ lbs. per square inch abs.
$P_k = P_j = 10$ lbs. per square inch abs.

$V_a = V_f = .12$ cu.ft.
$V_b = .4$ cu.ft.
$V_c = V_d = .8$ cu ft.
$V_g = V_l = .16$ cu.ft.
$V_i = V_j = 2$ cu.ft.
$V_e = .2$ cu.ft.
$V_k = .4$ cu.ft.

$(V_h - V_n) = (V_m - V_e)$.

The above may be expressed in initial pressure, etc., and in terms of cut-off, etc., but as the relation of the lettered points to these terms is shown on the diagram values for cut-off, etc., they will not be given here, as they may readily be found from values of the lettered points.

To locate point C:

$$P_c = P_b\frac{V_b}{V_c} = \frac{120\times.4}{.8} = 60 \text{ lbs. per sq.in.}$$

To locate point F:

$$P_f = \frac{P_e V_e}{V_f} = \frac{50\times.2}{.12} = 83.3 \text{ lbs. per sq.in.}$$

To locate point Q:

$$V_q = \frac{P_e V_e}{P_q} = \frac{10}{120} = .083 \text{ cu.ft.}$$

To locate point L:

$$P_l = \frac{P_k V_k}{V_l} = \frac{10 \times .4}{.16} = 25 \text{ lbs. per sq.in.}$$

To locate point N:

$$V_n = \frac{P_k V_k}{P_n} = \frac{4}{50} = .08 \text{ cu.ft.}$$

To locate point H:

$$(V_h - V_n) = (V_m - V_c), \quad \text{or} \quad V_h = V_m + V_n - V_c = .96 + .08 - .2 = .84 \text{ cu.ft.,}$$

since

$$P_m V_m = P_b V_b, = V_m = \frac{48}{50} = .96 \text{ cu.ft.}$$

To locate point I:

$$P_i = \frac{P_h V_h}{V_i} = \frac{50 \times .84}{2} = 21 \text{ lbs. per sq.in.}$$

Example 2. Find (a) the horse-power, (b) steam used per hour, and (c) receiver and release pressures for a 12- and 18×24-in. engine with infinite receiver, 6 per cent clearance in high-pressure cylinder, and 4 per cent clearance in low-pressure cylinder, when initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., cut-off in high-pressure cylinder is $\frac{1}{2}$, low-pressure cut-off is such as to give complete H.P. expansion, and compression is 15 per cent in high and complete in low.

(a) For complete high-pressure expansion the receiver pressure must be equal to the high-pressure release, and to maintain the receiver pressure constant the low-pressure cylinder must take as much steam per stroke as the high-pressure discharges. With initial pressure and cut-off as given, the release pressure for the high-pressure cylinder may be found from the relation $(\text{in pr.})(c_H + Z_H) = (\text{rel.pr.})_H(c_H + D_H)$ or $150 \times (.56) = (\text{rel.pr.})_H(1.04)$, or $(\text{rel.pr.})_H = 79.3$ lbs. Since there is 15 per cent compression in high-pressure cylinder there is exhausted each stroke 85 per cent of its volume. Also since compression in low-pressure cylinder is complete, the low-pressure clearance is full of steam at the receiver pressure at the beginning of the stroke. Hence the low-pressure displacement up to cut-off must equal $.85D_H$ or L.P. cut-off $= .85D_H$, divided by cylinder ratio, or $.85 \div 2.25 = .378$. As compression is complete, the per cent compression may be found from the relation $c_L \times (\text{rec.pr.}) = (c_L + X_L)(\text{bk.pr.})$, or $.04 \times 79.3 = (.04 + X_L)10$, or $X_L = .28$.

From Eq. (432), (m.e.p.) referred to low-pressure cylinder is obtained by dividing by 144 D_L, and on substituting the above values it becomes,

$$150(.1 + .06)\left(\frac{1}{2 \times 2.25}\right) \log_e 2 + 30(1 + .04) \log_e 2.64 - 30(.15 + .06)\left(\frac{2.64}{2.25}\right) \log_e \left(\frac{.15 + .06}{.06}\right)$$
$$- 10(.28 + .04) \log_e \left(\frac{.28 + .04}{.04}\right) + 2 \times 150(1 + .06)\left(\frac{1}{2 \times 2.25}\right) - 150 \times \frac{.06}{2.25}$$
$$- 30 \times \frac{2.64}{2.25}(1 + .06) + 2 \times 10(.28 + .04) - 30 \times 2.64 \times .04 - 10(1 + .04) = 60.5 \text{ lbs. per. sq.in.,}$$

hence

$$\text{I.H.P.} = 235.$$

(*b*) From Eq. (435) by substituting the above values

$$\text{u.ft. steam per hour per horse-power} = \frac{13{,}750}{60.5}\left[\left((.38+.04)-(.28+.04)\frac{10}{79.3}\right)\frac{79.3}{150}\right]=45.5,$$

r pounds per hour will be 3550.

(*c*) Release pressure for high-pressure cylinder has been shown to be 79.3 lbs. nd may be checked by Eq. (437), as follows:

$$(\text{rel.pr.})_H = 150\left(\frac{.5+.06}{1.06}\right)=79.3 \text{ lbs.}$$

Receiver pressure has already been shown to be equal to this quantity and may e checked by Eq. (436)

$$(\text{rec.pr.})_H = \frac{150\times(.5+.06)}{(.378+.04)2.25+(.28+.04)}+\frac{10\times(.28+.04)2.25}{(.378+.04)2.25+(.28+.04)}=79.3 \text{ lbs.}$$

Low-pressure release pressure is found from Eq. (438) to be

$$(\text{rel.pr.})_L=150\left[\frac{\frac{1+.06}{1+.04}\times\frac{1}{2\times2.25}}{1+\frac{(.15+.06)2.64}{(1+.04)2.25}}\right]+10\left[\frac{\frac{.28+.04}{1+.04}}{1+\frac{(.15+.06)2.64}{(1+.04)2.25}}\right]=30 \text{ lbs.}$$

Prob. 1. What will be the horse-power and steam used by the following engine or the data as given?

Engine 20 and 28×36 ins., running at 100 R.P.M., clearance 5 per cent in high ressure, 3 per cent in low. From cards H.P. cut-off = .3, L.P. = .4, H.P. compression, 1, L.P., .2. Gages show (in.pr.) to be 150 lbs., (r c.pr.) 60 lbs., (bk.pr.) 26 ins. Hg. (barometer = 30 ins.).

Prob. 2. What must be the cut-offs and the cylinder ratio of an engine to give qual work and complete expansion and ompression for 200 lbs. per square inch bsolute initial pressure and atmospheric exhaust, if clearance is 5 per cent in the igh and 3 per cent in the low-pressure cylinder? What will the horse-power for an ngine with a low-pressure cylinder 24×36 ins., running at 100 R.P.M. for this case?

Prob. 3. Should there be no compression, how would the results of Prob. 2 be ltered?

Prob. 4. What will be the total steam used by engines of Probs. 2 and 3?

Prob. 5. For an 11- and 19×24-in. engine with 5 per cent clearance in each ylinder, ½ cut-off in each cylinder, and 20 per cent compression in each cylinder, what vill be the horse-power and the steam consumption when the speed is 125 R.P.M., the nitial pressure 150 lbs. per square inch gage, and back pressure at atmosphere?

16. Compound Engine with Infinite Receiver. Exponential Law, with Clearance and Compression, Cycle XII. General Relation between Pressures, Dimensions and Work. Referring to Fig. 92, of the preceding section, which

will represent this cycle by a slight change of slope of the expansion and com pression lines, the high-pressure work may be expressed in terms of dimension ratios and pressures. Since this must contain receiver pressure as a factor and since that is not an item of original data, it is convenient first to state receiver pressure in terms of fundamental data:

$$V_m - V_e = V_h - V_n.$$

But

$$V_m = V_b\left(\frac{\text{in.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}} \quad \text{and} \quad V_n = V_k\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}}$$

Whence,

$$V_b\left(\frac{\text{in.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}} + V_k\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}} = V_h + V_e,$$

whence

$$(\text{rec.pr.}) = (\text{in.pr.})\left(\frac{V_b + V_k\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}}{V_h + V_e}\right)^s,$$

or in terms of dimensions and pressures,

$$(\text{rec.pr.}) = (\text{in.pr.})\left(\frac{Z_H + c_H + R_C(X_L + c_L)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}}{R_C(Z_L + c_L) + X_H + c_H}\right)^s. \quad . \quad (46$$

The high-pressure work may be stated as follows:

$$W_H = 144 D_H \left\{ (\text{in.pr.})\frac{(Z_H + c_H)}{s-1}\left[s - \left(\frac{Z_H + c_H}{1 + c_H}\right)^{s-1}\right] - (\text{rec.pr.})(1 - X_H) \right.$$

$$\left. - (\text{rec.pr.})\frac{(X_H + c_H)}{s-1}\left[\left(\frac{X_H + c_H}{c_H}\right)^{s-1} 1 - \right] - (\text{in.pr.})c_H \right\}. \quad . \quad (47$$

$$= 144 D_H\ (\text{in.pr}).\left\{\frac{Z_H + c_H}{s-1}\left[s - \left(\frac{Z_H + c_H}{1 + c_H}\right)^{s-1}\right] - c_H\right\}$$

$$- 144 D_H (\text{in.pr.})\left(\frac{Z_H + c_H + R_C(X_L + c_L)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}}}{R_C(Z_L + c_L) + X_H + c_H}\right)^s \times$$

$$\left\{\frac{X_H + c_H}{s-1}\left[\left(\frac{X_H + c_H}{c_H}\right)^{s-1} 1 - \right] + 1 - X_H\right\}. \quad (471)$$

$$W_L = 144 D_L \left\{ (\text{rec.pr.}) \frac{(Z_L + c_L)}{s-1} \left[s - \left(\frac{Z_L + c_L}{1 + c_L} \right)^{s-1} \right] - (\text{bk.pr.})(1 - X_L) \right.$$

$$\left. - (\text{bk.pr.}) \frac{X_L + c_L}{s-1} \left[\left(\frac{X_L + c_L}{c_L} \right)^{s-1} - 1 \right] - (\text{rec.pr.}) c_L \right\} \quad . \quad . \quad (472)$$

$$= 144 D_H R_C (\text{in.pr.}) \left(\frac{Z_H + c_H + R_C (X_L + c_L) \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}}}{R_C (Z_L + c_L) + X_H + c_H} \right)^s \times$$

$$\left\{ \frac{Z_L + c_L}{s-1} \left[s - \left(\frac{Z_L + c_L}{1 + c_L} \right)^{s-1} \right] - c_L \right\}$$

$$- 144 D_H R_C (\text{bk.pr.}) \left\{ \frac{X_L + c_L}{s-1} \left[\left(\frac{X_L + c_L}{c_L} \right)^{s-1} - 1 \right] + 1 - X_L \right\} \quad . \quad . \quad (473)$$

The expression for *total work* need not be written here, as it is simply the sum of Eqs. (470) and (472) or of (471) and (473), the former containing receiver pressure and the latter containing only dimensions, initial and back pressures and both, the exponent of the expansion, s,

The volume of high-pressure fluid supplied per cycle is $\overline{QB}$, Fig. 92, which may be expressed either in terms of high pressure or of low pressure points, thus;

$$\left. \begin{aligned} (\text{Sup.Vol.}) &= D_H \left[Z_H + c_H - (X_H + c_H) \left(\frac{\text{rec.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \right] \quad (a) \\ &= D_L \left[(Z_L + c_L) \left(\frac{\text{rec.pr.}}{\text{bk.pr.}} \right)^{\frac{1}{s}} - (X_L + c_L) \right] \left(\frac{\text{bk.pr.}}{\text{in.pr.}} \right)^{\frac{1}{s}} \quad (b) \end{aligned} \right\} \quad (474)$$

The following quantities will be indicated, and may be evaluated by substitution from the preceding:

$$(\text{m.e.p. ref. to L.P.}) = \frac{W}{144 D_L} = \frac{W}{144 D_H R_C} \quad . \quad . \quad . \quad . \quad . \quad (475)$$

$$\text{Work per cu.ft. fluid supplied} = \frac{W}{(\text{Sup.Vol.})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (476)$$

Consumption cu.ft. per hr. per I.H.P.

$$= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})} \frac{(\text{Sup.Vol.})}{D_L} \quad . \quad . \quad . \quad . \quad . \quad . \quad (477)$$

Equal division of *work* between high- and low-pressure cylinders requires that Eqs. (470) and (472), or (471) and (473) be made equal. The latter will

give an expression showing the required relation between dimensions and initial and final pressures, cut-off and compression in high- and low-pressure cylinders. *In this expression there are so many variables that an infinite number of combinations may be made to give equality of work.*

It is desirable to examine the results of assuming special conditions such as those of the previous section, the most important of which is that of *complete expansion and compression in both cylinders,* which is represented by Fig. 93.

$$W_H = 144(\text{in.pr.})Z_H D_H \frac{s}{s-1}\left[1-\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right]. \quad . \quad . \quad . \quad (478)$$

$$W_L = 144(\text{rec.pr.})Z_L D_L \frac{s}{s-1}\left[1-\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{s-1}{s}}\right], \quad . \quad . \quad . \quad (479)$$

but

$$Z_L D_L = Z_H D_H \left(\frac{\text{in.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}},$$

hence

$$W = 144(\text{in.pr}).D_H Z_H \frac{s}{s-1}\left\{\left[1-\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right] + \left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)\left(\frac{\text{in.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}}\left[1-\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{s-1}{s}}\right]\right\}$$

$$= 144(\text{in.pr.})D_H Z_H \frac{s}{s-1}\left\{1-\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}} + \left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}} - \left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{s-1}{s}}\right\}$$

$$= 144(\text{in.pr.})D_H Z_H \frac{s}{s-1}\left[1-\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (480)$$

The receiver pressure may be found as follows. In Fig. 93, $\overline{EC} = \overline{GH}$:

$$\overline{EC} = D_H\left[1+c_H-c_H\left(\frac{\text{in.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}}\right],$$

$$\overline{GH} = \overline{KI}\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}} = D_L\left[1+c_L-c_L\left(\frac{\text{rec.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{s}}.\right]\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}}$$

Equating

$$1+c_H-c_H\left(\frac{\text{in.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}} = R_C\left[1+c_L-c_L\left(\frac{\text{rec.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{s}}\right]\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}} \quad . \quad (481)$$

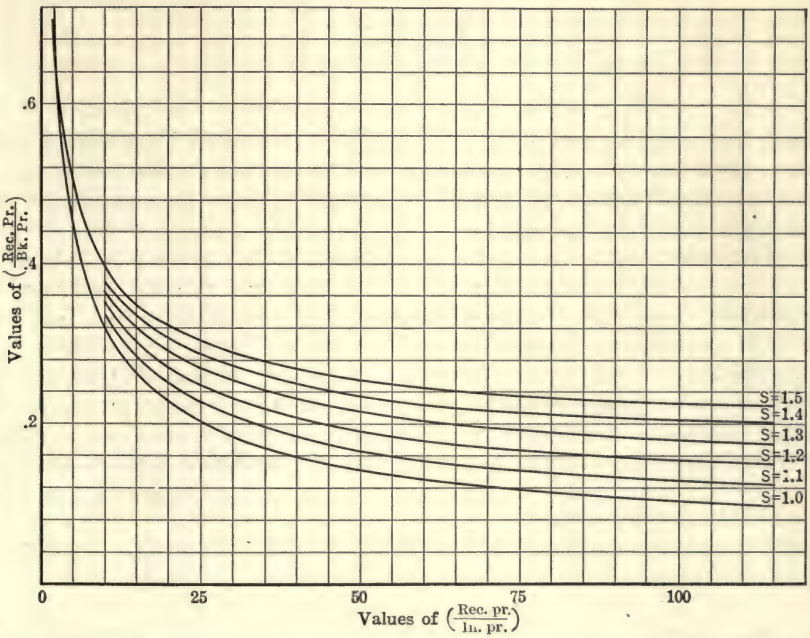
When this is solved for receiver pressure it results in an equation of the econd degree, which is somewhat cumbersome, and will not be stated here. Eq. (481) is, however, used later to find R_C.

If work is to be equally distributed between high- and low-pressure cylinders, rom Eqs. (478) and (479),

$$1-\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}=\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}-\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{s-1}{s}},$$

r

$$2\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}=1+\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}},$$



IG. 96.—Curves to Show Receiver Pressure to Give Equal Work Distribution when Expansion and Compression are Complete in both Cylinders of the Compound Engine with Infinite Receiver, with Clearance when Expansion and Compression are not Logarithmic.

ience, for equal division of work,

$$(\text{rec.pr.})=(\text{in.pr.})\left[\frac{1+\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{s-1}{s}}}{2}\right]^{\frac{s}{s-1}}, \quad . \quad . \quad . \quad . \quad (482)$$

which, if satisfied, will give equality of work in the two cylinders, for this case f perfect compression and expansion.

In Fig. 96, is given a set of curves for use in determining the value of the atio of (rec.pr.) to (in.pr.) as expressed by Eq. (482).

When (rec.pr.) has been found by Eq. (482) it is possible by means of (481) and the clearances to find R_C. The events of the stroke must have the following values to maintain complete and perfect compression and expansion.

$$Z_H = (1+c_H)\left(\frac{\text{rec.pr.}}{\text{in.pr.}}\right)^{\frac{1}{s}} - c_H; \quad \ldots \quad (483)$$

$$Z_L = (1+c_L)\left(\frac{\text{bk.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}} - c_L; \quad \ldots \quad (484)$$

$$X_H = c_H\left[\left(\frac{\text{in.pr.}}{\text{rec.pr.}}\right)^{\frac{1}{s}} - 1\right]; \quad \ldots \quad (485)$$

$$X_L = c_L\left[\left(\frac{\text{rec.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{s}} - 1\right]. \quad \ldots \quad (486)$$

Example. Find (*a*) the horse-power, (*b*) compressed air used per hour, and (*c*) receiver and relea e pressures for a 12- and 18×24-in. engine with infinite receiver, 6 per cent clearance in the high-pressure cylinder, and 4 per cent in the low-pressure cylinder, when initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., cut-off in high-pressure cylinder ½, low-pressure cut-off such as to give complete expansion in high-pressure cylinder, compression in high-pressure cylinder 15 per cent, and complete in low. Expansion such that $s = 1.4$.

(*b*) As in example of Section 15, receiver pressure equal high-pressure release pressure, and low-pressure volume at cut-off must equal volume of steam exhausted from high pressure. Release pressure may be found from relation $(\text{in.pr.})(c_H+Z_H)^s = (\text{rel.pr.}) - (c_H+D_H)^s$, or $150(.06+.5)^{1.4} = (\text{rel.pr.}) - (.06+1)^{1.4}$, or $(\text{rel.pr.}) = 60$ lbs. As in the previous example, the low-pressure cut-off is .38, and the low-pressure compression may be found from the relation $c_L{}^s(\text{rec.pr.}) = (c_L+X_L)^s(\text{bk.pr.})$, or $(.04)^{1.4}\times 60 = (.04+X_L)^{1.4}$ (10), or $X_L = .09$.

From the sum of equations (471) and (473) divided by $144 D_H R c$, and with the proper values substituted the following expression for (m.e.p.) results:

$$\frac{150}{2.25}\left\{\frac{.5+.06}{.4}\left[1.4-\left(\frac{.5+.06}{1+.06}\right)^{.4}\right]-.06\right\} - \frac{150}{2.25}\left\{\frac{.5+.06+2.25(.15+.06)\left(\frac{10}{150}\right)^{.71}}{2.25(.38+.04)+.15+.06}\right\}^{1.}$$

$$\left\{\frac{.15+.06}{.4}\left[\left(\frac{.15+.06}{.06}\right)^{.4}-1\right]+1-.15\right\} + 150\left\{\frac{.5+.06+2.25(.15+.04)\left(\frac{10}{150}\right)^{.71}}{2.25(.38+.04)+.15+.06}\right\}^{1.}$$

$$\left\{\frac{.38+.04}{.4}\left[1.4-\left(\frac{.38+.04}{1+.04}\right)^{.4}\right]-.04\right\}$$

$$-.10\left\{\frac{.09+.04}{.4}\left[\left(\frac{.09+.04}{.04}\right)^{.4}-1\right]+1-.09\right\} = 55 \text{ lbs. per sq.in.}$$

hence the horse-power is 214.

From Eq. (477) with proper values substitute,

$$\text{Cu.ft. per I.H.P. hr.} = \frac{13{,}750}{55} \times \left[(.38+.04)\left(\frac{60}{10}\right)^{.71} - (.09+.04)\left(\frac{10}{130}\right) \right]^{.71} = 50,$$

total steam per hour will be

$$50 \times 214 = 10700 \text{ cu.ft.}$$

(*c*) Release for the high-pressure cylinder has already been given as 60 lbs. and the ceiver pressure the same. The latter quantity may be checked by equation (469) d will be found to be the same. The low-pressure release pressure may be found m the relation $(\text{rec.pr.})(Z_L+c_L)^{1.4} = (\text{rel.pr.})_L(1+c_L)^{1.4}$, which on proper substitution ves

$$(\text{rel.pr.})_L = 60\left(\frac{.38+.04}{1.04}\right)^{\frac{1}{1.4}} = 27 \text{ lbs. per sq. inch}$$

Prob. 1. What will be the horse-power and steam used per hour by an 18- and ×30-in. engine with 5 per cent clearance in each cylinder and with infinite receiver nning on 100 lbs. per square inch gage initial pressure, and 5 lbs. per square inch solute back pressure, when the speed is 100 R.P.M. and the cut-off in high-pressure linder is $\frac{1}{2}$ and in low $\frac{4}{10}$?

NOTE: $s=1.3$ and $\delta=.2$.

Prob. 2. What must be the receiver pressure for equal work distribution when the itial pressure has the following values for a fixed back pressure of 10 lbs. per uare inch absolute? 200, 175, 150, 125, 100, and 75 lbs. per square inch gage?

Prob. 3. For the case of 150 lbs. per square inch gage initial pressure and 14 lbs. r square inch absolute back pressure, what will be the required high-pressure cylinder e for an air engine with a low-pressure cylinder 18×24 ins., to give equality of work, arance in both cylinders being 5 per cent?

Prob. 4. What will be the horse-power and air consumption of the above engine en running at a speed of 150 R.P.M., and under the conditions of perfect expansion d compression?

17. Compound Engine with Finite Receiver. Logarithmic Law, with earance and Compression, Cycle XIII. General Relations between Pressures, mensions, and Work when H.P. Exhaust and L.P. Admission are Inde-ndent. As this cycle, Fig. 97, is made up of expansion and compression lines ferred to the different origins together with constant pressure, and constant lume lines, the work for high- and low-pressure cylinders and for the cycle can set down at once. These should be combined, however, with the relation ted for the case of infinite clearance which might be termed the condition a steady state

$$[(PV) \text{ on H.P. expansion line}] - [(PV) \text{ on H.P compression line}]$$
$$= [(PV) \text{ on L.P. expansion line} - PV \text{ on L.P. compression line},]$$
$$P_b V_b - P_e V_e = P_h V_h - P_k V_k \quad . \quad . \quad . \quad . \quad . \quad . \quad (487)$$

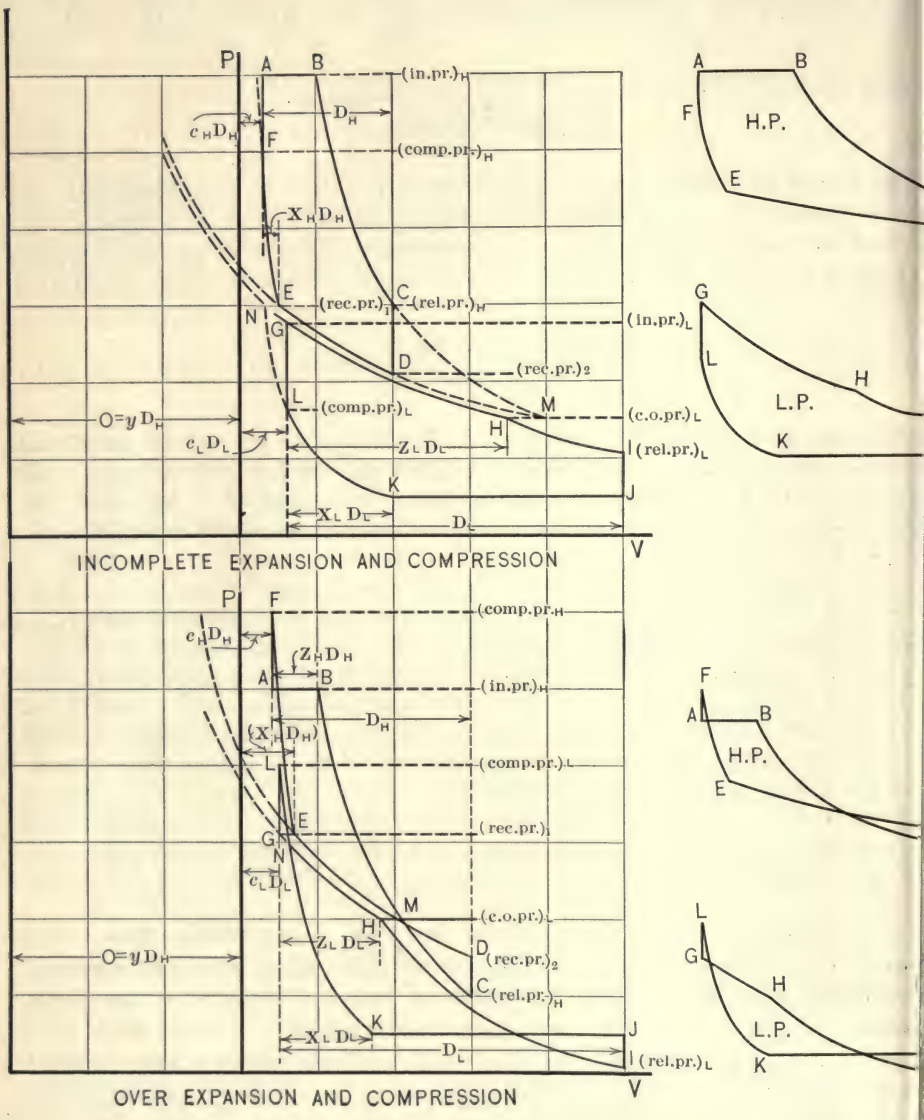


FIG. 97.—Work of Expansive Fluid in Compound Engine with Finite Receiver and w Clearance. Cycle XIII, Logarithmic Cycle XIV, Exponential Expansion, and Co pression.

Besides this there is a relation between H.P. exhaust and L.P. admission pressures, corresponding to the equality that existed for the infinite receiver, that may be set down as follows:

$$P_m V_m = P_b V_b, \quad \text{and} \quad P_m(V_m+O) = P_d(V_d+O);$$

$$\therefore \quad P_m V_m = P_b V_b = P_d(V_d+O) - P_m O, \quad \text{and} \quad P_m = P_h;$$

$$\therefore \quad P_d = \frac{P_b V_b + P_h O}{V_d + O}. \qquad (488)$$

Also

$$P_n V_n = P_k V_k, \quad \text{and} \quad P_n(V_n+O) = P_g(V_g+O);$$

$$\therefore \quad P_n V_n = P_k V_k = P_g(V_g+O) - P_n O, \quad \text{and} \quad P_n = P_e,$$

$$\therefore \quad P_g = \frac{P_k V_k + P_e O}{V_g + O}. \qquad (489)$$

These two expressions for the pressure at D and at G are not available in their present form, since they involve two unknown pressures—those at H and E, but two other equations of relation can be set down from which four equations, the four unknown pressures P_e, P_d, P_g and P_h, can be found. These other equations are

$$P_d(V_d+O) = P_e(V_e+O), \quad \text{or} \quad P_d = P_e\left(\frac{V_e+O}{V_d+O}\right), \qquad (490)$$

and

$$P_g(V_g+O) = P_h(V_h+O), \quad \text{or} \quad P_g = P_h\left(\frac{V_h+O}{V_g+O}\right). \qquad (491)$$

Equating (488) to (490),

$$\frac{P_b V_b + P_h O}{V_d + O} = P_e\left(\frac{V_e+O}{V_d+O}\right), \quad \text{or} \quad P_b V_b + P_h O = P_e(V_e+O),$$

and

$$P_e = \frac{P_b V_b + P_h O}{V_e + O},$$

Equating (489) to (491)

$$\frac{P_k V_k + P_e O}{(V_g+O)} = P_h\left(\frac{V_h+O}{V_g+O}\right), \quad \text{or} \quad P_k V_k + P_e O = P_h(V_h+O)$$

and

$$P_e = \frac{P_h(V_h+O) - P_k V_k}{O}.$$

Therefore

$$P_e = \frac{P_b V_b + P_h O}{V_e + O} = \frac{P_h(V_h+O) - P_k V_k}{O}.$$

$$P_b V_b O + P_h O^2 = P_h(V_h+O)(V_e+O) - P_k V_k(V_e+O);$$

$$P_h[(V_h+O)(V_e+O) - O^2] = P_b V_b O + P_k V_k(V_e+O),$$

$$P_h = \frac{P_b V_b O + P_k V_k(V_e+O)}{(V_h+O)(V_e+O) - O^2} = \frac{P_b V_b O + P_k V_k(V_e+O)}{V_h V_e + V_e O + V_h O}.$$

Therefore

$$P_h=\frac{P_bV_bO+P_kV_k(V_e+O)}{V_h(V_e+O)+V_eO} \quad (a)$$

Substitution will give

$$P_e=\frac{P_bV_b(V_h+O)+P_kV_kO}{(V_e+O)(V_h+O)-O^2} \quad (b)$$

$$P_g=\left[\frac{P_bV_bO+P_kV_k(V_e+O)}{V_h(V_e+O)+V_eO}\right]\left[\frac{V_h+O}{V_g+O}\right] \quad (c)$$

$$P_d=\left[\frac{P_bV_b(V_h+O)+P_kV_kO}{(V_e+O)(V_h+O)-O^2}\right]\left[\frac{V_e+O}{V_d+O}\right] \quad (d) \qquad (492)$$

It will be found that the use of these pressures is equivalent to the application of the equation of condition given in Eq. (487), for substitution of them reduces to an identity, therefore the work of the two cylinders can be set down by inspection in terms of point pressures and volumes and the above pressures substituted. The result will be the work in terms of the pressures and cylinder dimensions.

$$W_H=P_bV_b\left(1+\log_e\frac{V_c}{V_b}\right)-P_fV_f\left(1+\log_e\frac{V_e}{V_a}\right)-P_eV_e\log_e\frac{V_d+O}{V_e+O}-(P_a-P_f)V_a$$

$$=P_bV_b\left(1+\log_e\frac{V_c}{V_b}\right)-P_eV_e-P_eV_e\log_e\frac{V_e}{V_a}-P_eV_e\log_e\frac{V_d+O}{V_e+O}-P_aV_a+P_eV_e$$

$$=P_bV_b\left(1+\log_e\frac{V_c}{V_b}\right)-P_eV_e\log_e\left(\frac{V_e}{V_a}\right)\left(\frac{V_d+O}{V_e+O}\right)-P_aV_a.$$

Therefore

$$W_H=P_bV_b\left(1+\log_e\frac{V_c}{V_b}\right)-P_aV_a-\left[\frac{P_bV_b(V_b+O)+P_kV_kO}{(V_e+O)(V_h+O)-O^2}\right]V_e\log_e\left(\frac{V_e}{V_a}\right)\left(\frac{V_d+O}{V_e+O}\right) \qquad (4$$

$$W_L=P_gV_g\left(1+\log_e\frac{V_h+O}{V_g+O}\right)+P_hV_h\log_e\frac{V_i}{V_h}-P_iV_i\left(1+\log_e\frac{V_k}{V_i}\right)-\left(P_g-P_i\right)V_i-P_j(V_j-V_k)$$

$$=P_gV_g\log_e\left(\frac{V_h+O}{V_g+O}\right)+P_hV_h\log_e\frac{V_i}{V_h}-P_kV_k\log_e\frac{V_k}{V_i}-P_j(V_j-V_k).$$

Therefore

$$W_L=\left[\frac{P_bV_bO+P_kV_k(V_e+O)}{V_h(V_e+O)+V_eO}\left(\frac{V_h+O}{V_g+O}\right)\right]V_g\log_e\left(\frac{V_h+O}{V_g+O}\right)+\left[\frac{P_bV_bO+P_kV_k(V_e+O)}{V_h(V_e+O)+V_eO}\right]V_h\log_e\frac{V_i}{V_h}-P_kV_k\log_e\frac{V_k}{V_i}-P_j(V_j-V_k). \qquad (494)$$

Adding W_H and W_L

$$W=\left\{\begin{array}{l} P_bV_b\left(1+\log_e\frac{V_e}{V_b}\right)-P_kV_k\log_e\frac{V_k}{V_L}-P_aV_a-P_j(V_j-V_k) \\ -\left[\frac{P_bV_b(V_h+O)+P_kV_kO}{(V_e+O)(V_h+O)-O^2}\right]V_e\log_e\left(\frac{V_e}{V_a}\right)\left(\frac{V_d+O}{V_e+O}\right) \\ +\left[\frac{P_bV_bO+P_kV_k(V_e+O)}{V_h(V_e+O)+V_eO}\left(\frac{V_h+O}{V_g+O}\right)\right]V_g\log_e\frac{V_h+O}{V_g+O} \\ +\left[\frac{P_bV_bO+P_kV_k(V_e+O)}{V_h(V_e+O)+V_eO}\right]V_h\log_e\frac{V_t}{V_h}. \end{array}\right\} \quad (495)$$

While this Eq. (495) for the cyclic work is in terms of initial data, it is not of very much value by reason of its complex form. To show more clearly that only primary terms are included in it, the substitution of the usual symbols will be made.

$$W=144\times\left\{\begin{array}{l} (\text{in.pr.})(Z_H+c_H)D_H\left[1+\log_e\frac{1+c_H}{Z_H+c_H}\right] \\ -(\text{bk.pr.})(X_L+c_L)D_L\log_e\frac{X_L+c_L}{c_L} \\ -(\text{in.pr.})c_HD_H-(\text{bk.pr.})(1-X_L)D_L \\ -\frac{(\text{in.pr.})(Z_H+c_H)D_H[(Z_L+c_L)D_L+O]+(\text{bk.pr.})(X_L+c_L)D_LO}{[(X_H+c_H)D_H+O][(Z_L+c_L)D_L+O]-O^2}\times \\ (X_H+c_H)D_H\log_e\left(\frac{X_H+c_H}{c_H}\right)\left[\frac{(1+c_H)D_H+O}{(X_H+c_H)D_H+O}\right] \\ +\frac{(\text{in.pr.})(Z_H+c_H)D_HO+(\text{bk.pr.})(X_L+c_L)D_L[(X_H+c_H)D_H+O]}{(Z_L+c_L)D_L[(X_H+c_H)D_H+O]+(X_H+c_H)D_HO}\times \\ \left[\frac{(Z_L+c_L)D_L+O}{c_LD_L+O}\right]c_LD_L\log_e\left[\frac{(Z_L+c_L)D_L+O}{c_LD_L+O}\right] \\ +\frac{(\text{in.pr.})(Z_H+c_H)D_HO+(\text{bk.pr.})[(X_L+c_L)D_L][X_H+c_H)D_H+O]}{(Z_L+c_L)D_L[(X_H+c_H)D_H+O]+(X_H+c_H)D_HO}\times \\ (Z_L+c_L)D_L\log_e\left(\frac{1+c_L}{Z_L+c_L}\right) \end{array}\right\}. \quad (496)$$

Such equations as this are almost, if not quite, useless in the solution of problems requiring numerical answers in engine design, or in estimation of engine performance, and this fact justifies the conclusion that in cases of finite receivers graphic methods are to be used rather than the analytic for all design work. When estimates of power of a given engine are needed, this graphic work is itself seldom justifiable, as results of sufficient accuracy for all practical engine operation problems can be obtained by using the formulas derived for infinite receiver when reasonably, large and zero receivers when small and the pistons move together.

It might also be possible to derive an expression for work with an equivalent constant-receiver pressure, that would give the same total work and approximately the same work division as for this case, but this case so seldom arises that it is omitted here.

Inspection of the work equations makes it clear that any attempt to find equations of condition for equal division of work for the general case must be hopeless. It is, however, worth while to do this for one special case, that of complete expansion and compression in both stages, yielding the diagram Fig. 98. This is of value in drawing general conclusions on the influence of receiver size by comparing with the similar case for the infinite receiver.

By referring to Fig. 98, it will be seen by inspection that cylinder sizes, clearances and events of the stroke must have particular relative values in order

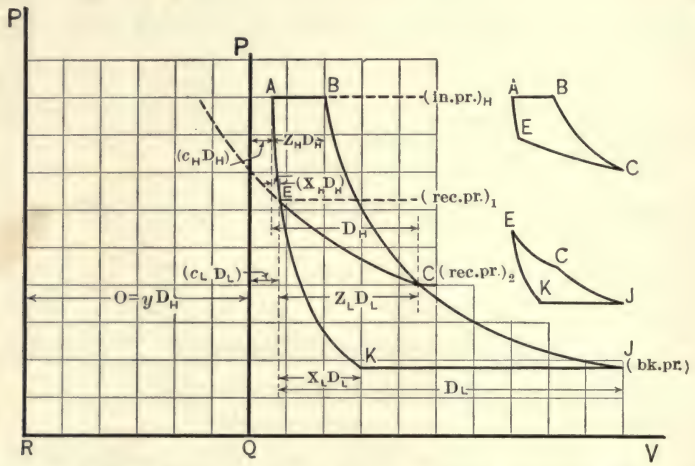


FIG. 98.—Special Case of Cycles XIII and XIV, Complete Expansion and Compression in both Cylinders of Compound Engine with Clearance and Finite Receiver.

to give the condition assumed, i.e., complete expansion and compression. It is, therefore, desirable to state the expressions for work in terms which may be regarded as fundamental. For this purpose are chosen, initial pressure (in.pr.), back pressure (bk.pr.); high-pressure displacement, D_H; cylinder ratio, R_C; high-pressure clearance, c_H; and ratio of receiver volume to high-pressure displacement, y. Call

$$\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right) = R_P.$$

It will be convenient first to find values of maximum receiver pressure (rec.pr.)$_1$, and minimum (rec.pr.)$_2$; high-pressure cut-off Z_H, and compression X_H; low-pressure clearance c_L, cut-off Z_L, and compression, X_L, in terms of these quantities. Nearly all of these are dependent upon the value of c_L and it will, therefore, be evaluated first.

From the points C and J, Fig. 98,

$$(\text{rec.pr.})_2 = (\text{bk.pr.})\frac{R_C(1+c_L)}{(1+c_H)}. \quad \ldots \quad (497)$$

From A and E,

$$(\text{rec.pr.})_1 = (\text{in.pr.})\frac{c_H}{R_C c_L}, \quad \ldots \quad (498)$$

and from E and C,

$$\frac{(\text{rec.pr.})_1}{(\text{rec.pr.})_2} = \frac{1+c_H+y}{R_C c_L+y}. \quad \ldots \quad (499)$$

Dividing Eq. (498) by Eq. (497) and equating to Eq. (499),

$$\frac{R_P c_H(1+c_H)}{R_C^2 c_L(1+c_L)} = \frac{1+c_H+y}{R_C c_L+y}.$$

Multiplying out and arranging with respect to c_L, the relation to be fulfilled in order that complete expansion and compression may be possible is,

$$c_L^2[R_C^2(1+c_H+y)]+c_L[R_C^2(1+c_H+y)-R_C R_P c_H(1+c_H)] \\ -[yR_P c_H(1+c_H)]=0. \quad (500)$$

This is equivalent to

$$c_L^2 l+c_L m-n=0, \quad \ldots \quad (501)$$

and the value of c_L is

$$c_L=\frac{(m^2+4ln)^{\frac{1}{2}}-m}{2l}. \quad \ldots \quad (502)$$

It is much simpler in numerical calculation to evaluate l, m, and n and insert their values in Eq. (502) than to make substitutions in Eq. (500), which would make a very cumbersome formula. $(\text{rec.pr.})_2$ and $(\text{rec.pr.})_1$ may now be evaluated from Eqs. (497) and (498) by use of the now known value of c_L.

High-pressure cut-off Z_H may be found from the relation of points B and J, Fig. 98,

$$R_P(Z_H+c_H)=R_C(1+c_L)$$

or

$$Z_H=\frac{R_C}{R_P}(1+c_L)-c_H. \quad \ldots \quad (503)$$

Low-pressure cut-off, Z_L from,

$$R_C(Z_L+c_L)=1+c_H,$$

or

$$Z_L=\frac{1+c_H}{R_C}-c_L. \quad \ldots \quad (504)$$

High-pressure compression, X_H,

$$X_H=c_L R_C-c_H. \quad \ldots \quad (505)$$

Low-pressure compression, X_L, by the use of points A and K,

$$R_C(X_L+c_L)=R_P c_H,$$

or

$$X_L=\frac{R_P}{R_C}c_H-c_L. \quad \ldots\ldots\ldots \quad (506)$$

If c_L is regarded as being part of the original data, though it is related to R_C, R_P, c_H and y as indicated in Eq. (488), the expressions for high- and low-pressure work and may be stated as follows:

$$W_H=144(\text{in.pr.})D_H\left\{\frac{R_C}{R_P}(1+c_L)\left[1+\log_e\frac{R_P}{R_C}\frac{(1+c_H)}{(1+c_L)}\right]\right.$$

$$\left.-\frac{R_C}{R_P}\frac{(1+c_L)}{(1+c_H)}(1+c_H+y)\log_e\frac{1+c_H+y}{R_C c_L+y}-c_H\left[1+\log_e\frac{c_L R_C}{c_H}\right]\right\}. \quad (507)$$

$$W_L=144(\text{in.pr.})D_H\left\{\frac{R_C}{R_P}\frac{(1+c_L)}{(1+c_H)}\left[(1+c_H+y)\log_e\frac{1+c_H+y}{R_C c_L+y}\right.\right.$$

$$\left.\left.+(1+c_H)\log_e R_C\frac{1+c_L}{1+c_H}\right]-\left[\frac{R_C}{R_P}(1+c_L)-c_H\right]-c_H\log_e\frac{R_P c_H}{R_C c_L}\right\} \quad (508)$$

Adding these two equations gives the total work of the cycle as follows:

$$W=144(\text{in.pr.})D_H R_C(1+c_L)\left\{\frac{1}{R_P}\left[1+\log_e\frac{R_P}{R_C}\frac{(1+c_H)}{(1+c_L)}\right]+\frac{1}{R_P}\log_e R_C\frac{1+c_L}{1+c_H}\right.$$

$$\left.-\frac{1+c_H+y}{R_P(1+c_H)}\log_e\frac{1+c_H+y}{R_C c_L+y}+\frac{1+c_H+y}{R_P(1+c_H)}\log_e\frac{1+c_H+y}{R_C c_L+y}\right\}$$

$$-144(\text{in.pr.})D_H\left\{c_H\left[1+\log_e\frac{c_L R_C}{c_H}\right]+c_H\log_e\frac{R_P c_H}{R_C c_L}+\frac{R_C}{R_P}(1+c_L)-c_H\right\}.$$

This, however, may be greatly simplified,

$$\log_e\frac{R_P(1+c_H)}{R_C(1+c_L)}+\log_e R_C\frac{1+c_L}{1+c_H}=\log_e R_P,$$

and

$$\log_e\frac{c_L R_C}{c_H}+\log_e\frac{R_P c_H}{R_C c_L}=\log_e R_P.$$

Hence

$$W=144(\text{in.pr.})D_H\left[\frac{R_C}{R_P}(1+c_L)-c_H\right]\log_e R_P.$$

$$=144(\text{in.pr.})D_H Z_H \log R_P. \quad \ldots\ldots\ldots \quad (509)$$

From this may be obtained mean effective pressure referred to the low-pressure cylinder, work per cubic foot supplied, and consumption per hour per indicated horse-power, all leading to the same results as were found for the case of complete expansion and compression with infinite receiver (Section 15,) and will not be repeated here.

To find the conditions of *equal division of work* between cylinders, equate Eqs. (507) and (508).

$$2\frac{R_C}{R_P}(1+c_L)-2c_H+\frac{R_C}{R_P}(1+c_L)\left[\log_e\frac{R_P}{R_C}\frac{(1+c_H)}{(1+c_L)}-\log_e\frac{R_C(1+c_L)}{(1+c_H)}\right]$$

$$-2\frac{R_C}{R_P}(1+c_L)\frac{1+c_H+y}{1+c_H}\log_e\frac{1+c_H+y}{R_Cc_L+y}+c_H\left[\log_e\frac{R_Pc_H}{R_Cc_L}-\log_e\frac{R_Cc_L}{c_H}\right]=0,$$

which may be simplified to the form,

$$1-\frac{1+c_H+y}{1+c_H}\log_e\frac{1+c_H+y}{R_Cc_L+y}+\log_e\frac{R^{\frac{1}{2}}{}_P(1+c_H)}{R_C(1+c_L)}+\frac{R_Pc_H}{R_C(1+c_L)}\left[\log_e\frac{R^{\frac{1}{2}}{}_Pc_H}{R_Cc_L}-1\right]=0. \tag{510}$$

This equation reduces to Eq. (376) of Section 11, when c_H and c_L are put equal to zero. In its present form, however, Eq. (510) it is not capable of solution, and it again becomes apparent that for such cases the graphical solution of the problem is most satisfactory.

Example 1. Method of calculating Diagram, Fig. 97.

Assumed data:

$P_a=P_b=120$ lbs. per square inch abs.
$P_m=P_h=30$ lbs. per square inch abs.
$P_j=10$ lbs. per square inch abs.

$V_j=V_i=2$ cu.ft.
$V_c=V_d=V_k=.8$ cu.ft.
$V_g=V_l=.24$ cu.ft.
$V_e=.2$ cu.ft.
$V_a=V_f=.12$ cu.ft.
$V_o=1.2$ cu.ft.
$V_b=.4$ cu.ft.

To locate point C:

$$P_c=\frac{P_bV_b}{V_c}=\frac{120\times.4}{.8}=60 \text{ lbs. per sq.in.}$$

To locate point M:

$$V_m=\frac{P_bV_b}{P_m}=\frac{120\times.4}{30}=1.6 \text{ cu.ft.}$$

To locate point D:

$$P_d(V_d+O)=P_m(V_m+O), \text{ or } P_d=30\frac{(1.6+1.2)}{.8+1.2}=42 \text{ lbs. per sq.in.}$$

To locate point E:

$$P_e=(V_e+O)=P_m(V_m+O), \quad \text{or} \quad P_e=\frac{2.8}{2.+1.2}\times 30=60 \text{ lbs. per sq.in.}$$

To locate point F:

$$P_f=\frac{P_eV_e}{V_f}=\frac{60\times .2}{.12}=100 \text{ lbs. per sq.in.}$$

To locate point L:

$$P_l=\frac{P_kV_k}{V_l}=\frac{10\times .8}{.24}33.3 \text{ lbs. per sq.in.}$$

To locate point N:

$$V_n=\frac{P_kV_k}{P_n}=\frac{10\times .8}{60}=.13 \text{ cu.ft.}$$

since $P_n=P_e$

To locate pcint G:

$$P_g(V_g+O)=P_n(V_n+O) \quad \text{or} \quad P_g=60\frac{(.13+1.2)}{(.24+1.2)}=55.5 \text{ lbs. per sq.in.}$$

To locate point H:

$$P_h(V_h+O)=P_g(V_g+O) \quad V_h=\frac{P_gV_g+P_gO-P_hO}{P_h}$$

or

$$V_h=\frac{.24\times 55.5+55.5\times 1.2-30\times 1.2}{30}=1.46 \text{ cu.ft.}$$

To locate point I:

$$P_i=\frac{P_hV_h}{V_i}=\frac{1.46\times 30}{2}=21.7 \text{ lbs. per sq.in.}$$

Example 2. Find the horse-power of a 12- and 18×24-in. engine, running at 125 R.P.M., with a receiver volume twice as large as the low-pressure cylinder, 6 per cent clearance in the high-pressure cylinder, 4 per cent in the low, when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, high-pressure cut-off $\frac{1}{2}$, low-pressure $\frac{3}{4}$, high-pressure compression 10 per cent, low 3 per cent. From Eq. (496) divided by $144D_L$, and with the values as given above, the (m.e.p.) is equal to following expression:

$$150\times .56\times\frac{1}{2.25}\left(1+\log_e\frac{1.06}{.56}\right)-10\times .34\log_e\frac{.34}{.04}-150\times .06\times\frac{1}{2.25}-10\times .7$$

$$-\frac{150\times .56[.79\times 2.25+4.5]+10(.34)2.25\times 4.5}{[.16+4.5][.79\times 2.25+4.5]-(4.5)^2}\times .16\times\frac{1}{2.25}\log_e\frac{.16}{.06}\times\frac{1.06\times 2.25+4.5}{.16\times 2.25+4.5}$$

$$+\frac{150\times .56\times 4.5+10\times .34\times 2.25[.16+2.25]}{.79\times 2.25[.16+2.25]+(.16\times 2.25)}\left[\frac{.79\times 2.25+4.5}{.04\times 2.25+4.5}\right].04\times 2.25\log_e\frac{.79\times 2.25+4.5}{.04\times 2.25+4.5}$$

$$+\frac{150\times .56\times 4.5+10(.34\times 2.25)[.16+2.25]}{.79\times 2.25[.16+4.5]+(.16\times 2.25)}\left[.79\log_e\frac{1.04}{.79}\right]=49.2 \text{ lbs., per sq.in.}$$

hence the horse-power will be 191.

Prob. 1. Find the work done in the high-pressure cylinder and in the low-pressure cylinder of the following engine under the conditions given.

Engine 14 and 30×28 ins., 100 R.P.M., 5 per cent clearance in each cylinder, high-pressure cut-off $\frac{3}{10}$, low-pressure cut-off $\frac{4}{10}$, high-pressure compression $\frac{2}{10}$, low-pressure compression $\frac{1}{10}$, initial pressure 100 lbs. per square inch gage, back pressure 5 lbs. per square inch absolute, and receiver volume 3 times the high-pressure displacement. Logarithmic expansion.

Prob. 2. The following data are available: initial pressure 200 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, engine 10×15×22 ins., with 5 per cent clearance in the high- and low-pressure cylinders, speed 100 R.P.M. What will be the cut-offs, and compression percentages to give complete expansion and compression. Logarithmic expansion?

Prob. 3. What will be the work done by the above engine working under these conditions?

Prob. 4. What must be the low-pressure clearance, cut-offs, and compression percentages, to give complete expansion and compression for a similar engine working under the same conditions as those of Prob. 2, but equipped with a receiver twice as large as the high-pressure cylinder?

18. Compound Engine with Finite Receiver, Exponential Law, with Clearance and Compression, Cycle XIV. General Relations between Pressures, Dimensions, and Work when H.P. Exhaust and L.P. Admission are Independent. It cannot be expected that the treatment of this cycle by formulas will give satisfactory results, since even with the logarithmic expansion law, Cycle XIII gave formulas of unmanageable form. For the computation of work done during the cycle, however, and for the purpose of checking pressures and work determined by graphical means, it is desirable to have set down the relations of dimensional proportions, initial and final pressures, and valve adjustments, to the receiver pressures, release pressures and work of the individual cylinders.

The conditions of a steady state, explained previously, require that (Fig. 97)

$$V_b - V_a = V_h\left(\frac{P_h}{P_b}\right)^{\frac{1}{s}} - V_k\left(\frac{P_k}{P_b}\right)^{\frac{1}{s}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (511)$$

which is the same as to say, that the quantity of fluid passing per cycle in the high-pressure cylinder must equal that passing in the low. Expressed in terms of dimensions,

$$D_H(c_H+Z_H) - D_H(c_H+X_H)\left[\frac{(\text{rec.pr.})_1}{(\text{in.pr.})}\right]^{\frac{1}{s}}$$
$$= D_H R_C(c_L+Z_L)\left[\frac{(\text{cut-off pr.})_L}{(\text{in.pr.})}\right]^{\frac{1}{s}} - D_H R_C(c_L+X_L)\left[\frac{(\text{bk.pr.})}{(\text{in.pr.})}\right]^{\frac{1}{s}}$$

or, rearranging, and using $R_P = \dfrac{(\text{in.pr.})}{(\text{bk.pr.})}$.

$$c_H+Z_H+R_C(c_L+X_L)\ \frac{1}{R_P^{\frac{1}{s}}} = (c_H+X_H)\left[\frac{(\text{rec.pr.})_1}{(\text{in.pr.})}\right]^{\frac{1}{s}}$$
$$+R_C(c_L+Z_L)\left[\frac{(\text{cut-off pr.})_L}{(\text{in.pr.})}\right]^{\frac{1}{s}}, \quad .(512)$$

an equation which contains two unknown pressures (rec.pr.)$_1$ and (cut off pr.)$_L$. To evaluate either, another equation must be found:

$$V_n = V_k\left(\frac{P_k}{P_n}\right)^{\frac{1}{s}},$$

where $P_n = P_c$, so that

$$V_n = R_C D_H (c_L + X_L)\left[\frac{(\text{bk pr.})}{(\text{rec.pr.})_1}\right]^{\frac{1}{s}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (513)$$

Hence

$$P_h = P_n\left(\frac{V_n + O}{V_h + O}\right)^s,$$

or

$$(\text{cut-off pr.})_L = (\text{rec.pr.})_1\left[\frac{y + R_C(c_L + X_L)\left[\frac{(\text{bk.pr.})}{(\text{rec.pr.})_1}\right]^{\frac{1}{s}}}{y + R_C(c_L + Z_L)}\right]^s$$

$$= \left[\frac{y(\text{rec.pr.})_1^{\frac{1}{s}} + R_C(c_L + X_L)(\text{bk.pr.})^{\frac{1}{s}}}{y + R_C(c_L + Z_L)}\right]^s, \quad (514)$$

which constitutes a second equation between (cut-off pr.)$_L$ and (rec.pr.)$_1$, which used with Eq. (512) makes it possible to solve for the unknown. By substitution in Eq. (512) and rearranging,

$$(\text{rec.pr.})_1 = (\text{bk.pr.})\left[\frac{R_P^{\frac{1}{s}}(c_H + Z_H)[y + R_C(c_L + Z_L)] + R_C y(c_L + X_L)}{(c_H + X_H)[y + R_C(c_L + Z_L)] + R_C y(c_L + Z_L)}\right]^s. \quad . \quad (515)$$

This expression is of great assistance even in the graphical construction of the diagram, as otherwise, with all events known a long process of trial and error must be gone through with. It should also be noted that when $s = 1$ this expression does not become indeterminate and can, therefore, be used to solve for maximum receiver pressure for Cycle XIII, as well as Cycle XIV.

Cut-off pressure of the low-pressure cylinder, which is same as the pressure at H or at M, Fig. 97, is now found most easily by inserting the value found by Eq. (515) for (rec.pr.)$_1$ in Eq. (514).

Enough information has been gathered now to set down the expressions for work.

$$W_H = 144 D_H \Bigg\{ (\text{in.pr.})\frac{c_H + Z_H}{s-1}\left[s - \left(\frac{c_H + Z_H}{1 + c_H}\right)^{s-1}\right] - c_H(\text{in.pr.})$$

$$- \frac{(c_H + X_H)}{s-1}(\text{rec.pr.})_1\left[\left(\frac{c_H + X_H}{c_H}\right)^{s-1} - 1\right]$$

$$- \frac{(y + c_H + X_H)}{s-1}(\text{rec.pr.})_1\left[1 - \left(\frac{y + c_H + X_H}{y + c_H + 1}\right)^{s-1}\right]\Bigg\}. \quad . \quad (51$$

$$W_L = 144 D_H \left\{ (\text{cut-off pr.})_L \frac{y+R_C(c_L+Z_L)}{s-1}\left[\left(\frac{y+R_C(c_L+Z_L)}{y+R_C c_L}\right)^{s-1}-1\right]\right.$$

$$+(\text{cut-off pr.})_L \frac{y+R_C(c_L+Z_L)}{s-1}\left[1-\left(\frac{y+R_C(c_L+Z_L)}{y+R_C(c_L+1)}\right)^{s-1}\right]$$

$$\left.-(\text{bk.pr.})\frac{R_C(c_L+X_L)}{s-1}\left[\left(\frac{c_L+X_L}{c_L}\right)^{s-1}-1\right]-(\text{bk.pr.})(1-X_L)R_C\right\}. \quad . \quad (517)$$

Addition of these two Eqs. (516) and (517) gives an expression for the total work W, and equating them gives conditions which must be fulfilled to give equality of work in the high- and low-pressure cylinders. Since these equations so obtained cannot be simplified or put into more useful form, there is no object in inserting them here, but if needed for any purpose they may be easily written. In finding the conditions of equal work, the volumes of $(\text{rec.pr.})_1$ and $(\text{cut-off pr.})_L$ must be inserted from Eqs. (514) and (515) in (516) and (517), in order to have terms in the two equations consist of fundamental data. This, however, increases greatly the complication of the formula.

After finding the total work of the cycle, the mean effective pressure referred to the low pressure is obtained by dividing by $144 \times D_L$.

To assist in finding the work per cubic foot supplied and consumption, and the cubic feet or pounds per hour per I.H.P. it is important to know the volume of fluid supplied per cycle,

$$(\text{Sup.Vol.}) = \overline{QB} = D_H\left[(c_H+Z_H)-(c_H+X_H)\left(\frac{(\text{rec.pr.})_1}{(\text{in.pr.})}\right)^{\frac{1}{s}}\right]. \quad . \quad (518)$$

Example. Find the horse-power of and compressed air steam used by a 12- and 18×24-in. engine running at 125 R.P.M., with a receiver volume twice as large as the low-pressure cylinder, 6 per cent clearance in the high-pressure cylinder, 4 per cent in the low, when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, high-pressure cut-off $\frac{1}{2}$, low-pressure cut-off $\frac{3}{4}$, high-pressure compression 10 per cent, low-pressure compression 30 per cent, and expansion and compression follow the law $PV^{1.4} = c$.

From Eq. (515) $(\text{rec.pr.})_1$ is found to be as follows when values for this problem are substituted:

$$(\text{rec.pr.})_1 = 10\left[\frac{15^{\cdot 7}(.56)[4.5+2.25(.79)]+4.5\times 2.25(.34)}{.16\,[4.5+2.25\times .79]+4.5\times 2.25\times .79}\right]^s = 81.7 \text{ lbs. sq.in. absolute,}$$

and by using this in Eq. (514)

$$(\text{cut-off pr.})_L = \left(\frac{4.5\times 81.7^{.71}+2.25\times .34\times 10^{.71}}{4.5+2.25\times .79}\right)^s = 53 \text{ lbs. sq.in. absolute.}$$

It is now possible by use of Eqs. (516) and (517) by addition and division by $144D_L$, to obtain (m.e.p.). Substituting the values found above and carrying out the process just mentioned.

$$(\text{m.e.p.}) = \frac{1}{2.25}\left\{ 150\times\frac{.56}{.4}\times\left[1.4-\left(\frac{.56}{1.06}\right)^{.4}\right]-.06\times150-\frac{.16}{.4}\times82\left[\left(\frac{.16}{.06}\right)^{.4}-1\right]\right.$$

$$-\frac{4.5+.16}{.4}\times82\left[1-\left(\frac{4.5+.16}{45+1.06}\right)^{.4}\right]+53\times\frac{4.5+2.25\times.79}{.4}\left[\left(\frac{4.5+2.25\times.79}{4.5+2.25+.04}\right)^{.4}-1\right]$$

$$+53\times\frac{4.5+2.25\times.79}{.4}\left[1-\left(\frac{4.5+2.25\times.79}{4.5+2.25\times1.04}\right)^{.4}\right]-10\times\frac{2.25\times.34}{.4}\left[\left(\frac{.34}{.04}\right)^{.4}-1\right]$$

$$\left.-10\times2.25\times.7\right\} = 51.5 \text{ lbs. sq.in.}$$

hence the horse-power will be 200.

By means of Eq. (518) the supply volume may be found. This gives upon substituting of the proper values:

$$(\text{Sup.Vol.}) = D_H\left[.56-16\times\left(\frac{82}{150}\right)^{.71}\right] = .46D_H.$$

$$\text{Cubic feet per hour per I.H.P.} = \frac{13{,}750}{(\text{m.e.p.})}\times\frac{\text{Sup.Vol.}}{D_L},$$

$$= \frac{13{,}750}{51.5}\times\frac{.46}{2.25} = 54.5,$$

hence the total volume of air per hour will be

$$54.5\times200 = 10900 \text{ cu.ft.}$$

Prob. 1. What will be the receiver pressure and L.P. cut-off pressure for a cross-compound compressed air engine with 5 per cent clearance in each cylinder, running on 100 lbs. per square inch gage initial pressure and atmospheric exhaust, when the high-pressure cut-off is $\frac{1}{4}$, low-pressure $\frac{3}{8}$, high-pressure compression 15 per cent, low 25 per cent, and $s = 1.4$. Receiver volume is twice the high-pressure cylinder volume.

Prob. 2. Find the superheated steam per hour necessary to supply a 14- and 21×28-in. engine with 5 per cent clearance in each cylinder and a receiver twice the size of the high-pressure cylinder when the initial pressure is 125 lbs. per square inch gage, back pressure 7 lbs. per square inch absolute, speed 100 R.P.M., high-pressure cut-off $\frac{1}{2}$, low-pressure $\frac{4}{15}$, high-pressure compression 15 per cent, low pressure 40 per cent and $s = 1.3$.

NOTE: $\delta = .3$.

Prob. 3. If the high-pressure cut-off is changed to $\frac{1}{4}$ without change of any other factor in the engine of Prob. 2, how will the horse-power, total steam per hour and steam per horse-power per hour be affected? If it is changed to $\frac{3}{4}$?

Prob. 4. A boiler capable of supplying 5000 lbs. of steam per hour at rated load furnishes steam for a 12- and 18×24-in. engine with 5 per cent clearance in each cylinder and running at 125 R.P.M. The receiver is three times as large as the high-pressure cylinder

the initial pressure 150 lbs. per square inch gage, back pressure 5 lbs per square inch absolute, the low-pressure cut-off fixed at $\frac{1}{2}$ and low-pressure compression fixed at 30 per cent. At what per cent of its capacity will boiler be working for these following cases, when $=1.2$ for all and 20% of the steam condenses during admission?

(*a*) high-pressure cut-off $\frac{1}{4}$, high-pressure compression 80 per cent,

(*b*) high-pressure cut-off $\frac{1}{2}$, high-pressure compression 20 per cent,

(*c*) high-pressure cut-off $\frac{3}{4}$, high-pressure compression 10 per cent.

NOTE: $\delta = .33$.

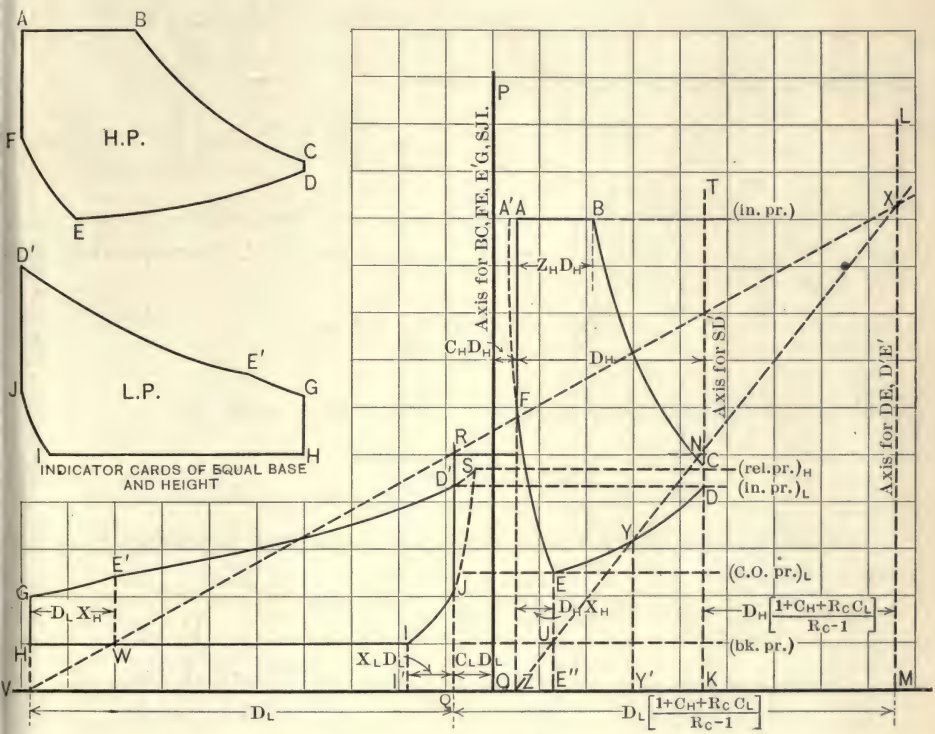


FIG. 99.—Work of Expansion in Compound Engine without Receiver and with Clearance. Cycle XV, Logarithmic Expansion; Cycle XVI, Exponential, High-pressure Exhaust and Low-pressure Admission Coincident.

19. Compound Engine without Receiver. Logarithmic Law, with Clearance and Compression, Cycle XV. General Relations between Pressures, Dimensions, and Work when H.P. Exhaust and L.P. Admission are Coincident. The graphical construction for this cycle has been described to some extent in connection with the first description of the cycle, given in Section 8, of this chapter, and is represented here by Fig. 99 in more detail.

To show that the expansion from D to E is the same as if volumes were measured from the axis ML, consider a point Y on DE. If the hypothesis is correct

$$P_d \times \overline{KM} = P_y \times (\overline{KM} + \overline{Y'K}). \quad . \quad . \quad . \quad . \quad . \quad . \quad (519)$$

The true volume when the piston is at the end D of the stroke, i $D_H(1+c_H+R_Cc_L)$, and at Y, the true volume is

$$D_H(1+c_H+R_Cc_L)-D_Hy+D_Ly,$$

where y is the fraction of the return stroke that has been completed in both cylinders when the point Y has been reached. Then

$$P_aD_H(1+c_H+R_Cc_L)=P_yD_H(1+c_H+R_Cc_L)+P_yD_H(R_C+1)y.$$

Dividing though by (R_C-1),

$$P_a\left[\frac{D_H+c_H+R_Cc_L}{R_C-1}\right]=P_y\left[D_H\frac{1+c+R_Cc_L}{R_C-1}+D_Hy\right]. \quad . \quad (520)$$

This equation may be observed to be similar in form to Eq. (519). Moreover, the last term within the bracket, D_Hy, is equal to the corresponding term $Y'K$, in Eq. (519), hence,

$$\overline{KM}=D_H\left[\frac{1+c_H+R_Cc_L}{R_C-1}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (521)$$

Similarly, the distance $\overline{QM}$, or equivalent volume at D' is

$$\overline{QM}=D_L\left[\frac{1+c_H+R_Cc_L}{R_C-1}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (522)$$

The following quantities will be evaluated preparatory to writing the expressions for work:

$$(\text{rel.pr.})_H=(\text{in.pr.})\frac{Z_H+c_H}{1+c_H}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (523)$$

$$V=D_HR_C(c_L+X_L)\frac{(\text{bk.pr.})}{(\text{rel.pr.})_H}=D_HR_C(c_L+X_L)\frac{(\text{bk.pr.})}{(\text{in.pr.})}\frac{1+c_H}{Z_H+c_H}. \quad . \quad (524)$$

$$(\text{in.pr.})_L=P_{a'}=(\text{rel.pr.})_H\frac{\overline{SC}}{\overline{DD'}}$$

$$=(\text{in.pr.})\left[\frac{Z_H+c_H}{1+c_H}\right]^{D_H\left[1+c_H+R_C(c_L+X_L)\frac{(\text{bk.pr.})}{(\text{in.pr.})}\frac{1+c_H}{Z_H+c_H}\right]}_{D_H[1+c_H+R_Cc_L]}$$

$$=(\text{in.pr.})\frac{Z_H+c_H+R_C(X_L+c_L)\frac{(\text{bk.pr.})}{(\text{in.pr.})}}{1+c_H+R_Cc_L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (525)$$

$$(\text{cut-off pr.})_L=(\text{in.pr.})_L\frac{\overline{KM}}{\overline{E''M}}=(\text{in.pr.})_L\left[\frac{1+c_H+R_Cc_L}{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}\right]$$

$$=(\text{in.pr.})\left[\frac{Z_H+c_H+R_C(X_L+c_L)\frac{(\text{bk.pr.})}{(\text{in.pr.})}}{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}\right] \quad . \quad . \quad . \quad . \quad . \quad (526)$$

The ratio of expansion from E' to G is equal to

$$\frac{(\text{cut-off pr.})_L}{(\text{rel.pr.})_L}=\frac{1+c_L}{(1-X_H)+c_L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (527)$$

Hence

$$W_H=144D_H(\text{in.pr.})\left\{(Z_H+c_H)\left[1+\log_e\frac{1+c_H}{Z_H+c_H}\right]\right.$$

$$-\left[\frac{Z_H+c_H+R_C(X_L+c_L)\dfrac{(\text{bk.pr.})}{(\text{in.pr.})}}{R_C-1}\right]\log_e\left[\frac{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}{1+c_H+R_Cc_L}\right]$$

$$\left.-\left[\frac{Z_H+c_H+R_C(X_L+c_L)\dfrac{(\text{bk. pr.})}{(\text{in.pr.})}}{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}\right](c_H+X_H)\log_e\frac{c_H+X_H}{c_H}-c_H\right\} \quad . \quad . \quad . \quad . \quad . \quad (528)$$

$$W_L=144D_L(\text{in.pr.})\left\{\left[\frac{Z_H+c_H+R_C(X_L+c_L)\dfrac{(\text{bk.pr.})}{(\text{in.pr.})}}{R_C-1}\right]\times\right.$$

$$\log_e\left[\frac{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}{1+c_H+R_Cc_L}\right]$$

$$\left.+\left[\frac{Z_H+c_H+R_C(X_L+c_L)\dfrac{(\text{bk.pr.})}{(\text{in.pr.})}}{1+c_H+R_Cc_{LH}+(R_C-1)(1-X_H)}\right](1+c_L-X_H)\log_e\left(\frac{1+c_L}{1+c_L-X_H}\right)\right\}$$

$$-144D_L(\text{bk.pr.})\left\{1-X_L+(X_L+c_L)\log_e\frac{X_L+c_L}{c_L}\right\}. \quad . \quad . \quad (529)$$

The *total work* found by adding W_H and W_L as given above, leads to the following:

$$W=144D_H(\text{in.pr.})\left\{(Z_H+c_H)\log_e\left(\frac{1+c_H}{Z_H+c_H}\right)\right.$$

$$+Z_H+\left[Z_H+c_H+R_C(X_L+c_L)\frac{(\text{bk.pr.})}{(\text{in.pr.})}\right]\log_e\left[\frac{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}{1+c_H+R_Cc_L}\right]$$

$$+\left[\frac{Z_H+c_H+R_C(X_L+c_L)\dfrac{(\text{bk.pr.})}{(\text{in.pr.})}}{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}\right]\left[R_C(1+c_L-X_H)\times\right.$$

$$\left.\left.\log_e\left(\frac{1+c_L}{1+c_L-X_H}\right)-(c_H+X_H)\log_e\left(\frac{c_H+X_H}{c_H}\right)\right]\right\}$$

$$-144D_L(\text{bk.pr.})\left\{1-X_L+(X_L+c_L)\log_e\left(\frac{X_L+c_L}{c_L}\right)\right\}. \quad . \quad . \quad (530)$$

This is the general expression for the work of the compound engine without receiver, with clearance and compression, when high-pressure exhaust and low-pressure admission are simultaneous and expansion and compression logarithmic, in terms of fundamental data regarding dimensions and valve periods.

From this the usual expressions for mean effective pressure, work per cubic foot supplied, and consumption per hour per I.H.P., may be easily written, provided the supply volume is known. This is given by

$$(\text{Sup.Vol.}) = \overline{A'B} = D_H\left[(Z_H+c_H)-(c_H+X_H)\cdot\frac{(\text{cut-off pr.})_L}{(\text{in.pr.})}\right]$$

$$= D_H\left\{Z_H+c_H-(c_H+X_H)\frac{Z_H+c_H+R_C(X_L+c_L)\dfrac{(\text{bk.pr.})}{(\text{in.pr.})}}{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}\right\}. \quad . \quad (531)$$

To find the conditions which must be fulfilled to give equal work in the two cylinders, equate Eqs. (528) and (529).

$$Z_H+(Z_H+c_H)\log_e\left(\frac{1+c_H}{Z_H+c_H}\right)$$

$$-\frac{R_C+1}{R_C-1}\left[Z_H+c_H+R_C(X_L+c_L)\frac{(\text{bk.pr.})}{(\text{in.pr.})}\right]\log_e\left[\frac{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}{1+c_H+R_Cc_L}\right]$$

$$-\left[\frac{Z_H+c_H+R_C(X_L+c_L)\dfrac{(\text{bk.pr.})}{(\text{in.pr.})}}{1+c_H+R_Cc_L+(R_C-1)(1-X_H)}\right]\left\{R_C(1+c_L-X_H)\log_e\left(\frac{1+c_L}{1+c_L-X_H}\right)\right.$$

$$\left.+(c_H+X_H)\log_e\frac{c_H+X_H}{c_H}\right\}+R_C\frac{(\text{bk.pr.})}{(\text{in.pr.})}\left\{1-X_L+(X_L+c_L)\log_e\left(\frac{X_L+c_L}{c_L}\right)\right\}=0. \quad (532)$$

These expressions are perfectly general for this cycle, and expressed in terms of fundamental data, but are so complicated that their use is very limited, as in the case of some of the general expressions previously derived for other cycles.

As in other cycles, it is desirable to investigate a special case, that of complete expansion and compression in both cylinders, Fig. 100. First it is necessary to determine what are regarded as fundamental data in this case, and then to evaluate secondary quantities in terms of these quantities. The following items are assumed to be known: (in.pr.), (bk.pr.), which is equal to (rel.pr.)$_L$, R_C, c_H, and c_L, and D_H, which are dimensions, and it is known that the pressure at the end of compression in L.P. is equal to (rel.pr.)$_H$.

Referring to the diagram, displacements, clearances, and the axis for the common expansion, ML, can all be laid out, and the location of the points A and G determined.

The points E and E' are at the end of the common expansion within the two cylinders, and at beginning of high-pressure compression and of separate low-pressure expansion, hence $p_e = p_e'$.

From the points A and E:

$$p_e = (\text{in.pr.})\frac{c_H}{X_H + c_H}.$$

From the points G and E',

$$p_{e'} = p_e = (\text{bk.pr.})\frac{1+c_L}{1+c_L-X_H},$$

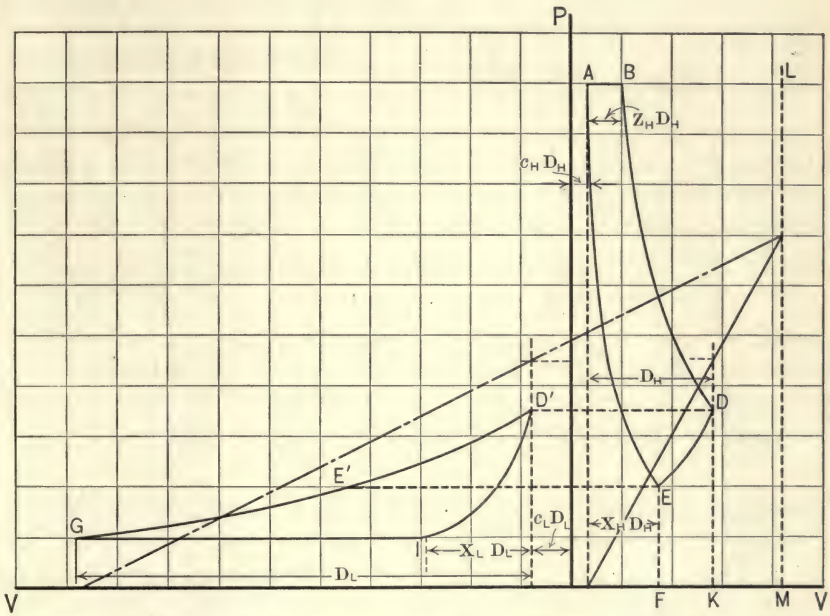


FIG. 100.—Special Case of Cycles XV and XVI. Complete Expansion and Compression in both Cylinders of Compound Engine without Receiver and with Clearance High-Pressure Exhaust and Low-Pressure Admission Coincident

and equating,

$$(\text{in.pr.})\frac{c_H}{X_H + c_H} = (\text{bk.pr.})\frac{1+c_L}{1+c_L-X_H},$$

whence

$$X_H = c_H\frac{(1+c_L)(R_P-1)}{1+c_L+c_H R_P}, \quad \ldots\ldots\ldots \quad (533)$$

where

$$R_P = \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)$$

Substituting the value of X_H in either of the expressions for p_e, which is the low-pressure cut-off pressure,

$$(\text{cut-off pr.})_L = p_e = (\text{bk.pr.})\left[\frac{1+c_L+R_P c_H}{1+c_L+c_H}\right] = p_{e'}. \quad \ldots\ldots \quad (534)$$

It may be noted here that the cylinder ratio does not enter into this, but only clearances and pressures. In the no-clearance case, it may be remembered that the point E or E' was not present, as it coincided with G.

Next, to find the high-pressure release pressure, p_d, by means of points E and D, and their relation to the axis ML, Fig. 100.

$$(\text{rel.pr.})_H = p_d = p_e \frac{\overline{FM}}{\overline{KM}} = (\text{bk.pr.}) \left[\frac{1+c_L+c_H R_C}{1+c_L+c_H} \right] \left[\frac{1-X_H+\dfrac{1+c_H+R_C c_L}{R_C-1}}{\dfrac{1+c_H+R_C c_L}{R_C-1}} \right]$$

$$= (\text{bk.pr.}) \frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} \quad \ldots \ldots \ldots \ldots \quad (535)$$

Knowing the release pressure of the high-pressure cylinder, it is possible to find the high-pressure cut-off and compression necessary to give the required performance.

$$Z_H = (1+c_H) \frac{(\text{rel.pr.})_H}{(\text{in.pr.})} - c_H = \frac{(1+c_H)}{R_P} \left[\frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} \right] - c_H, \quad (536)$$

$$X_L = c_L \frac{(\text{rel.pr.})_H}{(\text{bk.pr.})} - c_L = c_L \left[\frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} - 1 \right] \quad \ldots \quad (537)$$

The work of the two cylinders is as follows:

$$W_H = 144 D_H (\text{in.pr.}) \left\{ \frac{(1+c_H)}{R_R} \left[\frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} \right] \left[1+\log_e \frac{R_P(1+c_H+R_C c_L)}{R_C(1+c_L)+R_P c_H} \right] - c_H \right.$$

$$- \left[\frac{1+c_H+R_C c_L}{R_C-1} \right] \left[\frac{R_C(1+C_L)+R_P c_H}{1+c_H+R_C c_L} \right] \frac{1}{R_P} \log_e \left[\frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} \right] \left[\frac{1+c_L+c_H}{1+c_L+R_P c_H} \right]$$

$$\left. - c_H \log_e \frac{R_P(1+c_L+c_H)}{1+c_L+R_P c_H} \right\}$$

$$= 144 D_H (\text{in.pr.}) \left\{ \frac{(1+c_H)}{R_P} \frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} \left[1+\log_e \frac{R_P(1+c_H+R_C c_L}{R_C(1+c_L)+R_P c_L} \right] - c_H \right.$$

$$- \left[\frac{R_C(1+c_L)+R_P c_H}{R_P(R_C-1)} \right] \log_e \left[\frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} \right] \left[\frac{1+c_L+c_H}{1+c_L+R_P c_H} \right]$$

$$\left. - c_H \log_e \left[\frac{R_P(1+c_L+c_H)}{1+c_L+R_P c_H} \right] \right\} \quad \ldots \ldots \ldots \ldots \quad (538)$$

$$W_L = 144 D_L (\text{bk.pr.}) \left\{ \left[\frac{R_C(1+c_L)+R_P c_H}{(R_C-1)} \right] \log_e \left[\frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} \right] \left[\frac{1+c_L+c_H}{1+c_L+R_P c_H} \right] \right.$$

$$+ (1+c_L) \log_e \left[\frac{1+c_L+R_P c_H}{1+c_L+c_H} \right] - c_L \left[\frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} \right] \log_e \left[\frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} \right]$$

$$\left. - 1 + c_L \left[\frac{R_C(1+c_L)+R_P c_H}{1+c_H+R_C c_L} - 1 \right] \right\} \quad \ldots \quad (539)$$

These expressions, when added and simplified, give the following for total work per cycle,

$$W=144D_L(\text{bk.pr.})\left\{1-c_L\left[\frac{R_C(1+c_L)+R_Pc_H}{1+c_H+R_Cc_L}-1\right]\right\}\log_e R_P, \quad . \quad . \quad (540)$$

in which of course D_HR_C may be used instead of D_L and $\frac{(\text{in.pr.})}{R_P}$ instead of (bk.pr.) and then

$$W=144D_HZ_H(\text{in.pr.})\log_e R_P, \quad . \quad . \quad . \quad . \quad (541)$$

Z_H having the value of Eq. (536).

Equality of work the in high- and low-pressure cylinders results, if W^H Eq. (538) equals W_L, Eq. (539), or if

$$2W_H=W, \quad \text{or} \quad 2W_L=W,$$

all of which lead to equivalent expressions. Simplification of these expressions, however, does not lead to any direct solution, and hence the equations will not be given here.

Example. Find (*a*) the horse-power and (*b*) steam used per hour for a 12- and 18×24-in. tandem compound engine with no receiver, 6 per cent clearance in the high-pressure cylinder, and 4 per cent in the low, when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., high-pressure cut-off ½, high-pressure compression 15 per cent and low-pressure compression is complete.

(*a*) Since the low-pressure compression is complete, the pressure at end of compression must be equal to the release pressure of the high. This latter quantity may be found rom the relation $(\text{in.pr.})(Z_H+c_H)=(\text{rel.pr.})_H(1+c_H)$, or

$$(\text{rel.pr.})_H=150\frac{(.56)}{1.06}=79.3 \text{ lbs. per sq.in. absolute.}$$

Low-pressure compression may be found from the relation $(\text{rel.pr.})_H(c_L)=(\text{bk.pr.})(c_L+X_L)$, or $X_L=.28$. (m.e.p.) may be found from Eq. (540) divided by $144D_L$,which on substitution gives

$$\frac{150}{2.25}\left\{(.5+.06)\log_e\frac{1.06}{.56}\right.$$

$$+.5+\left[.5+.06+2.25(.28+.04)\frac{10}{150}\right]\log_e\left[\frac{1+.06+2.25\times.04+1.25\times.85}{1+.06+2.25\times.04}\right]$$

$$+\left[\frac{.5+.06+2.25(.28+.04)\frac{10}{150}}{1+.06+2.25\times.04+1.75+.85}\right]2.25(1.04-.15)\log_e\frac{1+.04}{1+.04-.15}$$

$$\left.-(.06+.15)\log_e\frac{.06+.15}{.06}\right\}-10\left\{1.28+(.28+.04)\log_e\frac{.28+.04}{.04}\right\}=69.7. \text{ lbs. sq. in.}$$

and the horse-power will be 271.

(*b*) Since the consumption in cubic feet per hour per horse-power is equal to

$$\frac{13{,}750}{(\text{m.e.p.})} \times \frac{\text{Sup.Vol.}}{D_L},$$

and supply volume is given by Eq. (531), this becomes

$$\frac{13{,}750}{69.7} \times \frac{1}{2.25} \left\{ .56 - .21 \left(\frac{.56 + 2.25(.32)\frac{10}{150}}{1.06 + .09 + 1.25 \times .85} \right) \right\} = 44,$$

hence the consumption per hour will be

$$44 \times 271 \times .332 = 4000 \text{ pounds.}$$

Prob. 1. A Vauclain compound locomotive has cylinders 18 and 30×42 ins., with 5 per cent clearance in each and runs on a boiler pressure of 175 lbs. per square inch gage and atmospheric exhaust. The steam pressure may be varied as may also the cut-off to a limited degree. For a speed of 200 R.P.M., a cut-off $\frac{3}{4}$ and 10 per cent compression in each cylinder, find how the horse-power will vary with the initial pressures of 175, 150, 125, and 100 lbs. gage.

Prob. 2. When the cut-off is reduced to $\frac{1}{2}$ in the above engine compression increases in the high-pressure cylinder to 20 per cent. For the case of 175 lbs. gage initial pressure find the change in horse-power.

Prob. 3. Find the steam used by the engine per hour for the first case of Prob. 1 and for Prob. 2.

Prob. 4. It is desired to run a 12- and 18×24-in. no-receiver engine with 5 per cent clearance in each cylinder, under the best possible hypothetical economy conditions for an initial pressure of 200 lbs. per square inch absolute and atmospheric exhaust. To give what cut-off and compression must the valves be set and what horse-power will result for 100 R.P.M.?

20. Compound Engine Without Receiver. Exponential Law, with Clearance and Compression, Cycle XVI. General Relations between Pressures, Dimensions, and Work, when H.P. Exhaust and L.P. Admission are Coincident. Again referring to Fig. 99, it may be observed that reasoning similar to that in Section 19 but using the exponential law, would show that the same formulas and graphical constructions will serve to locate the axes of the diagram, hence, as before,

$$\overline{KM} = D_H \frac{1 + c_H + R_C\, c_L}{R_C 1}, \quad \ldots\ldots\ldots \quad (542)$$

and

$$\overline{QM} = D_L \frac{1 + c_H + R_C\, c_L}{R_C - 1}. \quad \ldots\ldots\ldots \quad (543)$$

Release pressure in the high-pressure cylinder is

$$(\text{rel.pr.})_H = (\text{in.pr.}) \left(\frac{c_H + Z_H}{1 + c_H} \right)^s. \quad \ldots\ldots\ldots \quad (544)$$

Immediately after release the pressure is equalized in the high-pressure cylinder and the low-pressure clearance. The pressure after equalization, termed $(\text{in.pr.})_L$, is found by the relation of the volume at S and that at D', Fig. 99, measured from the axis $\dot{K}T$.

$$(\text{in.pr.})_L = (\text{rel.pr.})_H \left[\frac{1+c_H+R_C(c_L+X_L)\left(\frac{(\text{bk.pr.})}{(\text{rel.pr.})_H}\right)^{\frac{1}{s}}}{1+c_H+R_C c_L} \right]^s,$$

which, by means of Eq. (544) becomes

$$(\text{in.pr.})_L = (\text{bk.pr.}) \left[\frac{R^{\frac{1}{s}} P(c_H+Z_H)+R_C(c_L+X_L)}{1+c_H+R_C c_L} \right]^s \quad . \quad . \quad . \quad . \quad . \quad (545)$$

The expansion of the fluid goes on as it passes from the high-pressure cylinder to the greater volume in the low-pressure, as indicated by $D'E'$ and DE, and when the communicating valve closes, the pressure has become

$$(\text{cut-off pr.})_L = (\text{in.pr.})_L \left[\frac{\frac{1+c_H+R_C c_L}{R_C-1}}{\frac{1+c_H+R_C c_L}{R_C-1}+(1-X_H)} \right]^s,$$

which, by means of Eq. (545) reduces to

$$(\text{cut-off pr.})_L = (\text{bk.pr.}) \left[\frac{R^{\frac{1}{s}} P(c_H+Z_H)+R_C(c_L+X_L)}{1+c_H+R_C c_L+(1-X_H)(R_C-1)} \right]^s \quad . \quad . \quad . \quad (546)$$

After cut-off in the low pressure, expansion goes on in that cylinder alone to the end of the stroke, when release occurs at a pressure

$$(\text{rel.pr.})_L = (\text{cut-off pr.})_L \left(\frac{1+c_L-X_H}{1+c_L} \right)^s$$

or by substitution from equation (546),

$$(\text{rel.pr.})_L = (\text{bk.pr.}) \left[\frac{R^{\frac{1}{s}} P(c_H+Z_H)+R_C(c_L+X_L)}{1+c_H+R_C c_L+(1-X_H)(R_C-1)} \left(\frac{1+c_L-X_H}{1+c_L} \right) \right]^s . \quad (547)$$

In terms of these quantities the work of the high- and low-pressure cylinders can be written out as follows:

$$W_H = 144 D_H \left\{ (\text{in.pr.}) \frac{c_H+Z_H}{s-1} \left[s - \left(\frac{c_H+Z_H}{1+c_H} \right)^{s-1} \right] - c_H(\text{in.pr.}) \right.$$

$$- \frac{(\text{bk.pr.})}{s-1} \left[\frac{R^{\frac{1}{s}} P(c_H+Z_H)+R_C(c_L+X_L)}{R_C-1} \right]^s \left[1 - \left(\frac{1+c_H+R_C c_L}{1+c_H+R_C c_L+(1-X_H)(R_c-1)} \right)^{s-1} \right]$$

$$\left. - \frac{(\text{bk.pr.})}{s-1} (c_H+X_H) \left[\frac{R^{\frac{1}{s}} P(c_H+Z_H)+R_C(c_L+X_L)}{1+c_H+R_C c_L+(1-X_H)(R_C-1)} \right]^s \left[\left(\frac{c_H+X_H}{c_H} \right)^{s-1} - 1 \right] \right\} \quad (548)$$

and

$$W_L=144D_L\left\{\frac{(\text{bk.pr.})}{s-1}\left[\frac{R^{\frac{1}{s}}p(c_H+Z_H)+R_C(c_L+X_L)}{R_C-1}\right]^s\right.\cdot$$

$$\times\left[1-\left(\frac{1+c_H+R_Cc_L}{1+c_H+R_Cc_L+(1-X)(R_C-1)}\right)^{s-1}\right]$$

$$+\frac{(\text{bk.pr.})}{s-1}(1+c_L-X_H)\left[\frac{R^{\frac{1}{s}}p(c_H+Z_H)+R_C(c_L+X_L)}{1+c_H+R_Cc_L+(1-X_H)(R_C-1)}\right]^s\left[1-\left(\frac{1+c_L-X_H}{1+c_L}\right)^{s-1}\right]$$

$$\left.-\frac{(\text{bk.pr.})(c_L+X_L)}{s-1}\left[\left(\frac{c_L+X_L}{c_L}\right)^{s-1}-1\right]-(1-X_L)(\text{bk.pr.})\right\}. \quad (549)$$

These are general expressions for work of high- and low-pressure cylinders for this cycle, and from them may be obtained the total work of the cycle, mean effective pressure referred to the low-pressure cylinder, and by equating them may be obtained the relation which must exist between dimensions, events, and pressures to give equal division of work. It would, however, be of no advantage to state these in full here, as they can be obtained from the above when needed.

The supply volume, cubic feet per cycle, is represented by $\overline{A'B}$, Fig. 99, and its value is found by referring to points B and E as follows:

$$(\text{Sup.Vol.})=D_H\left[(c_H+Z_H)-(c_H+X_H)\left(\frac{(\text{cut-off pr.})_L}{(\text{in.pr.})}\right)^{\frac{1}{s}}\right],$$

$$=D_H\left[c_H+Z_H-\frac{(c_H+X_H)}{R^{\frac{1}{s}}p}\left(\frac{R^{\frac{1}{s}}p(c_H+Z_H)+R_C(c_L+X_L)}{1+c_H+R_Cc_L+(1-X_H)(R_C-1)}\right)\right]. \quad (550)$$

Work per cubic foot supplied is found from Eqs. (548), (549), and (550).

$$\text{Work per cu.ft. supplied}=\frac{W_H+W_L}{(\text{Sup.Vol.})}. \quad (551)$$

Consumption, cubic feet per hour per I.H.P., is found from mean effective pressure referred to L.P. cyl. and supply volume as follows:

Consumption, cu.ft. per hr. per I.H.P.

$$=\frac{13{,}750}{(\text{m.e.p. ref. to L.P.})}\frac{(\text{Sup.Vol.})}{D_L}. \quad (552)$$

This will give pounds consumption by introducing the factor of density.

Further than this, it will be found more practicable to use graphical methods instead of computations with this cycle.

Example. Find (*a*) the horse-power and (*b*) consumption of a 12- and 18×24-in. no-receiver engine having 6 per cent clearance in the high pressure cylinder and 4 per cent in the low when the initial pressure is 150 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., high-pressure cut-off $\frac{1}{2}$, high-pressure compression 15 per cent, and low-pressure compression is complete.

(*a*) The per cent of low-pressure compression may be found as in the Example of Section 19, using the value of s in this case of 1.4. Then

$$(\text{in.pr.})(Z_H+c_H)^{1.4}=(\text{rel.pr.})_H(1+c_H)^{1.4},$$

or

$$(\text{rel.pr.})_H=61.5 \text{ lbs. sq. inch absolute,}$$

and

$$(\text{rel.pr.})_H\times c_L{}^{1.4}=(\text{bk.pr.})(c_L+X_L)^{1.4},$$

or

$$X_L=.11$$

From the sum of Eqs. (548) and (549) divided by $144D_L$ and with proper values substituted, (m.e.p.) =48.5 lbs.; hence the horse-power is 189.

(*b*) From Eqs. (550) the value for (Sup. Vol.), which when multiplied by $\frac{13{,}750}{\text{m.e.p.}}$, and divided by D_L, gives cubic feet air per hour per I.H.P.

$$\frac{13{,}750}{48.5}\times\frac{1}{2.25}\left[.06+.5-\frac{.21}{(15)^{.7}}\;\frac{15^{.7}(.56)+2.25(.15)}{1+.06+2.25\times.04+(1-.15)(2.25-1)}\right]=.63 \text{ cu.ft. per hour per I.H.P.}$$

Prob. 1. If the locomotive of Prob. 1, Section 19, should be equipped with superheater so that the steam expanded in such a way that $s=1.3$, what would be the effect upon the horse-power for conditions of that problem and on the cylinder event pressures?

Prob. 2. A 30- and 42×54-in. no receiver steam pumping engine runs at 30 R.P.M. and has 3 per cent clearance in the high-pressure cylinder and 2 per cent in low. There is no compression in either cylinder. Initial pres ure is 120 lbs. per square inch gage, and back pressure 28 ins. of mercury (barometer reading 30 ins.). The steam is such that the expansion exponent is 1.25. What will be the horse-power of, and the steam used by the engine when the cut-off in the high is $\frac{1}{2}$?

Prob. 3. By how much would the power change if the cut-off were shortened to $\frac{3}{8}$ and then to $\frac{1}{4}$, and what would be the effect of these changes on the economy?

21. Triple-Expansion Engine with Infinite Receiver. Logarithmic Law. No Clearance, Cycle XVII. General Relations between Pressures, Dimensions and Work. Fig. 101 represents the cycle of the triple-expansion engine with infinite receiver, no clearance, showing one case of incomplete expansion in all cylinders, and another where overexpansion takes place in all cylinders.

The reasoning which follows applies equally well to either case, and to any combination of under or overexpansion in the respective cylinders.

It is desired to express the work of the respective cylinders and the total work in terms of dimensions, initial and back pressures, and the cut-off of the respective cylinders. To do this, it is convenient first to express the

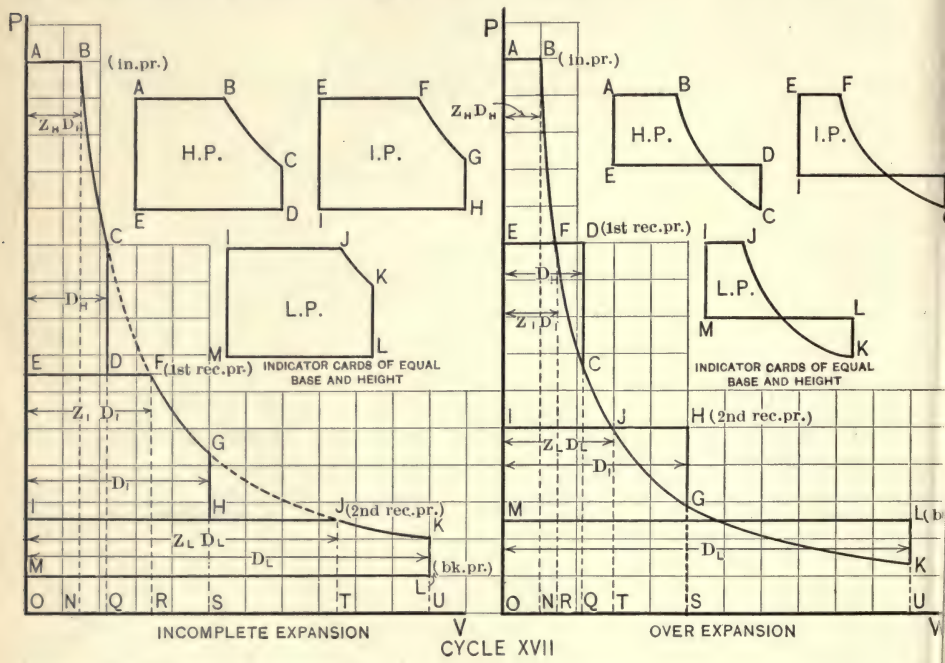


FIG. 101.—Work of Expansive Fluid in Triple-Expansion Engine with Infinite Receiver and Zero Clearance. Cycle XVII, Logarithmic Expansion.

first receiver pressure (1st rec.pr.) and *second receiver pressure* (2d rec.pr.) in terms of these quantities. The subscript I refers to the intermediate cylinde

$$P_f = P_b \frac{V_b}{V_f},$$

or

$$(\text{1st rec.pr.}) = (\text{in.pr.}) \frac{Z_H D_H}{Z_I D_I}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55$$

and

$$P_j = P_b \frac{V_b}{V_j},$$

or

$$(\text{2d rec.pr.}) = (\text{in.pr.}) \frac{Z_H D_H}{Z_L D_L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (55$$

Work of high-pressure cylinder is

$$W_H = P_b V_b \left(1+\log_e \frac{V_c}{V_b}\right) - P_d V_d,$$

$$= 144(\text{in.pr.}) D_H \left\{ Z_H \left(1+\log_e \frac{1}{Z_H}\right) - \frac{Z_H D_H}{Z_I D_I} \right\}. \quad . \quad . \quad (555)$$

Work of intermediate cylinder is

$$W_I = P_f V_f \left(1+\log_e \frac{V_g}{V_f}\right) - P_h V_h,$$

$$= 144(\text{in.pr.}) D_H \left\{ Z_H \left(1+\log_e \frac{1}{Z_I}\right) - \frac{D_I}{D_H} \frac{Z_H D_H}{Z_L D_L} \right\}$$

$$= 144(\text{in.pr.}) D_H \left\{ Z_H \left(1+\log_e \frac{1}{Z_I}\right) - \frac{Z_H D_I}{Z_L D_L} \right\}. \quad . \quad . \quad . \quad . \quad (556)$$

Work of low-pressure cylinder is

$$W_L = P_i V_j \left(1+\log_e \frac{V_k}{V_j}\right) - P_l V_l,$$

$$= 144(\text{in.pr.}) D_H Z_H \left(1+\log_e \frac{1}{Z_L}\right) - 144(\text{bk.pr.}) D_L. \quad . \quad . \quad (557)$$

The total work by addition is

$$W = 144(\text{in.pr.}) D_H Z_H \left\{ \left(1+\log_e \frac{1}{Z_H}\right) + \left(1+\log_e \frac{1}{Z_I}\right) + \left(1+\log_e \frac{1}{Z_L}\right) - \frac{D_H}{Z_I D_I} - \frac{D_I}{Z_L D_L} \right\} - 144(\text{bk.pr.}) D_L$$

$$= 144(\text{in.pr.}) D_H Z_H \left\{ 3+\log_e \frac{1}{Z_H Z_I Z_L} - \frac{D_H}{Z_I D_I} - \frac{D_I}{Z_L D_L} \right\} - 144(\text{bk.pr.}) D_L. \quad . \quad (558)$$

Mean effective pressure referred to the low-pressure cylinder is found by dividing W by $144 D_L$, and is therefore

(m.e.p. ref. to L.P.)

$$= (\text{in.pr.}) Z_H \frac{D_H}{D_L} \left\{ 3+\log_e \frac{1}{Z_H Z_I Z_L} - \frac{D_H}{Z_I D_I} - \frac{D_I}{Z_L D_L} \right\} - (\text{bk.pr.}). \quad . \quad (559)$$

Work done per cubic foot supplied is equal to W divided by the supp volume $\overline{AB}$ or $Z_H D_H$,

Work per cu.ft. supplied

$$= 144(\text{in.pr.}) \left\{ 3 + \log_e \frac{1}{Z_H Z_I Z_L} - \frac{D_H}{Z_I D_I} - \frac{D_I}{Z_L D_L} \right\} - 144(\text{bk.pr.}) \frac{D_L}{Z_H D_H}. \quad (56$$

The volume of fluid supplied per hour per indicated horse-power is

Consumption, cu.ft. per hr. per I.H.P.

$$= \frac{13{,}750}{(\text{m.e.p. ref. to L.P.})} \frac{Z_H D_H}{D_L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (56$$

The *weight* of fluid used per hour hour per indicated horse-power is course found by multiplying this volume Eq. (561) by the density of the flu used.

The conditions which will provide for *equal division of work* between th three cylinders may be expressed in the following ways:

$$W_H = W_I = W_L,$$

which is equivalent to, first:

$$W_H = W_I,$$

or

$$\log_e \frac{1}{Z_H} - \frac{1}{Z_I}\left(\frac{D_H}{D_I}\right) = \log_e \frac{1}{Z_I} - \frac{1}{Z_L}\left(\frac{D_I}{D_L}\right),$$

hence

$$\log_e \frac{Z_I}{Z_H} = \frac{1}{Z_I}\left(\frac{D_H}{D_I}\right) - \frac{1}{Z_L}\left(\frac{D_I}{D_L}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56$$

Similarly from $W_H = W_L$,

$$\log_e \frac{Z_L}{Z_H} = \frac{1}{Z_I}\left(\frac{D_H}{D_I}\right) - \frac{(\text{bk.pr.})}{(\text{in.pr.})}\left(\frac{D_L}{D_H}\right)\frac{1}{Z_7}. \quad . \quad . \quad . \quad . \quad . \quad (56$$

These two equations, (562) and (563), show the necessary relations betw

$$Z_H, \quad Z_I, \quad Z_L, \quad \left(\frac{D_I}{D_H}\right), \quad \left(\frac{D_L}{D_H}\right), \quad \text{and} \quad \left(\frac{(\text{in.pr.})}{(\text{bk.pr.})}\right),$$

in order that work shall be equally divided. Since there are six independ quantities entering (as above) and only two equations, there must be *f* of these quantities *fixed by conditions of the problem*, in order that the ot

two may be found. For instance, if the cylinder ratios, the pressure ratio, and one cut-off are known, the other two cut-offs may be found, though the solution is difficult.

Again, *if cut-offs are equal*, and the ratio of initial to back pressure is known, it is possible to find the cylinder ratios. This forms a special case which is of sufficient importance to require investigation.

If $Z_H = Z_I = Z_L$, Eq. (562) becomes

$$\frac{D_I}{D_H} = \frac{D_L}{D_I}, \quad \ldots \ldots \ldots \quad (564)$$

and Eq. (563) reduces to

$$\frac{D_I D_L}{D_H{}^2} = \frac{\text{(in.pr.)}}{\text{(bk.pr.)}}, \quad \ldots \ldots \ldots \quad (565)$$

but from Eq. (564),

$$\frac{D_L}{D_I}\,\frac{D_I}{D_H} = \frac{D_L}{D_H} = \left(\frac{D_I}{D_H}\right)^2,$$

and therefore

$$\frac{D_I D_L}{D_H{}^2} = \left(\frac{D_I}{D_H}\right)^3$$

and

$$\frac{D_I}{D_H} = \frac{D_L}{D_I} = \left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)^{\frac{1}{3}}, \quad \ldots \ldots \ldots \quad (566)$$

which, along with the condition assumed that

$$Z_H = Z_I = Z_L \quad \ldots \ldots \ldots \quad (567)$$

constitute *one set of conditions that will make work equal* in the three cylinders This is not an uncommon method of design, since by merely maintaining equal cut-offs, the work division may be kept equal.

The work done in any one cylinder under these conditions Eqs. (566) and (567) is then

$$W_H = W_I = W_L = 144(\text{in.pr.})D_H\left\{Z\left(1 + \log_e \frac{1}{Z}\right) - \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{3}}\right\} \quad \ldots \quad (568)$$

and the total work

$$W = 432(\text{in.pr.})D_H\left\{Z\left(1 + \log_e \frac{1}{Z}\right) - \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)^{\frac{1}{3}}\right\} \ldots \ldots \quad (569)$$

in which Z represents the cut-off in each cylinder, all being equal.

A special case of the triple-expansion engine with infinite receiver and n clearance which demands attention is that of *complete expansion* in a cylinders, represented by Fig. 102. Here

$$Z_L = \frac{D_I}{D_L} = \frac{(\text{bk.pr.})}{(\text{2d rec.pr.})}; \quad \ldots \quad (570$$

$$Z_I = \frac{D_H}{D_I} = \frac{(\text{2d rec.pr.})}{(\text{1st rec.pr.})}; \quad \ldots \quad (571$$

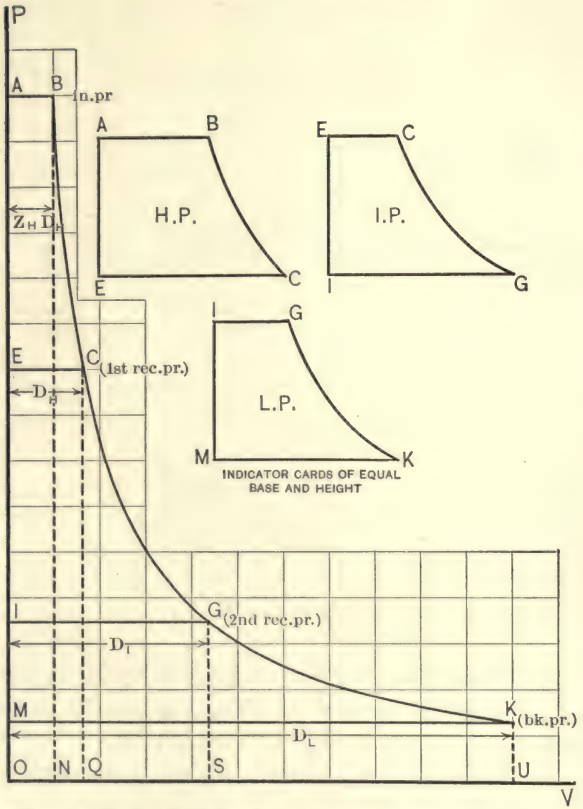


FIG. 102.—Special Case of Cycle XVIII Complete Expansion in Triple-expansion Engi with Infinite Receiver, Zero Clearance, Logarithmic Expansion.

and

$$Z_H = \frac{(\text{1st rec.pr.})}{(\text{in.pr.})} = \left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)\frac{D_L}{D_H}, \quad \ldots \quad (57$$

hence the receiver pressures are as follows:

$$(\text{1st rec.pr.}) = (\text{bk.pr.})\frac{D_L}{D_H}, \quad \ldots \quad (57$$

and

$$(2\text{d rec.pr.}) = (\text{bk.pr.})\frac{D_L}{D_I}. \quad \ldots\ldots\ldots \quad (574)$$

The work of the respective cylinders, expressed in terms of initial and back pressures and displacements is then,

$$W_H = 144(\text{in.pr.})D_H\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)\frac{D_L}{D_H}\left[1+\log_e\left(\frac{\text{in.pr.}}{\text{bk.pr.}}\right)\left(\frac{D_H}{D_L}\right)\right]-144\text{bk.pr.}\frac{D_L}{D_H}\ D_H$$

$$= 144(\text{bk.pr.})D_L \log_e\left(\frac{(\text{in.pr.})}{(\text{bk.pr.})}\frac{D_H}{D_L}\right) \quad \ldots\ldots\ldots \quad (575)$$

Similarly,

$$W_I = 144(\text{bk.pr.})D_L \log_e\left(\frac{D_I}{D_H}\right), \quad \ldots\ldots\ldots \quad (576)$$

and

$$W_L = 144(\text{bk.pr.})D_L \log_e \frac{D_L}{D_I}. \quad \ldots\ldots\ldots \quad (577)$$

Total work, by addition, is

$$W = 144(\text{bk.pr.})D_L\left[\log_e\left(\frac{(\text{in.pr.})}{(\text{bk.pr.})}\frac{D_H}{D_L}\right)+\log_e\frac{D_I}{D_H}+\log_e\frac{D_L}{D_I}\right]$$

$$= 144(\text{bk.pr.})D_L \log_e\left[\frac{(\text{in.pr.})}{(\text{bk.pr.})}\frac{D_H}{D_L}\frac{D_I}{D_H}\frac{D_L}{D_I}\right]$$

$$= 144(\text{bk.pr.})D_L \log_e \frac{(\text{in.pr.})}{(\text{bk.pr.})}. \quad \ldots\ldots\ldots \quad (578)$$

If for this special case of complete expansion *equality of work* is to be obtained, then from Eqs. (575), (576), and (577),

$$\frac{(\text{in.pr.})}{(\text{bk.pr.})}\frac{D_H}{D_L} = \frac{D_I}{D_H} = \frac{D_L}{D_I}, \quad \ldots\ldots\ldots \quad (579)$$

which is readily seen to be the same result as was obtained when all cut-offs were equalized, Eqs. (564) and (565). This case of complete expansion and equal work in all cylinders is a special case of that previously discussed where cut-offs are made equal. Hence for this case cut-offs are equal,

$$Z_H = Z_I = Z_L = \frac{D_H}{D_I} = \frac{D_I}{D_L} = \frac{D_L}{D_H}\frac{(\text{bk.pr.})}{(\text{in.pr.})}. \quad \ldots\ldots \quad (580)$$

Example. A triple-expansion engine 12- and 18- and 27×24-ins., with infinite receiver and no clearance, runs at 125 R.P.M. on an initial pressure of 150 lbs. per square inch absolute, and a back pressure 10 lbs. per square inch absolute.

If the cut-offs in the different cylinders, beginning with the high, are $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{3}{8}$, what will be (*a*) the horse-power, (*b*) steam consumed per hour, (*c*) release and receiver pressures?

(*a*) From Eq. (559)

$$(\text{m.e.p.}) = (\text{in.pr.}) Z_H \frac{D_H}{D_L} \left\{ 3 + \log_e \frac{1}{Z_H Z_I Z_L} - \frac{D_H}{Z_I D_I} - \frac{D_I}{Z_L D_L} \right\} - (\text{bk.pr.}),$$

$$= 150 \times .5 \times \frac{1}{5.06} \left\{ 3 + \log_e 14.2 - \frac{8}{3 \times 2.25} - \frac{8}{3 \times 2.25} \right\} - 10 = 39 \text{ lbs. per sq.in,}$$

hence

$$\text{I.H.P.} = \frac{39 \times 2 \times 573 \times 250}{33{,}000} = 338.$$

(*b*) From Eq. (561).

$$\text{Cubic feet per horse-power per hour} = \frac{13{,}750}{(\text{m.e.p.})} \frac{Z_H D_H}{D_L},$$

$$= \frac{13{,}750}{39} \times .5 \times \frac{1}{5.06} = 34.9,$$

hence total pounds per hour will be,

$$34.9 \times .338 \times .332 = 3920.$$

(*c*) From Eq. (553)

$$1\text{st (rec.pr.)} = (\text{in.pr.}) \frac{D_H Z_H}{Z_I D_I},$$

$$= 150 \times \frac{.5}{.375 \times 2.25} = 89 \text{ lbs. per sq.in absolute.}$$

From Eq. (554)

$$2\text{d (rec.pr.)} = (\text{in.pr.}) \frac{Z_H D_H}{Z_L D_L},$$

$$= 150 \times \frac{.5}{.375 \times 5.06} = 3.75. \text{ per sq.in absolute.}$$

High-pressure release pressure may be found from relation $(\text{in.pr.}) Z_H D_H = (\text{rel.pr.})_H D_H$ $\therefore$ $(\text{rel.pr.})_H = 75$ lbs. Similarly 1st $(\text{rec.pr.}) Z_I D_I = (\text{rel.pr.})_I D_I$, or $(\text{rel.pr.})_I = 33.4$. Similarly 2d $(\text{rec.pr.}) Z_L D_L = (\text{rel.pr.})_L D_L$, or $(\text{rel.pr.})_L = 14.8$.

Prob. 1. What would be the horse-power and steam used per hour by a 10- and 16- and 25×20-in. infinite receiver, no-clearance engine, running at 185 R.P.M. on an initial pressure of 180 lbs. per square inch gage and atmospheric exhaust. Cut-offs .4, .35, and .3.

Prob. 2. The following data are reported for a test of a triple engine:

Size 20×33×52×42 ins., speed 93 R.P.M., initial pessure 200 lbs. per square inch gage, back pressure one atmosphere, H.P. cut-off .5, horse-power 1600, steam per hsore

ower per hour 17 lbs. Check these results, using cut-offs in other cylinders to give pproximately even work distribution.

Prob. 3. What change in cylinder sizes would have to be made in the above engine o have equal work with a cut-off of $\frac{1}{2}$ in each cylinder, keeping the high pressure the ame size as before?

Prob. 4. What would be the horse-power of a triple-expansion engine whose low-ressure cylinder was 36×3 ins., when running on 150 lbs. per square inch absolute itial pressure and 10 lbs. per square inch absolute back pressure, with a cut-off in ach cylinder of .4 and equal work distribution? Make necessary assumptions.

Prob. 5. A triple engine $18 \times 24 \times 36 \times 30$ ins., running at 100 R. P.M. on an initial ressure of 200 lbs. per square inch absolute and back pressure of 20 lbs. per quare inch absolute, is to be run at such cut-offs as will give complete expansion in ll cylinders. What will these be, what receiver pressures will result, what horse-ower can be produced under these conditions, and how much steam will be needed er hour?

NOTE: δ for 200 lbs. = .437.

22. Multiple-Expansion Engine. General Case. Any Relation between Cylinder and Receiver. Determination of Pressure Volume-Diagram and Work, by Graphic Methods. It is possible to arrange multiple-expansion ngines in an almost infinite variety of ways with respect to the pressure-volume hanges of the fluid that take place in their cylinders and receivers. There ay be two or three cylinder compounds of equal or unequal strokes, pistons oving together by connection to one piston rod, or separate piston rods ith a common cross-head or even with completely independent main parts nd cranks at 0°, 180°, displaced with either one leading, or the pistons may ot move together, being connected to separate cranks at any angle apart, nd any order of lead. Moreover, there may be receivers of large or small ze, and there may be as a consequence almost any relation between H.P. ischarge to receiver and low-pressure receipt from it, any amount of fluid assing to correspond to engine load demands and consequently any relation f cut-offs, compressions, and receiver-pressure fluctuations. Triple and uadruple engines offer even greater varieties of combination of related factors, o that problems of practical value cannot be solved by analytical methods ith anything like the same ease as is possible by graphic means, and in me cases not at all.

These problems that demand solution are of two classes.

1. To find the work distribution and total work for cylinders of given imensions, clearances, receiver volumes and mechanical connection or move-ent relation, with given initial and back pressures, and given valve gear at y setting of that valve gear or at a variety of settings.

2. To find the cylinder relations to give any proportion of the total work any cylinder at any given valve setting or any fraction of initial pressure any value of release pressure or total number of expansions.

The essential differences between these two classes of problems is that in the rst the cylinder dimensions are given, while in the second they are to be found.

In general, however, the same methods will do for both with merely change in the order, and in what follows the dimensions of cylinder valve periods, receiver volume, initial and back pressures will be assume and the diagrams found. By working to scale these diagrams will give th work by evaluation of their area, by means either of cross-section paper directly on which strips can be measured and added, or by the planimeter. Thu will high- and low-pressure work be evaluated through the foot-pound equiva lent per square inch of diagram, and the total work or the equivalent mea effective pressure found by the methods of mean ordinates referred to th pressure scale of ordinates.

In the finding of the pressure-volume diagram point by point there is but on common principle to be applied, and that is that for a given mass the produc of pressure and volume is to be taken as constant (for nearly all steam prob lems, which is the almost sole application of this work) and *when two masse come together at originally different pressures and mix, the product of the resul ing pressure and the new volume, is equal to the sum of the PV products of th two parts before mixture.* At the beginning of operations in the high-pressur cylinder, a known volume of steam is admitted at a given pressure and it pressure and volume are easily traced up to the time when it communicate with the receiver in which the pressure is unknown, and there difficulty encountered, but this can be met by working from the other end of th series of processes. The low-pressure cylinder, having a known compressio volume at the back pressure, there will be in it at the time of opening t receiver a known volume, its clearance, at a known compression pressur The resulting receiver pressure will then be that for the mixture. These tw receiver pressures are not equal ordinarily, but are related by various compre sions and expansions, involving high- and low-pressure cylinder parti displacements, grouped with receiver volumes in various ways.

Take for an illustrative example the case of a two-cylinder, single-actin cross-compound engine with slide valves, cylinder diameters $12\frac{5}{8}$ and 20 in with 24 ins. stroke for both. High-pressure clearance is 10 per cent, lo pressure clearance 8 per cent. Receiver volume 4000 cu.ins. High-pressu crank following by 90°. Find the mean effective pressure for the high- and lo pressure cylinders, for a cut-off of 50 per cent in the high, and 60 per ce in the low, a compression of 10 per cent in the high and 20 per cent in the lo initial pressure 105 lbs. per square inch gage, back pressure 5 lbs. p square inch absolute, expansion according to logarithmic law.

On a horizontal line SZ, Fig. 103, lay off the distances

TU = low-pressure cylinder displacement volume in cubic inches to scale.

UV = low-pressure cylinder clearance volume in cubic inches to scale.

VW = receiver volume in cubic inches to scale.

WX = high-pressure cylinder clearance volume in cubic inches to scale.

XY = high-pressure cylinder displacement volume in cubic inches to scale

Through these points draw verticals produced above and below, $T'T''$, $U'U''$, $V'V''$, $W'W''$, $X'X''$, $Y'Y''$. Then will $W'W$ and WZ be PV coordinates for the high pressure diagram in the quadrant $W'WZ$, and $V'V$

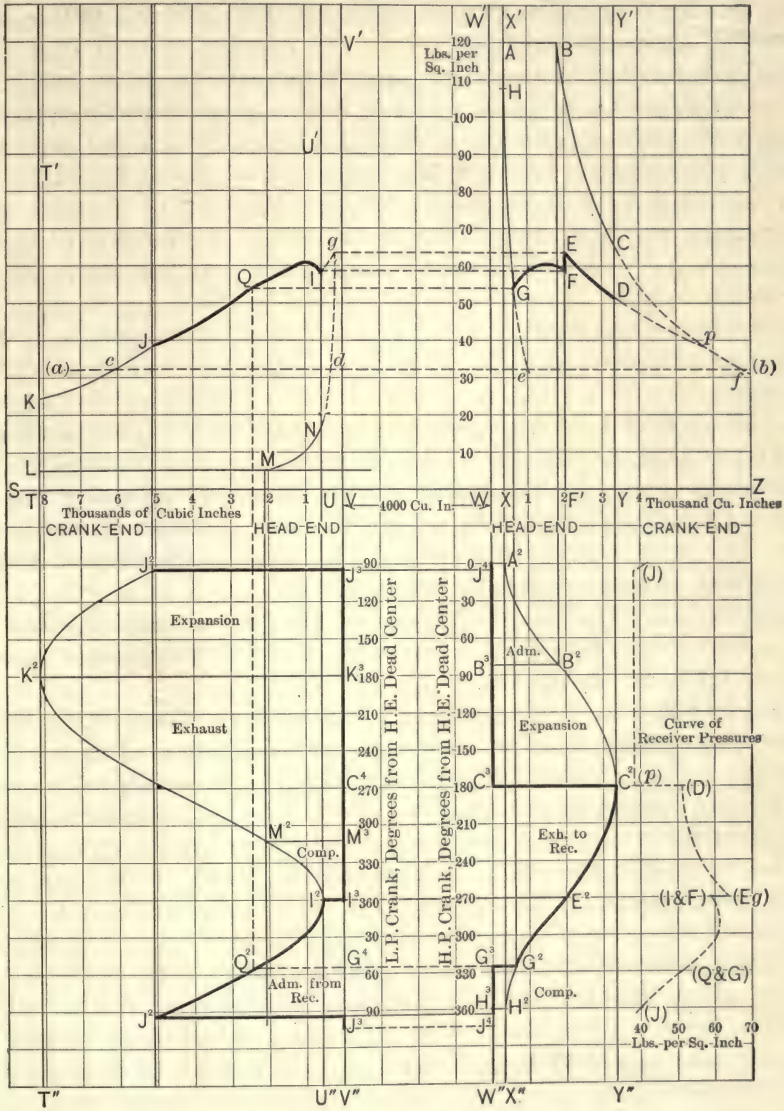


FIG. 103.—Graphical Solution of Compound Engine with Finite Receiver and with Clearance Illustrating General Method of Procedure for any Multiple Expansion Engine.

and VS the PV coordinates for the low pressure diagram in the reversed quadrant $V'VS$.

Lay off AB to represent the high-pressure admission at a height XA rep-

resenting absolute initial pressure; lay off LM at a height TL representing low-pressure exhaust at a constant absolute back pressure to the same scale.

Locate point B at the cut-off point $\overline{AB}=.50\overline{XY}$ on the initial pressure line, and drop a vertical BB^2 and draw similar verticals JJ^2, GG^2, MM^2, at suitable fractional displacements to represent L.P. cut-off, H.P. and L.P. compression volumes resprectively.

This operation will fix two other points besides the points A and L, B the H.P. cut-off at the initial pressure and M the low-pressure compression at the back pressure. Through the former draw an expansion line BC and through the latter a compression line MN, locating two more points, C and N, at the end of the outstroke of the high and instroke of the low.

At point C the H.P. cylinder steam releases to the receiver of unknown pressure, and at N, the L.P. cylinder steam is opened to both the receiver and high-pressure cylinder at unknown pressure and volume.

To properly locate these pressures and volumes from the previously known pressures and volumes in a simple manner, the construction below the line SZ is used.

Lay off on $W'W''$ the high-pressure crank angles 0–360°, and to the right of each lay off from the clearance line XX'' the displacement of the piston at the various crank angles for the proper rod to crank ratio, locating the curve $A^2B^2C^2E^2F^2G^2H^2$. This is facilitated by Table XIII at the end of the chapter, but may be laid out graphically by drawing the crank circle and sweeping arcs with the connecting rod as radius.

Opposite H.P. crank angle 270° locate L.P. crank angle 0°=360° and draw to left of the low-pressure clearance line UU'' the crank angle displacement curve for that piston.

It will be noted that steam volumes are given in the lower diagram by the distances from either of these curves toward the other as far as circumstances call for open valves. Thus H.P. cylinder volumes are distances from the H.P. displacement curve to WW'', but when H.P. cylinder is in communication with receiver, the volume of fluid is the distance from H.P. displacement curve to VV'', and when H.P. cylinder, receiver and L.P. cylinder are all three in communication the volume is given by the distance from H.P. displacement curve to L.P. displacement curve. This pair of displacement curves located one with respect to the other as called for by the crank angle relations, which may be made to correspond to any other angular relation, by sliding the low up or down with respect to high-pressure curve, serve as an easy means of finding and indicating the volumes of fluid occupying any of the spaces that it may fill at any point of either stroke.

On each curve locate the points corresponding to valve periods by the intersection of the curve with verticals to the upper diagram, such as BB^2. These points being located, the whole operation can be easily traced.

At H.P. cut-off (B) the volume of steam is B^3B^2. During H.P. expansion (B to C) steam in the high increases in volume from B^3B^2 to C^3C^2.

During H.P. release (C to D) the volume of steam in the high C^3C^2 is

added to the receiver volume C^4C^3, making the total volume C^4C^2. During H.P. exhaust (D to E) the steam volume C^4C^2 in H.P. and rec. is compressed to volume I^3E^2.

At L.P. admission (N) in low and (E) in high, the volume I^2I^3 is added, making the total volume I^2E^2 in high, low, and receiver.

During (I to Q) in low and (F to G) in high the volume I^2E^2 in high, low and receiver, changes volume until it becomes Q^2G^2 in high, low, and receiver.

At H.P. compression, G in the high, the steam divides to Q^2G^3 in low and receiver, while G^3G^2 remains in high and is compressed to H^3H^2, at the beginning of admission in the high. The former volume Q^2G^3, in low and receiver, expands to J^2J^4, at the moment cut-off occurs in the low, which divides the volume into, J^3J^4 in receiver, which remains at constant volume till high-pressure release, and the second part, J^2J^3 in the low, which expands in that cylinder to K^2K^3.

After low-pressure release the volume in low decreases from K^2K^3 to M^2M^3, when the exhaust valve closes and low-pressure compression begins.

During compression in low, the volume decreases from M^2M^3 to I^2I^3 which is the volume first spoken of above, which combines with I^3E^2, causing the drop in the high-pressure diagram from E to F.

The effects upon pressures, of the various mixings at constant volume between high, low, and receiver steam and the intermediate common expansions and compressions may be set down as follows:

At C, steam in high, at pressure P_c, mixes with steam in receiver at pressure P_j, resulting in high and receiver volume at pressure P_d.

From D to E there is compression in high and receiver resulting in pressure P_e.

At E steam in high and receiver at pressure P_e mixes with steam in low, at P_n, locating points I in low and F in high at same pressure.

From (F to G in high) and (I to Q in low) there is a common compression-expansion in high, low, and receiver, the pressures varying inversely as the total volume measured between the two displacement curves. At G in there begins compression in high alone to H.

In the low and receiver from Q to J there is an expansion and consequent fall in pressure from P_q to P_j.

After low-pressure cut-off at J the expansion takes place in low-pressure cylinder alone, to pressure P_k, when release allows pressure to fall (or rise) to exhaust pressure P_l.

When cut-off in low occurs at J the volume J^3J^4 is separated off in the receiver, where it remains at constant pressure P_j until high-pressure release at point C.

At the point M compression in low begins, increasing the pressure in low alone from P_m to P_n.

There are, it appears, plenty of relations between the various intermediate and common points, but not enough to fix them unless one be first established. One way of securing a starting point is to assume a compression

pressure P_g for the beginning of H.P. compression and draw a compression line HG through it, produced to some pressure line af, cutting low-pressure compression line at d. Then the H.P. intercept $(e\text{–}f)$ must be equal to the low-pressure intercept $(d\text{–}c)$; this fixes (c) through which a $PV = \text{const.}$ line intersects the L.P. cut-off volume at J.

Now knowing by this approximation the pressure at J, the pressure may by found at D, E, F, and at G. The pressure now found at G may differ considerably from that assumed for the point. If so, a new assumption for the pressure at G may be made, based upon the last figure obtained, and working around the circuit of pressures, J, D, E, F, and back to G should give a result fairly consistent with the assumption. If necessary, a third approximation may be made.

It might be noted that this is much the process that goes on in the receiver when the engine is being started, the receiver pressure rising upon each release from the high, closer and closer to the limiting pressure that is completely reached only after running some time.

These approximations may be avoided by the following computation, representing point pressures by P with subscript and volumes by reference to the lower diagram. P_j is the unknown pressure in receiver before high-pressure release and after low-pressure cut-off.

Pressure after mixing at D is then

$$\frac{P_j(C^4C^3)+P_c(C^3C^2)}{(C^4C^2)}=P_d.$$

The pressure at F, after mixing is

$$\frac{P_d\dfrac{(C^4C^2)}{(I^3E^2)}(I^3E^2)+P_n(I^2I^3)}{(I^2E^2)}=\frac{P_j(C^4C^3)+P_c(C^3C^2)+P_n(I^2I^3)}{I^2E^2}=P_f.$$

This pressure multiplied by $\dfrac{(I^2E^2)}{(Q^2G^2)}$ gives P_g, and this in turn multiplied by $\dfrac{(Q^2G^3)}{J^2J^4}$ will give P_j.

Writing this in full,

$$[P_j(C^4C^3)+P_c(C^3C^2)+P_n(I^2I^3)]\frac{(Q^2G^3)}{(Q^2G^2)(J^2J^4)}=P_j.$$

Solving for P_j,

$$P_j=\frac{[P_c(C^3C^2)+P_n(I^2I^3)](Q^2G^3)}{(Q^2G^2)(J^2J^4)-(C^4C^3)(Q^2G^3)}, \quad \ldots\ldots \quad (581$$

which is in terms of quantities all of which are measurable from the diagram. While this formula applies to this particular case only, the manner of obtaining it is indicative of the process to be followed for other cases.

When there are three successive cylinders the same constructions can be used, the intermediate diagrams taking the position of the low for the compound case, while the low for the triple may be placed under the high and off-set from the intermediate by the volume of the second receiver. In this case it is well to repeat the intermediate diagram. Exactly similar constructions apply to quadruple expansion with any crank angle relations.

Prob. 1. By means of graphical construction find the horse-power of a 12- and 18×24-in. single-acting cross-compound engine with 6 per cent clearance in each cylinder, if the receiver volume is 5 cu.ft., initial pressure 150 lbs. per square inch gage, back pressure 10 lbs. per square inch absolute, speed 125 R.P.M., high-pressure compression 30 per cent, low pressure 20 per cent, high pressure cut-off 50 per cent, low pressure 40 per cent, high-pressure crank ahead 70°, logarithmic expansion, and ratio of rod to crank 4.

Prob. 2. Consider the above engine to be a tandem rather than a cross-compound and draw the new diagrams for solution.

Prob. 3. A double-acting, 15- and 22×24-in. compound engine has the high-pressure crank ahead by 60°, and has 5 per cent clearance in the low-pressure cylinder, 10 per cent in the high, and a receiver 4 times as large as the high-pressure cylinder. What will be the horse-power when the speed is 125 R.P.M., initial pressure 150 lbs. per square inch absolute, back pressure 5 lbs. per square inch absolute, high-pressure cut-off $\frac{1}{2}$, low-pressure $\frac{1}{2}$, high-pressure compression 20 per cent, low-pressure 30 per cent, and ratio of rod to crank 5. Determine graphically the horse-power in each cylinder.

Prob. 4. Consider the engine of Prob. 3 to be a tandem compound and repeat the solution.

23. Mean Effective Pressure, Engine Power, and Work Distribution and their Variation, with Valve Movement and Initial Pressure. Diagram Distortion and Diagram Factors. Mechanical Efficiency. The indicated power developed by a steam engine is dependent upon three principal factors—piston displacement, speed, and mean effective pressure. The first, *piston displacement*, is dimensional in character, and, fixed for a given engine. *Speed* is limited by steam and inertia stresses, with which the present treatment is not concerned, or by losses due to fluid friction in steam passages, a subject that will be further considered under steam flow. *Mean effective pressure* is a third factor which is to be investigated, most conveniently by the methods laid down in the foregoing sections.

In these formulas for mean effective pressure, it will be observed that the terms entering are (*a*) initial pressure, (*b*) back pressure, (*c*) cut-off or ratio of expansion, (*d*) clearance, and (*e*) compression, for the single-cylinder engine. It is desirable to learn in what way the mean effective pressure varies upon changing any one of these factors.

Referring to Section 5, Eq. (262) for logarithmic expansion

$$\left.\begin{aligned}(\text{m.e.p.}) = (\text{in. pr.})\left[Z+(Z+c)\log_e \frac{1+c}{Z+c}\right] \quad &(\text{mean forward pressure})\\ -(\text{bk.pr.})\left[1-X+(X+c)\log_e \frac{X+c}{c}\right] \quad &(\text{mean back pressure})\end{aligned}\right\}. \quad (582)$$

it is seen that the mean effective pressure is the difference between a mean forward pressure and a mean back pressure. The former depends on initial pressure, cut-off, and clearance, and the latter on back pressure,

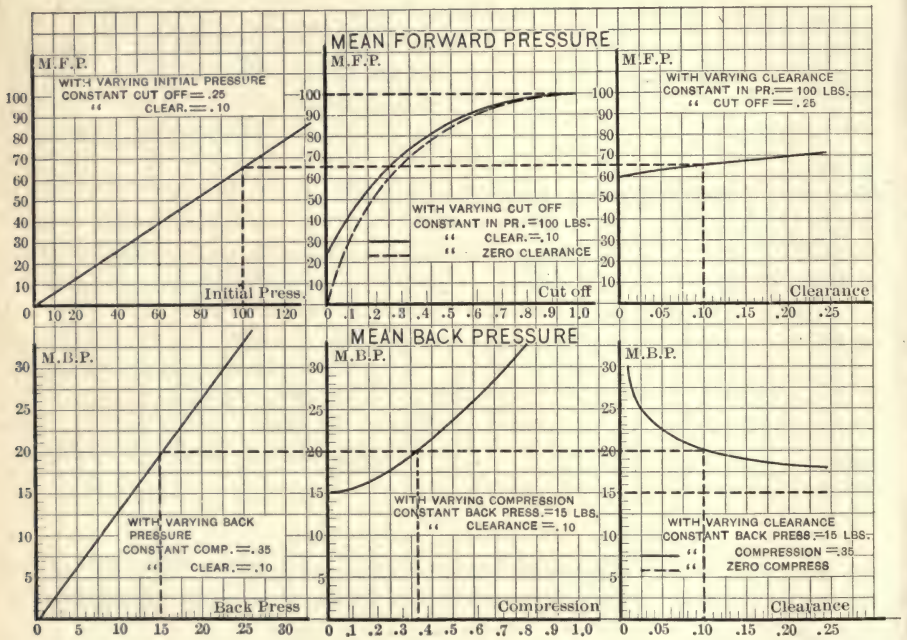


FIG. 104.—Curve to Show Variation of Mean Forward and Mean Back Pressure for Logarithmic Expansion and Compression in a Single Cylinder Engine with Clearance.

compression, and clearance. To study the effect of varying these terms it is most convenient to draw curves such as are shown in Fig. 104, and examine mean forward pressure and mean backward pressure separately.

Mean forward pressure is seen by inspection to vary in direct proportion to initial pressure. Cut-off, when short, gives a low mean forward pressure, but it is to be noted that zero cut-off will not give zero mean effective pressure so long as there is clearance, due to expansion of steam in the clearance space. Increasing the length of cut-off, or period of admission, increases mean forward pressure, but not in direct proportion, the (m.f.p.) approaching initial pressure as a limit as complete admission is approached.

Clearance has the tendency as it increases, to increase the mean forward pressure, though not to a great extent, as indicated by the curve Fig. 104.

Mean back pressure is usually small as compared to initial pressure, though a great loss of power may be caused by an increase of back pressure or compression. Back pressure enters as a direct factor, hence the straight line through the origin in the figure. So long as compression is zero, back pressure and mean back pressure are equal. When compression is not zero, there must be some clearance, and the ratio of (mean bk.pr.) to (bk.pr.) depends on both clearance and compression, being greater for greater compressions and for smaller clearances.

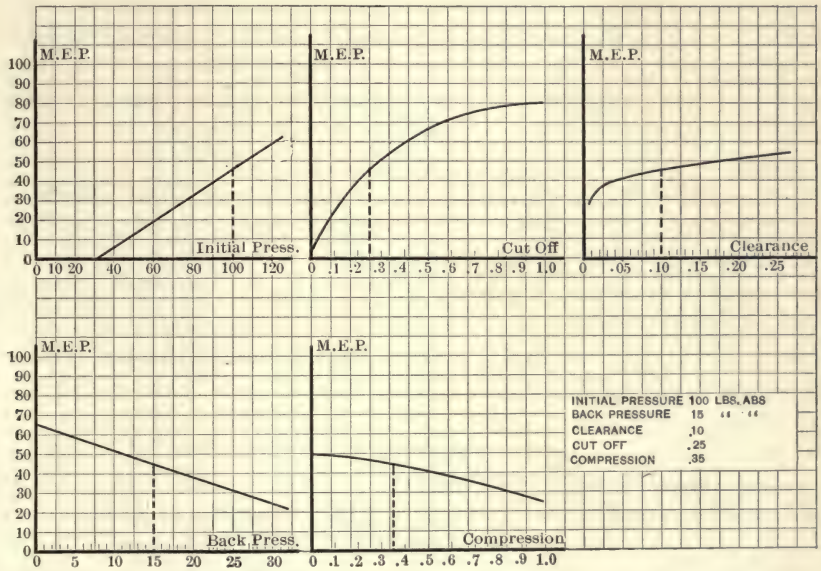


FIG. 105.—Curves to Show Variation of Mean Effective Pressure for Logarithmic Expansion and Compression in a Single Cylinder Engine with Clearance.

The mean effective pressures obtained by subtracting mean back from mean forward pressures in Fig. 104 are shown in curve form in Fig. 105.

The *multiple expansion engine* can not be so simply regarded. In a general way each cylinder may be said to be a simple engine, and subject to variations of mean effective pressure due to change in its own initial pressure and back pressure, clearance, cut-off and compression, which is true. At the same time these factors are interrelated in a way that does not exist in the simple engine. Consider, for instance, the high-pressure cylinder of a compound engine with infinite receiver, with clearance. An increase of high-pressure compression tends first to raise the mean back pressure according to the reasoning on simple engine, but at the same time the change has decreased the volume of steam passing to receiver. No change having been made

in the low-pressure cylinder, the volume admitted to it will remain the same as before, and the receiver pressure will fall, decreasing mean back pressure by a greater amount than compression increased it, and mean forward being the same as before, the increase of high-pressure compression has increased the mean effective pressure of the high-pressure cylinder. The only effect upon the low-pressure cylinder is that resulting from lowering its initial pressure, i.e., the receiver pressure. This results in a decrease of low-pressure mean effective pressure. Computation will show that the assumed increase of high-pressure compression decreases low-pressure work more than it increases high-pressure work, or in other words, decreased mean effective pressure referred to the low.

It is impracticable to describe all results of changing each of the variables for the multiple-expansion engine. Initial pressure and cut-off in the respective stages have, however, a marked influence upon receiver pressures and work distribution which should be noted. Power regulation is nearly always accomplished by varying initial pressure, i.e., throttling, or by changing cut-off in one or more cylinders.

The effect of decreasing initial pressure is to decrease the pressures on the entire expansion line and in all no clearance cycles, to decrease absolute receiver pressures in direct proportion with the initial pressure. Since back pressure remains constant, the result is, for these no-clearance cycles, that the mean effective pressures of all but the low-pressure cylinder are decreased in direct proportion to the initial pressure, while that of the low-pressure is decreased in a greater proportion. The same is true only approximately with cycles having clearance and compression.

The conditions giving equal work division have been treated in connection with the individual cycles, it may here be noted in a more general way that if high-pressure cut-off is shortened, the supply capacity of that cylinder is decreased, while that of the next cylinder remains unchanged. The result is that the decreased supply volume of steam will be allowed to expand to a lower pressure before it can fill the demand of the next cylinder than it did previously, i.e., the receiver pressure is lowered. Similarly shortening cut-off in the second cylinder will tend to increase receiver pressure. To maintain constant work division, there must be a certain relation between cut-offs of the successive cylinders, which relation can only be determined after all conditions are known, but then can be definitely computed and plotted for reference in operation.

So far, in discussing the steam engine, cycles only have been treated. These cycles are of such a nature that they can be only approached in practice, but since all conclusions have been arrived at through reasoning based on assumed laws or hypotheses, the term hypothetical may be applied to all these cycles. It is desirable to compare the actual pressure-volume diagram, taken from the indicator card of a steam engine, and the hypothetical diagram most nearly corresponding with the conditions.

In Fig. 106 is shown in full lines a pressure-volume diagram which ha

been produced from an actual indicator card taken from a simple non-condensing, four-valve engine having 5 per cent clearance.

Finding the highest pressure on the admission line $A'B'$ and the lowest pressure on the exhaust line $D'E'$, these pressures are regarded as (initial pressure) and (back pressure) and a hypothetical diagram constructed corresponding to Cycle III, with cut-off and compression at the same fraction of stroke as in the actual engine.

The first difference between the hypothetical and actual PV diagrams is that the point of release C' is not at the end of stroke, as was assumed for the hypothetical release, C, a difference which is intentional, since it requires time for pressure to fall after release to the exhaust pressure. This same fact may cause the corner of the diagram to be rounded instead of sharp as at D. Similarly, the point of admission F' is before the end of the return stroke has

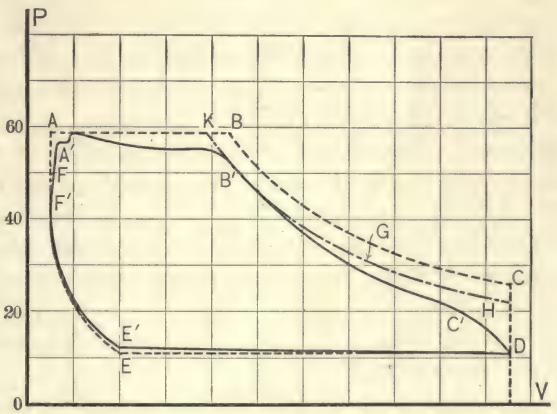


FIG. 106.—Diagram to Illustrate Diagram Factors.

been reached, and for a similar reason the corner A' may be rounded, though if release and admission are made sufficiently early the corners D' and A' will be sharp, approaching the hypothetical, H and A.

These differences, however, have little effect upon the area of the actual diagram, which is seen to be much smaller than the hypothetical. This deficiency of area is the net result of a large number of influences, only a few of which can be fully explained in connection with the pressure volume discussion.

Beginning with the point of admission, F', the line $F'A'B'$ represents the *period of admission*. The rounding at A' has been explained; the inclination of the line from A' toward B' is due in part to the frictional loss of pressure as the fluid passes at high velocity through ports and passages from steam chest to cylinder. As the stroke progresses, the linear velocity of the piston increases toward mid-stroke, requiring higher velocities in steam passages. The greater consequent friction causes pressure to fall in the cylinder. The resistance of pipes and valves leading to the engine have

an effect on the slope of this line. As cut-off is approached, this pressure fall becomes more rapid, due to the partial closure of the admission valve.

From B', the point of cut-off, to C', the point of release, is the *period of expansion*, during which the pressures are much lower than during the hypothetical expansion line BC, due principally to the lower pressure at the point of cut-off B' than at B. Hence, the frictional fall in pressure during admission has a marked effect upon the work done during expansion. The curve $B'C'$ rarely follows the law $PV = \text{const.}$ exactly, though it commonly gives approximately the same work area. During the first part of expansion, the actual pressure commonly falls below that indicated by this curve, but rises to or above it before the expansion is complete. This is largely due to condensation of steam on the cylinder walls at high pressures, and its reevaporation at lower pressure, to be studied in connection with a thermal analysis of the cycle. The curve of expansion may also depart from this very considerably, due to leakage, either inwardly, through the admission valve, or by piston from a region of higher pressure, or outwardly, through exhaust valve, or by piston into a region of lower pressure, or by drain, indicator, or relief valves, or by stuffing-boxes.

From the opening of the exhaust valve at the point of release, C', till its closure at compression E', is the *period of exhaust*. Pressures during this period, as during admission, are affected by frictional losses in the passages for steam, in this case the pressure in the cylinder being greater than that in exhaust pipe due to friction, by an increasing amount, as the velocity of the piston increases toward mid-stroke. Thus the line DE' rises above the line DE until the partial closure of the exhaust near the point of compression causes it to rise more rapidly.

At the point of compression E' the exhaust valve is completely closed and the *period of compression* continues up to admission at F'. Leakage condensation, and reevaporation affect this line in much the same way as they do the expansion, and often to a more marked degree, due to the fact that the volume in cylinder is smaller during compression than during expansion, and a given weight condensed, reevaporated, or added or removed by leakage will cause a greater change in pressure in the small weight present than if the change in weight had occurred to a large body of steam.

In the *compound engine* all these effects are present in each cylinder in greater or less degree. In addition, there are losses of pressure or of volume in the receivers themselves between cylinders, due to friction or condensation, and where especially provided for, reevaporation by means of reheating receivers. The effect of these changes in receivers is to cause loss of work between cylinders, and to make the discharge volume of one cylinder greater or less than the supply volume of the next, while these were assumed to be equal in the hypothetical cases.

The effect of all of these differences between the actual and hypothetical diagrams is to make the actual indicated work of the cylinder something less than that represented by the hypothetical diagram. Since these effects are

not subject to numerical calculations from data ordinarily obtainable, they are commonly represented by a single coefficient or *diagram factor* which is a ratio, derived from experiment, between the actual work and that indicated by hypothesis.

It is at once evident that there may be more than one hypothetical diagram to which a certain engine performance may be referred as a standard of comparison. When the heat analysis of the steam engine is taken up, a standard for comparison will be found there which is of great use. For determination of probable mean effective pressure, however, no method of calculation has been devised which gives better results than the computation of the hypothetical mean effective pressure from one of the standard hypothetical diagrams, and multiplying this by a diagram factor obtained by experiment from a similar engine, under as nearly the same conditions as can be obtained.

Such diagram factors are frequently tabulated in reference books on the steam engine, giving values for the factor for various types and sizes, under various conditions of running. Unfortunately, however, the exact standard to which these are referred is not stated. In this text it will be assumed, unless otherwise stated, that the diagram factor for an actual engine is the ratio of the mean effective pressure of the actual engine to that computed for Cycle I, without clearance or compression, logarithmic law, with cut-off at the same fraction of stroke as usual, initial pressure equal to maximum during admission in actual, and back pressure equal to minimum during exhaust of the actual engine.

This is selected as the most convenient standard of comparison for mean effective pressures, as it is frequently impossible to ascertain the clearance in cases where data are supplied. When it is possible to do so, however, closer approximation may be made to the probable performance by comparing the actual with that hypothetical diagram most nearly approaching the cycle, using same clearance, cut-off, and compression as are found in the actual.

Commercial cut-off is a term frequently used to refer to the ratio of the volume AK to the displacement, Fig. 106, in which the point K is found on the initial pressure line AB, by extending upward from the true point of cut-off B' a curve $PV = \text{const.}$

While the diagram factor represents the ratio of indicated horse-power to hypothetical, the output of power at the shaft or pulley of engine is less than that indicated in the cylinders, by that amount necessary to overcome mechanical friction among engine parts. If this power output at shaft or pulley of engine is termed *brake horse-power* (B.H.P) then the ratio of this to indicated horse-power is called the *mechanical efficiency*, E_m, of the engine

$$E_m = \frac{\text{(B.H.P.)}}{\text{(I.H.P.)}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (583)$$

The difference between indicated and shaft horse-power is the *power consumed by friction* (F.H.P.). Friction under running conditions consists

of two parts, one proportional to load, and the other constant and independent of load, or

$$(\text{F.H.P.}) = N[(\text{const.}) \times (\text{m.e.p.}) + (\text{const.})_2],$$

where N is speed, revolutions per minute. But $N \times (\text{const.})(\text{m.e.p.}) = (\text{I.H.P.})K_1$

and

$$(\text{F.H.P.}) = (\text{I.H.P.})K_1 + N(\text{const.})_2, \quad . \quad . \quad . \quad . \quad . \quad (584)$$

where K_1 and $(\text{const.})_2$ are constants to be determined for the engine, whose values will change as the conditions of the engine bearing-surfaces or lubrication alters. This value for (F.H.P.) may be used to evaluate E_m,

$$E_m = \frac{(\text{I.H.P.}) - (\text{F.H.P.})}{(\text{I.H.P.})} = 1 - K_1 - \frac{N(\text{const.})_2}{(\text{I.H.P.})}, \quad . \quad . \quad . \quad (585)$$

but indicated horse-power divided by speed is proportional to mean effective pressure, so that

$$E_m = 1 - K_1 - \frac{K_2}{(\text{m.e.p.})}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (586)$$

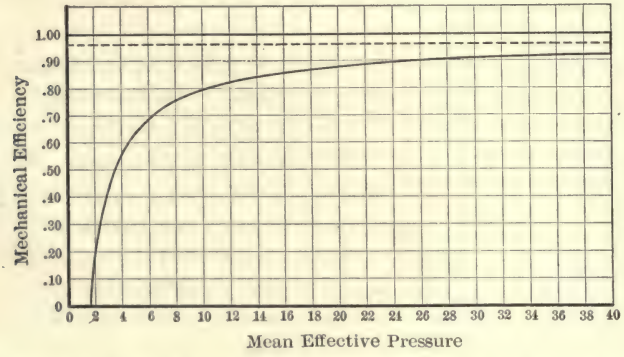


FIG. 107.—Diagram to Show Relation of Mechanical Efficiency and Mean Effective Pressure.

From this expression, speed has been eliminated, which agrees with general observation, that mechanical efficiency does not vary materially with speed. Values of the constants K_1 and K_2 may be ascertained if (m.e.p.) and E_m are known for two reliable tests covering a sufficient range, by inserting their values forming two simultaneous equations.

The numerical values of K_1 found in common practice are between .02 and .05, and for K_2 between 1.3 and 2, in some cases passing out of this range. In Fig. 107 is shown the form of mechanical efficiency curve when plotted on (m.e.p.) as abscissas, using $K_1 = .04$, $K_2 = 1.6$. It may be noted that at

higher (m.e.p.) the curve does not approach unity, but the value $(1-K_1)$ as a limit. The mechanical efficiency becomes zero for this case, at a mean effective pressure of about 1.67 lbs. per square inch, which is that just sufficient to keep the engine running under no load. For a given speed and size of cylinders, the abscissas may be converted into a scale of indicated horse-power.

Prob. 1. Assuming a back pressure of 10 lbs. per square inch absolute, a clearance of 8 per cent, a cut-off of 40 per cent, and compression of 20 per cent, show how (m.e.p.) varies with initial pressure over a range of 200 lbs., starting at 25 lbs.

Prob. 2. For an initial pressure of 150 lbs. per square inch absolute, show how (m.e.p.) varies with back pressure over a range of 30 lbs., starting at $\frac{1}{2}$ lb. per square inch absolute, keeping other quantities as in Prob. 1.

Prob. 3. For values of initial pressure, back pressure, etc., as given in Probs. 1 and 2, show how (m.e.p.) varies with clearance from 1 per cent to 15 per cent.

Prob. 4. For values of initial pressure, etc., as given in Probs. 1 and 2, show how (m.e.p.) will vary with cut-off from 0 to 1.

Prob. 5. For values of initial pressure, etc., as given in Probs. 1 and 2, show how (m.e.p.) will vary with compression for values from 0 to 50 per cent.

Prob. 6. A certain engine developing 675 I.H.P. at a speed of 151 R.P.M., delivered at the shaft 606 H.P. measured by an absorption dynamometer. A second test at 100 R.P.M. gave 150 I.H.P., and 114 shaft H.P. If this engine is to deliver 500 H.P. at the shaft at a speed of 150 R.P.M., what will be the I.H.P. and the mechanical efficiency?

Prob. 7. A compound Corliss engine, 25 and 52 ins. diameters, 60 ins. stroke, double-acting, was designed for 650 I.H P. at 63 R.P.M. It was found that at this speed and I.H.P. the mechanical efficiency was 91 per cent. When running with no load, the cylinders indicated 38.1 I.H.P. at 65 R.P.M. Find the probable mechanical efficiency when developing 300 I.H.P. at a speed of 64 R.P.M.

24. Consumption of the Steam Engine and its Variation with Valve Movement and Initial Pressure. Best Cut-off as Affected by Condensation and Leakage. The weight of steam used by a steam engine per hour divided by the indicated horse-power is said to be the water rate or steam consumption of that engine. It is almost needless to say that this is not a constant for a given engine, since it will change with any change of initial pressure, back pressure, or valve setting, leakage, or temperature conditions. Since there are at least two other uses of terms water rate or consumption, this may be termed the actual water rate, or *actual consumption*, the latter being a more general term which may refer to the weight of *fluid* used per hour per indicated horse-power, whatever the fluid may be, steam, air, carbon dioxide, or any other expansive fluid. The present discussion has special reference to steam.

From the hypothetical diagram, by computations such as are described for the various foregoing cycles, may be obtained a quantity representing the weight of fluid required to develop one horse-power for one hour, by the

performance of the hypothetical cycle. This may be termed the *hypothetica consumption* or for steam cycles the hypothetical water rate.

By the use of the actual indicator card, may be obtained, by methods stil to be described, the weight of fluid accounted for by volumes and pressure known to exist in the cylinder, this being called the *indicated consumption* o the engine or indicated water rate if the fluid be steam.

The heat analysis of the steam-engine cycle will lead to another standar of comparison which is of the greatest importance as a basis of determinin how nearly the actual performance approaches the best that could b obtained if the engine were to use all available energy possessed by the steam At present the object is to compare the actual and indicated performance wit that hypothetically possible with cylinders of the known size. Accordingl attention will be confined first to *hypothetical consumption*, and the quantitie upon which it is dependent.

For Cycle III, which is the most general for the single-expansion engine logarithmic law, the expression for consumption in pounds fluid per hour pe indicated horse-power, found in Section 5, Eq. (267), is as follows:

Hypothetical consumption, lbs. per hr. per I.H.P.

$$= \frac{13{,}750}{(\text{m.e.p.})}\left[(Z+c)-(X+c)\left(\frac{\text{bk.pr.}}{\text{in.pr.}}\right)\right]\delta_1,$$

in which the value of mean effective pressure itself depends upon (in.pr.) (bk.pr.), c, Z, and X. The density of the fluid at initial pressure, δ_1, is to b ascertained from tables of the properties of steam or of whatever fluid is used.

In Fig. 108 are the results of computations on the hypothetical steam con sumption, using mean effective pressures as plotted in Fig. 105. For each curv conditions are assumed to be as stated on the face of the diagram, varying onl one of the factors at a time.

Other conditions remaining unchanged, it may be noted that consumptio decreases for an increase of initial pressure, though not rapidly in the high pressure range.

Cut-off has a marked effect upon consumption, the minimum occurrin when cut-off is such as to give complete expansion. This occurs when

$$\frac{1+c}{Z'+c} = \frac{(\text{in.pr.})}{(\text{bk.pr.})},$$

or

$$Z' = (1+e)\frac{(\text{bk.pr.})}{(\text{in.pr.})} - c, \quad \ldots\ldots\ldots \quad (58'$$

which may be termed *hypothetically best cut-off*. In the case assumed in t diagram,

$$Z' = (1+.1)\frac{15}{100} - .1 = .065.$$

If clearance be varied, maintaining constant compression and cut-off, large clearance will give high consumption due to an excessive quantity of fluid required to fill the clearance space. Extremely small clearance leads to a high pressure at the end of compression, causing a loss of mean-effective pressure, and consequent high consumption. Between, the consumption has a minimum point, which is dependent for its location on both cut-off and compression.

Decreasing back pressure has a beneficial effect upon mean effective pressure and consumption. This would be still more marked in the figure if a case had been selected with a very short cut-off.

Compression, throughout the ordinary range of practice, has but slight effect upon consumption, indicated by the flat middle portion of the curve in Fig.

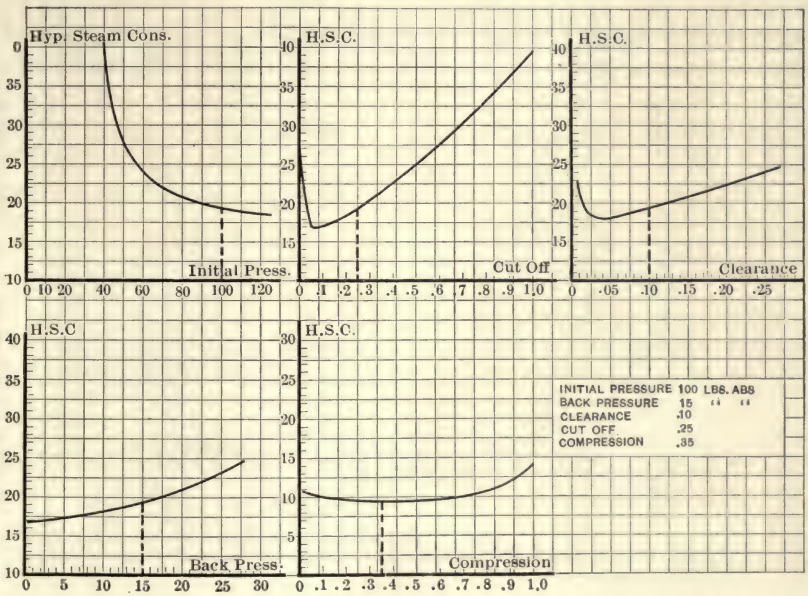


FIG. 108.—Curves to Show the Variation of Hypothetical Steam Consumption of Simple Engines, Logarithmic Expansion and Compression.

108. Very small or zero compression permits too much high-pressure steam to be admitted to the clearance space without doing work, and excessively large clearance causes pressures during compression to rise very high, thereby decreasing mean effective pressure; hence this curve of consumption rises at both ends.

Hypothetically, the best attainable consumption for given initial and back pressures is obtained when both expansion and compression are complete.

The *indicated consumption*, or, as it is frequently called for the steam engine, "steam accounted for by the indicator card" " or " indicated water rate," is determined from the indicator card as follows. Let Fig. 109 represent an indicator diagram. The points of cut-off and compression are located from the form of

the line, at the highest point on the expansion line and the lowest point on th compression line respectively. The fraction of the card lengths completed a cut-off,

$$Z=\frac{\overline{AB}}{\overline{AD}},$$

and the fraction of card length from point of compression to end of stroke,

$$X=\frac{\overline{AC}}{\overline{AD}},$$

are determined, the pressure at cut-off and compression measured by th proper vertical scale, and the corresponding densities, δ_1, and δ_2 respectively are ascertained from steam tables for dry saturated steam. Clearance, Cl, i known or ascertained by the form of the compression curve (Chap. I, Section 12

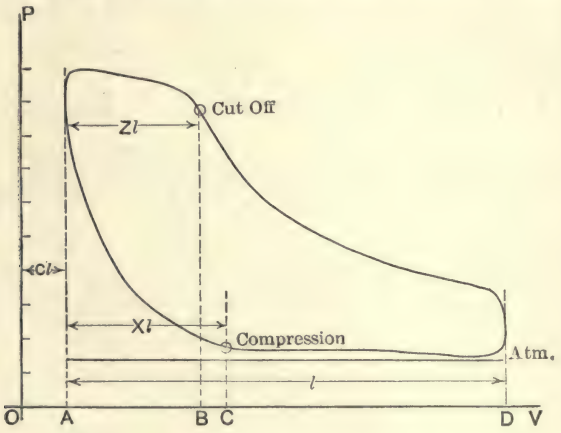


FIG. 109.—Diagram to Illustrate Method of Determining Indicated Water Rate of Ste Engine.

At the point of cut-off, the weight of dry saturated steam present in t cylinder is $D(Z+c)\delta_1$, and at compression the weight present is $D(X+c)$ on the assumption that the steam in the cylinder is of density δ_1 and δ_2 these two instants. Accepting this assumption, the weight of steam used cycle is

$$\text{Wt. steam per cycle}=w=[(Z+c)\delta_1-(X+c)\delta_2]D, \quad . \quad . \quad . \quad (5$$

The work per cycle

$$W=144D(\text{m.e.p.}),$$

and for n cycles per minute the indicated horse-power is

$$\text{I.H.P.}=\frac{144nD(\text{m.e.p.})}{33{,}000}$$

The *indicated consumption* is then, in pounds per hour per I.H.P.

$$60\frac{wn}{\text{I. H. P.}}=\frac{60\times 33{,}000\times D[(Z+c)\delta_1-(X+c)\delta_2]n}{144nD(\text{m.e.p.})},$$

or,

Ind. consumption, lbs. per hr. per I.H.P.

$$=\frac{13{,}750}{(\text{m.e.p.})}[(Z+c)\delta_1-(X+c)\delta_2], \quad . \quad . \quad . \quad . \quad . \quad . \quad (589)$$

which is the expression used to find indicated consumption for either simple- or multiple-expansion engines. In applying this to the *multiple-expansion engine* the terms Z, X and c are found for any one cylinder, and the mean effective pressure is referred to that cylinder. There may be, therefore, as many computations as there are expansion stages. For a compound engine, for instance, indicated consumption according to high-pressure card is found by inserting in formula Z, X and c for the high-pressure card, δ_1 and δ_2 for corresponding pressures, and for (m.e.p.) use

$$(\text{m.e.p. ref. to H.P.})=(\text{m.e.p.})_H+(\text{m.e.p.})_L\frac{D_L}{D_H}. \quad . \quad . \quad . \quad (590)$$

If the computation is done by means of events on the low-pressure card, the (m.e.p.) must be referred to the low.

$$(\text{m.e.p. ref. to L.P.})=(\text{m.e.p.})_H\frac{D_H}{D_L}+(\text{m.e.p.})_L. \quad . \quad . \quad . \quad . \quad (591)$$

In general for a multiple-expansion engine

$$(\text{m.e.p. ref. to cyl. A})=\Sigma(\text{m.e.p.})\frac{D}{D_A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (592)$$

It is often difficult and sometimes impossible to determine the point of cut-off and of compression on the indicator card. The expansion and compression lines are of very nearly hyperbolic in form and are usually recognizable. The highest point on the hyperbolic portion of the expansion line is regarded as cut-off, and the lowest point on the hyperbolic portion of the compression line, as the point of compression. It must be understood that by reason of the condensation and re-evaporation of steam in cylinders the weight of steam proper is not constant throughout the stroke, so that calculations like the above will give different values for every different pair of points chosen. The most correct results are obtained when steam is just dry and these points are at release and compression most nearly.

When under test of actual engines the steam used is condensed and weighed and the indicated horse-power determined, then the *actual steam consumption* or water rate can be found by dividing the weight of water used per hour in the form of steam by the indicated horse-power. This actual water rate is always greater than the water rate computed from the equation for indicated consumption. The reasons for the difference have been traced to (*a*) leakage in the engine, whereby steam weighed has not performed its share of work, to (*b*) initial condensation, whereby steam supplied became water before it could do any work, (*c*) variations in the water content of the steam by evaporation or condensation during the cycle, whereby the expansion and compression laws vary in unpredictable ways, affecting the work.

Estimation of probable water rate or steam consumption of engines cannot, therefore, be made with precision except for engines similar to those which have been tested, in all the essential factors, including, of course, their condition, and for which the deficiencies between actual and indicated consumptions have been determined. *This difference is termed the missing water*, and endless values for it have been found by experiments, but no value is of any use except when it is found as a function of the essential variable conditions that cause it. No one has as yet found these variables which fix the form for an empiric formula for missing water nor the constants which would make such a formula useful, though some earnest attempts have been made. This is no criticism of the students of the problem, but proof of its elusive nature, and the reason is probably to be found in the utter impossibility of expressing by a formula the leakage of an engine in unknown condition, or the effect of its condition and local situation on involuntary steam condensation and evaporation. It is well, however, to review some of these attempts to evaluate missing water so that steam consumption of engines may be estimated. After studying the many tests, especially those of Willans, Perry announced the following for non-condensing engines, in which the expansion is but little

$$\frac{\text{Missing water}}{\text{Indicated steam}} = m\frac{1+\frac{1}{Z}}{d\sqrt{N}}, \quad \ldots \ldots \quad (593$$

where d is the diameter of the cylinder in inches and N the number of revolutions per minute. This indicates that the missing steam or missing water has been found to increase with the amount of expansion and decrease with diameter of cylinder and the speed. Thermal and leakage conditions are met by the use of difference values of m, for there are given

$m=5$ for well-jacketed, well-drained cylinders of good construction with four poppet valves, that is, with minimum leakage and condensation.

$m=30$ or more for badly drained unjacketed engines with slid, valves, that is, with high leakage and condensation possibilities.

$m=15$ in average cases.

For condensing engines Perry introduces another variable—the initial pressure pounds per square inch absolute, p giving

$$\frac{\text{Missing water}}{\text{Indicated steam}} = \frac{120\left(1+\frac{1}{Z}\right)}{d\sqrt{np_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (594)$$

It might seem as if such rules as these were useless, but they are not, especially when a given engine or line of engines is being studied or two different engines compared; in such cases actual conditions are being analyzed rather than predictions made, and the analysis will always permit later prediction of considerable exactness, if the constants are fixed in a formula of the right empiric form. Similar study by Heck has resulted in a different formula involving different variables and constants, but all on the assumption that the discrepancies are due to initial condensation. He proposes an expression equivalent to

$$\frac{\text{Missing steam}}{\text{Indicated steam}} = \frac{.27}{\sqrt[3]{N}}\sqrt{\frac{S(x_2-x_1)}{p_1 Z}}, \quad . \quad . \quad . \quad . \quad . \quad (595)$$

in which N = R.P.M. of the engine;
d = diameter in inches;
L = stroke in feet;
S = the ratio of cylinder displacement surface in square feet to displacement in cubic feet.

$$S = \frac{2\times\frac{\pi\left(\frac{d}{12}\right)^2}{4}+\pi\frac{d}{12}L}{\frac{\pi\left(\frac{d}{12}\right)^2\times L}{4}} = \frac{2}{L}+\frac{48}{d},$$

The term (x_2-x_1) is a constant supposed to take into account the amount of initial condensation dependable on the difference between cylinder wall and live-steam temperature and is to be taken from a table found by trial as the difference between the x for the high pressure and x for the low pressure, both absolute, see Table XIV at the end of the Chapter.

In discussing the hypothetical diagrams, it was found that best economy was obtained with a cut-off which gives complete expansion. For other than hypothetical diagrams this is not true, which may be explained most easily by reference to the curves of indicated, and actual consumption, and missing steam, Fig. 110.

The curve ABC is the hypothetical consumption or water rate for a certain steam engine. Its point of best economy occurs at such a cut-off, B, that expan-

sion is complete. The curve *GHI* is computed by Heck's formula for missing water. The curve falls off for greater cut-offs. Adding ordinates of these two curves, the curve *DEF* for probable consumption is found. The minimum point in this curve, *E*, corresponds to a longer cut-off than that of *ABC*. Since cut-off *B* gave complete expansion, cut-off *E* must give incomplete expansion. In other words, due to missing steam, the condition which really gives least steam consumption per hour per indicated horse-power corresponds to a release pressure, which is greater than the back pressure.

It should be noted that the minimum point mentioned above will not be *best* cut-off, for the output of the engine is not *indicated*, but *brake* horse-power.

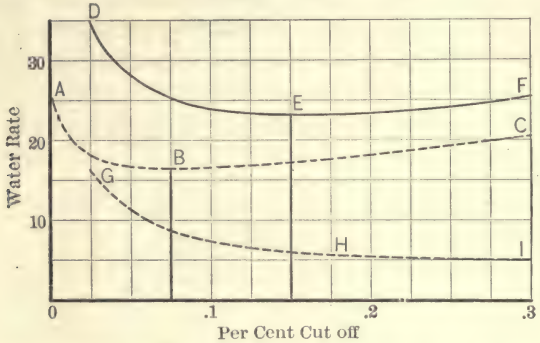


FIG. 110.—Diagram to Show Displacement of Best Cut-off Due to Effect of Missing Water from Point *B* for the Hypothetical Cycle to Some Greater Value *E*.

In Fig. 111 on cut-off as abscissa are plotted (*EFG*) consumption pounds per hour per I.H.P., and for the case assumed, (*OD*) the curve of mechanical efficiency, based on cut-off,

$$\frac{\text{(lbs. steam per hr.)}}{\text{I.H.P.}} \div \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{\text{(lbs. steam per hr.)}}{\text{B.H.P.}},$$

or, in other words,

$$\frac{\text{Consumption, lbs. per hr. per I.H.P.}}{E_m} = \text{Consumption, lbs. per hr. per B.H.P.} \quad (59$$

Due to the increasing value of E_m for greater cut-offs, the minimum point corresponds to a cut-off still longer than for the curve *EFG*, which itself was found in Fig. 110 to give a longer cut-off than that of the hypothetical curve.

Hence the best cut-off for economy of steam, where the net power at the shaft is regarded as the output, will be such as to give incomplete expansion or a release pressure above back pressure, this effect being caused by both missing steam and by frictional losses.

Prediction of actual consumption of steam engines as a general proposition is almost hopeless if any degree of accuracy worth while is desired, though the

effect on steam consumption of changing the value of any one variable can be pretty well determined by the previous discussion *qualitatively*, that is, in kind, though not quantitatively in amount. Probably the best attempt is that of Hrabak in German, which takes the form of a large number of tables developed from actual tests though not for engines of every class. These tables are quite extensive, being in fact published as a separate book and any abstraction is of no value.

There is, however, a sort of case of steam consumption prediction that can be carried out with surprising precision and that is for the series of sizes or line of engines manufactured by one establishment all of one class, each with about the same class of workmanship and degree of fit, and hence having *leakage* and *cylinder condensation* characteristics that vary consistently throughout the whole range. For such as these tables and curves of missing water

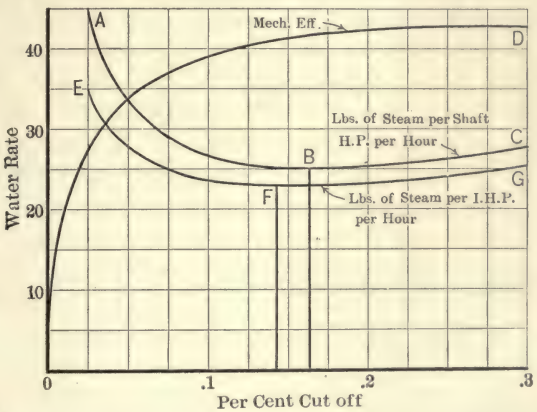


FIG. 111.—Diagram to Show Relation of Steam per Hour per I.H.P. and per B.H.P. to Cut-off.

can be made up and by the best builders are, for making guarantees of steam consumption for any service conditions that their engines are able to meet.

The practice of one firm making what is probably the best line of stationary engines in this country is of sufficient interest to warrant description. The primary data are curves of indicated water rate plotted to mean effective pressure for clearances of three or four per cent, and that mean effective pressure is chosen in any one specific case that will give the horse-power desired at the fixed speed for some one set of cylinder sizes avaliable. To this indicated water rate a quantity is added constituting the missing water which is made up of several parts as follows: The first is an addition representing condensation which is plotted in curve form as a function of (*a*) boiler pressure, (*b*) superheat in the steam, (*c*) piston speed, (*d*) the class of engine simple, compound or triple, with jacketed or unjacketed cylinders, and for cylinder ratios from 4 to 1, to 6 to 1 in the case of compounds. It is therefore a most complex quantity, the nature of the variations in which can only be indicated here.

For example, increase of piston speed decreases the condensation loss as does multiple expansion, and also jacketing, while increase of superheat in the steam also decreases it, but superheat has less effect in triple than in compounds and less in compound than in simple engines.

The next factor of correction is that covering leakage losses, also additive to indicated water rate and which with it and the condensation loss make up the probable steam consumption. The leakage decreases regularly with increase of piston speed, is less for large than for small engines, the change being rather fast from 50 to 200 horse-power and much slower later, being scarcely anything at all over 2000 horse-power.

Example 1. What cut-off will give the lowest indicated water rate for a 9×12-in. engine, with 5 per cent clearance and no compression when running non-condensing on an initial pressure of 100 lbs. per square inch gage, and what will be the value of this water rate? What steam will be used per hour per brake horse-power hypothetically? From Eq. (587)

$$Z' = (1+c)\frac{\text{bk.pr.}}{\text{in.pr.}} - c,$$

$$= (1+.05)\frac{15}{115} - .05 = 8.7 \text{ per cent},$$

and

$$(\text{m.e.p.}) = 115\left[.087 \times (.087+.05)\log_e \frac{1.05}{.137}\right] - 15 = 27.2 \text{ lbs. sq.in.}$$

Hence

$$\text{Steam per hour per I.H.P.} = \frac{13{,}750}{27.2}\left[.137 - .05 \times \frac{15}{115}\right] \times .262 = 17.2 \text{ lbs.}$$

From the curve of Fig. 107, assuming it to apply to the engine, for this value of (m.e.p.) mechanical efficiency is 90 per cent, hence from Eq. (596) the weight of steam per shaft horse-power per hour will be 19.1 pounds.

Prob. 1. Draw diagram similar to Fig. 108 for following case:

Initial pressure, 135 lbs. per square inch gage, back pressure 10 lbs. per square inch absolute, clearance 5 per cent, cut-off 30 per cent, compression 25 per cent.

Prob. 2. From indicator diagram shown in Fig. 106 find the indicated water rate of the engine from which it was taken.

Prob. 3. The indicated water rate of a 9×12-in. jacketed engine when running non-condensing at a speed of 250 R.P.M. with an initial pressure of 100 lbs. per square inch gage and ½ cut-off is 50 lbs. Using Perry's formula what will be the probable actual steam used by engine per horse-power hour?

Prob. 4. A 24×48-in. engine in good condition is found to have an indicated water rate of 25 lbs. when cut-off is ¼, initial pressure 100 lbs. per square inch gage, back pressure 10 lbs. per square inch absolute, and speed of 125 R.P.M. What will be the missing water, and the rate as found by Perry's formula and by Heck's?

Prob. 5. What will be the probable amount of steam used per hour by a 36×48-in. engine with 5 per cent clearance running at 100 R.P.M. on an initial pressure of 150 lbs. per square inch gage a back pressure of 5 lbs. per square inch absolute, ½ cut-off and 10 per cent compression?

Prob. 6. How will the amount of steam of Prob. 5 compare with that used by a 15×22×36-in engine with 5 per cent clearance in each cylinder, running at 100 R.P.M. on same pressure range with ¼ cut-off in high-pressure cylinder, ½ cut-off in low, and 10 per cent compression in each cylinder?

25. Variation of Steam Consumption with Engine Load. The Willans Line. Most Economical Load for More than One Engine and Best Load Division. However valuable it may be to the user of steam engines to have an engine that is extremely economical at its best load which, it should be noted, may have any relation to its *rated* horse-power, it is more important usually that the *form* of the economy load curve should be as flat as possible and always is this case when the engine must operate under a wide range of load. This being the case it is important to examine the real performance curves of some typical engines all of which have certain characteristic similarities as well as differences.

From the discussion of hypothetical and indicated water rates it appears that the curve of steam consumption (vertical) to engine load (horizontal) is always concave upward and always has a minimum point, not at the maximum load. Actual consumption curves are similar in general form, but as has been pointed out, the load at which the water rate is least corresponds to some greater mean effective pressure than that for the hypothetical, so the whole curve is displaced upwards and to the right by reason of cylinder condensation and leakage losses. This displacement may be so great as to prevent the curve rising again beyond the minimum point, in which case the least steam consumption corresponds to the greatest load. Just what form the actual water rate-load curve will take depends largely on the form of valve gear and type of governing method in use, by throttling initial pressure with a fixed cut-off or, by varying cut-off without changing initial pressure, with or without corresponding changes in the other valve periods.

Whenever the control of power is by throttling of the supply steam the curve is found to be almost exactly an hyperbola, so that (water rate×horse-power) plotted to horse-power is a straight line which being characteristic is much used in practical work and is known as the *Willans line.* All other engines, that is, those that govern on the cut-off, have Willans lines that are *nearly* straight, such curvature as exists being expressed by a second degree equation instead of one of the first degree.

Equations for Willans lines can always be found for the working range of load, that is, from about half to full load, though not for the entire range, except in unusual cases, and these equations are of very great value in predicting the best division of load between units, which is a fundamental step in deciding, how many and what sizes of engine to use in carrying a given load in industrial power plants.

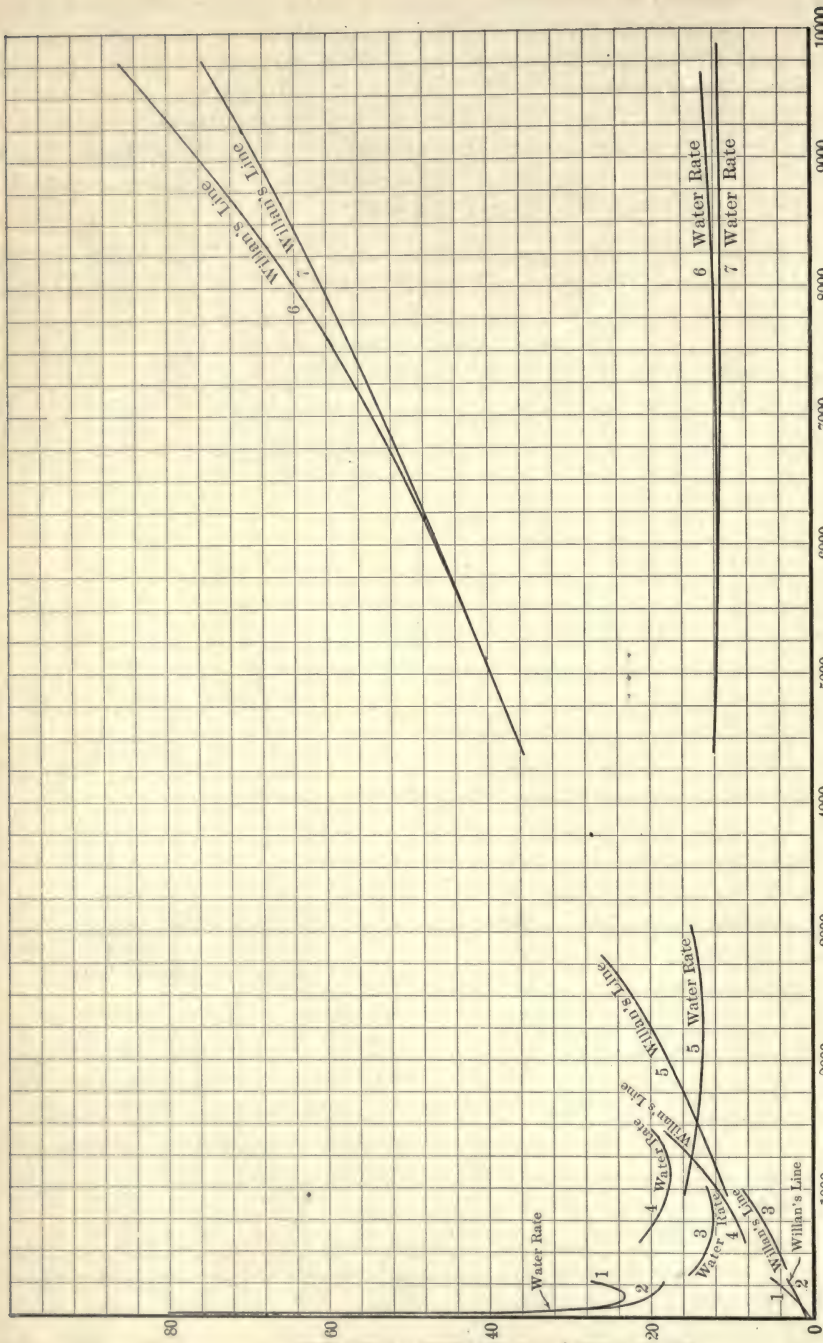
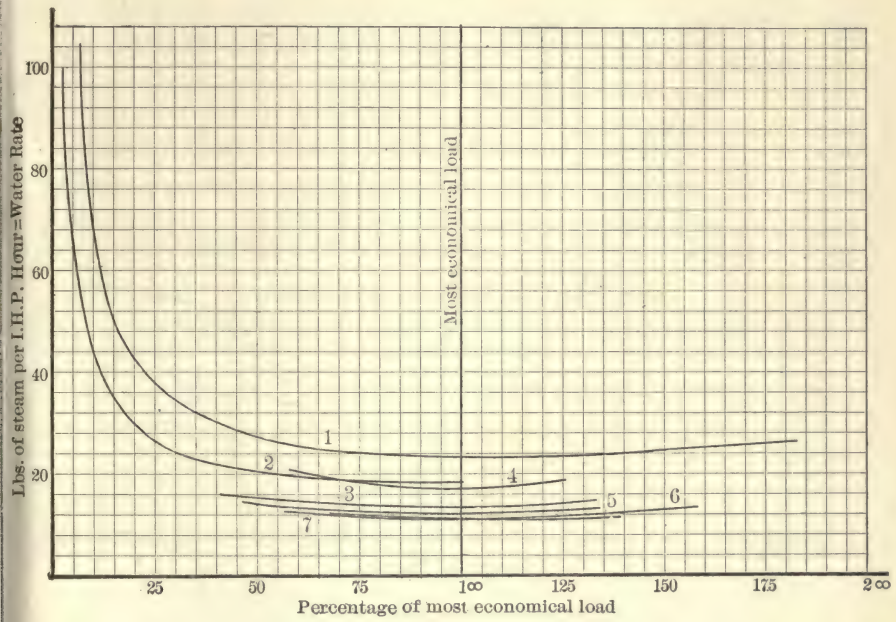


FIG. 112.— Typical Water Rate—Load Curves and Willan's Lines for Steam Engines.

Before taking up the derivation of equations some actual test curves will be examined and a number of these are grouped in Fig. 112 for engines of various sizes, simple and compound, up to 10,000 H.P., on which vertical distances represent pounds of steam per hour, per I.H.P. and horizontal .H.P. To show the essential similarity of the curves for engines of different ize more clearly, these are re-plotted in Fig. 113 to a new load scale based n best load of *each*, which is taken as unity. *This is evidently a function of ean effective pressure*, just what sort of function does not matter here. In



IG. 113.—Typical Water Rate—Lead Curves for Steam Engines Plotted to Practional Loads.

very case the Willans line is also plotted in Fig. 112, each line being numered to correspond to its water rate curve.

As there is a corresponding similarity of form for the water rate and Villans line of steam turbines, though the reasons for it will be developed ter, it must be understood that the mathematical analysis that follows applies both turbine and piston steam engines, and finally it makes no difference hat units are used for load, whether I.H.P., or B.H.P. or K.W. of a direct-nnected electric generator.

In Fig. 114 is shown the water-rate curve to a K.W. base for the 10,000 K.W. urtis steam turbine at the Chicago Edison, Fiske Street Station for which the llowing equation fits exactly:

$$\frac{Y}{P}=\frac{17.02}{P}+10.54+.156P,$$

where Y = pounds of steam per hour $\div 1000$;

P = load (in this case in K.W.) $\div 1000$;

$\frac{Y}{P}$ = pounds of steam per K.W. hr.

A similar equation fits fairly well the curve of Fig. 115, representing th 7000 H.P. piston engines of the Interborough Railway, Fifty-ninth Stre station, as well as the combined piston engine and low-pressure steam turbi

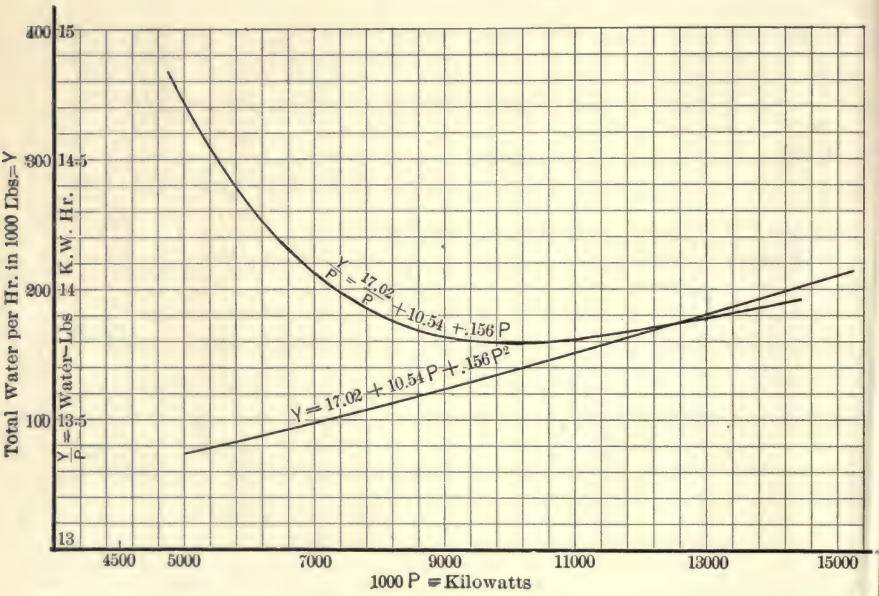


FIG. 114.—Performance of a 10000-K.W. Steam Turbine.

taking its exhaust steam, in the same station, but with different numeri constants, as below:

$$\text{Piston engine, } \frac{Y}{P} = \frac{36.75}{P} + .6 + 1.85P,$$

$$\text{Combined piston engine and turbine, } \frac{Y}{P} = \frac{89.4}{P} - 2.90 + .713P.$$

A third case of smaller size is shown in Fig. 116, representing the p formance of a 1000-K.W. Corliss piston engine driving a generator for wh the equation is

$$\frac{Y}{P} = \frac{9.8}{P} - .8 + 8.3P.$$

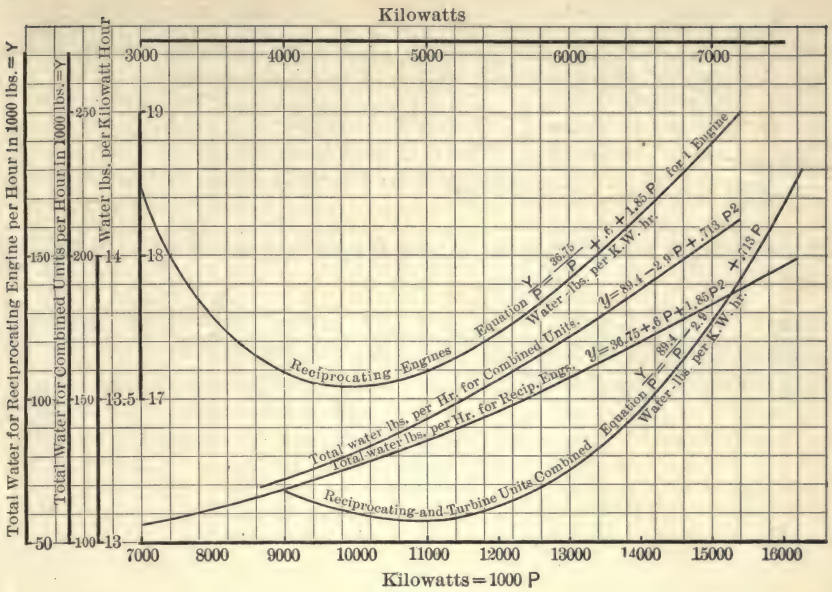


FIG. 115.—Performance of a 7000-H.P. Piston Engine alone and with a Low-pressure Steam Turbine.

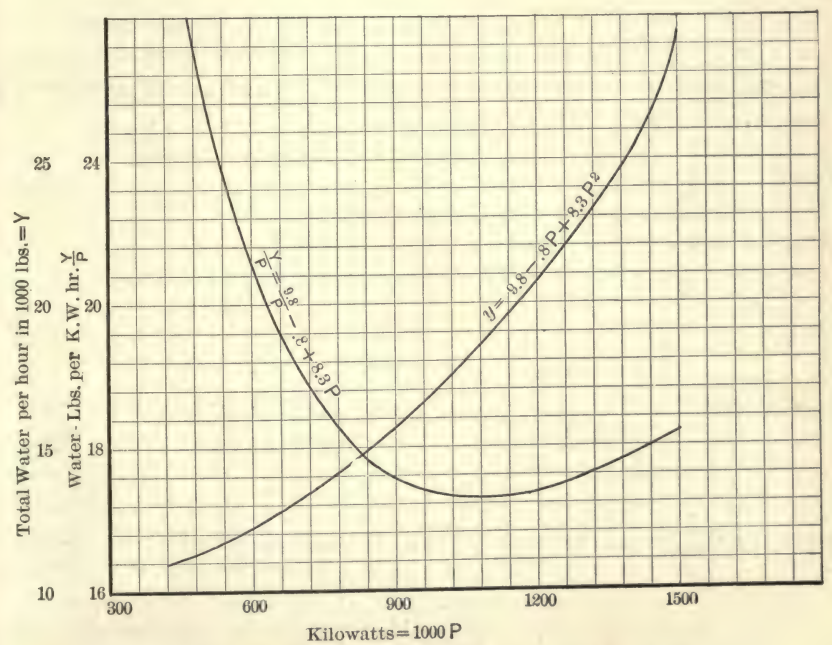


FIG. 116.—Performance of a 1000-K.W. Steam Turbine.

These illustrations could be multiplied indefinitely, but those given will suffice to establish the fact that the two following equations are fundamental *over the working range* of any steam engine of whatever type:

Water rate line, $$\frac{Y}{P}=\frac{A}{P}+B+CP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (597)$$

Water per hour, Willans line, $$Y=A+BP+CP^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (598$$

in which Y is the weight of steam per hour and P the engine load whether expressed in indicated or brake horse-power, or in kilowatts.

At the most economical load the water rate is a minimum, so that

$$\frac{d}{dP}\left(\frac{Y}{P}\right)=0=\frac{d}{dP}\left(\frac{A}{P}+B+CP\right)$$

whence the most economical load is

$$P'=\sqrt{\frac{A}{C}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (599)$$

Where the Willans line is straight, $C=0$, and the most economical load is the greatest load.

Two engines carrying the same load must divide it and some one proportion may be best. To find out, consider first any number of *similar engines*, that is, engines that have the same constants A, B, and C, denoting each case by subscripts. Then

Let P = total load;
" P_1, P_2, P_3, etc. = individual engine loads;
" Y = total water per hour;
" Y_1, Y_2, Y_3 = water per hour for each engine.

Then

$$\begin{aligned} Y&=Y_1+Y_2+Y_3+\ldots+Y_n \\ &=nA+B(P_1+P_2+P_3+\ldots P_n)+C(P_1^2+P_2^2+P_3^2+\ldots+P_n^2) \\ &=nA+BP+C(P_1^2+P_2^2+P_3^2+\ldots+P_n^2). \end{aligned}$$

Only the last term is variable and this is a minimum when

$$P_1=P_2=P_3=P_n.$$

Therefore for *similar engines, the best division of load is an equal division.*

When the engines are dissimilar it is convenient to first consider the case straight Willans lines for which $C=0$. Then for two such engines

$$\begin{aligned} Y &= A_1+A_2+B_1P_1+B_2P_2 \\ &= (A_1+A_2)+B_1(P-P_2)+B_2P_2 \\ &= (A_1+A_2)+B_1P+(B_2-B_1)P_2. \end{aligned}$$

At any given load P the first two terms together will be constant, and the ter per hour will be least when the last term is least. As neither factor n be zero, this will occur when P_2 is least.

Therefore for *two dissimilar engines the best division of load is that which ts the greatest possible share on the one with the smaller value of B, in its equation, vided each has a straight Willans line.*

Two dissimilar engines of whatever characteristics yield the equation,

$$\begin{aligned} Y &= A_1+A_2+B_1P_1+B_2P_2+C_1P_1^2+C_2P_2^2 \\ &= (A_1+A_2+B_1P+C_1P_2) \\ &\qquad +(B_2-2PC_1-B_1)P_2 \\ &\qquad +(C_1+C_2)P_2^2. \end{aligned}$$

Differentiation with respect to P_2, and solving for P_2, the load for the second gine that makes the whole steam consumption least, gives,

$$\begin{aligned} P_2 &= \frac{B_1-B_2}{2(C_1+C_2)}+\frac{C_1P}{(C_1+C_2)}. \quad \ldots\ldots\ldots \quad (600) \\ &= \text{constant}+\text{constant}\times P. \end{aligned}$$

Therefore, *the load division must be linear* and Eq. (600) gives the numerical ue, *when any two engines share a given load.*

This sort of analysis can be carried much further by those interested, but ce forbids any extension here. It is proper to point out, however, that means of it the proper *switch-in* points for each unit in a large power station n be accurately found, to give most economical operation on an increasing tion load.

26. Graphical Solution of Problems on Horse-power and Cylinder Sizes. e diagram for mean effective pressures in terms of initial and back ssure, clearance, compression and cut-off, Fig. 117, facilitates the solution Eq. (262) in Section 5. The mean effective pressure is the difference ween mean forward and mean back pressure. The former is dependent n clearance, cut-off and initial pressure. In the example shown on the re by letters and dotted lines, clearance is assumed 5 per cent, shown at Project horizontally to the point F, on the contour line for the assumed -off, 12 per cent. Project downward to the logarithmic scale for " mean

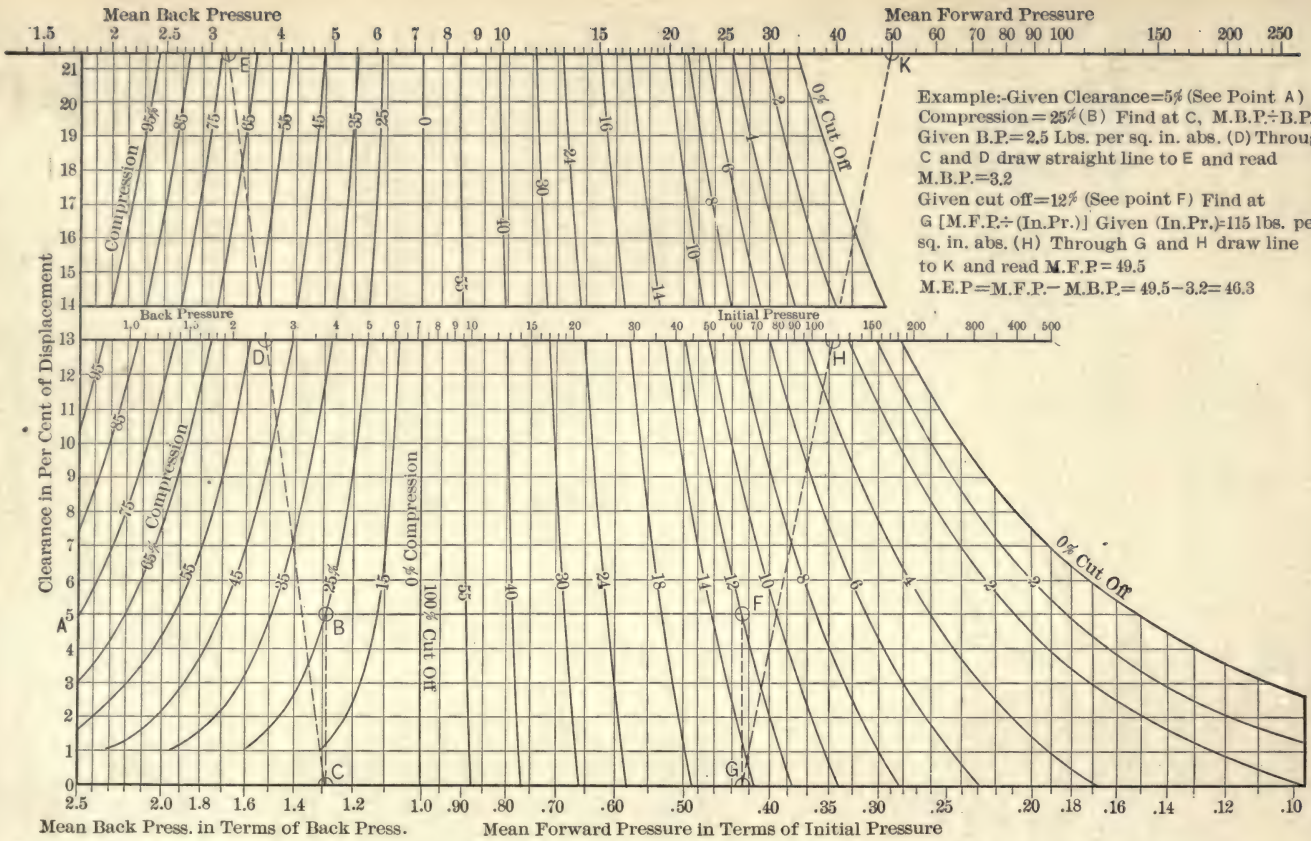


FIG. 117.—Graphical Determination of Mean Effective Pressure for Single Cylinder Engines with Clearance, Logarithmic Expansion and Compression

orward pressure in terms of initial pressure" to the point G. On the scale or "initial pressure" find the point H, representing the assumed initial pressure, 115 lbs. absolute. Through G and H a straight line is passed to the point on the scale for "mean forward pressure," where the value is read, n.f.p. = 49.5 lbs. absolute.

Mean back pressure is similarly dependent upon clearance, compression nd back pressure, and the same process is followed out by the points A, B, , D and E, reading the mean back pressure, 3.2 lbs. absolute at the point E. hen by subtraction,

$$(\text{m.e.p.}) = (\text{m.f.p.}) - (\text{m.b.p.}) = 49.5 - 3.2 = 46.3 \text{ lbs.}$$

Fig. 118 is arranged to show what conditions must be fulfilled in order to btain equal work with *complete expansion in both cylinders* in a compound ngine, finite receiver, logarithmic law, no clearance, Cycle VII, when low-ressure admission and high-pressure exhaust are not simultaneous. This is iscussed in Section 11, and the diagram represents graphically the conditions xpressed in Eqs. (376), (377), (378), (379).

To illustrate its use assume that in an engine operating on such a cycle, he volume of receiver is 1.5 times the high-pressure displacement, $1.5 = y$, then

$\frac{}{y} = .667$. Locate the point A on the scale at bottom of Fig. 118, corresponding o this value. Project upward to the curve marked "ratio of cut-offs" and at he side, C, read ratio of cut-offs

$$\frac{Z_H}{Z_L} = .572.$$

ext extending the line AB to its intersection D, with the curve GH, the point) is found. From D project horizontally to the contour line representing the iven ratio of initial to back pressure. In this case, initial pressure is assumed n times back pressure. Thus the point E is located. Directly above E at ie top of the sheet is read the cylinder ratio, at F,

$$R_C = \frac{D_L}{D_H} = 2.4.$$

If cylinder ratio and initial and final pressures are the fundamental data of ie problem, the ratio of cut-offs and ratio of high-pressure displacements receiver volume may be found by reversing the order.

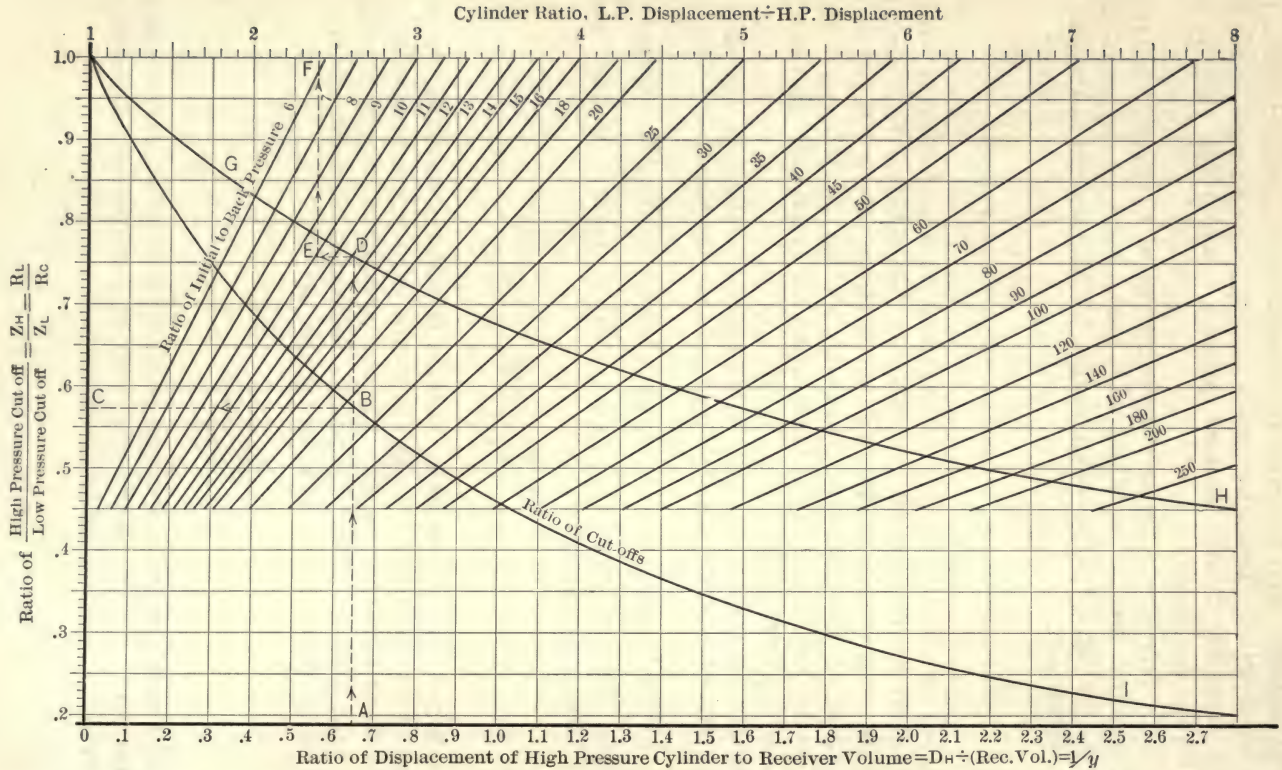


FIG. 118.—Relations for Equal Distribution of Work in Compound Engine with Finite Receiver and No Clearance when Expansion is Logarithmic, Complete in both Cylinders and High-Pressure Exhaust and Low-pressure Admission are Not

GENERAL PROBLEMS ON CHAPTER III.

Prob. 1. How much steam will be required to run a 14×18-in. double-acting engine with no clearance at a speed of 200 R.P.M. when initial pressure is 100 lbs. per square in. gage, back pressure 28 ins. of mercury (barometer reading 30 ins), and cut-off is $\frac{1}{2}$? What will be horse-power under these conditions?

NOTE: δ for 100 lbs. = .26, for 28 ins. = .0029.

Prob. 2. Draw the indicator cards and combined diagram for a compound steam engine without receiver, and with 3 per cent clearance in low pressure and 5 per cent in high, when initial pressure is 100 lbs. per square inch absolute, back pressure 10 lbs. per square inch absolute. high-pressure cut-off $\frac{1}{2}$, high-pressure compression $\frac{1}{10}$, and low-pressure compression $\frac{1}{8}$.

Prob. 3. A simple double-acting engine, 18×24 ins., is running at 100 R.P.M. on compressed air, the gage pressure of which is 80 lbs. The exhaust is to atmosphere. If the clearance is 6 per cent and cut-off $\frac{3}{4}$, and compression 10 per cent, what horse-power is being developed, the expansion being adiabatic, and how long can engine be run at rated load on 1000 cu.ft. of the compressed air?

Prob. 4. Will the work be equally distributed in a 12×18×24-in. engine with infinite receiver and no clearance when cut-off is $\frac{1}{2}$ in high pressure cylinder, and $\frac{3}{8}$ in low, expansion being logarithmic, initial pressure 150 lbs. per square inch absolute and back pressure atmosphere? What will be work in each cylinder?

Prob. 5. The receiver of a 15×20×22 in. engine is 4 times as large as high-pressure cylinder. What will be the horse-power, steam used per hour, and variation in receiver pressure for this engine, if clearance be considered, zero and initial pressure is 125 lbs. per square inch gage, back pressure 5 lbs. per square inch absolute, cut-offs $\frac{1}{2}$ and $\frac{3}{8}$ in high- and low-pressure cylinders respectively, and piston speed is 550 ft. per minute?

NOTE: δ for 125 lbs. = .315, for 5 lbs. = .014.

Prob. 6. At no load an engine having 7 per cent clearance, cuts off at 4 per cent of its stroke, while at full load it cuts off at 65 per cent of its stroke. At no load, compression is 40 per cent and at full load 5 per cent. What percentage of full-load horse-power is required to overcome friction, and what percentage of steam used at full load, is used on friction load, if initial pressure is constant at 100 lbs. per square inch gage, back pressure constant at 5 lbs. per square inch absolute, and expansion is logarithmic?

NOTE: δ for 100 lbs. = .262, for 5 lbs. = .014.

Prob. 7. The initial pressure on which engine is to run is 115 lbs. per square inch gage, and steam is superheated and known to give a value of $s = 1.3$. For an engine in which clearance may be neglected, work is to be equal, and expansion complete in both cylinders, when back pressure is 10 lbs. per square inch absolute. What must be the cut-offs and cylinder ratio to accomplish this when receiver is $3\frac{1}{2}$ times high pressure cylinder volume?

Prob. 8. A 12-in. and 18×24 ins. double-acting engine with zero clearance and infinite receiver operates on an initial pressure of 150 lbs. per square inch gage, and

a back pressure of 5 lbs. per square inch absolute. What will be the release and receiver pressures, horse-power, and steam consumption when speed is 150 R.P M., expansion logarithmic, and cut-off $\frac{1}{2}$ in each cylinder?

NOTE: δ for 150 lbs. = .367, for 5 lbs. = .014.

Prob. 9. If a third cylinder 24 ins. in diameter were added to engine of Prob. 8 and cut-off in this made $\frac{1}{2}$, how would horse-power, steam consumption, receiver and release pressures change?

Prob. 10. What would have to be size of a single cylinder to give same horse-power at same revolutions and piston speed as that of engine of Prob. 8 under same conditions of pressure and cut-off?

Prob. 11. With the high-pressure cut-off at $\frac{1}{2}$, and low and intermediate cut-offs at $\frac{3}{10}$, what will be the horse-power, water rate and receiver pressures of a $30 \times 48 \times 77 \times 72$-in. engine running at 102 R.P.M. on an initial pressure of 175 lbs. per square inch gage and a back pressure of 26 ins. of mercury (barometer reading 30 ins.), if the receiver be considered infinite and expansion logarithmic, clearance zero? What change in intermediate and low-pressure cut-offs would be required to give equal work distribution?

NOTE: δ for 175 lbs. = .419, for 26 ins. = .0058.

Prob. 12. If it had been intended to have all the cut-offs of the engine of Prob. 11, equal to $\frac{1}{2}$, what should have been the size of the intermediate and low-pressure cylinders to give equal work for same pressure range and same high-pressure cylinder?

Prob. 13. To attain complete expansion in all cylinders of the engine of Prob. 11, with the initial and back pressures as there given, what cut-offs would be required and what receiver pressures would result?

Prob. 14. A compound locomotive has no receiver, the high-pressure clearance is 8 per cent, and low-pressure clearance 5 per cent. The cylinders are 22 and 33×48 ins., high-pressure cut-off $\frac{3}{4}$, high- and low-pressure compression each 10 per cent, initial pressure 175 lbs. per square inch gage, back pressure one atmosphere, and expansion and compression logarithmic. What will be the horse-power at a speed of 40 miles per hour, the engine having 7-ft. driving wheels? At this speed, how long will a tank capacity of 45,000 gallons last?

NOTE: δ for 175 lbs. = .419, for 15 lbs. = .038.

Prob. 15. A superheater has been installed on engine of Prob. 14 and expansion and compression, now follow the law $PV^s = c$, when $s = 1.2$. What effect will this have on the horse-power and steam consumption?

Prob. 16. What will be the maximum receiver pressure work done in each cylinder and total work for a cross-compound engine 36 and 66×48 ins., running at 100 R.P.M. on compressed air of 100 lbs. per square inch gage pressure, exhausting to atmosphere if the high pressure cut-off is $\frac{1}{2}$, clearance 6 per cent, compression 20 per cent, low-pressure cut-off is $\frac{3}{8}$, clearance 4 per cent, compression 15 per cent, and receiver volume is 105 cu.ft.?

Prob. 17. A manufacturer gives the horse-power of a $42 \times 64 \times 60$-in. engine as 2020, when run at 70 R.P.M. on an initial pressure of 110 lbs. per square inch gage, atmospheric back pressure, and .4 cut-off in high-pressure cylinder. How does this value compare with that found on assumption of 5 per cent clearance in high, 4 per cent in low, and complete expansion and compression is each cylinder?

Prob. 18. A mine hoisting engine is operated on compressed air with a pressure of 150 lbs. per square inch absolute and exhausts to atmosphere. The cylinder sizes are $26 \times 48 \times 36$ ins., and clearance is 5 per cent in each. At the start the high-pressure

cut-off is 1 and low pressure $\frac{1}{2}$, while normally both cut-offs are $\frac{1}{2}$. The exhaust from high-pressure cylinder is into a large receiver which may be considered infinite. The compression is zero at all times. Considering the exponent of expansion to be 1.4, what will be the horse-power under the two conditions of cut-off given, for a speed of 100 R.P.M.?

Prob. 19. What must be ratio of cylinders in the case of a compound engine with infinite receiver, to give equal work distribution complete expansion and compression if the least clearance which may be attained is 5 per cent in the high-pressure cylinder, and 3 per cent in the low-pressure. The engine is to run non-condensing on an initial pressure of 125 lbs. per square inch gage, with expansion exponent equal to 1.3? What must be the cut-offs and compressions to satisfy these conditions?

Prob. 20. Assuming 7 per cent clearance in high-pressure cylinder and 5 per cent in low, infinite receiver, and no compression, how will the manufacturer's rating of 2100 H.P. check, for a $36 \times 6 \times 48$-in. engine running at 85 R.P.M. on an initial pressure of 110 lbs. per square inch gage, and a back pressure of 26 ins. vacuum, with .3 cut-off in high pressure cylinder?

Prob. 21. For a $25 \times 40 \times 36$-in. engine, with 5 per cent clearance, $\frac{3}{8}$ cut-off and 20 per cent compression in each cylinder, what will be horse-power for an initial pressure of 100 lbs. per square inch gage, and a back pressure of 17.5 lbs. per square inch absolute, with logarithmic expansion and compression?

Prob. 22. What must be the cylinder ratio and cut-off to give complete expansion in a no-clearance, 14 and 22×24-in. engine with no receiver and logarithmic expansion, when initial pressure is 100 lbs. per square inch gage, and back pressure 10 lbs. per square inch absolute? What will be the horse-power and steam used for these conditions at a speed of 150 R.P.M.?

NOTE: δ for 100 lbs. = .262, for 10 lbs. = .026.

Prob. 23. A $24 \times 20 \times 24$ in. engine with no receiver or clearance, runs on compressed air of 120 lbs. per square inch gage pressure, and exhausts to atmosphere. When running at a speed of 125 R.P.M., with high-pressure cut-off $\frac{1}{2}$, what horse-power will be developed and how many cubic feet of compressed air per minute will be required to run the engine, the expansion being adiabatic? Will the work be equally divided between the two cylinders?

Prob. 24. It is desired to run the above engine as economically as possible. What change in cut-off will be required, and will this cause a decrease or increase in horse-power and how much? How will the quantity of air needed be affected?

Prob. 25. A mill operates a cross-compound engine with a receiver 3 times as large as high-pressure cylinder, on an initial pressure of 125 lbs. per square inch gage, and a back pressure of 10 lbs. per square inch absolute. The engine may be considered as without clearance, and the expansion as logarithmic. As normally run the cut-off in high-pressure cylinder is $\frac{3}{8}$ and in low, $\frac{1}{4}$. It has been found that steam is worth 25 cents a thousand pounds. What must be charged per horse-power day (10 hours) to pay for steam if the missing water follows Heck's formula?

NOTE δ for 125 = .315, for 10 = .026.

Prob. 26. By installing a superheater the value of s in Prob. 25 could be changed to 1.3. The cost of st am would then be 30 cents a thousand pounds. From the effect on value of s alone would the installation of the superheater pay?

Prob. 27. When a $26 \times 48 \times 36$-in. cross-compound engine with a receiver volume of 35 cu.ft. and zero clearance, is being operated on steam of 125 lbs. per square inch

gage initial pressure, and atmospheric exhaust, is the work distribution equal, whe high-pressure cut-off is $\frac{5}{8}$ and low-pressure cut-off $\frac{1}{2}$? For these cut-offs what fluctuation in receiver pressure and what steam will be used per horse-power hour?

NOTE: δ for 125 = .315, for 15 lbs. = .038.

Prob. 28. To operate engine of Prob. 27 under most economical conditions, wha values must be given to the cut-offs, and what values will result for receiver pressure horse-power, and steam used per hour?

Prob. 29. What will be the horse-power and steam used by a 20×30×36-i engine with infinite receiver and no clearance, if expansion be such, that $s = 1.2$ high-pressure cut-off $\frac{1}{4}$, low-pressure cut-off $\frac{3}{8}$, initial pressure 100 lbs. per square inc gage, back pressure 3 lbs. per square inch absolute, and speed 100 R.P.M.

NOTE: δ for 100 lbs. = .262, for 3 lbs. = .0085.

Prob. 30. The following engine with infinite receiver and no-clearance, runs c steam which expands according to the logarithmic law. Cylinders 9, and 13× ins., initial pressure 125 lbs. per square inch gage, back pressure 5 lbs. per square inc absolute, high-pressure cut-off $\frac{1}{2}$, low-pressure $\frac{3}{8}$, speed 150 R.P.M. What will h horse-power and steam consumption hypothetical and probable?

NOTE: δ for 125 lbs. = .315, for 5 lbs. = .014.

Prob. 31. By graphical means find the (m.e.p.), of a 15×22×30-in. cros compound engine, with 5 per cent clearance in each cylinder, if the receiver volume 8 cu.ft., initial pressure 125 lbs. per square inch absolute, back pressure 10 lbs. p square inch absolute, high-pressure cut-off $\frac{1}{8}$, low-pressure $\frac{5}{16}$, high-pressure compre sion 40 per cent, low-pressure 20 per cent, high-pressure crank following 90°, logarithm expansion.

Prob. 32. Show by a series of curves, assuming necessary data, the effect (m.e.p.) of cut-off, back pressure, clearance, and compression.

Prob. 33. Show by curves, how the indicated, and actual water rate, of an 18×2 in. engine with 5 per cent clearance, and running at 125 R.P.M. on an initial pressu of 125 lbs. per square inch gage, and a back pressure of 10 lbs. per square inch absolu may be expected to vary with cut-off from $\frac{1}{20}$ to $\frac{3}{4}$.

TABLE XIII

PISTON POSITIONS FOR ANY CRANK ANGLE

FROM BEGINNING OF STROKE AWAY FROM CRANK SHAFT TO FIND PISTON POSITION FROM DEAD-CENTER MULTIPLY STROKE BY TABULAR QUANTITY

Crank Angle.	$\frac{l}{r}=4$	$\frac{l}{r}=4.5$	$\frac{l}{r}=5$	$\frac{l}{r}=5.5$	$\frac{l}{r}=6$	$\frac{l}{r}=7$	$\frac{l}{r}=8$	$\frac{l}{r}=9$
5	.0014	.0015	.0015	.0016	.0016	.0016	.0017	.0019
10	.0057	.0059	.0061	.0062	.0063	.0065	.0067	.0076
15	.0128	.0133	.0137	.0140	.0142	.0146	.0149	.0170
20	.0228	.0237	.0243	.0248	.0253	.0260	.0265	.0302
25	.0357	.0368	.0379	.0388	.0394	.0405	.0413	.0468
30	.0513	.0531	.0545	.0556	.0565	.0581	.0592	.0670
35	.0698	.0721	.0740	.0754	.0767	.0787	.0801	.0904
40	.0910	.0939	.0962	.0981	.0997	.1022	.1041	.1170
45	.1152	.1187	.1215	.1237	.1256	.1286	.1308	.1468
50	.1416	.1458	.1491	.1518	.1541	.1576	.1607	.1786
55	.1713	.1759	.1828	.1827	.1853	.1892	.1922	.2132
60	.2026	.2079	.2122	.2157	.2186	.2231	.2295	.2500
65	.2374	.2431	.2477	.2514	.2545	.2594	.2630	.2886
70	.2730	.2794	.2844	.2885	.2929	.2973	.3013	.3290
75	.3123	.3187	.3239	.3282	.3317	.3372	.3414	.3705
80	.3516	.3586	.3642	.3687	.3725	.3784	.3828	.4132
85	.3944	.4013	.4068	.4113	.4151	.4210	.4254	.4564
90	.4365	.4437	.4495	.4547	.4580	.4641	.4686	.5000
95	.4816	.4885	.4940	.4985	.5022	.5081	.5126	.5436
100	.5253	.5323	.5378	.5424	.5461	.5520	.5564	.5868
105	.5711	.5775	.5828	.5870	.5905	.5961	.6002	.6294
110	.6150	.6214	.6265	.6306	.6340	.6393	.6530	.6710
115	.6600	.6657	.6703	.6740	.6771	.6820	.6856	.7113
120	.7026	.7080	.7122	.7157	.7186	.7231	.7265	.7500
125	.7449	.7495	.7533	.7563	.7588	.7628	.7658	.7868
130	.7844	.7885	.7920	.7947	.7969	.8004	.8030	.8214
135	.8223	.8258	.8286	.8308	.8327	.8357	.8379	.8535
140	.8570	.8600	.8623	.8642	.8658	.8682	.8703	.8830
145	.8889	.8913	.8931	.8946	.8958	.8978	.8993	.9096
150	.9173	.9191	.9204	.9216	.9226	.9241	.9252	.9330
155	.9420	.9432	.9452	.9451	.9457	.9468	.9476	.9531
160	.9625	.9633	.9640	.9645	.9650	.9656	.9661	.9698
165	.9787	.9792	.9796	.9799	.9802	.9805	.9809	.9829
170	.9905	.9908	.9909	.9911	.9912	.9913	.9915	.9924
175	.9976	.9977	.9977	.9977	.9978	.9978	.9979	.9981
180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE XIV

VALUES OF x FOR USE IN HECK'S FORMULA FOR MISSING WATER

Absolute Steam Pressure.	x	Absolute Steam Pressure.	x	Absolute Steam Pressure.	x
0	170	70	297.5	165	393
1	175	75	304	170	397
2	179	80	310	180	405
3	183	85	316	185	409
4	186	90	321.5	190	413
6	191	95	327	195	416.5
8	196	100	332.5	200	420
10	200	105	338	210	427
15	210	110	343	220	434
20	220	115	348	230	441
25	229	120	353	240	447.5
30	238	125	358	250	454
35	246	130	362.5	260	460.5
40	254	135	367	270	467
45	262	140	371.5	280	473
50	269.5	145	376	290	479
55	277	150	380.5	300	485
60	284	155	385		
65	291	160	389		

TABLE XV

SOME ACTUAL ENGINE DIMENSIONS

SIMPLE

7 × 9	7¾×15	16 ×18	15¼×24	24 ×36
8 × 9	8¾×15	16½×18	16 ×24	26 ×36
9 × 9	12 ×15	17 ×18	18 ×24	26⅔×36
5½×10	13 ×15	17½×18	20 ×24	28 ×36
6½×10	14 ×15	18 ×18	22 ×24	14 ×42
8 ×10	14⅓×15	19 ×18	24 ×24	15 ×42
9 ×10	15 ×15	20 ×18	16½×27	16 ×42
10 ×10	16 ×15	29 ×19	17½×27	18 ×42
11 ×10	17⅓×15	12 ×20	10 ×30	20 ×42
9½×10½	11 ×16	14 ×20	12 ×30	22 ×42
10½×10½	12 ×16	18 ×20	16 ×30	24 ×42
7¾×12	13 ×16	19 ×20	18 ×30	26 ×42
8 ×12	14¼×16	28 ×20	18¾×30	28 ×42
8¾×12	15 ×16	21 ×20	20 ×30	18 ×48
9 ×12	15¼×16	22 ×20	24 ×30	20 ×48
10 ×12	16 ×16	12 ×21	22 ×33	22 ×48
11 ×12	17 ×16	13 ×21	24 ×33	24 ×48
11½×12	18 ×16	18¾×21	10 ×36	26 ×48
12 ×12	18½×17	20 ×21	12 ×36	28 ×48
12½×12	23 ×17	20 ×22	14 ×36	24 ×54
13 ×12	26 ×17	18 ×24	16 ×36	26 ×54
14 ×12	10 ×18	10 ×24	18 ×36	28 ×54
10 ×14	11 ×18	12 ×24	20 ×36	28 ×60
11 ×14	15 ×18	14¼×24	22 ×36	

TABLE XV.—*Continued*

COMPOUND

NOTES: 1 to run condensing or non-condensing on initial pressure of 100–150.
2 to run condensing or non-condensing on initial pressure of 100.
3 to run condensing or non-condensing on initial pressure of 125.
4 to run condensing or non-condensing on initial pressure of 90–100.
5 to run condensing or non-condensing on initial pressure of 110–130.
6 to run condensing or non-condensing on initial pressure of 140–160.
7 to run condensing or non-condensing on initial pressure of 125

4¼– 8 × 6		13 –18 ×14	1	11 –19 ×18	3	14¼–26 ×22	4	22 –38 ×33	3
6 –10 × 6		13 –20 ×14	1, 2	12 –21 ×18	3	18 –32 ×22	5	24 –42 ×33	3
7 –13 × 8		7¾–13½×15	3	13 –22½×18	3	10 –17½×24	3	18¾–32½×36	3
6 –12 ×10	1	9 –15½×15	3	14 –24 ×18	1	11 –19 ×24	3	20 –36 ×36	3
7 –12 ×10	1	11 –19 ×15	4	15¼–26½×18	3	12 –18 ×24	7	26½–46 ×36	3
8 –12 ×10	1	13 –19 ×15	5	16 –24 ×18	1	13 –20 ×24	7	28½–50 ×36	3
8½–15½×10		7¾–13½×16	3	16 –26 ×18		14 –22 ×24	7	14¼–25 ×42	3
7 –14 ×10	1	9 –15½×16	3	16½–28½×18	3	16½–28½×24	3	15¼–26½×42	3
8 –14 ×10	1	10 –17½×16	3	8 –12 ×20	7	17½–30½×24	3	18½–32½×42	3
9½–15 ×11	2	11 –19 ×16	3	9 –14 ×20	7	22 –38 ×24	3	20 –36 ×42	3
7¾–13½×12	3	11 –22 ×16	1	16 –28 ×20	1	24 –42 ×24	3	16½–28½×48	3
9 –15½×12	3	12 –21 ×16	3	17 –30 ×20	1	12 –21 ×27	3	17½–30½×48	3
19 –14 ×12	1	13 –22 ×16	1	18 –28 ×20	1	13 –22½×27	3	22 –38 ×48	3
10 –16 ×12	1	13 –22½×16	3	19 –30 ×20	1	16½–28½×27	3	24 –42 ×48	3
10 –18 ×12	1	14 –22 ×16	2	19 –30 ×22	1	17½–30½×27	3	18⅛–32½×54	3
11 –16 ×12	1	14¼–25 ×16	3	9 –15½×21	3	14¼–25 ×30	3	20 –36 ×54	3
9 –18 ×14	1	15 –22 ×16	1	12 –21 ×21	3	15¼–26½×30	3	26½–46 ×54	3
10 –18 ×14	1	15¼–26½×16	3	13 –22½×21	3	18¾–32½×30	3	28½–50 ×54	3
10 –17½×14	3	16 –25 ×16	2	14¼–26½×21	3	20 –36 ×30	3	22 –38 ×60	3
11 –19 ×14	3	13 –23 ×17	4, 6	15¼–28½×21	3	28½–50 ×30	3	24 –42 ×60	3
11 –18 ×14	1	15 –26 ×17	4, 6	18¾–32½×21	3	30 –54 ×30	3	30 –54 ×60	3
12 –18 ×14	1	16½–29 ×17	4	20 –36 ×21	3	16½–28½×33	3	32½–57 ×60	3
12 –20 ×14	1	9 –15⅝×18	3	13 –23 ×22	5	17½×30½×33	3	34 –60 ×60	3

TRIPLE

NOTE: All condensing and to run of initial pressure as given.

SIZE.	*p*	SIZE.	*p*	SIZE.	*p*
10 –15½–26×15	200	27 –43 $\left\{\begin{matrix}51\\-51\end{matrix}\right.$×39	180	30–50–82×48	180
11 –18 –30×20	250			25–41–68×48	190
12 –20 –34×24	180	18 –28½–48×40	180	27–45–75×54	190
12 –19 –32×24	190	22 –37 –63×42	180	28–45–72×54	185
	175	22 –38 –64×42	185	28–46–75×54	180
12¾–22 –36×24	180	32½–53 –$\left\{\begin{matrix}61\\61\end{matrix}\right.$×48	265	29–47–83×54	160
14 –23 –28×26	190			32–52–92×50	200
18 –29 –47×30	200	35 –57 –$\left\{\begin{matrix}66\\66\end{matrix}\right.$×48	265	34–56–100×60	200
16½–24 –41×30	180			35–58 $\left\{\begin{matrix}69\\69\end{matrix}\right.$×60	190
18 –30 –50×30	200	36 –57 –$\left\{\begin{matrix}76\\76\end{matrix}\right.$×48	295	34–57–104×63	200
16 –25½–43×30	190				
16½–24 –41×30	180	28 –45 –72×48	180		

CHAPTER IV

HEAT AND MATTER. QUALITATIVE AND QUANTITATIVE RELATIONS BETWEEN THE HEAT CONTENT OF SUBSTANCES AND THEIR PHYSICAL-CHEMICAL STATE,

1. Substances and Heat Effects Important in Engineering. It has been shown in preceding chapters concerned with work in general and with the determination of quantity of work that may be done in the cylinders by, or on expansive fluids that

(*a*) Fluids originally at low may be put in a high-pressure condition by the expenditure of work;

(*b*) Fluids under high pressure may do work in losing that pressure.

That work may be done, fluids under pressure are necessary and that the greatest amount of work may be done per unit of fluid the fluid itself must be expansive, that is, it must be a gas or a vapor. Gases or vapors under pressure are, therefore, prerequisites to the economical use of fluids for the doing of work, and that this work may be done at the expense of heat or derived from heat, it is only necessary that the heat be used to create the necessary primary condition of high pressure in vapors and gases. There are two general ways of accomplishing this purpose—first, to apply the heat to a boiler supplied with liquid and discharging its vapor at any pressure as high as desired or as high as may be convenient to manage; second, to apply the heat to a gas confined in a chamber, raising its pressure if the chamber be kept at a fixed volume, which is an intermittent process, or increasing the fluid volume if the size of the chamber be allowed to increase, the fluid pressure being kept constant or not, and this latter process may be intermittently or continuously carried out.

These two processes are fundamental to the steam and gas engines that are the characteristic prime movers or power generators of engineering practice, utilizing heat energy, and with the exception of water-wheels the sole commercially useful sources of power of the industrial world. Thus, the heating of gases and the evaporation of liquids are two most important thermal processes to be examined together with their inverse, cooling and condensation, and necessarily associated in practical apparatus with the heating and cooling of solid containers or associated liquids. From the power standpoint, the effects of heat on solids, liquids, gases and vapors, both without change of state and with change of state are fundamental, and the substances to be studied as heat carriers do not include the whole known chemical world, but only those that are cheap enough to be used in engineering practice or otherwise essential thereto. These substances of supreme importance are, of course, air and water, with all their

ysical and chemical variations, next the fuels, coal, wood, oil, alcohol and mbustible gases, together with the chemical elements entering into them d the chemical compounds which mixed together may constitute them.

Probably next in importance from the standpoint of engineering practice e the substances and thermal processes entering into mechanical refrigeration and ice making. There are but three substances of commercial importance re—ammonia, pure and in dilute aqueous solution, carbonic acid and air. he process of heating or cooling solids, liquids, gases and vapors, together ith solidification of water into ice, evaporation and condensation, fundamental power problems, are also of equal importance here, but there is added an ldditional process of absorption of ammonia vapor in water and its discharge om the aqueous solution.

Many are the practical applications of heat transfer or transmission, some which call into play other substances than those named. In the heating buildings there is first combustion with transfer of heat to water in boilers, ow of the hot water or steam produced to radiators and then a transfer of heat the air of the room; in feed-water heaters, heat of exhaust steam warms ater on its way to the boilers; in economizers, heat of hot flue gases is transrred to boiler feed water; in steam superheaters, heat of hot flue gases is transrred to steam previously made, to raise its temperature, steam pipes, boiler rfaces and engine cylinders transfer heat of steam to the air which is opposed y covering and lagging, in steam engine condensers heat of exhaust steam is ansferred to circulation water; in cooling cold storage rooms and making ice, solution of calcium or sodium chloride in water is circulated through pipes nd tanks and is itself kept cool in brine coolers in which the brine transfers he heat absorbed in the rooms and tanks, to the primary substance ammonia carbonic acid and evaporates it.

While evaporation and condensation as processes are fundamental to the achinery and apparatus of both power and refrigeration, they also are of nportance in certain other industrial fields. In the concentration of lutions to promote crystallization such, for example, as sugar, evaporaon of the solution and condensation of the distillate are primary processes s also is the case in making gasolene and kerosene from crude oil, in the making f alcohol from a mash, and many other cases found principally in chemical anufacture. These are examples of evaporation and condensation in which ttle or no gases are present with the vapor but there are other cases in which gas is present in large proportion, the thermal characteristics of which are ifferent as will be seen later. Among these processes are: the humidification r moistening of air with water in houses and factories to prevent excessive kin evaporation of persons breathing the air, excessive shrinkage of wood-work nd to facilitate the manufacturing processes like tobacco working and thread pinning. Conversely, air may be too moist for the purpose, in which case it s dried by cooling it and precipitating its moisture as rain or freezing it out as e, and this is practiced in the Gayley process of operating blast furnaces, where xcess of moisture will on dissociating absorb heat of coke combustion and reduce

the iron output per ton of coke, and in the factories where, for example, collodion is worked, as in the manufacture of photographic films, with which moisture seriously interferes. Of course, humidification of air by water is accomplished only by evaporation of water, and evaporation of water is only to be accomplished by the absorption of heat, so that humidification of air by blowing it over water or spraying water into it must of necessity cool the water, and this is the principle of the cooling tower or cooling pond for keeping down the temperature of condenser circulating water, and likewise the principle of the evaporative condenser, in which water cooler and steam condenser are combined in one. The same process then, may serve to cool water if that is what is wanted, or to moisten air, when dry air is harmful, and may also serve to remove moisture from solids like sand, crystals, fabrics, vegetable or animal matter to be reduced to a dryer or a pulverized state.

There are some important examples of humidification in which the substances are not air and water, and one of these is the humidification of air by gasolene or alcohol vapor to secure explosive mixtures for operating gas engines. Here the air vaporizes enough of the fuel, humidifying or carburetting itself to serve the purpose, sometimes without heat being specifically added and sometimes with assistance from the hot exhaust. A somewhat similar action takes place in the manufacture of carburetted water gas when the water gas having no illuminating value is led to a hot brick checkerwork chamber supplied with a hydrocarbon oil, the vapor of which humidifies the gas, the heat of vaporization being supplied by the hot walls and regularly renewed as the process is intermittent. Of course, in this case some of the vapors may really decompose into fixed gases, a peculiar property of the hydrocarbon fuels, both liquid and gaseous, and frequently leaving residues of tar, or soot, or both.

Finally, among the important processes there is to be noted that of gasification of solid and liquid fuels in gas producers and vaporizers, a process also carried on in blast furnaces in which it is only an accidental accompaniment and not the primary process. Some of the actions taking place in gas producers are also common to the manufacture of coal gas, and coke, in retorts, beehive and by-product ovens.

From what has been said it should be apparent that engineers are concerned not with any speculations concerning the nature of heat but only with the kind and quantity of effect that heat addition to, or abstraction from, substances may be able to produce and not for all substances either. While this interest is more or less closely related to philosophic inquiry, having for its object the development of all embracing generalizations or laws of nature, and to the relation of heat to the chemical and physical constitution of matter, subject matter of physical chemistry, the differences are marked, and a clearly defined field of application of laws to the solution of numerical problems dealing with identical processes constitutes the field of engineering thermodynamics.

It is not possible or desirable to take up and separately treat every single engineering problem that may rise, but on the contrary to employ the scientific methods of grouping thermal processes or substance effects into types.

Prob. 1. Water is forced by a pump through a feed-water heater and economizer) a boiler where it is changed to steam, which in turn passes through a superheater) a cylinder from which it is exhausted to a condenser. Which pieces of apparatus ave to do with heat effects and which with work? Point out similarities and ifferences of process.

Prob. 2. Air is passed over gasolene in a carburetter; the mixture is compressed, urned and allowed to expand in a gas engine cylinder. Which of the above steps have) do with heat effects and which with work effects?

Prob. 3. In certain types of ice machines liquid ammonia is allowed to evaporate, ie vapor which is formed being compressed and condensed again to liquid. Which of iese steps is a work phase and which a heat phase? Compare with Problem 1.

Prob. 4. When a gun is fired what is the heat phase and what is the work hase? Are they separate or coincident?

Prob. 5. Air is compressed in one cylinder, then it is cooled and compressed to igher pressure and forced into a tank. The air in the tank cools down by giving p heat to the atmosphere. From the tank it passes through a pipe line to a heater nd then to an engine from which it is exhausted to the atmosphere. Which steps n the cycle may be regarded as heat and which as work phases? Compare with roblem 2.

2. Classification of Heating Processes. Heat Addition and Abstraction with, or without Temperature Change. Qualitative Relations. That heat will pass from a hot to a less hot body if it gets a chance is axiomatic, so that a body acquiring heat may be within range of a hotter one, the connection between hem being, either immediate, that is they touch each other, or another body nay connect them acting as a heat carrier, or they may be remote with no more provable connection than the hypothetic ether as is the case with the sun and earth. A body may gain heat in other ways than by transfer from a hotter body, for example, the passage of electrical current through a conductor will eat it, the rubbing of two solids together will heat both or perhaps melt one, he churning of a liquid will heat it, the mixing of water and sulphuric acid will roduce a hotter liquid than either of the components before mixture, the absorp- ion by water of ammonia gas will heat the liquid. All these and many other similar examples that might be cited have been proved by careful investigation partly experimental, and partly by calculation based on various hypotheses to e examples of transformation of energy, mechanical, electrical, chemical, into he heat form. While, therefore, bodies may acquire heat in a great many different concrete ways they all fall under two useful divisions:

(*a*) By transfer from a hotter body;

(*b*) By transformation into heat of some other energy manifestation.

One body may be said to be hotter than another when it feels so to the sense of touch, provided neither is too hot or too cold for injury to the tissues, or more generally, when by contact one takes heat from the other. Thus ideas of heat can scarcely be divorced from conceptions of temperature and the definition of one will involve the other. As a matter of fact temperature as indicated by any instrument is merely an arbitrary number located by some- body on a scale, which is attached to a substance on which heat has some *visible*

effect. Temperature is then a purely arbitrary, though generally accepted number indicating some heat content condition on a scale, two points of which have been fixed at some other conditions of heat content, and the scale space between, divided as convenient. Examination of heat effects qualitatively will show how thermometers might be made or heat measured in terms of any handy effect, and will also indicate what is likely to happen to any substance when it receives or loses heat. Some of the more common heat effects of various degrees of importance in engineering work are given below:

Expansion of Free Solids. Addition of heat to free solids will cause them to expand, increasing lengths and volumes. Railroad rails and bridges are longer in summer than winter and the sunny side of a building becomes a little higher than the shady side. Steam pipes are longer and boilers bigger hot, than cold and the inner shell of brick chimneys must be free from the outer to permit it to grow when hot without cracking the outer or main supporting stack body. Shafts running hot through lack of lubrication or overloading in comparatively cool bearing boxes may be gripped tight enough to twist off the shaft or merely score the bearing.

Stressing of Restrained Solids. A solid being heated may be restrained in its tendency to expand, in which case there will be set up stresses in the material which may cause rupture. Just as with mechanically applied loads, bodies deform in proportion to stress up to elastic limit, as stated by Hooke's law, so if when being heated the tendency to expand be restrained the amount of deformation that has been prevented determines the stress. A steam pipe rigidly fixed at two points when cold will act as a long column in compression and buckle when hot, the buckling probably causing a leak or rupture. If fixed hot, it will tend to shorten on cooling and being restrained will break something. Cylinders of gas engines and air compressors are generally jacketed with water and becoming hot inside, remaining cold outside, the inner skin of the metal tends to expand while the outer skin does not. One part is, therefore, in tension and the other in compression, often causing cracks when care in designing is not taken and sometimes in spite of care in large gas engines.

Expansion of Free Liquids. Heating of liquids will cause them to expand just as do solids, increasing their volume. Thus, alcohol or mercury in glass tubes will expand and as these liquids expand more than the glass, a tube which was originally full will overflow when hot, or a tube of very small bore attached to a bulb of cold liquid will on heating receive some liquid; the movement of liquid in the tube if proportional to the heat received will serve as a thermometer. If the solid containing the liquid, expanded to the same degree as the liquid, there would be no movement. Two parts of the same liquid mass may be unequally hot and the hotter having expanded will weigh less per cubic foot that is, be of less density. Because of freedom of movement in liquids the lighter hot parts will rise and the cooler heavy parts fall, thus setting up a circulation the principle of which is used in hot water heating systems, the hot water from the furnace rising to the top of the house through one pipe and cooling on its downward path through radiators and return pipe. In general then, liquid

decrease in density on heating and increase in density on cooling, but a most important exception is water, which has a point of maximum density just above the freezing-point, and if cooled below this becomes not heavier but lighter. Consequently, water to be cooled most rapidly should be cooled at first from the top and after reaching this point of maximum density, from the bottom, if it is to be frozen.

Rise of Pressure in Confined Liquids. When liquids are restrained from expanding under heating they suffer a rise of pressure which may burst the containing vessel. For this reason, hot water heating systems have at their highest point, open tanks, called expansion tanks, which contain more water when the system is hot than when cold, all pipes, radiators and furnaces being constantly full of water. Should this tank be shut off when the water is cold something would burst, or joints leak, before it became very hot.

Expansion of Free Gases. Just as solids and liquids when free expand under heating, so also do gases and on this principle chimneys and house ventilation systems are designed. The hot gases in a chimney weigh less per cubic foot than cooler atmospheric air; they, therefore, float as does a ship on water, the superior density of the water or cold gas causing it to flow under and lift the ship or hot gas, respectively. Similarly, hot-air house furnaces and ventilating systems having vertical flues supplied with hot air can send it upward by simply allowing cold air to flow in below and in turn being heated flow up and be replaced.

Rise of Pressure in Confined Gases. Gases when restrained from expanding under heat reception will increase in pressure just as do liquids, only over greater ranges, and as does the internal stress increase in solids when heated under restraint. It is just this principle which lies at the root of the operation of guns and gas engines. Confined gases are rapidly heated by explosive combustion and the pressure is thus raised sufficiently to drive projectiles or pistons in their cylinders.

Melting of Solids. It has been stated that solids on being heated expand but it should be noted that this action cannot proceed indefinitely. Continued heating at proper temperatures will cause any solid to melt or fuse, and the previously rising temperature will become constant during this change of state. Thus, melting or fusion is a process involving a change of state from solid to liquid and takes place at constant temperature. The tanks or cans of ice-making plants containing ice and water in all proportions retain the same temperature until all the water becomes ice, provided there is a stirring or circulation so that one part communicates freely with the rest and provided also the water is pure and contains no salt in solution. Impure substances, such as liquid solutions, may suffer a change of temperature at fusion or solidification. For pure substances, melting and freezing, or fusion and solidification, are constant temperature heat effects, involving changes from solid to liquid, or liquid to solid states.

Boiling of Liquids. Ebullition. Continued heating of solids causes fusion, and similarly continued heating of liquids causes boiling, or change of state from

liquid to vapor, another constant temperature process—just what temperature, will depend on the pressure at the time. So constant and convenient is this temperature pressure relation, that the altitude of high mountains can be found from the temperature at which water boils. The abstraction of heat from a vapor will not cool it, but on the contrary cause condensation. Steam boilers and ammonia refrigerating coils and coolers are examples of evaporating apparatus, and house heating radiators and steam and ammonia condensers of condensing apparatus.

Evaporation of Liquids; Humidification of Gases. When dry winds blow over water they take up moisture in the vapor form by evaporation at any temperature. This sort of evaporation then must be distinguished from ebullition and is really a heat effect, for without heat being added, liquid cannot change into vapor; some of the necessary heat may be supplied by the water and some by the air. This process is general between gases and liquids and is the active principle of cooling towers, carburetters, driers of solids like wood kilns. The chilling of gases that carry vapors causes these to condense in part. As a matter of fact it is not necessary for a gas to come into contact to produce this sort of evaporation from a liquid, for if the liquid be placed in a vacuum some will evaporate, and the pressure finally attained which depends on the temperature, is the vapor pressure or vapor tension of the substance, and the amount that will so evaporate is measured by this pressure and by the rate of removal of that which formed previously.

Evaporation of Solids. Sublimation. Evaporation, it has been shown, may take place from a liquid at any temperature, but it may also take place directly from the solid, as ice will evaporate directly to vapor either in the presence of a gas or alone. Ice placed in a vacuum will evaporate until the vapor tension is reached, and it is interesting to note that the pressure of vapors above their solids is not necessarily the same as above their liquids at the same temperature, though they merge at the freezing-point. This is the case with ice-water-water vapor.

Change of Viscosity. Heating of liquids may have another effect measured by their tendency to flow, or their viscosity. Thus, a thick oil will flow easier when heated, and so also will any liquid. If, therefore, the time for a given quantity to flow through a standard orifice under a given head or pressure be measured, this time, which is the measure of viscosity, will be less for any liquid hot, than cold, for the same liquid. Viscosity then decreases with heat addition and temperature rise.

Dissociation of Gases. When gases not simple are heated and the heating continued to very high temperatures, they will split up into their elements or perhaps into other compound gases. This may be called decomposition or, better, dissociation, and is another heat effect. Thus, the hydrocarbon C_2H_4 will split up with solid carbon soot C and the other hydrocarbon CH_4 and steam H_2O into hydrogen and oxygen. This is not a constant temperature process but the per cent dissociated increases as the temperature rises.

Dissociation of Liquids. Similar to the dissociation of gases receiving heat a

high temperature is the decomposition of some liquids in the liquid state, notably the fuel and lubricating oils, or hydrocarbons which are compounds of H and C in various proportions, each having different properties. Sometimes these changes of H and C groupings from the old to the new compounds under the influence of heating will be at constant and at other times at varying temperatures; sometimes the resulting substances remain liquid and sometimes soot or C separates out, and this is one of the causes for the dark color of some cylinder oils.

Absorption of Gases in Liquids. Liquids will absorb some gases quite freely; thus, water will absorb very large quantities of ammonia, forming aqua ammonia. Addition of heat will drive off this gas so that another heat effect is the expulsion of gases in solution. Use is made of this industrially in the absorption system of ammonia refrigeration.

Solubility of Solids in Liquids. The heating of liquids will also affect their solubility for solid salts; thus, a saturated solution of brine will deposit crystals on heat abstraction and take them back into solution on heat addition. Certain scale-forming compounds are thrown down on heating the water intended for boilers, a fact that is made use of in feed-water heating purifiers; for these salts increase of temperature reduces solubility. In general then heat addition affects the solubility of liquids for solid salts.

Chemical Reaction. Combustion. If oxygen and hydrogen, or oxygen and carbon, be heated in contact, they will in time attain an ignition temperature at which a chemical reaction will take place with heat liberation called combustion, and which is an exothermic or heat-freeing reaction. Another and different sort of reaction will take place if CO_2 and carbon be heated together, for these will together form a combustible gas, CO, under a continuation of heat reception. This is an endothermic or heat-absorbing reaction. Neither of these will take place until by heat addition the reaction temperature, called ignition temperature for combustion, has been reached.

Electrical and Magnetic Effects. Two metals joined together at two separate points, one of which is kept cool and the other heated, will be found to carry an electric current or constitute a thermo-electric couple. Any conductor carrying an electric current will on changing temperature suffer a change of resistance so that with constant voltage more or less current will flow; this is a second electrical heat effect and like the former is useful only in instruments indicating temperature condition. A fixed magnet will suffer a change of magnetism on heating so that heat may cause magnetic as well as electric effects.

These heat effects on substances as well as some others of not so great engineering importance may be classified or grouped for further study in a variety of ways, each serving some more or less useful purpose.

Reversible and Non-reversible Processes. There may be reversible and non-reversible thermal processes, when the process may or may not be considered constantly in a state of equilibrium. For example, as heat is applied to boiling water there is a continuous generation of vapor in proportion to the

heat received; if at any instant the heat application be stopped the evaporation will cease and if the flow of heat be reversed by abstraction, condensation will take place, indicating a state of thermal equilibrium in which the effect of the process follows constantly the direction of heat flow and is constantly proportional to the amount of heat numerically, and in sign, of direction. As an example of non-reversible processes none is better than combustion, in which the chemical substances receive heat with proportional temperature rise until chemical reaction sets in, at which time the reception of heat has no further relation to the temperatures, because of the liberation of heat by combustion which proceeds of itself and which cannot be reversed by heat abstraction. Even though a vigorous heat abstraction at a rate greater than it is freed by combustion may stop combustion or put the fire out, no amount of heat abstraction or cooling will cause the combined substances to change back into the original ones as they existed before combustion. The effect of heat in such cases as this is, therefore, non-reversible.

Constant and Variable Volume or Density. When gases, liquids or solids are heated they expand except when prevented forcibly from so doing, and as a consequence they suffer a reduction of density with the increase of volume; this is, of course, also true of changing liquids to their vapors. It should be noted that all such changes of volume against any resistance whatever, occur with corresponding performance of some work, so that some thermal processes may directly result in the doing of work. Heating accompanied by no volume change and during which restraints are applied to keep the volume invariable, cannot do any work or suffer any change of density, but always results in change of pressure in liquids, gases and vapors and in a corresponding change of internal stress in solids.

Constant and Variable Temperature Processes. Another useful division, and that most valuable in the calculation of relations between heat effect and heat quantity, recognizes that some of the heating processes and, of course, cooling, occur at constant temperature and others with changing temperature. For example, the changes of state from liquid to solid, and solid to liquid, or freezing and fusion, are constant temperature processes in which, no matter how much heat is supplied or abstracted, the temperature of the substance changing state is not affected, and the same is true of ebullition and condensation, or the changing of state from liquid to vapor, and vapor to liquid. These latter constant-temperature processes must not be confused with evaporation, which may proceed from either the solid or liquid state at any temperature whether constant or not.

Prob. 1. From the time a fire is lighted under a cold boiler to the time steam first comes off, what heat effects take place?

Prob. 2. What heat effects take place when a piece of ice, the temperature of which is 20° F., is thrown onto a piece of red-hot iron?

Prob. 3. What heat effects must occur before a drop of water may be evaporated from the ocean, and fed back into it as snow?

Prob. 4. What heat changes take place when soot is formed from coal or oil?

Prob. 5. In a gas producer, coal is burned to CO_2, which is then reduced to CO. team is also fed to the producer, and H and O formed from it. Give all the heat fects which occur.

Prob. 6. By means of what heat effects have you measured temperature changes, r have known them to be measured?

Prob. 7. When the temperature changes from 40° F. to 20° F., give a list of all eat effects you know that commonly occur for several common substances. Do the me for a change in the reverse direction.

Prob. 8. If a closed cylinder be filled with water it will burst if the temperature e lowered or raised sufficiently. What thermal steps occur in each case?

Prob. 9. If salt water be lowered sufficiently in temperature, a cake of fresh ice id a rich salt solution will be formed. State the steps or heat effects which occur uring the process.

3. Thermometry Based on Temperature Change Heat Effects. Thermometer and Absolute Temperature Scales. Those thermal processes in which eat addition or abstraction is followed as a result by a corresponding and more r less proportional temperature change, are quite numerous and important oth in engineering practice and as furnishing a means for thermometer-mak-ng, and temperature definition and measurement. According to Sir William homson "every kind of thermoscope must be founded on some property f matter continuously varying with the temperature" and he gives the fol-owing:

(*a*) Density of fluid under constant pressure.
(*b*) Pressure of a fluid under a constant volume envelope.
(*c*) Volume of the liquid contained in a solid holder (ordinary mercury or spirit thermometer).
(*d*) Vapor pressure of a solid or liquid.
(*e*) Shape or size of an elastic solid under constant stress.
(*f*) Stress of an elastic solid restrained to constant size.
(*g*) Density of an elastic solid under constant stress.
(*h*) Viscosity of a fluid.
(*i*) Electric current in a thermo-couple.
(*j*) Electric resistance of a conductor.
(*k*) Magnetic moment of a fixed magnet.

Any, or all of these—pressure, volume, shape, size, density, rate of flow, agnetic or electrical effects, may be measured, and their measure constitutes measure of temperature indirectly, so that instruments incorporating these mperature effects to be measured, are also thermometers.

Any temperature-indicating device may be called a thermometer, though ose in use for high temperatures are generally called pyrometers, which dicates the somewhat important fact that no thermometer is equally useful r all ranges of temperature. Practically all thermometers in use for tempera-ures short of a red heat, depend on certain essential relations between the density volume, the pressure and temperature of a fluid, though metals are used in me little-used forms in which change of size is measured, or change of shape

of a double metallic bar, often brass and iron, consisting of a piece of e fastened to the other to form a continuous strip. The two metals are expan by the temperature different amounts causing the strip to bend under heati There are also in use electric forms for all temperatures, and these are only reliable ones for high temperatures, both of the couple and resista types except one dependent on the color of a high temperature body, bl when cold. That most useful and common class involving the inter pendence of pressure and temperature, or volume and temperature, of a fl is generally found in the form of a glass bulb or its equivalent, to wh is attached a long, narrow glass tube or stem which may be open or closed the end; open when the changes of fluid volume at constant pressure are be observed and closed when changes of contained fluid pressure at const restrained volume are to be measured as the effect of temperature chan For the fluid there is used most commonly a liquid alone such as mercury, a gas alone such as air; though a gas may be introduced above mercury there may be used a liquid with its vapor above. When the fluid is a liqu such as mercury, in the common thermometer, the stem is closed at the end that the mercury is enclosed in a constant-volume container or as nearly so the expansion or deformation of the glass will permit, which is not filled w mercury, but in which a space in the stem is left at a vacuum or filled wit gas under pressure, such as nitrogen, to resist evaporation of the mercury high temperatures. Gas-filled mercury thermometers, as the last form is call are so designed that for the whole range of mercury expansion the press of the gas opposing it does not rise enough to offer material resistance to expansion of mercury or to unduly stress the glass container. It should noted that mercury thermometers do not measure the expansion of merc alone, but the difference between the volume of mercury and the glass envel but this is of no consequence so long as this difference is in proportion to expansion of the mercury itself, which it is substantially, with proper g composition, when the range is not too great. Such thermometers indi temperature changes by the rise and fall of mercury in the stem, and any num cal value that may be convenient can be given to any position of the merc or any change of position. Common acceptance of certain locations of the s number, however, must be recognized as rendering other possible ones unne sary and so undesirable. Two such scales are recognized, one in use with m units, the centigrade, and the other with measurements in English units, Fahrenheit, both of which must be known and familiar, because of the frequ necessity of transformation of numerical values and heat data from one sys to the other. To permit of the making of a scale, at least two points must fixed with a definite number of divisions between them, each called one *de* The two fixed points are *first*, the position of the mercury when the thermom is in the vapor of boiling *pure* water at sea level, or under the standard at pheric pressure of $29.92'' = 760$ mm. of mercury absolute pressure, *second*, the position of the mercury when the thermometer is surrounde melting ice at the same pressure. These are equivalent to the boiling- or

densation, and melting- or freezing-points, of pure water at one atmosphere pressure. The two accepted thermometer scales have the following characteristics with respect to these fixed points and division between them:

THERMOMETER SCALES

	Pure Water Freezing-point. at one atm. pr.	Pure Water Boiling-point. at one atm. pr.	Number of Equal Divisions Between Freezing and Boiling.
Centigrade scale.....	0	100	100
Fahrenheit scale.....	32	212	180

From this it appears that a degree of *temperature change* is on the centigrade scale, $\frac{1}{100}$ of the linear distance between the position of the mercury surface at the freezing- and boiling-points of water, and on the Fahrenheit scale, $\frac{1}{180}$ of the same distance. From this the relation between a degree temperature change for the two scales can be given.

$$\left.\begin{array}{c}\text{One degree } \textit{temperature} \\ \textit{change} \text{ centigrade}\end{array}\right\} = \frac{180}{100} = \frac{9}{5} \left\{\begin{array}{c}\text{of one degree } \textit{temperature} \\ \textit{change} \text{ Fahrenheit;}\end{array}\right.$$

or

$$\left.\begin{array}{c}\text{One degree } \textit{temperature} \\ \textit{change} \text{ Fahrenheit}\end{array}\right\} = \frac{5}{9} \left\{\begin{array}{c}\text{of one degree } \textit{temperature} \\ \textit{change} \text{ centigrade.}\end{array}\right.$$

It is also possible to set down the relation between scale readings, for when the temperature is 0° C., it is 32° F., and when it is 100° C. it is (180+32) = 212° F., so that

$$\text{Temperature Fahrenheit} = 32 + \frac{9}{5} \text{ (Temperature centigrade),}$$

or

$$\text{Temperature centigrade} = \frac{5}{9} \text{ (Temperature Fahrenheit} - 32).$$

For convenience of numerical work tables are commonly used to transform temperatures from one scale to the other and such a transformation is shown in a curve, Fig. 119, and in Table XXIX at end of the Chapter.

By reason of the lack of absolute proportionality between temperature and effect, other fixed points are necessary, especially at high temperatures, and the following of Table XVI have been adopted by the U. S. Bureau of Standards and are considered correct to within 5° C., at 1200° C.

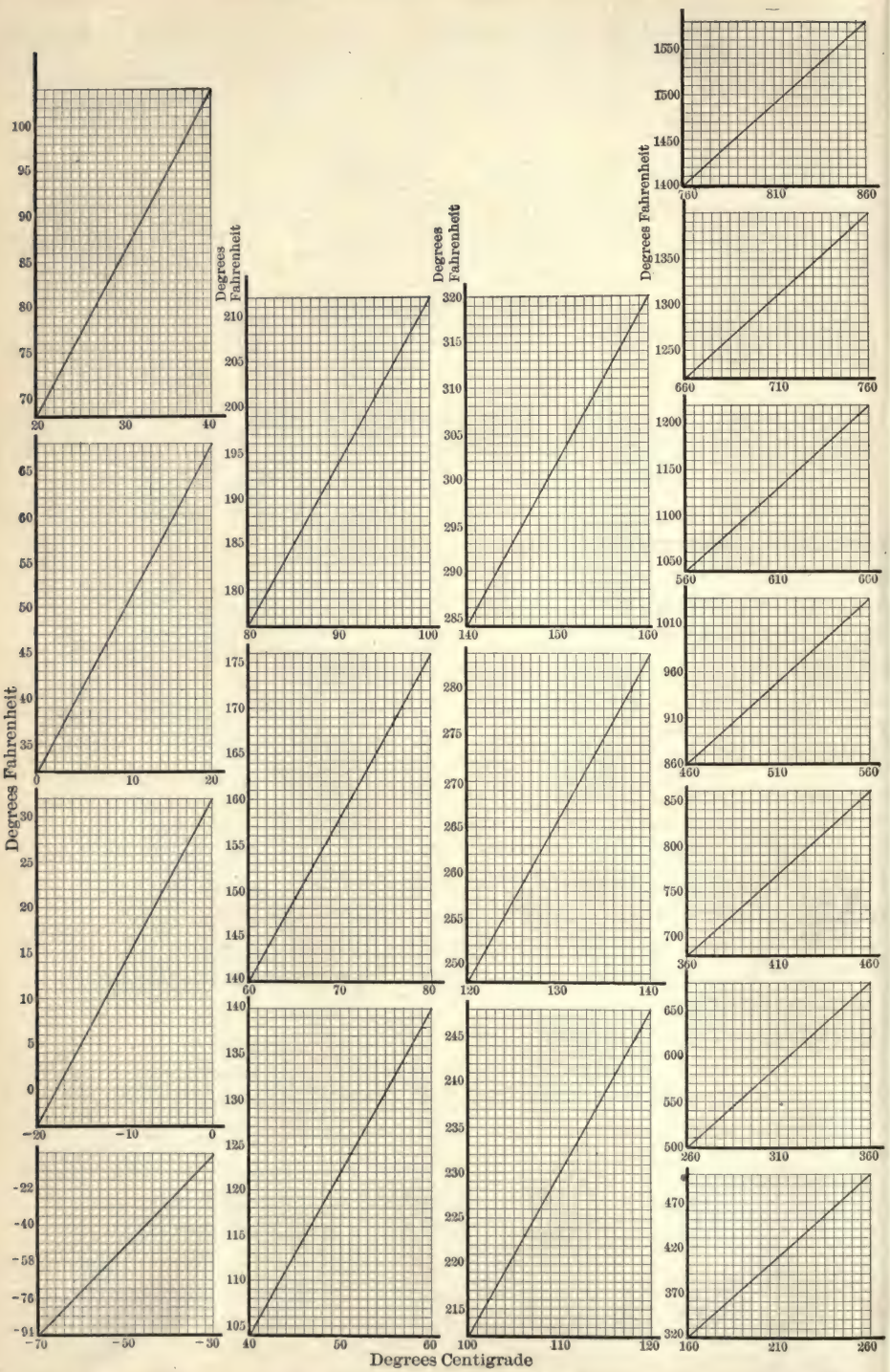


FIG. 119.—Graphical Relation between Centigrade and Fahrenheit Thermometer Scales.

TABLE XVI

FIXED TEMPERATURES

U. S. BUREAU OF STANDARDS

Temperature, ° C.	Temperature, ° F.	Determined by the Point at which
232	449	Liquid tin solidifies
327	621	Liquid lead solidifies
419.4	787	Liquid zinc solidifies
444.7	832.5	Liquid sulphur boils
630.5	1167	Liquid antimony solidifies
658	1216	Liquid aluminum, 97.7% pure, solidifies
1064	1947	Solid gold melts
1084	1983	Liquid copper solidifies
1435	2615	Solid nickel melts
1546	2815	Solid palladium melts
1753	3187	Solid platinum melts

Thermometers in which a liquid and its vapor exist together, depend on a property to be noted in detail later, the relation of vapor pressure to temperature and its independence of the volume of vapor. So long as any vapor exists above the liquid the temperature will depend only on the pressure of that vapor so that such thermometers will indicate temperature by the pressure measurement, after experimental determination of this pressure-temperature relation of vapors. Conversely, temperature measurements of vapors by mercury thermometers will lead to pressure values, and at the present time some steam plants are introducing mercury thermometers on the boilers and pipe lines, in place of the proverbially inaccurate pressure gages.

Gas thermometer, is the name generally applied to the class in which the fluid is a gas, whether air, hydrogen, nitrogen or any other, and whether the pressure is measured for a fixed contained volume, or the volume measured when acted on by a constant pressure. These gas thermometers are so bulky as to be practically useless in ordinary engineering work and are only employed as standards for comparison and for tests of extraordinary delicacy in investigation work. They give much larger indications than mercury thermometers because the changes of gas volume under constant pressure are far greater than for mercury or any other liquid. Regnault was the first to thoroughly investigate air thermometers and reported that the second form, that of constant gas volume with measurement of pressure, was most useful.

Using the centigrade scale, fixing freezing point at 0° C., and making the corresponding pressure p_0, atmospheric at this point, and reading at 100° C. another pressure p_{100}, he found experimentally a relation between these two pressures and the temperature corresponding to any other pressure p, as given by the empiric formula,

$$t = 100\frac{p - p_0}{p_{100} - p_0}. \quad \ldots\ldots\ldots \quad (601)$$

He also determined the pressure at the boiling-point to be related to the pressu at the freezing-point, by

$$p_{100} = 1.3665\ p_0,$$

which on substitution gives

$$t = 100\frac{p - p_0}{.3665 p_0} = 272.85\left(\frac{p}{p_0} - 1\right). \quad . \quad . \quad . \quad . \quad . \quad (60$$

This constant, 272.85, the reciprocal of which is .003665, is, of course, the pressu increase factor per degree C. rise of temperature for a gas held at consta volume, received extended investigation and it *was found that it had abo the same value applied to the other type of thermometer in which gas volum are measured at constant pressure.* This was true even when the pressu used was anything from 44 to 149 cm. of mercury, though it is reporte that for 44 cm. pressure the value 272.98, and for 149 cm. pressure, the valu 272.7, seemed closer. For hydrogen it was found that the constant was sub stantially the same as for air, while for carbonic acid it was 270.64, and whi the hydrogen thermometers agreed with the air over the whole scale, showin proportional effects, this was hardly true of carbonic acid. Such uncertaint in the behavior of these thermometers and in the fixing of the constants w traced to the glass in some cases, but there still remained differences charg able only to the gases themselves. Comparison of the air with mercury the mometers showed that there was not a proportional change with the temperatu and that temperatures on the two, consistently departed.

Examination of Eq. (602), giving the relation between two temperatures an the corresponding gas pressures, will show a most important relation. If in E (602), the pressure be supposed to drop to zero and it is assumed th the relations between pressure and temperature hold, then when $p =$ $t = -272.85$. *This temperature has received the name of the absolute zer* and may be defined as the temperature at which pressure disappears or becom zero at constant volume, and correspondingly, at which the volume also di appears, since it was found that similar relations existed between volume a temperature at constant pressure. Calling temperature on a new scale begi ning 272.85° below the centigrade zero by the name *absolute* temperatur then

$$\left\{\begin{matrix}\text{Absolute temperature}\\ \text{centigrade}\end{matrix}\right\} = 272.85 + \left\{\begin{matrix}\text{Scale temperature}\\ \text{centigrade}\end{matrix}\right\}$$

As this constant or absolute temperature of the centigrade scale zero, is experimental value, it is quite natural to find other values presented by diff ent investigators, some of them using totally different methods. One of th methods is based on the temperature change of a gas losing pressure witho doing work, generally described as the porous plug experiment, and the resu

s the Joule-Thomson effect, and another is based on the coefficient of expansion f gases being heated. Some of these results agreed exactly with Regnault's alue for hydrogen between 0° C. and 100° C. for which he gave −273° C. = -491.4° F. Still other investigations continued down to the last few years ielded results that tend to change the value slightly to between −491.6° F., nd −491.7° F., and as yet there is no absolute agreement as to the exact value. n engineering problems, however, it is seldom desirable or possible to work ɔ such degrees of accuracy as to make the uncertainty of the absolute zero a natter of material importance, and for practical purposes the following values ıay be used with sufficient confidence for all but exceptional cases which are ɔ be recognized only by experience.

$$\text{Absolute Temperature } (T) \left\{ \begin{matrix} \text{Centigrade} = 273 \\ \text{Fahrenheit} = 460 \end{matrix} \right\} + \text{Scale Temperature } (t).$$

When great accuracy is important it is not possible at present to get a better 'ahrenheit value than 459.65, the mean of the two known limits of 459.6 and 59.7, though Marks and Davis in their Steam Tables have adopted 459.64, which s very close to the value of 459.63 adopted by Buckingham in his excellent Bulletin of U. S. Bureau of Standards and corresponding to 273.13 on the entigrade scale.

These experiments with the gas thermometers, leading to a determination f temperature as a function of the pressure change of the gas held at constant olume, or its volume change when held at constant pressure, *really supply a efinition of temperature* which before meant no more than an arbitrary number, nd furnished a most valuable addition to the generalization of relations between eat content of a body and its temperature or physical state.

A lack of proportionality between thermometer indication and temperature, as already been pointed out, and it is by reason of this that two identical ther- ıometers, or as nearly so as can be made, with absolute agreement between vater boiling- and freezing-points, will not agree at all points between, nor will he best constructed and calibrated mercury thermometers agree with a similarly ood gas thermometer.

The temperature scale now almost universally adopted as standard is that f the constant volume hydrogen gas thermometer, on which the degree F. s one one-hundred-and-eightieth part of the change in pressure of a fixed olume of hydrogen between melting pure ice, and steam above boiling pure vater, the initial pressure of the gas at 32° being 100 cm. = 39.37 ins. Hg. A ıercury in glass thermometer indication is, of course, a measure of the proper- ies of the mercury and glass used, and its F. degree of temperature is defined n parallel with the above as one one-hundred-and-eightieth part of the volume f the stem between its indications at the same two fixed points. A comparison f the hydrogen thermometer and two different glasses incorporated in mercury hermometers is given below, Table XVII, from the Bulletin of the U. S. Bureau f Standards, v. 2, No. 3, by H. C. Dickinson, quoting Mahlke, but it must be

remembered that other glasses will give different results and even differen thermometers of the same glass when not similarly treated.

TABLE XVII

FAHRENHEIT TEMPERATURES BY HYDROGEN AND MERCURY THERMOMETERS

Temperature by Hydrogen Thermometer.	Difference in Reading by Mercury in Jena 59″ Glass.	Difference in Reading by Mercury in 16″ Jena Glass.	Temperature by Hydrogen Thermometer.	Difference in Reading by Mercury in Jena 59″ Glass.	Difference in Reading by Mercury in 16′ Jena Glass.
32	0	0	617	+10.6	
212	0	0	662	+16.6	
302		− .18	707	+18.7	
392	+1.3	+ .072	752	+24.6	
428		+ .39	797	+28.2	
464		+ .83	842	+38.3	
500		+1.79	887	+41.4	
536		+2.4	932	+50.0	
572		+3.53			

Useful comparisons of air, hydrogen, nitrogen, and other gases, with alcoh and mercury, in various kinds of glass, are given in the Landolt-Börnstei Meyerhoffer, and in the Smithsonian Physical Tables, but are seldom neede for engineering work.

One sort of correction that is often necessary in mercury thermomet work is that for stem immersion. Thermometers are calibrated as a rule wit the whole stem immersed in the melting ice or the steam, but are ordinari used with part of the stem exposed and not touching the substance whose ter perature is indicated. For this condition the following correction is reco mended by the same Bureau of Standards Bulletin:

$$\text{Stem correction} = .000088\ n\ (t - t_1)^\circ \text{ F}.$$

When n = number of degrees exposed;

t = temperature indicated Fahrenheit degrees;

t_1 = mean temperature of emergent stem itself, which must necessari be estimated and most simply by another thermometer next it, and entirely free from the bath.

Prob. 1. What will be the centigrade scale and absolute temperatures, for t following Fahrenheit readings? −25°, 25°, 110°, 140°, 220°, 263° scale, and 30 460°, 540°, 710°, 2000° absolute.

Prob. 2. What will be the Fahrenheit scale and absolute temperatures, for the follo ing centigrade readings? −20°, 10°, 45°, 80°, 400°, 610° scale, and 200°, 410°, 65 810°, 2500° absolute.

Prob. 3. By the addition of a certain amount of heat the temperature of a quantity of water was raised 160° F. How many degrees C. was it raised?

Prob. 4. To bring water from 0° C. to its boiling-point under a certain pressure required a temperature rise of 150° C. What was the rise in Fahrenheit degrees?

Prob. 5. For each degree rise Fahrenheit, an iron bar will increase .00000648 of its length. How much longer will a bar be at 150° C. than at 0° C.? At 910° C. absolute than at 250° C. absolute?

Prob. 6. The increase in pressure for SO_2 for a rise of 100° C. is given as .3845 at constant volume. What would have been absolute zero found by Regnault had he used SO_2 rather than air?

Prob. 7. A thermometer with a scale from 40° F. to 700° F. is placed in a thermometer well so that the 200° mark is just visible. The temperature as given by the thermometer is 450°. If the surrounding temperature is 100° F., what is true temperature in the well?

4. Calorimetry Based on Proportionality of Heat Effects to Heat Quantity. Units of Heat and Mechanical Equivalent. Though it is generally recognized from philosophic investigations extending over many years, that heat is one manifestation of energy capable of being transformed into other forms such as mechanical work, electricity or molecular arrangement, and derivable from them through transformations, measurements of quantities of heat can be made without such knowledge, and were made even when heat was regarded as a substance. It was early recognized that equivalence of heat effects proved effects proportional to quantity; thus, the melting of one pound of ice can cool a pound of hot water through a definite range of temperature, and can cool two pounds through half as many degrees, and so on. The condensation of a pound of steam can warm a definite weight of water a definite number of degrees, or perform a certain number of pound-degrees heating effect in water. So that taking the pound-degree of water as a basis the ratio of the heat liberated by steam condensation to that absorbed by ice melting can be found. Other substances such as iron or oil may suffer a certain number of pound degree changes and affect water by another number of pound-degrees. The unit of heat quantity might be taken as that which is liberated by the condensation of a pound of steam, that absorbed by the freezing of a pound of water, that to raise a pound of iron any number of degrees or any other quantity of heat effect. The heat unit generally accepted is, in metric measure, the calorie, or the amount to raise one kilogramme of pure water one degree centigrade, or in English units, the British thermal unit, that necessary to raise one pound of water one degree Fahrenheit. Thus, the calorie is the kilogramme degree centigrade, and the British thermal unit the pound degree Fahrenheit, and the latter is used in engineering, usually abbreviated to B.T.U. There is also occasionally used a sort of cross unit called the centigrade heat unit, which is the pound degree centigrade.

The relation between these is given quantitatively by the conversion table at the end of this Chapter, Table XXX.

All the heat measurements are, therefore, made in terms of equivalent water heating effects in pound degrees, but it must be understood that a water pound degree is not quite constant. Careful observation will show that the

melting of a pound of ice will not cool the same weight of water from 200° F. to 180° F., as it will from 60° F. to 40° F., which indicates that the heat capacity of water or the B.T.U. per pound-degree is not constant. It is, therefore, necessary to further limit the definition of the heat unit, by fixing on some water temperature and temperature change, as the standard, in addition to the selection of water as the substance, and the pound and degree as units of capacity. Here there has not been as good an agreement as is desirable, some using 4° C. = 39.4° F. as the standard temperature and the range one-half degree both sides; this is the point of maximum water density. Others have used 15° C. = 59° F. as the temperature and the range one-half degree both sides; still others, one degree rise from freezing point 0° C. or 32° F. There are good reasons, however, for the most common present-day practice which will probably become universal, for taking as the range and temperatures, freezing-point to boiling-point, and dividing by the number of degrees. The heat unit so defined is properly named the mean calorie or mean British thermal unit; therefore,

$$\text{Mean calorie} = \frac{1}{100} \text{ (amount of heat to raise 1 Kg. water from 0° C. to 100° C.)}$$

$$\text{Mean B.T.U.} = \frac{1}{180} \text{ (amount of heat to raise 1 lb. water from 32° F. to 212° F.)}$$

In terms of the heat unit thus defined, the amount of heat per degree temperature change is variable over the scale, but only in work of the most accurate character is this difference observed in engineering calculations, but in accurate work this difference must not be neglected and care must be exercised in using other physical constants in heat units reported by different observers, to be sure of the unit they used in reporting them. It is only by experience that judgment can be cultivated in the selection of values of constants in heat units reported for various standards, or in ignoring differences in standards entirely. The great bulk of engineering work involves uncertainties greater than these differences and they may, therefore, be ignored generally.

By various experimental methods, all scientifically carried out and exetndin over sixty years, a measured amount of work has been done and entirely converted into heat, originally by friction of solids and of liquids, for the determination of the foot-pounds of work equivalent to one B. T. U., when the conversion is complete, that is, when all the work energy has been converted in heat. This thermo-physical constant is the *mechanical equivalent of he* Later, indirect methods have been employed for its determination by calcul tion from other constants to which it is related. All of these experimen have led to large number of values, so that it is not surprising to find doubt to the correct value and different values are used even by recognized autho ties. The experiments used include:

(*a*) Compression and expansion of air; Joule.
(*b*) Steam engine experiments, comparing heat in supplied and exhausted steam; Hirn.
(*c*) Expansion and contraction of metals; Edlund and Haga.
(*d*) Specific volume of vapor; Perot.
(*e*) Boring of metals; Rumford and Hirn.
(*f*) Friction of water; Joule and Rowland.
(*g*) Friction of mercury; Joule.
(*h*) Friction of metals; Hirn, Puluj, Sahulka.
(*i*) Crushing of metals; Hirn.
(*j*) Heating of magneto electric currents; Joule.
(*k*) Heating of disk between magnetic poles; Violle.
(*l*) Flow of liquids (water and mercury) under pressure; Hirn, Bartholi.
(*m*) Heat developed by wire of known absolute resistance; Quintus Icilius, Weber, Lenz, Joule, Webster, Dieterici.
(*n*) Diminishing the heat contained in a battery when the current produces work; Joule, Favre.
(*o*) Heat developed in, and voltage of Daniell cells; Weber, Boscha, Favre and Silberman, Joule.
(*p*) Combination of electrical heating and mechanical action by stirring water; Griffiths.
(*q*) Physical constants of gases.

The results of all of these were studied by Rowland in 1880, who himself experimented also, and he concluded that the mechanical equivalent of heat was nearly

778.6 ft.-lbs. = 1 B.T.U., at latitude of Baltimore,

or

774.5 ft.-lbs. = 1 B.T.U., at latitude of Manchester

with the following corrections to be added for other latitudes.

Latitude.....	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
Ft.-lbs.....	1.62	1.50	1.15	.62	.15	−.75	−1.41	−1.93	−2.30	−2.43

Since that time other determinations have been made by Reynolds and Morby, using mechanical, and Griffiths, Schuste and Gannon, Callendar and Barnes, using electrical transformation into heat. Giving these latter determinations equal weight with those of Joule and Rowland, the average is

1 small calorie at 20° C. (nitrogen thermometer) = 4.181×10^7 ergs.

On the discussion of these results by Smith, Marks and Davis accept and use the mean of the results of Reynolds, and Morby and Barnes, which is

$$1 \text{ mean calorie} = 4.1834 \times 10^7 \text{ ergs},$$

$$= 3.9683 \text{ B.T.U.}$$

$$1 \text{ mean B.T.U.} = 777.52 \text{ ft.-lbs.},$$

when the gravitational constant is 980.665 cm. sec^2, which corresponds to 32.174 lbs., and is the value for latitude between 45° and 46°.

For many years it has been most common to use in engineering calculations the round number 778, and for most problems this round number is still the best available figure, but where special accuracy is needed it is likely that no closer value can be relied upon than anything between 777.5 and 777.6 for the above latitude.

Example. To heat a gallon of water from 60° F. to 200° F. requires the heat equivalent of how many foot-pounds?

$$1 \text{ gallon} = 8.33 \text{ lbs.},$$

$$200° \text{ F.} - 60° \text{ F.} = 140° \text{ F. rise},$$

$$8.33 \times 140 = 1665 \text{ pound-degrees},$$

$$= 1665 \text{ B.T.U.}$$

$$= 778 \times 1665, \text{ ft.-lbs.}$$

$$= 90{,}800 \text{ ft.-lbs.}$$

Prob. 1. A feed-water heater is heating 5000 gallons of water per hour from 40° F. to 200° F. What would be the equivalent energy in horse-power units?

Prob. 2. A pound of each of the following fuels has the heating values as given. Change them to foot-pounds.

Average bituminous coal, 14,000 B.T.U. per lb., Average kerosene, 18,000 B.T.U. per lb.
" small anthracite, 12,000 B.T.U. per lb., " alcohol, 10,000 B.T.U. per lb.
Average gasolene, 20,000 B.T.U. per lb.

Prob. 3. A cubic foot of each of the following gases yields on combusture, the number of heat units shown. Change them to foot-pounds.

Natural gas (average), 880 B.T.U. per cu.ft., Carburetted water gas, 700 B.T.U. per cu.ft.
Coal gas, 730 B.T.U. per cu.ft., Mond gas, 150 B.T.U. per cu.ft.
Blast furnace gas, 100 B.T.U. per cu.ft.

Prob. 4. A pool contains 20,000 cu.ft. of water and must be warmed from 40° to 70° F. How much work might be done with the equivalent energy?

Prob. 5. How many calories and how many centigrade heat units would be required in Prob. 4?

Prob. 6. In the course of a test a man weighing 200 lbs. goes up a ladder 25 ft. high, every 15 minutes. If the test lasted 12 hours how many B.T.U. did he expend?

Prob. 7. A reservoir contains 300 billion gallons of water which are heated each year from 39° F. to 70° F. What is the number of foot-pounds of work equivalent?

Prob. 8. A pound of water moving at the rate of 450 ft. per second is brought to rest, so that all of its energy is turned into heat. What will be the temperature rise?

Prob. 9. For driving an automobile 30 horse-power is being used. How long will a gallon of average gasolene, sp.gr. = .7, last, if 10% of its energy is converted into work?

Prob. 10. Power is being absorbed by a brake on the flywheel of an engine. If the engine is developing 50 horse-power how many B.T.U. per minute must be carried off to prevent burning of the brake?

5. Temperature Change Relation to Amount of Heat, for Solids, Liquids, Gases, and Vapors, not Changing State. Specific Heats. Provided gases do not decompose, vapors condense, liquids freeze or evaporate, and solids melt, under addition or abstraction of heat, there will always be the same sort of relation between the quantity of heat gained or lost and the temperature change for all, differing only in degree. As the reception of heat in each case causes a temperature rise proportional to it and to the weight of the substances, this constant of proportionality once determined will give numerical relations between any temperature change and the corresponding amount of heat. Making the weight of the substance unity, which is equivalent to the consideration of one pound of substance, the constant of proportionality may be defined as the quantity of heat per degree rise, and as thus defined is the *specific heat* of the substance. Accordingly, the quantity of heat for these cases is equal to the product of specific heat, temperature rise and weight of substance heated.

The heat, as already explained, may be added in two characteristic ways: (*a*) at constant volume or density, or (*b*) at constant pressure. It might be expected that by reason of the increase of volume and performance of work under constant pressure heating, more heat must be added to raise the temperature of one pound, one degree, than in the other case where no such work is done, and both experimental and thermodynamic investigations confirm this view. There are, therefore, two specific heats for all substances, capable of definition:

(*a*) The specific heat at constant volume, and

(*b*) The specific heat at constant pressure.

These two specific heats are quite different both for gases and for vapors, which suffer considerable expansion under constant pressure heating, but for solids and liquids, which expand very little, the difference is very small and is to be neglected here. As a matter of fact, there are no cases of common engineering practice involving the specific heat of liquids and solids under constant volume, and values for the specific heats of liquids and solids are always without further definition to be understood as the constant pressure values.

Let C, be the specific heat of solids and liquids suffering no change of state.
" C_p, be the specific heat of gases and vapors at constant pressure and suffering no change of state.
" C_v, be the specific heat of gases and vapors at constant volume and suffering no change of state.
" t_2 and t_1, be the maximum and minimum temperatures for the process.
" w, be the weight in pounds.

Then will the heat added, be given by the following equation, if the temperature rise is exactly proportional to the quantity of heat, or in other words, *if the specific heat is constant.*

$$Q = Cw(t_2 - t_1), \text{ for solids and liquids} \quad (603)$$

$$Q = C_v w(t_2 - t_1), \text{ for gases and vapors (not near condensation) when volume is constant.} \quad (604)$$

$$Q = C_p w(t_2 - t_1), \text{ for gases and vapors (not near condensation) when pressure is constant.} \quad (605)$$

When, however, the specific heat is variable, as is the case for many substances, probably for all, the above equation cannot be used except when the specific heat *average* value, or mean specific heat is used. If the variation is irregular this can be found only graphically, but for some substances the variation is regular and integration will give the mean value. It has been the custom to relate the specific heat to the temperature above the freezing-point of water, expressing it as the sum of the value at 32° F., and some fraction of the temperature above this point to the first and second powers, as in Eq. (606).

$$\text{Specific heat at temperature } (t) = a + b(t-32) + c(t-32)^2 \quad (606)$$

In this equation a is the specific heat at 32°, while b and c are constants different for different substances, c being generally zero for liquids.

When this is true, the heat added is related to the temperature above 32° by a differential expression which can be integrated between limits

$$Q = \int_{t_1-32}^{t_2-32} [a + b(t-32) + C(t-32)^2]dt$$

$$= a[(t_2-32) - (t_1-32)] + \frac{b}{2}[(t_2-32)^2 - (t_1-32)^2] + \frac{c}{3}[(t_2-32)^3 - (t_1-32)^3]. \quad (607)$$

Usually the heats are calculated above 32° so that the heats between any two temperatures will be the difference between the heats from 32° to those two temperatures. In this case $t_1 = 32°$, and, $t_2 = t$, whence

$$\text{B.T.U. per lb., from 32° to } t, = \left[a + \frac{b}{2}(t-32) + \frac{c}{3}(t-32)^2\right](t-23). \quad (608)$$

For this range of temperature 32° to t, the quantity of heat may be expressed as the product of a mean specific heat and the temperature range or

$$\text{Heat from } 32° \text{ to } t = (\text{mean sp. heat from } 32° \text{ to } t°) \times (t-32). \quad . \quad (609)$$

Comparing Eq. (608) with Eq. (609) it follows that

$$\left\{ \begin{matrix} \text{Mean specific heat} \\ \text{from } 32° \text{ F. to } t° \text{ F.} \end{matrix} \right\} = a + \frac{b}{2}(t-32) + \frac{c}{3}(t-32)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (610)$$

The coefficient of $(t-32)$ in the mean specific heat expression, is half that in the expression for specific heat *at* t, and the coefficient of $(t-32)^2$, is one-third. This makes it easy to change from specific heat *at* a given temperature above 32°, to the mean specific heat from 32° to the temperature in question.

The specific heats of some substances are directly measured, but for some others, notably the gases, this is too difficult or rather more difficult than calculation of values from other physical constants to which they are related.

It often happens that in engineering work the solution of a practical problem requires a specific heat for which no value is available, in which case the general law of specific heats, known as the law of Dulong and Petit, for definite compounds may be used as given in Eq. (611).

$$(\text{Specific heat of solids}) \times (\text{atomic weight}) = 6.4. \quad . \quad . \quad . \quad (611)$$

This is equivalent to saying that all atoms have the same capacity for heat, and while it is known to be not strictly true, it is a useful relation in the absence of direct determinations. Some values, experimentally determined for the specific heats of solids, are given in Table XXXI at the end of this Chapter, together with values calculated from the atomic weights to show the degree of agreement. The atomic weights used are those of the International Committee on Atomic Weights (Jour. Am. Chem. Soc., 1910). When the specific heat of a solid varies with temperature and several determinations are available, only the maximum and minimum are given with the corresponding temperatures, as these usually suffice for engineering work.

To illustrate this variability of specific heat of solids, the values determined for two samples of iron are given in Figs. 120 and 121, the former showing the variation of the mean specific heat as determined by Oberhoffer and Harker from 500° F. up, and the latter the amount of heat per pound of iron at any temperature above the heat content at 500° F., which is generally called its *total heat* above the base temperature, here 500° F.

It is extremely probable that the specific heats of liquids all vary irregularly with temperature so that the constant values given in Table XXXII at the end of the Chapter must be used with caution. This is certainly the case for water, and is the cause of the difficulty in fixing the unit of heat, which is best solved by the method of means. In Fig. 122 are shown in curve form the values for the

specific heats of water at temperatures from 20° F. to 600° F., as accepted by Marks and Davis after a critical study of the experimental results of

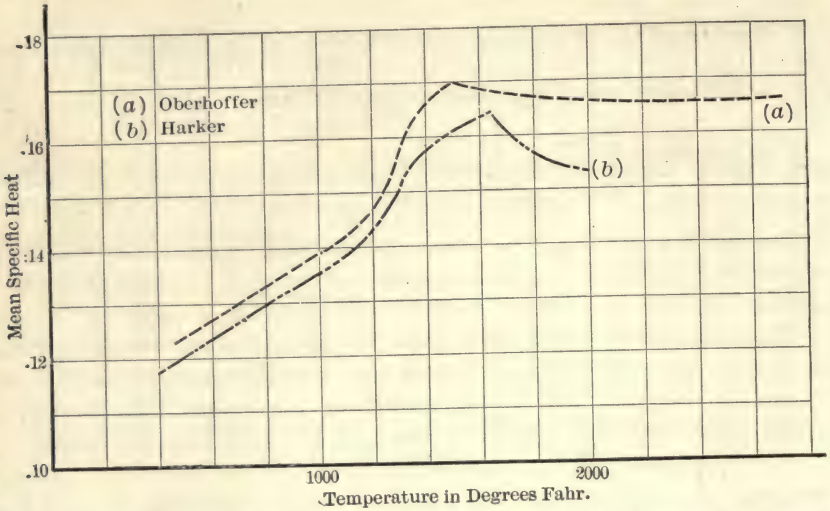


FIG. 120.—Mean Specific Heat of Iron above 500° F., Illustrating Irregular Variations not Yielding to Algebraic Expression.

Barnes and Dieterici and adjustment of the differences. The integral curve is plotted in Fig. 123 which, therefore, gives the heat of water from 32°F. to any

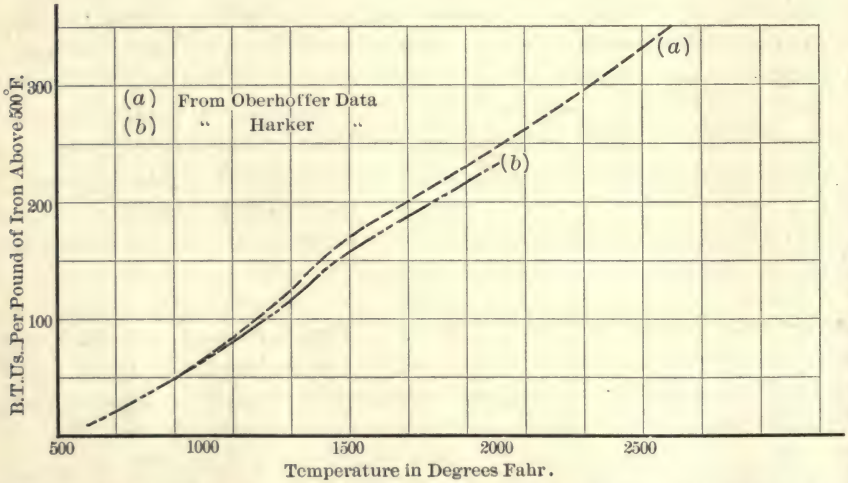


FIG 121.—Total Heat of Iron above 500° F., Illustrating its Approximation to a Straigl Line Relation in Spite of Wide Variation in Specific Heat Given in Fig. 120.

temperature up to the highest used in steam practice and which is designate in steam tables, summarizing all the properties of water and steam, as th

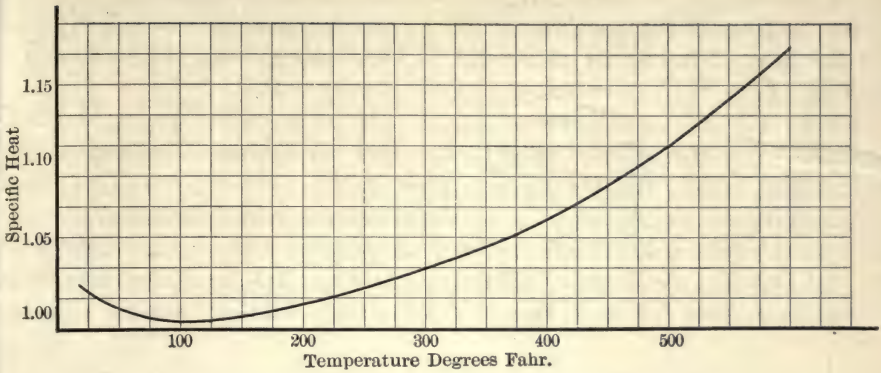


FIG. 122.—Specific Heat of Water at Various Temperatures.

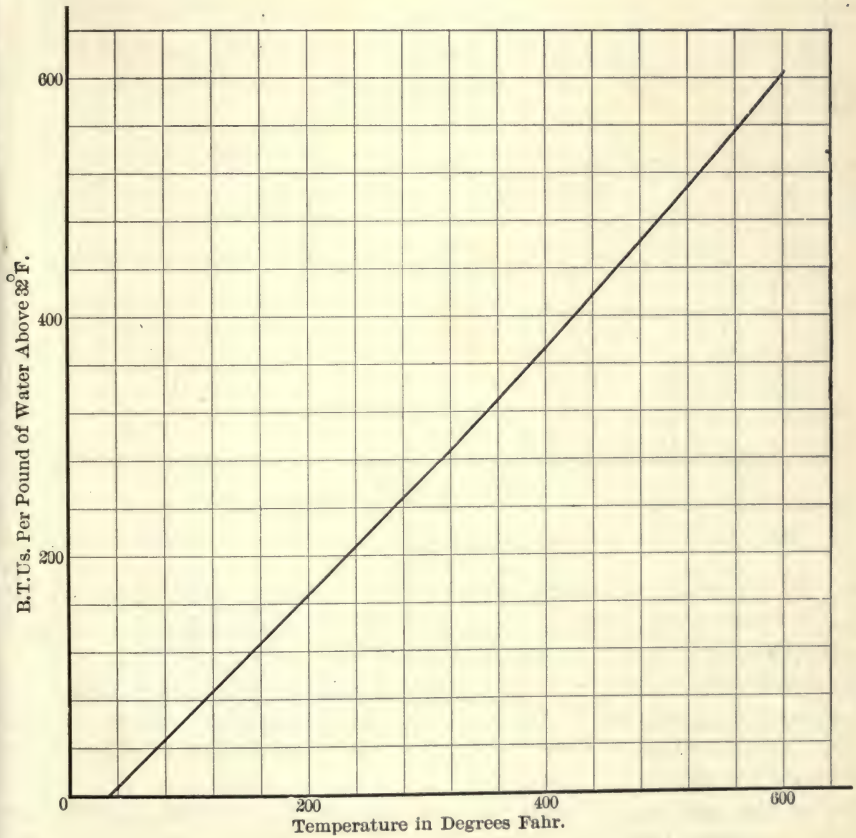


FIG. 123.—Total Heat of Water from 32° F., to any Temperature, *the Heat of the Liquid* at that Temperature above 32° F.

heat of the liquid. For the purpose of comparison, the mean specific heat of water is given in Fig. 124 from 32° F. to any temperature which is obtained from the heat of the liquid above 32° F. by dividing it by the temperature above 32° F.

In the table of specific heats of liquids there is a column giving the value calculated from the atomic weights to show at a glance the degree with which liquids satisfy the Dulong and Petit law.

Variability of specific heat is especially noticeable in liquids that are solutions with different amounts of dissolved substance, in which case the specific heat varies with the density and temperature. Problems of refrigeration involve four cases of this kind: (*a*), calcium, and (*b*), sodium chloride,

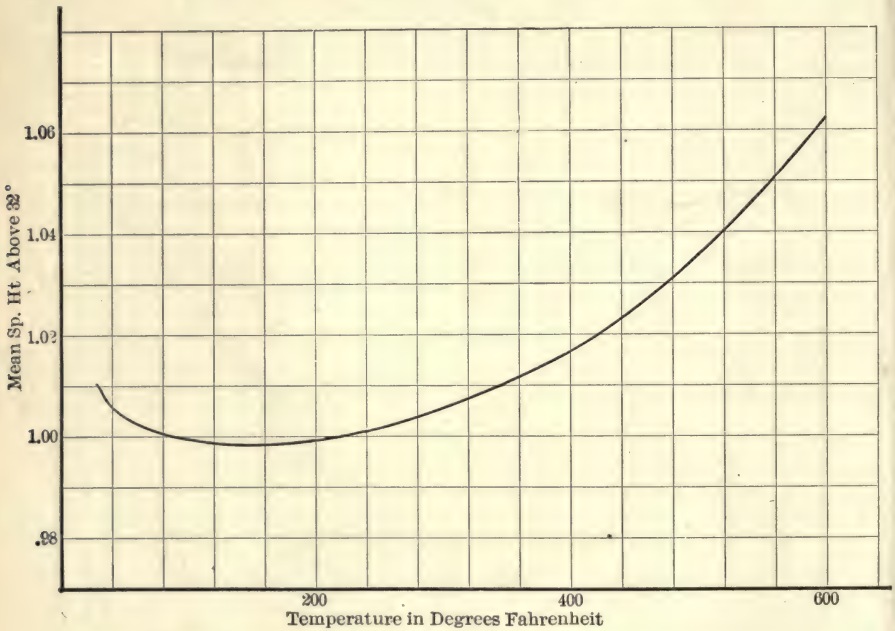


FIG. 124.—Mean Specific Heat of Water from 32° to any Temperature.

brines, the densities of which vary considerably but which are used with but little temperature range, seldom over 20° F. and often not over 5° F., (*c*), anhydrous ammonia and (*d*), carbonic acid.

As the density of brines is often reported on the Baumé scale and liquid fuels always so, a comparison of this with specific gravities is given in Table XXXIII in connection with the specific heat tables at the end of this Chapter to facilitate calculation.

One of the best-known solutions so far as accuracy of direct experimental data is concerned, is calcium brine, results for which, from 35° C. to 20° C. given below, are from U. S. Bureau of Standards Bulletin by Dickinson, Mueller and George, for densities from 1.175 to 1.250. For chemically pure

calcium chloride in water, it was found that the following relation between density D, and specific heat C, at 0° C.,

$$D=2.8821-3.6272C+1.7794C^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (612)$$

and these results plotted in Fig. 125 show the specific heat variation with temperature to follow the straight line law very nearly. This being the case the mean specific heat for a given temperature range is closely enough the arithmetical mean of the specific heat *at* the two limiting temperatures. To the figure are also added dotted, the specific heats for some commercial brines, not pure calcium chloride, but carrying magnesium and sodium chloride of density 1.2.

It might be conveniently noted here that the relation between freezing-point and density for pure calcium chloride by the same bulletin is given in Table XVIII below:

TABLE XVIII

FREEZING-POINT OF CALCIUM CHLORIDE

U. S. BUREAU OF STANDARDS

Density of Solution.	Per cent $CaCl_2$ by Wt.	Freezing-point, ° C.	Freezing-point, ° F.
1.12	14.88	− 9	15.8
1.14	16.97	−13	8.6
1.16	19.07	−16	3.2
1.18	21.13	−20	− 4.0
1.20	23.03	−24	−11.2
1.22	24.89	−29	−20.2
1.24	26.77	−34	−29.2
1.26	28.55	−40	−40.0

Other values for the specific heats of brines as commonly used are given in Table XIX, the accuracy of which is seriously in doubt and which may be checked by more authoritative values at different points where determinations have been made.

Anhydrous ammonia liquid, has a variable specific heat with temperature, but the experimental values are too few to make its value and law quite certain. Several formulas have been proposed, however, that tend to give an impression of accuracy not warranted by the facts though quite convenient in preparing tables.

Authority	Specific heat of NH_3 liquid at t° F.	
Zeuner	$1.0135+.00468\ (t-32)$	(a)
Dieterici	$1.118\ +.001156\ (t-32)$	(b)
Wood	$1.1352+.00438\ (t-32)$	(c)
Ledoux	$1.0057+.00203\ (t-32)$	(d)

. . (613)

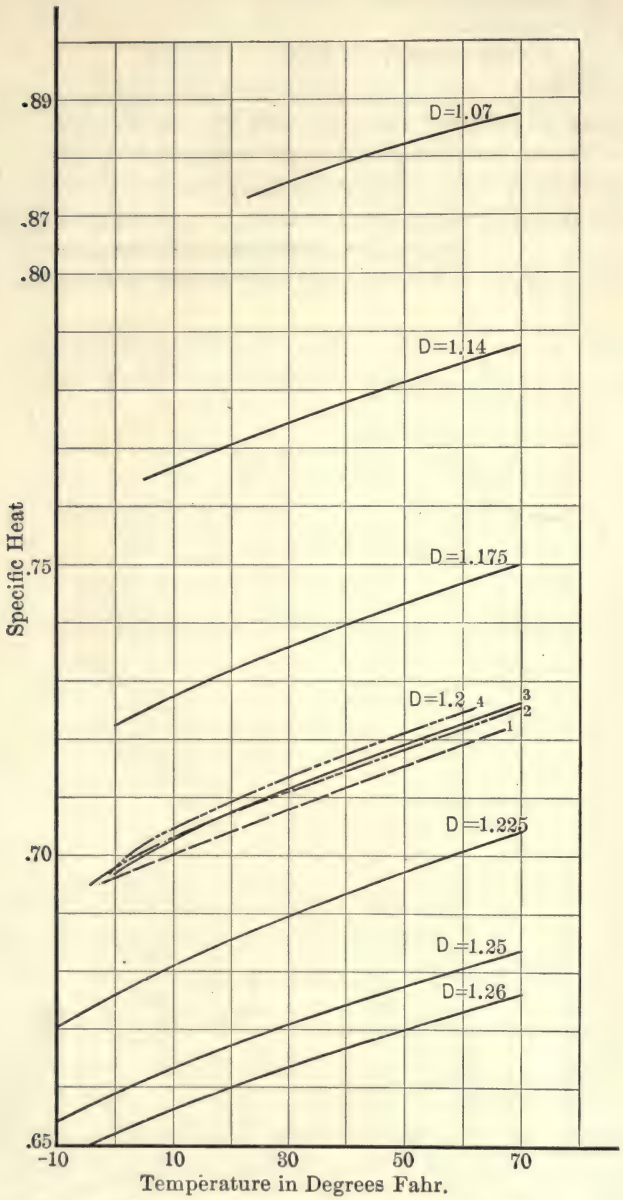


FIG. 125.—Specific Heat of Calcium Chloride Brine of Various Densities D at Temperatures $-10°$ F. to $+70°$ F.

TABLE XIX

SPECIFIC HEAT OF SODIUM CHLORIDE BRINE

Density, Bé	Sp.gr.	Per cent NaCl by Wt.	Sp. Heat.	Temp. F.	Authority.
1	1.007	1	.992	−0	Common
.....	.....	1.6	.978	64.4	Thomsen
.....	.....	4.9	.995	66−115	Winkelmann
5	1.037	5.0	.960	−0	Common
10	1.073	10.0	.892	−0	Common
.....	.....	10.3	.892	59−120	Teudt
.....	.....	10.3	.912	59−194	Teudt
.....	.....	11.5	.887	61−126	Marignac
.....	.....	12.3	.871	64.4	Winkelmann
15	1.115	15.0	.892	−0	Common
.....	.....	18.8	.841	63−125	Teudt
.....	.....	18.8	.854	68−192	Teudt
19	1.150	20.0	.829		Common
.....	.....	24.3	.7916	64−68	Winkelmann
.....	.....	24.5	.791	64	Thomsen
23	1.191	25	.783		Common

From these expressions the mean specific heat follows by halving the coefficient of $(t-32)$ F., and these were determined and plotted to scale, together with some direct experimental values of Drewes, in Fig. 126. Giving greatest weight to Drewes and Dieterici, a mean curve shown by the solid line is located as the best probability of the value for liquid anhydrous and it has the Eq. (614).

$$\left\{\begin{array}{l}\text{Mean specific heat of anhydrous}\\ \text{liquid } NH_3 \text{ from } 32^\circ \text{ F. to } t^\circ \text{ F.}\end{array}\right\} = 1.07 + .00056(t-32) \quad . \quad . \quad . \quad (614)$$

From this value the heat of liquid ammonia above 32° F. has been determined and is presented graphically in Fig. 127 from which, and the equation, the tabular values at the end of the Chapter were determined.

Ammonia dissolved in water, giving an aqueous solution as used in the absorption refrigerating system, has a nearly constant specific heat so closely approximating unity as shown by Thomsen, who gives

3 per cent NH_3 in water solution, sp.ht., =.997, at 66° F.
1.8 per cent NH_3 in water solution, sp.ht., =.999, at 66° F.
.9 per cent NH_3 in water solution, sp.ht., =.999, at 66° F.,

that it is customary in these calculations to ignore any departure from unity, the value for water.

Liquid carbonic acid, another important substance in engineering, especially in mechanical refrigeration, is less known as to its specific heat than is ammonia, and that is much too uncertain. There is probably nothing better available at present for the necessary range than the results of Amagat and Mollier, reported by Zeuner for the heat of the liquid, which are reproduced in Fig. 128, and used in the table at the end of this Chapter.

It is, however, with gases that the most complex situation exists with respect to specific heats. As has already been pointed out, gases may be heated at

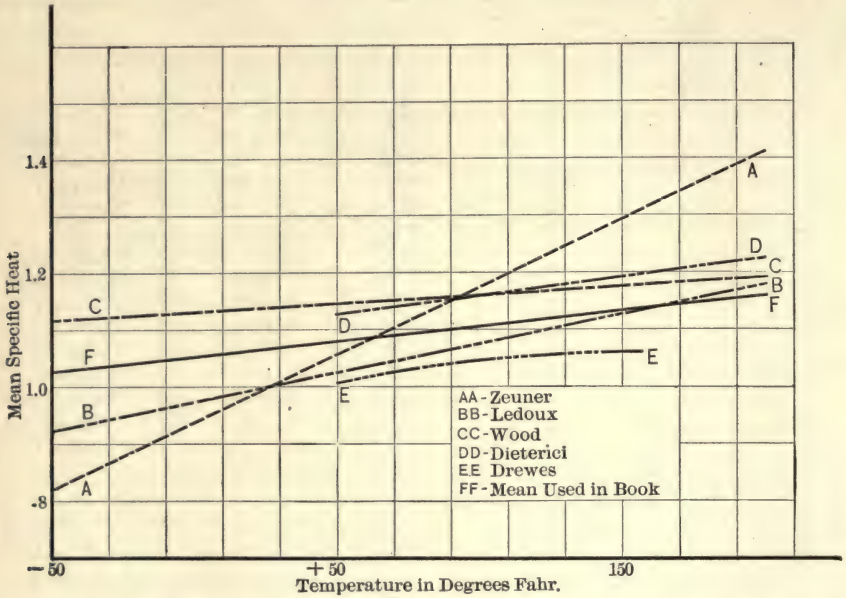


FIG. 126.—Mean Specific Heat of Liquid Anhydrous Ammonia from −50° F. to 150° .F

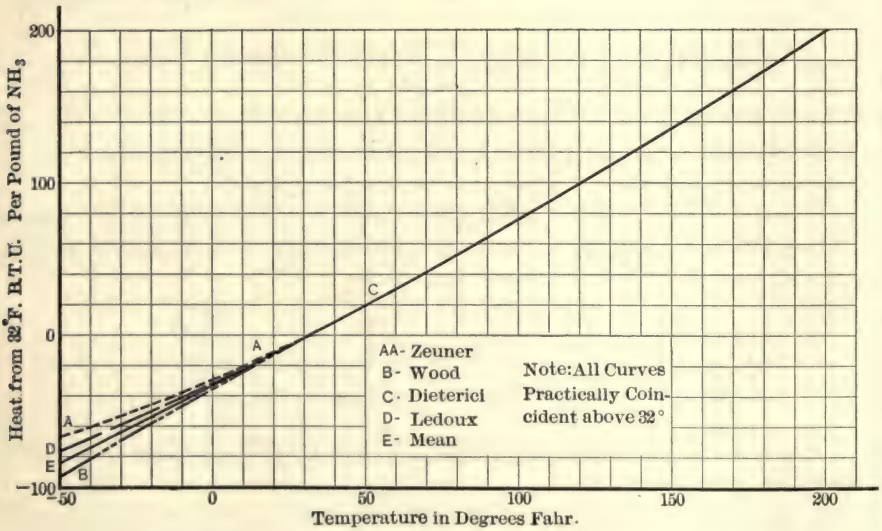


FIG. 127.—Heat of Liquid Anhydrous Ammonia above −50° F.

constant volume, doing no external work while being heated, or at constant pressure, in which latter case work is done by expansion of the gas against the resisting constant pressure. Therefore, there must be two different specific

heats for each gas, one C_p at constant pressure and the other C_v at constant volume, the difference between them representing the heat equivalent of the

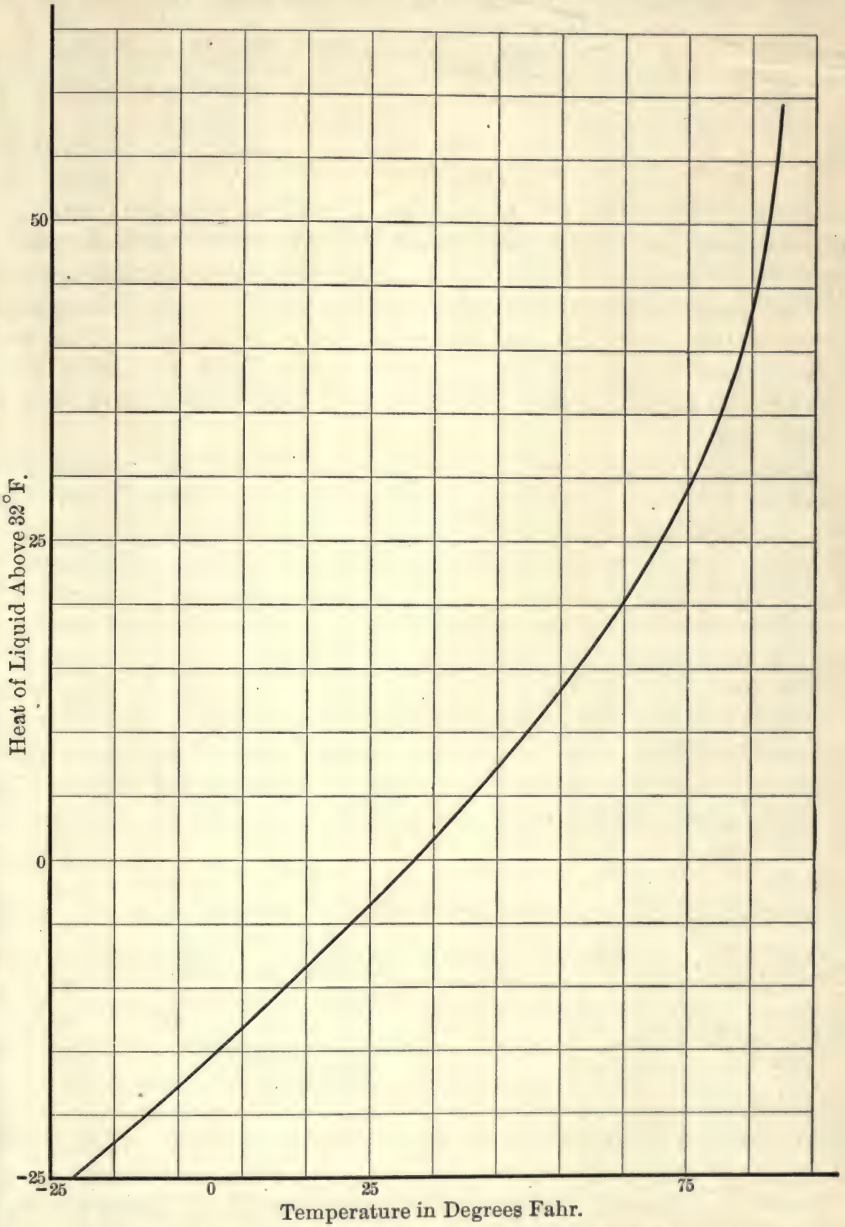


FIG. 128.—Heat of Liquid Carbonic Acid above 32° F.

work of expansion done during the rise of temperature. Most experimental determinations of the specific heats of gases have been made at constant pressure

and the constant volume value found from established relations between it and other physical constants. These relations most commonly used are two, Eq. (615) connecting the difference with a constant R and the other Eq. (616)

$$777.52(C_p - C_v) = R, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (615)$$

$$\frac{C_p}{C_v} = \gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (616)$$

connecting their ratio to a constant γ. These constants have each a special significance that may be noted here and proved later, thus R is the ratio of the PV product of a pound of gas to the absolute temperature, and γ the particular value taken by the general exponent s in $PV^s = c$, when the expansion represented takes place with no heat addition or abstraction, i.e., adiabatic, it is also a function of the velocity of sound in gases. Table XXXIV at the end of this Chapter gives some authentic values, with those adopted here designated by heavy type.

Variability of specific heats of gases and vapors is most marked and of some engineering importance, because so many problems of practice involve highly heated gases and vapors, the most common being superheated steam and the active gases of combustion in furnaces, gas producers and explosive gas engines. In fact, with regard to the latter it may be regarded as quite impossible with even a fair degree of accuracy to predict the temperature that will result in the gaseous products from the liberation of a given amount of heat of combustion. The first fairly creditable results on the variability of the specific heats of gases of combustion at high temperatures were announced by Mallard and LeChatelier, Vieille and Berthelot, all of whom agree that the specific heat rises, but who do not agree as to the amount. A general law was proposed by LeChatelier giving the specific heat as a function of temperature by an equation of the following form:

Specific heat at $t°$ F., $(V = C), = C_v = a + b(t - 32)$, ... (617)

in which a = specific heat at constant volume at 32° F. This yields,

$$\left\{\begin{matrix}\text{B.T.U. per lb. from } 32° \\ \text{F., to } t° \text{ F., } (V = C),\end{matrix}\right\} = Q_{(32 \text{ to } t)} = \left[a + \frac{b}{2}(t - 32)\right](t - 32). \quad . \quad . \quad . \quad . \quad (618)$$

$$\left\{\begin{matrix}\text{Mean specific heat from} \\ 32° \text{ F., to } t° \text{ F., } (V = C),\end{matrix}\right\} = C'_v = a + \frac{b}{2}(t - 32). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (619)$$

The specific heat at constant pressure is obtained by adding a constant to the value for constant volume according to

$$C_p = C_v + \frac{R}{777.52}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (620)$$

whence

$$\text{Specific heat at } t^\circ \text{ F., } (P=C), = C_p = a + \frac{R}{777.52} + b(t-32), \quad . \quad . \quad . \quad . \quad . \quad (621)$$

$$\left\{\begin{matrix}\text{B. T. U. per lb. from} \\ 32^\circ \text{ F., to } t^\circ \text{ F., } (P=C),\end{matrix}\right\} = Q_{32 \text{ to } t} = \left[a + \frac{R}{777.52} + \frac{b}{2}(t-32)\right](t-32) \quad . \quad . \quad (622)$$

$$\left\{\begin{matrix}\text{Mean specific heat from} \\ 32^\circ \text{ F., to } t^\circ \text{ F., } (P=C),\end{matrix}\right\} = C'_p = a + \frac{R}{777.52} + \frac{b}{2}(t-32). \quad . \quad . \quad . \quad . \quad . \quad (623)$$

The values of these constants have been determined by LeChatelier, Clerk, Callender, and Holborn and Austin, from which the following values are selected.

TABLE XX

SPECIFIC HEAT CONSTANTS, GASES.

Gas.	a	$a+\frac{R}{777.52}$	b	$\frac{b}{2}$	Authority.
CO_2	.1477	.1944	.000097	.0000484	LeChatelier
CO_2	.....	.2010	.0000824	.0000412	Holborn and Austin
N_2	.170	.2404	.0000484	.0000242	LeChatelier
N_2	.....	.2350	.000021	.0000105	Holborn and Austin to 2606° F.
N_2	.....	.2350	.0000208	.0000104	Callender 1544° F. to 2440° F.
O_2	.1488	.2125	.0000424	.0000212	LeChatelier
H_2O	.3211	.....	.000122	.000061	LeChatelier
Air	.....	.2431	.000135	.0000675	Callender (1544° F. to 2440° F.)

For purposes of comparison the following curves are plotted, showing all these results of specific heat at constant volume, at temperature t° F., the total heat above 32° F. per pound of gas, and the mean specific heat from 32° F. to t° F. in Fig. 129.

Probably there is now more known of the specific heat of superheated steam than of any common gaseous substance, and it is likely that other substances will be found in time to have somewhat similar characteristics. Pure computation from the laws of perfect gases indicates that the specific heat of gases or superheated vapors must be either a constant, or a function of temperature only, and this is what prompted the form of the LeChatelier formula. Bold experimentation on steam, disregarding the law, or rather appreciating that superheated steam is far from a perfect gas, principally by Knobloch and Jacob and by Thomas, showed its specific heat to be a function of both pressure and temperature. Results were obtained that permitted the direct solution of problems of heat of superheat, or the heat per pound of vapor at any temperature above that at which it was produced, or could exist in contact with the liquid from which it came. Critical study of various results by Marks and Davis led them to adopt the values of Knobloch and Jacob with slight modifi-

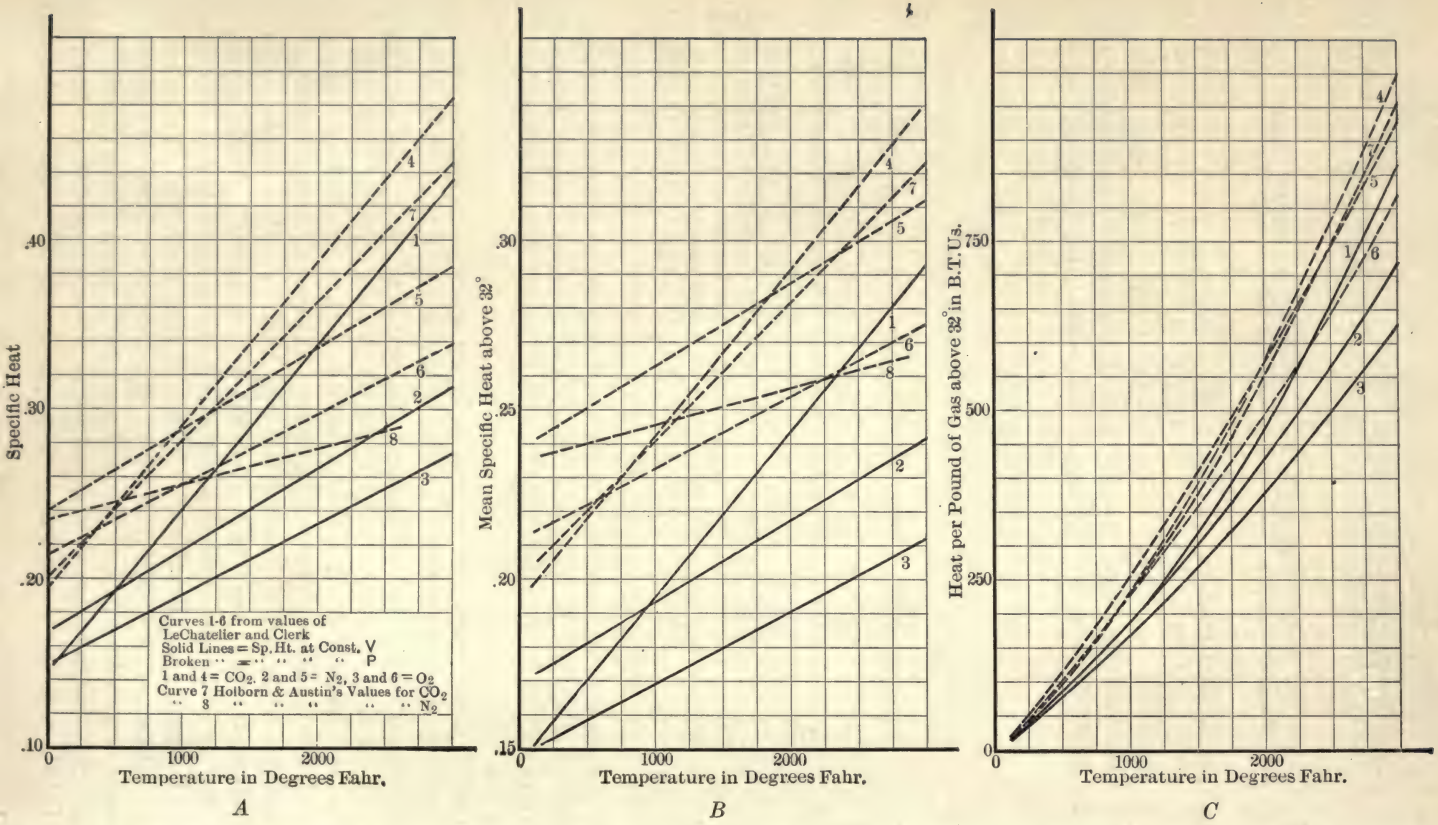


FIG. 129.—Specific Heat of Gases at, Mean Specific Heat from 32° F. to, and Total Heat per Pound from 32° F. to, Various Temperatures.

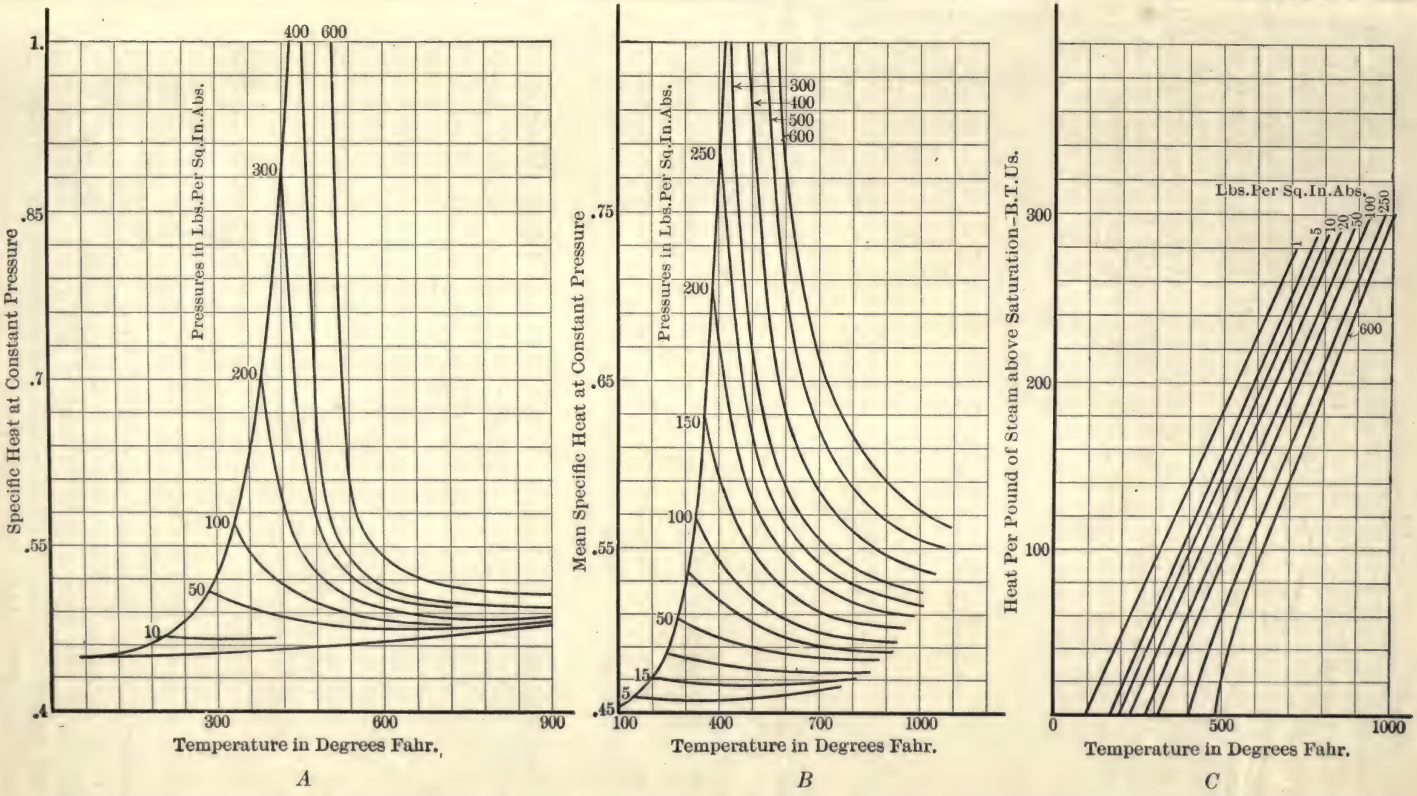


FIG. 130.—Specific Heat of Superheated Steam at, Mean Specific Heat from Saturation Temperature to, and Heat of Superheat per pound from Saturation Temperature to, Various Temperatures.

cations, for which evidence was in existence, raising the specific heats at low pressures and temperatures, and their conclusions are adopted in this work. In Fig. 130 is shown (A) the Marks and Davis modification of the C_p curve of Knobloch and Jacobs, the integral of which (C) gives the heat of superheat from any temperature of steam generation to actual steam temperature, while (B) shows the values for the mean specific heat above the temperature of saturation for the particular pressure in question.

When substances of the same class are mixed so that w_1, w_2, w_3, etc., lbs. of the different substances having specific heats C_1, C_2, C_3, etc., or C_{p1}, C_{p2}, C_{p3}, etc., or C_{v1}, C_{v2}, C_{v3}, etc., then the specific heat of the mixture is given by

$$C=\frac{C_1w_1+C_2w_2+C_3w_3+\text{etc.}}{w_1+w_2+w_3+\text{etc.}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (624)$$

$$C_v=\frac{C_{v1}w_1+C_{v2}w_2+C_{v3}w_3+\text{etc.}}{w_1+w_2+w_3+\text{etc.}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (625)$$

$$C_p=\frac{C_{p1}w_1+C_{p2}w_2+C_{p3}w_3+\text{etc.}}{w_1+w_2+w_3+\text{etc.}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (626)$$

Example. If 5 lbs. of olive oil at a temperature of 100° F., 10 lbs. of petroleum at a temperature of 150° F., and 50 lbs. of water at 50° F. are mixed together, what will be the resultant temperature and how much heat will be required to heat the mixture 100° above this temperature?

Sp. ht. of olive oil = .4,
Sp. ht. petroleum = .511,
Sp. ht. water = 1.000.

Let x = the final temp. The heat given up by the substances falling in temperature is equal to that gained by those rising, hence

$$50(x-50)\times1=5(100-x)\times.4+10(150-x)\times.511,$$

$$50x-2500=200-2x+766-5.11x,$$

$$57.11x=3466, \quad \text{or,} \quad x=60.7° \text{ F.},$$

$$\text{Sp.ht. of mixture}=\frac{C_1w_1+C_2w_2+C_3w_3}{w_1+w_2+w_3}, \text{ from Eq. (611)},$$

$$=\frac{5\times.4+10\times.511+50\times1}{5+10+50}=\frac{57.11}{65}=.8786.$$

whence the heat required will be $65\times.8786=57$ B.T.U.

Prob. 1. To change a pound of water at 32° F. to steam at 212° F. requires 1150.4 B.T.U's. If the same amount of heat be given to a cubic foot of each of the following substances at 32° F., what will be final temperature in each case? (a) copper; (b) iron; (c) mercury; (d) clay; (e) stone.

Prob. 2. How many pounds of the following substances could be warmed 10° F. by the heat required to raise 100 lbs. of water from 40° F. to 200° F.?

(*a*) Ethyl alcohol from 100° F.;

(*b*) Sea water from 60° F., (density = 1.045);

(*c*) Glycerine from 60° F;

(*d*) Tin from 480° F.

Prob. 3. If 150 lbs. of water at 200° F. are added to a tank containing 200 lbs of petroleum at 70° F., what will be the resultant temperature, neglecting any heat absorbed or given up by the tank itself?

Prob. 4. To melt 1 lb. of ice requires 144 B.T.U. How much would this lower the temperature of 1 lb. of the following substances (1) at constant pressure; (2) at constant volume; (*a*) air; (*b*) oxygen; (*c*) ammonia; (*d*) hydrogen; (*e*) nitrogen?

Prob. 5. What would be the specific heats of the following mixture? Hydrogen 3 lbs., oxygen 1 lb., nitrogen 7 lbs., carbon dioxide 20 lbs., carbon monoxide 10 lbs.?

Prob. 6. Air is approximately 77 per cent N_2, and 23 per cent O_2 by weight. By means of the specific heats of the components, find its specific heats at constant pressure, and at constant volume.

Prob. 7. By means of the specific heats, find the values of R and γ most correct at atmospheric temperature (60° F.) for, hydrogen, air, carbon dioxide, carbon monoxide and nitrogen.

Prob. 8. How much water could be heated from 40° F. to 60° F. by the heat needed to superheat 10 lbs. of steam at 200 lbs. per square inch absolute to 700° F.?

Prob. 9. A building containing 250,000 cu. ft. of space is heated by a hot-water system. Considering the air to change eight times per hour, how many pounds of water per hour must be circulated if the drop in temperature of the water is from 200° to 100° and the temperature of the outside air is 30° F. while that of the room is 60° F. neglecting wall conducted heat?

Prob. 10. How much heat would be required to warm a pound of liquid CO_2 from zero to 80° F.? Compare with water and ammonia.

6. Volume or Density Variation with Temperature of Solids, Liquids, Gases and Vapors, Not Changing State. Coefficients of Expansion. Coefficients of Pressure Change for Gases and Vapors. Solids increase in length or in any linear dimension, a certain fraction of their original length for each degree temperature rise and the expansion is usually assumed to be in proportion to temperature rise. The relation between original and final length can be set down in an equation involving the coefficient of expansion.

Let a = coefficient of linear expansion = fractional increase in length per degree.

" l_1 and t_1 = original length or any other linear dimension and the corresponding temperature;

" l_2 and t_2 = length which l_1 becomes after heating and the corresponding temperature.

Then

$$\text{Increase in length} = l_2 - l_1 = a l_1 (t_2 - t_1), \quad \ldots\ldots (627)$$

$$\text{New length } l_2 = l_1 + a l_1 (t_2 - t_1),$$

$$= l_1 [1 + a(t_2 - t_1)]. \quad \ldots\ldots (628)$$

Solids, of course, expand cubically and the new volume will be to the old as the cubes of the linear dimension.

Let α = coefficient of volumetric expansion;
" v_1 = original volume;
" v_2 = final volume after heating.

Then when the temperature rises one degree,

$$\frac{v_2}{v_1} = \left(\frac{l_2}{l_1}\right)^3 = (1+a)^3 = 1+3a+3a^2+a^3 = 1+\alpha \quad . \quad . \quad . \quad . \quad (629$$

If a is small, and it is generally less than $\frac{1}{10000}$, then the square and cube can be neglected in comparison with the first power, whence

$$1+\alpha = 1+3a \quad \text{and} \quad \alpha = 3a.$$

so that the coefficient of volumetric expansion may be taken as sensibly equal to three times the coefficient of linear expansion, and similarly, the coefficient of surface expansion as twice the coefficient of linear expansion.

Liquids, by reason of the fact that they must always be held in solid containers, may be said to have no linear expansion, and therefore, although the expansion may be one direction only, the amount is due to the total change of volume rather than the change of length along the direction of freedom to expand. The same is true of gases, so that for gases and liquids only coefficients of volumetric expansion are of value and these are given in the Tables XXXV, XXXVI, XXXVII, and XXXVIII at the end of this Chapter. With liquids and gases it is usual to take the volume at 0° C. or 32° F. and 29.92 ins. Hg pressure as a standard, and the coefficient gives the increase as a fraction of this, per degree departure from the freezing-point. This is the universal practice with gases.

It appears that the coefficients of expansion for solids are quite different from one another, ranging from over 15×10^{-4} for wax, to $.085\times10^{-4}$ for Jena normal glass, a range of over two hundred and sixty times. Determinations of the value at various temperatures for any one substance indicate a variation with temperature, which proves that proportionality of increase of dimensions to temperature rise, does not hold true, a fact which has led to formulas of the form

$$l_2 = l_1[1+x(t_2-t_1)+y(t_2-t_1)^2],$$

the value of which is dependent on the determination of the constant and verification of correctness of form, which has not by any means been conclusively done. For most engineering work the constant values nearest the temperature range will suffice except for certain liquids, vapors, and gases. A more marked tendency to follow such a law of variation with temperature is found with liquids and coefficients for some are given in the standard physical tables.

The two important liquids, mercury and water, have been separately studied in greater detail and the latter exhibits a most important exception to the rule. For mercury, according to Broch

$$v_2 = v_1(1 + .000455t + 54 \times 10^{-12}t^2 + 602 \times 10^{-14}t^3), \quad . \quad . \quad . \quad (630)$$

which exhibits a refinement of value only in instrument work such as barometers and thermometers. Water, as already mentioned, has its maximum density at 39.1° F. and expands with both fall and rise of temperature. Its expansion is given by a similar formula by Scheel, as follows:

$$v_2 = v_1(1 - .03655 \times 10^{-3}t + 2.625 \times 10^{-6}t^2 - 1.161t^3). \quad . \quad . \quad (631)$$

Most commonly the expansion of water is not considered in this way, but by comparing densities at varying temperatures, and all sets of physical tables contain values which in this work are significant only as affecting the change of volume in turning water to steam *and such values as are needed are* incorporated in the steam tables later.

The study of the expansion of gases and vapors at constant pressure, and rise of pressure at constant volume, per degree has perhaps been fairly complete and is of greatest significance, because from it most of the important laws of thermodynamics have been derived. This work may be said to have started with the Regnault air and gas thermometer work, already described. Some of the authentic values collected in the Landolt, Börnstein, Myerhoffer, and Smithsonian Physical Tables, are given at the end of this Chapter, where α_v is the coefficient of pressure change at constant volume, and α_p the coefficient of expansion, or volume change at constant pressure.

The remarkable thing about the coefficients for these gases and vapors is the approach to constancy for most of the gases, not only of the coefficients of expansion for $P = c$ nor the similar constancy of the coefficients of pressure rise for $V = c$, but more remarkable than either of these is the similarity of the two constant coefficients. These facts permit of the generalizing of effect when $P = c$, and when $V = c$, and of the announcement of a law by means of which all such problems can be solved instead of applying separate coefficients for every substance and every different temperature necessary for solids and liquids where, for example, the maximum coefficient was over 260 times as great as the least. The average coefficient for all gases, applying both to pressures and volumes, is the same as enters into the gas thermometer work and its best value is found to be

$$\left.\begin{aligned} \alpha &= \frac{1}{491.52} = .002034, \text{ per degree F.} \\ \alpha &= \frac{1}{273.13} = .003661, \text{ per degree C.} \end{aligned}\right\}, \quad . \quad . \quad . \quad . \quad (632)$$

and approximately

$$\left.\begin{aligned} \alpha &= \frac{1}{492} = .00203, \text{ per degree F.} \\ \alpha &= \frac{1}{273} = .00366, \text{ per degree C.} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (633)$$

These are the same as the reciprocals of the absolute temperature of the ice-melting point, and are but expressions of conditions for reduction of the volume and pressure at the ice-melting temperature to zero by constant pressure and constant volume abstraction of heat respectively, and by stating the amount of reduction per degree give by implication the number of degrees for complete reduction.

Example. The rails on a stretch of railroad are laid so that they just touch when the temperature is 120° F. How much total space will there be between the rails per mile of track at 0° F.?

For wrought iron a will be nearly the same for Bessemer steel = .00000648.

Hence the linear reduction in 5280 ft. for a change of 120° F. will be

$$5280 \times 120 \times .00000648 = 4.1 \text{ ft.}$$

Prob. 1. A steam pipe is 700 ft. long when cold (60° F.), and is anchored at one end. How much will the other end move, if steam at a temperature of 560° F. is turned into the pipe?

Prob. 2. A copper sphere is one foot in diameter at 50° F. What must be the diameter of a ring through which it will pass at a temperature of 1000° F.?

Prob. 3. A hollow glass sphere is completely filled with mercury at 0° F. What per cent of the mercury will be forced out if the temperature rises to 300° F.?

Prob. 4. A room 100 ft. × 50 ft. × 10 ft. is at a temperature of 40° F. The temperature rises to 70° F. How many cubic feet of air have been forced from the room?

Prob. 5. The air in a pneumatic tire is at a pressure of 90 lbs. per square inch gage and at a temperature of 50° F. Due to friction of the tire on the ground in running, the temperature rises to 110° F. What will be the pressure?

Prob. 6. A brick lighthouse is approximately 200 ft. high. Should it be exactly this at 0° F., what would it be at 100° F.?

Prob. 7. Show that if a glass tube is rigidly held at each end by brackets attached to an iron tank it will break if the tank is warmed.

Prob. 8. From Eq. (618) find the density of water at 60° F., 100° F., 212° F., and compare with the values in the steam tables.

Prob. 9. A drum containing CO_2 gas at a pressure of 250 lbs. per square inch gage is raised 100° F. above its original temperature. What will be the new pressure?

7. Pressure, Volume and Temperature Relations for Gases. Perfect and Real Gases. Formulating the relations between the pressure change at constant volume and the volume change at constant pressure,

Let P and V be the simultaneous pressure and volume of gas;

" t be its scale temperature at the same time, F.;

" T be its absolute temperature at the same time, F.

Then at constant volume the pressure reached at condition (a) after heating from 32° F. is given by

$$P_a - P_{32°} = \frac{1}{492} P_{32} \times (t_a - 32).$$

$$\therefore \quad P_a = P_{32°}\left[1 + \frac{t_a - 32}{492}\right].$$

Similarly for another temperature t_b, the pressure will be

$$P_b = P_{32°}\left[1 + \frac{t_b - 32}{492}\right].$$

Whence

$$\frac{P_a}{P_b} = \frac{1 + \frac{t_a - 32}{492}}{1 + \frac{t_b - 32}{492}} = \frac{492 - 32 + t_a}{492 - 32 + t_b} = \frac{t_a + 460}{t_b + 460},$$

or

$$\frac{P_a}{P_b} = \frac{T_a}{T_b}, \text{ for } V \text{ constant}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (634)$$

Similarly

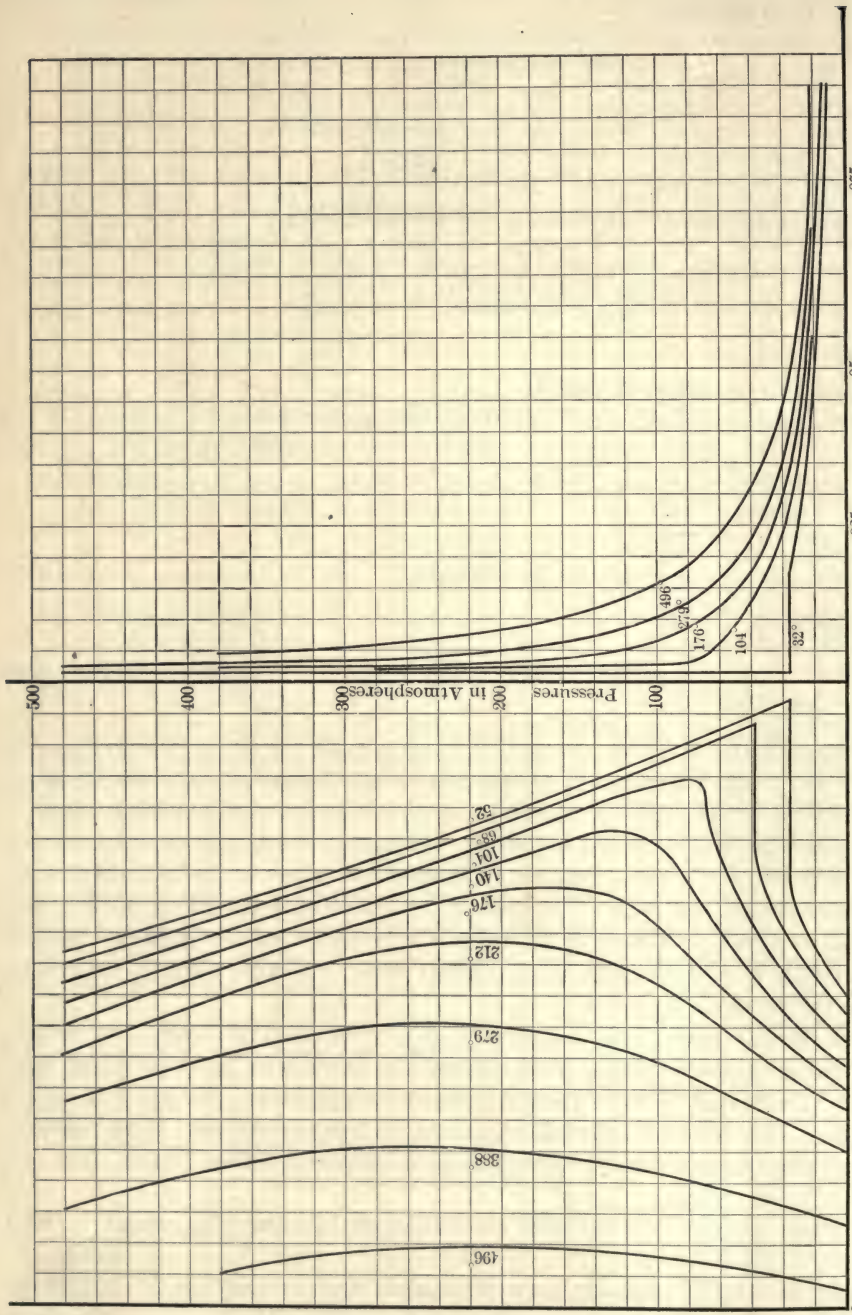
$$\frac{V_a}{V_b} = \frac{T_a}{T_b}, \text{ for } P \text{ constant}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (635)$$

Both Eqs. (634) and (635) are true, for no gas all the time, but very nearly true for all, under any range of change, and *a hypothetical gas is created for which it is exactly true all the time, known as a perfect gas,* about which calculations can be made as would be impossible for real gases and yet the results of which are so close to what would be the result with real gases, as to be good enough for engineering practice. Therefore, with a mental reservation as a guard against too great confidence in the work, all real gases will be assumed perfect and to follow Eqs. (634) and (635) except when experience shows the results are too far wrong to be useful.

These laws, known by the names both of Charles and Gay-Lussac, are closely associated with another also doubly named as Boyle's or Mariotte's and likewise an idealization of experimental observations known to be nearly true for all gases. This is to the effect that so long as temperatures are kept constant the pressures of gases vary inversely as their volume, or that,

$$\frac{P_a}{P_b} = \frac{V_b}{V_a}, \quad \text{and,} \quad P_a V_a = P_b V_b = \text{constant, for } T \text{ constant} \quad . \quad (636)$$

Study of the PV product, for various gases has revealed a good deal on the general properties of matter, especially as to the transition from one state to another. This is most clearly shown by curves which may be plotted in two



PV Products for Carbon Dioxide.

ways. To coordinates of pressure and volume a family of equilateral hyperbolas one for each temperature, would represent the true $PV=C$ or *isothermal* relation and any variation in the constancy of the product would be shown by its departure from the hyperbola. Still more clearly, however, will the departure appear when the product PV is plotted against pressures, constancy of product would require all lines to be straight and inconstancy appear by departures from straight lines. To illustrate, the data from Young for carbon dioxide are plotted both ways in Fig. 131, from 32° F. to 496° F., the values of PV at 32° and 1 atm. are taken as unity on one scale. It appears that up to the temperature of 88° F, known as the critical temperature, each isothermal plotted to P and PV coordinates consists of three distinct parts:

(*a*) a curved line sloping to the right and upwards;

(*b*) a straight line nearly or exactly horizontal;

(*c*) a nearly straight line sloping upward rapidly and to the left.

In this region then the isothermals are discontinuous, and this is caused by the liquification or condensation of the gas, during which increase of pressure, produces no change of volume, provided the temperature is low enough. It also appears that each PV line has a minimum point and these minima joined result in a parabola. At the end of this Chapter are given in Table XXXIX the values of PV at three different temperatures and various pressures for oxygen, hydrogen, carbon dioxide and ammonia, in terms of the values at 32° and 1 atm. for further comparison and use. Further study along these lines is not profitable here and the topic while extremely interesting must be dropped with the observation, that except near the point of condensation or liquefaction, gases or vapors, which are the same thing except as to nearness to the critical state, follow the Boyle law closely enough for engineering purposes.

None of these approximate laws, Eqs. (634), (635) and (636) can be considered as general, because each assumes one of the variables to be constant, but a general law inclusive of both of these follows from further investigation of a fixed mass of gas suffering all sorts of pressure volume and temperature changes, such as occur in the cylinders of compressors and gas engines. A table of simultaneous experimental values of pressure, volume, and temperature, for any gas will reveal the still more general relation inclusive of the preceding three as follows:

$$\frac{P_a V_a}{T_a}=\frac{P_b V_b}{T_b}=\frac{PV}{T}=C_g, \quad \ldots\ldots\ldots \quad (637)$$

in which C_g is approximately constant for any one gas and assumed constant for perfect gases in all calculations. For twice the weight of gas at the same pressures and temperatures C_g would be twice as large, so that taking a constant R for one pound, and generally known as the "gas constant," and introducing a weight factor w, the general characteristic equation for the perfect gas is,

$$PV=wRT \quad \ldots\ldots\ldots\ldots \quad (638)$$

This general law may be derived from the three primary laws by imagining in Fig. 132, two points, A and B, in any position and representing any two states of the gas. Such points can always be joined by three lines, one constant

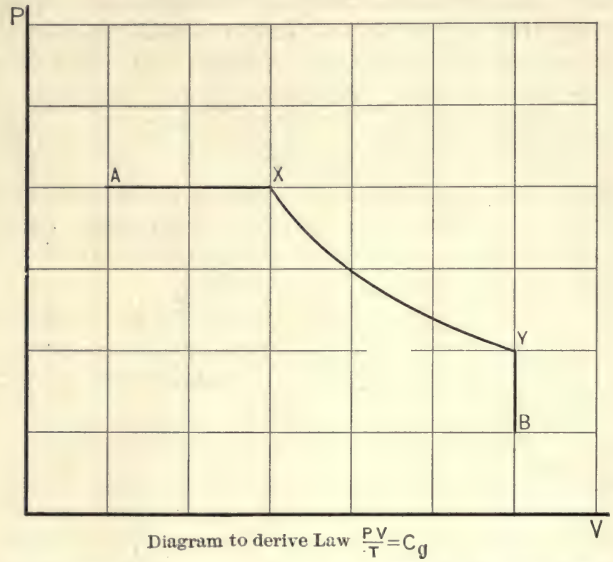


FIG. 132.—Curve of Continuous Relation between P, V, and T for Gases.

pressure A to X, one constant temperature X to Y, and the other constant volume Y to B. For these the following relations hold, passing from A to B

$$V_a = V_x \frac{T_a}{T_x}.$$

But

$$V_x = V_y \frac{P_y}{P_x},$$

and

$$T_x = T_y,$$

whence

$$V_a = V_y \frac{P_y}{P_a} \frac{T_a}{T_y},$$

or

$$\frac{P_a V_a}{T_a} = \frac{P_y V_y}{T_y}.$$

Passing to B,

$$P_y = P_b \frac{T_y}{T_b}.$$

But

$$V_y = V_b,$$

$$\therefore \quad \frac{P_a V_a}{T_a} = \frac{P_y V_y}{T_y} = \frac{P_b \frac{T_y}{T_b} V_b}{T_y} = \frac{P_b V_b}{T_b},$$

or in general

$$\frac{PV}{T} = \text{constant.} = wR$$

when the weight of gas is w lbs.

For numerical work, the values of R must be fixed experimentally by direct measurement of simultaneous pressure, volume, and temperature, of a known weight of gas or computed from other constants through established relations. One such relation already mentioned but not proved is

$$R = 777.52(C_p - C_v). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (639)$$

It is extremely unlikely that the values of R found in both ways by a multitude of observers under all sorts of conditions should agree, and they do not, but it is necessary for computation work that a reasonable consistency be attained and that judgment in use be cultivated in applying inconsistent data. In the latter connection the general rule is to use that value which was determined by measurement of quantities most closely related to the one being dealt with. Thus, if R is to be used to find the state of a gas as to pressure, volume, and temperature, that value of R determined from the first method should be selected, but the second when specific heats or Joule's equivalent are involved. Of course, a consistency could be incorporated for a perfect gas, but engineers deal with real gases and must be on guard against false results obtained by too many hypotheses or generalizations contrary to the facts. Accordingly, two values of R are given in Table XL, at the end of this chapter, one obtained from measurements of specific heats at constant pressure and determinations of the ratio of specific heats unfortunately not always at the same temperature and generally by different people, and the other by direct measure of gas volume at standard 32° F. temperature and 1 atm. pressure. These measurements are separately reported in Sections (5) and (8), respectively.

If a gas in condition A, Fig. 133, expand in any way to condition B, then it has been shown that

$$P_a V_a^{\ s} = P_b V_b^{\ s},$$

in which s has any value and which becomes numerically fixed only when the process and substance are more definitely defined. Comparing the temperatures at any two points A and B, it follows that

$$T_a = \frac{P_a V_a}{wR}, \quad \text{and} \quad T_b = \frac{P_b V_b}{wR},$$

whence

$$\frac{T_b}{T_a}=\frac{P_bV_b}{P_aV_a}.$$

But

$$P_aV_aV^{s-1}=P_bV_bV^{s-1},$$

and

$$\frac{P_bV_b}{P_aV_a}=\left(\frac{V_a}{V_b}\right)^{s-1},$$

whence

$$\frac{T_b}{T_a}=\left(\frac{V_a}{V_b}\right)^{s-1}, \quad \ldots\ldots\ldots\ldots \quad (640)$$

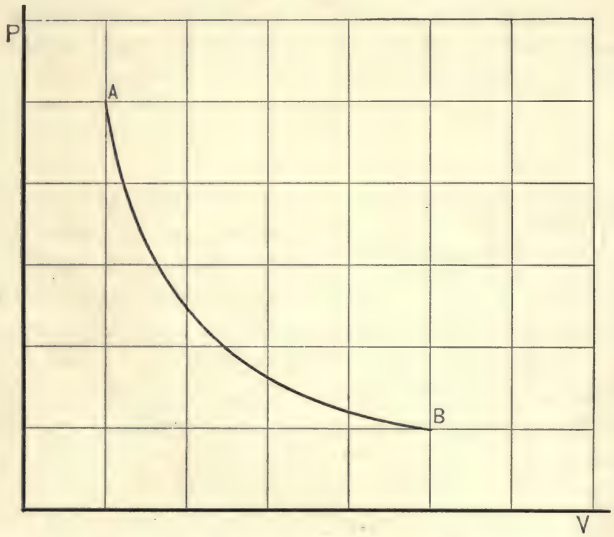


FIG. 133.—Expansion or Compression of Gas between A and B, Causing a Change of Temperature.

and

$$\frac{V_b}{V_a}=\left(\frac{T_a}{T_b}\right)^{\frac{1}{s-1}}, \quad \ldots\ldots\ldots\ldots \quad (641)$$

Eqs. (640) and (641) give the relation between temperatures and volumes.

But

$$\frac{V_a}{V_b}=\frac{T_aP_b}{P_aT_b}=\frac{T_bP_a}{T_aP_b},$$

which, substituted in above, gives

$$\frac{T_b}{T_a}=\left(\frac{T_a}{T_b}\right)^{s-1}\left(\frac{P_b}{P_a}\right)^{s-1},$$

or

$$\left(\frac{T_b}{T_a}\right)^{s}=\left(\frac{P_b}{P_a}\right)^{s-1},$$

and

$$\frac{T_b}{T_a}=\left(\frac{P_b}{P_a}\right)^{\frac{s-1}{s}}, \quad \ldots \ldots \ldots \ldots \quad (642)$$

or

$$\frac{P_b}{P_a}=\left(\frac{T_b}{T_a}\right)^{\frac{s}{s-1}}, \quad \ldots \ldots \ldots \ldots \quad (643)$$

Eqs. (629) and (630), give the relation between pressures and temperatures. It is convenient to set down the volume and pressure relations again to complete the set of three pairs of most important gas equations.

$$\frac{V_b}{V_a}=\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}} \quad \ldots \ldots \ldots \ldots \quad (644)$$

$$\frac{P_b}{P_a}=\left(\frac{V_a}{V_b}\right)^{s} \quad \ldots \ldots \ldots \ldots \quad (645)$$

These are perfectly general for any expansion or compression of any gas, but are of value in calculations only when s is fixed either by the gas itself or by the thermal process as will be seen later.

Example. A pound of air has a volume of 7.064 cu.ft. at a pressure of two atmospheres and a temperature of 100° F. Find the value of R for air from the data; also the final volume and temperature if expansion occurs so that $s=1.4$ until the pressure becomes $\frac{1}{2}$ an atmosphere.

$$PV=wRT, \quad \text{or} \quad 2116\times 2\times 7.064=1\times R\times 560, \quad \text{or} \quad R=53.38,$$

$$\frac{T_1}{T_2}=\left(\frac{P_1}{P_2}\right)^{\frac{s-1}{s}}=(4)^{\frac{.4}{1.4}}=1.49.$$

$$\therefore \quad T_2=T_1\div 1.49=\frac{560}{1.49}=352 \text{ abs.}=-108^\circ \text{ F.}$$

$$\frac{V_2}{V_1}=\left(\frac{P_1}{P_2}\right)^{\frac{1}{s}}=2.7, \quad \text{or} \quad V_2=2.7\ V_1=19.1 \text{ cu.ft.}$$

Prob. 1. A perfect gas is heated in such a way that the pressure is held constant. If the original volume was 10 cu.ft. and the temperature rose from 100° F. to 400° F., what was the new volume?

Prob. 2. The above gas was under a pressure of 100 lbs. per square inch gage at the beginning of the heating. If the volume had been held constant what would have been the pressure rise?

Prob. 3. A quantity of air, 5 lbs. in weight, was found to have a volume of 50 cu.ft. and a temperature of 60° F. What was the pressure?

Prob. 4. A cylinder holding 12 cu.ft. has a pressure of 250 lbs. per square inch gage, and the temperature is 50° F. What would be the weight of its contents were it filled with (*a*) CO_2; (*b*) NH_3; (*c*) Oxygen; (*d*) Hydrogen?

Prob. 5. At a pressure of 14.696 lbs. per square inch and a temperature of melting ice, one pound of air has a volume of 12.387 cu.ft. From the data find the value of R for air. The specific heats of air are given by one authority as $C_p = .2375$ and $C_v = .1685$. Find R from the data and see how the two values obtained compare.

Prob. 6. At the same pressure and temperature as in Prob. 5, a pound of the following substances has a volume as shown. From the data and the values of specific heats, find R by the two methods.

Substance.	Cu.ft. per lb.	C_p.	C_v.
Hydrogen.....	178.93	3.409	2.412
Carbon dioxide....	8.15	.217	.1535
Oxygen.....	11.21	.2175	.1551
Nitrogen.....	12.77	.2438	.1727

Prob. 7. 5 cu.ft. of gas at a pressure of 3 atmospheres absolute and a temperature of 50° F. expand to atmospheric pressure. What will be the final volume and temperature, if $s = 1.35$?

Prob. 8. 1000 cu.ft. of gas at atmospheric pressure and 60° F. are compressed into a tank of 100 cu.ft. capacity. What will be the pressure in the tank and the temperature of the gas at the end of the process, if the gas is CO_2 and the compression adiabatic?

Prob. 9. What will be the final volume, pressure and temperature, if a pound of air at atmospheric pressure (14.7 lbs. per square inch) and a temperature of 60° F. be compressed adiabatically until its absolute temperature is six times its original value?

8. Gas Density and Specific Volume and its Relation to Molecular Weight and Gas Constant. The density of a gas is best stated for engineering purposes as the weight of a cubic foot, but as this becomes less on rise of temperature or decrease of pressure it is necessary to fix a standard condition for reporting this important physical constant. It is best to take one atmosphere 760 mm. or 29.92 ins. of mercury as the pressure, and 0° C. = 32° F. as the standard temperature, though it is in some places customary in dealing with commercial gases, such for example as those used for illumination, to take the temperature at 60° F. and illuminating gas at this condition is often known among gas men as standard gas. In this work, however, the freezing-point and standard atmosphere will be understood where not specifically mentioned as the conditions for reporting gas density and its reciprocal, the specific volume of gases or the cubic feet per pound. The chart, Fig. 134, shows the relation of volume and density at any pressure and temperature to the volume and density under standard conditions.

These constants have been pretty accurately determined by many investigators, whose figures, to be sure, do not agree absolutely, as is always the case in experimental work, but the disagreement is found only in the last significant figures. Some selected values of reliable origin are reported at the end of this Chapter in Table XLI for the important gases and these may be used in computation work.

It often happens in dealing with gases and especially superheated vapors that a value is needed for which no determination is available, so that general

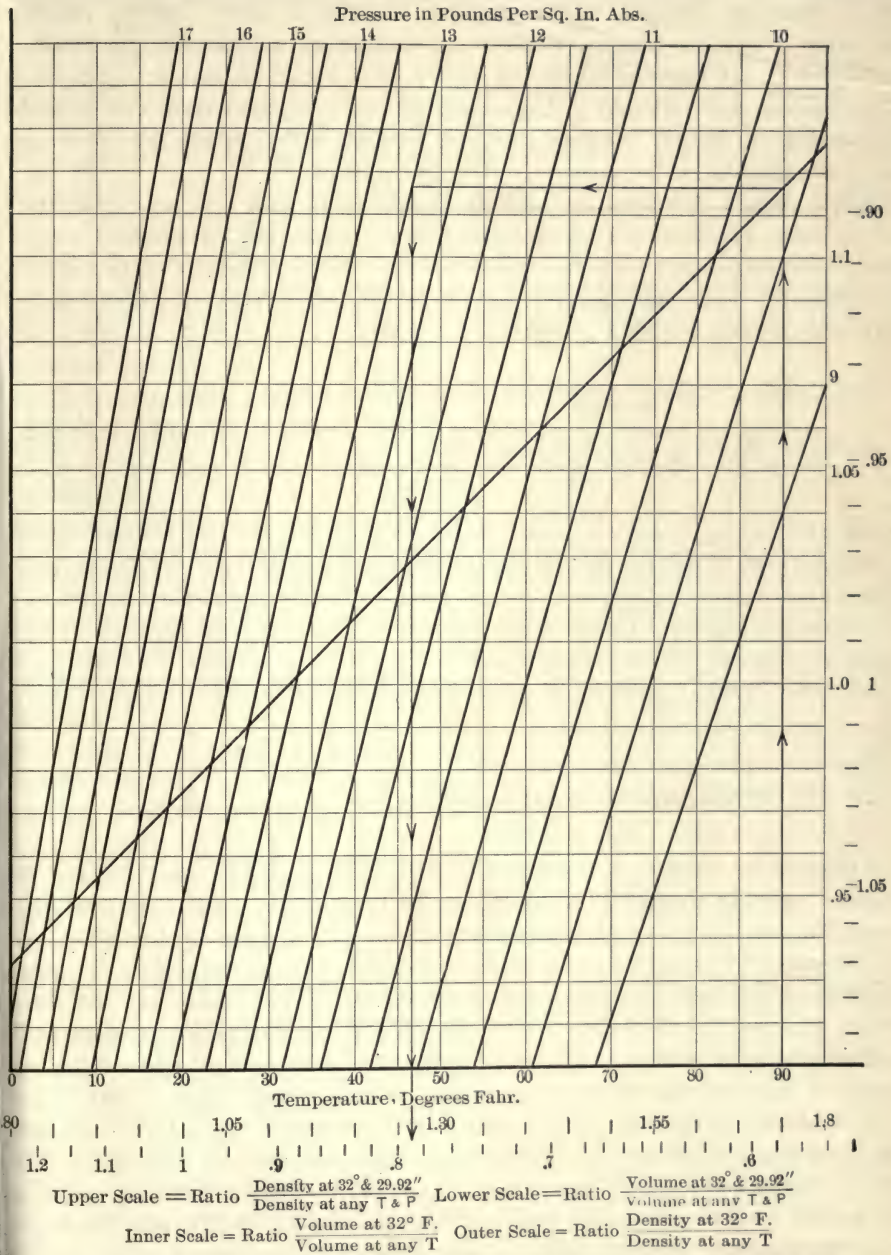


FIG. 134.—Equivalent Gas Densities At Different Pressures and Temperatures.

ws of density or specific volumes of substances are necessary to permit the eeded constant to be estimated. These relations may be applied to vapors

as well as to gases even though the standard conditions are those for the liquid state, on the assumption that all gases and vapors will expand under temperature, or contract under pressure rise, to the same degree, retaining the same relative relations between all substances as exist at the standard atmosphere and freezing-point. A vapor thus reported below its point of condensation and assumed to have reached that condition from one of higher temperature at which it exists as vapor is often called steam gas, or alcohol gas, for example in the case of water and alcohol.

Such general relations between the densities of gases as are so desirable and useful in practical work have been found by studying the manner in which gases chemically combine with respect to the volume relations before and after the reaction. Following several experimenters, who reported observed relations, Gay-Lussac stated a general law, as follows:

> *When two or more gaseous substances combine to form a compound, the volumes of the combining gases bear a simple ratio to each other and also to that of the compound when it is also a gas.*

He also attempted to derive some relation between this law and Dalton's atomic combining law, which states that, in combining chemically, a simple numerical relation exists between the number of atoms of different elements which unite to form a compound. This was not successful, but Avagadro later found the expected relation by assuming that it is a particle, or a number of atoms, or a molecule, that is important in combining, and the law stated is as follows:

> *Equal volumes of different gases measured at the same pressure and temperature contain the same number of molecules.*

It is possible by analysis of these two laws to get a relation between the volume of gases and the weights of their molecules because the molecular relation o Avagadro, combines with the combining law of Gay-Lussac to define the rela tion between the number of combining molecules. At the same time the weigh relations in chemical reactions, based on atomic weights, may be put into similar molecular form, since the weight of any one substance entering is th product of the number of its molecules present and the weight of the molecul Applying the relation between the number of molecules derived previously there is fixed a significance for the weight of the molecule which for simple gase like hydrogen and oxygen is twice the atomic weight and for compound gase like methane and carbon dioxide, is equal to the atomic weight. Applyin this to the Avagadro law, the weights of equal volumes of different gases mu be proportional to their molecular weights, as equal volumes of all contain t same number of molecules.

Putting this in symbolic form and comparing any gas with hydrogen, as its density, because it is the lightest gas of all and has well determined chara

teristics, requires the following symbols, denoting hydrogen values by the subscript H.

Let m = molecular weight of a gas,

" δ = density in lbs. per cu.ft. $= \frac{1}{V}$,

then

$$\frac{\delta_1}{\delta_2} = \frac{m_1}{m_2}, \quad \ldots \ldots \ldots \quad (646)$$

and

$$\frac{\delta_1}{\delta_H} = \frac{m_1}{m_H}. \quad \ldots \ldots \ldots \quad (647)$$

But as the molecular weight of hydrogen is for engineering purposes equal to 2 closely enough and hydrogen weighs .00562 lb. per cu.ft. $= \delta_H$, at 32° F and 29.92 ins. Hg,

$$\text{Lbs. per cu.ft.} = \delta_1 = .00281\ m_1 \quad \ldots \ldots \quad (648)$$

To permit of evaluation of Eq. (648) it is necessary that there be available a table of molecular weights of gases and the atomic weights of elements from which they are derived, and the values given at the end of this Chapter in Table XLII are derived from the international table. As atomic weights are purely relative they may be worked out on the basis of any one as unity, and originally chemists used hydrogen as unity, but for good reasons that are of no importance here, the custom has changed to $\frac{1}{16}$ the value for oxygen as unity. These atomic weights are not whole numbers but nearly so, therefore, for convenience and sufficient accuracy the nearest whole number will be used in this work and hydrogen be taken as unity except where experience shows it to be undesirable.

The reciprocal expression to Eq. (648) can be set down, giving the specific volume of a gas or its cubic feet per pound at 32° F. and 29.92 ins. Hg., as follows:

$$\text{Cu.ft. per lb.} = \frac{1}{\delta_1} = \frac{1}{.00281} m_1 = \frac{355.87}{m_1}. \quad \ldots \ldots \quad (649)$$

This is a most important and useful conclusion as applied to gases and vapors for which no better values are available, and in words it may be stated as follows:

The cubic feet per pound of any gas or vapor at 32° and 29.92 ins. Hg, is equal to 355.87 divided by its molecular weight,

or

The molecular weight of any gas or vapor in pounds, will occupy a volume of 355.87 cu.ft. at 32° and 29.92 ins. Hg.

The approach to truth of these general laws is measured by the values given for specific volume and density at the end of this Chapter (*a*) experimentally derived and, (*b*) as derived from the hydrogen value by the law.

Another and very useful relation of a similar nature is derivable from what has been established, connecting the gas constant R with molecular weights. The general law $PV=wRT$ when put in the density form by making $\delta=\frac{w}{V}$ becomes

$$\frac{P}{\delta}=RT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (650)$$

Whence, comparing gases with each other and with hydrogen at the same pressure and temperature

$$\frac{\frac{P_1}{\delta_1}}{\frac{P_2}{\delta_2}}=\frac{\delta_2}{\delta_1}=\frac{R_1}{R_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (651)$$

$$\frac{\delta_1}{\delta_H}=\frac{R_H}{R_1}, \quad \text{and} \quad \delta_1=\frac{\delta_H R_H}{R_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (652)$$

which indicate that the *densities of gases are inversely proportional to the gas constants*, or the density of any gas is equal to the density of hydrogen times the gas constant for hydrogen divided by its own.

Inserting the values of density at 32° and 29.92 ins. Hg and of the gas constant for hydrogen, it follows that for any gas

$$\text{Lbs. per cu.ft.}=\delta_1=\frac{4.2998}{R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (653)$$

the reciprocal of which gives the specific volume at 32° F. and 29.92 ins. Hg, or

$$\text{Cu.ft. per lb.}=V_1=\frac{R}{4.2998}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (654)$$

Example. 1. Explanation and use of Chart, Fig. 134. This diagram is for the purpose of finding the cubic feet per pound, or pounds per cubic foot, of a gas at 32° F. and a pressure of 29.92 ins. of Hg, if its volume or weight per cubic foot be known at any pressure and temperature. The curves depend upon the fact that the pounds per cubic foot (δ) vary directly as the pressure and inversely as the temperature. That is

$$\delta_{32°,\ 29.92''}=\delta_{TP}\frac{T}{492}\frac{29.92}{P}.$$

The line of least slope is so drawn that for any temperature on the horizontal scale its value when divided by 492 may be read on the vertical scale. The group of lines with the greater slope is so drawn that for any value on the vertical scale this quantity times $\frac{29.92}{P}$ may be used on the horizontal scale. That is, the vertical scale gives the

atio of densities as affected by temperature for constant pressure, while horizontal cale gives the ratio as affected by both temperature and pressure. A reciprocal cale is given in each case for volume calculations.

To find the pounds per cubic foot of gas at 32° F. and 29.92 ins. of mercury when s value is known for 90° and 13 lbs. per sq.in. On the temperature scale, pass ertically until the temperature line is reached, then horizontally until the curve or 13 lbs. absolute is reached. The value on the scale below is found to be 1.265, o that the density under the standard conditions is 1.265 of the value under known onditions. Had it been required to find the cubic feet per pound the process would be recisely the same, the value being taken from the lower scale, which for the example eads .79, or, the cubic feet per pound under standard conditions is 79 per cent of he value under conditions assumed.

Example 2. By means of the molecular weight find the density of nitrogen at 2° F. and 29.92 ins. Hg, and the cubic feet per pound for these conditions.

From Eq. (646)

$$\frac{\delta_1}{\delta_H} = \frac{m_1}{m_H}, \quad \text{or} \quad \delta_1 = \frac{28 \times .00562}{2}.$$

Hence δ for nitrogen = .07868 pounds per cu.ft. and,

$$\frac{1}{\delta} = \frac{1}{.07868} = 12.709. \text{ cu.ft. per lb.}$$

Prob. 1. Taking the density of air from the table, find the value of R for air, by neans of Eq. (653) and compare its value with that found in Section 7.

Prob. 2. Compare the density of carbon monoxide when referred to 32° F. and 60° F. as the standard temperature, as found both ways.

Prob. 3. By means of their molecular weights find the density of oxygen, nitrogen nd carbon dioxide at 32° F. and 29.92 ins. Hg.

Prob. 4. What are the cubic feet per pound of acetylene, methane and ammonia t 32° F. and 29.92 ins. Hg?

Prob. 5. An authority gives the following values for R. Compare the densities ound by this means with the densities for the same substance found by the use of he molecular weights.

Oxygen	48.1
Hydrogen	764.0
Carbon monoxide	55.0

Prob. 6. What will be the volume and density under standard conditions, of a as which contains 12 cu.ft. per pound at a temperature of 70° F. and a pressure of 6 lbs. per square inch absolute?

Prob. 7. What will be the difference in volume and density of a gas when con-idered at 60° and 29.92 ins. of Hg, and at 32° F. and 29.92 ins. of Hg?

9. Pressure and Temperature Relations for Vapor of Liquids or Solids. Vaporization, Sublimation and Fusion Curves. Boiling- and Freezing-points or Pure Liquids and Dilute Solutions. Saturated and Superheated Vapors. Substances may exist in one of three states, solid, liquid or gas, the latter being enerally called vapor when, at ordinary temperatures the common state is that

of liquid or solid, or when the substance examined is near the point of liquefaction or condensation, and just which state shall prevail at any time depends on thermal conditions. Within the same space the substance may exist in two of these three states or even all three at the same time under certain special conditions. These conditions may be such as to gradually or rapidly make that part in one state, turn in to another state, or may be such as to maintain the relative amounts of the substance in each state constant; conditions of the latter sort are known as conditions of equilibrium. These are experimental conclusions, but as in other cases they have been concentrated into general laws of which they are but special cases. The study of the conditions of equilibrium, whether of physical state or chemical constitution, is the principal function of physical chemistry, in the pursuit of which the Gibbs phase rule is a controlling principle. According to this rule each possible state is called a phase, and the number of variables that determine which phase shall prevail or how many phases may exist at the same time in equilibrium for one chemical substance like water, is given by the following relation, which is but one of the conclusions of this general principle of equilibrium.

Number of undefined variables = 3 − number of phases.

Now it is experimentally known that if water be introduced into a vacuum chamber some of it will evaporate to vapor and that, therefore, water and its vapor may coexist or the number of phases is two, but this does not state how or when equilibrium will be attained. The rule above, however, indicates that for this case there can be but one undefined or independent variable and, of course, since the pressure rises more when the temperature is high than when low, the two variables are pressure and temperature, of which accordingly only one is free or independent, so that fixing one fixes the other. In other words, when a vapor and its liquid are together the former will condense or the latter evaporate until either pressure or temperature is fixed, and fixing the one the other cannot change, so that the conditions of equilibrium are indicated by a curve to coordinates P and T, on one side of which is the vapor state and on the other that of liquid. Such a curve is the *vapor pressure-temperature curve* of the substance, sometimes called its *vapor tension curve*, and much experimental information exists on this physical property of substances, all obtained by direct measurement of simultaneous pressures and temperatures of a vapor above its liquid, carefully controlled so that the pressure or the temperature is at any time uniform throughout.

The conditions of equilibrium between vapor and liquid, defined by the vapor tension curve extend for each substance over a considerable range of pressure and temperature, but not indefinitely, nor is the range the same for each. At the high-pressure and temperature end a peculiar interruption takes place due to the expansive effect of the temperature on the liquid and the compressive effect of the pressure on the vapor, the former making liquid less dense and the latter making vapor more dense, the two densities become equal at some pressure and temperature. The point at which this occurs is the "*critical point*" at which the equilibrium between liquid and vapor that previously existed

nds and there is no longer any difference between vapor and liquid. This oint is a most important one in any discussion of the properties of matter, nd while difficult to exactly locate, has received much experimental attention, nd some of the best values are given below in Table XXI for the pressure, ensity, and temperature defining it, for the substances important in engineering

TABLE XXI

THE CRITICAL POINT

Substance.	Symbol.	Critical Temp.		Critical Pressures.		Critical Density Water at 4°C = 1.	Authority.	Critical vol. Cu.ft. per Lb.	Authority.
		0° C.	0° F.	Atm.	Lbs. per Sq.in.				
Hydrogen.....	H_2	−243.5	−390.1	20	294	.....	Olszewski		
Oxygen.....	O_2	−118.[1]	−180.4	50[1]	735	.65[2]	[1] Wroblewski [2] Dewar		
Nitrogen.....	N_2	−146.[1]	−232.8	35.[1]	515	.44[2]	[1] Olszewski [2] Wroblewski		
Ammonia.....	NH_3	+130.0	266.	115.	1690	.....	Dewar		
Ammonia.....	NH_3	+131.0	267.8	113.	1660	.....	Vincent and Chappuis		
Carbon dioxide...	CO_2	+ 31.35	88.43	72.9	1070	.464	Amagat		
Carbon dioxide...	CO_2	+ 30.92[1]	87.67	77.[1]	1130	.45[2]	[1] Andrews [2] Cailletet and Mathias		
Water.....	H_2O	+358.1	676.4	.....	....	.429	Nadejdini		
Water.....	H_2O	+364.3	687.7	194.61	2859	.....	Batteli	26.8	Nadejdini
Water.....	H_2O	+365.0	689.	200.5	2944	.....	Cailletet and Colardeau	13.	Batteli
Water.....	H_2O	+374.	705.2	.....	....	.....	Traube and Teichner		
Water.....	H_2O	+374.6	706.3	.....	3200	.....	Holborn and Baumann		
Water.....	H_2O	+374.5	706.1	.....	3200	.....	Marks		

To illustrate this discussion there is presented the vapor tension curves of ater, ammonia and carbon dioxide to a large scale in chart form derived om the tabular data both at the end of this Chapter, while a small scale dia- ram for water is given in Fig. 135. These data are partly direct experimental eterminations and partly corrections obtained by passing a smooth urve representing an empiric equation of relation between pressure and tem- erature, through the major part of the more reliable experimental points. hese pressure-temperature points are very accurately located for water, the rst good determinations having been made by Regnault in 1862 and the last y Holborn and Baumann of the German Bureau of Standards in the last year. he data presented are those of Regnault corrected by various investigations y means of curve plotting, and empiric equations by Wiebe, Thiessen and chule, and those of various later observers, including Battelli, Holborn, Hen- ing, Baumann, Ramsay and Young, Cailletet and Colardeau, somes eparately, ut all together as unified by Marks and Davis in their most excellent steam

tables, and later by Marks alone for the highest temperatures 400° F. to the critical point, which he accepts as being located at 706.1° F. and 3200 lbs. square

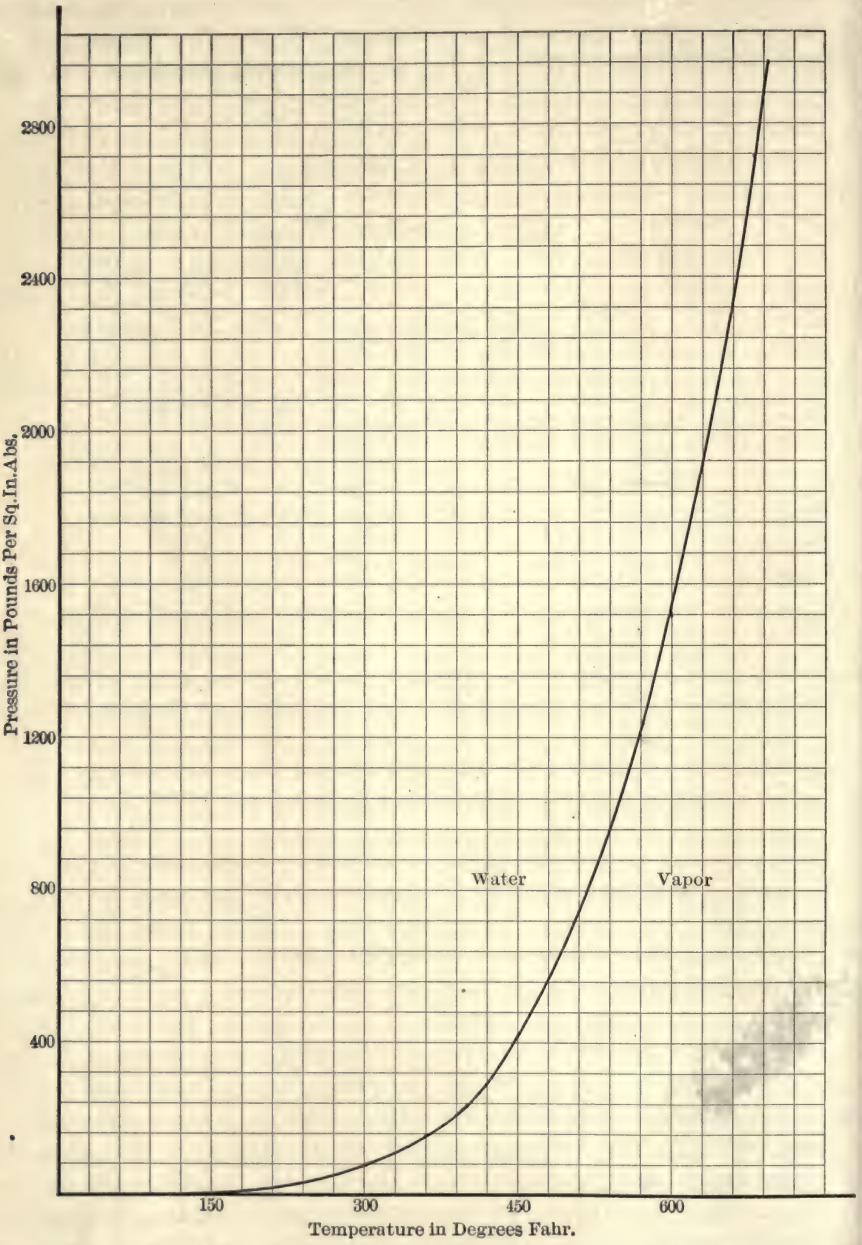


FIG. 135.—Vapor of Water, Pressure-temperature Curve over Liquid (Water).

inch. In calculations the values of Marks and Davis, and Marks, will b accepted and used.

Carbon dioxide and ammonia are by no means as well known as steam, and the original data plotted, while representing the best values obtainable, must be accepted with some uncertainty. A smooth curve Figs. (139) and (140) has been drawn for each through the points at locations that seem most fair, for both these substances and the values obtained from it are to be used in calculations; these curves have been located by the same method as used by Marks in his recent paper and described herein later. The equalized values are given in the separate table at the end of the Chapter with others for latent

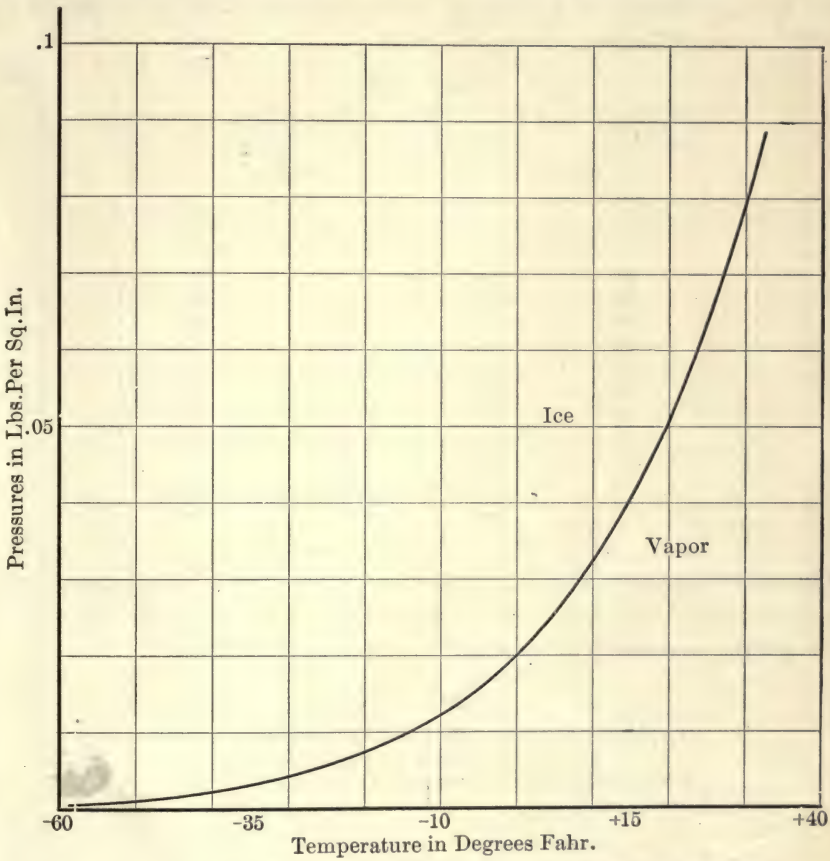


FIG. 136.—Vapor of Water, Pressure-temperature Curve over Solid (Ice).

heats and volumes, but while consistent each with the other are probably but little more correct than values reported by others which are inconsistent.

The curves and the equivalent tabular data are most useful in practical work, as they indicate the temperature at which the vapor exists for a given pressure, either as formed during evaporation or as disappearing during condensation, or the other way round, they indicate the pressure which must be maintained to evaporate or condense at a given temperature.

Just as the vapor-liquid curves indicate the conditions of equilibrium between

vapor and its liquid, dividing the two states and fixing the transition pressure or temperature from one to the other, so also does a similar situation exist with respect to the vapor-solid relations. In this case the curve is that of "*sublimation*" and indicates the pressure that will be developed above the solid by direct vaporization at a given temperature in a closed chamber. In Fig. 136 is plotted a curve of sublimation of vapor-ice, based on Juhlin's data, Table XXII, which indicates that the line divides the states of ice from that of vapor so that at a constant pressure, decrease of temperature will cause vapor to pass directly to ice and at constant temperature a lowering of pressure will cause ice to pass directly to vapor.

TABLE XXII

JUHLIN'S DATA ON VAPOR PRESSURE OF ICE

Temperature.		Pressure.	
C.	F.	Min. Hg.	Lbs. sq.in.
−50	−58.	.050	.001
−40	−40.	.121	.0023
−30	−22.	.312	.006
−20	− 4.	.806	.0156
−15	5.	1.279	.0247
−10	14.	1.999	.0386
− 8	17.6	2.379	.0459
− 6	21.2	2.821	.0544
− 4	24.8	3.334	.0643
− 2	28.4	3.925	.0758
− 0	32.	4.602	.0888

Likewise the liquid, water, may pass to the solid, ice, by lowering temperature at a fixed pressure as indicated in Fig. 137, plotted from data by Tamman, Table XXIII, and which becomes then the curve of "*fusion.*"

TABLE XXIII

TAMMAN'S DATA ON FUSION PRESSURE AND TEMPERATURE OF WATER-ICE

Temperature.		Pressure.	
C.	F.	Kg. sq.cm.	Lbs. sq.in.
0	32.	1	1423
− 2.5	27.5	336	4779.
− 5.	23.	615	8747.4
− 7.5	18.5	890	13658.8
−10.0	14.	1155	16428.
−12.5	9.5	1410	20055.
−15.	5.	1625	23113.
−17.5	.5	1835	26100.
−20.	− 4.	2042	27044.
−22.1	− 7.8	2200	31291.

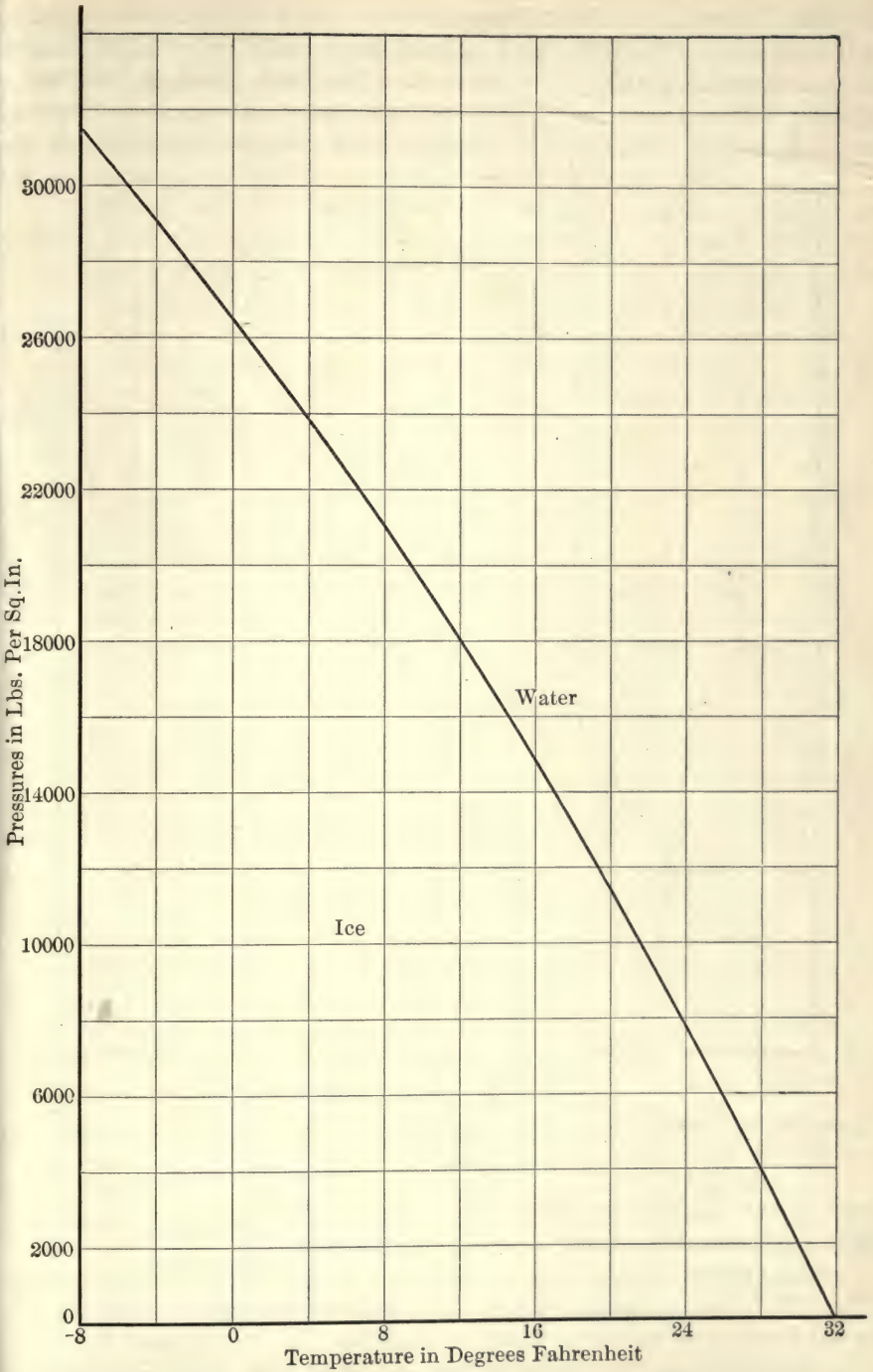


FIG. 137.—Water—Ice, Pressure-temperature Curve.

These three curves plotted to the same scale meet at a point located at a pressure of 4.6 mm. Hg. = .18 ins. Hg., and temperature +.0076° C. = 32.01° F., ordinarily taken at 32° F., which point is named the triple point, as indicated in Fig. 138. The fact that the vapor pressure for water extends below freezing-point and parallels more or less that of ice indicates the condition of supercooled

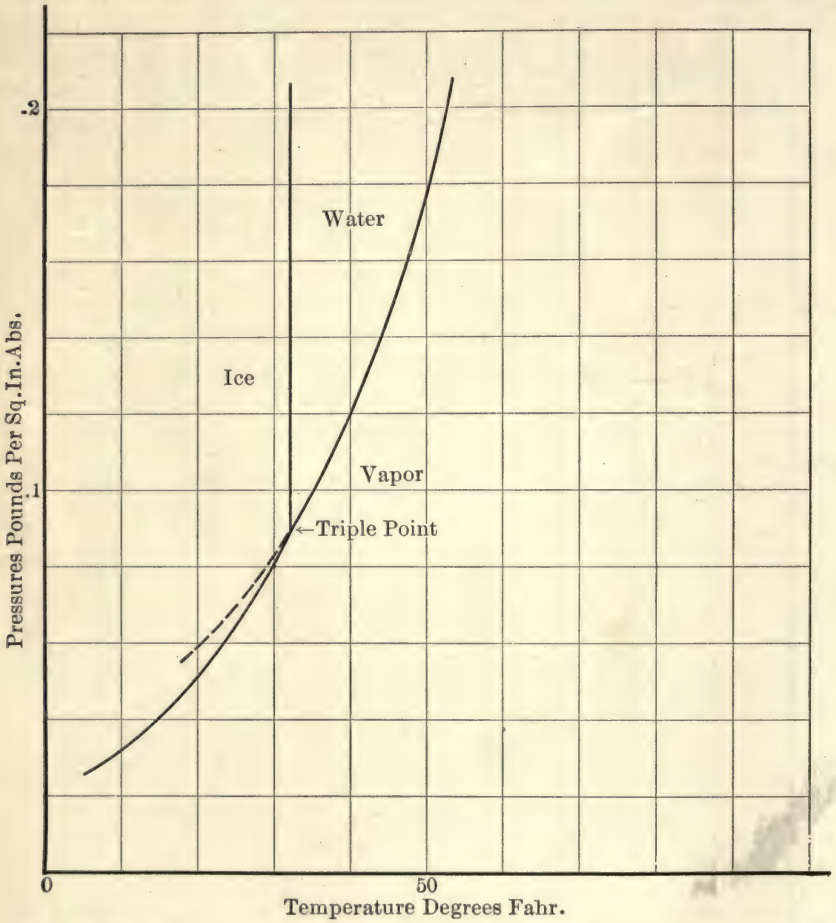


FIG. 138.—Water Vapor—Water—Ice, Combined Curves of Pressure-temperature Relation. The *Triple Point.*

water, one of unstable equilibrium instantly dispelled by the introduction of a little ice at the proper stable state for this temperature.

Ordinary engineering work is not concerned with the entire range indicated in Fig. 138 for any substance, but with the higher temperature ranges for some and the low for others, with transition from solid to liquid state for metals and similar solids and the transition from liquid to vapor for a great many, of which water comes first in importance, then the refrigerating fluids, ammonia

and carbon dioxide, and last certain fuels like alcohol and the petroleum oils with their distillates and derivatives.

Melting-points, or the fusion temperature of such solids as are important, are usually given for only one pressure, the standard atmosphere, as in ordinary practice these substances are melted only at atmosphere pressure, and some such values are given at the end of the Chapter in Table XLIII.

This is not the case, however, for *boiling-points*, which must be defined a little more closely before discussion. The vapor pressure curves indicate that as the temperature of a liquid rises, the pressure rises also if the substance is enclosed, but if the pressure were relieved by opening the chamber to a region of lower pressure and kept constant, then the temperature would no longer rise and boiling or ebullition would take place. The boiling-point then is the highest temperature to which the liquid and its vapor could rise under the existing pressure. When not otherwise defined the term boiling-point must be taken to mean the temperature of ebullition for atmospheric pressure of 29.92 ins. Hg, and values for several substances are given at the end of this Chapter in Table XLIV.

Vapor having the temperature required by the pressure of the pressure-temperature curve is known as *saturated vapor*, and this may be defined as vapor having the lowest temperature at which it could exist as vapor, under the given pressure. Vapors may, however, be superheated, that is, have higher temperatures than saturated vapors at the same pressure, but cannot so exist for long in the presence of liquid. Superheating of vapors, therefore, implies isolation from the liquid, and the *amount* of *superheat* is the number of degrees excess of temperature possessed by the vapor over the saturation temperature for the pressure. In steam power plant work, especially with turbines, it is now customary to use steam with from 75° F. to 150° F. of superheat, and it might be noted that all so-called gases like oxygen and nitrogen are but superheated vapors with a great amount of superheat.

It has already been mentioned that the saturated vapor pressure-temperature curve of direct experiment is seldom accurate as found, but must be corrected by empiric equations or smooth average curves, and many investigators have sought algebraic expressions for them. These equations are quite useful also in another way, since they permit of more exact evaluation of the rate of change of pressure with temperature, which in the form of a differential coefficient is found to be a factor in other physical constants. One of these formulas for steam as adopted by Marks and Davis in the calculation of their tables is given in Eq. (655), the form of which, was suggested by Thiessen:

$$(t+459.6)\log\frac{p}{14.7}=5.409(t-212°)-3.71\times10^{-10}[(689-t)^4-477^4], \quad (655)$$

in which t = temperature F.; and p = pressure lbs. sq.in.

This represents the truth to within a small fraction of one per cent up to 400° F., but having been found inaccurate above that point Professor Marks has

very recently developed a new one, based on Holborn and Baumann's high temperature measurements, which fits the entire range, its agreement with the new data being one-tenth of 1 per cent, and with the old below 400° F., about one-fifth of 1 per cent, maximum mean error. It appears to be the best ever found and in developing it the methods of the physical chemists have been followed, according to which a pressure is expressed as a fraction of the critical pressure and a temperature a fraction of the critical temperature. This gives a relation between *reduced* pressures and temperatures and makes use of the principle of *corresponding states* according to which bodies having the same reduced pressure and temperature, or existing at the same fraction of their critical are said to be in equivalent states. The new Marks formula is given in Eqs. (656) and (657), the former containing symbols for the critical $\left\{\begin{matrix}\text{pressure } p_c \\ \text{temperature } T_c \text{ abs.}\end{matrix}\right\}$ and the latter giving to them their numerical values, in pressure pounds per square inch, and temperature absolute F.

$$\log \frac{p_c}{p} = 3.006854\left(\frac{T_c}{T} - 1\right)\left[1 + .0505476\frac{T}{T_c} + .629547\left(\frac{T}{T_c} - .7875\right)^2\right], \quad (656)$$

$$\log p = 10.515354 - 4873.71T^{-1} - .00405096T + .000001392964T^2. \quad (657)$$

As the method used in arriving at this formula is so rational and scientific, it has been adopted for a new determination, from old data to be sure, of the relations between p and T for ammonia and carbon dioxide, so important as substances in refrigeration, especially the former. According to this method if p_c and T_c are the critical pressures and temperatures, both absolute, and p and T those corresponding to any other point, then according to Van der Waals,

$$\log \frac{p_c}{p} = f\left(\frac{T_c}{T} - 1\right). \quad (658)$$

Accordingly, the logarithm of the critical divided by any other pressure, is to be plotted against the quantity [(critical temperatur edivided by the temperature corresponding to the pressure) − 1], and the form of curve permits of the determination of the function, after which the values of the critical point are inserted. This has been done for NH_3 and CO_2 with the result for NH_3

$$\log \frac{p_c}{p} = .045 + 2.75\left(\frac{T_c}{T} - 1\right) + .325\left(\frac{T_c}{T} - 1\right)^2, \quad (659)$$

which on inserting the critical constants,

$\left.\begin{matrix}p_c = 114 \text{ atm.} = 1675.8 \text{ lbs. per square inch} \\ T_c = 727.4° \text{ F. absolute}\end{matrix}\right\}$ which are the Vincent and Chappuis values,

becomes,

$$\log p = 5.60422 - 1527.54T^{-1} - 171961T^{-2} \quad . \quad . \quad . \quad . \quad (660)$$

For CO_2 it was found that

$$\log \frac{p_c}{p} = .038 + 2.65\left(\frac{T_c}{T} - 1\right) + 1.8\left(\frac{T_c}{T} - 1\right)^3, \quad . \quad . \quad . \quad . \quad (661)$$

which on inserting the critical constants,

$p_c = 77$ atm. $= 1131.9$ lbs. per square inch
$T_c = 547.27°$ F. abs.
} which are Andrews' values,

becomes,

$$\log p = 7.46581 - 4405.765T^{-1} + 1617501.366T^{-2} - 257086165.8706T^{-3}. \quad (662)$$

Curves showing the relation of reduced and actual temperatures and pressures are given in Fig. 139 for ammonia and in Fig. 140 for carbon doixide.

For the past half century far more time and effort have been devoted to making other formulas of relation of p to T for saturated vapor not only for steam, but also for other vapors, than would have sufficed for accurate experimental determination, and as these help not at all they are omitted here. Equations of physical relations can be no better than the data on which they are based, and for the substances ammonia and carbonic acid the charts or formulas must be used with a good deal of suspicion.

In all engineering calculations requiring one of these constants even for steam no one is justified in using a formula; the nearest tabular or chart value must be employed and it will be as accurate as the work requires. Time is at least as important as accuracy, if not more so, for if too much time is required to make a calculation in commercial work, it will not be made because of the cost, indirect and approximate methods being substituted.

It is sometimes useful in checking the boiling-point of some substance little known, to employ a relation between boiling-points of different substances at the same pressure applied to a substance well-known.

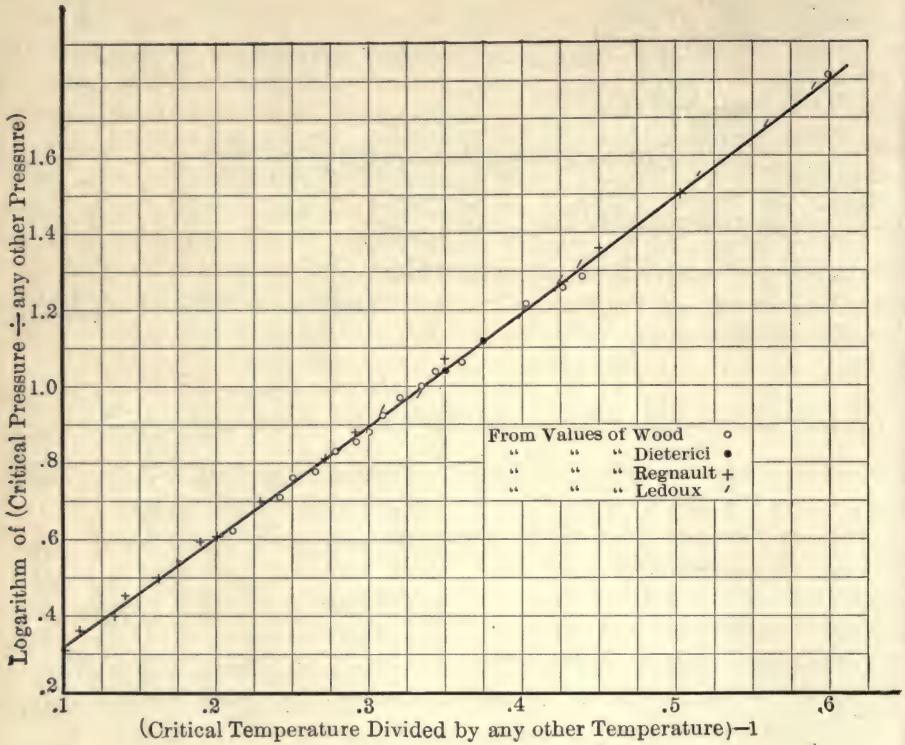
Let T_a and T_b be absolute temperatures of boiling for substances A and B under same pressure;

" T_a' and T_b' be absolute temperature of boiling for substances A and B under some other pressure.

Then,

$$\frac{T_a'}{T_b'} = \frac{T_a}{T_b} + c(T_b' - T_b). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (663)$$

Such equations as this are useful in finding the saturation curve of other substances from that for water, which is now so well established, when enough points are known for the other substances to establish the constant c. Also the ratio $\frac{T_a'}{T_b^1}$ plotted against the temperature difference $T_b' - T_b$ should give



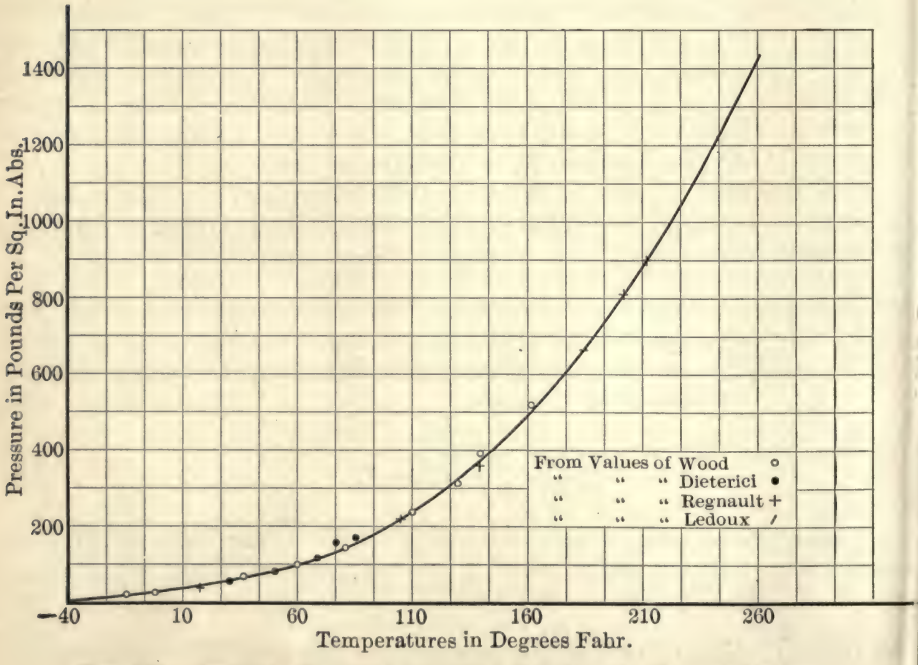
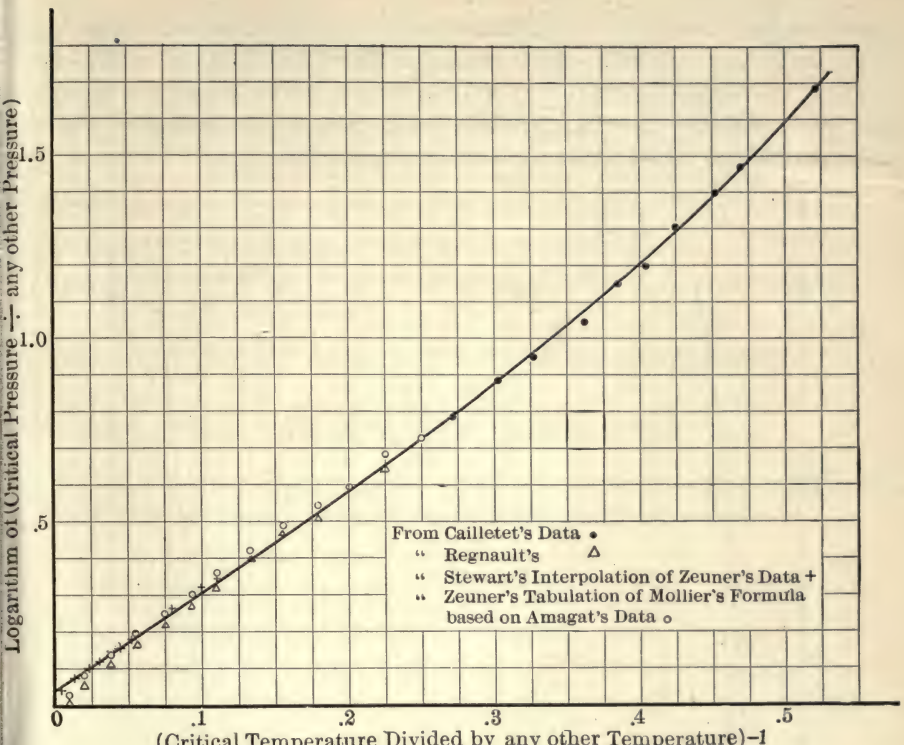
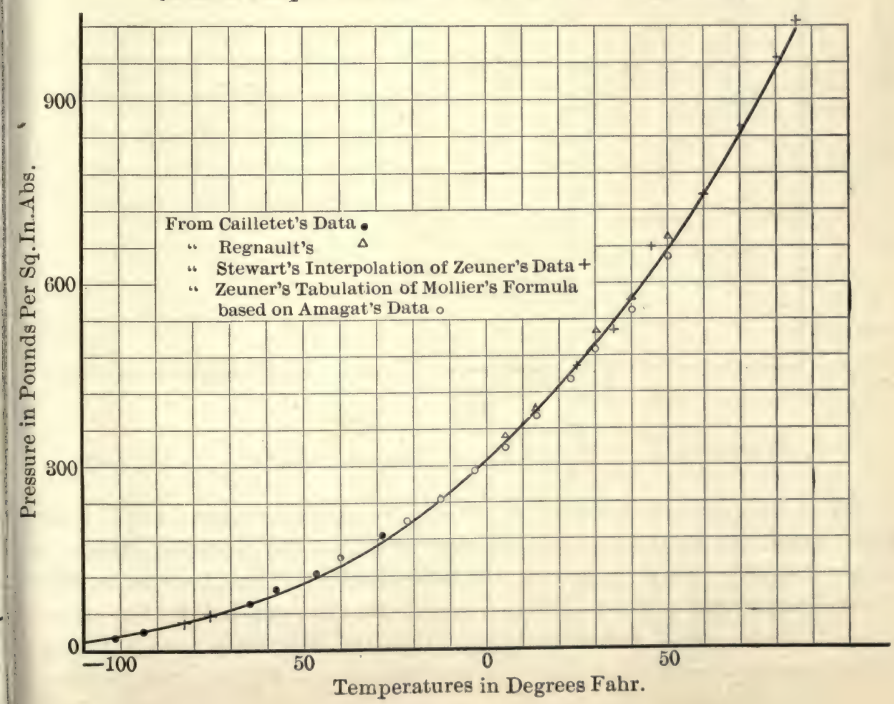


FIG. 139.—Ammonia Pressure-temperature Relations, for Saturated Vapor.


Logarithm of (Critical Pressure ÷ any other Pressure)
1.5
1.0
.5
0
.1
.2
.3
.4
.5
From Cailletet's Data •
" Regnault's △
" Stewart's Interpolation of Zeuner's Data +
" Zeuner's Tabulation of Mollier's Formula based on Amagat's Data o
(Critical Temperature Divided by any other Temperature)−1


Pressure in Pounds Per Sq. In. Abs.
900
600
300
0
From Cailletet's Data •
" Regnault's △
" Stewart's Interpolation of Zeuner's Data +
" Zeuner's Tabulation of Mollier's Formula based on Amagat's Data o
−100
50
0
50
Temperatures in Degrees Fahr.

a straight line, and if the line is not straight the experimental values may b wrong or the law untrue. This procedure has been followed in Fig. 141, i checking the curves for CO_2 and NH_3 against those for water, but it is impos sible to say whether the discrepancies for CO_2 are due to a failure of the law o bad experimental values, probably both, as the law holds poorly for water itself.

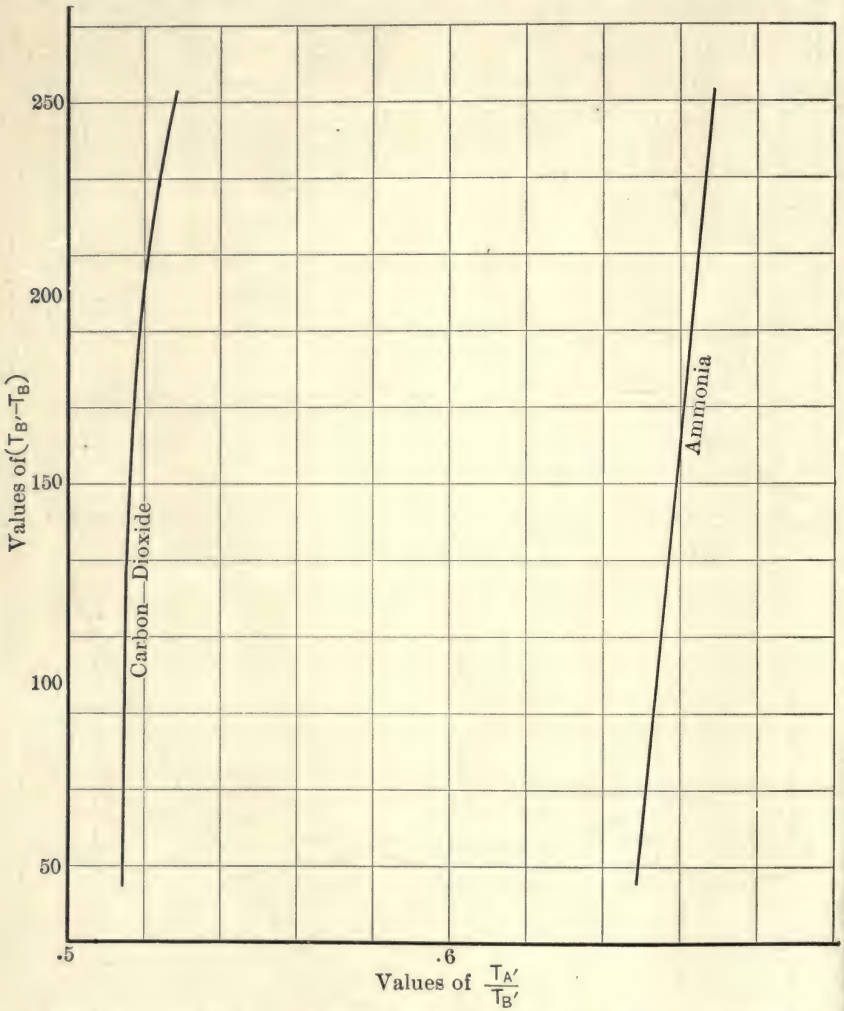


FIG. 141.—Curves for CO_2 and NH_3 to Check the Linear Relation Eq. (663).

All of the preceding refers, of course, to pure substances, but in practi work there are frequently encountered problems on solutions where la differences may exist compared to the pure liquids. Thus, for salts in water is well known that addition of a salt lowers the freezing-point, that more s lowers it more, and it was first thought that the depression was in proporti to the amount dissolved. This being found to be untrue, recourse was h

again to molecular relations by Raoult, who announced the general law that *the molecular depression of the freezing-point is a constant.*

$$\text{Molecular lowering of freezing-point } E' = \frac{\Delta t m}{w} = \text{const.}, \quad . \quad . \quad (664)$$

in which

Δt = depression of freezing-point in degrees F.;
w = weight dissolved in 100 parts of solvent;
m = molecular weight of substance dissolved.

From Eq. (664) the freezing-point for brines may be found as follows:

$$\text{Freezing-point of aqueous solutions} = 32° - (\text{const.}) \times \frac{w}{m} \quad . \quad . \quad . \quad (665)$$

As examples of the degree of constancy of the " constant " the following values Table XXIV, taken from Smithsonian Tables are given:

TABLE XXIV
LOWERING OF FREEZING POINTS

Salt.	g. Mol. / 1000 g. H_2O	Molecular Lowering.	Authorities.
NaCl	.004	3.7	
	.01	3.67	
	.022	3.55	Jones
	.049	3.51	Loomis
	.108	3.48	Abegg
	.232	3.42	Roozeboon
	.429	3.37	
	.7	3.43	
NH_4Cl	.01	3.6	
	.02	3.56	
	.035	3.5	
	.1	3.43	Loomis
	.2	3.4	
	.4	3.39	
$CaCl_2$	.01	5.1	
	.05	4.85	
	.1	4.79	
	.508	5.33	
	.946	5.3	Arrhenius
	2.432	8.2	Jones-Getman
	3.469	11.5	Jones-Chambers
	3.829	14.4	Loomis
	.048	5.2	Roozeboon
	.153	4.91	
	.331	5.15	
	.612	5.47	
	.788	6 34	

Just as the pressure of dissolved substances in liquids lowers the freezing-point, so also does it lower the vapor pressure at a given temperature or raise the boiling-point at a given pressure. Investigation shows that a similar formula expresses the general relation:

$$\text{Molecular rise of boiling-point} = E = \frac{\Delta t m}{w} = \text{constant} = 5.2, \quad . \quad (666)$$

when water is the solvent.

From Eq. (666) the rise of the boiling-point is found to be

$$\text{Rise of boiling-point} = 5.2\frac{m}{w}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (667)$$

When liquids are mixed, such as is the case with all fuel oils and with denatured alcohol, the situation is different than with salts in solution, and these cases fall into two separate classes: (*a*) liquids infinitely miscible like alcohol and water or like the various distillates of petroleum with each other, and (*b*) those not miscible, like gasolene and water.

The vapor pressure for miscible liquid mixtures is a function of the pressure of each separately and of the molecular per cent of one in the other when there are two. This rule, which can be symbolized, is no use in engineering work, because in those cases where such mixtures must be dealt with there will be generally more than two liquids, the vapor pressure characteristic and molecular per cent of each, or at least some of which will be unknown.

When, however, the two liquids in contact or in fact any number are non-miscible they behave in a very simple manner with respect to each other, in fact are quite independent in action. Each liquid will evaporate until its own vapor pressure is established for the temperature, as if the other were not there, and the vapor pressure for the mixture will be the sum of all the separate ones. On the other hand the boiling-point will be the temperature at which all the vapor pressures together make up the pressure of say the atmosphere, and this is necessarily lower than the highest and may be lower than the lowest value for a single constituent. This action plays a part in vaporizers and carburettors using alcohol and petroleum products. To permit of some approximations, however, a few vapor tension curves for hydrocarbons and alcohols are given later in the Section on vapor-gas mixtures, and data on the vapor pressure and temperature relations of ammonia-water solution are given in the section on the solution of gases in liquids.

Example 1. Through how many degrees has ammonia vapor at a pressure of 50 lbs. per square inch absolute been superheated, when it is at the temperature at which steam is formed under a pressure of 100 lbs. per square inch absolute?

From the curve of pressure and temperature of steam the temperature is 328° F. for the pressure of 100 lbs. From the similar curve for ammonia vaporization occurs under a pressure of 50 lbs. at a temperature of 22° F. Hence, superheat = 328 − 22 = 306°F.

Prob. 1. Three tanks contain the following liquids together: water, ammonia, and carbon dioxide respectively, and at a temperature of 30° F. What pressure exists in each tank? If the temperature rises to 70° F. how much will the pressure rise in each?

Prob. 2. The pressure exerted by water vapor in the atmosphere when saturated, is that due to the temperature and is independent of the pressure of the air. The total pressure read by a barometer is the sum of the air pressure and the water vapor pressure. What is the pressure due to each under a saturated condition for temperatures of 50° F., 100° F., 150° F., and 200° F., the barometer in each case being 29.92 inches of Hg?

Prob. 3. In order to secure a sufficiently high rate of heat transfer the steam in a radiator must be at a much higher temperature than the room to be warmed. If it is to be 150° above room temperature what must be its pressure for room temperatures of 50° F., 60° F., 70° F., 80° F., and 125° F.?

Prob. 4. In one type of ice machine ammonia gas is condensed at a high pressure and evaporated at a low pressure. What is the least pressure at which gas may be condensed with cooling water of 70° F., and what is the highest pressure which may be carried in the evaporating coils to maintain a temperature in them of 0° F.?

Prob. 5. Should carbon dioxide be substituted in the above machine what pressures would there be in the condensing coils, and in the evaporating coils?

Prob. 6. How many degrees of superheat have the vapors of water, ammonia and carbon dioxide at a pressure of 15 atmospheres and a temperature of 500° F.?

Prob. 7. Change the following pressures in pounds per square inch absolute to reduced pressures for water, ammonia, and carbon dioxide, 15 lbs., 50 lbs., 100 lbs., 500 lbs.

Prob. 8. At the temperature of melting ice what will be the vapor pressure of ammonia and carbon dioxide? At the temperature of melting tin what will be the pressure of water vapor? At this same temperature how many degrees of superheat would ammonia vapor under 100 lbs. pressure have, and how many degrees superheat would carbon dioxide vapor have under 1000 lbs. pressure?

Prob. 9. If 10 lbs. of common salt, NaCl, be dissolved in 100 lbs. of water, what will be the boiling point of the solution at atmospheric pressure, what the freezing-point?

10. Change of State with Amount of Heat at Constant Temperature. Latent Heats of Fusion and Vaporization. Total Heats of Vapors. Relation of Specific Volume of Liquid and of Vapor to the Latent Heat. As previously explained, a liquid boils or is converted into a vapor at constant temperature when the pressure on the surface is constant. Then *during the change of state the amount of heat added is indirect proportion to the amount of vapor formed.* The amount of vapor to convert a pound of liquid into vapor at any one steady temperature, is the *latent heat of vaporization* some values for which are given at the end of this chapter in Table XLV, and it must be understood that this latent heat is also the amount given up by the condensation of a pound of vapor. Latent heat is not the same for different pressures or temperatures of vaporization but is intimately associated with the volume change in the transition from the liquid to the vapor state. That this should be so, is clear on purely rational grounds because there is necessarily external mechanical work done

in converting the liquid to the vapor, since this is accompanied by a change of volume against the resisting pressure at which the conversion takes place. Thus, if

V_V = specific volume of the vapor in cubic feet per pound;
V_L = specific volume of the liquid in cubic feet per pound;
P = pressure of vaporization lbs. per sq.ft. absolute.

Then

$$\left\{\begin{array}{l}\text{Mechanical external work done during vaporization of 1 lb.}\end{array}\right\} = P(V_V - V_L) \text{ ft.lbs.} \quad . \quad . \quad . \quad . \quad (668)$$

Of course, at high temperatures the volume of a pound of liquid is greater than at low because of its expansion with temperature rise, and under the corresponding higher pressures the volume of a pound of vapor is less, because of the compressional effect of the pressure, than at low pressures, so that as pressures and temperatures rise the difference $V_V - V_L$ becomes less and disappears at the critical point where it is zero. The latent heat being thus associated with a factor that becomes less in the higher ranges of temperature and pressure may be expected, likewise to become less unless some other factor tends to increase. All the energy of vaporization making up the latent heat may be said to be used up in (*a*) doing external work as above, or (*b*) overcoming attraction of the molecules for each other. As at the critical point there is no molecular change and no external work, the latent heat becomes zero at this point.

This relation between latent heat and volume change was formulated by Clausius and Clapeyron, but Eq. (669) is generally known as Clapeyron's equation:

Let L = latent heat;
" J = mechanical equivalent of heat = 778, or better 777.52, in such cases as this;
" T = absolute temperature of vaporization;
" $\frac{dP}{dT}$ = rate of increase of vapor pressure per degree change of corresponding temperature.

Then

$$L = \frac{T}{777.52}\frac{dP}{dT}(V_V - V_L) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (656$$

This formula is used to calculate latent heat from the specific volumes of vapor and liquid and from the curvature of the saturation curve when they are known but as these volumes are especially difficult to measure, direct experimental determination of the latent heat should be depended upon to get numerical values wherever possible. The formula will then be useful for the inverse process of calculating specific volumes from latent heats or as a means of

checking experimental values of both, one against the other. It is, however, ust as useful to calculate latent heats from the specific volumes, and $\frac{dP}{dT}$ of the vapor curve, when the latent heats are less positively determined than he volumes or densities.

Another simpler relation of a similar general character exists and is useful n estimating latent heats approximately for some little known substances ike, for example, the liquid fuels, and in the use of which accurate physical lata are badly needed. Despretz announced that

$$\frac{L}{V_V - V_L}$$

s nearly constant for all substances, and this was simplified by Ramsay and Trouton on the assumption, first, that the volume of the liquid is very small t ordinary temperatures and may be neglected, in comparison with the volume f the vapor, and second, that the volume of the vapor is inversely proportional o the molecular weight m and directly proportinal to absolute temperatures o that (Trouton's law)

$$\left.\begin{aligned} m\frac{L}{T} &= \text{constant} = C \\ \text{or} \qquad L &= c\frac{T}{m} \end{aligned}\right\}, \quad \ldots\ldots\ldots \quad (670)$$

he constant c is given the following values by Young:

CO_2	$c = 21.3$
NH_3	$c = 23.6$
Hydrocarbons	$c = 20.21$
Water and alcohols	$c = 26 \cdot 0$

For such substances as water and steam, the properties of which must be ccurately known, general laws like the above are of no value compared with, irect experimental determination except as checks on its results, and even hese checks are less accurate than others that are known.

These experimental data are quite numerous for water, but as generally nade include the *heat of liquid* water from some lower temperature to the oiling-point. The amount of heat necessary to warm a pound of liquid from emperature 32° F. to some boiling-point, and to there convert it entirely into apor is designated as the *total heat* of the dry saturated vapor above the origi- al temperature. This is, of course, also equal to the heat given up by the con- ensation of a pound of dry saturated vapor at its temperature of existence and y the subsequent cooling of the water to some base temperature taken univer- ally now as 32° F. in engineering calculations.

From observations by Regnault and formulated by him in 1863 the present knowledge of the total heat of water may be said to date. He gave the expression, Eq. (671), in which the first term is the latent heat at 32° and one atmospheric pressure:

$$\text{Total heat per pound dry saturated steam} = H = 1091.7 + .305(t - 32). \quad (671)$$

This was long used as the basis of steam calculations, but is now to be discarded in the light of more recent experimental data, the best of it based on indirect measurements by Grindley, Griessmann, Peake, who observed the behavior of steam issuing from an orifice, together with the results of Knobloch and Jacob and Thomas on specific heats of superheated steam, and in addition on direct measurements by Dieterici, Smith, Griffiths, Henning, Joly. All this work has been recently reviewed and analyzed by Davis, who accepts 1150.3 B.T.U. as the most probable value of the total heat under the standard atmosphere and the following formula as representing total heats from 212° up to 400° F.

$$H = 1150.3 + .3745(t - 212) - .000550(t - 212)^2 \quad . \quad . \quad . \quad (672)$$

The Davis curve containing all the important experimental points and the accepted line, extended dotted from 212° to 32°, is presented in Fig. 142.

From the total heats given by this formula the latent heat is obtainable by subtraction, according to the relation,

$$\text{Latent heat } (L) = \text{total heat of vapor above } 32° \text{ F. } (H) - \text{heat of liquid from } 32° \text{ F. to boiling point } (h), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (673)$$

in which the heat of the liquid is computed from a mean curve between Dieterici's and Regnault's values, having the equation $h = .9983 - .0000288\ (t - 32) + .0002133(t - 32)^2$. This is the basis of the values for latent and total heats in the Marks and Davis steam tables referred to, and accepted as the best obtainable to-day. From these tables a pair of charts for latent heat and total heat of dry saturated steam are given at the end of this Chapter.

The specific volume and density of dry saturated steam, given in the chart and table are calculated, as this seems to promise more exact results than direct experiment, the method of calculation involving three steps:

(*a*) From the pressure-temperature equation the ratio of $\frac{dp}{dT}$ is found by differentiation as follows:

$$\log p = 10.515354 - 4873.71T^{-1} - .00405096T + .0000013929647^2,$$

whence

$$\frac{dp}{dT} = \left(\frac{4873.71}{T^2} - .00405096 + .000002785928T\right) p.$$

(*b*) From the latent heats the difference between specific volume of vapor and liquid, $(V_V - V_L)$ is calculated by substituting (*a*) in Clapeyron's equation.

(*c*) From the Landolt, Börnstein, Myerhoffer tables for density of water the volume V_L is taken, whence by addition the volume of the vapor V_V is found, $V_V = (V_V - V_L) + V_L$.

For ammonia and carbonic acid there are no data available on total heats by either direct measure or by the orifice expansion properties, and very few

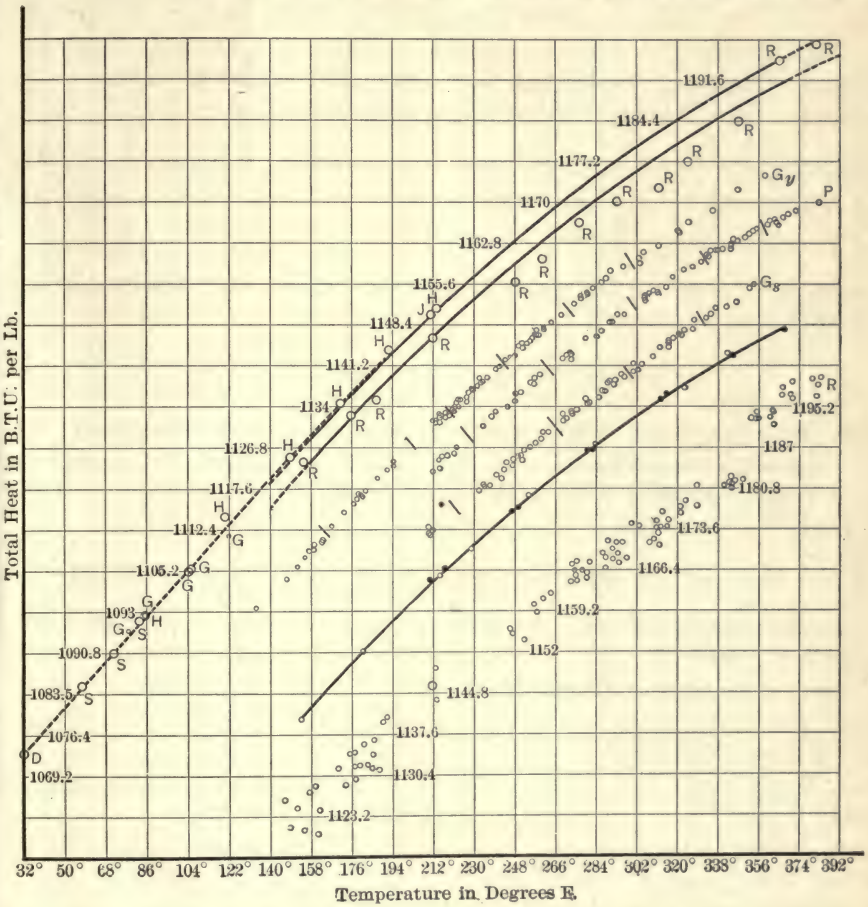


FIG. 142.—Total Heat of Dry Saturated Steam above 32° F. (Davis).

determinations of the latent heat itself, so that the process that has proved so satisfactory with steam cannot be directly followed with these substances. Accordingly, a process of adjustment has been used, working from both ends, beginning with the pressure temperature relations on the one hand and specific volumes of liquid and vapor on the other, the latent heat is determined by Clapeyron's equation and where this does not agree with authentic values an adjustment of both latent heat and specific volume is made.

This process is materially assisted by the so-called Cailletet and Mathias law of mean diameter of the curves of density of liquid and vapor, which are given in Figs. 143, 144 and 145, for water, ammonia and carbon dioxide, on which the points are marked to indicate the source of information.

On each of these curves the line BD is the line of mean density, its abscissa being given by the following general equation,

$$s=\tfrac{1}{2}\left(\frac{1}{V_V}+\frac{1}{V_L}\right)=a+bt+ct^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (674)$$

Of course, this mean density line passes through the critical volume B. For these three cases this Eq. (674) is found to have the form,

$$\left.\begin{array}{lll} \text{For water. . . . . } & s=28.7-.015(t-300)-.000015(t-300)^2. & (a) \\ \text{For ammonia. . . . . } & s=20-.022(t-30). & (b) \\ \text{For carbonic acid.. } & s=33.1-.0219(t+20)-.00016(t-20)^2. & (c) \end{array}\right\} \quad . \quad (675)$$

A more exact equation for water has been determined by Marks and Davis in their steam tables and is

$$s=28.424-.01650(t-320)-.0000132(t-320)^2. \quad . \quad . \quad . \quad (676)$$

From the smooth curve, which has the above equation, the volumes and densities of liquid and vapor that are accepted have been derived, and are presented in chart form on a large scale and in tabular form at the end of the Chapter, the values for water being those of Marks and Davis.

From these volume differences and the $\frac{dp}{dT}$ relation the latent heats have been calculated and the newly calculated points are compared with experimental values in Fig. 146.

The total heats are obtained by adding to the latent heat the heat of liquid above 32° from −50° F. up to the critical point for CO_2 and to 150° F. for NH_3, which include the working range for refrigeration. These liquid heats have already been determined in Section 5 in discussing specific heats.

Charts and tables at the end of this Chapter give the final values of total heat, heat of liquid, latent heat, specific volume and density of dry saturated vapor based on large-scale plottings, without equations beyond that for the pressure-temperature relations for saturated vapor, and the results are believed to be as reliable as it is possible to get them without more experimental data.

The properties of dry-saturated steam are given in Table XLVII, and charts, A, B, C, D, E, F; the properties of superheated steam, in Table XLVIII; dry-saturated ammonia vapor in Table XLIX, and Charts G, H, I, J, K, L; and dry-saturated carbon dioxide vapor in Table L, and Charts M, N, O, P, Q, R.

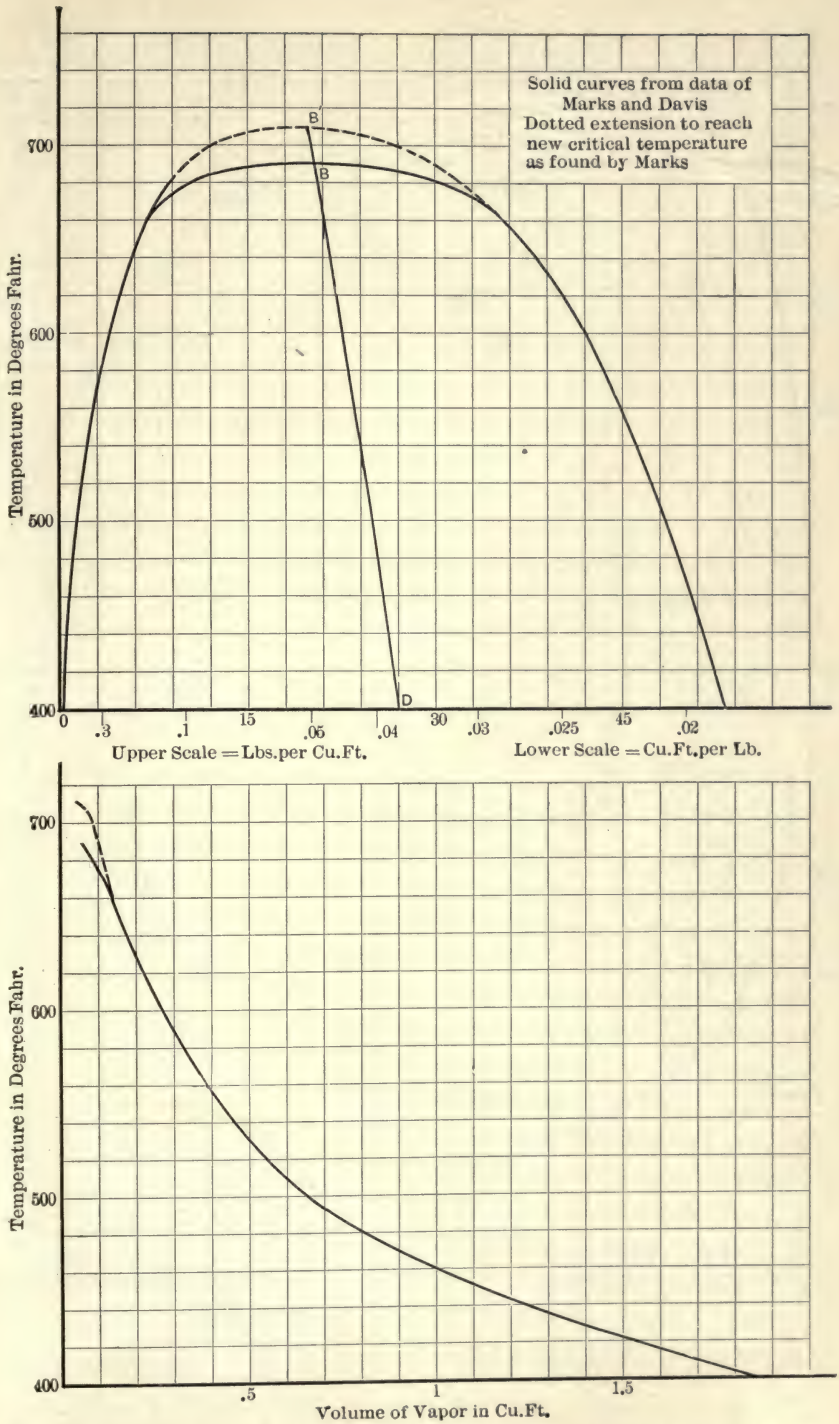


FIG. 143.—Specific Volume and Density of Liquid and Dry Saturated Vapor of Water.

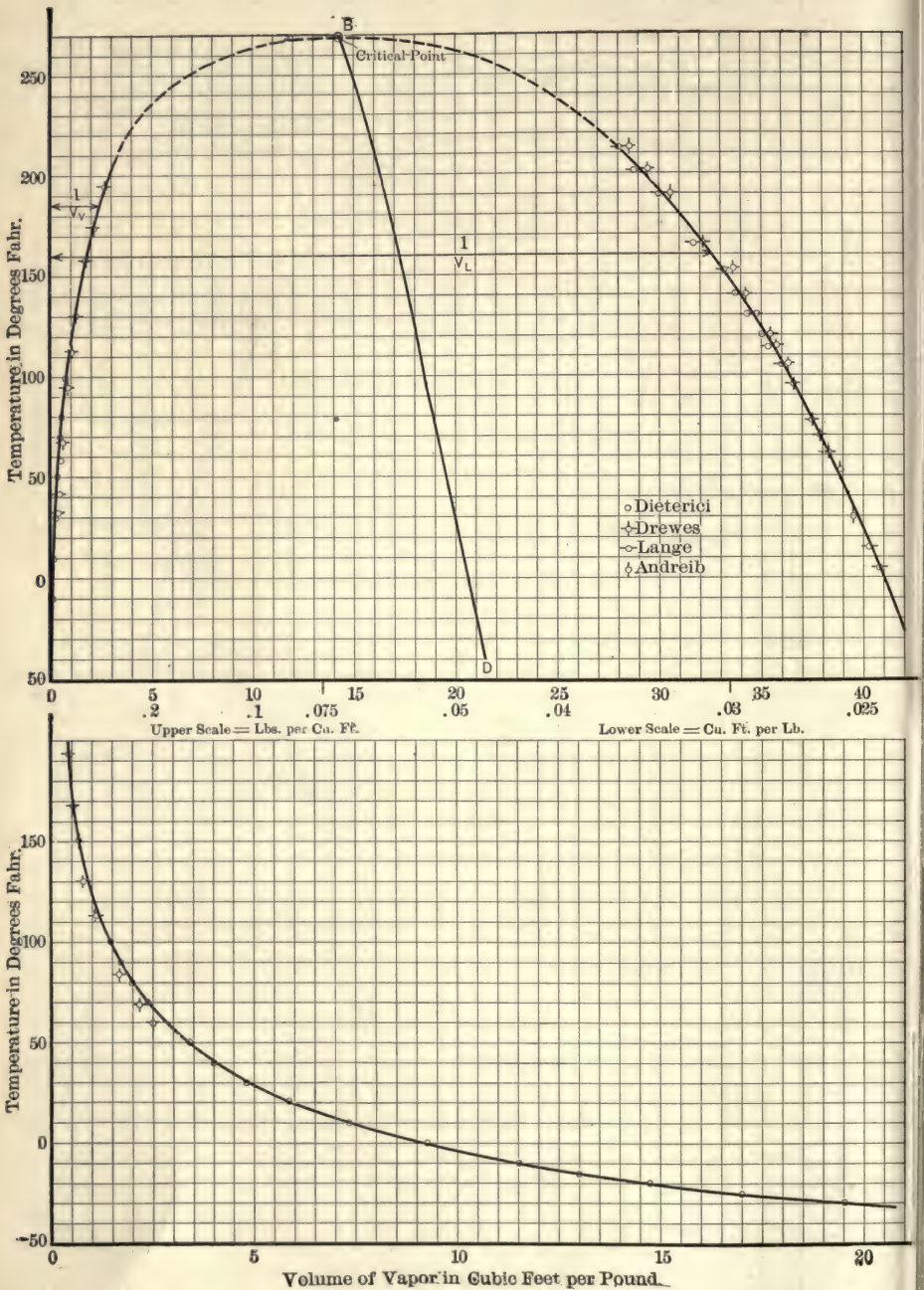


FIG. 144.—Specific Volume and Density of Liquid and Dry Saturated Vapor of Ammoni

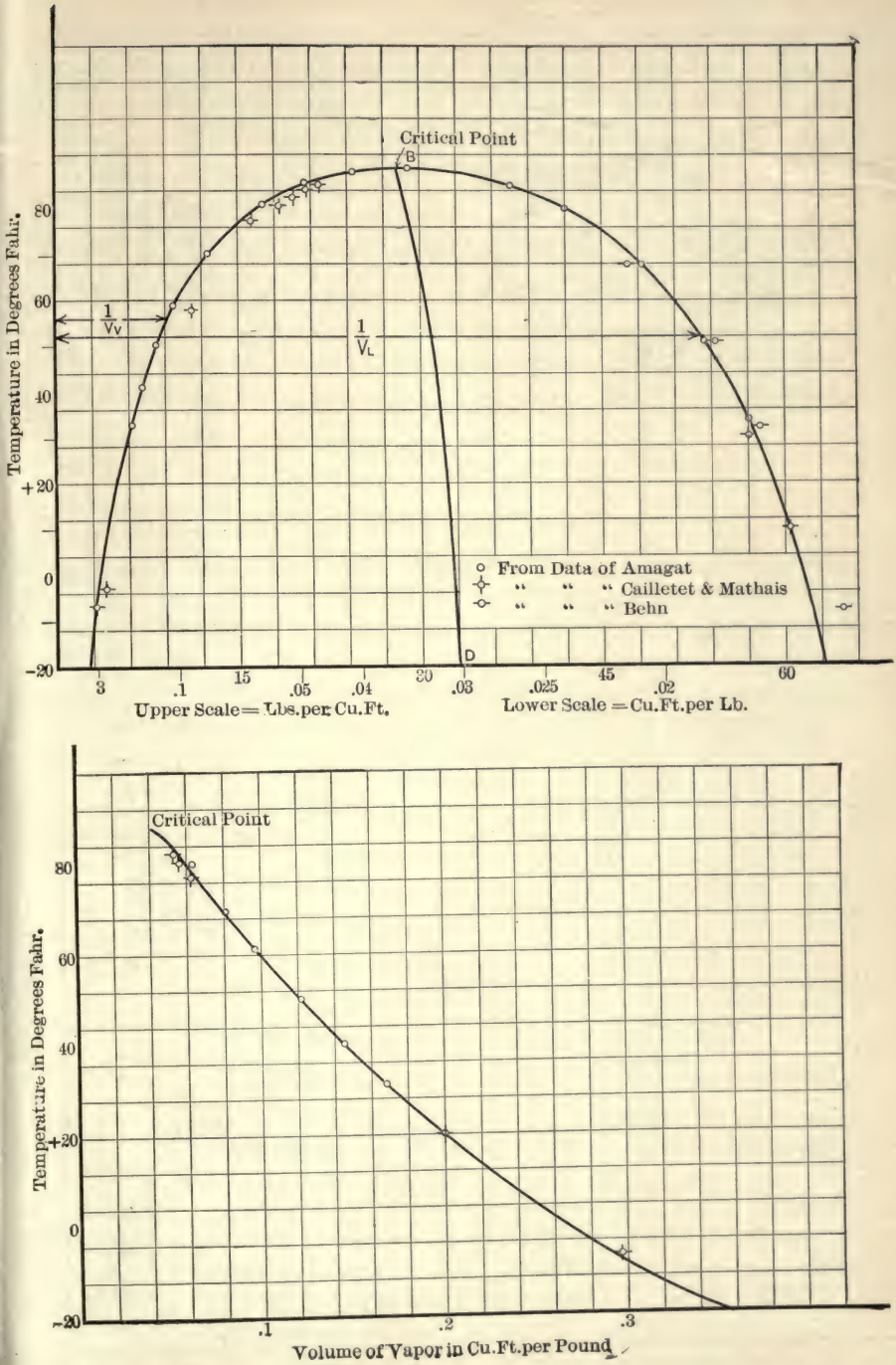
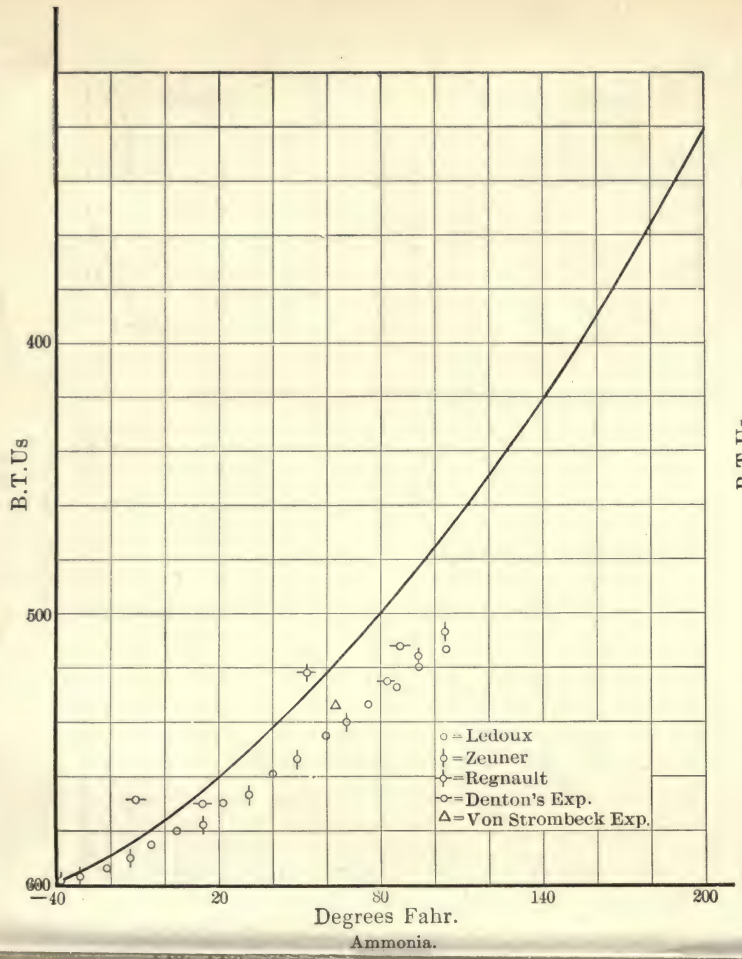
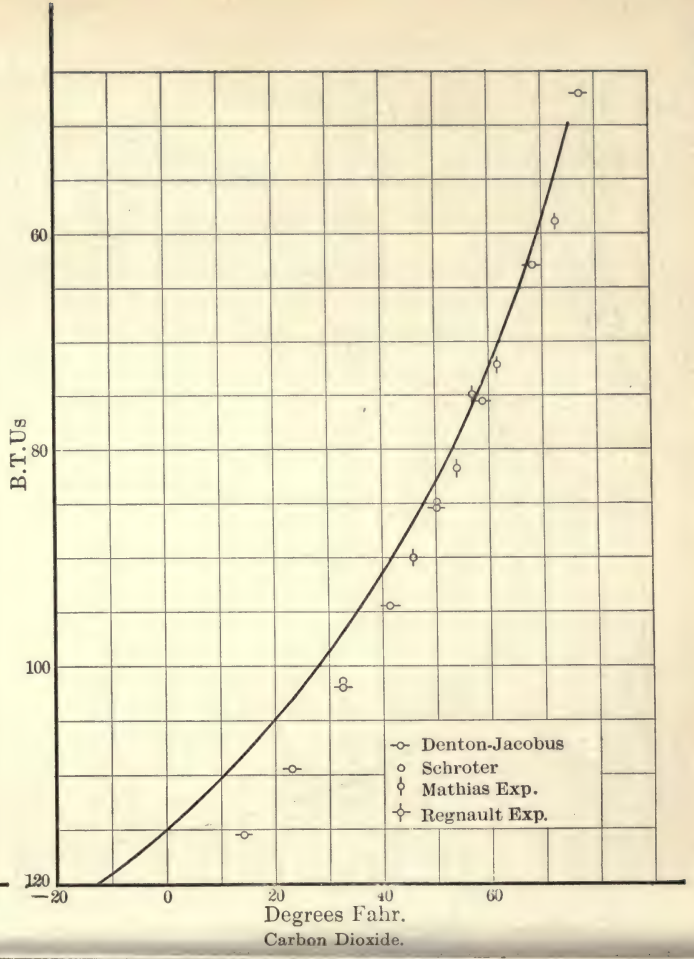


FIG. 145.—Specific Volume and Density of Liquid and Dry Saturated Vapor of Carbon Dioxide.



Ammonia.



Carbon Dioxide.

The volumes of dry-saturated steam determined from the tables when compared with their pressures show that there exists an approximate relation of the form for steam,

$$p(V_V - V_L)^{1.0646} = \text{constant} = 497, \quad . \quad . \quad . \quad . \quad . \quad (677)$$

when pressures are in pounds per square inch and volumes are in cubic feet. This curve plotted to PV coordinates is called the saturation curve for the vapor. It is useful in approximate calculations of the work that would be done by steam expanding so that it remains dry and saturated or the work required to compress vapor such as ammonia under the same conditions. But as the specific volume of liquid is generally negligible it may be written as one of the general class

$$PV^s = \text{constant}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (678)$$

for which $s = 1.0646$ and constant $= 497$.

This curve supplies a means for computing the work for wet vapors (not too wet) as well as dry, provided only that they at no time become superheated or change their quality, by using for V some fraction of the true specific volume representing the *dryness*. The very fact that a great volume of vapor may be formed from an insignificant volume of liquid makes the saturation curve a useful standard of comparison with actual expansion and compression lines for wet vapors.

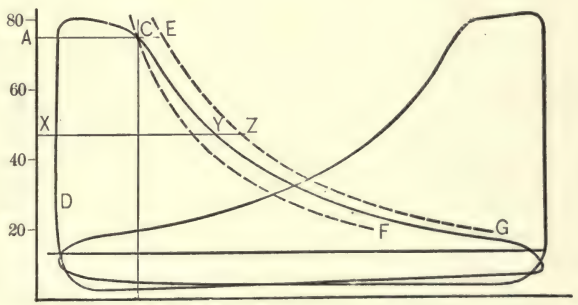


FIG. 147.—Comparison of Steam Expansion Line of an Indicator Card with the Saturation Line for both Dry Saturated Vapor and for Vapor Constantly Wet at the Initial Value.

Plotting the saturation curve beside an actual cylinder expansion or compression curve will show the quality of vapor at all times and also give a measure of evaporation and condensation taking place during the process. In Fig. 147 is shown a set of diagrams taken from a simple Corliss engine, 18×24 ins. with 4 per cent clearance, a $2\frac{1}{2}$-inch piston rod and tail rod, running at 150 R.P.M., to which have been added lines of zero pressure and volume by the method explained in Chapter I. The discharge from the condenser per hour for a constant load of the value to give the above cards was 2600 lbs. Allow-

ing for the rods, the displacement volumes of each end of the cylinder will be 5990 cu.ins., and since the clearance volume is 4 per cent, the steam volume will be 239.6 cu.ins. From the left-hand card it will be seen that the cut-off was at point C, 16.5 per cent of stroke, hence the volume at C is $(.165\times 5990)+239.6=1228$ cu.ins. It will also be seen from the card, that the pressure at C was 73.5 lbs. per square inch absolute. From the curves or the tables at the end of the Chapter 1 cu.ft. of dry steam at this pressure weighs .1688 lb. and hence the weight of steam in this end of cylinder was $\frac{1228}{1728}\times.1668$ or .1185 lb. at cut-off. From the card it will also be seen that at the end of the exhaust stroke, denoted by the point D, the pressure was 30 lbs., at which the weight of 1 cu.ft. of dry steam is .0728 lb., hence the weight of steam in the cylinder was $\frac{239.6}{1728}\times.0728=.01010$ lb., and the amount admitted was $.1185-.0101=.1084$ lb.

In as much as the two ends of cylinder are identical and as the cards from both ends are practically the same, it may be assumed that the same weight of steam was in each end, or that $.1084\times2=.2168$ lb. are accounted for by the card per revolution, or $.2168\times150\times60=1950$ lbs. per hour. There is then the difference to otherwise account for, of $2600-1950=650$ lbs. per hour, which can only have been lost by condensation. 2600 lbs. per hour is $2600\div(150\times60\times2)=.1442$ lb. per stroke, which with the .0101 lb. left from previous stroke would make .1543 lb. in the cylinder at cut-off, and if it were all steam its volume would be 1581 cu.ins., denoted by point E on diagram. The ratio of $\overline{AC}$ to $\overline{AE}$ gives the amount of actual steam present in the cylinder at cut-off, to the amount of steam and water. The saturation curves CF and EG are drawn through C and E from tabular values and represent in the case of CF the volumes which would have been present in the cylinder at any point of stroke had the steam and water originally present expanded in such a way as to keep the ratio or dryness constant, and in case of EG, volumes at any point of the stroke if all the steam and water originally present had been in form of steam and had remained so throughout the stroke. Just as the ratio of $\overline{AC}$ to $\overline{AE}$ shows per cent of steam present at cut-off, so does the ratio of distances of any points Y and Z, from the volume axis denote the per cent of steam present at that particular point of the stroke. By taking a series of points along the expansion curve it is possible to tell whether evaporation or condensation is occurring during expansion. In this case the ratio,

$$\frac{\overline{AC}}{\overline{AE}}=.795, \text{ and } \frac{\overline{XY}}{\overline{XZ}}=.86.$$

Hence, it is evident that evaporation is occurring since the percentage of steam is greater in the second case.

For some classes of problems it is desirable that the external mechanical work be separated from the latent heat, and for this reason latent heat is given in three ways:

(*a*) External latent heat,

(*b*) Internal latent heat,

(*c*) Latent heat total.

The external latent heat in foot-pounds is the product of pressure and volume change, or expressing pressures in pounds per square inch,

$$\text{External latent heat} = \frac{144}{J} p(V_V - V_L). \quad . \quad . \quad . \quad . \quad . \quad . \quad (679)$$

This is sometimes reduced by neglecting V_L as insignificantly small as it really is for most problems which are limited to temperatures below 400° for saturated vapor, in which case,

$$\text{External latent heat} = \frac{144}{J} p V_V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (680)$$

In all cases

$$\left.\begin{aligned} \text{Internal latent heat} &= L - (\text{Ext. Lat. Ht.}) \qquad (a) \\ &= L - \frac{144}{J} p(V_V - V_L) \qquad (b) \\ &= L - \frac{144}{J} p V_V \qquad (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (681)$$

or

Fusion and freezing are quite similar to vaporization and condensation in that they are constant temperature processes with proportionality between the amount of substance changing state and the amount of heat exchanged. They are different in as much as little or no volume change occurs. As there is so little external work done it may be expected that there is little change in their latent heats with temperature and pressure, but as a matter of fact it makes very little difference in most engineering work just how this may be, because practically all freezing and melting takes place under atmospheric pressure. There does not appear to be any relation established between heats of fusion like those for vaporization that permit of estimates of value from other constants, so direct experimental data must be available and some such are given for a few substances at the end of this Chapter in Table XLVI. As a matter of fact such laws would be of little use, and this is probably reason enough for their non-discovery.

Example 1. Pigs of iron having a total weight of 5 tons and a temperature of 1000° F. are cooled by immersing them in open water at a temperature of 60° F. If one-half of the water is evaporated by boiling, how much must there have been originally?

The iron must have been cooled to the final temperature of the water, which must have been 212° F. Also the heat given up by the iron will be the

product of its weight, specific heat and temperature difference, or, considering the mean specific heat to be .15,

$$10{,}000 \times (2000 - 212) \times .15 = 2{,}682{,}000 \text{ B.T.U.}$$

The heat absorbed by the water in being heated, considering its specific heat as unity will be its weight times its temperature change and, since one-half evaporates, the heat absorbed in evaporating it will be half its weight times the latent heat, or

$$W[(212 - 60) + \tfrac{1}{2} \times 970] = 637W \text{ B.T.U.}$$

These expressions for heat must be equal, hence

$$W = \frac{2{,}682{,}000}{637} = 4210 \text{ lbs.}$$

Example 2. A tank of pure water holding 1000 gallons is to be frozen by means of evaporating ammonia. The water is originally at a temperature of 60° F. and the ice is finally at a temperature of 20° F. The ammonia evaporates at a pressure of one atmosphere and the vapor leaves the coils in a saturated condition. How many pounds of ammonia liquid will be needed, how many cubic feet of dry saturated vapor will be formed, and how much work will be done in forming the vapor?

The heat to be removed is the sum of that to cool the water, the latent heat of fusion of ice, and that to cool the ice, or for this case

$$[(60 - 32) + 144 + .5(32 - 20)] \times 8333,$$

8333 being the weight of 1000 gallons of water. Hence the B.T.U. abstracted amount to 1,466,608.

Each pound of ammonia in evaporating at atmospheric pressure absorbs 594 B.T.U. as latent heat and, therefore, 2470 lbs. are needed. At this pressure each pound of vapor occupies 17.5 cu.ft., hence there will be 43,200 cu.ft. of vapor. At this same pressure the volume of a pound of liquid is .024 cu.ft., so that the work done per pound in evaporating the ammonia is 37,000 ft.-lbs. and the total work is 915×10^6 ft.-lbs.

Prob. 1. How much ice would be melted at 32° F. with the heat necessary to boil away 5 lbs. of water at atmospheric pressure, the water being initially at the temperature corresponding to the boiling-point at this pressure?

Prob. 2. What is the work done during the vaporization of 1 lb. of liquid anhydrous ammonia at the pressure of the atmosphere?

Prob. 3. From the tables of properties of anhydrous ammonia check the value of the constant in Trouton's law given as 23.6 by Young.

Prob. 4. As steam travels through a pipe some of it is condensed on account of the radiation of heat from the pipe. If 5 per cent of the steam condenses how much heat per hour will be given off by the pipe when 30,000 lbs. of steam per hour at a pressure of 150 lbs. per square inch absolute is passing through it?

Prob. 5. Brine having the specific heat of .8 is cooled by the evaporation of ammonia in coils. If the brine is lowered 5° F. by ammonia evaporating at a pressure of

bs. per square inch gage, the vapor escaping at brine temperature, how many pounds f brine could be cooled per pound of ammonia?

Prob. 6. Steam from an engine is condensed and the water cooled down to a emperature of 80° F. in a condenser in which the vacuum is 28 ins. of Hg. How many ounds of cooling water will be required per pound of steam if the steam be initially 0 per cent wet?

Prob. 7. A pound of water at a temperature of 60° F. is made into steam at 100 bs. per square inch gage pressure. How much heat will be required for this, and what vill be the volumes at (*a*) original condition; (*b*) just before any steam is made; (*c*) after ll the water has been changed to steam?

Prob. 8. A sand mold weighs 1000 lbs. and 100 lbs. of melted cast iron are poured nto it. Neglecting any radiation losses and assuming the iron to be practically at ts freezing temperature how much of the iron will solidify before the mold becomes f the same temperature as the iron?

Prob. 9. How many pounds of ice could be melted by heat given up by freezing 0 lbs. of lead?

11. Gas and Vapor Mixtures. Partial and Total Gas and Vapor Pressures. Volume, Weight, and Gas Constant Relations. Saturated Mixtures. Humidity. One of the characteristic properties of gases distinguishing them from liquids, nd which also extends to vapors with certain limitations is that of *infinite expansion*, according to which no matter how the containing envelope or volume f the expansive fluid may vary, the space will be filled with it at some pres- ure and the weight remain unchanged except when a vapor is brought to ondensation conditions, or the pressure lowered on the surface of a liquid vhich will, of course, make more vapor. A given weight of gas or vapor (within imits) will fill any volume at some pressure peculiar to itself, and two gases, wo vapors, or a vapor and gas, existing together in a given volume, will fill it t some new pressure which is the sum of the pressures each would exert sepa- ately at the same temperature (if *non-miscible*). This fact, sometimes des- gnated as Dalton's Law, permits of the derivation of equations for the rela- ion of any one pressure, partial or total, to any other total or partial, in erms of the weights of gas or vapor present, and the gas constants R. It lso leads to equations for the various constituent and total weights in erms of partial and total pressures and gas constants. Such equations sup- ly a basis for the solution of problems in humidification and drying of air, in arburetion of air for gasolene and alcohol engines, or of water gas for illumina- ion, and are likewise useful as check relations in certain cases of gas mixtures uch as the atmospheric mixture of nitrogen and oxygen, producer gas or gase- us combustibles in general.

Let w_1, w_2 and w_x be the respective weights of the constituents of a mixture;

" $w_m = \Sigma w$ be the weight of the mixture;

" P_1, P_2, P_x be the respective partial pressures of the constituents;

" $P_m = \Sigma P$ be the pressure of the mixture;

" R_1, R_2, R_x be the respective gas constants;

" R_m be the gas constant for the mixture.

Then if w_1 lbs. of one, and w_2 lbs. of another gas or vapor at temperature T_m occupy the volume V_m cubic feet together,

$$\left.\begin{aligned} V_m P_1 &= w_1 R_1 T_m, \quad \ldots\ldots\ldots (a) \\ \text{and} \qquad V_m P_2 &= w_2 R_2 T_m, \quad \ldots\ldots\ldots (b) \end{aligned}\right\} \quad (682)$$

whence

$$V_m(P_1+P_2) = (w_1R_1+w_2R_2)T_m, \quad \ldots\ldots\ldots (683)$$

or, in general,

$$\Sigma P = \Sigma(wR)T_m \quad \ldots\ldots\ldots (684)$$

Or putting

$$\Sigma P = P_m, \quad \ldots\ldots\ldots (685)$$

and

$$\Sigma(wR) = R_m w_m, \quad \text{or} \quad R_m = \frac{\Sigma(wR)}{w_m}, \quad \ldots (686)$$

then

$$P_m V_m = w_m R_m T_m, \quad \ldots\ldots\ldots (687)$$

so that the *mixture will behave thermally quite the same as any one gas with such exceptions as may be due to a different gas constant* R_m.

Dividing Eq. (682*a*) by Eq. (683) or (684) gives

$$\frac{P_1}{P_m} = \frac{w_1R_1}{w_1R_1+w_2R_2} = \frac{w_1R_1}{\Sigma(wR)}, \quad \ldots\ldots\ldots (688)$$

which gives the ratio of any partial pressure to that for the mixture in terms of the individual weights and gas constants. Hence

$$\frac{P_1}{P_m} = \frac{w_1R_1}{w_mR_m}, \quad \ldots\ldots\ldots (689)$$

which gives the *ratio of any partial pressure to that for the mixture in terms of its own weight and gas constant and those for the mixture.*

It is possible to express the ratio of weights as a function of gas constants alone which will permit of a third expression for the partial pressures in terms of gas constants without involving any weights. For two gases

$$w_1 = w_m - w_2.$$

Whence

$$\frac{w_1}{w_m} = 1 - \frac{w_2}{w_m}, \quad \ldots\ldots\ldots (690)$$

But from Eq. (686)

$$w_1R_1+w_2R_2=w_mR_m,$$

or

$$\frac{w_2}{w_m}=\frac{1}{R_2}\left(R_m-\frac{w_1}{w_m}R_1\right),$$

so that

$$\frac{w_1}{w_m}=1-\frac{w_2}{w_m}=1-\frac{1}{R_2}\left(R_m-\frac{w_1}{w_m}R_1\right),$$

or

$$\frac{w_1}{w_m}\left(1-\frac{R_1}{R_2}\right)=1-\frac{R_m}{R_2},$$

and

$$\frac{w_1}{w_m}=\frac{R_2-R_m}{R_2-R_1}, \quad \ldots\ldots\ldots \quad (691)$$

which is the ratio of partial to total weights in terms of gas constants. On substitution in Eq. (689),

$$\frac{P_1}{P_m}=\frac{R_1}{R_m}\left(\frac{R_2-R_m}{R_2-R_1}\right), \quad \ldots\ldots\ldots \quad (692)$$

which *gives the ratio of partial pressures of two gases or vapors to that for the mixture in terms of the individual gas constants and that for the mixture,* and a similar expression can be found for more than two gases. The ratio of any one partial, to the total weight can also be found from Eq. (689) in terms of its gas constant and partial pressure, and the mixture gas constant and pressure, from Eq. (691) in terms of the gas constants for the constituents and for the mixture. This *ratio of partial to total mixture weight gives the fractional composition by weight.*

It is sometimes necessary to know the volume relations in a mixture of two gases existing at the same pressure or two vapors or a vapor and gas, such, for example, as air and water vapor. In this case two different volumes existing together at a common temperature and pressure together make up a mixture volume equal to their sum. Using similar symbols

$$\left.\begin{aligned} P_mV_1&=w_1R_1T_m \\ P_mV_2&=w_2R_2T_m \end{aligned}\right\}, \quad \ldots\ldots\ldots \quad (693)$$

where V_1 and V_2 are the volumes occupied by the two constituents respectively when at a mixture pressure P_m and temperature T_m, whence for the mixture

$$P_m(V_1+V_2)=(w_1R_1+w_2R_2)T_m \quad \ldots\ldots \quad (694)$$

or

$$P_m\Sigma(V)=\Sigma(wR)T_m \quad \ldots\ldots\ldots \quad (695)$$

These Eqs. (694) and (695) are identical in form with (683) and (684) except that V replaces P, and V, P, so that all equations just derived also apply to volumes as the volume proportion will be identical with pressure proportions. For convenience of reference these may be set down.

From Eq. (688),

$$\frac{V_1}{V_m}=\frac{w_1R_1}{\Sigma wR}, \quad \ldots \ldots \ldots \quad (696)$$

which gives the *ratio of any partial volume, to that for the mixture in terms of the individual weights and gas constants.*

From Eq. (689)

$$\frac{V_1}{V_m}=\frac{w_1R_1}{w_mR_m}, \quad \ldots \ldots \ldots \quad (697)$$

which gives the *ratio of any partial volume to that of the mixture in terms of its own weight and gas constant and those for the mixture.*

From Eq. (692)

$$\frac{V_1}{V_m}=\frac{R_1}{R_m}\frac{(R_2-R_m)}{(R_2-R_1)}, \quad \ldots \ldots \ldots \quad (698)$$

which gives the *ratio of any partial volume to that of the mixture in terms of the individual gas constants and that for the mixture.*

The volumetric composition of air is given by Eq. (697) or its equal numerically Eq. (692), and since the partial pressure of oxygen and nitrogen in air are 78.69 per cent and 21.31 per cent, these are its volumetric per cents.

When one of the constituents is a vapor, all the preceding applies, provided the condition of the vapor is such that at the temperatures assumed it is not near the condition of condensation, but then the relations become more definite since the *partial pressure of the vapor is fixed by the temperature.* In practical work with gas and vapor mixtures the failure of the perfect gas laws near the condensation condition is ignored and they are assumed to be true for the very good reason that there is no other way as good, to get numerical results.

All liquids, and many, if not all solids will, if placed in a vacuum chamber, evaporate until the pressure has reached a certain value depending on the temperature, at which time the liquid and its vapor are in equilibrium, and evaporation may be said either to cease or proceed at a rate exactly equal to the rate at which vapor condenses, or more precisely, at equilibrium the weight of vapor in the vapor form remains constant. The weight of vapor that will rise over a liquid in a given space depends on the temperature and pressure which are related

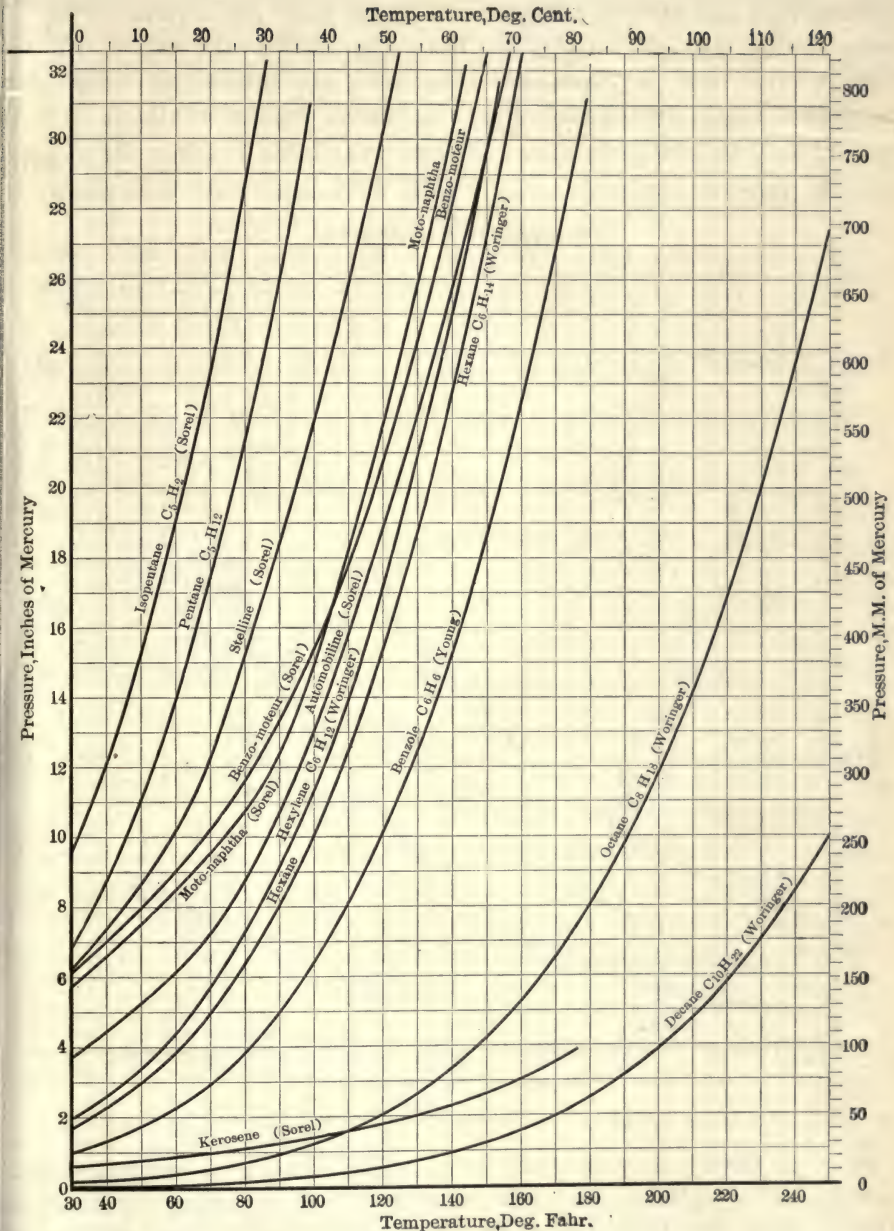


FIG. 148.—Vapor Pressure of Hydrocarbons and Light Petroleum Distillates of the Gasolene Class.

in the so-called vapor tension or vapor pressure tables and curves, such as shown in Figs. 148, 149 and 150, for some liquid fuels or as given in the previous section for water. At any fixed temperature the vapor will continue to rise until it exerts its own vapor pressure for the temperature the process being often described as evaporation without ebullition. If the liquid or solid be introduced into a chamber containing dry gas the evaporation will proceed precisely the same as in the vacuum until the pressure has

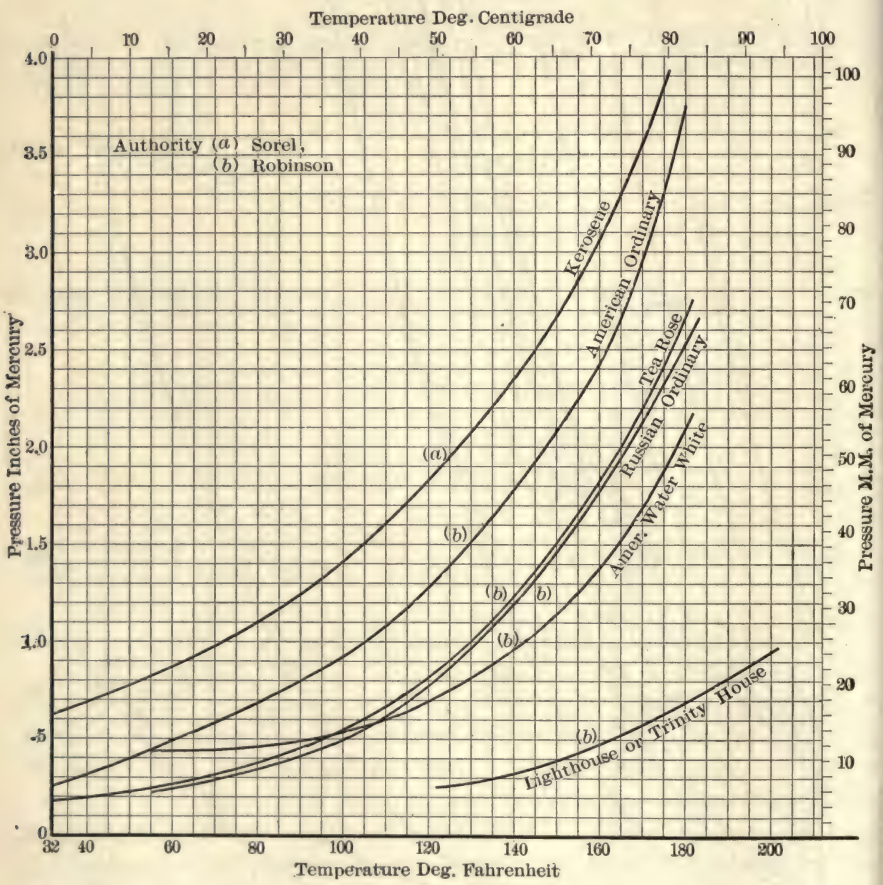


FIG. 149.—Vapor Pressure of Heavy Petroleum Distillates of the Kerosene Class.

risen by an amount corresponding to the vapor pressure for the temperature because each substance exerts the pressure it would if alone occupying the volume; when they both occupy the same volume the pressure will be their sum and equal to the pressure of the gas alone added to the vapor pressure for the same temperature. There is one important practical condition, and that is, time enough for the completion of the process of evaporation which proceeds very slowly toward the end. If time enough is allowed the vapor pressure will establish itself and the *gas is said to be saturated*, and this is an important

special case of gas-vapor mixtures. It is the condition in which the gas may be said to carry the maximum weight of vapor possible for the total pressure

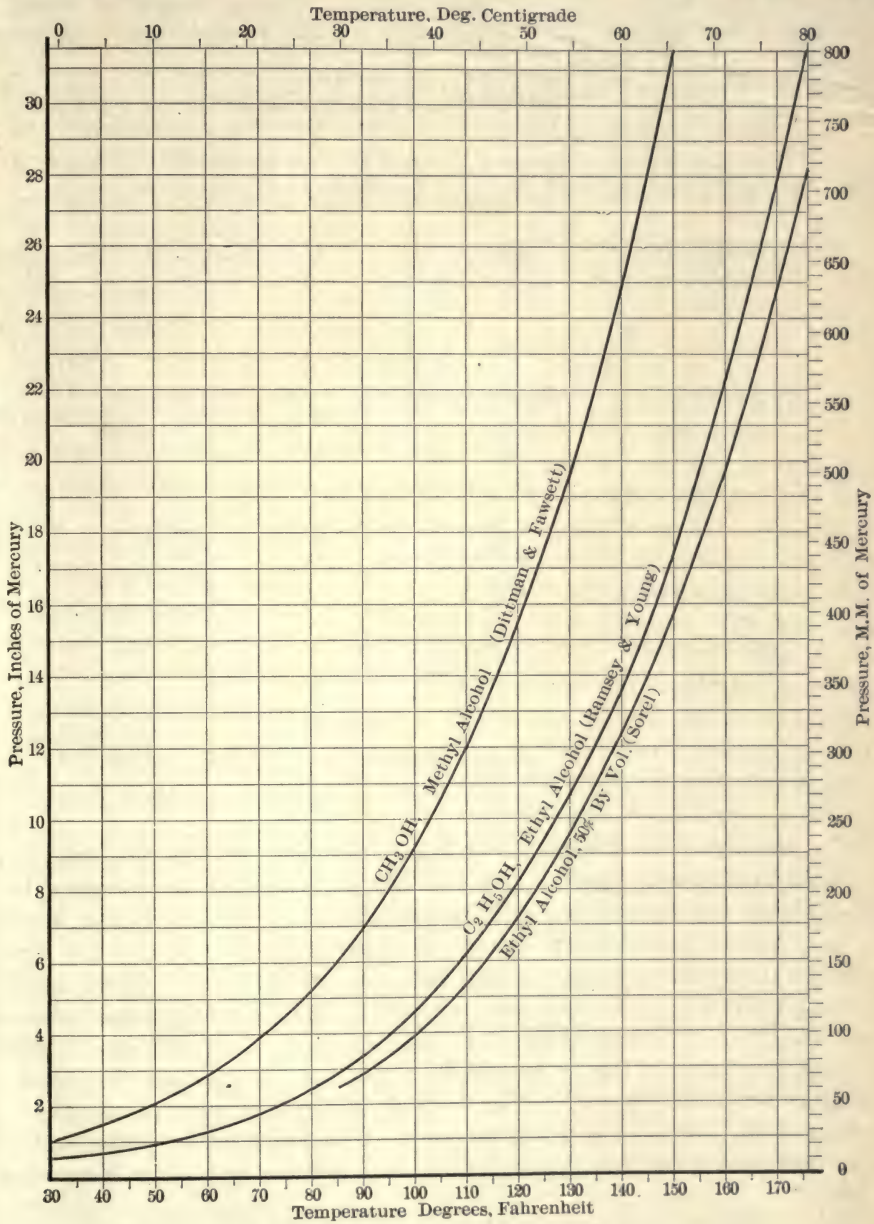


FIG. 150.—Vapor Pressure of the Alcohols.

and temperature. The gas in contact with the liquid may carry less vapor if it has not been in contact long enough at the given temperature, and a gas

no longer in contact with the liquid may carry less, because, (*a*) of insufficient time of original contact; (*b*) of condensation of some it originally carried; (*c*) of a rise of temperature after leaving the liquid. To all such general cases the equations above apply without change, but for the special case of saturated mixtures they have a simpler form.

Let P_v be the vapor pressure of the liquid for temperature T, which is the partial pressure of the vapor in a saturated vapor gas mixture;

" P_g be the partial pressure of the gas at same temperature.

Then for a gas saturated with vapor at temperature T, Eq. (689),

$$\left(\frac{\text{Weight vapor}}{\text{Weight mixt.}}\right)=\left(\frac{\text{Vapor pressure}}{\text{Mixture press.}}\right)\times\left(\frac{R \text{ for mixture}}{R \text{ for vapor}}\right) \quad . \quad . \quad . \quad (699)$$

But according to Eq. (646) and Eq. (651),

$$\left(\frac{R \text{ for vapor}}{R \text{ for mixt.}}\right)=\left(\frac{\text{Density of mixture}}{\text{Density of vapor}}\right)=\left(\frac{\text{Mol. wt. of mixt.}}{\text{Mol. wt. of vapor}}\right), \quad . \quad (700)$$

whence

$$\frac{w_v}{w_m}=\frac{P_v}{P_m}\frac{m_v}{m_m}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (701)$$

Also

$$\frac{w_g}{w_m}=\frac{P_g}{P_m}\frac{m_g}{m_m}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (702)$$

and

$$\frac{w_v}{w_g}=\frac{P_v}{P_g}\frac{m_v}{m_g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (703)$$

The presence of water vapor in the atmosphere, and problems connected with it, constitute a specific case of vapor-gas mixture, subject to the foregoing laws. This subject has been given most attention by the United States Weather Bureau; tables have been prepared for ready computation and for certain problems for which only experimental data or empiric formulas afford solution.

Air is said to be "*saturated with moisture*" when it contains the saturated vapor of water. It might be better to say that the *space* is saturated since the presence or absence of the air has no effect upon the water vapor other than imposing its temperature or imparting heat to the water vapor, and also that the air retards the diffusion of water particles. The weight of saturated aqueous vapor per cubic foot depends only on the temperature, and not on the presence of air.

If the space contains only a certain fraction of the weight of aqueous vapor corresponding to saturation, that fraction is called the "*relative humidity*" or degree of saturation, and the corresponding percentage, the per cent of saturation. If air containing saturated water vapor be cooled ever so little,

some of the vapor will be condensed and appear in the liquid form. If air is cooled at constant pressure, from a given initial condition, the degree of saturation approaches unity, and finally reaches that value at a temperature called the "*dew point*" corresponding to the initial condition. At this temperature the condition of saturation has been reached and any further cooling will cause the precipitation of liquid water. as occurs in the formation of dew, rainclouds or fog.

A space or body of air carrying water vapor in smaller quantity than that of saturation, in reality contains superheated steam. If the vapor density and the temperature of the mixture be known, the degree of superheat may be ascertained from the temperature of the mixture, and the temperature corresponding to saturated water vapor having a pressure equal to the partial pressure of the vapor in the mixture.

Humidity of atmospheric air is ordinarily determined by an instrument called the psychrometer, which consists merely of two thermometers, one with a bulb exposed directly to the air and the other covered with a piece of wick which is kept moist with water. The two are mounted together so that they can be whirled or swung about in the air until a stable condition has been reached. The dry-bulb thermometer should record the temperature of the air. The wet-bulb thermometer will record something lower than the air temperature, dependent upon the rate at which evaporation takes place, since the process of evaporation abstracts heat. Were there no other influence, this process of evaporation would continue till the temperature of the wet bulb became that of the dew point. The temperature of the wet-bulb thermometer never falls to the dew point, however, because of conduction of heat between the cold bulb and the warmer surrounding air. From extensive experiments conducted by the U. S. Weather Bureau, Professor Ferrel has devised the following formula for the vapor pressure, h in ins. of mercury corresponding to given readings of the wet- and dry-bulb thermometers, t_d and t_w degrees F.

$$h = h' - .000367 h_b (t_d - t_w)\left(1 - \frac{t_w - 32}{1571}\right), \quad . \quad . \quad . \quad . \quad . \quad (704)$$

where h_b is barometric height in inches, after applying all corrections, and h' is pressure of saturated water vapor, in inches of mercury, corresponding to the temperature t_w.

The relations shown by this formula are expressed graphically in much more convenient form in the curves of Figs. 151 and 152, devised by Prof. H. L. Parr. The use of the curves is best illustrated by an example: f the dry-bulb reading is 75° F. and the wet-bulb 65° F., find the dew point. The difference of wet- and dry-bulb temperatures is 10°. From 10° at the top of he diagram (*B*) Fig. 151 project downward, and from 75° air temperature at the eft of diagram project to the right to the intersection, where the dew point is ead by interpolation between the contour curves at (*C*) to be 59.5° F. These urves are drawn for a barometric pressure of 29.92 ins. (standard) and will

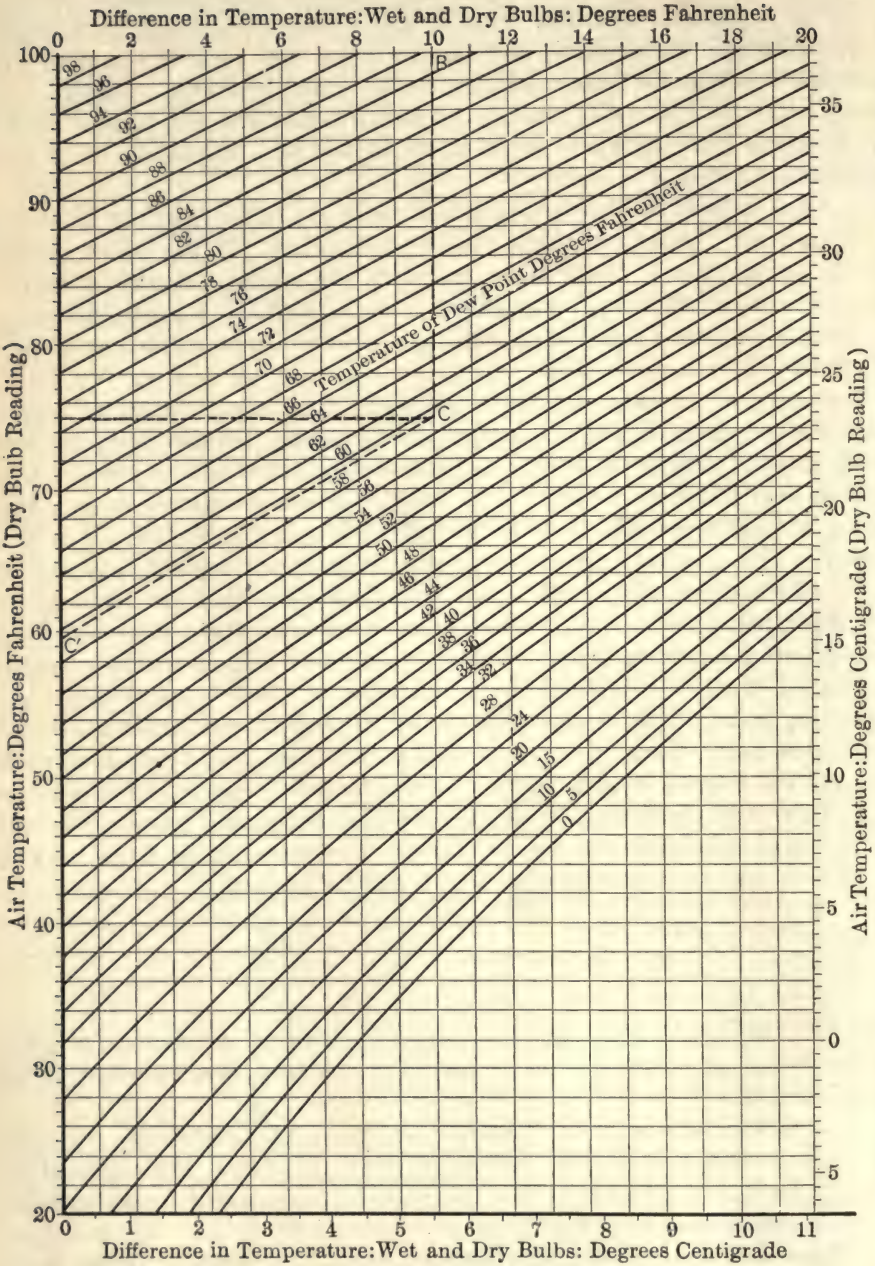


FIG. 151.—Relation between Wet and Dry Bulb Psychrometer Readings and Dew Point for Air and Water Vapor.

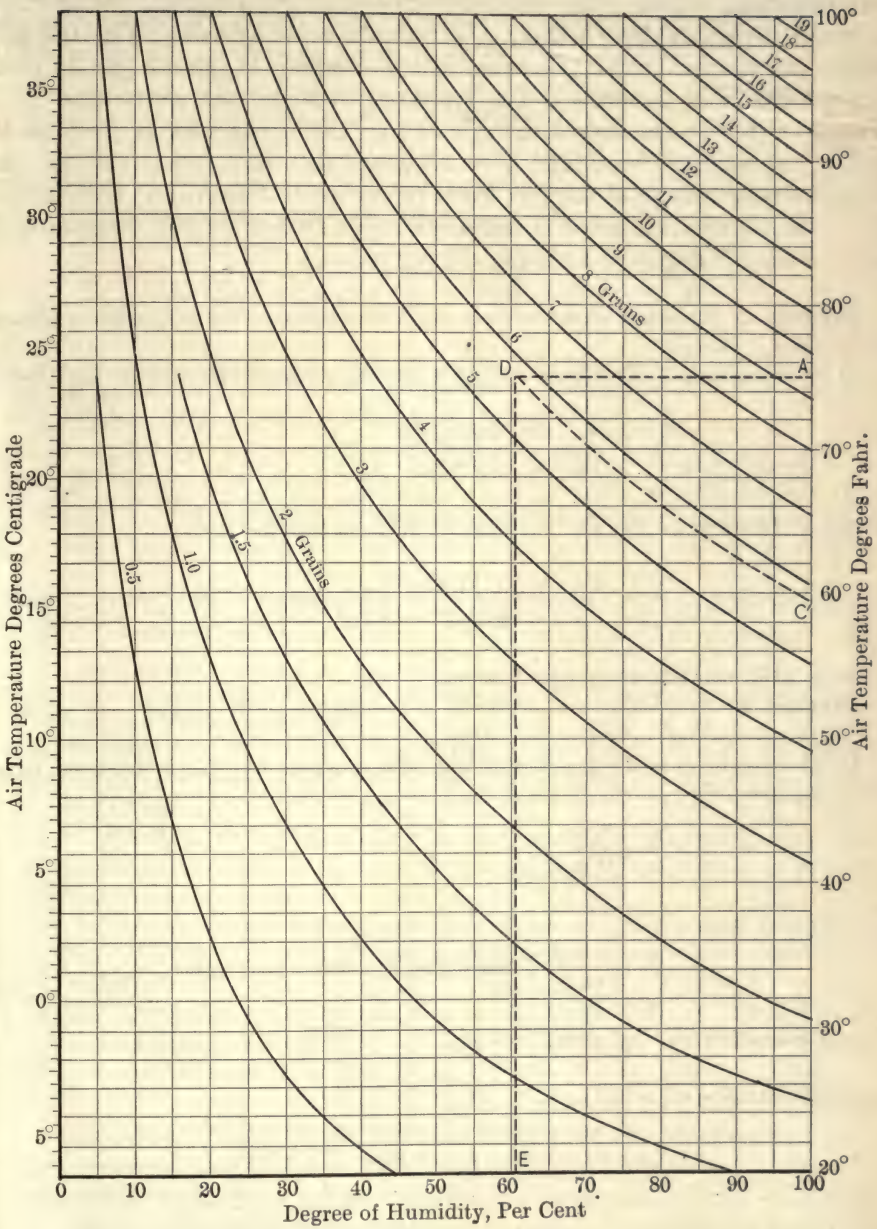


FIG. 152.—Relation between Humidity and Weight of Moisture per Cubic Foot of Saturated Air.

not apply correctly, when the barometer is not equal to this, though with fair approximation, so long as the difference in barometer is not great. Where there is much departure the original the formula must be used. Fig. 152 gives weight of aqueous vapor per cubic foot of mixture, in grains ($\frac{1}{7000}$ lb.) and also the degree of humidity. The temperature of the dew point 59.6° F, is located at (C') on the right-hand side of Fig. 152. Interpolation between the ends of the contours for weight, gives 5.6 grains per cubic foot. On the same scale the temperature of the air, F., is represented at point (A) 75°, projecting to the intersecting point D and down to the bottom of the diagram gives on the scale for degree of humidity, 60 per cent.

Example 1. By means of the relation of gas constants find the proportion of nitrogen and oxygen in the air.

R for nitrogen is 54.92 and for oxygen 48.25 and for air 53.35. From Eq. (698)

$$\frac{V_0}{V_m}=\frac{R_0}{R_m}\frac{(R_N-R_m)}{(R_N-R_0)},$$

which, on substituting the above values for R_N, R_0 and R_m gives

$$\frac{V_0}{V_m}=\frac{48.25}{53.35}\frac{(54.92-53.35)}{(54.92-48.25)}=.213,$$

or air is 21.3 per cent oxygen by volume.

Example 2. At what temperature will air containing $\frac{1}{2}$ lb. of water per pound of dry air at atmospheric pressure be saturated?

If the vapor pressure be known, the temperature may be found from tables. From Eq. (701)

$$\frac{w_v}{w_g}=\frac{P_v m_v}{P_g m_g},$$

or

$$P_v=\frac{P_g m_g w_v}{m_v w_g}, \quad \text{and} \quad P_v+P_g=760 \text{ mm. of Hg.}$$

For air $m_g=28.88$ and for water $m_v=18$,

hence substituting those values

$$P_v=\frac{(760-P_v)(28.88\times.5)}{18\times1},$$

or

$$P_v=\frac{750\times7.22}{16.22}=337 \text{ mm. Hg},$$

which corresponds to a temperature of 172° F.

Example 3. A pound of alcohol requires 9.06 lbs. of air for a proper combustible mixture for gas engines. At what temperature will these proportions contitute a saturated mixture?

From Eq. (701)

$$P_v = \frac{P_g m_g W_v}{m_v W_g}.$$

For alcohol $m_v = 46$, for air $m_g = 28.88$, and $P_v + P_g = 760$ mm. of Hg for atmospheric pressure. Substituting these values in the above equation

$$P_v = \frac{(760 - P_v)28.88}{46 \times 9.06} = 49 \text{ mm. Hg.}$$

From the curve of vapor tension of alcohol, the temperature corresponding to 49 mm. of Hg is about 72° F.

Prob. 1. Air at 80 per cent lumidity, atmospheric pressure and 70° F, is cooled to 40° F. How much water will be thrown down per 1000 cu.ft. of moist air.

Prob. 2. The same air is compressed adiabatically to five atmospheres, and again cooled to 40° F. at this pressure. How much moisture per 1000 cu.ft. of moist air will be separated out when the tenperature becomes 70° F, and beeu much at 40° F.

Prob. 3. What will be the weight of water in a pound of air and water vapor if the value for R for the mixture is taken as 55.25, for air as 53.35 and for water vapor as 91?

Prob. 4. At what temperature will air containing its own weight of water vapor be saturated at atmospheric pressure?

Prob. 5. An internal combustion engine uses a saturated mixture of air and gasolene vapor in which ratio of air to gasolene is 15.3. Considering the gasolene to be hexane, at what temperature will the mixture be?

Prob. 6. Should kerosene regarded as decane, $C_{10}H_{22}$, be substituted for gasolene, in the above problem what would be the change in temperature of mixture, assuming it still to be saturated?

Prob. 7. Air containing moisture equal to one per cent of the weight of the air alone is at a temperature of 150° F. How much is the water vapor superheated? What is the humidity?

Prob. 8. The reading of a dry bulb of a psychrometer is 90° F. and of the wet bulb 70° F. By means of curves of Figs. 151 and 152, find the dew point, relative humidity, and grains of water per cubic foot of air.

12. Absorption of Gases by Liquids and Adsorption or Occlusion by Solids. Relative Volumes and Weights with Pressure and Temperature. Heats of Absorption and of Dilution. Properties of Aqua Ammonia. In the attainment of high vacuua in steam condensers, separate removal of considerable quantities of non-condensible gases is found necessary by means of dry vacuum pumps, a fact that proves in a practical way the freedom with which the boiler water had absorbed gases. These gases for otherwise pure water are carbon dioxide and air, probably mainly air, but may include many others, notably the

products of organic decomposition especially when condensing water is taken from the neighborhood of sewers, as is generally the case when power plants are located on city water fronts. To a very much greater extent, however, is ammonia soluble in water, the latter being capable of taking up about 1000 volumes of ammonia at 0° and one atmosphere, against about 30 volumes of air, and one-fiftieth of a volume of hydrogen. It is the freedom of solution of ammonia in water that makes the process useful as a means of removing anhydrous ammonia from the cooling coils in mechanical refrigerating plants, as a substitute for the mechanical removal by piston compressors.

In all cases the solubility of gases in liquids decreases with rise of temperature, a fact associated with the separation of gases from boiler feed-water during their heating in feed-water heaters, economizers and the boiler itself. This property is also depended upon to free the aqua ammonia that has absorbed its ammonia charge from the cooling coils, of the amount so taken up, by heating the solution in a separate chamber from which the ammonia vapor escapes to the ammonia condenser to become liquid anhydrous while the weak liquor returns to the absorber for a new charge. To permit of the calculation of the quantity of liquor to be circulated, in order that a given amount of anhydrous ammonia may be absorbed from the cooling coils and delivered later by heating to high temperature to a condenser, requires accurate data on the maximum possible ammonia content of solutions at various temperatures and pressures. Rise of temperature always will reduce the gas content of the solution if originally saturated, but the volume dissolved is independent of pressure for slightly soluble gases like nitrogen or hydrogen, the weight dissolved, of course, being greater and directly proportional to pressure at a given temperature by reason of the increased density.

This law of independence of volume and pressure or proportionality of weight to pressure, is known as Henry's law, and is hardly true for gases as freely soluble as ammonia, probably due to some action between water and the gas, equivalent to that studied by Thomsen for solids, which tend to form hydrates of various kinds. For such gases as are soluble by weight in proportion to pressure, it is not the total pressure of the solution that is significant, but the partial pressure of the gas alone, so that the amount of mixed gases like air dissolved in water will depend, for the oxygen part, on the specific solubility of oxygen and its partial pressure in the air, which is approximately one-fifth that of the air, and the same is true for the nitrogen. Thus, in examining the solubility conditions for ammonia in water, while in practice the total pressure only is known, it is to the separate pressure of the ammonia that the amount dissolved must be referred in any attempt to establish a law of relation.

Just as gases dissolve in liquids, so are they absorbed by solids, though in this case the process is described as one of *adsorption* or occlusion. This phenomenon is now being studied in connection with coal, which it is found more or less freely absorbs air, the oxygen of which under comparatively low temperatures unites with the coal causing spontaneous combustion if the heat is *conserved*

as in a pile or its liberation in any way accelerated by heating or otherwise. Most investigations of adsorption or occlusion have been made with charcoals,

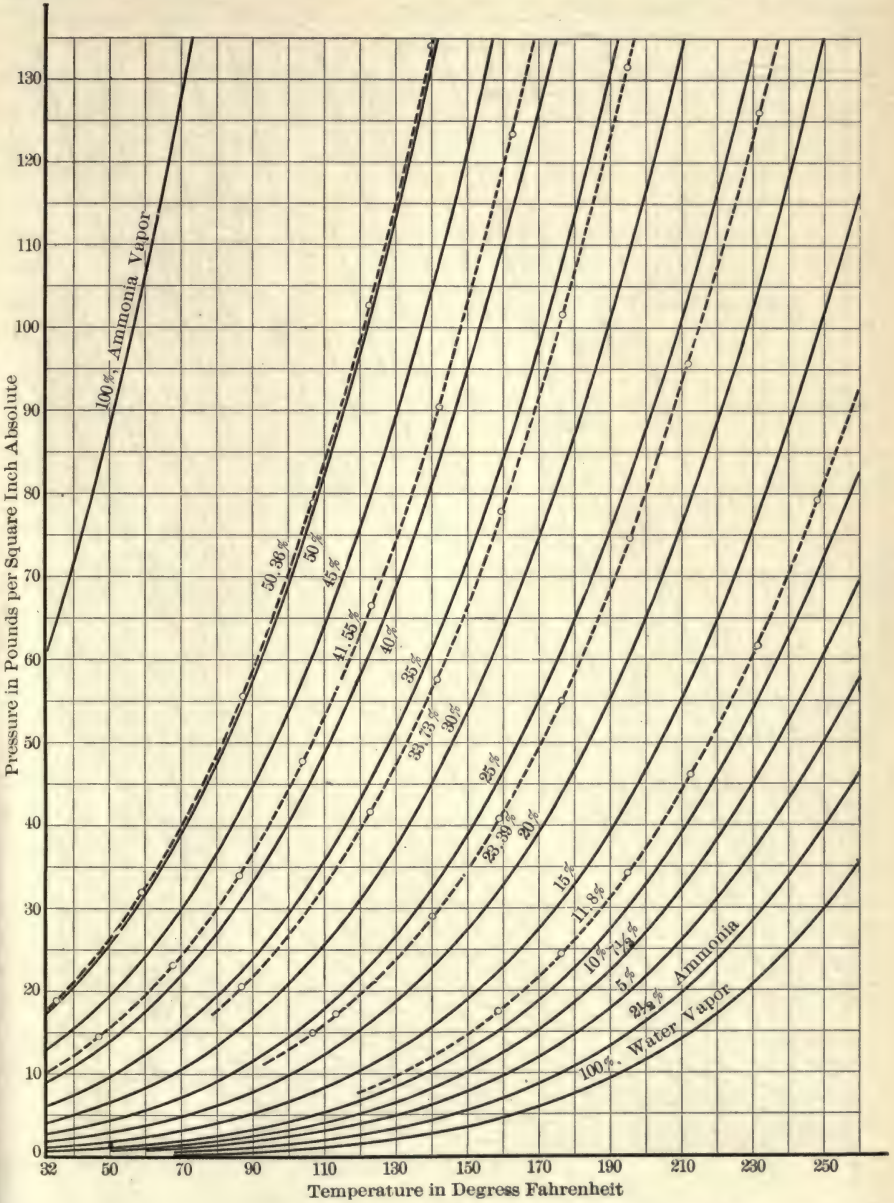


FIG. 153.—Ammonia-water Solutions, Relation between Total Pressure and Temperature (Dotted Lines Mollier Data).

the more dense varieties of which have greater adsorptive power than others. The quantity of different gases adsorbed is believed by Dewar to be the same in volume per unit of charcoal when each is held at its own condensation tem-

perature. The quantity increases with rise of pressure but not in proportion, and decreases rapidly with rise of temperature and a curve showing the tem-

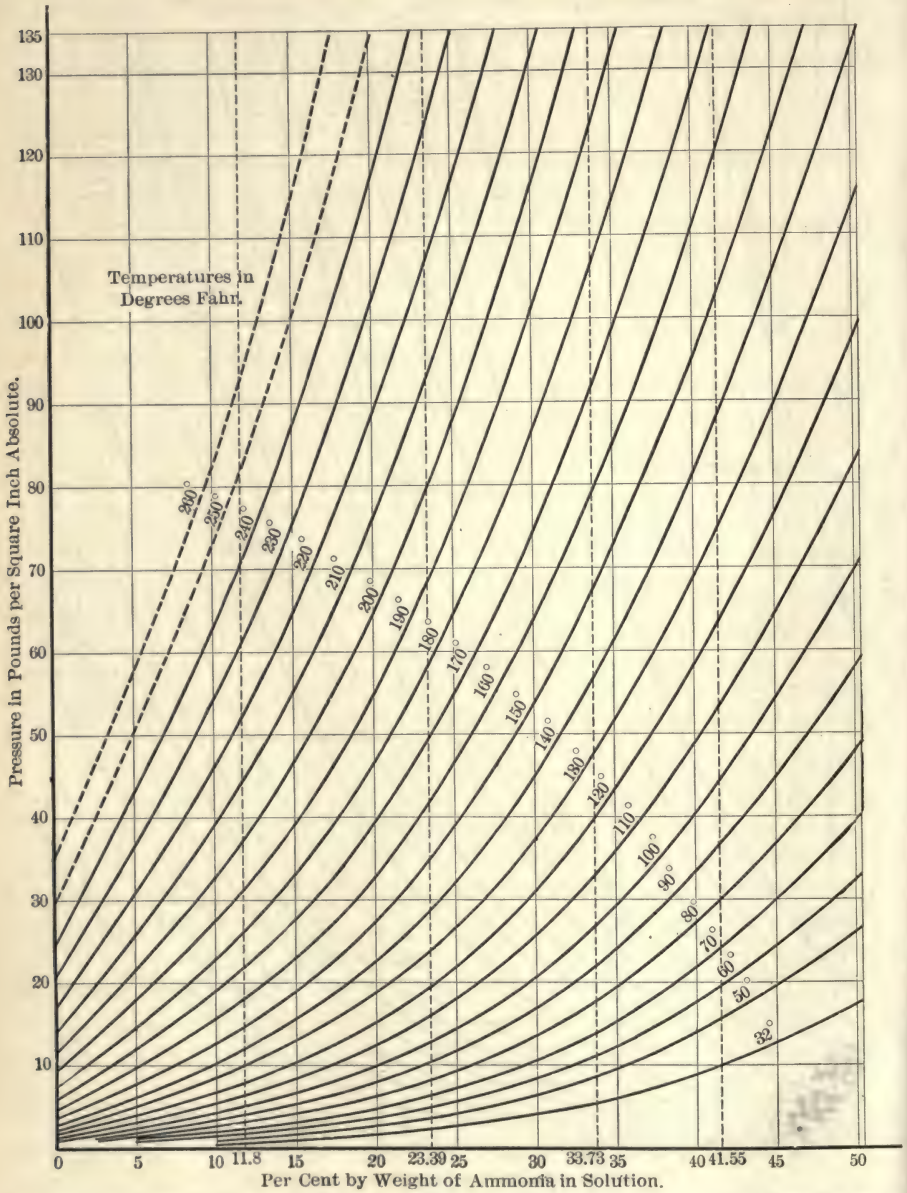


FIG. 154.—Ammonia-water Solutions, Relation between Total Pressure and Per Cent NH_3 in Solution.

perature and pressure at which equal volumes are adsorbed is similar in form to a vapor tension curve.

In the establishment of the properties of aqueous solutions of ammonia

progress was for many years very slow and the early experimental data of Abegg and Rosenfeld, 1903, for weak solutions and low pressures, that of Roscoe and Dettman, 1859, for highly concentrated solutions under low pressures at

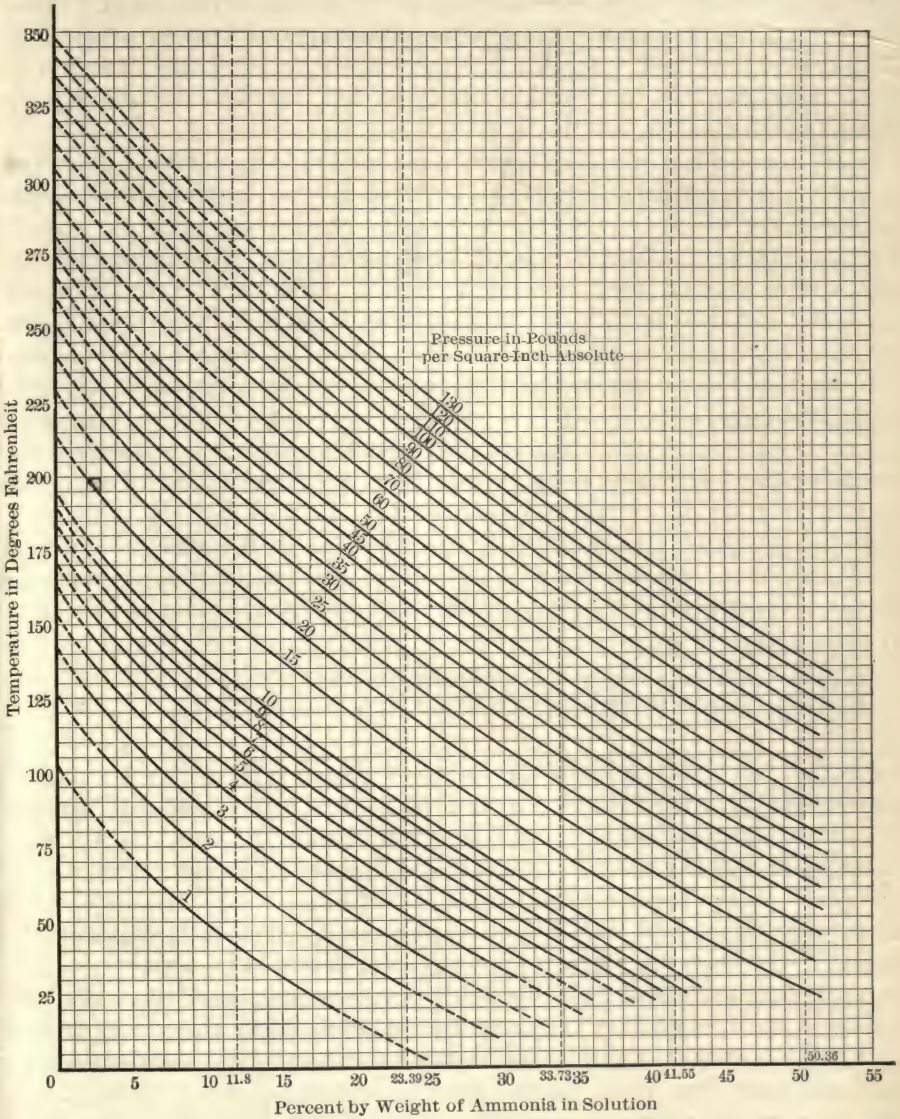


FIG. 155.—Ammonia-water Solutions, Relation between Temperature and Per Cent NH_3 in Solution.

32° F., together with that of Sims, 1862, for four temperatures from 32° F. to 212° F., failing to check and having many gaps, has been more recently supplemented by Perman, 1903, and Mollier, 1908. With these new data it has been possible to graphically fill in the data of the unexplored region with reasonable,

though not yet satisfactory accuracy. The results are given in three diagrams, grouping the three variables of pressure, temperature and concentration in pairs, Fig. 153, giving pressure-temperature, Fig. 154, pressure-concentration and Fig. 155, temperature-concentration, from which can be read off with reasonable accuracy any quantity needed in calculations and from which Table LI, has been prepared. In this table the lower numbers are new, and the upper those as given by Starr several years ago and since used by engineers engaged in refrigeration, as standards.

These data refer to the equilibrium conditions of the solution, and in using them for practical problems care must be taken to avoid applying them to other conditions, for example to solutions that are not homogeneous, or in which there has not been sufficient time for the establishment of equilibrium. Over the surface of such solutions there exists a mixture of water vapor and ammonia, each exerting its own partial pressure and the sum of the partial pressures making up the pressure of the solution; it must not be assumed, however, that the partial pressures are those of the pure substances, as this is a true solution and not a simple mixture. Moreover, there is no certainty that it is always a solution of just ammonia in water, as there is a reasonably good possibility that hydrates of ammonia may form, which would further complicate the relative pressures of the two constituents.

It is from Perman that the most accurate data on the composition of the equilibrium vapor mixture have come, and he calculated the partial pressures from the composition of the mixture determined by analysis and using Dalton's law,

$$\frac{\text{Partial pressure of ammonia}}{\text{Total pressure}} = \frac{\text{Volume of ammonia}}{\text{Total vapor volume}}.$$

No success has yet been met with, in attempting to express a general relation between partial pressures and the two variables, pressure and temperatures, so Perman's values are given as found in Table LII, and for the unexplored region it is not possible to do better than make a guess at a needed value. As a check on the Perman values and to assist in estimating needed values for other ranges the sum of the two partial pressures is given and compared with the accepted values for the total pressure, and it should be noted that as the partial pressure relations give the volumetric composition of the vapor mixture, it is unfortunate that the data are not extended to at least 35 per cent concentration to cover the solutions in the generator of the absorption refrigerating machine, from which both ammonia and water-vapor are discharged in as yet unknown relative amounts and which must be separated as completely as possible before condensation.

Any change in the ammonia content of a solution is thermal in character and is, therefore, accompanied by heat changes. When water absorbs ammonia heat is liberated, as is also the case when ammonia in solution is diluted with more water, the latter being really a further absorption in the fresh water of the ammonia already contained in its solution being diluted by it. Likewise,

when ammonia is absorbed by an ammonia solution heat is also liberated, but heat is absorbed by a solution from which ammonia is escaping, as in evaporation of liquids. There are three cases of the exothermic process, each with an endothermic inverse and these are:

(*a*) Absorption of ammonia by pure water.

(*b*) Absorption of ammonia by ammonia solution.

(*c*) Dilution by water of an ammonia solution.

Data on the amounts of heat liberated in these cases are not sufficient to establish firmly any general law of change, but are sufficient to give an approximation. The first important fact in this connection is that the heat liberated per pound of ammonia when pure water absorbs ammonia depends on the amount of water. One pound of ammonia absorbed in a little water gives out a little heat, more is liberated when more water is present, but when the amount of water is large, put at 200 times the weight of ammonia by Thomsen, the *heat of absorption is constant* and does not increase beyond this point. It may easily be, however, that the point is reached with fifty water weights, or that some heat continues to be generated for any amount of water to infinity, but so small in quantity as to be impossible to measure in the great weight of liquid present. For example, if one B.T.U. were liberated in 100 lbs. of solution, the temperature rise would be somewhere near to $^1/_{100}°$ F., and to detect this in the presence of radiation and conduction influences and make allowance for the heat of stirring would be difficult.

For engineering purposes, however, it may be accepted that the heat of absorption of a pound of ammonia is constant if the weight of water is large, and its value was fixed at 926 B.T.U. per lb. by Favre and Silberman and later redetermined by Thomsen at 8430 calories per gram molecule of NH_3, which is equivalent to $\left(\frac{9}{5} \times \frac{8430}{17}\right) = 893$ B.T.U. per pound. This value is accepted and defined as the heat of *complete absorption* for the want of a better term, and in view of the desirability of distinguishing it from the heat of absorption in *small* amounts of water or in solutions already containing appreciable amounts of ammonia, which will be designated as heat of *partial absorption.*

Experiments have further established another important relation between the heats of dilution of solutions and their original strength. According to this, solutions behave like ammonia itself with respect to pure water and liberate a little heat when a little water is added, more with larger amounts, attaining a constancy for very large amounts of water. Thus a solution of a given ammonia strength, it may be assumed, will always liberate the same amount of heat when diluted with water, if the total amount of water after dilution exceeds two hundred times the weight of ammonia present or thereabouts. This heat per pound of NH_3 contained will be designated as the heat of *complete dilution,* and defined as the heat liberated when a solution containing 1 lb. of ammonia in a given amount of water is completely diluted with water, or brought to the condition of 200 lbs. or more of water per pound of ammonia contained.

There is a rational relation between these three heats, that of *complete absorption*, which is a constant, that of *complete dilution*, which depends only on the original ratio of ammonia to water, and that of *partial absorption*, which is a function of the character of solution receiving it, or if pure water the amount. This relation is

$$\left\{\begin{matrix}\text{Heat of}\\ \text{complete}\\ \text{absorption}\end{matrix}\right\} = \left\{\begin{matrix}\text{(Heat of partial absorption)}\\ \text{(+Heat of complete dilution)}\end{matrix}\right\} = 893 \text{ B.T.U. per lb.NH}_3. \quad (705)$$

Numerical values for heats of partial absorption are entirely lacking, but Berthelot has given ten values for the heats of complete dilution for solutions containing from 1 lb. of water per pound of ammonia, to a little over one hundred, but at only one temperature, 57° F. Up to Thomsen's time these figures seem to have been the sole dependence for engineering calculation; he, however, added three more figures for more concentrated solutions, giving not the heats of complete dilution, but those of partial dilution, that is, the heat liberated when the water content is increased from one original value to three different final values not corresponding to the state of complete dilution. However, these may be used to check and correct the Berthelot figures and are especially useful because they cover the doubtful range of his determination and the range of ammonia strengths.

Berthelot's results are given in the following Table XXV and plotted in Fig. 156, from which he derives a general law of relation given by the following formula Eq. (706), which is also plotted to show its agreement with the points, and which is an equilateral hyperbola asymptotic to axes of H and w.

TABLE XXV

BERTHELOT'S DATA ON HEATS OF COMPLETE DILUTION OF AMMONIA SOLUTIONS

Original Solution, 1 Lb. Ammonia in (w) Lbs. Water.	When Completely Diluted gives	Original Solution, 1 Lb. Ammonia in (w) Lbs. Water.	When Completely Diluted gives
1.04	136	3.76	34
1.06	134	6.11	22
1.13	124 B.T.U. per lb. ammonia	10.06	2 B.T.U. per lb. ammonia
1.98	51	57.39	0
3.18	41	116.47	0

$$\text{Heat of complete dilution, B.T.U. per lb. NH}_3 = \frac{142.5}{w}\text{(Berthelot)} \quad . \quad . \quad (706)$$

The agreement, it will be noted, is not very good for larger values of w than 4 or 5, which is unfortunate, as commercial ammonia lies between one part ammonia to nine parts water, or $w = 9$, and one part ammonia to 39

parts water, or $w=39$. Nevertheless engineers have been using this formula in these doubtful ranges for some years.

By means of the few but probably accurate figures given by Thomsen and experimentally determined by him it is possible to check this practice. He measured not the heat of complete dilution as did Berthelot, but the heats of partial dilution, and the manner in which his data merge into those of Berthelot make the combined results of doubly great value because of the difference in method. Thomsen added a limited amount of water to a solution of ammonia containing 3.39 lbs. water per pound of ammonia and measured the heat, which, of course, was not the heat of complete dilution, with the following results:

THOMSEN'S DATA ON HEATS OF PARTIAL DILUTION OF AMMONIA SOLUTIONS

Original Solution. 1 Lb. Ammonia in (w) Lbs. Water.	Final Solution. 1 Lb. Ammonia in (w) Lbs. Water.	B.T.U. per Lb. Ammonia Absorbed by the Ammonia Solution.
$w=\begin{cases}3.39\\3.39\\3.39\end{cases}$	$w=\begin{cases}19.27\\29.86\\56.33\end{cases}$	$\begin{matrix}+34\\+37\\+40\end{matrix}$

These results have been fitted into those of complete dilution by the relations of Eqs. (707) and (708), and by the introduction of one assumption.

$$\left\{\begin{matrix}\text{Heat of complete dilu-}\\\text{tion of original am-}\\\text{monia solution per}\\\text{lb. } NH_3\end{matrix}\right\}=\left\{\begin{matrix}\text{Heat of partial dilution}\\\text{from original to some}\\\text{greater water content}\\\text{per lb. } NH_3.\end{matrix}\right\}+\left\{\begin{matrix}\text{Heat of complete}\\\text{dilution of the}\\\text{new solution per}\\\text{lb. } NH_3.\end{matrix}\right\} \quad (707)$$

or

$$\left\{\begin{matrix}\text{Heat of complete dilu-}\\\text{tion of new solution}\\\text{per lb. } NH_3\end{matrix}\right\}=\left\{\begin{matrix}\text{Heat of complete dilu-}\\\text{tionof original solution}\\\text{per lb. } NH_3.\end{matrix}\right\}-\left\{\begin{matrix}\text{Heat of partial dilution}\\\text{from original to final}\\\text{solution per lb. } NH_3.\end{matrix}\right\} \quad (708)$$

If the heat of complete dilution of the original solution containing 3.39 lbs. water per pound ammonia be taken from the Berthelot equation its value is 42, so assuming this to be correct it may be introduced in Eq. (708) as a constant, giving with the Thomsen figures the following:

$$\left\{\begin{matrix}\text{Heat of complete dilution}\\\text{of new solution, per lb.}\\NH_3.\end{matrix}\right\}=42-\left\{\begin{matrix}\text{Heat of partial dilution}\\\text{from original to final}\\\text{solution per lb. } NH_3.\end{matrix}\right\}$$

Heat of complete dilution per pound NH_3 with

$$(w=19.27)=42-34\ (\text{Thomsen})=8;$$
$$(w=29.86)=42-37\ (\text{Thomsen})=5;$$
$$(w=56.33)=42-40\ (\text{Thomsen})=2.$$

These three new points are also plotted and agree well with the Berthelot equation, better even than the original Berthelot points themselves, so that Thomsen's partial dilution figures seem to confirm Berthelot's complete dilution data and the curve of Fig. 156 and Eq. (706) may be taken as truly represen-

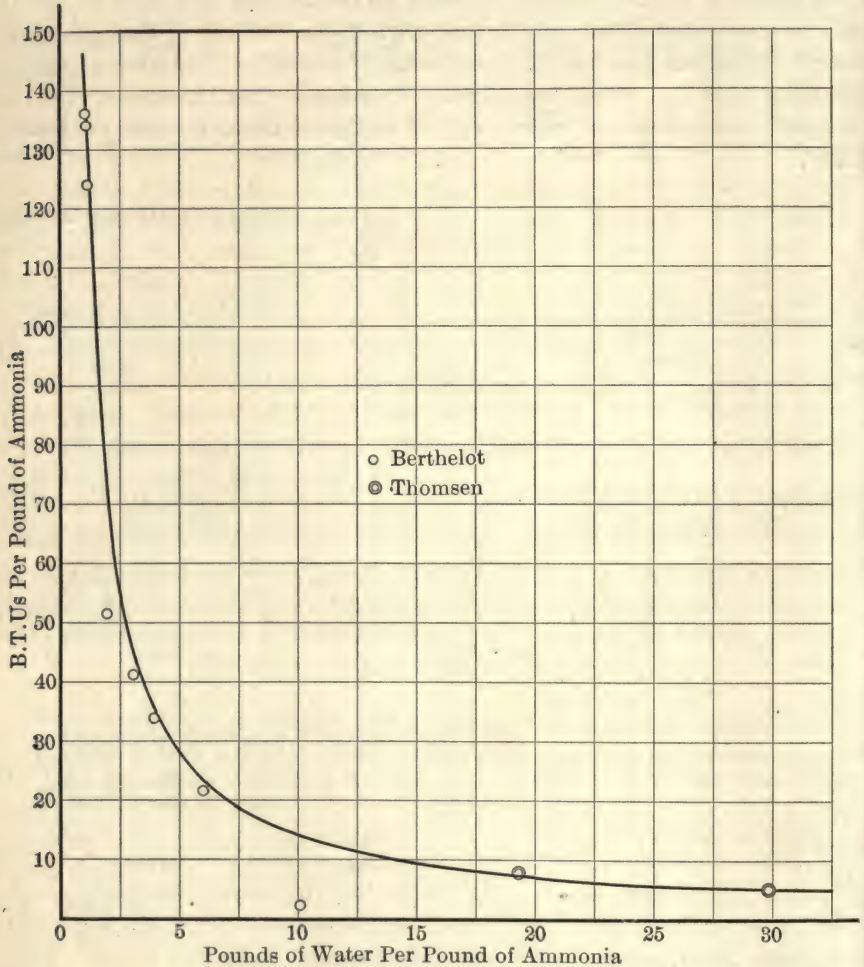


FIG. 156.—Heat of Complete Dilution of Ammonia—Water Solutions, by Excess Water.

tative of the heat of complete dilution of ammonia solutions and indirectly of course, heats of partial dilutions as well.

Heats of absorption are more often needed in practical problems than heat of complete or partial dilution, but these heats follow on the assumption that th*heat of complete absorption, must be equal to the sum of the heat of partial absorp tion, and the heat of complete dilution of the solution so formed.*

Hence

$$\left\{\begin{array}{l}\text{Heat of partial absorption}\\ \text{in } w \text{ lbs. water B.T.U.}\\ \text{per lb. } NH_3.\end{array}\right\} = \left\{\begin{array}{l}\text{Heat of complete}\\ \text{absorption in ex-}\\ \text{cess water B.T.U.}\\ \text{per lb. } NH_3.\end{array}\right\} - \left\{\begin{array}{l}\text{Heat of complete dilution by}\\ \text{excess water of solution}\\ \text{with } w \text{ lbs. water per lb.}\\ NH_3, \text{ B.T.U. per lb. } NH_3.\end{array}\right\}$$

or

$$\left\{\begin{array}{l}\text{Heat of partial absorption in water}\\ \text{B.T.U. per lb. } NH_3 \text{ absorbed}\end{array}\right\} = 893 - \frac{142.5}{w}. \quad \ldots\ldots\ldots \quad (709)$$

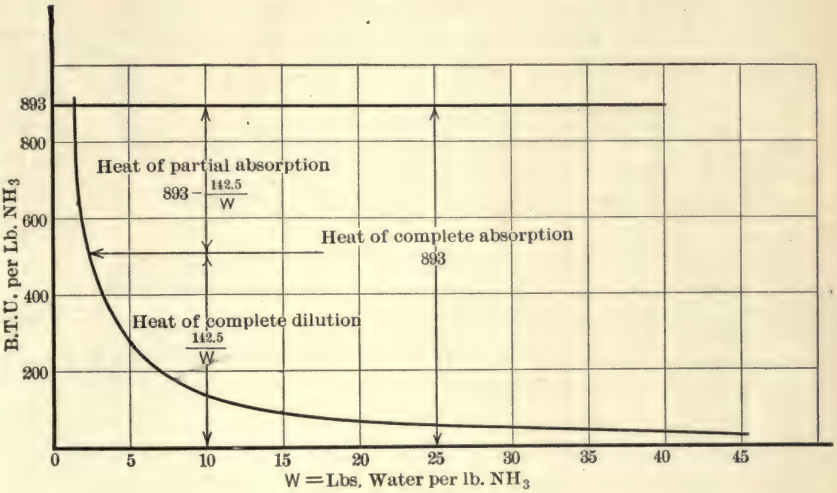


FIG. 157.—Ammonia-water Solutions, Relation between Heats of $\left\{\begin{array}{l}\text{Partial Absorption}\\ \text{Complete Absorption}\\ \text{Complete Dilution}\end{array}\right\}$ Shown Graphically.

It is interesting to note that the relation between these three quantities, heat of partial absorption in limited amounts of water, heat of complete absorption in excess water and heat of complete dilution of the solution in excess water, can be shown graphically in Fig. 157, where the designations are self-explanatory.

Most important are the heats liberated when ammonia is absorbed not by water but by weak ammonia solutions themselves, and these heats of partial solution of ammonia in ammonia solutions are obtainable from the data already established by a comparatively simple relation. In this case there are two different solutions in question, an original one which becomes the second one on receiving more ammonia. If the water received all the ammonia contained in the second solution a certain quantity of heat would be liberated and it must be equal to the total amount liberated by absorption of the first ammonia in the water, and by the absorption of the second ammonia in the resulting solu-

tion, whence this last quantity is obtainable by differences between the heats of partial solution of ammonia in water alone.

Let w = lbs. water per lb. ammonia in original solution which, therefore, consists of $w+1$ lbs. solution, 1 lb. of ammonia and w lbs. of water.

" A = lbs. ammonia added per lb. ammonia already present, making new solution containing w lbs. of water and $A+1$ lbs. of ammonia or $\frac{w}{A+1}$ lbs. water per lb. ammonia in $w+A+1$ lbs. of solution.

Then

$$\left\{\begin{array}{l}\text{Heat of partial absorption of}\\ \text{original 1 lb. ammonia in } w\\ \text{lbs. water, B.T.U.}\end{array}\right\} = 893 - \frac{142.5}{w},$$

$$\left\{\begin{array}{l}\text{Heat of partial absorption of}\\ \text{all } (A+1) \text{ lbs. of ammonia}\\ \text{in } w \text{ lbs. of water, B.T.U.}\end{array}\right\} = \left(893 - \frac{142.4}{\frac{w}{A+1}}\right)(A+1),$$

Whence

$$\left\{\begin{array}{l}\text{Heat of partial}\\ \text{absorption of}\\ A \text{ lbs. } NH_3 \text{ by}\\ \text{solution con-}\\ \text{taining 1 lb.}\\ NH_3 \text{ in } w \text{ lbs.}\\ \text{water, B.T.U.}\end{array}\right\} = \left(893 - \frac{142.5}{\frac{w}{A+1}}\right)(A+1) - \left(893 - \frac{142.5}{w}\right) = A\left[893 - \frac{142.5}{w}(A+2)\right] \quad (710)$$

Therefore

$$\left\{\begin{array}{l}\text{Heat of partial absorption of } A \text{ lbs.}\\ NH_3 \text{ by solution containing 1 lb.}\\ NH_3 \text{ in } w \text{ lbs. water B.T.U. per lb.}\\ NH_3\end{array}\right\} = 893 - \frac{142.5}{w}(A+^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (711)$$

As ammonia solution strengths are often given in terms of per cent of ammonia present by weight and the heat of absorption in terms of changes in the per cent strength, the following conversion factors are useful:

$$\text{Per cent ammonia in original solution} = S_1 = \frac{100}{w+1},$$

$$\text{Per cent ammonia in final solution} = S_2 = \frac{100(A+1)}{w+A+1}.$$

Whence

$$\left.\begin{array}{l} w = \dfrac{100 - S_1}{S_1}, \quad (a)\\ A = \dfrac{100}{S_1}\left(\dfrac{S_2 - S_1}{100 - S_2}\right) (b)\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (712)$$

These on substitution in Eq. (711) give the heat of absorption per pound of ammonia absorbed to change the per cent NH_3 from S_1 to S_2.

$$\left\{\begin{array}{l}\text{Heat of partial absorption of 1 lb.}\\ \text{of } NH_3 \text{ in a solution containing}\\ \text{1lb. } NH_3 \text{ in } w \text{ lbs. water, B.T.U.}\end{array}\right\} = 893 - 142.5\left(\frac{S_1}{100-S_1}+\frac{S_2}{100-S_2}\right). \quad . \quad (713)$$

When, however, the absorbed ammonia is to be driven off from the solution by heating it, the discharge will consist partly of ammonia vapor and partly water vapor, so the heat of liberation of a given amount of ammonia from solution will be in practice that for the ammonia and equal to what would be liberated by its absorption, but also in addition the heat of vaporization of the water vapor. When, as in absorption refrigerating machine generators, the discharged vapors meet incoming solution and are thereby partly condensed, practically all except perhaps 2 per cent of the heat of vaporization of water vapor is returned and the net heat of ammonia liberation is not materially different from the value for absorption. If this is not done a correction must be introduced for the water vapor.

To assist in the solution of problems on the amount of non-condensible gases to be handled by dry vacuum pumps serving steam condensers, and on the change in the composition of gases when scrubbed by water in the course of cooling and cleaning after manufacture, a table of solubilities of various gases in water is added at the end of this Chapter in Tables LIII and LIV, as compiled from various sources and reported in the Smithsonian physical tables. The numbers in the tables are volumes of *standard* gas, that is, gas measured at 32° F. and 1 atm. pressure, per unit volume of water, though they are at a different volume as absorbed or when absorbed at the temperature given. The pressure of the solution is in every case 29.92 ins. Hg., absolute and this is the combined pressure of both the gas and the water vapor.

Example. In the absorber of an ice machine of the absorption type, a weak solution of ammonia in water takes up the ammonia vapor coming from the refrigerating coils, the heat found being removed by water. The weak liquor, as it is called, is continuously supplied and the rich liquor continually pumped away to the generator where, by heating, some of the ammonia vapor is driven off to the condenser. Assuming the action in the absorber to be merely one of ammonia dissolving in a weak solution and that no water vapor leaves the generator, what will be the heat produced in the absorber and needed in the generator per pound of ammonia for the following assumed conditions:

Weak solution, 15 per cent NH_3; strong solution, 30 per cent NH_3; temperature in absorber, 80° F.

From Eq. (713), the heat of absorption per lb. of ammonia absorbed will be

$$893 - 142.5\left(\frac{S_1}{100-S_1}+\frac{S_2}{100-S_2}\right),$$

where S_1 and S_2 are the per cents of ammonia in weak and rich solutions, respectively, or

$$Q = 893 - 142.5\left(\frac{15}{85} + \frac{30}{70}\right) = 807. \text{ B.T.U. per lb. } NH_3 \text{ absorbed}$$

Prob. 1. Ammonia is being absorbed by water at a temperature of 100° F. The solution contains 10 per cent of ammonia. If the total pressure is 15 ins. of Hg, what part of this is due to foreign gases, what part to ammonia and what part to water vapor?

Prob. 2. How many cubic feet and how many pounds of the following gases can be separately dissolved in 1000 gallons of pure water at atmospheric pressure and a temperature of 50° F.? Air, carbon dioxide, and hydrogen. How would the results be changed if the pressure were doubled? If the temperature were doubled?

Prob. 3. When either water or ammonia is added to an ammonia solution, heat is evolved. Explain why this is so.

Prob. 4. Ammonia is being absorbed by a stream of running water, there being 5 lbs. of water per pound of ammonia. What will be the heat developed per pound of ammonia liquor formed?

Prob. 5. Ten pounds of the above solution receives an additional pound of ammonia. How much heat is generated by this action?

Prob. 6. A solution containing 10 per cent of ammonia receives an addition of another 10 per cent. What was the amount of heat developed per pound of ammonia and per pound of solution when the second portion of the ammonia was added?

Prob. 7. The pressure in the absorber of an ammonia absorption machine is one atmosphere and the temperature is 80° F. What is the maximum per cent of ammonia which can be absorbed by the water?

Prob. 8. The generator is working under a pressure of 125 lbs. per square inch gage and the heat is supplied by a steam coil in which the pressure is 30 lbs. per square inch gage. What per cent of ammonia will be left in solution after passing through the generator and about how much steam must condense per lb. NH_3 discharged?

13. Combustion and Related Reactions. Relative Weights and Volumes of Substances and Elements, before and after Reaction. Not only may matter assume the three states of solid, liquid and vapor separately, in pairs simultaneously, or even all three together with various accompanying or causal temperature, pressure, or heat content, conditions, without chemical change of the matter itself, as already discussed, but matter itself may change in kind. As Mellor puts it—"matter appears to be endowed with properties in virtue of which two or more dissimilar substances, when brought into close contact give rise to other forms of matter possessing properties quite distinct from the original substance." "The process of change is called a chemical reaction. Chemical changes are assumed to be characterized by molecular rearrangement according to which molecules of elements may divide into atoms and the separated atoms of one combine with those of another element, to form a molecule of a new substance to be called a compound. Similarly, the molecules of compounds may split and reassociate, part of one, with part of another, to make a new compound or a single compound may split up into its elements which

remain separated, the last case being generally termed dissociation. All three classes of change of substance are classifiable as chemical reactions, and there is really no very rigid line to be drawn between the sub-classes of reaction known as combination and dissociation except when applied to the same substances, in which case one is the reverse of the other. The complete or partial destruction of a substance as such is commonly termed decomposition as, for example, when hydrocarbon constituents of coal volatile, or liquid fuel, are changed by excessively high temperature into free carbon, and other hydrocarbons or even free hydrogen.

Every chemical change whether of combination or dissociation is accompanied by a heat change of the system or group of substances. When the reaction is such that heat is set free tending to raise the temperature of the reacting mass unless it be carried away as liberated, the reaction is classed as *exothermic*. On the contrary, those reactions that are accompanied by heat absorption, tending to lower the temperature, unless heat be added to supply the absorption, are classed as *endothermic*.

It appears then that every reaction *tends* to change the temperature of the system, causing it to rise or fall according as the reaction is exothermic or endothermic, except in the one case where several reactions occur simultaneously, in which all the exothermic, set free just enough heat to supply what is required for the endothermic.

The most important reaction in engineering is combustion, defined as the chemical reaction between fuels and the oxygen of the air, which is exothermic or heat liberating, and the source of practically all the heat used in engines for conversion into work. Combustion is often classed as an oxidation process, and is thus distinguished from another important engineering class of related reactions, reduction or the reverse of oxidation, the most prominent case of which is the change from carbon dioxide to carbon monoxide in gas producers, and in a lesser degree in furnaces, according to which there is a reduction of oxygen content per molecule, and which process is endothermic. The formation of carbon monoxide directly from carbon by its oxidation, is sometimes defined as partial combustion of carbon or incomplete oxidation because the product, carbon monoxide, may further oxidize or burn to carbon dioxide by taking up more oxygen. The substances produced by the partial combustion or partial oxidation of one fuel element may also be considered as the result of the dissociation or reduction or deoxidation of the substances produced by its complete combustion. Ordinarily, partial combustion and reduction are considered as reverse processes producing the same substances by (*a*) exothermic reaction of the primary substance in partial combustion and (*b*) the endothermic deoxidizing reaction of the products of complete exothermic reactions of the same primary substances. It is also common to consider only the reaction of a substance or so-called fuel element with oxygen, as combustion and other processes whether involving oxygen or not, as related reactions. Thus, carbon combining with oxygen to form carbonic acid is complete, and carbon combining with oxygen to form carbon monoxide incomplete combustion of carbon, while carbon

monoxide and steam reacting can hardly be considered as combustion, and is best classified as a related reaction.

The number of elements entering into combustion and related reactions is small, but the number of possible substances and reactions between them is amazingly large, and the prediction of just what reactions will take place between various groupings or mixtures of these substances extremely difficult and in some cases quite impossible. The study of possible reactions has become the province of physical chemistry, especially when the conditions controlling the result are also subjects of study. These conditions include the temperature, pressure, electrical state and the mutual relation of the elementary compounds present, and the relation of these various conditions to the primary and resultant substances, and the intermediate, successive, simultaneous or parallel reactions constitute the subject matter of the study of chemical equilibrium. If chemical equilibrium were better understood than it is, it would be possible to predict the resultant from primary substances for specified conditions, but at the present time this is impossible even though some of the greatest scientists the world has ever known, have devoted their lives to the study and many volumes of specific results have been published. Even if the exact prediction of the direction in which reaction would proceed in an ordinary complex system and the extent to which it would go in that direction, were made possible by a more complete thermochemistry than now exists, it would not be of much use in engineering because *it is seldom possible to define the conditions that are present or to be met.* For example, in gas producers, solid coal, air and steam are the primary materials and the product or result of their mutual reaction is a combustible gas. Engineers would like very much to be able to predict and control the exact composition of this gas, but this is not possible nor will it in all probability ever be possible, because it would first be necessary to fix the chemical and thermal character of the coal, air and steam, to fix their relative quantities, to maintain an absolutely uniform fuel bed as to size, porosity and, quality with some other conditions equally elusive.

It must be understood, therefore, that while the possibility or even probability of certain reactions taking place may be known, it is quite impossible to predict just what will happen, or what products will result, when a given group of primary substances mutually, react so that many important problems of combustion in boiler furnaces and gas producers cannot be solved except by approximation.

The approximation takes the form of a calculation which is exact, based on an hypothesis which does not represent the facts of the case. In other words, engineering calculations about combustion are always to be prefaced by a statement that certain substances are going to change completely or within a given degree to certain others, whether they will or not. Furthermore, the substances must be defined chemically by their symbols, and then will it be possible to calculate the relative weights and volumes of the various substances that can so react, the corresponding relative weights and volumes of the products, and the heat liberated or absorbed, but not otherwise. It is quite important

that too much confidence be not put in these results, which are no more correct than is the assumption of what substances are to be formed.

From the study of chemical equilibrium a few principles of guidance have been developed that help a little but not very much. For example, Van't Hoff's "*law of movable equilibrium*" says that when the temperature of a system is raised that reaction takes place which is accompanied by absorption of heat and conversely. Another similar law is to the effect that a rise of pressure in a system in equilibrium causes that reaction that is accompanied by a reduction of volume. There are more of this sort but they are entirely too general to make it possible to avoid the procedure adopted by engineers of assuming the kind of reaction, and then calculating quantities and heats that should and would accompany it, if it did take place and, however, crude this may look it is very useful, and in most cases good enough or rather as good as the knowledge of other conditions to be met.

Carbon and hydrogen are the only chemical elements of fuel value, and all commercial fuels, including wood, peat, lignite, bituminous and anthracite coals, charcoal, coke, crude petroleum oils with their distillates, gasolene, kerosene, and their residues, tars, heavy oils, alcohols, benzole a bituminous coal product, natural gas as well as blast furnace, carburetted and uncarburetted water gas, coal gas, producer gas, and oil gas, are *compounds*, *mixtures* and *mixtures of compounds* of these fuel elements, with oxygen in some cases. The one exception is sulphur, which exists in many solid, liquid and gaseous fuels as an impurity, but which also has some small fuel value.

This being the case, the number of products to be formed by the *complete* combustion of fuel is also small, and includes only carbon dioxide and water vapor, with the nitrogen carried by the air, and a small amount of sulphur compounds, often ignored.

The process of reaction, whether combustion or one of the related ones, is to be described by the usual chemical equation which has the additional significance of showing the relative weights involved directly, because,

(*a*) The total number of atoms of each chemical element on each side of the equation must be the same.

(*b*) The sum of the products of the atomic weight of each atom and the number present, must be the same on each side of the equation.

This is the same as saying, (*a*) that the total weight of hydrogen in the products must be the same as the total weight of hydrogen in the original mixture and so also for the other elements, and (*b*) the total weight of the original mixture of reacting substances must be the same as that of the products.

Natural fuels while sometimes consisting of simple elements like carbon or hydrogen alone, or simple compounds like carbon monoxide or methane alone, are more often mixtures. Their reaction equations are then to be derived from combinations of the equations representing the reactions of elements and compounds with oxygen or with each other. To facilitate this the following table of some characteristic reactions of simple substances is inserted:

SOME COMBUSTION AND RELATED REACTIONS OF FUEL ELEMENTS AND COMPOUNDS

Combustion of fuelel ements

$$C + O_2 = CO_2 \quad (1)$$

$$2H_2 + O_2 = 2H_2O \quad (2)$$

Combustion of Chemical Compounds

Product of partial carbon combustion

$$2CO + O_2 = 2CO_2 \quad (3)$$

Hydrocarbons

Paraffin series C_nH_{2n+2}:

Methane $\quad CH_4 + 2O_2 = CO_2 + 2H_2O \quad (4)$

Ethane $\quad C_2H_6 + \frac{7}{2}O_2 = 2CO_2 + 3H_2O \quad (5)$

General C_nH_{2n+2} $\quad \left(\frac{3n+1}{2}\right)O_2 = nCO_2 + (n+1)H_2O \quad (6)$

Olefines or Ethylene series C_nH_{2n}:

Ethylene $\quad C_2H_4 3O_2 = 2CO_2 + 2H_2O \quad (7)$

Propylene $\quad C_3H_6 + \frac{9}{2}O_2 = 3CO_2 + 3H_2O \quad (8)$

General $\quad C_nH_{2n} + \frac{3n}{2}O_2 = nCO_2 + nH_2O \quad (9)$

Alcohols

Monatomic alcohols $C_nH_{2n+1}OH$:

Methyl alcohol $\quad CH_3OH + \frac{3}{2}O_2 = CO_2 + 2H_2O \quad (10)$

Ethyl alcohol $\quad C_2H_5OH + 3O_2 = 2CO_2 + 3H_2O \quad (11)$

General $\quad C_nH_{2n+1}OH + \frac{3n}{2}O_2 = nCO_2 + (n+1)H_2O \quad (12)$

Related Reactions, Incomplete Combustion, Decomposition, Dissociation.

$$2C + O_2 = 2CO \quad (13)$$

$$CO_2 + H_2 = CO + H_2O \quad (14)$$

$$CH_4 + CO + H_2 = C_2H_4 + H_2O \quad (15)$$

$$C + CO_2 = 2CO \quad (16)$$

$$CO + H_2O = CO_2 + H_2 \quad (17)$$

$$2CO_2 = 2CO + O_2 \quad (18)$$

$$2H_2O = 2H_2 + O_2 \quad (19)$$

$$CH_4 = C + 2H_2 \quad (20)$$

$$C_2H_4 = C + CH_4 \quad (21)$$

$$C_6H_{14} = C_2H_4 + C_4H_{10} \quad (22)$$

$$2CH_4 = C_2H_2 + 3H_2 \quad (23)$$

$$3C_2H_2 = C_2H_4 + 2C_2H_2 \quad (24)$$

All these and any other reactions are characterized by definite weigh relations which are given directly by the reaction equation by introducing th weight of each element from a table of atomic weights, and for this purpos the nearest whole number is close enough. For example, the complete com bustion of hexane, C_6H_{14}, the main constituent of gasolene, to carbon dioxid and water, is given by,

$$2C_6H_{14} + 19O_2 = 12CO_2 + 14H_2O.$$

or

$$C_6H_4 + \tfrac{19}{2}O_2 = 6CO_2 + 7H_2O$$

This may be interpreted as follows, taking the atomic weight of C = 12, of H = 1, and of O = 16.

$$\left.\begin{array}{l}(6\times12=72)\text{ lbs. carbon}\\ +(14\times1=14)\text{ lbs. hydrogen}\\ +(9\tfrac{1}{2}\times2\times16=304)\text{ lbs. oxygen}\end{array}\right\}\text{make}\left\{\begin{array}{l}(6\times12=72)\text{ lbs. carbon}\\ +(6\times2\times16=192)\text{ lbs. oxygen}\\ +(7\times2\times1=14)\text{ lbs. hydrogen}\\ +(7\times16=112)\text{ lbs. oxygen}\end{array}\right.$$

or

$$\left.\begin{array}{l}(72+14=86)\text{ lbs. hexane}\\ +(304)\text{ lbs. oxygen}\end{array}\right\}\text{make}\left\{\begin{array}{l}(72+192=264\text{ lbs. carbon dioxide}\\ +(14+112=126)\text{ lbs. water vapor}\end{array}\right.$$

This general weight relation may be simplified somewhat by considering the weight of each one of the substances as unity, thus yielding the following different, but equivalent interpretations:

$$\left(1\text{ lb. hexane}+\frac{304}{86}\text{ lbs. oxygen}\right)\text{ make }\left(\frac{264}{86}\text{ lbs. car. diox.}+\frac{126}{86}\text{ lbs. water.}\right)$$

$$\left(1\text{ lb. oxy.}+\frac{86}{304}\text{ lbs. hexane}\right)\text{ make }\left(\frac{264}{304}\text{ lbs. car. diox.}+\frac{126}{304}\text{ lbs. water.}\right)$$

$$\left(1\text{ lb. carb. diox.}+\frac{126}{264}\text{ lbs.water}\right)\begin{array}{l}\text{result from com-}\\ \text{plete combus-}\\ \text{tion of}\end{array}\left(\frac{86}{264}\text{ lbs. hex.}+\frac{304}{264}\text{ lbs. oxy.}\right)$$

$$\left(1\text{ lb. water}+\frac{264}{126}\text{ lbs. car. diox.}\right)\begin{array}{l}\text{result from com-}\\ \text{plete combus-}\\ \text{tion of}\end{array}\left(\frac{86}{126}\text{ lbs. hex.}+\frac{304}{126}\text{ lbs. oxy.}\right)$$

It might also be said from the same data that,

1 lb. hexane completely burned with oxygen yields 390 lbs. products.

1 lb. oxygen will completely burn $\frac{86}{304}$ lbs. hexane and yield 390 lbs. products.

This particular example can be analyzed in still another way, yielding a general expression for the combustion of an analyzed fuel.

Fuel analyses are reported in two ways: (*a*) Proximate, giving per cent of each independent compound, or separately determined constituent substance; (*b*) ultimate, giving the per cent of each chemical element. Applying this distinction to the original mixture of hexane and oxygen, and its products the proximate analyses are:

$$\text{Original mixture}\left\{\begin{array}{lll}\text{Hexane} & =\frac{86}{390} & =22.1\%\text{ by weight.}\\ \text{Oxygen} & =\frac{304}{390} & =77.9\%\text{ by weight.}\\ \text{Total} & & 100.00\%\end{array}\right.$$

Products	Carbon dioxide	$=\frac{264}{390}$	= 67.6 % by weight.
	Water vapor or liquid	$=\frac{126}{390}$	= 23.4 % by weight.
	Total		100.00%

Using the ultimate analysis method of designation for mixtures and products

Original mixture	Carbon (in hexane)	$=\frac{72}{390}$	= 18.4% by weight.
	Hydro. (in hexane)	$=\frac{14}{390}$	= 3.6 % by weight.
	Oxygen (free)	$=\frac{304}{390}$	= 78.0 % by weight.
	Total		100.00%

Products	Carbon (in CO_2)	$=\frac{72}{390}$	= 18.4 % by weight.
	Hydrogen (water)	$=\frac{14}{390}$	= 3.6 % by weight.
	Oxygen (in CO_2)	$=\frac{192}{390}$	$=\frac{304}{390}$ = 78.0 % by weight.
	Oxygen (water)	$=\frac{112}{390}$	
	Total		100.00%

Neither combustion or its related reactions take place with oxygen alone, but with air containing oxygen, 23.2 per cent, and nitrogen, 76.8 per cent by weight, each pound of oxygen carrying with it $\frac{76.8}{23.2}=3.31$ lbs. of nitrogen or existing in 4.31 lbs. of air. The nitrogen is generally considered neutral, though it may form compounds with hydrogen, such as ammonia directly, or with oxygen, such as nitrous oxide if conditions are right. If neutral, it has the effect of changing the weight of the mixture by an amount depending on the proportion of oxygen that came from air.

$$\left\{\begin{array}{l}\text{(Weight of mixture with oxygen alone)}\\ \quad +3.31\text{ (wt. oxygen present)}\end{array}\right\} = \text{(weight of mixture with air)},$$

or

$$\left\{\begin{array}{l}\text{(weight of nitrogen to be added when}\\ \text{air is used instead of oxygen)}\end{array}\right\} = 3.31\text{ (wt. of oxygen present)}.$$

This will add in the present case $304 \times 3.31 = 1006$ lbs. of nitrogen in the combustion of one molecule or 86 lbs. of hexane, the sum of 304 lbs. of oxygen and 1006 lbs. of nitrogen, giving 1310 lbs., the weight of air required, and raising the total weight of the mixture to 1396 lbs. Of course, the per cent of the various constituents of the mixture and products is now changed, but the amount of change is quickly computed. All these relative numbers can be conveniently given in tables of conversion factors, such a table for hexane being given below, as a type form, useful in every-day work when it is necessary to make repeated calculations with some one fuel.

CONVERSION FACTORS FOR WEIGHTS IN THE COMPLETE COMBUSTION OF HEXANE WITH AIR TO WATER AND CARBON DIOXIDE

Original Mixture.	Final Mixture.	Hexane C_6H_{14}.	Oxygen O_2.	Nitrogen N_2.	Air.	Carbon Dioxide CO_2.	Water H_2O.
1396.	1396.	86.	304.	1006.	1310.	264.	126.
1.	1.	.0616	.218	.720	.938	.189	.091
16.22	16.22	1.	3.54	11.68	15.22	3.07	1.46
4.59	4.59	.283	1.	3.31	4.31	.87	.41
1.387	1.387	.085	.302	1.	1.302	.262	.125
1.066	1.066	.066	.232	.768	1.	.202	.096
5.29	5.29	.326	1.15	3.81	4.96	1.	.48
11.08	11.08	.683	2.42	7.98	10.40	2.10	1.

The average analysis of pure air is given by Parkes as being $O_2 = 20.96$ per cent; $CO_2 = .04$ per cent; $N_2 = 76.00$ per cent by volume, giving $O_2 = 23.20$ per cent; $CO_2 = .06$ per cent; $N_2 = 76.74$ per cent by weight. Regarding the CO_2 as being negligible, the relation may be used for the purpose of computations of this sort, $O_2 = 23.2$ per cent and $N_2 = 76.8$ per cent. On the assumption that any nitrogen present came from air and not from any other compound, such as ammonia, it must have been associated with oxygen in the proportion 3.31 to 1 of O_2. The weight of oxygen on both sides must be equal and the weight of air on one side must be 4.31 times the weight of nitrogen. Of the nitrogen that is present $\frac{1}{3.31} \times$ (the weight of nitrogen present) must have come from air and the rest from the other substances.

A much-used weight relation is, for the weight of air and products per pound of fuel, and on the assumption that

H = part of hydrogen by weight per pound fuel,
C = part of carbon by weight per pound fuel,

$$\text{Weight of air per lb. of fuel} = C \times \frac{32}{12} \times 4.31 + H \times 8 \times 4.31 = 11.49C + 34.46H, \quad (71)$$

Weight of products per lb. fuel $= 3\frac{2}{3}$C lbs. $CO_2 + 9$H lbs. $H_2O + \frac{32}{12} \times 3.31$C lbs. N $+ 8 \times 3.31$H lbs. N

$= 3.67$C in form of $CO_2 + 9$H in form of $H_2O + 8.82$ C in form of N

$= 12.49$C $+ 35.46$H, ... (715)

for complete combustion in air with no more air supplied than enters into the reaction

Most of the practical problems concerning the relative amounts of substances involved in combustion and reduction processes are concerned with gases, at least on one side of the equation and sometimes on both. Direct combustion and the gasification of fuels in producers and complete combustion in boiler furnaces always yields gas mixtures, the composition of which is always volumetrically determined by analysis, while the explosive mixtures or primary working substances of gas engines are gaseous initially and remain so after combustion. It is quite necessary, therefore, to transform the weight relations of the reaction equation into another form yielding volumes. There are three ways of doing this, all equivalent and all yielding the same result if the constants used are consistent, and if the gases and vapors follow the Avagadro law, on which the most useful of the methods depends.

1. The volume at standard conditions of any substance can be found from the weight present by multiplying that weight by the specific volume of the substances, in cubic feet per pound, at standard condition of 1 atm. at 32° F.

2. The molecular weight in pounds of any gaseous or vapor substance assumed to follow Avagadro's law occupies 358 cu.ft.

3. The volumetric relations are given directly by the coefficients of the substance in the chemical equation when the substances are gaseous and all enter into the reaction, and when each is expressed in terms of molecules present.

The first method needs no explanation or further development and the second and third are really one. *If in the reaction equations there be written for any substance its molecular weight* $\times 358$, *the product will be the volume of that substance in the reaction in cubic feet at* 32° *F. and standard atmospheric pressure of* 29.92 *ins.* Hg, if the substance could so exist and even if not, the relation between the volumes will hold at any other simultaneous pressure and temperature for all. When the substances all are gaseous the coefficient of the symbol, representing the number of molecules, indicates the relative volume at once. Thus, for complete combustion of *carbon* with oxygen alone

$$C + O_2 = CO_2$$

or (one molecule of carbon) + (one molecule) of oxygen makes (one molecule of CO_2) or 12 lbs. carbon + 368 cu.ft. oxygen makes 358 cu.ft. of CO_2. One pound of carbon requires 29.8 cu.ft. of oxygen and will make 29.8 cu.ft. of CO_2 at 32° F. and 29.92 ins. Hg.

Also for *hydrogen*

$$2H_2 + O_2 = 2H_2O,$$

or (two molecules of hydrogen)+(one molecule of oxygen) makes (two molecules of water); or 2 cu.ft. of hydrogen+1 cu.ft. oxygen makes 2 cu.ft. gaseous water; or 4 lbs. hydrogen requires 358 cu.ft. of oxygen, hence 1 lb. of hydrogen requires 89.5 cu.ft. of oxygen.

For *hexane* there was written

$$C_6H_{14}+9\tfrac{1}{2}O_2=7H_2O+6CO_2,$$

or 1 cu.ft. of hexane requires $9\frac{1}{2}$ cu.ft. of oxygen and makes 7 cu.ft. of gaseous water and 6 cu.ft. of carbon dioxide. It thus appears that the volumetric analysis of gaseous mixtures or products is given simply by the ratio of the number of molecules of each substance to the total number of molecules present.

When the oxygen is to be derived from the air each cubic foot of oxygen will carry with it $\frac{79.04}{20.96}=3.771$ cu.ft. of nitrogen which is to be added, or is to be found in 4.771 cu.ft. of air.

The air requirements for various fuels chemically are given in Table XXVI below in pounds and cubic feet of air per pound of fuel and per cubic foot of fuel (standard), for the exact combining proportions:

TABLE XXVI

AIR REQUIRED FOR COMBUSTION FOR VARIOUS SUBSTANCES

(Combustion complete in every case except for C burning to CO)

Substance		1 Lb. of Substance Requires Air		1 Cu. Ft. of Substance (Standard) Requires Air	
		Lbs.	Cu.Ft. Standard	Lbs.	Cu.Ft. Standard.
Carbon,	C to CO_2	11.55	143.10		
Carbon,	C to CO	5.77	71.55		
Hydrogen,	H_2	34.64	429.19	.193	2.39
Carbon monoxide,	CO	2.47	30.6	.193	2.39
Sulphur,	S	4.32	53.52		
Methane,	CH_4	17.32	214.59	.774	9.59
Ethane,	C_2H_6	16.16	200.22	1.354	16.73
Ethylene,	C_2H_4	14.85	183.99	1.157	14.34
Acetylene,	C_2H_2	13.32	165.07	.964	11.95
Propane,	C_3H_8	15.75	195.14	1.929	23.90
Propylene,	C_3H_6	14.85	183.99	1.736	21.51
Allylene,	C_3H_4	13.86	172.73	1.543	19.12
Butane,	C_4H_{10}	15.53	192.42	2.508	31.07
Butylene,	C_4H_8	14.85	183.99	2.315	28.68
Pentylene,	C_5H_{10}	14.85	183.99	2.890	35.85
Hexane,	C_6H_{14}	3.53	43.79	3.66	45.45
Benzole,	C_6H_6	13.32	165.07	2.89	35.84
Heptane,	C_7H_{16}	15.24	188.85	4.243	52.58
Methyl alcohol,	CH_3OH	6.49	80.47	.58	7.17
Ethyl alcohol,	C_2H_5OH	9.04	111.96	1.17	14.34

Prob. 1. Complete the following equations for combustion with oxygen and with air:

$$CH_4 + 2H_2 + -O_2 =$$

$$C_6H_6 + -O_2 =$$

$$C + 3H_2 + 4S + -O_2 =$$

Prob. 2. What will be the weight of the air needed in each of the cases of Prob. 1 and what will be the weight of the products, the volumetric composition?

Prob. 3. When 1 lb. of propylene is burned how much air is needed and what is formed? What is the proportion of each product by weight? What would be the approximate volume and volumetric composition of the products at a temperature of 60° F.?

Prob. 4. After combustion the products of a combustible mixture were found to be 17 cu.ft. of nitrogen, 3 cu.ft of carbon dioxide and water, equivalent to 6 cu.ft. of water vapor. What was the composition prior to combustion and what was the volume at 60° F.?

Prob. 5. 5 lbs. of carbon dioxide, 4 lbs. of oxygen, and 2 lbs. of water would result from the combustion of how much carbon and hydrogen in what amount of oxygen, and what would be the complete weight of the products if the combustion had taken place in air?

Prob. 6. When steam is passed through a bed of hot coke the coke takes the oxygen from the water to form CO and the resulting gas is called water gas. For 10 lbs. of carbon how much gas by weight could be made and how much steam would be required?

Prob. 7. A sample of producer gas gave upon analysis the following per cubic foot.

$H_2 = .18$ cu.ft.
$CO = .25$ cu.ft.
$CH_4 = .03$ cu.ft.
$CO_2 = .07$ cu.ft.
$N_2 = .47$ cu.ft.

What will be the cubic feet of air needed to burn a cubic foot and what will be the composition of the products?

14. Heats of Reaction. Calorific Power of Combustible Elements and Simple Chemical Compounds. B.T.U. Per Pound and Per Cubic Foot. Combustion of the so-called fuels is always exothermic and the heat set free per pound of fuel is its calorific power. This heat of reaction may also be expressed per unit weight or per unit volume of any other element or substance entering into the reaction, but all are derivable from the calorific power by the weight and volume relations. Each is more directly useful in some particular calculation, for example, the B.T.U. per cubic foot of combustible gaseous mixture is an important factor in the mean effective pressure of gas engines, and to the B.T.U. per pound of products is the temperature rise during combustion proportional. To facilitate boiler calculations and reduce the large number of B.T.U. per pound of fuel, its equivalent evaporative power from water at 212° F. to dry saturated steam at the same temperature is often given and is the B.T.U. per pound divided by 970.4. This value is described as

equivalent evaporative power, the number of evaporation units, or as evaporative power from and at 212° F.

All calorific powers can be determined exactly only, by direct experimental observations of burning the fuel in water cooled calorimeters, and the heat of combustion is directly given by the pound-degrees of the water with a correction for the metal and other parts simultaneously heated, called the water equivalent of the instrument, and also for radiation during test. It is assumed in calorimeter tests that all the substances are brought to their original temperature after combustion, if not, another correction must be made for the residual heat remaining in the hot products, ash or instrument parts, and which has not been imparted to the calorimeter.

It should be noted that whenever hydrogen burns to superheated water vapor in a calorimeter, the products in cooling will somewhere reach the saturation temperature for the pressure existing. When this point is reached water vapor will condense and the calorimeter water will receive the latent heat of condensation of the water produced by combustion. If, for example, the combustion proceeded at atmospheric pressure as in all gas calorimeters, this point would be 212° F. and as the condensation proceeds at this temperature there may be two calorific powers reported for products of combustion cooled to 212° F., the first for products condensed and the second for the products not condensed. The former will be greater than the latter by 970.4 B.T.U. per pound of water condensed and not per pound formed because all will not condense at any temperature. Of course, this same difference exists when the products are cooled below 212°, but there is in this case another difference inasmuch as the substances giving up heat to the calorimeter water are not all gaseous, but a liquid, some fixed gases and some water vapor, a little of which continues to condense for each degree drop, and this is in contrast to the conditions above 212°, where nothing but gases and superheated vapor, which behaves the same as gases, are being cooled.

It is thus apparent that not only may there be two calorific powers for fuels containing hydrogen, one *high* and the other *low*, but that the exact measure of the difference between them per pound of hydrogen burned or per pound of water formed is a question of how far or to what temperature the cooling be carried. In view of this possibility of more than one interpretation, it is sufficiently close for engineering work of the usual routine to accept as the difference, 970.4×weight of water vapor formed and this has the advantage of being definite and possible to calculate, even though it may not be exactly in accordance with actual conditions. Since 1 lb. of hydrogen makes 9 lbs. of water in combustion, the weight of water is nine times that of the hydrogen, therefore, it should be accepted that

$$\left\{\begin{array}{c}\text{Difference between high}\\ \text{and low calorific power}\end{array}\right\} = 970.4 \times 9 \times (\text{weight of hydrogen per pound fuel}).$$

$$= 8734 \times \left\{\begin{array}{c}(\text{weight of hydrogen}\\ \text{per pound fuel})\end{array}\right\} \quad . \quad . \quad . \quad . \quad (716)$$

Engineers have been forced to take this difference into consideration more particularly since the greatly increased use of hydrocarbon gaseous and liquid fuels and the alcohols, the former for burning both under boilers and in internal combustion engines and the latter in gas engines alone. In both these classes of apparatus the products of combustion leave quite hot, always above 212° F., so that it may be said with some propriety that they did not receive the full heat of combustion as reported by the calorimeter for products reduced to initial temperature. On the other hand, it may with equal propriety be contended that they have received even less than the low value because the products are discharged hotter than 212° F., but that they should receive no credit for the difference which is to be charged against them because they are unable to utilize low temperature heat. In this book the most commonly accepted practice will be adopted and this is to charge against boilers and all open or atmospheric pressure fires or furnaces the high calorific power, and against explosive combustion or that taking place in closed chambers at pressures considerably in excess of atmosphere, the low value as previously defined. To do otherwise is to enter a controversy already ten years old and still without end.

Unless otherwise defined *calorific power* shall be understood to mean the heat liberated by combustion obtained by burning 1 lb. of fuel at constant pressure in free oxygen, both fuel and oxygen being at equal temperature before, and products reduced to that same temperature after combustion. This is the high value, but not so named unless for special reasons. Of course, constant volume combustion would yield a different value which some authorities insist on applying to gas engine explosions, but this is in the nature of hair-splitting in engineering, where conditions of service are always somewhat undefined and practical factors of performance introduced as an allowance.

Since the mean specific heat of the original fuel and oxygen may not be the same as that of the products, it will make some difference what the base temperature of the experiment is. This effect will be greatest in the case of hydrogen burning to water liquid or vapor, and the fact is illustrated best by a specific computation. Julius Thomsen gives the experimental heat of combustion of hydrogen as being 61,200 B.T.U. per pound hydrogen, when the original hydrogen and oxygen are at a temperature of 18° C. or 64° F., and the resulting water brought to the same temperature. Since 9 lbs. of water are formed per pound hydrogen the heat obtained from the calorimeter, in case the product, water, were finally at 212° would be less than the above by the difference between heat of the liquid at 212° and at 64° for the 9 lbs. Then per pound of H_2 (H_2 and O_2 at 64° F. burning to liquid water at 64° F.) gives 61,200 B.T.U.; (H_2 and O_2 at 64° F. burning to liquid water at 212° F.) gives $61{,}200 - 148 \times 9 = 59{,}868$ B.T.U. If, however, the original substances H_2 and O_2 are at 212° instead of 64°, more heat would be removed from the calorimeter, by the amount of heat necessary to raise the temperature of 1 lb. H_2 and 8 lbs. O_2 from 64° to 212° F. Or (H_2 and O_2 at 212° F. burning to liquid water at 212° F.) gives $59{,}868 + 148(1 \times 3.4 + 8 \times .217) = 60{,}626$ B.T.U.

per lb. H_2. If the final products of combustion be vapor of water at 212° instead of liquid, the (H_2 and O_2 at 212° F. burning to water vapor at 212° F.) gives $60{,}626 - 9 \times 970.4 = 51{,}892$ B.T.U.

The temperature 212° F. has no significance when the pressure is other than atmospheric, and in the products of combustion there are nearly always other substances than water vapor present, so that very rarely is the partial pressure of the resulting vapor equal to standard atmosphere. Nevertheless, to conform to the practice of regarding the difference between high and low value as being equal to the product of the weight of water vapor formed by the latent heat at 212°, and at the same time to be consistent with scientific information available, it will be regarded that the expression " high value " for heat of combustion of hydrogen refers to the figure 60,626 B.T.U. per pound hydrogen, as derived above, while the term " low value " refers to the figure 51,892 B.T.U. per pound hydrogen, both being referred to an arbitrary base temperature of 212° F.

Investigations of the thermo-chemists have shown that the heat of combustion is not only determined by the chemical elements that burn, in kind and amount as might be expected, but also on their molecular structure, whether a single chemical element is involved or a chemical compound. In other words, assuming carbon and hydrogen to be the fuel elements in a fuel, its calorific power is not equal to the sum of the products of these two calorific powers into their respective partial weights. There is a difference and the difference is not the same for different compounds but follows a simple law for the compounds of one series, like the hydrocarbons of the paraffine series, for example. Furthermore, for carbon alone there are several different calorific powers, that for soft, porous carbon being at one end and that for crystalline diamond at the other. Fortunately, these differences are not to be considered in engineering as two successive samples of the same natural fuel will differ from each other more than would warrant fine corrections like that for the carbon. The molecular structure as a cause of difference is, however, useful in explaining the lack of agreement between a calorimeter test which is always conclusive and a calculation based on chemical composition and calorific power of the elements.

As combination and dissociation are inverse processes the heat liberated by the combustion of hydrogen to form water which may be regarded as the heat of water formation must be equal to the heat necessarily absorbed in separating the water into hydrogen and oxygen, whence the heat of dissociation of a compound formed by combustion is equal to the heat of combustion of its fuel element. This is important in high temperature work in which steam is often dissociated as hydrogen is burned and where in successive steps steam dissociation and hydrogen combustion may follow each other. In high temperature combustion as, for example, gas engine explosions, it is quite possible that before the fuel is burned entirely the temperature has risen to the value at which dissociation of products takes place and, therefore, combustion or combination will be retarded until heat be carried away

When this occurs, all the heat of combustion does not enter into the raising of temperature indefinitely but only partly so, the rest acting as liberated to maintain a high temperature as heat is being abstracted or otherwise being disposed of.

As the calorific power of a commercial fuel is a function of the calorific power of the carbon, hydrogen and sulphur it contains, it is important that their calorific powers be firmly established, and some of the more authoritative values are given in the Table LV at the end of this chapter. In the case of carbon, which may burn to carbon monoxide or to carbon dioxide, it is important that the corresponding reaction heats be *consistent.* This will be the case when the heat of combustion of 1 lb. of carbon burning to CO, added to the heat of combustion of the amount of CO containing a pound of carbon, be together equal to the heat of combustion of carbon to CO_2 directly. One lb. of C is contained in $\frac{28}{12}=2.33$ lbs. of carbon monoxide according to atomic weights compared to O = 16, or $\frac{11.99+15.88}{11.99}=2.324$, according to atomic weights compared to H = 1.

Therefore, for atomic weight O = 16.

$$\left\{\begin{matrix}\text{B.T.U. per lb. C}\\ \text{burning to CO}\end{matrix}\right\}+\left\{\begin{matrix}2.333\times\text{B.T.U. per lb.}\\ \text{CO burning to } CO_2\end{matrix}\right\}=\left\{\begin{matrix}\text{B.T.U. per lb. C}\\ \text{burning to CO}\end{matrix}\right\}$$

or for atomic weights H = 1.

$$\left\{\begin{matrix}\text{B.T.U. per lb. C}\\ \text{burning to CO}\end{matrix}\right\}=\left\{\begin{matrix}2.324\times\text{B.T.U. per lb.}\\ \text{CO burning to } CO_2\end{matrix}\right\}=\left\{\begin{matrix}\text{B.T.U per lb. C}\\ \text{burning to } CO_2\end{matrix}\right\}.$$

This is but a special form of the general thermo-chemical law. *The heat of combustion of a compound* (CO) *together with the heat of formation of that compound from its elements* (C *and* O) *must be equal to the heat of combustion of the elements* (C *and* O) *direct to the final products* CO_2 *or the heat of formation of products directly from the elements.*

Taking the Favre and Silberman value for the complete combustion of carbon to carbon dioxide, which is accepted by Julius Thomsen one of the greatest of the thermochemists as 14,544 B.T.U. per pound C, and Thomsen's own value for the heat of combustion of CO to CO_2 as 4369 B.T.U. per pound CO, the heat of combustion of the compound must be (using atomic weights C = 12 and O = 16 because Thomsen did so in reporting his value)

$$\text{B.T.U. per pound C burning to CO}=\left\{\begin{matrix}(\text{B.T.U. perpound C burning to } CO_2)\\ -2.333\times(\text{B.T.U. per pound CO burning to } CO_2),\end{matrix}\right.$$

$$=14{,}544-10{,}193=4351.$$

From the above it is clear that the heat of combustion of CO burning to CO_2 is 10,193 B.T.U. per pound of *carbon.*

It is evident from the general law that the heat of combustion of compounds like the hydrocarbons and alcohols cannot be computed from the heat of combustion of the carbon and hydrogen they contain, without correcting for the heats of formation of these compounds from their elements. These heats are, however, pretty well determined for some of the definite compounds that are constituents of natural fuels; and among these the most important are the hydrocarbons. For these Thomsen has determined many of the heats of formation of the compound and finds some positive and others negative, so that in the former cases the heat of combustion of the compound would be that calculated for its hydrogen and carbon less the heat of formation of the compound, and in the latter cases plus the heat of formation. These heats of formation are generally small as indicated by the following table:

HEAT OF FORMATION IN TERMS OF HEAT CIRCULATION

Methane CH_4	+10.2%	of its heat of	combustion
Ethane C_2H_6	+ 7.6%	"	"
Ethylene C_2H_4	− .8%	"	"
Acetylene C_2H_2	−15.4%	"	"
Propane C_3H_8	+ 6.5%	"	"
Benzene C_6H_6	− 1.6%	"	"
Trimethyl methane $CH:(CH_3)_3=C_4H_{16}$	+ 6.2%	"	"
Tetramethyl methane $C:(CH_3)_4=C_5H_{12}$	+ 5.6%	"	"
Propylene $CH_2:CH:CH_3=C_3H_6$	− .7%	"	"
Trimethylene C_3H_6	− .7%	"	"
Isobutylene $CH_2:C:(CH_3)_2=C_4H_8$	+ 1.6%	"	"
Isoamylene $C_2H_4+C:(CH_3)_2=C_5H_{10}$	+ 2.2%	"	"
Diallyl $C_3H_5:C_3H_5=C_6H_{10}$	− .1%	"	"
Allylene $CH:C:CH_2=C_3H_3$	− 8.6%	"	"

Such figures as these are not of very much use in the practical work of dealing with the volatile of coals or liquid fuels because it is not possible to tell precisely what hydrocarbons are present though some of the above may be and others for which these heats are unknown. For all such practical work approximate methods are used and some of them are surprisingly accurate, as will be seen from further discussion. Fig. 158 shows curves plotted for several series of hydrocarbons, giving the heat of formation of the products of combustion. This is the heat which would be obtained by the burning of that amount of hydrogen and carbon in the free state which exists in the combined state in the hydrocarbon. For instance, if the hydrocarbon is represented by the symbol C_xH_y, the heat of formation of the products is (high value)

$$\frac{12\times 14{,}544x+60{,}626y}{12x+y},$$

which is not the heat of combustion of the hydrocarbon, since such combustion requires the supply of heat of formation of the hydrocarbon itself, either posi-

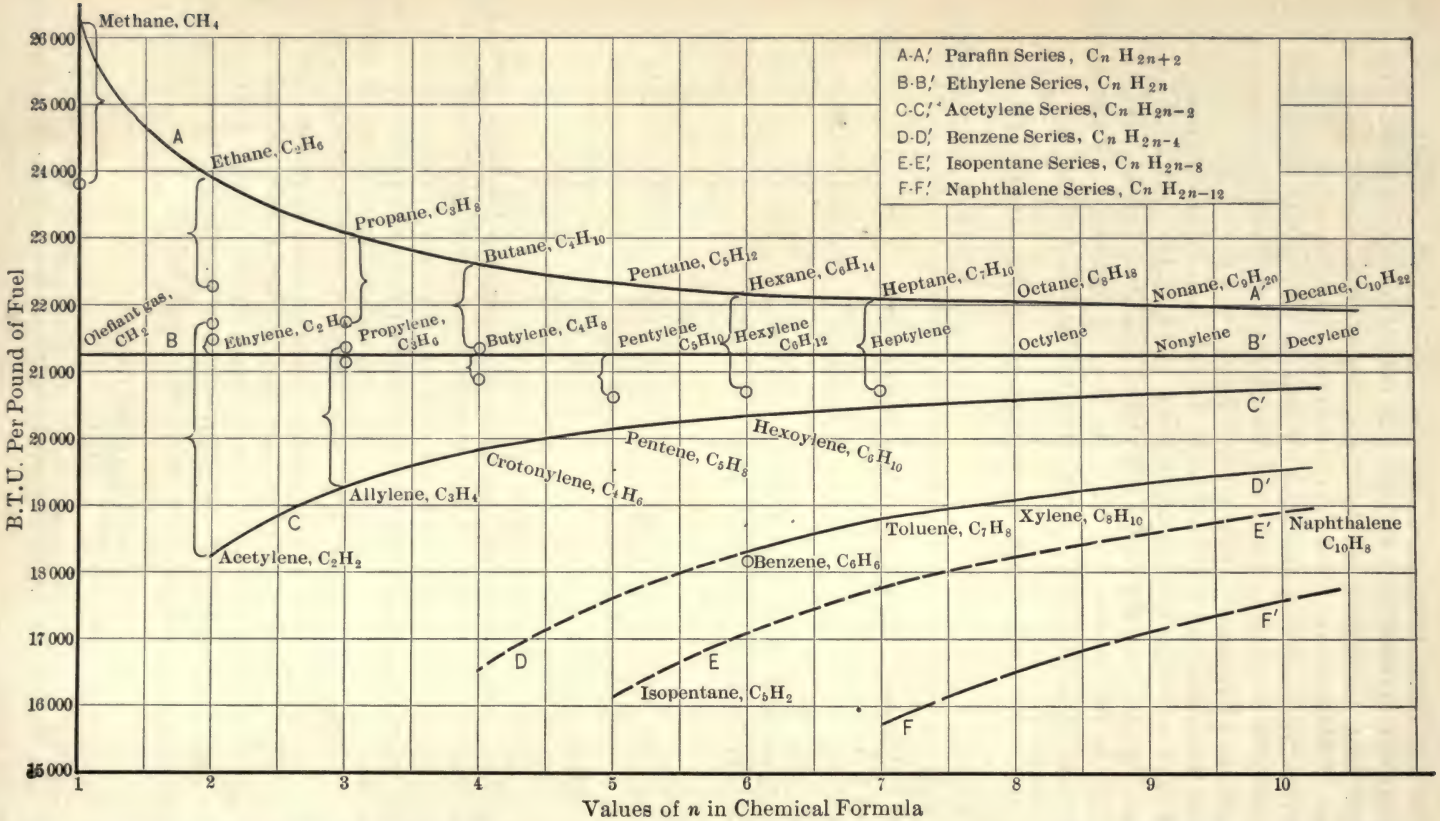


FIG. 158.—Heat of Formation of the Products (Curves) for Combustion of Hydrocarbons, Compared with Heats of Combustion (Points), and the Heat of Formation of the Compound (Differences).

ive or negative. The experimental values of heat of combustion of a small number of these compounds are indicated by points, and the difference between point and the curve, shown by brackets, is this heat of formation of the hydrocarbon. These curves bring out the remarkable fact that however complex the hydrocarbons are as to molecular constitution, the heats of combustion of the majority of them do not vary very much and the ethylenes have a constant heat except as the heats of formation vary a little.

Calculations dealing with the combustion of gases and vapors are often more conveniently carried through, on a volume than on a weight basis. In doing this, it is convenient to have available the volumetric calorific powers of the standard gases and vapors reduced to 32° F., and 29.92 ins. Hg absolute pressure even if the particular compound is a liquid under these conditions, for the proportionality will hold for higher temperatures as if it were a vapor or gas. Gases reported for these conditions of pressure and temperature are termed *standard* and the reduction to this standard from any other pressure and temperature or to any desired pressure and temperature is to be made on the assumption of perfect gas behavior.

Let $P_{29.92}$ = absolute pressure in inches of mercury = (29.92), of standard
" gas or vapor;
" $V_{32/29.92}$ = volume in cubic feet per pound of standard gas;
" $d_{32/29.92}$ = density in pounds per cubic foot of standard gas;
" (B.T.U. per cubic foot)$_{32/29.92}$ = $\dfrac{\text{B.T.U. per lb.}}{V_{32/29.92}}$ = B.T.U. per cubic foot standard gas;
" P = any other pressure in inches of mercury at which the gas may be under study;
" t = any other Fahrenheit temperature;
" V and d = corresponding specific volume and density for P and t;
" (B.T.U. per cubic foot) = corresponding volumetric heating power for P and t.

Then

$$\frac{PV}{(t+460)} = \frac{P_{29.92} \times V_{32/29.92}}{292} = \frac{29.92 \times V_{32/29.92}}{492}.$$

Hence

$$V = \left(\frac{t+460}{492}\right)\left(\frac{29.92}{P}\right) V_{32/29.92},$$

$$d = \left(\frac{492}{t+460}\right)\left(\frac{P}{29.92}\right) d_{32/29.92},$$

and

$$(\text{B.T.U. per cu.ft.}) = \frac{\text{B.T.U. per lb.}}{\left(\frac{t+460}{492}\right)\left(\frac{29.92}{P}\right)V_{32/29.92}}$$

$$= \frac{(\text{B.T.U. per cu.ft.})_{32/29.92}}{\left(\frac{t+460}{492}\right)\left(\frac{29.92}{P}\right)} \quad . \quad . \quad . \quad (717)$$

The values for the (B.T.U.per cubic foot)$_{32/29.92}$, that is, of standard gases given in the table at the end of the Chapter, are derived from the B.T.U. values per pound reported, both high and low values, and from the standard volumes of the pound taken from table at the end of the Chapter, the direct experimental determination being used wherever it is available otherwise, as calculated from the molecular weights.

A consistency is desirable whenever not seriously in conflict with facts, and the molecular relations of density are necessary in dealing with liquid fuels and the gasification of solid ones; the values per cubic foot for hydrogen and carbon monoxide are often derived in this way from the pound value instead of making use of direct calorimetric values given in the table. This is always done for the vapors.

According to the formulas

$$2H_2 + O_2 = 2H_2O$$

and

$$2CO + O_2 = 2CO_2,$$

it appears that the calorific power of hydrogen per cubic foot standard must be

$$\left.\begin{aligned} \text{B.T.U. per cu.ft. hydrogen} &= \frac{2\times 60626}{358} = 339 \text{ high} \quad (a) \\ &= \frac{2\times 51892}{358} = 290 \text{ low} \quad (b) \end{aligned}\right\} \quad . \quad . \quad . \quad (718)$$

Then if a molecule of CO has the same volume as a molecule of H_2

$$\text{B.T.U. per cu.ft. carbon monoxide} = \frac{28\times 4369}{358} = 342. \quad . \quad . \quad (719)$$

For the hydrocarbons the same procedure can be put in a general form, neglecting the heats of formation of the compounds.

Thus, for the paraffines C_nH_{2n+2} taking the molecular weight of C = 12,

$$\left.\begin{aligned}\text{Wt. C in compound} &= \frac{12n}{12n+2n+2} = \frac{6n}{7n+1} \text{ per lb. fuel} \quad (a)\\ \text{Wt. H in compound} &= \frac{2n+2}{12n+2n+2} = \frac{n+1}{7n+1} \text{ per lb. fuel} \quad (b)\end{aligned}\right\} . \quad (720)$$

$$\left.\begin{aligned}\text{B.T.U. per lb. fuel} &= \frac{14544\times 6n+60626(n+1)}{7n+1} = \frac{147890n+60626}{7n+1} \text{(high)} \quad (a)\\ &= \frac{14544\times 6n+51892(n+1)}{7n+1} = \frac{139156n+51892}{7n+1} \text{(low)} \quad (b)\end{aligned}\right\} \quad (721)$$

$$\text{Also cu.ft. vapor (st'd) per lb.} = \frac{358}{12n+2n+2} = \frac{358}{14n+2} . \quad . \quad . \quad (722)$$

Therefore

$$\left.\begin{aligned}\text{B.T.U. per cu.ft. vapor} &= \frac{\frac{147890n+60626}{7n+1}}{\frac{358}{14n+2}} = \frac{147890n+60626}{179} \text{(high)} \quad (a)\\ &= \frac{\frac{139731n+51872}{7n+1}}{\frac{358}{14n+2}} = \frac{139159n+5246}{179} \text{(low)} \quad (b)\end{aligned}\right\} . \quad (723)$$

For the ethylene series C_nH_{2n}

$$\left.\begin{aligned}\text{Weight C in compound} &= \frac{12n}{12n+2} = \frac{6}{7} = .857 \text{ per lb. fuel} \quad (a)\\ \text{Weight H in compound} &= \frac{2n}{12n+2} = \frac{1}{7} = .143 \text{ per lb. fuel} \quad (b)\end{aligned}\right\} . \quad (724)$$

Hence

$$\left.\begin{aligned}\text{B.T.U. per lb.} &= .857\times 14544 + .143\times 60626 = 21134 \text{ (high)} \quad (a)\\ &= .857\times 14544 + .143\times 51892 = 19885 \text{ (low)} \quad (b)\end{aligned}\right\} . \quad . \quad (725)$$

$$\text{Cu.ft. vapor (std) per lb.} = \frac{358}{14n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (726)$$

Therefore

$$\left.\begin{aligned}\text{B.T.U. per cu.ft. vapor (std.)} &= \frac{.857\times 14544 + .143\times 60626}{\frac{358}{14n}} = 827n \text{(high)} \quad (a)\\ &= \frac{.857\times 14544 + .143\times 51892}{\frac{358}{14n}} = 778n \text{(low)} \quad (b)\end{aligned}\right\} \quad (727)$$

These results are plotted to a base of n in Fig. 159, and illustrate beautifully how the calorific power varies with the position of the hydrocarbon in the series with the paraffine series at the top as the highest of all. These values are the basis of work on coal and oil gas and liquid fuels to be taken up later. To the calculated values on the curve of Fig. 159 may be added the available direct determination for comparison from Fig. 158 and the Slaby empiric formula

$$(\text{B.T.U. per cubic foot}) = 112 + 18880 \times (\text{pounds per cubic foot.}) \quad . \quad (728)$$

The calorific power of a power of a commercial fuel or lighting gas per cubic foot is the sum of the volumetric per cents of its constituents into the calorific

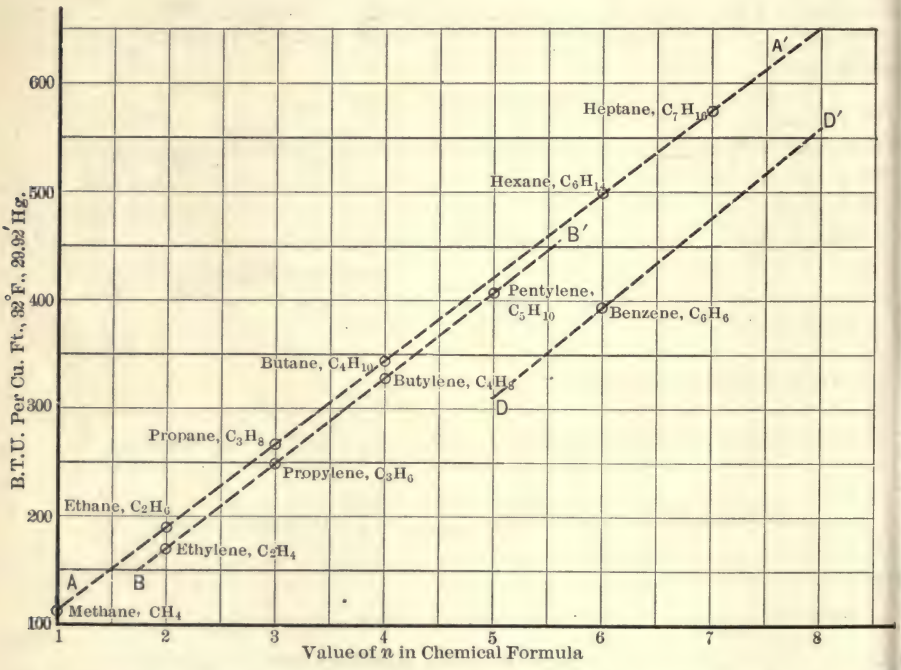


FIG. 159.—Heat of Formation of Products for Combustion of Hydrocarbons, in B.T.U. per cu.ft. at Standard Conditions.

power of those constituent gases which include carbon monoxide, hydrogen methane, ethylene, ethane and sometimes some higher hydrocarbons, the nature of which is assumed, like benzene. Coals and oils cannot be so treated, but the methods used here with varying degrees of approximation will be taken up later. Of course, in all cases an estimate may be made from the combustible chemical elements if they are known in amount, by guessing at their molecular condition.

Example 1. A certain fuel consists of 5 per cent hydrogen, 8 per cent oxygen 1 per cent sulphur, and 87 per cent carbon, by weight. Find the heat of formation

of the products of complete combustion, if all of the oxygen in the fuel is originally combined with hydrogen in form of moisture.

.08 lb. oxygen with $\frac{.08}{8} = .01$ lb. hydrogen constitute .09 lb. water originally in fuel, leaving $.05 - .01 = .04$ lb. hydrogen to be burned.

$$\begin{aligned} .04 \times 60626 &= 2425 \text{ B.T.U.} \\ .01 \times 3998 &= 40 \text{ B.T.U.} \\ .87 \times 14544 &= 12653 \text{ B.T.U.} \end{aligned}$$

Heat of formation of products = 15118 B.T.U. high value

Prob. 1. A certain fuel contains 20 per cent H_2 and 80 per cent C by weight. If the heat of formation of the compound was 8 per cent of the heat of formation of the products, what are the high and low values of the heat of combustion?

Prob. 2. What would be the heat of combustion for the reactions given in Problem 1, Section 13?

Prob. 3. It is known that a cubic foot of natural gas under standard conditions contains 980 B.T.U.'s. What would be its calorific power at 70° F. and at an altitude of 8000 ft.?

Prob. 4. A sample of producer gas gave the following analysis per cubic foot. What would be its high and low heating value per cubic foot?

$$H_2 = .18;\ CO = .25;\ CH_4 = .03;\ CO_2 = .07;\ N_2 = .47.$$

Prob. 5. What would be the cost per 100,000 cu.ft. of standard gas and per bbl. of oil to give the same cost per B.T.U. as coal at $4.00 per ton, when analyses are as follows?

Coal.	Oil.	Gas.
$C = .75$	$C_5H_{12} = .10$,	$N_2 = .04$,
$CH_4 = .07$,	$C_6H_{14} = .30$,	$H_2 = .02$,
Moisture = .08,	$C_9H_{20} = .50$,	$CH_4 = .94$,
Ash = .10,	$C_{13}H_{28} = .10$.	

Analysis of coal and oil by weight, gas by volume.

Prob. 6. An evaporation of 7.7 lbs. of water from and at 212° F. was obtained with the following coal. What per cent of the heat of formation of products was delivered to the steam?

$$C = .758;\ H_2 = .049;\ O_2 = .085;\ S = .023;\ Ash = .085.$$

Prob. 7. An internal combustion engine gives a horse-power hour with a fuel consumption of .5 lb. of hexane, while another engine uses 8 cu.ft. of methane during the same period and develops the same power. What are the respective efficiencies?

Prob. 8. By burning a pound of the oil, the analysis for which by weight is given below, 500 lbs. of water were heated 40° F. in a calorimeter. What was the heat of formation of the compounds?

$$C_{10}H_{22} = 10 \text{ per cent};\ C_8H_{18} = 80 \text{ per cent};\ C_6H_{14} = 10 \text{ per cent}.$$

15. Heat Transmission Processes. Factors of Internal Conduction, Surface Resistance, Radiation and Convection. Coefficients of Heat Transmission. When air supports combustion, the gaseous products may be said to be heated directly, and to define the process, the term "internal combustion heating" is generally applied, and likewise when a liquid cools by evaporation from its surface due to a lesser surface vapor pressure than corresponds to the temperature, it might be said to be directly or internally cooled. With such exceptions as these it is universally necessary in dealing with heat in commercial apparatus to transmit or transfer it from one place or one substance to another place or substance. Thus, the heat for boiling water in steam boilers must come through tubes and plates from the fire or hot products of combustion, and a similar transfer takes place in closed feed-water heaters in economizers, steam superheaters, in all heated surfaces of boiler, pipe and engine parts, in surface condensers for steam, ammonia or carbon dioxide, in steam and hot-water radiators, through the walls of buildings outward when heated during the winter and inward when cooled for cold storage below the atmosphere, likewise in the brine coolers, ice cans and other parts of mechanical refrigerating systems, in apparatus for concentrating solutions in chemical manufacture or in distillation, in the cooling of gas engine and compressor cylinders by water jackets and cooling air between compressor cylinders, or drying and superheating steam between cylinders of multiple expansion engines, or preheating of compressed air for air engines. Thus, the practical application of heat for the doing of a thing desired, or the abstraction of undesired heat, generally though not always involves transmission, and most often through solid containers of great variety ranging from the thin brass tubes of surface condensers to the iron or steel pipe of boiler walls or ammonia evaporating coils, to steam pipe covering or the brick wall of a dwelling house.

That the movement of heat from one place or substance to another of lower temperature follows some law of flow was long ago recognized, even before anything was known of the transformation of heat into work. Old as is the realization of the existence of heat flow laws, there is, however, no general acceptance of what are those laws except in a few special cases, and the reason for this situation, which seems somewhat queer after all these years of experimental study and formulation of theories, becomes clear after even a brief analysis of what must really happen in the course of an actual case of transmission. In spite, however, of the absence of such laws as will permit of reasonably exact calculation of the relations between quantity of flow, the dimensions and kind of surfaces and quantities and kinds of material exchanging heat, with their corresponding changes of temperature or state, engineers must design apparatus that is needed, as best they can, and for this purpose there has been developed a method of approximation with which is used a series of coefficients developed by experience for the more common and recurrent cases. The treatment of this subject will, therefore, be divisible into two parts, first the examination of such laws as are fairly well fixed, and second, the development of an approximate method of treatment for the practical everyday problems that the fundamental laws do not solve.

Heat may pass from one substance or place to another in three essentially different ways and every practical case includes one or all of these and *no others*.

1. Heat may flow along a bar or wire or along any line in a solid body from a high to a low temperature point, by simple molecular communication. Each molecule heats the next without any discontinuity and with a regular continuous fall of temperature from the high to the low temperature point. This is *internal conduction* of heat, and the conducting capacity of nearly all the common substances is pretty accurately determined and represented by coefficients of thermal conductivity, some selected values for which are given at the end of this Chapter in Table LVI. Of course, by internal conductivity only, heat could not flow beyond the limits of one piece of substance, but at the boundary it may be communicated to the next body and so on pass any number of dividing surfaces or through any number of bodies. This is a most common mode of *conduction* about which there is no exact information comparable with that for *internal* conduction.

2. Heat may flow from a hot body through space to a distant colder body entirely without reference to the separating medium, and the best example is the passage of the sun's heat to the earth. This mode of transmission is defined as *radiation* and heat so received by the cold, or discharged by the hot body, as radiant heat. The capacity for radiation of heat for different substances is practically unknown, though there is *fair* though not good agreement as to the relation of heat quantity to the two controlling temperatures. There is practically nothing known of the radiating capacity of any but originally black bodies brought to high temperature conditions or incandescence.

3. Heat may get from one place or substance to another by being carried as a *charge* on a particle that is moved bodily between the two places. Thus, a particle of water heated at the bottom of a vessel becomes less dense and floating in the colder or more dense, actually carries its heat to the top as is also the case in chimneys where the flotation tendency produces the necessary draft. This is *convection* communication of heat and takes place only in liquids and gases, where in action it is generally associated most intimately with radiation and conduction. There may be a movement of particles of liquid or gas not caused by differences in density characteristic of convection, but by pumps or fans, and which is quite effective in carrying or transmitting heat, but this is an externally forced circulation of heated particles rather than a self-caused heat transfer.

Transmission of heat by internal conduction takes place according to a law of proportionality to temperature difference, but as the conductivity of substances depends on the temperature itself there must be involved a temperature term. This gives for solids a comparatively simple law which is apparently also applicable to liquids and gases, but for gases variations enter that have been the subject of lengthy study by such eminent physicists as Maxwell, Clausius, Kelvin, Tyndall, and some hundreds or so less famous but good men without acceptable solution.

For solids and liquids the conductivity is given by the following Eq. (729):

$$K_t = K_{32}[1+x(t-32)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (729)$$

in which

K_t = B.T.U. per hour per square foot of cross section per degree F. difference in temperature between the points under consideration, per inch of path, when the body has the temperature $t°$ F.;

K_{32} = B.T.U per hour per square foot per degree F. per inch of thickness at 32° F.;

t = temperature degrees F.;

x = a constant.

This conductivity *at* any temperature is not directly useful except in comparing the conductivity of substances, but in computing the amount of heat flowing between a high temperature and a low temperature point the mean conductivity for this range must be determined. As conductivity is a straight line function of temperature the mean conductivity between the temperature is the arithmetical mean of the conductivities at these temperatures, hence

Let t_h = high temperature degrees F.;

" t_l = low temperature degrees F.;

" $K_{(t_h - t_l)}$ = mean conductivity between t_h and t_l.

Then

$$K_{(t_h-t_l)} = \frac{K_{32}[1+x(t_h-32)]+K_{32}[1+x(t_l-32)]}{2}$$

$$= K_{32}\left(1+\frac{x}{2}[(t_h-32)+(t_l-32)]\right). \quad . \quad . \quad . \quad . \quad . \quad (730)$$

From this mean conductivity expression Eq. (730) the amount of heat that will flow through a body by *internal conduction* is given by Eq. (731)

$$Q = Al(t_h-t_1)K(t_h-t_l),$$

$$= AlK_{32}\left(1+\frac{x}{2}[(t_h-32)-(t_l-32)]\right)(t_h-t_l), \quad . \quad . \quad . \quad (731)$$

where

Q = B.T.U. per hour;

A = cross-section of conducting path in square feet;

l = length of conducting path in inches.

Applying expressions like this to solids is a direct and safe proceeding if the high and low temperatures are taken *within* the conducting body itself and not in

some external substance like a liquid or gas in contact. The same expression and limitation applies to liquids *if there is no convection flow*, in which case the heat flow will be the algebraic sum of the conduction and convection parts. For gases there enters likewise a radiation factor as a third disturber as well as a more general and important doubt as to the value of the expressions at all for gases except as a means comparing them with liquids and solids. In no case can these equations be applied to heat flow across a surface boundary or joint in a solid, or from solid to liquid to gas.

Inspection of the table of conductivities reveals some most interesting and valuable relations—thus, silver is about the head of the list as a conductor and carbon dioxide at the foot with an enormous difference between them, silver being able to conduct about 35,000 times as much heat as carbon dioxide for equal temperature differences. In fact, the substances group themselves naturally into good and bad conductors, all the gases being in the latter class and many solids, like felt and wool. These solids are the natural heat insulators and their insulating value, seemingly dependent on their porosity, at least to some degree, has led to the theory that the gases confined between fibers or granules of solids are strong contributors to the insulating values of otherwise solid bodies. Liquids fall into an intermediate class, being universally better conductors than gases, not so good as the metals, which as a class are the best conductors, but of the same order as the group of solids generally considered as insulators.

As the relative conductivity of solids, liquids and gases, or their reciprocals the resistances to heat, are most useful in explaining or even predicting the relative characteristics of the common heat transfer paths in commercial apparatus, it is important that a table be developed from existing data. These data are of two sorts, first, the absolute internal conductivities of the table and second, direct experimental measurement of relative conducting power, which is determined by different methods than the absolute value. These relative conductivities (Table LVII) are similarly useful in selecting from conflicting values like those for iron, that which is most probable, though, of course, commercial irons being alloys merging into the steels must have small differences depending on composition, but so small as to be negligible for most engineering purposes. Thus, comparing the common metals with silver, Wiedermann and Franz find the following numbers as representing the conductivity, silver being 100, lead 8.5, iron 11.9, steel 11.9, copper 73.6, zinc 28.1, tin 15.2, of course, all at the same temperature, which is most conveniently fixed at 0° C. = 32° F. Applying these ratios to the absolute conductivity of silver by Weber, 1.096, the most closely corresponding of the direct measurements when there are more than one is found and marked in the Table LVI by an arrow ←. For liquids, the value for water being 100, the following relative values are given: Benzole 28.8, by Weber; ethyl alcohol absolute 30.9; 90 per cent, 32.05; 50 per cent, 54.29; by Henneberg, and methyl alcohol 27.37 by De Heen. For gases with air taken as 100, hydrogen is fixed at 701, oxygen 102, carbon dioxide 62, methane 139, ethylene 74 by Stefan, hydrogen at 710, carbon dioxide 59, carbon monoxide

98, by Kundt and Warberg, ammonia 91.7 and illuminating gas 267 by Plank. Where these ratios do not check, adjustment has been made in the table of relative conductivities with carbon dioxide as unity for all and resistances with silver unity for all, so that all numbers, are whole numbers, and to avoid conflict of testimony only two significant numbers are retained for large differences and three for small.

It is extremely likely that the conductivity of gases does not follow the linear law with respect to temperature that seems satisfactory for solids, and it is from Clark Maxwell that the best suggestion for a substitute has come. He predicted from the kinetic theory of gases that the conductivity is proportional to the product of the coefficient of gas viscosity and specific heat at constant volume or

$$K = \mu C_v \times (\text{a constant}).$$

Dalby from a study of values of μ finds it can be put proportional to the three-quarter power of the absolute temperature T so that,

$$K = cC_v\,(T)^{\frac{3}{4}},$$

or

$$\frac{K_T}{K_{32}} = \left(\frac{T^\circ \text{ F.}}{492}\right)^{\frac{3}{4}},$$

approximately if specific heats are assumed constant. This more than doubles the conductivity value for 32° F., at the temperature 930° F. This is an important addition because the gases of furnaces are very hot indeed while the tubes, plates and water are comparatively cool.

Conductivity beyond the boundaries of a body is known to suffer a rapid decrease or heat flow to encounter a resistance through joints in metal bars or between plates at point of contact. Boiler plates offer appreciable resistance and boiler seams are forbidden in contact with fires by certain inspection laws, because experience shows the plates frequently burn, which they would not do were there not a large joint resistance. It is also known that several layers of thin boards make better insulation than equal thickness of single boards for refrigerated boxes and rooms. Whether this joint resistance is due tc a thin layer of poor conducting air or is a strictly separate phenomenon is not known, but most carefully made accurately fitted joints in experimental bars in which there could not be over one ten-thousandth of an inch air layer gave measurable joint resistance, smaller, of course, than a bad joint. It is probably true that there is a real joint or surface resistance of unknown character at the boundary of every body and likewise a fluid film of some thickness as well, though probably not uniform. When heat passing from fluid to plate or plate to fluid, encounters a film of fluid adhering to the plate and for all practical purposes sticking to it, the heat resistance is materially increased. Inspection

of the table indicates that, taking the relative conductivity of iron at about 6000, a layer of carbon dioxide of $\frac{1}{6000}$ of an inch thick will offer as much resistance to the passage of heat as a one-inch thickness of metal, and taking water as 40, relative conductivity, a layer of water $\frac{40}{6000} = \frac{1}{150}$ of an inch thick would offer the same resistance, both in addition to the separate joint resistance.

The existence of such heat-resisting films of gas and liquid in heat transfer plates is conclusively proved, so that the problem of *estimating heat flow from a body on the other side involves an estimate of the thickness of the adhering fluid film, which must be the purest sort of guess.* In addition to the film and joint resistance difficulty, there is another when dealing with boiler, condenser, feed-water heater, economizer, superheater and similar tubes and plate surfaces, and that is the scale, grease, soot, rust or dirt layer that is always present in some condition, of density and thickness and also highly resistant as belonging in the heat insulator group. Furthermore, such a layer on both sides involves another joint resistance, so that the practical everyday problem of deciding on how much surface to allow for a given heat flow or how much heat can flow past a given surface, is not one that can be solved by the laws of conduction even if data on all substances were available, and probably never will be, because it requires first an hypothesis fixing the thickness of fluid film, the joint resistance, the kind and thickness of fouling or layer of dirt. Problems of this character are to be solved in another way by means of coefficients of transmission found by experiment and applied to the same kind of apparatus as that on which the experiment was made and covering all resistances together, though each cannot be separated from the rest.

There is no doubt now as to the existence and supreme importance of the surface films, especially on the gas side of plates receiving heat from or delivering it to gases or involving the presence of gases such as deposit on steam condenser tubes or intermittent layers of vapor on boiler surfaces when ebullition is taking place, nor can there be longer any doubt about the *reduction of film thickness and resistence by fluid agitation.* This agitation generally is the result of velocity increase of the fluid over the surface, but such data as exist on this effect do not permit of a determination of actual film thickness or resistance, but rather of overall coefficients of transmission to be considered later.

If a series of resistances to heat flow as reciprocals of conductivity are known, the whole heat flow temperature relations, can be set down algebraically; the real practical difficulty arises from their numerical evaluation.

Let l_1, l_2, etc. = thickness of each portion of plate or film along the path of heat flow in inches;

" K_1, K_2, etc. = mean or constant coefficients of conductivity of each material;

" ρ_1, ρ_2, etc. $= \frac{1}{K_1}, \frac{1}{K_2}$, etc., thermal resistances of each material;

" σ_1, σ_2, etc. = surface resistances;

" t_h and t_l = high and low temperatures at the limits of the path.

Then

$$Q=\frac{A(t_h-t_l)}{\Sigma\rho l+\Sigma\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (732)$$

For the whole complex heat flow path there can be introduced a general overall coefficient U inclusive of all internal and boundary resistances such that

$$Q=AU(t_h-t_l), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (733)$$

where

$$U=\frac{1}{\Sigma\rho l+\Sigma\sigma}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (734)$$

Where U = B.T.U. per hour per square foot, per degree difference of temperature between any two points usually in practical problems taken in two separated fluids, one receiving heat from the other.

Direct evaluation of this coefficient of transmission U is the usual practical procedure, inclusive of all separate conductivities or thermal resistances together with radiation and convection influences without separately evaluating each.

Heat flow by radiation alone has been experimentally studied and reduced to laws by a number of skillful physicists, and while they do not agree, the so-called Stefan and Bolzmann law is now quite generally accepted. According to this law heat will be radiated from an incandescent " black " body according to Eq. (735).

$$\text{B.T.U. per hour radiated per square foot} = 16\times10^{-10}(T_1^4-T_2^4), \quad (735)$$

where

T_1 = absolute temperature F of the " black " radiating body,

and

T_2 = absolute temperature F of the receiving body.

A black body is defined as one capable of absorbing all heat rays received, neither reflecting or transmitting any, and for practical purposes may be assumed to be any porous carbon, that is all carbon except the crystalline. Bodies not " black," like hot gases or fire brick or boiler plates, do reflect some heat received, and fail to radiate to the same extent as does a black body. For them absorption and radiation increase with darkness (when cold) and roughness, which is one reason for nickel plating and polishing surfaces from which radiation is to be resisted.

For various substances the following figures, Table XXVII, give numerical values on relative radiation or absorption and reflecting power the sources of which are uncertain, but which have been for a number of years quoted by handbooks. These figures are probably wrong, but are given in the absence of any better, which even if available would be of but little real use.

TABLE XXVII

RADIATION COEFFICIENTS

	Radiating and Absorbing Powers.	Reflecting Power.
Porous carbon (black body)...	1.00	0.00
Glass.....	.90	.10
Ice.....	.85	.15
Polished cast iron.....	.25	.75
Wrought iron polished.....	.23	.77
Steel polished.....	.19	.81
Brass polished.....	.07	.93
Copper hammered.....	.07	.93
Silver polished.....	.03	.97

For the black body itself the following curves, Fig. 160, are calculated from the Stefan and Bolzmann law for various temperatures of the radiator up to 6000° absolute F. to a heat absorber at 500, 600, 700, 800, 900 and 1000° absolute. The very great quantities of heat that may thus pass, independent entirely of conduction, convection and mechanical carrying by gases and equally independent of dead gas films from an incandescent coal fire to boiler plates or furnace interiors is at first surprising. It must not be assumed, however, that all this will be taken up directly by the plates of a firebox, for example, as they will reflect possibly a quarter to a half of what is so received, sending it out again to other walls, the reflection being especially severe when the heat ray strikes at a small angle, when nearly all the heat will be immediately reflected. However, these quantities are significant of the great steam-forming capacity of enclosed fireboxes common to locomotives and to the Scotch marine boiler. Dalby estimates that assuming the hydrocarbon volatile gases which on combustion separate out solid carbon are themselves equal to an incandescent black body and filling the firebox so that area of radiating surface equals that of absorption, each square foot of firebox can evaporate from this source alone about 134 evaporation units, equivalent roughly to $4\frac{1}{2}$ boiler horse-power. Experiments all prove the firebox capacity for evaporating greater than all the rest of the boiler as ordinarily made.

To compare the Stephan and Bolzmann law of fourth power of temperature with other proposals, Prof. Callender has plotted all on one sheet, which is reproduced in Fig. 161, with the scale changed to English units by Prof. Dalby. Ordinarily the heat radiated is not separated from that otherwise transmitted, but

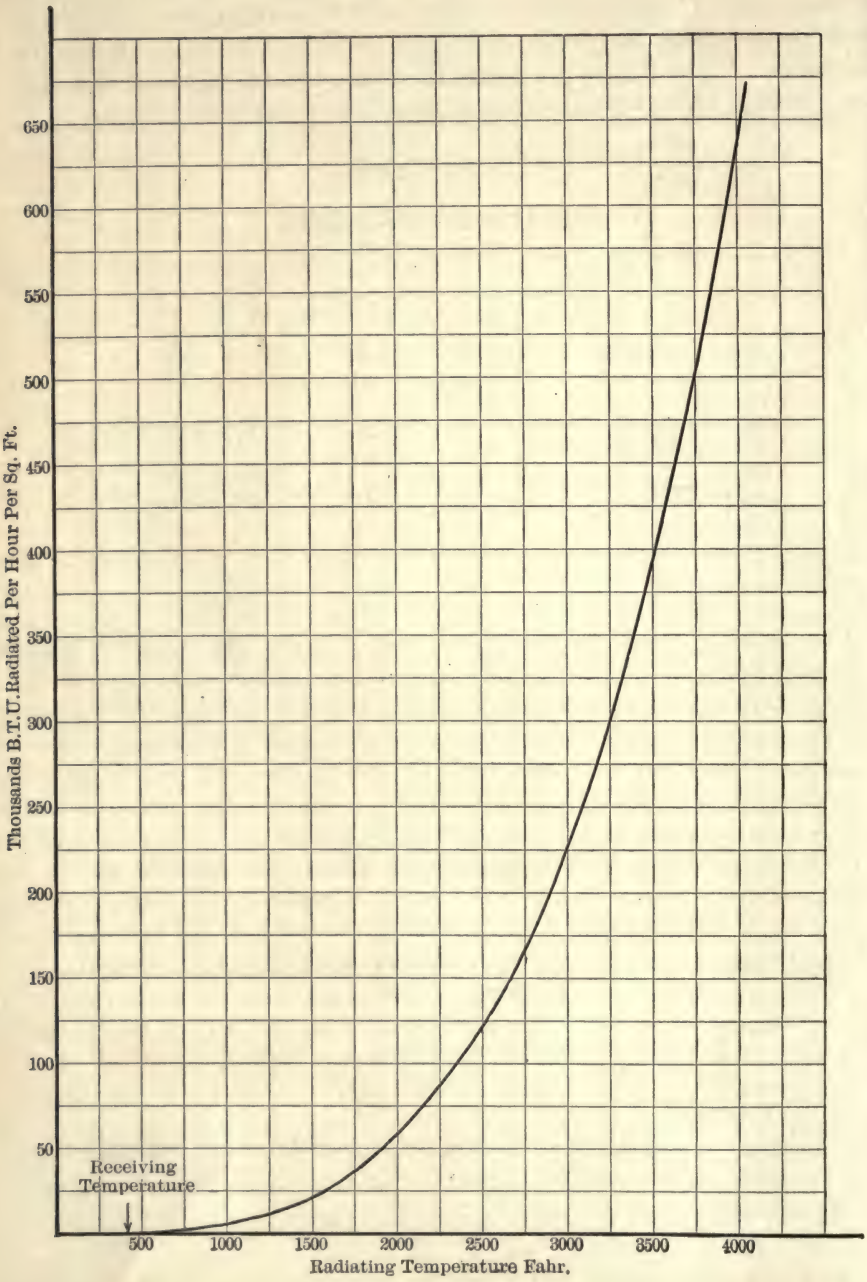


FIG. 160.—Heat Radiation from "Black Body" According to Stefan and Bolzmann Law.

the whole transmission determined from general experimental coefficients where known.

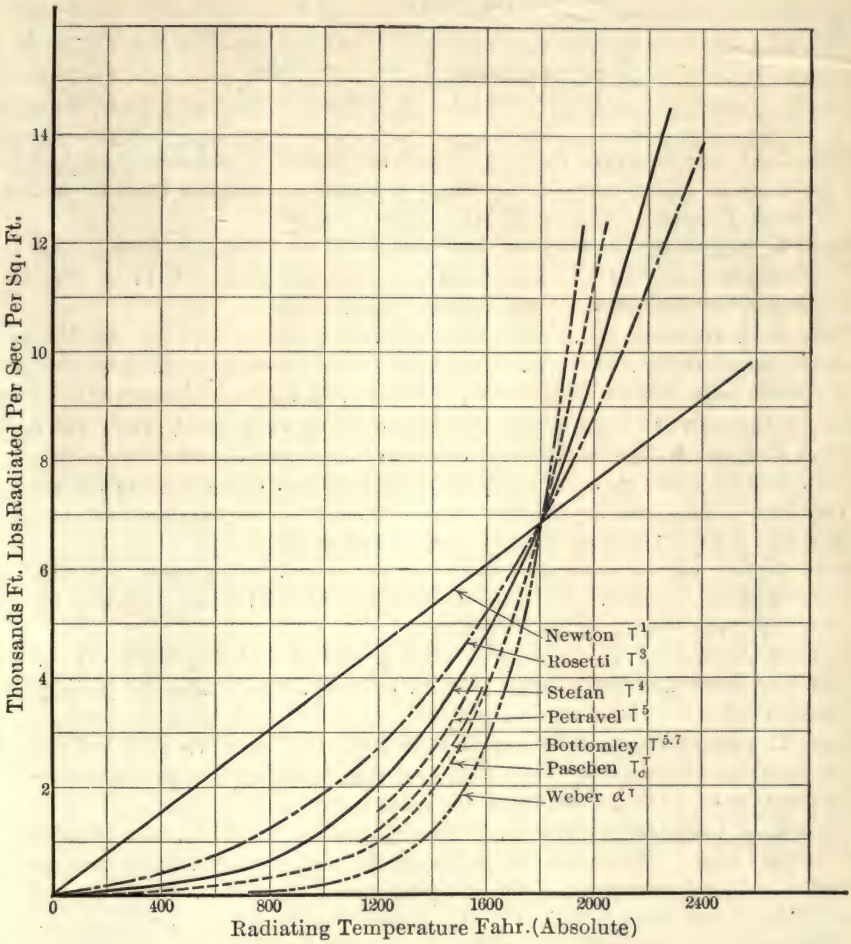


FIG. 161.—Comparison of Heat Radiating Laws.

Example. The wall of a house consists of 6 ins. of brickwork lined with 1 in. of plaster. The junction of the plaster to the brick being equivalent to 10 ins. of plaster. The windows are of glass $\frac{1}{8}$ in. thick and have a film of moisture on them .003 in. thick, the junction of the water and glass being equivalent to 3.5 ins. of glass. How does the coefficient of heat transfer for the walls compare with that of the windows?

From Eq. (734),

$$U = \frac{1}{\Sigma \rho l + \Sigma \sigma}$$

For the wall

$$U = \frac{1}{\frac{1}{.8} \times 6 + \frac{1}{2} \times 10 + 1}$$

For the window

$$\rho = 5 \text{ for the glass,}$$
$$= 3.5 \text{ for the water.}$$

Hence

$$U = \frac{1}{\frac{1}{5}\left(\frac{1}{8}+3.5\right)+\frac{1}{3.5}\times.003} = 1.38.$$

Prob. 1. A bar of copper and one of glass are heated by the same source of heat. How much longer will it take for the end of the glass one furthest from the source of heat to reach a uniform temperature than that of copper?

Prob. 2. Neglecting joint effects, how much will the heat lost from a pipe $\frac{1}{4}$ in. thick carrying steam at 500° F. and running through a room at 50° F. be reduced by the application of a plaster of Paris coating 2 ins. thick?

Prob. 3. A standard 2 in. boiler tube has a thickness of .095 in. On the inside of the tube is a layer of scale $\frac{1}{16}$ in. thick which may be considered as plaster of Paris; on the outside of the tube is $\frac{1}{8}$ in. coating of soot having the same conductivity as wool. Neglecting the resistance due to the junction of these substances, what will be the difference in heat transmitted through the above tube and a new clean one?

Prob. 4. The walls of a room are as follows: 6 ins. concrete, 6 ins. sawdust, 3 ins. concrete. What will be the heat lost per square foot per hour for an inside temperature of 40° F. and an outside temperature of 90° F.?

Prob. 5. The copper tubes of a condenser are .05 in. in thickness. On each side is a water film .01 in. thick. How will the heat transferred through this tube compare with that of a dry one in contact with air?

Prob. 6. A tin tube .05 in. thick is silver plated, the plating being .01 in. thick and the joint resistance being equal to 1 in. of silver. How does the plating affect the value of U?

Prob. 7. Considering steel to have a radiating power equal to .2 of carbon, what will be the value of the radiant heat of a steel sphere having 10 sq.ft. of surface at a the temperature of 1500° F. when the receiving body is at 50° F.?

Prob. 8. A boiler has 1700 sq.ft. of heating surface of which 100 are exposed to direct radiant heat. The remainder is heated by hot gases of average temperature of 1000° F. The temperature of the radiating fire is 3000° F. and the temperature of the water in the boiler is 350° F. If the surface not subject to radiation consists of iron tubes .1 in. thick with .1 in. scale and the junction of scale to plate is equal to an inch of iron, what is the relative importance of each part of the boiler?

16. Heat Transmission between Separated Fluids. Mean Temperature Differences, Coefficients of Transmission. Heat flow, in terms of B.T.U. per hour transmitted, is usually taken to be proportional to the temperature difference when one fluid is giving up heat to another separated by plates or tubes, as for internal conduction, though experimental determinations have indicated the existence of other relations. For example, Grashof, Rankine and others have announced the square of the mean temperature difference between the two fluids as the correct function, while Orrok, recently experimenting with surface condensers, reports a seven-eighths power instead of the first power. The temperatures that are measurable in engineering work are those in the body of fluid, gas or water entering or leaving a coil or pipe or the tem-

perature in a condenser or boiling mass at some selected points. It is not possible to determine outside of the laboratory the skin temperature of a boiler tube, the mean temperature of a kettle shell or even the temperature of the fluid next the cooling or heating surface. Exact scientific analysis of heat flow laws would involve those and many other quantities, hence the practical necessity for approximate calculations and the absence of any generally inclusive law or equation. Each class or case must be studied by itself though in the light of relations to others.

Mathematical analysis and experimental observations both in the laboratory on special apparatus and in the field on standard working equipment has led to the following generalizations beside the proportionality of flow to some power of the mean temperature difference between the giving and receiving fluid. *The quantity of heat transmitted is found to be proportional to the velocity of the fluid to some power when one of the fluids is in motion and to some power of each not necessarily the same when both move.* Thus, for the case of water in the tubes of surface condensers and feed-water heaters, the one-third power has been used by Joule and Ser, the one-half power by Hageman, Josse and Orrok, and the first power by Stanton. The heat flow has been related to the steam velocity at the condensing surface by Hausbrand and Ser as a function of its one-half power, but found to be independent for such conditions as exist in surface condensers and feed-water heaters by Orrok, which corresponds to the zero power. *Air or other gas in motion giving up heat does so at a rate, proportional not to velocity according to Jordan and Reynolds, but to the weight passing per unit of cross-section of gas stream directly,* a conclusion that may reconcile the discrepancies noted previously where the velocities used by different experimenters might be the same,whereas temperature, density and channel cross-section were in all cases different. When steam condenses, the heat flow is strongly controlled by the collection of gaseous matter on the surfaces, and data by Smith and others show that in ordinary apparatus this may affect the heat flow as much as 50 per cent.

For the purposes of calculation the cases are grouped into types, for example, surface condensers and feed-water heaters are typical of heat transmission from condensing steam to water entering at a low and leaving at a higher temperature, but still lower than that of the steam. If the water temperatures are fixed and the amount of heat to be exchanged likewise, the problem becomes one of finding the necessary surface. On the other hand the problem may be set to find the rise of water-temperature for a given amount of surface.

A different situation exists, for example, in steam superheaters in which steam temperature rises by heat received from hot gases, the temperature of which is falling. In this case, given the surface and the initial temperatures of the gases and the steam the final temperature of both may be required for a given amount of surface or inversely the object sought may be the surface necessary to accomplish a given rise of steam or given fall of gas temperature.

In all these practical cases of transfer what is sought is the relation between amount of surface and the corresponding heat flow or its equivalent

in quantity of substance heated or cooled, vapors condensed or liquids evaporated. The temperatures of the substance are not always uniform throughout the mass or over the heating surface and ordinarily change as the substance passes over the surface. Therefore, while the quantity of heat passing is proportional to some power of the temperature difference at any instant, the instantaneous temperature differences being indeterminate, it is necessary to establish a *mean* temperature difference for the whole period of thermal contact of the substance in question.

In addition to the fixing of a *mean* temperature difference for each typical kind of fluid flow to which heat flow is related, it is also necessary in the solution of numerical problems to have the *constant* of *proportionality* for each class apparatus or typical transfer. This constant of proportionality is known as the coefficient of heat transmission and is defined as the B.T.U. per hour, transmitted through one square foot of surface or cross-section of heat flow path, per degree F. difference in temperature between the two substances measured in the body of each and not at the surfaces of contact. Accordingly,

Let Q = B.T.U. per hour transmitted;

" U = coefficient of heat transmission = B.T.U. per hour per square foot per degree F. measured in the substances;

" t_m = mean temperature difference for whole period of transfer;

" A = square feet of surface through which transfer is taking place or the cross-sectional at area of the heat flow path in square feet.

Then

$$Q = AUt_m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (736)$$

There is a definite relation between the mean temperature difference and the initial and final temperatures of the fluids exchanging heat, and in establishing this relation five different cases must be recognized as follows:

CASE I. The substance giving up heat may be at a constant temperature and the substance receiving heat suffer a rise of temperature.

CASE II. The substance giving up heat may suffer a drop in temperature and the substance receiving heat remain at a constant temperature.

CASE III. Both substances may remain at a constant temperature.

CASE IV. Both substances may change temperature, that giving up heat falling and that taking the heat rising by *parallel flow*, the final temperatures of both tending to become equal.

CASE V. Both substances may change temperature by *counter current flow*, i.e. in opposite directions, the *final* temperature of one substance tending to become equal to the *initial* temperature of the other.

For each of these cases there is a different relation between the initial and final temperatures of both substances and the mean temperature difference and these algebraic relations will be derived after a further examination of the typical cases to show how these five are inclusive of all ordinary conditions.

Constant temperatures characterize the two cases of condensing vapors and evaporating liquids, though not strictly so, because it is well known that in a boiler or condenser the water on the vapor side is not quite constant in temperature, but for the purposes of such calculations as these it must be taken so, or no calculation at all would be possible. Substances changing temperature regularly and indefinitely may be liquid or gaseous. With these distinctions the following classification of heat transfer cases can be made with reference (*a*) to the kind of thermal change taking place in the substances;

CLASSIFICATION OF TYPICAL HEAT TRANSFER APPARATUS

Thermal Action in Substances		Class of Relation between temperatures and mean temperature differences.	Examples of Standard Thermal Apparatus.
Giving up Heat.	Receiving Heat.		
Liquid Cooling	Liquid warming	Case IV or V	Aqua ammonia exchanger. Water coils of ammonia absorber. Brine water cooler.
	Gas warming	Case IV or V	Hot-water house-heating radiator; cooling tower. Automobile radiator.
	Liquid boiling	Case II	Ammonia and carbon dioxide brine cooler. Evaporator with hot liquid coils or jacket.
Gas cooling	Liquid warming	Case IV or V	Brine coils in cold storage rooms. Economizers. Cylinder jackets. Forecooler ammonia condenser. Brine coil air dryer or cooler. Compressor intercooler.
	Gas warming	Case IV or V	Dense air machine coils in cold storage rooms. Steam superheaters. Compressed air engine preheater.
	Liquid boiling	Case II	Steam boiler. Ammonia or carbon dioxide direct expansion coils in cold storage rooms.
Vapor condensing	Liquid warming	Case I	Exhaust steam boiler feed-water heater. Steam, ammonia and carbon dioxide condenser.
	Gas warming	Case I	Steam radiator and tempering coils. Steam piping radiation. Multiple expansion engine reheating receiver. Dry air condenser.
	Liquid boiling	Case III	Exhaust steam vac. evaporator. Multiple effect evaporator. Coil and jacketed evap. kettle. Aqua ammonia generator. Evaporative condenser.

(*b*) type of relation between initial, final temperatures and mean temperature difference, and (*c*) specific examples of apparatus in which each of the type actions take place which, therefore, are grouped as similar or dissimilar.

It might be noted in the case of ice cans submerged in brine that before freezing begins the case is one of heat of water cooling to brine warming, and after freezing begins, constant temperature latent heat of freezing to brine warming, but as the brine is not allowed to rise more than a few degrees the process is nearly one of constant temperature on both sides nd is not listed above because it is a peculiar case without parallel. A similar complexity exists with respect to reheating receivers of multiple expansion engines in which the heat of condensing high pressure steam is added to steam passing between cylinders and where if the working steam is wet it is first dried at constant temperature and afterward warmed or superheated to a temperature approaching that of the live steam. Still other cases of possible complexity are the cooling tower and evaporative condensers in which more than one action may take place and in which the specific construction exerts a strong or controlling influence.

It is structure entirely that determines whether Case IV for parallel flow or Case V for counter-current flow shall apply to the transfer between liquids and gases, one to the other in which both substances suffer temperature change. In some constructions it is quite impossible to fix the flow relations and this is in general true when a large mass of substance is under treatment as in a tank, instead of in coils pipes or flowing between guiding partitions, and in these cases it is necessary to assume the nearest typical case as representative, which, of course, requires some good judgment. Derivations of mean differences will first be based on *proportionality of heat transmitted to the first power of the temperature difference.*

Case I. Mean temperature difference, for constant high temperature source to rising temperature cold substance. In Fig. 51 the temperature of the two substances is plotted vertically, and the surfaces over which the flow causes temperature change, horizontally for the conditions specified as Case I.

Let t_h = constant temperature of the hot substance degrees F.;
" t_c = any momentary temperature of the cold substance degrees F.;
" t_{c_1} = initial temperature of the cold substance degrees F.;
" t_{c_2} = final temperature of the cold substance degrees F.;
" w_c = pounds of cold substance flowing per hour over surface.
" C_c = specific heat of cold substance;
" A = surface in square feet;
" t_m = mean temperature difference between hot and cold substance;
" U = coefficient of heat transfer = B.T.U. per hour per square feet per degree F.

Then will the heat transmitted per hour through the elementary surface dA be given by $U(t_h - t_c)dA$, and as the temperature rise for this period will

be dt_c, the heat transmitted per hour is also given by $C_c w_c dt_c$ which is the amoune taken up by the cold substance. These two quantities must be equal, whenct

$$dA = \frac{C_c w_c}{U} \frac{dt_c}{t_h - t_c}.$$

If, as is here assumed to be the case, the hot substance be at a constant temperature, t_h is independent of t_c and the coefficient of heat transfer U like-

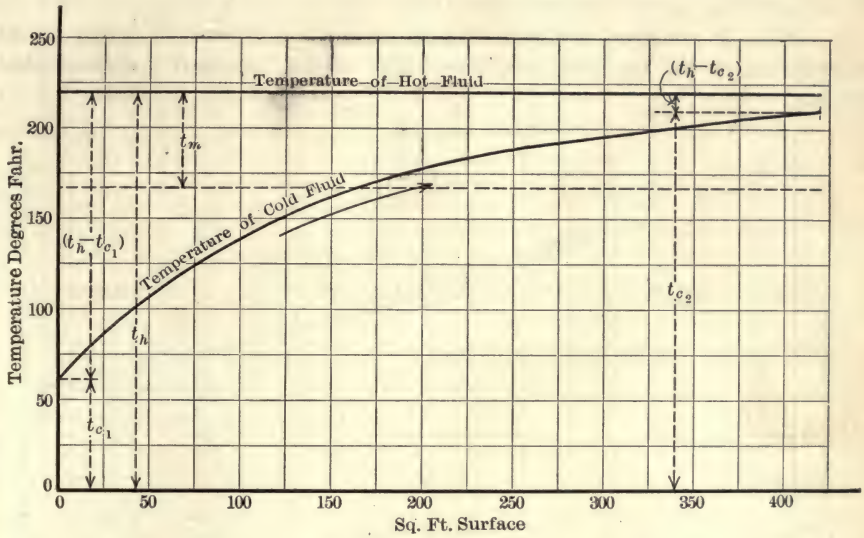


FIG. 162.—Heat Transfer from Constant Temperature Source to Rising Temperature Fluid, Case I.

wise, then will the relation between temperature rise of the cold substance and the surface traversed be given by,

$$A = \frac{C_c w_c}{U} \int_{t_{c_1}}^{t_{c_2}} \frac{dt_c}{(t_h - t_c)},$$

or

$$A = \frac{C_c w_c}{U} \log_e \left(\frac{t_h - t_{c_1}}{t_h - t_{c_2}} \right), \quad \ldots \ldots \quad (737)$$

For the whole period of transfer if U is independent of the temperature difference

$$t_m U A = C_c w_c (t_{c_2} - t_{c_1}),$$

or

$$t_m = \frac{C_c w_c}{UA} (t_{c_2} - t_{c_1}). \quad \ldots \ldots \quad (738)$$

Substitution in Eq. (738) of the value of UA from Eq. (737) in terms of temperature differences gives the mean temperature difference

$$t_m = \frac{(t_{c_2} - t_{c_1})}{\log_e \left(\frac{t_h - t_{c_1}}{t_h - t_{c_2}} \right)}, \quad \ldots \ldots \ldots \quad (739)$$

This value is indicated by the distance from the high temperature line t_h to the dotted line on the diagram Fig. 162.

Case II. A constant temperature of the cold substance T_c with a falling temperature of the hot one constitutes the second case and is represented in Fig. 163.

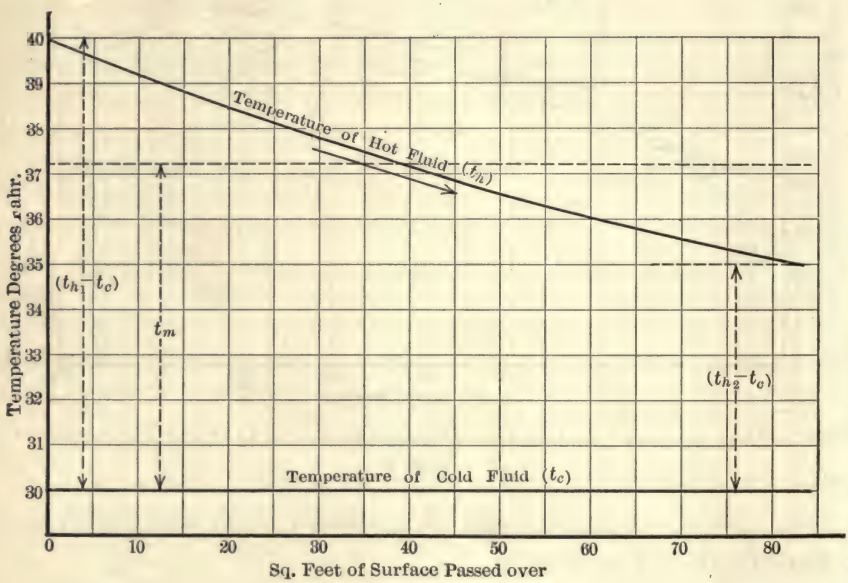


FIG. 163.—Heat Transfer from a Falling Temperature Source to a Constant Low Temperature Fluid, Case II.

With the use of similar symbols and as indicated on the diagram,

$$dA = \frac{C_h w_h}{U} \frac{dt_h}{(t_h - t_c)},$$

$$A = \frac{C_h w_h}{U} \int_{t_{h_2}}^{t_{h_1}} \frac{dt_h}{(t_h - t_c)}, \quad \ldots \ldots \ldots \quad (740)$$

$$t_m UA = C_h w_h (t_{h_1} - t_{h_2}),$$

$$t_m = \frac{(t_{h_1} - t_{h_2})}{\log_e \left(\frac{t_{h_1} - t_c}{t_{h_2} - t_c} \right)}. \quad \ldots \ldots \ldots \quad (741)$$

Case III. When both temperatures are constant the relations shown in Fig. 164 are most simple since the mean temperature difference is constant and the heat transmitted per hour is directly proportional to the surface in action.

$$t_m = t_h - t_c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (742)$$

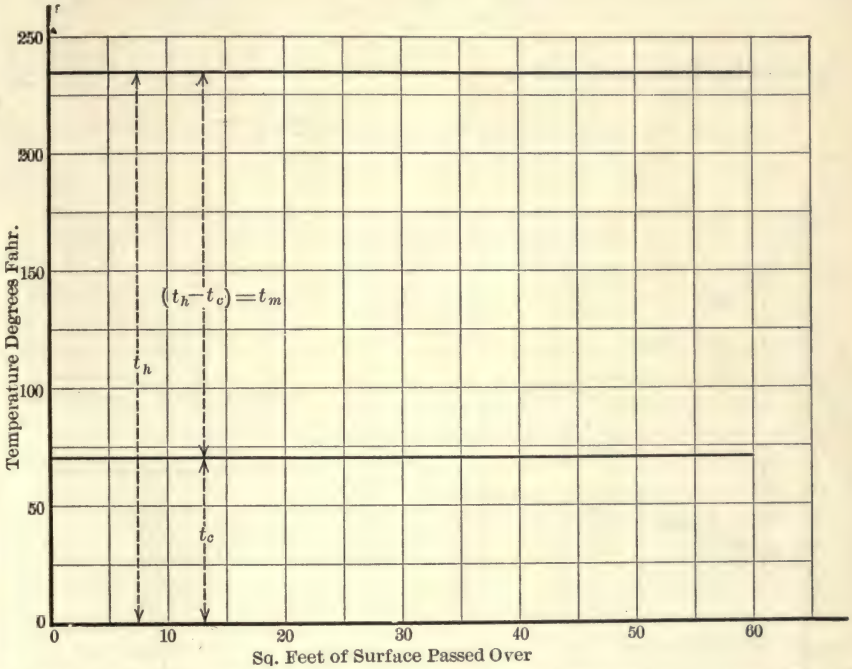


FIG. 164.—Heat Transfer between Two Fluids Each at a Constant Temperature, Case III.

Case IV. Parallel flow conditions of two substances both changing temperature are indicated in Fig. 165 which, therefore, represents diagrammatically the fourth case. After passage over the elementary surface dA, the cold substance will rise in temperature an amount dt_c and the temperature of hot substance fall correspondingly dt_h; therefore, the heat transmitted per hour is given by

$$U(t_h - t_c)dA = C_c w_c dt_c = C_h w_h dt_h.$$

It appears, therefore, that the relation between surface and temperature change can be given two forms, as follows,

$$A = \frac{C_c w_c}{U} \int_{t_{c_1}}^{t_{c_2}} \frac{dt_c}{(t_h - t_c)} = \frac{C_h w_h}{U} \int_{t_{h_2}}^{h_1} \frac{dt_h}{(t_h - t_c)}.$$

These are really identical because t_h is a function of t_c at any moment since the heat gained by the cold body must be equal to that lost by the hot one, which fact fixes the relation between temperatures.

Hence after any interval of time or surface traversed

$$C_h w_h(t_{h_1} - t_h) = C_c w_c(t_c - t_{c_1}).$$

Therefore

$$t_h = t_{h_1} + \frac{C_c w_c}{C_h w_h}(t_{c_1} - t_c).$$

Subtracting from each side t_c,

$$t_h - t_c = t_{h_1} + \frac{C_c w_c}{C_h w_h} t_{c_1} - \left(1 + \frac{C_c w_c}{C_h w_h}\right) t_c.$$

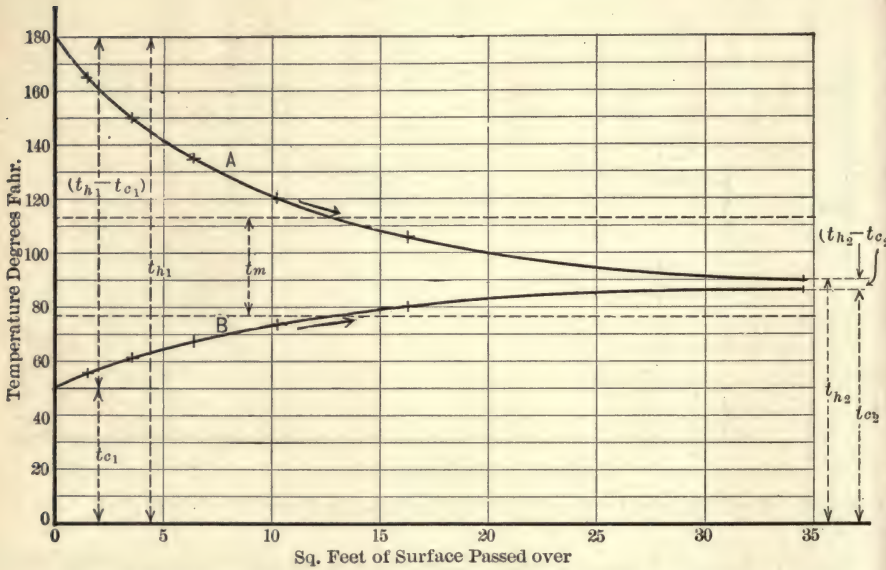


FIG. 165.—Heat Transfer between Fluids, Each Changing Temperature, Parallel Flow.

CASE IV.

Substituting this in the differential equation where the variable is t_c, gives

$$A = \frac{C_c w_c}{U} \int_{t_{c_1}}^{t_{c_2}} \frac{dt_c}{\left[t_{h_1} + \frac{C_c w_c}{C_h w_h} t_{c_1} - \left(1 + \frac{C_c w_c}{C_h w_h}\right) t_c\right]}$$

$$= \left[\frac{C_c w_c}{U\left(1 + \frac{C_c w_c}{C_h w_h}\right)}\right] \log_e \left[\frac{t_{h_1} + \frac{C_c w_c}{C_h w_h} t_{c_1} - \left(1 + \frac{C_c w_c}{C_h w_h}\right) t_{c_1}}{t_{h_1} + \frac{C_c w_c}{C_h w_h} t_{c_1} - \left(1 + \frac{C_c w_c}{C_h w_h}\right) t_{c_2}}\right],$$

Therefore

$$A = \left[\frac{C_c w_c}{U\left(1 + \frac{C_c w_c}{C_h w_h}\right)}\right] \log_e \left(\frac{t_{h_1} - t_{c_1}}{t_{h_2} - t_{c_2}}\right), \quad \ldots \ldots \quad (742$$

This can be put in a slightly different form by the relation

$$1+\frac{C_c w_c}{C_h w_h}=1+\frac{t_{h_1}-t_{h_2}}{t_{c_2}-t_{c_1}}=\frac{(t_{c_2}-t_{c_1})+(t_{h_1}-t_{h_2})}{t_{c_2}-t_{c_1}},$$

which may be inserted in Eq. (743) where the problem indicates a necessity for it.

For the whole transfer,

$$t_m UA=C_c w_c(t_{c_2}-t_{c_1})=C_h w_h(t_{h_1}-t_{h_2}),$$

whence

$$t_m=\frac{C_c w_c(t_{c_2}-t_{c_1})}{UA}=\frac{C_c w_c(t_{c_2}-t_{c_1})}{\left[\frac{C_c w_c}{\frac{(t_{c_2}-t_{c_1})+(t_{h_1}-t_{h_2})}{(t_{c_2}-t_{c_1})}}\right]\log_e\left(\frac{t_{h_1}-t_{c_1}}{t_{h_2}-t_{c_2}}\right)}.$$

Therefore

$$t_m=\frac{(t_{c_2}-t_{c_1})+(t_{h_1}-t_{h_2})}{\log_e\left(\frac{t_{h_1}-t_{c_1}}{t_{h_2}-t_{c_2}}\right)}=\frac{(t_{h_1}-t_{c_1})-(t_{h_2}-t_{c_2})}{\log_e\left(\frac{t_{h_1}-t_{c_1}}{t_{h_2}-t_{c_2}}\right)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (744)$$

Case V. Counter-current flow conditions for this case are indicated on Fig. 166, which shows a rising temperature of the cold body to a value approaching that of the initial temperature of the hot one, instead of its final temperature as in the case of parallel flow. Beginning with the entrance of the cold body into the system its temperature will have risen dt_c degrees from t_{c_1}, after passage over the elementary surface dA, and in the same time the hot body will have fallen dt_h degrees to the value t_{h_2}. Therefore, as for parallel flow

$$A=\frac{C_c w_c}{U}\int_{t_{c_1}}^{t_{c_2}}\frac{dt_c}{(t_h-t_c)}=\frac{C_h w_h}{U}\int_{t_{h_2}}^{t_{h_1}}\frac{dt_h}{(t_h-t_c)}.$$

Elimination of one variable t_h by introducing its relation to t_c is done in a similar manner but by terms differently involved. After any appreciable interval

$$C_h w_h(t_h-t_{h_2})=C_c w_c(t_c-t_{c_1}).$$

Therefore

$$t_h=t_{h_2}+\frac{C_c w_c}{C_h w_h}(t_c-t_{c_1}),$$

and

$$t_h-t_c=t_{h_2}-\frac{C_c w_c}{C_h w_h}t_{c_1}+\left(\frac{S_c w_c}{S_h w_h}-1\right)t_c.$$

Substitution in the differential equation where the variable is t_c gives,

$$A = \frac{C_c w_c}{U} \int_{t_{c_1}}^{t_{c_2}} \frac{dt_c}{\left[t_{h_2} - \frac{C_c w_c}{C_h w_h} t_{c_1} + \left(\frac{C_c w_c}{C_h w_h} - 1 \right) t_c \right]}$$

$$= \left[\frac{C_c w_c}{U \left(\frac{C_c w_c}{C_h w_h} - 1 \right)} \right] \log_e \left[\frac{t_{h_2} - \frac{C_c w_c}{C_h w_h} t_{c_1} + \left(\frac{C_c w_c}{C_h w_h} - 1 \right) t_{c_2}}{t_{h_2} - \frac{C_c w_c}{C_h w_h} t_{c_1} + \left(\frac{C_c w_c}{C_h w_h} - 1 \right) t_{c_1}} \right].$$

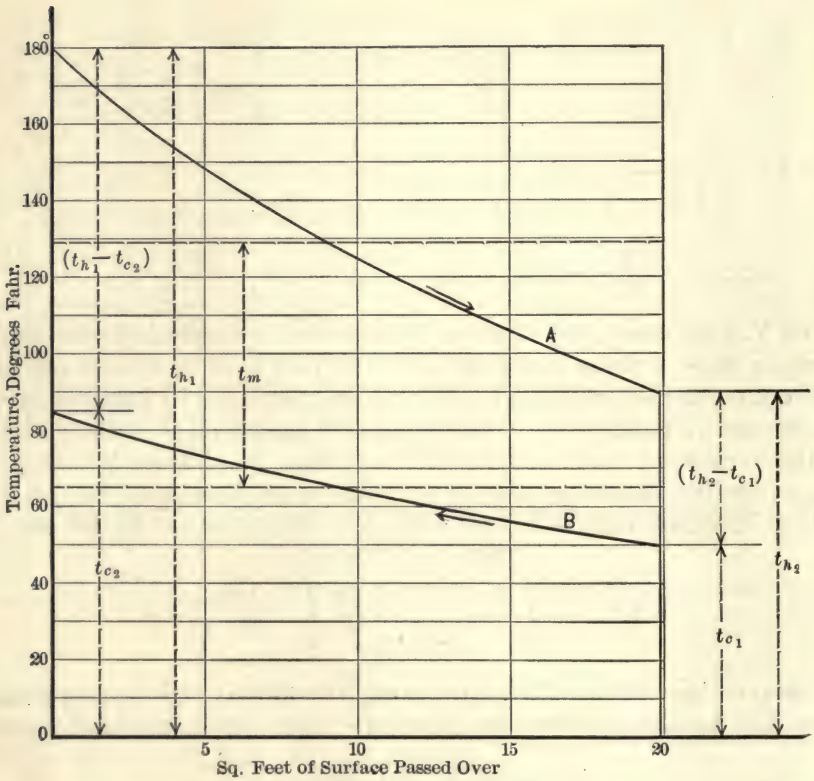


FIG. 166.—Heat Transfer between Fluids, Each Changing Temperature, Counter-current Flow. Case V.

Therefore

$$A = \left[\frac{C_c w_c}{U \left(\frac{C_c w_c}{C_h w_h} - 1 \right)} \right] \log_e \left(\frac{t_{h_1} - t_{c_2}}{t_{h_2} - t_{c_1}} \right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (745)$$

The weight, specific heat ratio can be eliminated by the relation between it and temperatures which is given by

$$\frac{C_c w_c}{C_h w_h} - 1 = \frac{t_{h_1} - t_{h_2}}{t_{c_2} - t_{c_1}} - 1 = \frac{(t_{h_1} - t_{h_2}) - (t_{c_2} - t_{c_1})}{t_{c_2} - t_{c_1}}.$$

For the whole transfer

$$t_m UA = C_c w_c (t_{c_2} - t_{c_1}) = C_h w_h (t_{h_1} - t_{h_2}).$$

Whence

$$t_m = \frac{C_c w_c (t_{c_2} - t_{c_1})}{UA} = \frac{C_c w_c (t_{c_2} - t_{c_1})}{\left[\frac{(t_{h_1} - t_{h_2}) - (t_{c_2} - t_{c_1})}{(t_{c_2} - t_{c_1})}\right] \log_e \left(\frac{t_{h_1} - t_{c_2}}{t_{h_2} - t_{c_1}}\right)}$$

$$= \frac{(t_{h_1} - t_{h_2}) - (t_{c_2} - t_{c_1})}{\log_e \left(\frac{t_{h_1} - t_{c_2}}{t_{h_2} - t_{c_1}}\right)},$$

$$t_m = \frac{(t_{h_1} - t_{c_2}) - (t_{h_2} - t_{c_1})}{\log_e \frac{(t_{h_1} - t_{c_2})}{t_{h_2} - t_{c_1}}}. \quad \ldots \ldots \ldots \ldots \quad (746)$$

All of these equations for mean temperature difference are identical when properly interpreted, however different they may look in terms of the specific symbols. Inspection will show that the numerator is in every case the initial temperature difference less the final temperature difference, while the denominator is only the hyperbolic logarithm of the ratio of initial to final temperature differences.

$$\text{(Mean temp. diff.)} = \frac{\text{(Initial temp. diff.)} - \text{(Final temp. diff.)}}{\log_e \left(\frac{\text{Initial temp. difference}}{\text{Final temp. difference}}\right)}. \quad (747)$$

By the adoption of coefficients of transfer U, or rather numerical values for U, for the common cases of practice, it becomes possible to calculate relation between quantities of substance heated or quantity of heat transmitted and surface but only approximately. The coefficients used in most engineering work allow a good safe margin in the form of sufficient surface so the apparatus will always be big enough to perform the required service, but economy in the use of material demands care in design to bring to bear as large a coefficient as possible by whatever means are available and which means are confined to the use of high velocities of fluid or equivalent means of fluid film reduction and cleanliness. In Table XXVIII are given some average values of U that have been in common use in designing apparatus. These are presented not as accurately determined values but as average value illustrating the most important fact that in all cases where gases are involved the coefficient is ever so much less than when it is not.

TABLE XXVIII

COEFFICIENTS OF HEAT TRANSFER

AVERAGE PRACTICE

Thermal Action in Substances.		B.T.U. per Hour per Square Foot per Degree.	Apparatus.
Giving Up Heat.	Receiving Heat.		
Liquid cooling.....	Liquid warming	50–75	Liquid heat exchangers, aqua ammonia water and beer coolers, ammonia absorber cooling coils
	Gas warming	2–6	Hot-water radiators and cooling tower surfaces, depending on air velocity and character of water surface
	Liquid boiling	100 10–20 30–50	Shell brine coolers with circulator; tank brine coolers without circulator; double pipe brine coolers depending on velocity and hot liquid evaporators
Gas cooling.....	Liquid warming	2–5	Brine coolers in cold storage rooms depending on air circulation. Air coolers with water or brine coils; economizers
	Gas warming	2–4	Steam superheaters
	Liquid boiling	2–5	Direct expansion ammonia coils in cold storage rooms depending on air circulation. Steam boilers
Vapor condensing....	Liquid warming	150–350 1000	Feed-water heaters and steam condensers depending on water velocity and removal of air on steam side. Experimental feed-water heater high velocity
	Gas warming	2–4	Steam radiators and pipes
	Liquid boiling	400–600	Vacuum evaporators with condensing exhaust steam depending on viscosity of solution

Example 1. Case I, Fig. 162. A feed-water heater, with exhaust steam at 220° F. and a water supply at 60° F. and $U=200$, specific heat $=1$, water 300,000 lbs. per hour, which in round numbers corresponds roughly to the feed of a 100 H.P. boiler. Compute the surface to give the feed water a final temperature of 90°, 120°, 150°, 180°, 210° and find t_m.

$$A=\frac{C_c w_c}{U}\log_e\left(\frac{t_h-t_{c_1}}{t_h-t_{c_2}}\right)$$

For $t_c=90°$ F., $A=\frac{30000}{200}\log_e\left(\frac{220-60}{220-90}\right)$ sq.ft.

$$=150\log_e\frac{160}{130}=150\log_e 1.23=150\times.207=31.05 \text{ sq.ft.},$$

" $t_{c_2}=120°$ F., $A=150\log_e\frac{160}{100}=150\log_e 1.6=150\times.47=70.5$ sq.ft.,

" $t_{c_2}=150°$ F., $A=150\log_e\frac{160}{70}=150\times.833=124.95$ sq.ft.,

" $t_{c_2}=180°$ F., $A=150\log_e\frac{160}{40}=150\times1.386=207.90$ sq.ft.,

" $t_{c_2}=210°$ F., $A=150\log_e\frac{160}{10}=150\times2.77=415.5$ sq.ft.,

$$t_m=\frac{t_{c_2}-t_{c_1}}{\log_e\left(\frac{t_h-t_{c_1}}{t_h-t_{c_2}}\right)}=\frac{210-60}{\log_e\left(\frac{160}{10}\right)}=\frac{150}{2.77}=54 \text{ degrees mean temperature difference.}$$

Example 2. Case II, Fig. 163. An ammonia brine cooler where the evaporation takes place at 30° F. and the brine enters the coils at 40° and leaves at 35° F., a total fall in temperature of 5° F. Take the specific heat of the brine at .68 and assume 17,700 lbs. per hour, which about corresponds to one ton of ice per 24 hours. Take $U=100$. Find the surface corresponding to 39° F., 38°, 37°, 36° and 35° F.

$$A=\frac{C_h w_h}{U}\log_e\frac{(t_{h_1}-t_c)}{(t_{h_2}-t_c)}$$

$$=\frac{.68\times17700}{100}\log_e\frac{(40-30)}{(t_{h_2}-30)}$$

$$=120\log_e\frac{10}{(t_{h_2}-30)},$$

For $t_{h_2}=39°$ F., $A=120\log_e\frac{10}{9}=12.6$ sq.ft.,

" $t_{h_2}=38°$ F., $A=120\log_e\frac{10}{8}=26.8$ sq.ft.

$$\text{For } t_{h_2}=37^\circ \text{ F.}, \quad A=120 \log_e \frac{10}{7}=42.9 \text{ sq.ft.},$$

$$\text{“ } \quad t_{h_2}=36^\circ \text{ F.}, \quad A=120 \log_e \frac{10}{6}=61.4 \text{ sq.ft.}$$

$$\text{“ } \quad t_{h_2}=35^\circ \text{ F.}, \quad A=120 \log_e \frac{10}{5}=83.2 \text{ sq.ft.},$$

$$t_m=\frac{t_{h_1}-t_{h_2}}{\log_e\left(\frac{t_{h_1}-t_c}{t_{h_2}-t_c}\right)}=\frac{40-35}{\log_e\left(\frac{10}{5}\right)}=7.21 \text{ degrees mean temperature difference.}$$

Example 3. Case III, Fig 164. Assume that exhaust steam at a temperature of 235° F. is used to supply heat to an evaporator in which alcohol is being evaporated at a temperature of 70° F. So long as the pressures and the quality of the alcohol remain constant, these temperatures will remain the same. What will be the surface required if 3,600,000 B.T.U. are to be supplied to the evaporating fluid per hour? Assume $U=400$.

$$(t_h-t_c)=t_m=235-70=165 \text{ degree temperature difference.}$$

Heat transfer per square foot $=165\times400=60{,}000$ B.T.U. per hour.

$$\text{Required surface}=\frac{3600000}{60000}=60 \text{ sq.ft.}$$

Example 4. Case IV, Fig 165. Water leaving a still at a temperature of 180° F. passes through a cooler before entering ice tanks in which it is to be made into ice. Let this cooler be assumed to have a parallel flow of distilled water and of cooling water. The cooling water enters at a temperature of 50° F. The cooler is to be capable of cooling 10 tons of water per day from a temperature of 180° F. to 90° F. using for cooling $2\frac{1}{2}$ lbs. of cooling water per pound of distilled water. Take $\overline{U}=60$.

Per hour, $w_h=834$ lbs. per hour; $w_c=2080$ lbs.

$$C_h=1; \quad C_c=1.$$

Find the required cooling surface for each 15° fall of temperature of the hot water. Heat necessary to cool 20,000 lbs. through 15° will heat 50,000 lbs. of water $\frac{2}{5}\times15^\circ=6^\circ$, which is accordingly the corresponding increment for t_c as shown in tabular form.

t_h	t_c	t_h-t_c
180	50	130
165	56	109
150	62	88
135	68	67
120	74	46
105	80	25
90	86	4

Substitution in Eq. (743) gives the following expression for area corresponding to a given temperature difference, $t_{h_2}-t_{c_2}$

$$A=\left[\frac{1\times2080}{60\left(1+\frac{1\times2080}{1\times834}\right)}\right]\log_e\left(\frac{180-50}{t_{h_2}-t_{c_2}}\right)=9.9\times\log_e\left(\frac{130}{t_h-t_c}\right).$$

For $t_{h_2}=165°$ F., $A=9.9\log_e\frac{130}{109}=1.74$ sq.ft.,

For $t_{h_2}=150°$ F., $A=9.9\log_e\frac{130}{88}=3.88$ sq.ft.,

For $t_{h_2}=135°$ F., $A=9.9\log_e\frac{130}{67}=6.56$ sq.ft.,

For $t_{h_2}=120°$ F., $A=9.9\log_e\frac{130}{46}=10.30$ sq.ft.,

For $t_{h_2}=105°$ F., $A=9.9\log_e\frac{130}{25}=16.32$ sq.ft.,

For $t_{h_2}=90°$ F., $A=9.9\log_e\frac{130}{4}=34.5$ sq.ft.

The mean temperature difference is

$$t_m=\frac{(t_{h_1}-t_{c_1})-(t_{h_2}-t_{c_2})}{\log_e\left(\frac{t_{h_1}-t_{c_1}}{t_{h_2}-t_{c_2}}\right)}\quad\frac{130-4}{\log_e\left(\frac{130}{4}\right)}=\frac{126}{3.48}=36 \text{ degrees.}$$

Example 5. Case V. Fig. 166. Assume a problem similar to that for Case IV except that the flow is counter current instead of parallel, i.e., into a counter-current cooler flows 834 lbs. per hour of distilled water at a temperature of 180° F., which is to be cooled to 90° upon leaving. Cooling water is supplied at 50° F. at the rate of 2080 lbs. per hour. How much cooling surface is required to cool by intervals of 15°? What is the mean temperature difference when cooling to 90°?

$C_h=1$; $S_c=1$; $w_h=834$; $w_c=2080$. Assume $U=60$; $t_{c_1}=50°$. Since for any given interval, with counter flow

$$C_hw_h(t_{h_1}-t_{h_2})=C_cw_c(t_{c_2}-t_{c_1}),$$

$$t_{c_2}=t_{c_1}+(t_{h_1}-t_{h_2})\frac{C_hw_h}{w_cC_c}$$

$$=50+(180-90)\frac{834\times1}{2080\times1}$$

$$=86°\text{ F. as shown in the table.}$$

Next, let $t_{h_2}=165°$ F., and $t_{c_1}=86-(180-165)\frac{834}{2080}=80°$ F.

For each successive fall for t_h of 15°, there is a fall of 6° for t_c since the cold fluid is flowing in the opposite direction and the change of temperature of the one is due to the change of temperature of the other.

t_h	t_c	$(t_h - t_c)$
180	86	94°
165	80	85
150	74	76
135	68	67
120	62	58
105	56	49
90	50	40

From Eq. (745) the area from the point of entrance of the hot liquid to the point at which temperature has fallen 15° is given below.

$$\text{For } t_{h_2} = 165° \text{ F.}, \quad A = \frac{1 \times 2080}{30\left(\frac{2080}{834} - 1\right)} \log_e \left(\frac{180 - 86}{165 - 80}\right)$$

$$= 2.31 \times \log_e \frac{94}{85} = 2.33 \text{ sq.ft.}$$

" $t_{h_2} = 150°$ F., $A = 23.1 \log_e \frac{94}{76} = 4.94$ sq.ft.,

" $t_{h_2} = 135°$ F., $A = 23.1 \log_e \frac{94}{67} = 7.82$ sq.ft.,

" $t_{h_2} = 120°$ F., $A = 23.1 \log_e \frac{94}{58} = 11.15$ sq.ft.,

" $t_{h_2} = 105°$ F., $A = 23.1 \log_e \frac{94}{49} = 15.1$ sq.ft.,

" $t_{h_2} = 90°$ F., $A = 23.1 \log_e \frac{94}{40} = 19.8$ sq.ft.

The mean temperature difference is

$$t_m = \frac{C_h w_h (t_{h_1} - t_{h_2})}{UA} = \frac{834 \times 90}{60 \times 19.8} = 63.1 \text{ degrees,}$$

or by Eq. (746)

$$t_m = \frac{94 - 40}{\log_e \left(\frac{94}{40}\right)} = \frac{54}{1855} = 63.1 \text{ degrees, as previously.}$$

Prob. 1. In the manufacture of condensed milk, the water is evaporated at a low pressure by the use of steam. If the pressure in the vacuum kettle is such that the temperature is 90° F., the latent heat per pound will be about 1100 B.T.U.'s. On the assumption that this condition remains the same during the entire process, that $U=300$, and that the heat is derived from steam at a temperature of 215° F., how much surface will be required to evaporate 2 tons of water per hour?

Prob. 2. The rate of heat transfer in a surface condenser is 400. The vacuum desired calls for a temperature in the condenser of 100° F. The cooling water amounts to 1,000,000 lbs. per hour, enters at 50° F. and leaves at 95° F. How much surface will be needed if the heater is a five-pass heater, and what will be the temperature at the end of each pass.

Prob. 3. The boiler for a steam-heating plant runs at a pressure of 5 lbs. per square inch gage, and evaporates a thousand pounds of steam per hour. The products of combustion have an initial temperature of 1000° F., a final temperature of 400° F. and a specific heat of .25. Assuming 1000 B.T.U.'s needed per pound of steam and that U is 2.5, how much surface will be needed?

Prob. 4. The oil in an electric transformer is cooled by running it through annular tubes through the inside of which is circulated cold water in a counter-current direction. 1000 lbs. of oil per hour must be cooled from 150° F. to 80° F. while there is available 5000 lbs. of water per hour at a temperature of 50° F. The specific heat of the oil may be taken as .4, and U as 50. At what temperature will the water leave at the end of the cooler and how much surface will be needed for every 10° of cooling? If it were desired to cool oil down to 70° F. how much additional surface would be required?

Prob. 5. To reduce the quantity of moisture in air it is passed over coils containing cold brine and thereby lowered below the dew point. Disregarding the heat to be removed from the moisture what must be the area of coils to lower 500,000 cubic feet of air per hour from 70° F. to 20° F.? The air and brine pipes enter the cooling duct at the same end. Using the following data, what will be the required surface? Specific heat of air .25, of brine .7, brine enters at −20° F. and leaves at 15° F. What will be the mean temperature difference?

Prob. 6. Solve Prob. 5 for counter-current flow.

17. Variation in Coefficient of Heat Transmission Due to Kind of Substance, Character of Separating Wall and Conditions of Flow. Nothing could well be more striking than the fluctuations in the accepted workable values of U for different conditions of surface, kinds of substance and rate of flow, ranging as they do from about $U=2$, to somewhere near $U=1000$. It is quite natural in view of the uncertainty that must accompany the selection of the correct value for a given practical problem that many investigators have sought to explain by experiment and analysis the variations that exist with a view to associating all values by a general law. This ultimate aim appears, however, to be quite hopeless of attainment, and even if it were not without hope it would probably be useless because, any general law must include terms to account for the conditions of surface, the gas content of liquids or vapors, something equivalent to the thickness or resistance of fluid films and other things equally intermediate by a computer engaged in predicting what will happen in an apparatus for which these things must first be evaluated but cannot be. About all that one could reasonably expect in this connection is a relation of

U to the *definable* variables in each characteristic class of heat transfer cases with perhaps a few general principles, and some of these principles are fairly well established, though none quite beyond the limits of controversy.

Among the general principles, one already cited appears most important of them all, that when a gas is either giving or receiving heat the coefficient of heat transfer is very much lower than when a liquid is involved as such, or as changing state with its vapor. *So very high is the resistance and so very low the value of U when a gas is involved, that the main resistance to flow is always on the gas side unless most extraordinary means are used to change it.* As transfer always proceeds from one substance to another and through a third generally, it may be considered as a three-stage operation, of which the first step is the giving up of heat by the hot one, the second its transmission proper and the third its absorption by the cooler body. The rate with which it will pass while a function of all the heat resistances encountered is practically *controlled by the one single highest resistance when that one is much larger than the others.* Thus, in a given case the rate of heat exchange may be limited by the ability of the fluid on one side or the other to give or to take and in the great majority of cases this is so, for a steam boiler the water can take up heat faster than the hot gases can give it, except possibly for surfaces receiving radiant heat, while in a steam condenser the limit is imposed by the ability of the circulating water to take the heat, and the latter is also the case for steam feed-water heaters as well. Therefore, even if means were available to increase the heat-absorbing capacity of water next the heating surface of boilers it would be of no value until the gases had been first caused to give up heat at a very much faster rate than they do. These cited, are plain cases, but there are others in which conditions are not so clear such, for example, as water heat exchangers and steam superheaters where in each case the same fluid and fluid conditions exist on each side and where it is impossible without further information to state just where the principal resistance is, that is on which side.

The usual plan of experimenters seeking to relate the coefficient of heat transfer or the hourly transfer of heat to some prime variable has been to vary every condition that might produce a change, one at a time, plot the results to coordinates and seek an equation for the curve. Practically all that is known of the conditions that control the value of U has been derived in this manner with the exception of one result announced by Osborne Reynolds and derived by mathematical analysis based on the kinetic theory of gases and on certain hypotheses with regard to the condition of affairs at the metal surface.

Some of these results will be reviewed to illustrate the complexity of the relations involved and the almost complete hopelessness of any attempt to generalize much beyond the following principles in addition to those of a single controlling resistance and the almost universally higher value of gas resistance over liquid.

The transmission per hour is said to be proportional to the temperature difference of the fluids directly according to most experimenters, so that

the coefficient in B.T.U. per hour per square foot per degree is independent of temperature difference, however much it may vary with other things. However, Werner, Grashof and Weiss give the hourly rate as proportional to the square of the temperature difference or the value of U as proportional to temperature difference to the first power, while Orrok, experimentally studying heat transfer in steam condensers, reports the hourly rate as proportional to the seven-eighth power of the temperature difference and U inversely as its one-eighth power.

When the heat transmitted per hour is directly proportional to temperature difference, the value of U, being the B.T.U. per square foot per degree mean temperature difference, is independent of the temperature difference and the mean temperature difference t_m is a function of the initial and final as determined in the last section. Should the heat flow per hour be proportional to any but the first power of the temperature difference then the value of U will be itself a function of temperature difference, and the mean temperature difference a different function of the actual temperatures than those previously derived. This can be shown by analysis for one case as follows, a similar method being applicable to other kinds of flow and cases of transfer though not worked out here because of the uncertainty of the value of the exponent.

Consider the case of flow of heat from condensing vapor at a constant temperature to a liquid or gas with rising temperature which, of course, includes surface condensers and feed-water heaters. Then, an element of surface dA there will pass $U(t_h-t_c)dA$, B.T.U. per hour, which is equal to the product of weight per hour, specific heat and temperature rise of the substance receiving the heat, whence,

$$dA=\frac{Cw}{U}\frac{dt_c}{t_h-t_c}.$$

If now the heat per hour be proportional to the nth power of the temperature difference, then U is proportional to the $(n-1)$ power of the temperature difference or,

$$U=K(t_h-t_c)^{n-1}.$$

Therefore

$$A=\frac{Cw}{K}\int_{t_{c_1}}^{t_{c_2}}(t_h-t_c)^{-n}dt_c=\frac{Cw}{K(1-n)}[(t_h-t_c)^{1-n}-(t_h-t_{c_2})^{1-n}], \quad (748)$$

which is the relation between the surface and the temperatures resulting from the passage over that surface. The mean temperature difference is given by the relation

$$t_m U_m A=Cw(t_{c_2}-t_{c_1}),$$

in which U_m is the average value of U for the whole heat exchange.

Whence

$$t_m = \frac{Cw(t_{c_2} - t_{c_1})}{U_m A} = \frac{Cw(t_{c_2} - t_{c_1})}{U_m \frac{Cw}{K(1-n)}[(t_h - t_{c_1})^{1-n} - (t_h - t_{c_2})^{1-n}]}$$

$$= \frac{K(1-n)}{U_m} \frac{(t_{c_2} - t_{c_1})}{(t_h - t_{c_1})^{1-n} - (t_h - t_{c_2})^{1-n}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (749)$$

It appears from this that the mean temperature difference is a function of the temperature rise of the water or gas being heated, of the $(1-n)$ power of the initial and final temperature differences and of the mean value of the coefficient of heat transfer which is itself now a variable. This mean value of U can be found by integration, but is not, because, first the resulting expression is very complicated for practical use, and second the completed expression is hardly worth while in spite of its apparent accuracy, because of the uncertainty in the value of n and the real value of U, so that the real result is not and cannot be accurate.

Both hourly rate and the value of U assumed independent of temperature difference, that is, for $n = 1$, are found to be more clearly a function of flow conditions which are defined by different experimenters in different ways, none of which seem to quite reach the heart of the matter. For example, all observers agree in assigning higher values to U when the flow is vigorous than when it is sluggish, that is, *U is found to increase in some manner with rate of flow in all cases but one, and that is when the increased rate of flow takes place in the fluid on that side of the surface where the resistance is but a small fraction of the total.* Thus, for hot gas warming water, increase in flow of the water does not sensibly increase the value of U, but increase in the rate of gas flow will do so. This is one source of discrepancy in the reports of experiments not conducted in such a way as to control both rates when seeking to relate heat flow to one of them, many experimenters reporting results for variations in one rate of fluid flow without making sure that the rate of flow of the other fluid was either constant or of negligible resistance. This point is clearly demonstrated by the following experimental data, comparing one set of experiments with another.

The first series of experiments was conducted by Mr. W. D. Monks in the mechanical laboratories of Columbia University, under the writer's direction, on the transfer of heat from hot gases to water under varying conditions of water flow with the gas flow intentionally neglected, except as was necessary to control gas temperature. The metal walls were those of a 2-in. tube $88\frac{3}{4}$ ins. long, set vertically in a brick flue and heated outside by the products from a gas fire. The water flow was controlled partly by varying velocity in the tube as set and partly by varying the area of the water passage in the tube, while surface conditions of the tube on the gas side were also varied by applying in one case cast-iron rings of the Foster steam superheater elements. The first series was run with a perfectly plain tube

supplying varying quantities of water while holding the gas temperature constant, then the process was repeated with a higher gas temperature, later a plugged 1-in. standard pipe was inserted as a core and the same repeated, and finally a helical strip of metal substituted to give the water a rotary movement. Some of the results are plotted in Fig. 167, showing the relation of U to water velocity up to 160 ft. per minute, and these are representative. They show a small increase in U with water velocity as might be expected, as there is surely

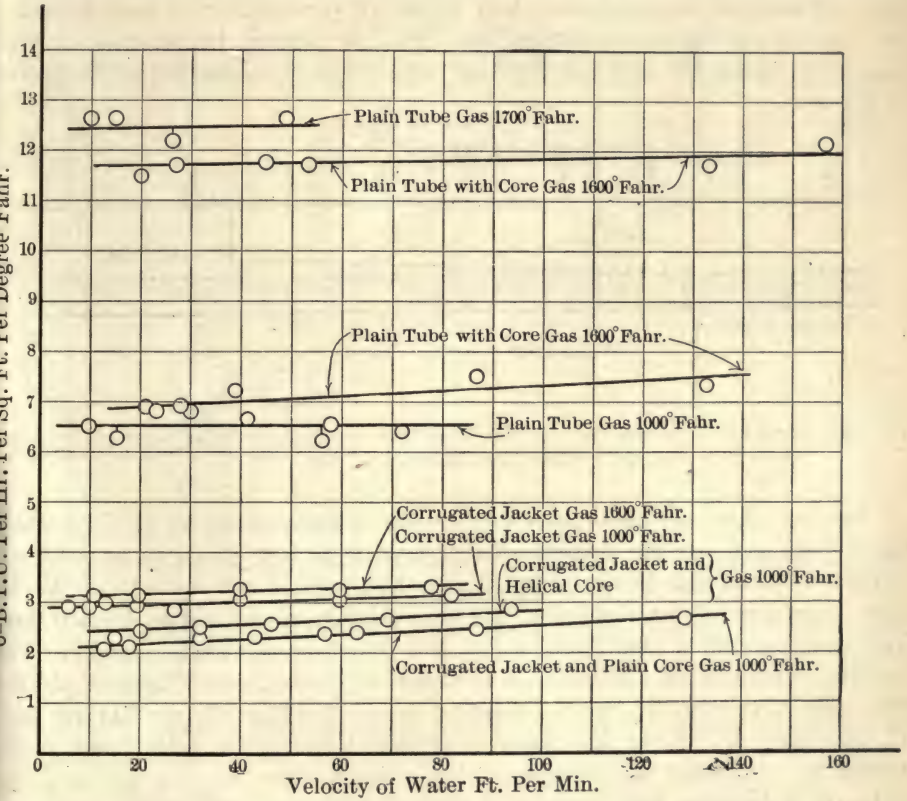


FIG. 167.—Variation of Heat Transfer Coefficient with Velocity of Water when Water Receives Heat from a Gas, Illustrating Substantial Constancy and Proving the *Controlling Resistance to be on the Gas Side.*

some water resistance but hardly enough to be worth while, for example, the increase along any one of the curves is less than 1 B.T.U. per hour per square foot per degree, and all values for whatever water velocity or tube condition lie between $U=2$, and $U=13$. There are two influences affecting U more than water velocity, and these are gas temperatures and the outside condition of the tube. For the plain tube the use of gas temperatures from 1000° F. to 1700° F. had the effect of approximately doubling the value of U, but it is not possible to say whether this effect is due inherently to temperature or to

increased gas velocity which accompanies it, or to some other related quantity such as perhaps gas density. In every case the jacketed tube gave much lower values than the unjacketed, calculating surface as that exposed to the water and not that exposed to the gas, or *about the same when calculating surface as that exposed to the gas, illustrating well the constancy of gas film resistance for equal gas flow and temperature conditions.* The whole series demonstrates absolutely the controlling character of the resistance on the gas side and leads to the conclusion that whatever variations in U may be found are due to varying film resistance, and may be related to velocity of fluid directly or indirectly, and the situation appears to be somewhat as illustrated in Fig. 168.

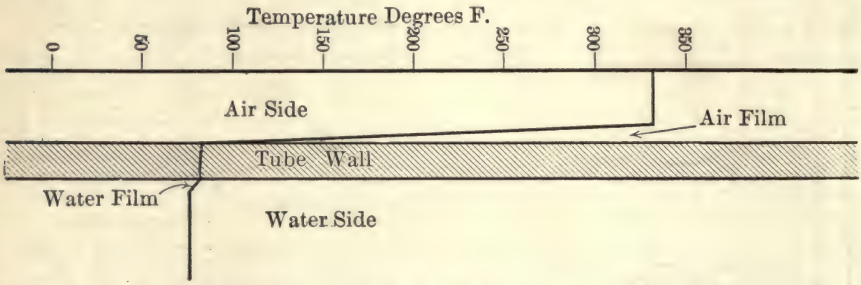


FIG. 168.—Probable Relations between the Heat Resistances or Drops in Temperature when Heat Passes from Hot Gas to Water.

Another series of experiments carried out in great detail by H. P. Jordan, three years ago, on the transmission of heat from hot air to water confirms the above conclusions but extends them, as the amount and velocity of the air were accurately determined and the transmission phenomenon divided into into two parts, first from air to metal and second from metal to water. The superior resistance on the gas side is shown by the curve of Fig. 169, plotted from one experiment, by the temperature drops, which on the air side was 246° F. with only 6.7° F. on the water side, that through the metal itself being negligible. In every case the rate of transmission from the hot gas to metal is found to increase with increase of flow, and here a most striking relation is demonstrated by an appropriate selection of the prime variable representing rate of flow. *When this variable is taken as pounds of air per squre foot of area of cross-section of air passage per second, the B.T.U. per hour per square foot per degree difference between air and metal relation, is linear and the curve a straight* line as shown in Fig. 169, for one series. These lines all have the equation

$$U = A + B\frac{w}{a}, \quad \ldots \ldots \ldots \quad (750)$$

where U = B.T.U. per square feet per degree difference between gas and metal;
w = pounds air per second;
a = square foot cross-section area of air pasage.

In the above equation A and B are factors not the same numerically for different circumstances and found by Jordan to depend on the following conditions: Thus A, which is the zero flow value of U appears to be independent of both cross-section of channel and temperature, but probably fixed by the condition of the surface as to cleanliness or smoothness, while B, which measures the rate of increase in U with flow, seems to be dependent on both the air temperature and on the dimensions of the air passage and given by

$$B = c + c'q + c''t_m,$$

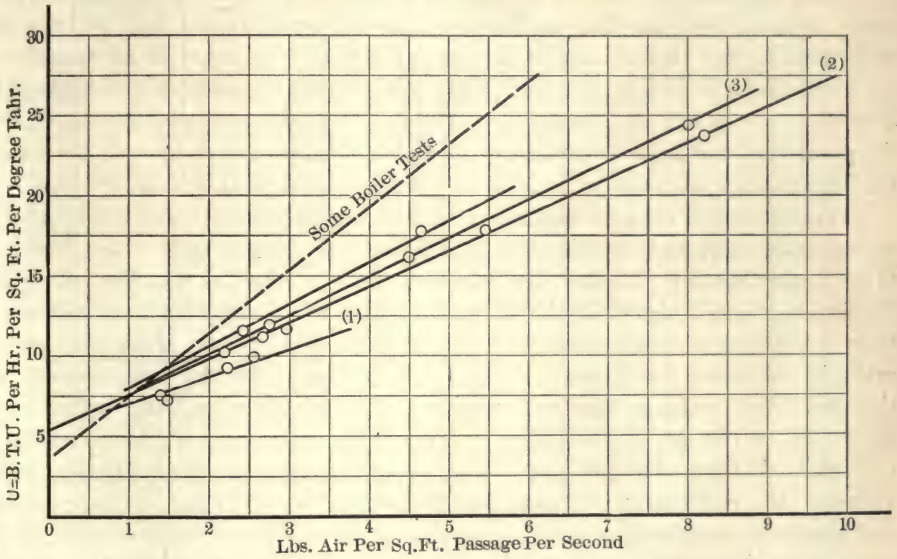


FIG. 169.—Variation of Heat Transfer Coefficient with Rate of Flow per sq.ft. of Stream Cross-section of the Gas (Air), when Water Receives Heat from a Gas, Illustrating Linear Relation between U and Gas Mass Flow.

where c, c' and c'' are constant, t_m the arithmetical mean of the air and metal temperature, and

$$q = \text{mean hydraulic depth in inches} = \frac{\text{area of flow of channel.}}{\text{perimeter of cooling surface}}. \quad . \quad (751)$$

Substituting the values of the constants and factors he found that

$$\frac{U}{3600} = .0015 + [.000506 - .00045q + .00000165t_m]\frac{w}{a}.$$

This gives the coefficient of heat transfer in terms of all variables investigated, including weight of flow of gas in pounds per second, cross-section of channel, shape of channel or mean hydraulic depth and temperature of air and metal. The laws are summarized by Jordan as follows:

(*a*) For a constant mass flow $\left(\frac{w}{a} = c\right)$ the rate of transfer is proportional to temperature difference directly;

(*b*) For a given temperature difference ($t_m = c$) the rate of transfer increases with speed by a linear law;

(*c*) For a given rate of flow and temperature difference the rate of transfer increases with the value of the temperature;

(*d*) The rate of transfer depends on the condition of surface;

(*e*) The rate of transfer depends on the size of channel, and the smaller the ratio, $\frac{\text{area}}{\text{perimeter}} = q$, the greater the rate of transfer.

These experiments, as well as another series by Dr. T. E. Stanton, fourteen years ago, on the transfer from water to water through metal, may be regarded as confirming the theoretical predictions on the laws of transfer as announced by Prof. Osborne Reynolds nearly forty years ago. *He gave on purely theoretic grounds a linear law of relation between rate of transfer per hour per degree and the product of density and velocity of fluid, but this product is equal to the weight per second per square foot of cross-section or the mass flow as it may be called.*

The conclusion that the coefficient of heat transmission when a gas is present on one side or the other increases in a linear law manner with mass flow is a most important one, because the greatest practical use of this sort of transmission is made in the steam boiler, the laws for which have for over a century been a subject of controversy and doubt, from which some order and agreement is beginning to appear. In the case of the boiler the temperatures on the two sides of the surface are unknown, greatest variations and uncertainty are found on the gas or flame side. This being the case it is quite impossible to find, or if found for one case, to use in engineering practice, any particular value of the coefficient per hour per degree per square foot. *It is possible, comparatively easy, and entirely practicable, to discuss and use the rate of transmission per hour per square foot, and this evidently in any case will also bear a linear relation to the quantity of gas per hour being discharged through the flues or tubes.* This is proved by all accurate tests on boilers in existence and which will be taken up in the next Chapter, as the subject is important enough to warrant a more detailed treatment.

The next important class of heat transfer cases in view of practical applications in standard engineering apparatus is the transfer from condensing steam to moving water, typical of exhaust steam feed-water heaters, surface condensers, steam-heated kettles and similar equipment. This has long been a subject of experimental investigation and there are available many test results, all of which do not, however, agree in detail, though all do agree in assigning increasing values to U with water velocity, usually making it proportional to some power of the water velocity, Ser and Joule the one-third power, Hegeman the one-half power and Stanton the first power. In some of the work attempts have been made to relate U to the steam velocity or mass flow of steam approaching the condensing surface as well, but these are in the main unsuccessful. *One great source of variation in U noticed by some and ignored by others as peculiar to this class, is the collection of non-condensible gases in bubbles and films on the condensing surface, preventing actual vapor-metal*

contact and interposing great heat resistance. In general it may be said that for this class the principal resistance is on the water side and the steam can condense as fast as the water is able to carry heat away so that increase of water velocity always increases U by decreasing the film thickness and corresponding heat resistance. The value of U is always higher than for cases where gases are present, because liquid film resistances are always less than those

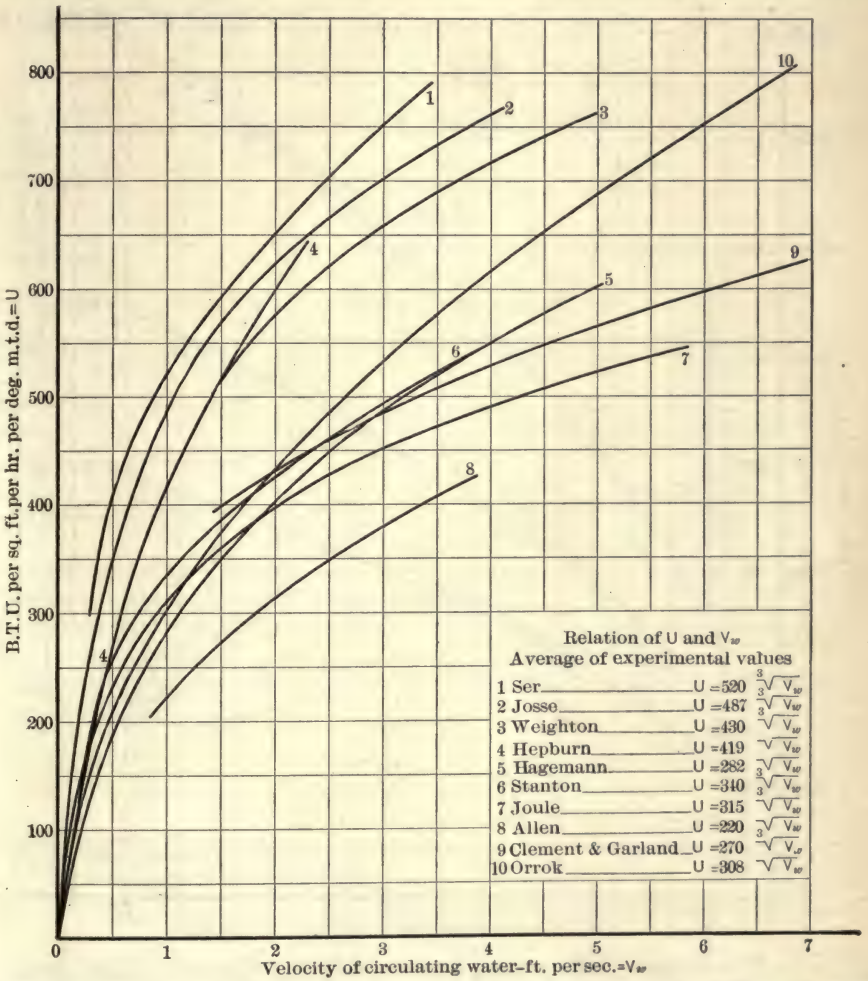


FIG. 170.—Variation in Heat Transfer Coefficient with Water Velocity, when Water Receives Heat from Condensing Steam (Orrok).

for gases of equal thickness. When gases collect on the condensing side the principal resistance may no longer rest in the water film, but may be transferred to the vapor side, in which case increase of water velocity results in little or no gain.

In no series of experiments has more care been exercised to get accurate data with a full understanding beforehand of the difficulties and interferences

to be overcome than those of Orrok made a year or so ago. In this series extraordinary precautions were taken to exclude air and other gases, and steam generated from a constant mass of water in a closed system was condensed under vacuum on tubes of different compositions and surface conditions supplied with water of varying velocity. The results reported are consistent and reliable as no others have been, probably for this reason, but are not applicable

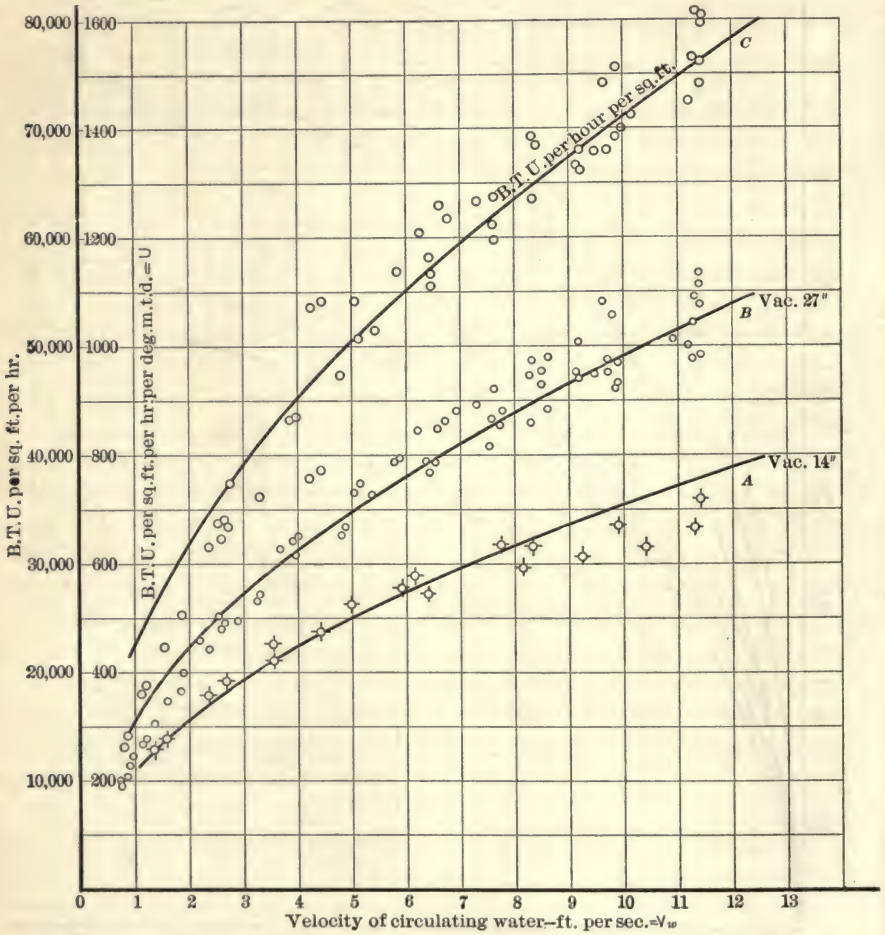


FIG. 171.—Heat Transfer Coefficient as a Function of Water Velocity for Steam Condensers, Showing Curve of Accepted Law and Experimental Points by Orrok.

to commercial steam generated by fresh supplies of water carrying gases in solution, as all water supplies do, without correction or allowance for the increase of heat resistance due to collection of gas films on condensing surfaces.

The curves of Fig. 170, reproduced from Orrok's paper, give a graphic summary of the value of U in terms of water velocity as determined by various experimenters and includes his own result. These crossing lines of different curvature and slope prove conclusively the complex nature of the case under

examination and the fact that U cannot be regarded as a function of water velocity alone. More in detail Orrok's results for the two vacua 15 and 27 ins. Hg are given in Fig. 171, curves (A) and (B), being the values of U and (C), that of B.T.U. per square foot per hour or $U \times t_m$ and it should be noted how widely distributed are the points about the curve in spite of great care in testing. All these curves, as located in the bands of points, follow the square root of velocity law having the equations,

$$\left.\begin{aligned} U &= 308\sqrt{V_w} \text{ for 27 ins. vacuum.} \quad (a) \\ U &= 224\sqrt{V_w} \text{ for 15 ins. vacuum.} \quad (b) \end{aligned}\right\} \quad . \quad (752)$$

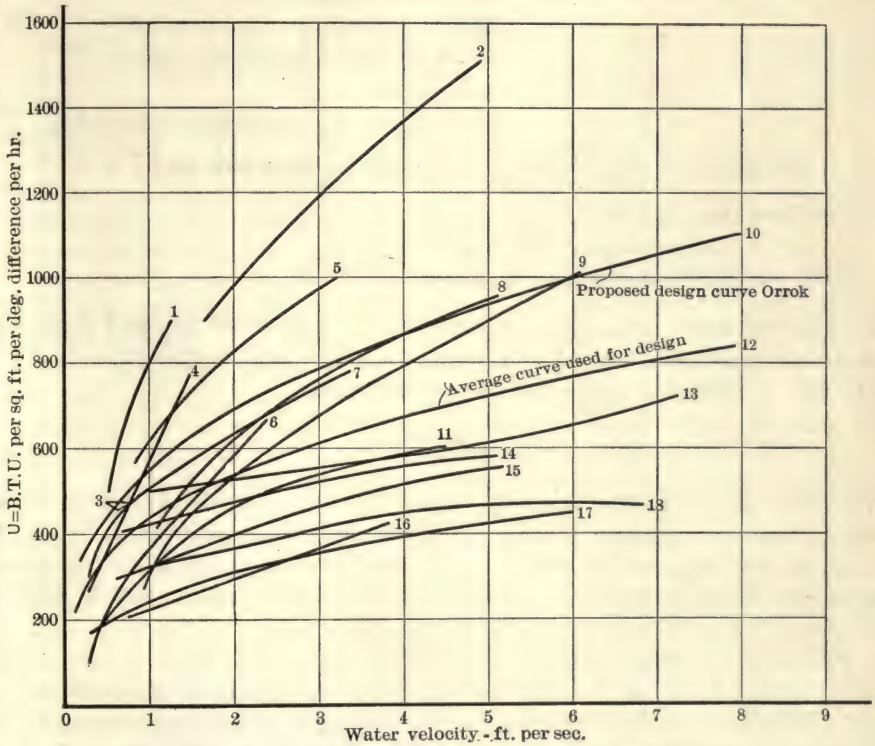


FIG. 172.—Comparison of Values of U for Surface Condensers.

These are compared with the work of the more reliable experimenters whose numbers are available for recalculation of results in Fig. 172. The effect of steam velocity, amount of air present in the steam as represented by the partial pressure of the air and of the nature of the tube were also investigated, the results being summed up in the following equation,

$$U = K\frac{C'\rho^5\mu\sqrt{V_w}}{t_m^{\frac{1}{4}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (753)$$

Where C' is a cleanliness coefficient varying from 1.00 to .5;

" μ is material coefficient =
- 1.00 for copper;
- .98 for admiralty metal;
- .97 for admiralty aluminum lined;
- .92 for admiralty black oxidized;
- .87 for admiralty aluminium bronze;
- .80 for cuprous nickel;
- .79 for tin or admiralty lead lined;
- .75 for zinc;
- .74 for monel metal;
- .63 for Shelby steel;
- .55 for admiralty badly corroded;
- .47 for admiralty inside vulcanized;
- .25 for glass;
- .17 for admiralty vulcanized both sides;

" ρ = ratio of $\frac{\text{Partial pr. steam}}{\text{Total pressure}}$ and varying from 1.00 to 0.

" V_d = water velocity feet per second;

" t_m = mean temperature difference;

" K = a constant = 630 approximately.

For high vacuum condensers the standard vacuum is 28 ins. and guarantees are usually based on this and on a circulating water inlet temperature of 70° F. with 20° F. allowable rise. For these conditions

$$U = 435C'\left(\frac{P_s}{P_t}\right)^5.$$

The effect of air or other gases deposited on the condensing surface on the rate of heat transmission was investigated most thoroughly by J. A. Smith, about six years ago, and his results are the best available. He finds that enough gases to be equivalent to a partial pressure of $\frac{1}{20}$ in. Hg will at 90° F. decrease the transmission 25 per cent, and $\frac{3}{20}$ in. Hg, 50 per cent.

Probably the most complex of the heat transmission cases so far as concerns the variability of U is that in which the two fluids are the same in kind and both suffer a change of temperature, and these include heat passage from liquid to liquid, or gas to gas. *For all of these it is quite impossible to fix off-hand the location of the controlling resistance, as it may be on either side, and with variations in flow may pass alternately from one side to the other,* as in liquid heat exchangers, steam superheaters and similar apparatus. As there are no available data covering a sufficient range of conditions, about the best that can be done is to analyze such cases by trial with assumed metal temperature, taking Jordan's data for air relating U to mass flow and Orrok's for water relating U to the square root of the water velocity. These cases are not so common or so important in practice, which probably accounts for lack of data, but they are quite important enough to warrant some trouble in establishing experimentally the needed laws.

18. GENERAL PROBLEMS ON CHAPTER IV

Prob. 1. What will be the final temperature of all the substances if 10 lbs. of iron at a temperature of 800° F. and 25 lbs. of stone at a temperature of 1000° F. are thrown into a tank containing 1000 lbs. of water at 40° F. and 10 lbs. of ice?

Prob. 2. A room has a volume of 60,000 cu.ft. and the air in it is changed ten times per hour. The entering air is warmed by steam coils from a temperature of 20° F. to 60° F. The steam enters the coils with a pressure of 3 lbs. per square inch gage and containing 10 per cent of moisture. The condensed steam leaves at a temperature of 100° F. How many pounds of steam per hour will be required to heat the room?

Prob. 3. How many cubic feet will a pound of saturated ammonia vapor contain at −15° C. and what will be the pressure at this temperature? What will be the absolute F. and C. temperature corresponding to a pressure of 100 lbs. per square inch gage pressure for steam, ammonia vapor, and carbon dioxide vapor?

Prob. 4. If a glass jar held exactly $\frac{1}{10}$ of a gal. of water at 32° F. what would be its capacity at 100° F. Should it be full of water at 60° F. how much more could be added when the temperature was 39° F. and how much would have spilled over when the temperature had risen to 100° F.?

Prob. 5. The unit elongation of an iron bar is $25 \times 10^6 \times$unit stress. What will be the force exerted by an iron beam of 7.5 sq.ins. cross-section when heated from 0° F. to 100° F.?

Prob. 6. A room full of air is heated from 20° F. to 100° F. How much air was forced out upon being heated?

Prob. 7. An ammonia refrigerating machine has a condenser in which ammonia vapor at a pressure of 150 lbs. per square inch absolute and at a temperature of 225° F. is converted into liquid by means of water supplied at 70° F. passed over the tubes of the condenser. If each pound of water suffers a rise in temperature of 20° F. and the liquid ammonia is cooled to 60° F., how many pounds of water per pound of ammonia will be needed?

Prob. 8. To produce one ton of refrigeration in twenty-four hours it is necessary to absorb 200 B.T.U. per minute. What will be the capacity of a machine in tons, in the coils of which 100 lbs. of liquid ammonia are evaporated per minute? The ammonia enters the coils at a temperature of 60° F., the pressure in the coils is 25 lbs. per square inch absolute and the vapor leaves the coils at a temperature of 20° F.

Prob. 9. A thermometer placed in a well in a pipe carrying saturated steam reads 350° F. when half the scale (50° F. to 400° F.) is exposed. What will be the difference between the pressure due to reading as corrected for exposed stem as advised by the Bureau of Standards and that due to the reading as taken?

Prob. 10. With a boiler efficiency of 70 per cent, 1 lb. of a certain grade of coal made 7.5 lbs. of steam at 100 lbs. per square inch gage from feed water at 60° F., the steam having 50° of superheat. If it were possible to convert 30 per cent of the heat of the steam directly into work, what would be the thermal efficiency referred to the coal and the water rate of the engine?

Prob. 11. A volumetric analysis of boiler flue gases gave the following result: Carbon dioxide = 12 per cent; carbon monoxide = 2 per cent; oxygen, 7 per cent; nitrogen 79 per cent; what would be the error in the calculation of the sensible heat carried up

the stack on the assumption that the specific heat was the same as that of air rather than its proper value? What would be the total heat carried up the stack for 1000 lbs. of flue gases for a room temperature of 80° F. and a stack temperature of 480° F.?

Prob. 12. The water level in a hot water tank is shown by a gage glass. The glass shows the level to be 6 ft., but the temperature of the water in glass is 80° F., while that in tank is 210° F. What is the true height of water in tank?

Prob. 13. The mean temperature of the gases in a chimney is 350° F., while the outside air is 40° F. For a stack 150 ft. high what would be maximum draft possible? In another similar stack the gases are used to warm water in tubes at the base of the chimney and the average temperature is reduced to 280° F. What would be the loss of draft?

Prob. 14. A thousand cubic feet of air are compressed from one to 10 atmospheres. If half is so compressed that the heat is removed sufficiently fast to prevent any rise in temperature and the other half in such a way that no heat is removed, what will be the sum of the volumes at end of process before and after mixing?

Prob. 15. The barometer reading at a certain time is 29.5 ins. of Hg, the temperature is 60° F. and the air is saturated with water vapor. What is the pressure of air alone and the water content of the air? How much water will be lost by cooling to 20° F.?

Prob. 16. Ammonia vapor leaves the coils of an ice plant at a temperature of 10° F., at which temperature it is superheated 10°. What pressure was there in the coils and how much heat per pound NH_3 was taken up if the liquid NH_3 had originally a temperature of 60° F.

Prob. 17. Carbon dioxide gas is being compressed into a cylinder at a temperature of 60° F. At what pressure will condensation occur and how much heat must be removed per pound of stuff to cause this action?

Prob. 18. Steam is being generated at a pressure of 100 lbs. per square inch gage and at the rate of 100 lbs. per minute. What horse-power is being developed in overcoming the external resistance to the formation of the steam?

Prob. 19. The following is an analysis by volume of a sample of water gas. How much air would be needed to burn 1 cu.ft. of it and what would be the products resulting from such combustion by weight and by volume and the heat of combustion of the gas per pound and per cubic foot?

$$CO = 25.3;\ H_2 = 9.2;\ CH_4 = 3.1;\ C_2H_4 = .8;\ CO_2 = 3.4;\ N_2 = 58.$$

Prob. 20. Upon analysis a pound of Pocahontas coal yielded the following results by weight. $C = 84.9$; $H = 4.2$; $O = 2.8$. What would result upon burning a pound of this coal in heat developed and composition of products?

Prob. 21. Why should the specific heat at constant volume differ from that at constant pressure and what is the relation between them for any assumed gas?

Prob. 22. The walls of an ice storage room are composed of material through which the rate of heat transfer is .5 B.T.U. per square foot per degree difference per hour. If the room is $100 \times 100 \times 15$ ft., what must be the pounds of ammonia evaporated at a pressure of 15 lbs. per square inch absolute to maintain a temperature of 25° F. within when the temperature without is 80° F.? The ammonia liquid enters the coils at 60° F. and the vapor leaves in a saturated condition. How much coil surface is needed?

Prob. 23. A boiler is supplied with fuel, the ultimate analysis of which showed 70 per cent C, 15 per cent H, and the remainder ash and moisture. A similar boiler

s supplied with natural gas containing 99 per cent of CH_4. Assuming that both have n efficiency of 70 per cent, and that the heats of combustion of the fuels equal the eats of formation of the products, how many pounds of steam at 100 lbs. per square ich absolute pressure could be made from feed water at 60° F. per pound of each fuel?

Prob. 24. A boiler horse-power is given as the heat equivalent 34.5 lbs. of water vaporated from and at 212° F. To develop 1000 H.P. with a feed-water tempera- ire of 200° F. and a steam pressure of 200 lbs. per square inch gage would require how any feet of heating surface based on a transfer rate of 3 and a mean gas temperature f 800° F.?

Prob. 25. At a given time the dry-bulb thermometer reads 75° F. and a wet-bulb hermometer reads 60° F. What is the relative humidity and how much would the mperature have to fall to produce rain?

Prob. 26. An ammonia absorption machine is producing 10 tons of ice per day om water at 50° F., the ice being cooled to 25° F. The brine is cooled by evaporation ammonia at a pressure of 10 lbs. per square inch gage. How much water will be quired to remove the heat from the absorption if each pound is allowed to rise in mperature 20° F.? What will be the maximum per cent of ammonia in the lution in the absorber if the pressure be 10 lbs. gage and the temperature)° F.?

Prob. 27. The following data were taken during a test of a feed-water heater; hat was the rate of heat transfer per square foot of surface per degree mean temperature fference per hour? What was the boiler horse-power equivalent of the heat? Heat- g surface 11 sq.ft. Steam pressure in heater 2 lbs. per square inch gage. Steam)% wet. Pounds of water heated per hour 3800. Temperature of inlet ater 44° F. Temperature of water at end of successive passes, 78°, 106°, 133°, 154°, 0°, 180°, 202°.

Prob. 28. Fuel containing 70 per cent C, 15 per cent H and 15 per cent ash is rned under a boiler. 20 per cent of the heat developed goes to the stack. If there twice as much air supplied as is chemically needed, what will be the temperature the flue gases leaving the boiler and its volumetric analysis cold?

Prob. 29. The gases from above boiler pass through an economizer in which the w of fluids may be considered as counter current. 3000 lbs. of water per hour are ated from a temperature of 50° F. Assuming 4 lbs. of gases per pound of water d the specific heat of the former to be .25, what will be the surface required for ery 20° rise from inlet temperature to 230°?

NOTE: Let $U=3.5$.

Prob. 30. 400 lbs. of coal are burned on a boiler grate per hour and during the ne time 4000 lbs. of water are passed through an economizer. If the water is raised temperature 200° and the gas is cooled 300°, what is the weight of flue gas per pound coal burned, assuming the sp.ht. of the gases to be .25?

Prob. 31. A pound of coal contains 14,000 heat-units. If 70 per cent of these are sorbed by the boiler, how many pounds of steam may be made per pound of coal a pressure of 150 lbs. gage from feed-water at a temperature of 60° F.?

Prob. 32. 60,000 cu.ft. of air are taken into a building per hour. Assuming that air is at 40° F. and the humidity to be 7 per cent, how many pounds of moisture l the air contain?

Prob. 33. An ammonia refrigerating machine is required to remove 1,000,000 B.T.U. r hour from a cold-storage room. The liquid ammonia coming to the expansion valve s a temperature of 60°. The pressure in the vaporizing coils is 10 lbs. per square

inch absolute and the temperature in the coils is 25° F. How many cubic feet of ammonia vapor per minute must the compressor handle?

Prob. 34. The temperature of the outside air is 40° F. The average temperature of the gases in a chimney is 350° F. The gases may be considered as 80 per cent nitrogen, 10 per cent oxygen, and 10 per cent carbon dioxide. How high must be the stack to give a hypothetical draught of 1 inch of water?

Prob. 35. By the use of a feed-water heater the temperature of the water fed to a boiler is raised from 50° to 200°. The pressure in the boiler is 100 lbs. gage and the steam has 5 per cent moisture. What is the percentage of heat saved by the heater?

Prob. 36. Steam is generated in a boiler at a pressure of 100 lbs. gage from feed-water having a temperature of 60° F. When the steam leaves the boiler it has 5 per cent moisture, and is carried to a separately fired superheater from which it leaves with 100° superheat. What per cent of the total heat in the superheated steam comes from the boiler and what per cent from the superheater?

Prob. 37. A volume of 50 cu.ft. contains 5 lbs. of hydrogen and 10 lbs. of nitrogen at a temperature of 500° absolute. Find the pressure due to the hydrogen and to the nitrogen.

Prob. 38. A mixture of air and saturated water-vapor has a temperature of 55° F. The total pressure is 1 atmosphere. What is the pressure due to the air and due to the moisture?

Prob. 39. A wet bulb hygrometer gives readings of 70° and 48°. What is the humidity?

Prob. 40. A certain illuminating gas is .8 as heavy as air. What will be its value of R?

Prob. 41. The specific heat of a certain gas at constant pressure is 2.8. The value for $R=700$. Find the value of its specific heat at constant volume. Also the value of γ.

Prob. 42. The temperature in a condenser is 120° F. The vacuum shows 23 ins. of Hg with the barometer standing at 30 ins. What is the pressure of the air in the condenser?

Prob. 43. A pound of air is cooled from atmospheric pressure and 32° F. to −310° F. If the cooling is at constant volume, what will be the final pressure?

TABLE XXIX

TEMPERATURES, CENTIGRADE AND FAHRENHEIT

C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
−40	−40.	26	78.8	92	197.6	158	316.4	224	435.2	290	554	950	1742
−39	−38.2	27	80.6	93	199.4	159	318.2	225	437.	300	572	960	1760
−38	−36.4	28	82.4	94	201.2	160	320.	226	438.8	310	590	970	1778
−37	−34.6	29	84.2	95	203.	161	321.8	227	440.6	320	608	980	1796
−36	−32.8	30	86.	96	204.8	162	323.6	228	442.4	330	626	990	1814
−35	−31.	31	87.8	97	206.6	163	325.4	229	444.2	340	644	1000	1832
−34	−29.2	32	89.6	98	208.4	164	327.2	230	446.	350	662	1010	1850
−33	−27.4	33	91.4	99	210.2	165	329.	231	447.8	360	680	1020	1868
−32	−25.6	34	93.2	100	212.	166	330.8	232	449.6	370	698	1030	1886
−31	−23.8	35	95.	101	213.8	167	332.6	233	451.4	380	716	1040	1904
−30	−22.	36	96.8	102	215.6	168	334.4	234	453.2	390	734	1050	1922
−29	−20.2	37	98.6	103	217.4	169	336.2	235	455.	400	752	1060	1940
−28	−18.4	38	100.4	104	219.2	170	338.	236	456.8	410	770	1070	1958
−27	−16.6	39	102.2	105	221.	171	339.8	237	458.6	420	788	1080	1976
−26	−14.8	40	104.	106	222.8	172	341.6	238	460.4	430	806	1090	1994
−25	−13.	41	105.8	107	224.6	173	343.4	239	462.2	440	824	1100	2012
−24	−11.2	42	107.6	108	226.4	174	345.2	240	464.	450	842	1110	2030
−23	− 9.4	43	109.4	109	228.2	175	347.	241	465.8	460	860	1120	2048
−22	− 7.6	44	111.2	110	230.	176	348.8	242	467.6	470	878	1130	2066
−21	− 5.8	45	113.	111	231.8	177	350.6	243	469.4	480	896	1140	2084
−20	− 4.	46	114.8	112	233.6	178	352.4	244	471.2	490	914	1150	2102
−19	− 2.2	47	116.6	113	235.4	179	354.2	245	473.	500	932	1160	2120
−18	− 0.4	48	118.4	114	237.2	180	356.	246	474.8	510	950	1170	2138
−17	+ 1.4	49	120.2	115	239.	181	357.8	247	476.6	520	968	1180	2156
−16	3.2	50	122.	116	240.8	182	359.6	248	478.4	530	986	1190	2174
−15	5.	51	123.8	117	242.6	183	361.4	249	480.2	540	1004	1200	2192
−14	6.8	52	125.6	118	244.4	184	363.2	250	482.	550	1022	1210	2210
−13	8.6	53	127.4	119	246.2	185	365.	251	483.8	560	1040	1220	2228
−12	10.4	54	129.2	120	248.	186	366.8	252	485.6	570	1058	1230	2246
−11	12.2	55	131.	121	249.8	187	368.6	253	487.4	580	1076	1240	2264
−10	14.	56	132.8	122	251.6	188	370 4	254	489.2	590	1094	1250	2282
− 9	15.8	57	134.6	123	253.4	189	372.2	255	491.	600	1112	1260	2300
− 8	17.6	58	136.4	124	255.2	190	374.	256	492.8	610	1130	1270	2318
− 7	19.4	59	138.2	125	257.	191	375.8	257	494.6	620	1148	1280	2336
− 6	21.2	60	140.	126	258.8	192	377.6	258	496.4	630	1166	1290	2354
− 5	23.	61	141.8	127	260.6	193	379.4	259	498.2	640	1184	1300	2372
− 4	24.8	62	143.6	128	262.4	194	381.2	260	500.	650	1202	1310	2390
− 3	26.6	63	145.4	129	264.2	195	383.	261	501.8	660	1220	1320	2408
− 2	28.4	64	147.2	130	266.	196	384.8	262	503.6	670	1238	1330	2426
− 1	30.2	65	149.	131	267.8	197	386.6	263	505.4	680	1256	1340	2444
0	32.	66	150.8	132	269.6	198	388.4	264	507.2	690	1274	1350	2462
+ 1	33.8	67	152.6	133	271.4	199	390.2	265	509.	700	1292	1360	2480
2	35.6	68	154.4	134	273.2	200	392.	266	510.8	710	1310	1370	2498
3	37.4	69	156.2	135	275.	201	393.8	267	512.6	720	1328	1380	2516
4	39.2	70	158.	136	276.8	202	395.6	268	514.4	730	1346	1390	2534
5	41.	71	159.8	137	278.6	203	397.4	269	516.2	740	1364	1400	2552
6	42.8	72	161.6	138	280.4	204	399.2	270	518.	750	1382	1410	2570
7	44.6	73	163.4	139	282.2	205	401.	271	519.8	760	1400	1420	2588
8	46.4	74	165.2	140	284.	206	402.8	272	521.6	770	1418	1430	2606
9	48.2	75	167.	141	285.8	207	404.6	273	523.4	780	1436	1440	2624
10	50.	76	168.8	142	287.6	208	406.4	274	525.2	790	1454	1450	2642
11	51.8	77	170.6	143	289.4	209	408.2	275	527.	800	1472	1460	2660
12	53.6	78	172.4	144	291.2	210	410.	276	528.8	810	1490	1470	2678
13	55.4	79	174.2	145	293.	211	411.8	277	530.6	820	1508	1480	2696
14	57.2	80	176.	146	294.8	212	413.6	278	532.4	830	1526	1490	2714
15	59.	81	177.8	147	296.6	213	415.4	279	534.2	840	1544	1500	2732
16	60:8	82	179.6	148	298.4	214	417.2	280	536.	850	1562	1510	2750
17	62.6	83	181.4	149	300.2	215	419.	281	537.8	860	1580	1520	2768
18	64.4	84	183.2	150	302.	216	420.8	282	539.6	870	1598	1530	2786
19	66.2	85	185.	151	303.8	217	422.6	283	541.4	880	1616	1540	2804
20	68.	86	186.8	152	305.6	218	424.4	284	543.2	890	1634	1550	2822
21	69.8	87	188.6	153	307.4	219	426.2	285	545.	900	1652	1600	2912
22	71.6	88	190.4	154	309.2	220	428.	286	546.8	910	1670	1650	3002
23	73.4	89	192.2	155	311.	221	429.8	287	548.6	920	1688	1700	3092
24	75.2	90	194.	156	312.8	222	431.6	288	550.4	930	1706	1750	3182
25	77.	91	195.8	157	314.6	223	433.4	289	552.2	940	1724	1800	3272

TABLE XXIX—*Continued*

TEMPERATURES, FAHRENHEIT AND CENTIGRADE

F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.
−40	−40.	26	− 3.3	92	33.3	158	70.	224	106.7	290	143.3	360	182.2
−39	−39.4	27	− 2.8	93	33.9	159	70.6	225	107.2	291	143.9	370	187.8
−38	−38.9	28	− 2.2	94	34.4	160	71.1	226	107.8	292	144.4	380	193.3
−37	−38.3	29	− 1.7	95	35.	161	71.7	227	108.3	293	145.	390	198.9
−36	−37.8	30	− 1.1	96	35.6	162	72.2	228	108.9	294	145.6	400	204.4
−35	−37.2	31	− 0.6	97	36.1	163	72.8	229	109.4	295	146.1	410	210.
−34	−36.7	32	0.	98	36.7	164	73.3	230	110.	296	146.7	420	215.6
−33	−36.1	33	+ 0.6	99	37.2	165	73.9	231	110.6	297	147.2	430	221.1
−32	−35.6	34	1.1	100	37.8	166	74.4	232	111.1	298	147.8	440	226.7
−31	−35.	35	1.7	101	38.3	167	75.	233	111.7	299	148.3	450	232.2
−30	−34.4	36	2.2	102	38.9	168	75.6	234	112.2	300	148.9	460	237.8
−29	−33.9	37	2.8	103	39.4	169	76.1	235	112.8	301	149.4	470	243.3
−28	−33.3	38	3.3	104	40.	170	76.7	236	113.3	302	150.	480	248.9
−27	−32.8	39	3.9	105	40.6	171	77.2	237	113.9	303	150.6	490	254.4
−26	−32.2	40	4.4	106	41.1	172	77.8	238	114.4	304	151.1	500	260.
−25	−31.7	41	5.	107	41.7	173	78.3	239	115.	305	151.7	510	265.6
−24	−31.1	42	5.6	108	42.2	174	78.9	240	115.6	306	152.2	520	271.1
−23	−30.6	43	6.1	109	42.8	175	79.4	241	116.1	307	152.8	530	276.7
−22	−30.	44	6.7	110	43.3	176	80.	242	116.7	308	153.3	540	282.2
−21	−29.4	45	7.2	111	43.9	177	80.6	243	117.2	309	153.9	550	287.8
−20	−28.9	46	7.8	112	44.4	178	81.1	244	117.8	310	154.4	560	293.3
−19	−28.3	47	8.3	113	45.	179	81.7	245	118.3	311	155.	570	298.9
−18	−27.8	48	8.9	114	45.6	180	82.2	246	118.9	312	155.6	580	304.4
−17	−27.2	49	9.4	115	46.1	181	82.8	247	119.4	313	156.1	590	310.
−16	−26.7	50	10.	116	46.7	182	83.3	248	120.	314	156.7	600	315.6
−15	−26.1	51	10.6	117	47.2	183	83.9	249	120.6	315	157.2	610	321.1
−14	−25.6	52	11.1	118	47.8	184	84.4	250	121.1	316	157.8	620	326.7
−13	−25.	53	11.7	119	48.3	185	85.	251	121.7	317	158.3	630	332.2
−12	−24.4	54	12.2	120	48.9	186	85.6	252	122.2	318	158.9	640	337.8
−11	−23.9	55	12.8	121	49.4	187	86.1	253	122.8	319	159.4	650	343.3
−10	−23.3	56	13.3	122	50.	188	86.7	254	123.3	320	160.	660	348.9
− 9	−22.8	57	13.9	123	50.6	189	87.2	255	123.9	321	160.6	670	354.4
− 8	−22.2	58	14.4	124	51.1	190	87.8	256	124.4	322	161.1	680	360.
− 7	−21.7	59	15.	125	51.7	191	88.3	257	125.	323	161.7	690	365.6
− 6	−21.1	60	15.6	126	52.2	192	88.9	258	125.6	324	162.2	700	371.1
− 5	−20.6	61	16.1	127	52.8	193	89.4	259	126.1	325	162.8	710	376.7
− 4	−20.	62	16.7	128	53.3	194	90.	260	126.7	326	163.3	720	382.2
− 3	−19.4	63	17.2	129	53.9	195	90.6	261	127.2	327	163.9	730	387.8
− 2	−18.9	64	17.8	130	54.4	196	91.1	262	127.8	328	164.4	740	393.3
− 1	−18.3	65	18.3	131	55.	197	91.7	263	128.3	329	165.	750	398.9
0	−17.8	66	18.9	132	55.6	198	92.2	264	128.9	330	165.6	760	404.4
+ 1	−17.2	67	19.4	133	56.1	199	92.8	265	129.4	331	166.1	770	410.
2	−16.7	68	20.	134	56.7	200	93.3	266	130.	332	166.7	780	415.6
3	−16.1	69	20.6	135	57.2	201	93.9	267	130.6	333	167.2	790	421.1
4	−15.6	70	21.1	136	57.8	202	94.4	268	131.1	334	167.8	800	426.7
5	−15.	71	21.7	137	58.3	203	95.	269	131.7	335	168.3	810	432.2
6	−14.4	72	22.2	138	58.9	204	95.6	270	132.2	336	168.9	820	437.8
7	−13.9	73	22.8	139	59.4	205	96.1	271	132.8	337	169.4	830	443.3
8	−13.3	74	23.3	140	60.	206	96.7	272	133.3	338	170.	840	448.9
9	−12.8	75	23.9	141	60.6	207	97.2	273	133.9	339	170.6	850	454.4
10	−12.2	76	24.4	142	61.1	208	97.8	274	134.4	340	171.1	860	460.
11	−11.7	77	25.	143	61.7	209	98.3	275	135.	341	171.7	870	465.6
12	−11.1	78	25.6	144	62.2	210	98.9	276	135.6	342	172.2	880	471.1
13	−10.6	79	26.1	145	62.8	211	99.4	277	136.1	343	172.8	890	476.7
14	−10.	80	26.7	146	63.3	212	100.	278	136.7	344	173.3	900	482.2
15	− 9.4	81	27.2	147	63.9	213	100.6	279	137.2	345	173.9	910	487.8
16	− 8.9	82	27.8	148	64.4	214	101.1	280	137.8	346	174.4	920	493.3
17	− 8.3	83	28.3	149	65.	215	101.7	281	138.3	347	175.	930	498.9
18	− 7.8	84	28.9	150	65.6	216	102.2	282	138.9	348	175.6	940	504.4
19	− 7.2	85	29.4	151	66.1	217	102.8	283	139.4	349	176.1	950	510.
20	− 6.7	86	30.	152	66.7	218	103.3	284	140.	350	176.7	960	515.6
21	− 6.1	87	30.6	153	67.2	219	103.9	285	140.6	351	177.2	970	521.
22	− 5.6	88	31.1	154	67.8	220	104.4	286	141.1	352	177.8	980	526.7
23	− 5.	89	31.7	155	68.3	221	105.	287	141.7	353	178.3	990	532.2
24	− 4.4	90	32.2	156	68.9	222	105.6	288	142.2	354	178.9	1000	537.8
25	− 3.9	91	32.8	157	69.4	223	106.1	289	142.8	355	179.4	1010	543.3

TABLE XXX

HEAT AND POWER CONVERSION TABLE

Calorie Kilo °C.	B.T.U. Lb. °F.	Lb. ° C.	Kilo °F.	Calorie per Lb.	B.T.U. per Lb.	B.T.U. per Kilo.	Calorie per Kilo.
1.	3.9683	2.2046	1.8	1.	3.9683	8.7483	2.2046
.252	1.	.5556	.4536	.252	1.	2.2046	.5807
.4536	1.8	1.	.8165	.1143	.4536	1	.252
.5556	2.2046	1.2261	1.	.4536	1.8	3.9683	1.

Calorie per Cu. Ft.	B.T.U. per Cu. Ft.	Calorie per Liter.	B.T.U. per Liter.
1.	3.9683	.0353	.1402
.252	1.	.0089	.0353
28.317	112.37	1.	3.9683
7.136	28.317	.252	1.

Ft.-Lb.	B.T.U.	Calorie.	Cent. Heat Unit, At.	H.P. Sec.	H.P. Min.	H.P. Hour.
1	1.268×10^{-3}	$.324\times10^{-3}$	$.18\times10^{-3}$	1.818×10^{-3}	$.303\times10^{-4}$	5.05×10^{-7}
777.5	1	.252	.5556	1.414	2.356×10^{-2}	3.927×10^{-4}
3086	3.9683	1	2.2046	5.61	9.35×10^{-2}	1.558×10^{-3}
1399.5	1.8	.4536	1	2.545	4.24×10^{-2}	$.707\times10^{-3}$
550	.7074	.1783	.3931	1	1.67×10^{-2}	2.777×10^{-4}
3.3×10^{4}	42.44	10.695	23.578	60	1	1.67×10^{-2}
1.98×10^{6}	2545	641	1.413×10^{3}	3600	60	1

TABLE

SPECIFIC HEATS

Class.	Substance.	Atomic Weight H = 1.	Specific Gravity.	Authority.
Elements	Aluminum	26.9	2.57	Mallet
	Carbon (amorphous)	11.99	.....	.....
	Carbon graph.	11.99	2.10–2.32	Smithsonian Tables
	Copper (cast)	63.07	8.8–8.95	Smithsonian Tables
	Iron (pure)	55.41	7.85	Smithsonian Tables
	Iron (pure)	55.41	7.85	Smithsonian Tables
	Lead (cast)	205.46	11.37	Reich
	Mercury	198.5	14.18	Mallet
	Nickel	58.21	8.65	Smithsonian Tables
	Tin (cast)	118.1	7.29	Mathiessen
	Zinc (cast)	64.88	7.05	Smithsonian Tables
Common substances	Bronze	.....	8.75–9	Smithsonian Tables
	Brass	.....	7.8–8.6	Smithsonian Tables
	Brick work, Masonry	.....	1.84–2.3	Smithsonian Tables
	Butter	.....	.865	Smithsonian Tables
	Clay	.....	1.80–2.6	Smithsonian Tables
	Coal	.....	1.2–1.5	Smithsonian Tables
	Wood	.....	.4–1.2	Smithsonian Tables
	Glass	.....	2.4–2.8	Smithsonian Tables
	Ice	.....	.9	Smithsonian Tables
	Cast Iron	.....	6.8–7.5	*
	Wrought Iron	.....	7.4–7.9	*
	Marble	.....	2.5–2.8	Smithsonian Tables
	Steel	.....	7.7–7.9	*
	Sand	.....	1.45–1.6	Smithsonian Tables
	Stone	.....	2.1–3.4	*

* Kent's Mechanical Engineers' Pocketbook.

XXXI

OF SOLIDS

Specific Heat.	At Temperature.		Specific Heat Calculated from Atomic Weights.	Authority.
	C.	F.		
.2089	0	32	.238	Bontschew
.2226	20–100	68–212	.....	Bontschew
.2739	500	932	.....	Bontschew
.241	0	32	.52	Olsen
.1138	−50	−58	.52	Weber
.1605	+11	52	.....	Weber
.467	977	1795	.....	Weber
.310	16–1000	61–1832	.....	Dewar
.0924	17	62	.102	Naccari
.0985	300	572	.....	Naccari
.1162	0	32	.117	Olsen
.1091	15	59	.....	Naccari
.1376	300	572	.....	Naccari
.1765	500	392	.117	Pionchon
.218	720–1000	1328–1832	.....	Pionchon
.1989	1000–1200	1832–2192	.....	Pionchon
.0299	15	59	.031	Naccari
.0324	200	392	.....	Naccari
.0319	−78 to −40	−108 to −40	.0323	Regnault
.1084	21–99	69–210	.11	Voigt
.1233	500	932	.....	Tilden
.1608	1000	1832	.....	Pionchon
.0545	0–100	32–212	.052	Bunsen
.0538	16–197	69–387	.....	Spring
.0915	18	64	.099	Naccari
.0996	200	392	.....	Naccari
.0935	0–100	32–212	.....	Bunsen
.0858	15–98	59–208	.....	Regnault
.0939	.....	.....	.....	Regnault
About .2	.....	.....	.....	*
.55	.....	.....	.....	Siebel
.197	.....	.....	.....	Regnault
.2–.241	.....	.....	.....	Regnault
.45–.65	.....	.....	.....	*
.16–.18	.....	.....	.....	Regnault
.504	.....	.....	.....	Regnault
.1298	.....	.....	.....	Regnault
.1138	.....	.....	.....	Regnault
.21	.....	.....	.....	Regnault
.1165–.1175	.....	.....	.....	Regnault
.195	.....	.....	.....	*
.2–.22	.....	.....	.....	*

* Kent's Mechanical Engineers' Pocketbook.

TABLE XXXII

SPECIFIC HEATS OF LIQUIDS

Class.	Substance.	Atomic Weight, H = 1.	Specific Gravity.	Authority.	Specific Heat.	At. Temperature.		Specific Heat Calculated from At. Wt.	Authority.
						° C.	° F.		
Elements:	Bismuth	206.3	10.	Vincentini-Omodei	.0363	280–380	536–716	.031	Person
"	Lead	205.5	10.6	Vincentini-Omodei	.0356	310	590	.0311	Spring
					.041	360	680	.....	Spring
"	Mercury	198.5	13.5	Regnault	.0335	0	32	.0322	Olsen
"	Tin	118.1	6.97	Smithsonian Tables	.0637	250–350	482–662	.....	Person
					.0758	1100	2012	.0541	Pionchon
Common substances:	Alcohol, ethyl	....	.79	Smithsonian Tables	.5053	−20	−4	.....	Regnault
					.6479	40	104	.....	Regnault
"	Alcohol, methyl	....	.808	Smithsonian Tables	.590	5–10	41–50	.....	Regnault
"	Benzene	....	.899	Smithsonian Tables	.3402	10	50–104	.....	De Heen
					.4233	40	.....	.....	Deruyts
"	Glycerine	....	1.255	Smithsonian Tables	.576	15–50	59–122	.....	Emo
"	Vegetable oil	....	about .9	Smithsonian Tables	about .4	0–10	32–50	.....	Wachsmuth
									Weber
"	Petroleum	....	.88	Kent	.511	21–58	70–136	.....	Pagliani
"	Kerosene	....	.8–.82	Kent	.4747–.4766	10–20	50–68	.....	Gill
					.4903–.4997	20–30	68–96	.....	Gill
"	Gasolene	....	.68–.7	Kent	.5332–.5375	10–20	50–68	.....	Gill
					.5032–.555	20–30	68–98	.....	Gill
"	Aqua ammonia	....	.9–1	Starr	.....	.....	.....	.....	
"	Sea water	....	1.0043	.....	.98	17.5	63.5	.....	Thomsen
"	Sea water	....	1.0235	.....	.938	17.5	63.5	.....	Thomsen
"	Sea water	....	1.0463	.....	.903	17.5	63.5	.....	Thomsen

TABLE XXXIII

BAUMÉ SPECIFIC GRAVITY SCALE

Specific gravities are for 60° F. referred to water at same temperature as unity, at which temperature it weighs 62.34 lbs. per cubic foot.

Tabular results are calculated from:

$$\text{Degrees Baumé} = \begin{cases} \left(145 - \dfrac{145}{\text{specific gravity}}\right) & \text{for liquids lighter than water.} \\ \left(\dfrac{140}{\text{specific gravity}} - 130\right) & \text{for liquids heavier than water.} \end{cases}$$

RELATION BETWEEN SPECIFIC GRAVITY AND BAUME

Specific Gravity	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
	Degrees Baumé.									
.60	103.33	99.51	95.81	92.22	88.75	85.38	82.12	78.95	75.88	72.90[1]
.70	70.00	67.18	64.44	61.78	59.19	56.67	54.21	51.82	49.49	47.22[1]
.80	45.00	42.84	40.73	36.68	36.67	34.71	32.79	30.92	29.09	27.30[1]
.90	25.56	23.85	22.17	20.54	18.94	17.37	15.83	14.33	12.86	11.41[1]
1.00	10.00									
1.00	0.00	1.44	2.84	4.22	5.58	6.91	8.21	9.49	10.74	11.97[2]
1.10	13.18	14.37	15.54	16.68	17.81	18.91	20.00	21.07	22.12	23.15[2]
1.20	24.17	25.16	26.15	27.11	28.06	29.00	29.92	30.83	31.72	32.60[2]
1.30	33.46	34.41	35.15	35.98	36.79	37.50	38.38	39.16	39.93	40.68[2]
1.40	41.43	42.16	42.89	43.60	44.31	45.00	45.68	41.36	47.03	47.68[2]
1.50	48.33	48.97	49.60	50.23	50.84	51.45	52.05	52.62	53.23	53.80[2]

Adapted from Smithsonian Tables No. 65.

[1] Specific gravity less than 1.00 particularly useful for liquids fuel, oils, and alcohols.
[2] Specific gravities greater than 1.00 particularly useful for non-freezing brines.

TABLE

SPECIFIC HEATS OF GASES:

Substance.	C_p	At Temperature.		Authority.	C_v
		° C.	° F.		
Hydrogen, H_2.	3.3996	−28–+9	−18.4–15.8	Regnault	2.4219
	3.409	12–198	53.6–388.4	Regnault	
	3.410	21–100	70–212	Wiedeman	
Oxygen, O_2.	.2175	13–207	55–405	Regnault	.1603
	.2240	20–440	68–824	Holborn-Austin	
	.2300	20–630	68–166	Holborn-Austin	
Nitrogen, N_2.	.2438	0–200	22–392	Regnault	.1715
	.2419	20–440	68–824	Holborn-Austin	
	.2464	20–630	68–1166	Holborn-Austin	
	.2497	20–800	68–1472	Holborn-Austin	
Air.	.2377	−30–+10	32-50	Regnault	
	.2374	0–100	32-212	Regnault	
	.2375	0–200	32–392	Regnault	.1703
	.2366	20–440	68–824	Holborn-Austin	
	.2429	20–630	68–1166	Holborn-Austin	
	.2430	20–800	68–1472	Holborn-Austin	
	.2389	20–100	68–212	Wiedeman	
Ammonia, NH_3.	**.5202**	23–100	73–212	Wiedeman	.4011
	.5356	27–200	80–392	Wiedeman	
	.5125	24–216	75–421	Regnault	
Carbon diox., CO_2.	.1843	−28–+7	—18–45	Regnault	
	.2025	15–100	59–212	Regnault	.1558
	.2169	11–214	52–417	Regnault	
Carbon monoxide.	**.2425**	23–99	74–210	Wiedeman	.1734
	.2426	26–198	79–388	Wiedeman	
Methane, CH_4.	.5929	18–208	64–406	Regnault	.4505
Benzole, C_6H_6.	**.2990**	34–115	93–239	Wiedeman	.2131
	.3325	35–180	95–356	Wiedeman	
	.3754	116–218	241–424	Regnault	
Ethylene, C_2H_4.	**.4040**	10–202	50–396	Regnault	.3404

XXXIV

RATIOS AND DIFFERENCES

Determined from	$C_p - C_v$	$777.52(C_p - C_v) = \left(\frac{PV}{T}\right)$ in ft.-lbs.	$C_p \div C_v = \gamma$
Wiedeman $C_p = 3.41$ and $\frac{C_p}{C_v} = 1.408$ at 4°−16° C. by Lummer and Pringsheim	.9881	768.267	1.408
Holborn and Austin $C_p = .2240$ and $\frac{C_p}{C_v} = 1.3977$ at 5° to 14° C.	.0637	49.528	1.3977
Holborn and Austin $C_p = .2419$ and $\frac{C_p}{C_v} = 1.41$ by Cazin	.0704	54.737	1.4105
Wiedeman $C_p = .2389$ and $\frac{C_p}{C_v} = 1.4025$ at 5° to 14° C. by Lummer and Pringsheim	.0686	53.338	1.4028
Wiedeman $C_p = .5202$ and mean of $\left(\frac{C_p}{C_v} = 1.3172 \text{ at } 0° \text{ C. and } \frac{C_p}{C_v} = 1.2770 \text{ at } 100° \text{ C.}\right) = 1.2971$ by Wüllner	.1191	92.603	1.2969
Regnault $C_p = .2025$ and $\frac{C_p}{C_v} = 1.2995$ by Lummer and Pringsheim	.0467	36.310	1.2997
Wiedeman $C_p = .2425$ and mean of $\left(\frac{C_p}{C_v} = 1.4032 \text{ at } 0° \text{ C. and } \frac{C_p}{C_v} = 1.3946 \text{ at } 100° \text{ C.}\right) = 1.3989$ by Wüllner	.0691	53.726	1.3985
Regnault $C_p = .5929$ $\frac{C_p}{C_v} = 1.316$ at 30° C. by Müller	.1424	110.719	1.316
Wiedeman $C_p = .2990$ and $\frac{C_p}{C_v} = 1.403$ at 60° C. by Pagliani	.0859	66.789	1.4031
Regnault $C_p = .4040$ and $\frac{C_p}{C_v} = 1.1870$ at 100° C. by Wüllner	.0636	49.450	1.1867

TABLE XXXV

COEFFICIENT OF LINEAR EXPANSION OF SOLIDS

Substance.	$\alpha \times 10^4$ per degree C.	At Temp. C.	$\alpha \times 10^4$	At Temp. F.	Authority.
Aluminum. .	.2313–.3150	40–600	.1285–.175	104–1112	Fizeau and Le Chatelier
Antimony. .	.0882–.1692	40	.0311–.094	104	Fizeau
Carbon coke	.054	40	.03	104	"
Carbon graphite. .	.0786	40	.0437	104	"
Copper.....	.1678	40	.0932	104	"
Iron.....	.1061–.1210	40	.059–.0672	104	"
Steel.....	.1095–.1322	40	.06085–.0735	104	"
Lead.....	.2924	40	.0975	104	"
Nickel.....	.1279	40	.071	104	"
Platinum. . .	.0899	40	.05	104	"
Tin.....	.2234	40	.1241	104	"
Zinc.....	.2918	40	.1621	104	"
Brasses and bronze....	.17–.21	0–900	.0889–.1167	32–1652	Limits of determination
Rubber....	.770	16.7–25.3	.4278	62–77.5	Kohlrausch
Glass.....	.058–.0897	0–100	.3222–.0498	32–212	Limits of determination
Solder.....	.2508	0–100	.1338	32–215	Smeaton
Ice.....	.375	−20 to −1	.2083	−4–30.2	Brunner
Paraffin....	1.0662–4.7707	0–16; 38–49	.5921; 2.6505	32–60.8 100.4–120	Rodwell
Porcelain. . .	.0413	20–790	.023	68–145.4	Braun
Wood.....	.0325–.0614	2–34	.0181–.0341	35.6–93.2	Limits of determination
Wax.....	2.300–15.227	10–26; 43–57	1.278 8.46	50–78.8 109.4–134.6	Kopp
Concrete. . .	.1430	.....	.0795	.....	Clark
Masonry. . .	.046–.089	.....	.0256–.0494	.....	Clark

TABLE XXXVI

COEFFICIENT OF CUBICAL EXPANSION OF LIQUIDS

Substance.	$\alpha \times 10^2$ per ° C.	At Temp C.	$\alpha \times 10^2$ per ° F.	At Temp. F.	Authority.
Alcohol (methyl)	.1433	−38–+70	.0796	−36–158	Pierre
Benzene	.1385	11–81	.0770	32–178	Kopp
Bromine	.1168	−7–+60	.0649	19–140	Pierre
Calcium chloride, $CaCl_2$, 5.8 per cent.	.0506	18–25	.0281	64–77	Decker
Calcium chloride, $CaCl_2$, 40.9 per cent.	.0510	17–24	.0283	63–75	Decker
Ether	.2150	−15–+38	.1195	5–100	Pierre
Hydrochloric acid, $HCl+6.25\ H_2O$	.0489	0–30	.0272	32–86	Marignac
Hydrochloric acid, $HCl+50\ H_2O$	.0933	0–30	.0519	32–86	Marignac
Mercury	.0179	.....	.0099		
Olive oil	.0742	.....	.0412	.....	Spring
Phenol, C_6H_6O	.0899	3–157	.0500	97–314	Pinette
Petroleum, Sp.gr. .8467	.1039	24–120	.0577	75–248	Frankenheim
Sodium chloride, NaCl, 1.6 per cent.	.1067	.....	.0593	.....	Marignac
Sulphuric acid, H_2SO_4	.0489	0–30	.0272	32–86	Marignac
Sulphuric acid, H_2SO_4	.0799	0–30	.0444	32–86	Marignac

TABLE XXXVII

COEFFICIENT OF VOLUMETRIC EXPANSION OF GASES AND VAPORS AT CONSTANT PRESSURE

(Heated without change of state.)

Substance.	Pressure (Cm Hg)	$\alpha_p \times 100$ per Deg. C.	$\alpha_p \times 100$ per Deg. F.	Authority.
Air	76	.3671	.2040	Regnault
Air	256	.3693	.2055	Regnault
Hydrogen	76	.36613	.2034	Regnault
Hydrogen	254	.36616	.20342	Regnault
Carbon dioxide	76	.3710	.2060	Regnault
Carbon dioxide	252	.3845	.2135	Regnault
Carbon dioxide 0°–64°	17.1 atm.	.5136	.2855	Andrews
Carbon dioxide 84°–100°	17.1 atm.	.4747	.2635	Andrews
Carbon dioxide 0°–7.5°	24.81 atm.	.7000	.38885	Andrews
Carbon dioxide 64°–100°	24.81 atm.	.5435	.3020	Andrews
Carbon dioxide 0°–64°	24.81 atm.	.6204	.3446	Andrews
Carbon dioxide 0°–7.5°	34.49 atm.	.1097	.6100	Andrews
Carbon dioxide 0°–64°	34.49 atm.	.8450	.470	Andrews
Carbon dioxide 0°–100°	34.49 atm.	.6514	.362	Andrews
Carbon monoxide	76	.3669	.204	Regnault
Nitrous oxide	76	.3719	.2065	Regnault
Sulphur dioxide	76	.3903	.217	Reganult
Sulphur dioxide	98	.3980	.2465	Regnault
Water vapor (steam) 0°–119	1 atm.	.4187	.23261	Hirn
Water vapor 0°–141°	1 atm.	.4189	.23272	Hirn
Water vapor 0°–162°	1 atm.	.4071	.22617	Hirn
Water vapor 0°–200°	1 atm.	.3938	.21878	Hirn
Water vapor 0°–247°	1 atm.	.3799	.2111	Hirn

TABLE XXXVIII

COEFFICIENT OF PRESSURE RISE OF GASES AND VAPORS AT CONSTANT VOLUME

(Heated without change of state.)

Substance.	Pressure (Cm Hg)	$\alpha_v \times 100$ per Deg. C.	$\alpha_v \times 100$ per Deg. F.	Authority.
Air	.6	.3767	.20915	Meleander
Air	1.6	.3703	.2057	Meleander
Air	10.0	.3663	.2035	Meleander
Air	26.0	.3660	.20335	Meleander
Air	37.6	.3662	.20345	Meleander
Air	75.0	.3665	.20360	Meleander
Air	76–83	.3670	.20370	Magnus
Air	11–15	.3648	.20265	Regnault
Air	17–24	.3651	.20285	Regnault
Air	37–51	.3658	.20320	Regnault
Air	76	.3665	.20360	Regnault
Air	200	.3690	.205	Regnault
Air	2000	.3887	.206	Regnault
Air	10000	.4100	.22775	Regnault
Air	76	.3671	.20395	Rowland
Air	1 atm.	.3670	.20290	Jolly
Carbon dioxide	1 atm.	.3706	.2059	Jolly
Carbon dioxide	1 atm.	.3726	.2070	Meleander
Carbon dioxide	76–104	.3686	.20475	Regnault
Carbon dioxide	174	.3752	.2085	Regnault
Carbon dioxide	793	.4252	.2361	Regnault
Carbon dioxide 0°–64°	16.4 atm.	.4754	.2641	Andrews
Carbon di xide 64°–100°	16.5 atm.	.4607	.256	Andrews
Carbon dioxide 0°–64°	25.87 atm.	.5728	.3182	Andrews
Carbon dioxide 64°–100°	25.87 atm.	.5406	.30035	Andrews
Carbon dioxide 0°–64°	33.53	.6973	.38740	Andrews
Carbon dioxide 64°–100°	33.53	.6334	.35190	Andrews
Carbon monoxide	1 atm.	.3667	.2037	Regnault
Hydrogen	1 atm.	.3669	.20353	Regnault
Hydrogen	1 atm.	.3656	.2031	Jolly
Nitrogen	1 atm.	.3668	.20375	Regnault
Nitrous oxide	1 atm.	.3676	.20410	Regnault
Nitrous oxide	1 atm.	.3705	.206	Jolly
Oxygen	1 atm.	.3674	.2041	Jolly
Sulphur dioxide, SO_2	1 atm.	.3845	.21350	Jolly

TABLE

COMPRESSIBILITY OF GASES BY THEIR ISOTHERMALS. VALUES OF PV AT AND AT 1 ATMOSPHERE

Pressure in Atmosphere.		1	100	200	300	400	500	600
Oxygen at	32° F.	1.000	.9265	.9140	.9624	1.0516	1.1560	1.2690
	211.1	.....	.....	1.4	1.4529	1.532	1.622	1.7202
	391.1	.....	.....	1.819	1.8849	1.96	2.05	2.142
Air at	32° F.	1.000	.9730	1.010	1.0974	1.2144	1.3400	1.4700
	210.92	.....	.....	1.472	1.551	1.668	1.7825	1 908
	392.72	.....	.....	1.886	1.9866	2.096	2.211	2.3298
Nitrogen at	32° F.	1.000	.9910	1.0390	1.1358	1.2568	1.3900	1.5258
	211.1	.....	.....	1.4890	1.5903	1.7060	1.8275	1.9548
	391.28	.....	.....	1.9064	2.1045	2.1324	2.2575	2.3838
Hydrogen at	32° F.	1.000	.....	1.1380	1.2090	1.2828	1.3565	1.4322
	210.74	.....	.....	1.5134	1.5858	1.6588	1.7310	1.8036
	393.5	.....	.....	1.884	1.956	2.030	2.105	2.1762
Carbon dioxide at	32° F.	1.000	.202	.....	.559	.....	.891	
	212.	.....	1.03	.....	.890	.....	1.201	
	388.9	.....	1.582	.....	1.493	.....	1.678	
NH_3 at	32° F.	1.000						
	211.28	.....	.....	.9290	.8625	.832	.7450	.5850
	362.48	.....	.....	.9750	.9555	.9380	.8875	.8700

Calculated from Smithsonian Tables Nos. 55 and 58, reporting Amagat's results

TABLE XL

VALUES OF THE GAS CONSTANT R

	Determined from Specific Heats by $R=777.52(C_p-C_v)$ Section 5.	Determined from Volume of One Lb. at 32° F. and 29.92 ins. Hg. Section 8.	Authority for Specific Volume.
Hydrogen, H_2	768.267	765.893	Rayleigh
Oxygen, O_2	49.528	48.244	Rayleigh
Nitrogen, N_2	54.737	55.981	Rayleigh
Air	53.338	53.332	Rayleigh and Leduc
Ammonia, NH_3	92.603	90.467	Leduc
Carbon dioxide, CO_2	36.310	35.084	Rayleigh
Carbon monoxide, CO	53.726	55.135	Leduc
Methane, CH_4	110.719	96.200	Thomson
Benzole, C_6H_6	66.789	Liquid at 32°	
Ethylene, C_2H_4	49.450	54.153	Saussure

XXXIX

VARIOUS PRESSURES AND TEMPERATURES; THE VALUE OF *PV* AT 32° F TAKEN AS 1.00.

700	800	900	1000		
1.3853 1.827 2.2414	1.5032 1.9336 2.3432	1.6200 2.0412 2.4462	1.7350 2.151	Critical point { Pressure 50 atm. / Temperature 180.4° F. }	Wroblewski
1.6016 2.0328 2.4514	1.7344 2.1592 2.5752	1.8630 2.2896 2.7	1.992 2.415 2.828	Critical point { Pressure 39 atm. / Temperature 220° F. }	Olszewski
1.6618 2.086 2.5123	1.7920 2.22 2.64	1.9341 2.3544 2.7765	2.0680	Critical point { Pressure 35 atm. / Temperature 230.8° F. }	Olszewski
1.5043 1.876 2.2484	1.5776 1.9552 2.32	1.6488 2.1096 2.3913	1.7200 2.093	Critical point { Pressure 20 atm. / Temperature 390.1° F. }	Dewar
....	....	....	1.656 1.999	Critical point { Pressure 27 atm. / Temp. +87.66° F. }	Andrews
.... .8715	.... .9000	.... .9315	.... .95	Critical point { Pressure 115 atm. / Temp. +266° F. }	Dewar

and Table 62 Roth's results; also Table 218 reporting miscellaneous data.

TABLE XLI

DENSITIES OF GAS AT ONE ATMOSPHERE = 29.92″ Hg AND 32° F., COMPARING EXPERIMENTAL VALUES WITH COMPUTED VALUES FROM MOLECULAR WEIGHTS

Gas.	Sp.Gr. Air = 1.	Lbs. per Cu.ft. Exptl.	Cu.ft. per Lb.	Authority.	Molecular Weight Exact. H = 2.	Lbs. Cu.ft. from Exact Molecular Weight.	Molecular Weight Approx. H = 2.	Lbs. Cu.ft. from Approx. Molecular Weight.
Hydrogen, H_4 . .	.0696	.005621	177.9093	Rayleigh	2.	.....	2	
Oxygen, O_2.	1.053	.08922	11.208	Rayleigh	31.76	.08926	32	.08993
Nitrogen, N_2 . . .	.9673	.07829	12.773	Rayleigh	27.80	.07813	28	.07869
Air.	1.000	.08071	12.390	Rayleigh and Leduc				
Ammonia, NH_3. .	.597	.04758	21.017	Leduc	16.9	.04750	17	.04778
Carbon dioxide CO_2.	1.5291	.12269	8.1506	Rayleigh	43.75	.12295	44	.12366
Carbon monoxide, CO	.9672	.07807	12.8090	Leduc	27.87	.07833	28	.07869
Methane, CH_4 . .	.5576	.04470	22.349	Thomson	15.99	.04494	16	.04497
Benzole, C_6H_6. . .	....	Liquid						
Ethylene, C_2H_4. .	.9852	.07951	12.578	Saussure	27.98	.07862	28	.07868
Ethane, C_2H_6. . .	1.075	.08379	11.9354	Kolbe	29.98	.08426	30	.08431
Butane, C_4H_{10}. . .	2.01	.16194	6.1751	Frankland	57.96	.16289	58	.16301

Computed from data reported in Smithsonian Tables, Nos. 71 and 276.

TABLE XLII

INTERNATIONAL ATOMIC WEIGHTS

Selected from Report of the International Committee on Atomic Weights, *Journal Amer. Chem. Soc.*, 1910.

Substance.	Symbol.	Atomic Weight, O = 16.	Atomic Weight, H = 1.
Aluminum	Al	27.1	26.9
Calcium	Ca	40.09	39.77
Carbon	C	12.00	11.99
Chlorine	Cl	35.46	35.19
Copper	Cu	63.57	63.07
Hydrogen	H	1.008	1.00
Iron	Fe	55.85	55.41
Lead	Pb	207.10	205.46
Magnesium	Mg	24.32	24.13
Manganese	Mn	54.93	54.49
Mercury	Hg	200.00	198.50
Nickel	Ni	58.68	58.21
Nitrogen	N	14.01	13.90
Oxygen	O	16.00	15.88
Platinum	Pt	195.00	193.40
Potassium	K	39.10	38.79
Silicon	Si	28.30	28.20
Sodium	Na	23.00	22.82
Sulphur	S	32.07	31.82
Tin	Sn	119.00	118.10
Zinc	Zn	65.37	64.88

TABLE XLIII

MELTING OR FREEZING-POINTS (AT 29.92 HG)

Selected from Landolt, Börnstein, Meyerhoff, and Smithsonian Physical Tables.

Class.	Substance.	Symbols.	Freezing-point.		Authority.
			C.	F.	
Elements:	Hydrogen	H	−258.9	−432	Travers, 1902
	Oxygen	O	−230	−382.5	General
	Nitrogen	N	−210.5	−347	Fischer-Alt
	Chlorine	Cl	−102	−151.5	Olszewski
	Mercury	Hg	− 38.85	− 38	Vincentini and Omodei, 188
	Bromine	Br	− 7.3	45.2	Van der Plaats, 1886
	Phosphorus	P	44.2	111.5	Helff, 1893
	Potassium	K	62.5	144.5	Holt and Sims, 1894
	Sodium	Na	97	206.5	Kurnakow and Puschin, 190
	Sulphur	S	113.5–119.5	236–247	Depending on form of S

TABLE XLIII—*Continued*

MELTING OR FREEZING-POINTS (AT 29.92 Hg)

Selected from Landolt, Börnstein, Meyerhoff, and Smithsonian Physical Tables

Class.	Substance.	Symbols.	Freezing-point.		Authority.
			C.	F.	
Elements:	Tin	Sn	231.5	451	Kurnakow and Puschin, 1902
	Bismuth	Bi	269.2	517	Callendar, 1899
	Cadmium	Ca	321	610	Kurnakow and Puschin, 1902
	Lead	Pb	326.9	621	Holborn and Day
	Zinc	Zn	419	787	Holborn and Day
	Antimony	Sb	624	1154	Fay and Ashley
	Magnesium	Mg	632.6	1171	Heycock and Neville, 1895
	Aluminum	Al	657.3	1217	Holborn and Day
	Silver	Ag	961	1651	Holborn and Day
	Gold	Au	1063	1947	Roberts and Austin
	Copper	Cu	1083	1892	Roberts and Austin
	Manganese	Mn	1225	2232	Day-Sosman
	Silicon	Si	1420	2592	General
	Nickel	Ni	1450	2647	Carnelley, Pictet, 1879
	Cobalt	Co	1490	2813	General
	Chromium	Cr	1505	2792	General
	Iron	Fe	1600	2912	Roberts and Austin
	Platinum	Pt	1755	3192	Mean of three
	Tungsten	W	2950	5347	Waidner–Burgess, Waterburg
Inorganic compounds	Ammonia	NH_3	− 75.5	−104	Ladenburg and Krugel, 1900
	Calcium chloride	$CaCl_2$	780	1454	Ruff and Plato, 1903
	Carbon monoxide	CO	−203	−331.5	Wroblewski, Olszewski (mean)
	Carbon dioxide	CO_2	− 57	70.8	General
	Sodium chloride	NaCl	820	1510	Ruff and Plato, 1903
	Sulphur dioxide	SO_2	− 76	−105	Faraday, 1845
	Zinc chloride	$ZnCl_2$	262	504	Braun, 1875
	Air	.....	−1922	−314	Wroblewski, 1884
					LIQUID DENSITY
Hydrocarbon constituents of liquid and gaseous fuel	Ethane	C_2H_6	−171.4	−276.5	.446 at 32° F.
	Nonane	C_9H_2O	− 51	− 59.8	.733 at 32° F.
	Decane	$C_{10}H_{22}$	− 31	− 23.8	.745 at 32° F.
	Undecane	$C_{11}H_{24}$	− 26	− 14.8	.756 at 32° F.
	Dodecane	$C_{12}H_{26}$	− 12	10.4	.765 at 32° F.
Paraffine series, C_nH_{2n+2}	Tridecane	$C_{13}H_{28}$	− 6	21.2	.771 at 32° F.
	Tetradecane	$C_{14}H_{30}$	+ 5	41	.775 at 40° F.
	Pentadecane	$C_{15}H_{32}$	10	50	.776 at 10° C.
	Hexadecane	$C_{16}H_{34}$	18	64.4	.775 at 18° C.
	Heptadecane	$C_{17}H_{36}$	22	71.6	.777 at 22° C.
	Octadecane	$C_{18}H_{38}$	28	82.4	.777 at 28° C.
	Nonadecane	$C_{19}H_{40}$	32	89.6	.777 at 32° C.
Ethylene Series, C_nH_{2n}	Ethylene	C_2H_4	−169	−272	.610
	Ethyl alcohol	C_2H_5OH	−130	−202	.806 at 32° F.

TABLE XLIV

BOILING-POINTS (AT 29.92 HG)

Selected from Landolt, Börnstein, Meyerhoff, and Smithsonian Physical Tables.

Class.	Substance.	Symbol.	Boiling-point.		Authority.
			C.	F.	
Elements	Hydrogen	H	−252.5	−412	Dewar, 1901
	Oxygen	O	−182.7	−297	Holborn, 1901
	Nitrogen	N	−194.4	−318	Olszewski
	Chlorine	Cl	− 33.6	− 28.5	Regnault
	Mercury	Hg	357	674	Crafts–Regnault
	Bromine	Br	61.1	142	Mean of Thorpe, van d Plaats
	Phosphorus	P	287	558	Schrötter, 1848
	Potassium	K	712	1372	Perman, Ruff, and Johan sen
	Sodium	Na	750	1382	Perman, Ruff, and Johan sen
	Sulphur	S	444.7	837	Rothe, 1903
	Tin	Sn	2270	4118	Greenwood
	Bismuth	Bi	1430	2607	Barus, Greenwood
	Cadmium	Cd	782	1440	Barus, 1894
	Lead	Pb	1525	2777	Greenwood
	Zinc	Zn	918	1686	Berthelot
	Antimony	Sb	1440	2622	Greenwood
	Magnesium	Mg	1120	2047	Greenwood
	Aluminum	Al	1800	3272	Greenwood
	Silver	Ag	1955	3552	Greenwood
	Copper	Cu	2310	4192	Greenwood
	Manganese	Mn	1900	3452	Greenwood
	Chromium	Cr	2200	3992	Greenwood
	Iron	Fe	2450	4442	Greenwood
Inorganic compounds	Ammonia	NH_3	− 38.5	−37.4	Regnault, 1863
	Carbon monoxide	CO	−191.5	−313	Mean of Wroblewski a Olszewski
	Carbon dioxide	CO_2	− 79.1	110.5	Villard and Jarry
	Sulphur dioxide	SO_2	− 10.8	12.6	Regnault, 1863
	Zinc chloride	$ZnCl_2$	730	1347	Freyer and Meyer
	Air	.....	−192.2	314	Wroblewski
		.....	−191.4	312.5	Olszewski

TABLE XLIV—*Continued*

BOILING-POINTS (AT 29.92 HG)

Selected from Landolt, Börnstein, Meyerhoff, and Smithsonian Physical Tables.

Class.	Substance.	Symbol.	Boiling-point.		Authority.
			C.	F.	
Hydrocarbon constituents of liquid and gaseous fuels	Methane	CH_4	−165	−265	Young
	Ethane	C_2H_6	− 93	−135	Ladenberg
	Propane	C_3H_8	− 45	− 49	Young, Hamlen
	Butane	C_4H_{10}	+ 1	33.8	Butlerow, Young
	Pentane	C_5H_{12}	36.3	973	Thorpe, Young
	Hexane	C_6H_{14}	69	156.2	Schorlemmer
	Heptane	C_7H_{16}	98.4	209.1	Thorpe, Young
	Octane	C_8H_{18}	125.5	257.9	Thorpe, Young
Paraffine series, C_nH_{2n+2}	Nonane	C_9H_{20}	150	302	Kraft
	Decane	$C_{10}H_{22}$	173	343.4	Kraft
	Undecane	$C_{11}H_{24}$	195	384	Kraft
	Dodecane	$C_{12}H_{26}$	214	417.2	Kraft
	Tridecane	$C_{13}H_{28}$	234	453.2	Kraft
	Tetradecane	$C_{14}H_{30}$	252	485.6	Kraft
	Pentadecane	$C_{15}H_{32}$	270	518	Kraft
	Hexadecane	$C_{16}H_{34}$	287	548.6	Kraft
	Heptadecane	$C_{17}H_{36}$	303	577	Kraft
	Octadecane	$C_{18}H_{38}$	317	602	Kraft
	Nonadecane	$C_{19}H_{40}$	330	626	Kraft
Ethylene series, C_2H_{2n}	Ethylene	C_2H_4	−103	−153.4	Olszewski
	Propylene	C_3H_6	− 50.2	− 58.5	Ladenburg-Krügel
	Butylene	C_4H_8	+ 1	33.8	Sieben
	Amylene	C_5H_{10}	36	96.8	Wagner
	Hexylene	C_6H_{12}	69	156.2	Wreden
	Heptylene	C_7H_{14}	96–99	205–210	Morgan
	Octylene	C_8H_{16}	122–123	251–255	Möslinger
	Nonylene	C_9H_{18}	140–142	284–288	Beilstein
	Decylene	$C_{10}H_{20}$	175	347	Beilstein
	Acetylene	C_2H_2	− 85	−121	Villard
	Methyl alcohol	CH_3OH	66	150.8	.....
	Ethyl alcohol	C_2H_5OH	78	172.4	.....
	Naphthas	Mixture	.....	424 app.	General
	Benzines	Mixture	.....	177 app.	General

TABLE XLV

LATENT HEAT OF VAPORIZATION AT ONE ATMOSPHERE PRESSURE

Selected from Landolt, Börnstein, Meyerhoff, and Smithsonian Physical Tables.

Substance.	Symbol.	Cal. per Kg.	B.T.U. per Lb.	C.	F.	Authority.
Ammonia	NH_3	294.21	530	7.8	4.6	Regnault
	.....	291.32	524.45	11.04	51.87	Regnault
	.....	297.38	535	16.0	60.8	Regnault
	.....	296.5	534	17	62.6	Strombeck
Water	H_2O	535.9	964.6	100	212	Andrews
	.....	532.0	957.6	100	212	Schall
Benzol	C_6H_6	109.	196	0	32	Regnault
	.....	132.1	238	100	212	Regnault
	.....	154.5	278	210	410	Regnault
Air	.....	44.02	79.3	.....	.....	Shearer
	.....	45.4	81.7	.....	.....	Shearer
Oxygen	O	58.0	106.1	−188	−306.4	Shearer
	.....	60.9	109.8	.....	.....	Estreicher
Nitrogen	N	49.83	89.6	.....	.....	Shearer
Carbon dioxide	CO_2	72.23	130	−25	− 13	Cailletet
	.....	57.48	103.2	0	32	Matthias
	.....	56.25	10 .3	0	32	Chappuis
	.....	50.76	91.5	6.5	43.7	Matthias
	.....	31.80	57.2	22.4	72.3	Matthias
	.....	14.40	25.9	29.85	85.7	Matthias
	.....	11.60	20.9	30	86	Cailletet
	.....	3.72	6.7	30.82	87.4	Matthias
Alcohol, methyl	CH OH	267.48	482	64.5	148.	Wirtz
Alcohol, ethyl	C_2H_5OH	206.4	372	78	172.4	Schall
Alcohol+5% water	.....	214.25	386	78.4	173.1	Brix
Decane	$C_{10}H_{22}$	60.83	109.5	159.45	319	Louguinine
Hexylene	C_6H_{12}	87.3	157.1	68	154.4	Mabery
	.....	.....	.....	70	158	Goldstein
Octane	C_8H_{16}	71.1	128	125	257	Goldstein

TABLE XLVI

LATENT HEATS OF FUSION

Selected from Landolt, Börnstein, Meyerhoff, and Smithsonian Physical Tables.

Substance.	Symbol.	Cal. per Kg.	B.T.U .per Lb.	C.	F.	Authority.
luminum. . .	Al	239.4	432	625	1157	Pionchon
ead.	Pb	5.37	9.66	362.2	619.2	Person
on.	Fe	6.0	10.8	1000–1050	1832–1922	Pionchon
opper.	Cu	43.0	77.4	. . . . .	. . . . .	Richards
ickel.	Ni	4.64	8.35	. . . . .	. . . . .	Pionchon
nc.	Zn	28.1	50.5	415	779	Person
n.	Sn	14.25	25.65	233	451.4	Person
mmonia. . . .	NH_3	108.1	195	−75	−102	Massol
e-water.	H_2O	79.25	142.5	0	32	Person and Regnault
	. . . . .	79.06	142.2	0	32	Regnault
	. . . . .	79.24	142.5	0	32	Desains
	. . . . .	79.91	143.9	0	32	Smith
	. . . . .	80.025	144.3	0	32	Bunsen
nzol.	C_6H_6	30.08	55.5	5.3	41.6	Fisher

TABLE XLVII

PROPERTIES OF SATURATED STEAM

(Condensed from Marks and Davis's Steam Tables and Diagrams, 1909, by permission o the publishers, Longmans, Green & Co.)

Gauge Pressure Pounds per Sq.in.	Absolute Pressure Pounds per Sq.in.	Temperature, Fahrenheat.	Total Heat Above 32° F.		Latent Heat, $L = H - h$ Heat-units	Volume, Cu. Ft. in 1 Lb. of Steam.	Weight of 1 Cu. Ft. Steam, Pound.	Entropy of the Water.	Entropy of Evaporation.
			In the Water, h Heat-units	In the Steam, H Heat-units					
29.74	0.0886	32	0.00	1073.4	1073.4	3294	0.000304	0.0000	2.1832
29.67	0.1217	40	8.05	1076.9	1068.9	2438	0.000410	0.0162	2.1394
29.56	0.1780	50	18.08	1081.4	1063.3	1702	0.000587	0.0361	2.0865
29.40	0.2562	60	28.08	1085.9	1057.8	1208	0.000828	0.0555	2.0358
29.18	0.3626	70	38.06	1090.3	1052.3	871	0.001148	0.0745	1.9868
28.89	0.505	80	48.03	1094.8	1046.7	636.8	0.001570	0.0932	1.9398
28.50	0.696	90	58.00	1099.2	1041.2	469.3	0.002131	0.1114	1.8944
28.00	0.946	100	67.97	1103.6	1035.6	350.8	0.002851	0.1295	1.8505
27.88	1	101.83	69.8	1104.4	1034.6	333.0	0.00300	0.1327	1.8427
25.85	2	126.15	94.0	1115.0	1021.0	173.5	0.00576	0.1749	1.7431
23.81	3	141.52	109.4	1121.6	1012.3	118.5	0.00845	0.2008	1.6840
21.78	4	153.01	120.9	1126.5	1005.7	90.5	0.01107	0.2198	1.6416
19.74	5	162.28	130.1	1130.5	1000.3	73.33	0.01364	0.2348	1.6084
17.70	6	170.06	137.9	1133.7	995.8	61.89	0.01616	0.2471	1.5814
15.67	7	176.85	144.7	1136.5	991.8	53.56	0.01867	0.2579	1.5582
13.63	8	182.86	150.8	1139.0	988.2	47.27	0.02115	0.2673	1.5380
11.60	9	188.27	156.2	1141.1	985.0	42.36	0.02361	0.2756	1.5202
9.56	10	193.22	161.1	1143.1	982.0	38.38	0.02606	0.2832	1.5042
7.52	11	197.75	165.7	1144.9	979.2	35.10	0.02849	0.2902	1.4895
5.49	12	201.96	169.9	1146.5	976.6	32.36	0.03090	0.2967	1.4760
3.45	13	205.87	173.8	1148.0	974.2	30.03	0.03330	0.3025	1.4639
1.42	14	209.55	177.5	1149.4	971.9	28.02	0.03569	0.3081	1.4523
lbs. gauge	14.70	212	180.0	1150.4	970.4	26.79	0.03732	0.3118	1.4447
0.3	15	213.0	181.0	1150.7	969.7	26.27	0.03806	0.3133	1.4416
1.3	16	216.3	184.4	1152.0	967.6	24.79	0.04042	0.3183	1.4311
2.3	17	219.4	187.5	1153.1	965.6	23.38	0.04277	0.3229	1.4215
3.3	18	222.4	190.5	1154.2	963.7	22.16	0.04512	0.3273	1.4127
4.3	19	225.2	193.4	1155.2	961.8	21.07	0.04746	0.3315	1.4045
5.3	20	228.0	196.1	1156.2	960.0	20.08	0.04980	0.3355	1.3965
6.3	21	230.6	198.8	1157.1	958.3	19.18	0.05213	0.3393	1.3887
7.3	22	233.1	201.3	1158.0	956.7	18.37	0.05445	0.3430	1.3811
8.3	23	235.5	203.8	1158.8	955.1	17.62	0.05676	0.3465	1.3739
9.3	24	237.8	206.1	1159.6	953.5	16.93	0.05907	0.3499	1.3670
10.3	25	240.1	208.4	1160.4	952.0	16.30	0.0614	0.3532	1.3604
11.3	26	242.2	210.6	1161.2	950.6	15.72	0.0636	0.3564	1.3542
12.3	27	244.4	212.7	1161.9	949.2	15.18	0.0659	0.3594	1.3483
13.3	28	246.4	214.8	1162.6	947.8	14.67	0.0682	0.3623	1.3425
14.3	29	248.4	216.8	1163.2	946.4	14.19	0.0705	0.3652	1.3367
15.3	30	250.3	218.8	1163.9	945.1	13.74	0.0728	0.3680	1.3311
16.3	31	252.2	220.7	1164.5	943.8	13.32	0.0751	0.3707	1.3257
17.3	32	254.1	222.6	1165.1	942.5	12.93	0.0773	0.3733	1.3205
18.3	33	255.8	224.4	1165.7	941.3	12.57	0.0795	0.3759	1.3153
19.3	34	257.6	226.2	1166.3	940.1	12.22	0.0818	0.3784	1.3101
20.3	35	259.3	227.9	1166.8	938.9	11.89	0.0841	0.3808	1.3066

TABLE XLVII—*Continued*

Gauge Pressure Pounds per Sq.in.	Absolute Pressure Pounds per Sq.in.	Temperature, Fahrenheat.	Total Heat Above 32° F.		Latent Heat, $L = H - h$ Heat-units	Volume, Cu. Ft. in 1 Lb. of Steam.	Weight of 1 Cu. Ft. Steam, Pound.	Entropy of the Water.	Entropy of Evaporation.
			In the Water, h Heat-units	In the Steam, H Heat-units					
21.3	36	261.0	229.6	1167.3	937.7	11.58	0.0863	0.3832	1.3014
22.3	37	262.6	231.3	1167.8	936.6	11.29	0.0886	0.3855	1.2969
23.3	38	264.2	232.9	1168.4	935.5	11.01	0.0908	0.3877	1.2925
24.3	39	265.8	234.5	1168.9	934.4	10.74	0.0931	0.3899	1.2882
25.3	40	267.3	236.1	1169.4	933.3	10.49	0.0953	0.3920	1.2841
26.3	41	268.7	237.6	1169.8	932.2	10.25	0.0976	0.3941	1.2800
27.3	42	270.2	239.1	1170.3	931.2	10.02	0.0998	0.3962	1.2759
28.3	43	271.7	240.5	1170.7	930.2	9.80	0.1020	0.3982	1.2720
29.3	44	273.1	242.0	1171.2	929.2	9.59	0.1043	0.4002	1.2681
30.3	45	274.5	243.4	1171.6	928.2	9.39	0.1065	0.4021	1.2644
31.3	46	275.8	244.8	1172.0	927.2	9.20	0.1087	0.4040	1.2607
32.3	47	277.2	246.1	1172.4	926.3	9.02	0.1109	0.4059	1.2571
33.3	48	278.5	247.5	1172.8	925.3	8.84	0.1131	0.4077	1.2536
34.3	49	279.8	248.8	1173.2	924.4	8.67	0.1153	0.4095	1.2502
35.3	50	281.0	250.1	1173.6	923.5	8.51	0.1175	0.4113	1.2468
36.3	51	282.3	251.4	1174.0	922.6	8.35	0.1197	0.4130	1.2432
37.3	52	283.5	252.6	1174.3	921.7	8.20	0.1219	0.4147	1.2405
38.3	53	284.7	253.9	1174.7	920.8	8.05	0.1241	0.4164	1.2370
39.3	54	285.9	255.1	1175.0	919.9	7.91	0.1263	0.4180	1.2339
40.3	55	287.1	256.3	1175.4	919.0	7.78	0.1285	0.4196	1.2309
41.3	56	288.2	257.5	1175.7	918.2	7.65	0.1307	0.4212	1.2278
42.3	57	289.4	258.7	1176.0	917.4	7.52	0.1329	0.4227	1.2248
43.3	58	290.5	259.8	1176.4	916.5	7.40	0.1350	0.4242	1.2218
44.3	59	291.6	261.0	1176.7	915.7	7.28	0.1372	0.4257	1.2189
45.3	60	292.7	262.1	1177.0	914.9	7.17	0.1394	0.4272	1.2160
46.3	61	293.8	263.2	1177.3	914.1	7.06	0.1416	0.4287	1.2132
47.3	62	294.9	264.3	1177.6	913.3	6.95	0.1438	0.4302	1.2104
48.3	63	295.9	265.4	1177.9	912.5	6.85	0.1460	0.4316	1.2077
49.3	64	297.0	266.4	1178.2	911.8	6.75	0.1482	0.4330	1.2050
50.3	65	298.0	267.5	1178.5	911.0	6.65	0.1503	0.4344	1.2024
51.3	66	299.0	268.5	1178.8	910.2	6.56	0.1525	0.4358	1.1998
52.3	67	300.0	269.6	1179.0	909.5	6.47	0.1547	0.4371	1.1972
53.3	68	301.0	270.6	1179.3	908.7	6.38	0.1569	0.4385	1.1946
54.3	69	302.0	271.6	1179.6	908.0	6.29	0.1590	0.4398	1.1921
55.3	70	302.9	272.6	1179.8	907.2	6.20	0.1612	0.4411	1.1896
56.3	71	303.9	273.6	1180.1	906.5	6.12	0.1634	0.4422	1.1872
57.3	72	304.8	274.5	1180.4	905.8	6.04	0.1656	0.4437	1.1848
58.3	73	305.8	275.5	1180.6	905.1	5.96	0.1678	0.4449	1.1825
59.3	74	306.7	276.5	1180.9	904.4	5.89	0.1699	0.4462	1 1801
60.3	75	307.6	277.4	1181.1	903.7	5.81	0.1721	0.4474	1.1778
61.3	76	308.5	278.3	1181.4	903.0	5.74	0.1743	0.4487	1.1755
62.3	77	309.4	279.3	1181.6	902.3	5.67	0.1764	0.4499	1.1730
63.3	78	310.3	280.2	1181.8	901.7	5.60	0.1786	0.4511	1.1712
64.3	79	311.2	281.1	1182.1	901.0	5.54	0.1808	0.4523	1.1687
65.3	80	312.0	282.0	1182.3	900.3	5.47	0.1829	0.4535	1.1665
66.3	81	312.9	282.9	1182.5	899.7	5.41	0.1851	0.4546	1.1644
67.3	82	313.8	283.8	1182.8	899.0	5.34	0.1873	0.4557	1.1623
68.3	83	314.6	284.6	1183.0	898.4	5.28	0.1894	0.4568	1.1602

TABLE XLVII—*Continued*

Gauge Pressure Pounds per Sq.in.	Absolute Pressure Pounds per Sq.in.	Temperature, Fahrenheat.	Total Heat Above 32° F.		Latent Heat, $L = H - h$ Heat-units	Volume, Cu. Ft. in 1 Lb. of Steam.	Weight of 1 Cu. Ft. Steam, Pound.	Entropy of the Water.	Entropy of Evaporation.
			In the Water, h Heat-units	In the Steam, H Heat-units					
69.3	84	315.4	285.5	1183.2	897.7	5.22	0.1915	0.4579	1.1581
70.3	85	316.3	286.3	1183.4	897.1	5.16	0.1937	0.4590	1.1561
71.3	86	317.1	287.2	1183.6	896.4	5.10	0.1959	0.4601	1.1540
72.3	87	317.9	288.0	1183.8	895.8	5.05	0.1980	0.4612	1.1520
73.3	88	318.7	288.9	1184.0	895.2	5.00	0.2001	0.4623	1.1500
74.3	89	319.5	289.7	1184.2	894.6	4.94	0.2023	0.4633	1.1481
75.3	90	320.3	290.5	1184.4	893.9	4.89	0.2044	0.4644	1.1461
76.3	91	321.1	291.3	1184.6	893.3	4.84	0.2065	0.4654	1.1442
77.3	92	321.8	292.1	1184.8	892.7	4.79	0.2087	0.4664	1.1423
78.3	93	322.6	292.9	1185.0	892.1	4.74	0.2109	0.4674	1.1404
79.3	94	323.4	293.7	1185.2	891.5	4.69	0.2130	0.4684	1.1385
80.3	95	324.1	294.5	1185.4	890.9	4.65	0.2151	0.4694	1.1367
81.3	96	324.9	295.3	1185.6	890.3	4.60	0.2172	0.4704	1.1348
82.3	97	325.6	296.1	1185.8	889.7	4.56	0.2193	0.4714	1.1330
83.3	98	326.4	296.8	1186.0	889.2	4.51	0.2215	0.4724	1.1312
84.3	99	327.1	297.6	1186.2	888.6	4.47	0.2237	0.4733	1.1295
85.3	100	327.8	298.3	1186.3	888.0	4.429	0.2258	0.4743	1.1277
87.3	102	329.3	299.8	1186.7	886.9	4.347	0.2300	0.4762	1.1242
89.3	104	330.7	301.3	1187.0	885.8	4.268	0.2343	0.4780	1.1208
91.3	106	332.0	302.7	1187.4	884.7	4.192	0.2336	0.4798	1.1174
93.3	108	333.4	304.1	1187.7	883.6	4.118	0.2429	0.4816	1.1141
95.3	110	334.8	305.5	1188.0	882.5	4.047	0.2472	0.4834	1.1108
97.3	112	336.1	306.9	1188.4	881.4	3.978	0.2514	0.4852	1.1076
99.3	114	337.4	308.3	1188.7	880.4	3.912	0.2556	0.4869	1.1045
101.3	116	338.7	309.6	1189.0	879.3	3.848	0.2599	0.4886	1.1014
103.3	118	340.0	311.0	1189.3	878.3	3.786	0.2641	0.4903	1.0984
105.3	120	341.3	312.3	1189.6	877.2	3.726	0.2683	0.4919	1.0954
107.3	122	342.5	313.6	1189.8	876.2	3.668	0.2726	0.4935	1.0924
109.3	124	343.8	314.9	1190.1	875.2	3.611	0.2769	0.4951	1.0895
111.3	126	345.0	316.2	1190.4	874.2	3.556	0.2812	0.4967	1.0865
113.3	128	346.2	317.4	1190.7	873.3	3.504	0.2854	0.4982	1.0837
115.3	130	347.4	318.6	1191.0	872.3	3.452	0.2897	0.4998	1.0809
117.3	132	348.5	319.9	1191.2	871.3	3.402	0.2939	0.5013	1.0782
119.3	134	349.7	321.1	1191.5	870.4	3.354	0.2981	0.5028	1.0755
121.3	136	350.8	322.3	1191.7	869.4	3.308	0.3023	0.5043	1.0728
123.3	138	352.0	323.4	1192.0	868.5	3.263	0.3065	0.5057	1.0702
125.3	140	353.1	324.6	1192.2	867.6	3.219	0.3107	0.5072	1.0675
127.3	142	354.2	325.8	1192.5	866.7	3.175	0.3150	0.5086	1.0649
129.3	144	355.3	326.9	1192.7	865.8	3.133	0.3192	0.5100	1.0624
131.3	146	356.3	328.0	1192.9	864.9	3.092	0.3234	0.5114	1.0599
133.3	148	357.4	329.1	1193.2	864.0	3.052	0.3276	0.5128	1.0574
135.3	150	358.5	330.2	1193.4	863.2	3.012	0.3320	0.5142	1.0550
137.3	152	359.5	331.4	1193.6	862.3	2.974	0.3362	0.5155	1.0525
139.3	154	360.5	332.4	1193.8	861.4	2.938	0.3404	0.5169	1.0501
141.3	156	361.6	333.5	1194.1	860.6	2.902	0.3446	0.5182	1.0477
143.3	158	362.6	334.6	1194.3	859.7	2.868	0.3488	0.5195	1.0454
145.3	160	363.6	335.6	1194.5	858.8	2.834	0.3529	0.5208	1.0431
147.3	162	364.6	336.7	1194.7	858.0	2.801	0.3570	0.5220	1.0409

TABLE XLVII—*Continued*

Gauge Pressure Pounds per Sq.in.	Absolute Pressure Pounds per Sq.in.	Temperature, Fahrenheat.	Total Heat Above 32° F.		Latent Heat, $L = H - h$ Heat-units	Volume, Cu. Ft. in 1 Lb. of Steam.	Weight of 1 Cu. Ft. Steam, Pound.	Entropy of the Water.	Entropy of Evaporation.
			In the Water, h Heat-units	In the Steam, H Heat-units					
149.3	164	365.6	337.7	1194.9	857.2	2.769	0.3612	0.5233	1.0387
151.3	166	366.5	338.7	1195.1	856.4	2.737	0.3654	0.5245	1.0365
153.3	168	367.5	339.7	1195.3	855.5	2.706	0.3696	0.5257	1.0343
155.3	170	368.5	340.7	1195.4	854.7	2.675	0.3738	0.5269	1.0321
157.3	172	369.4	341.7	1195.6	853.9	2.645	0.3780	0.5281	1.0300
159.3	174	370.4	342.7	1195.8	853.1	2.616	0.3822	0.5293	1.0278
161.3	176	371.3	343.7	1196.0	852.3	2.588	0.3864	0.5305	1.0257
163.3	178	372.2	344.7	1196.2	851.5	2.560	0.3906	0.5317	1.0235
165.3	180	373.1	345.6	1196.4	850.8	2.533	0.3948	0.5328	1.0215
167.3	182	374.0	346.6	1196.6	850.0	2.507	0.3989	0.5339	1.0195
169.3	184	374.9	347.6	1196.8	849.2	2.481	0.4031	0.5351	1.0174
171.3	186	375.8	348.5	1196.9	848.4	2.455	0.4073	0.5362	1.0154
173.3	188	376.7	349.4	1197.1	847.7	2.430	0.4115	0.5373	1.0134
175.3	190	377.6	350.4	1197.3	846.9	2.406	0.4157	0.5384	1.0114
177.3	192	378.5	351.3	1197.4	846.1	2.381	0.4199	0.5395	1.0095
179.3	194	379.3	352.2	1197.6	845.4	2.358	0.4241	0.5405	1.0076
181.3	196	380.2	353.1	1197.8	844.7	2.335	0.4283	0.5416	1.0056
183.3	198	381.0	354.0	1197.9	843.9	2.312	0.4325	0.5426	1.0038
185.3	200	381.9	354.9	1198.1	843.2	2.290	0.437	0.5437	1.0019
190.3	205	384.0	357.1	1198.5	841.4	2.237	0.447	0.5463	0.9973
195.3	210	386.0	359.2	1198.8	839.6	2.187	0.457	0.5488	0.9928
200.3	215	388.0	361.4	1199.2	837.9	2.138	0.468	0.5513	0.9885
205.3	220	389.9	363.4	1199.6	836.2	2.091	0.478	0.5538	0.9841
210.3	225	391.9	365.5	1199.9	834.4	2.046	0.489	0.5562	0.9799
215.3	230	393.8	367.5	1200.2	832.8	2.004	0.499	0.5586	0.9758
220.3	235	395.6	369.4	1200.6	831.1	1.964	0.509	0.5610	0.9717
225.3	240	397.4	371.4	1200.9	829.5	1.924	0.520	0.5633	0.9676
230.3	245	399.3	373.3	1201.2	827.9	1.887	0.530	0.5655	0.9638
235.3	250	401.1	375.2	1201.5	826.3	1.850	0.541	0.5676	0.9600
245.3	260	404.5	378.9	1202.1	823.1	1.782	9.561	0.5719	0.9525
255.3	270	407.9	382.5	1202.6	820.1	1.718	0.582	0.5760	0.9454
265.3	280	411.2	386.0	1203.1	817.1	1.658	0.603	0.5800	0.9385
275.3	290	414.4	389.4	1203.6	814.2	1.602	0.624	0.5840	0.9316
285.3	300	417.5	392.7	1204.1	811.3	1.551	0.645	0.5878	0.9251
295.3	310	420.5	395.9	1204.5	808.5	1.502	0.666	0.5915	0.9187
305.3	320	423.4	399.1	1204.9	805.8	1.456	0.687	0.5951	0.9125
315.3	330	426.3	402.2	1205.3	803.1	1.413	0.708	0.5986	0.9065
325.3	340	429.1	405.3	1205.7	800.4	1.372	0.729	0.6020	0.9006
335.3	350	431.9	408.2	1206.1	797.8	1.334	0.750	0.6053	0.8949
345.3	360	434.6	411.2	1206.4	795.3	1.298	0.770	0.6085	0.8894
355.3	370	437.2	414.0	1206.8	792.8	1.264	0.791	0.6116	0.8840
365.3	380	439.8	416.8	1207.1	790.3	1.231	0.812	0.6147	0.8788
375.3	390	442.3	419.5	1207.4	787.9	1.200	0.833	0.6178	0.8737
385.3	400	444.8	422	1208	786	1.17	0.86	0.621	0.868
435.3	450	456.5	435	1209	774	1.04	0.96	0.635	0.844
485.3	500	467.3	448	1210	762	0.93	1.08	0.648	0.822
535.3	550	477.3	459	1210	751	0.83	1.20	0.659	0.801
585.3	600	486.6	469	1210	741	0.76	1.32	0.670	0.783

TABLE XLVIII

PROPERTIES OF SUPERHEATED STEAM

(Condensed from Marks and Davis's Steam Tables and Diagrams)

v = specific volume in cubic feet per pound, h = total heat, from water at 32° F. in B.T.U. per pound, n = entropy, from water at 32°.

Pressure Absolute, Pounds per Sq.in.	Temp. Sat. Steam.	Degrees of Superheat.									
		0	20	50	100	150	200	250	300	400	500
20	228.0	v 20.08	20.73	21.69	23.25	24.80	26.33	27.85	29.37	32.39	35.40
		h 1156.2	1165.7	1179.9	1203.5	1227.1	1250.6	1274.1	1297.6	1344.8	1392.2
		n 1.7320	1.7456	1.7652	1.7961	1.8251	1.8524	1.8781	1.9026	1.9479	1.9893
40	267.3	v 10.49	10.83	11.33	12.13	12.93	13.70	14.48	15.25	16.78	18.30
		h 1169.4	1179.3	1194.0	1218.4	1242.4	1266.4	1290.3	1314.1	1361.6	1409.3
		n 1.6761	1.6895	1.7089	1.7392	1.7674	1.7940	1.8189	1.8427	1.8867	1.9271
60	292.7	v 7.17	7.40	7.75	8.30	8.84	9.36	9.89	10.41	11.43	12.45
		h 1177.0	1187.3	1202.6	1227.6	1252.1	1276.4	1300.4	1324.8	1372.2	1420.0
		n 1.6432	1.6568	1.6761	1.7062	1.7342	1.7603	1.7849	1.8081	1.8511	1.8908
80	312.0	v 5.47	5.65	5.92	6.34	6.75	7.17	7.56	7.95	8.72	9.49
		h 1182.3	1193.0	1208.8	1234.3	1259.0	1283.6	1307.8	1331.9	1379.8	1427.9
		n 1.6200	1.6338	1.6532	1.6833	1.7110	1.7368	1.7612	1.7840	1.8265	1.8658
100	327.8	v 4.43	4.58	4.79	5.14	5.47	5.80	6.12	6.44	7.07	7.69
		h 1186.3	1197.5	1213.8	1239.7	1264.7	1289.4	1313.6	1337.8	1385.9	1434.1
		n 1.6020	1.6160	1.6358	1.6658	1.6933	1.7188	1.7428	1.7656	1.8079	1.8468
120	341.3	v 3.73	3.85	4.04	4.33	4.62	4.89	5.17	5.44	5.96	6.48
		h 1189.6	1201.1	1217.9	1244.1	1269.3	1294.1	1318.4	1342.7	1391.0	1439.4
		n 1.5873	1.6016	1.6216	1.6517	1.6789	1.7041	1.7280	1.7505	1.7924	1.8311
140	353.1	v 3.22	3.32	3.49	3.75	4.00	4.24	4.48	4.71	5.16	5.61
		h 1192.2	1204.3	1221.4	1248.0	1273.3	1298.2	1322.6	1346.9	1395.4	1443.8
		n 1.5747	1.5894	1.6096	1.6395	1.6666	1.6916	1.7152	1.7376	1.7792	1.8177
160	363.6	v 2.83	2.93	3.07	3.30	3.53	3.74	3.95	4.15	4.56	4.95
		h 1194.5	1207.0	1224.5	1251.3	1276.8	1301.7	1326.2	1350.6	1399.3	1447.9
		n 1.5639	1.5789	1.5993	1.6292	1.6561	1.6810	1.7043	1.7266	1.7680	1.8063
180	373.1	v 2.53	2.62	2.75	2.96	3.16	3.35	3.54	3.72	4.09	4.44
		h 1196.4	1209.4	1227.2	1254.3	1279.9	1304.8	1329.5	1253.9	1402.7	1451.4
		n 1.5543	1.5697	1.5904	1.6201	1.6468	1.6716	1.6948	1.7169	1.7581	1.7962
200	381.9	v 2.29	2.37	2.49	2.68	2.86	3.04	3.21	3.38	3.71	4.03
		h 1198.1	1211.6	1229.8	1257.1	1282.6	1307.7	1332.4	1357.0	1405.9	1454.7
		n 1.5456	1.5614	1.5823	1.6120	1.6385	1.6632	1.6862	1.7082	1.7493	1.7872
220	389.9	v 2.09	2.16	2.28	2.45	2.62	2.78	2.94	3.10	3.40	3.69
		h 1199.6	1213.6	1232.2	1259.6	1285.2	1310.3	1335.1	1359.8	1408.8	1457.7
		n 1.5379	1.5541	1.5753	1.6049	1.6312	1.6558	1.6787	1.7005	1.7415	1.7792
240	397.4	v 1.92	1.99	2.09	2.26	2.42	2.57	2.71	2.85	3.13	3.40
		h 1200.9	1215.4	1234.3	1261.9	1287.6	1312.8	1337.6	1362.3	1411.5	1460.5
		n 1.5309	1.5476	1.5690	1.5985	1.6246	1.6492	1.6720	1.6937	1.7344	1.7721
260	404.5	v 1.78	1.84	1.94	2.10	2.24	2.39	2.52	2.65	2.91	3.16
		h 1202.1	1217.1	1236.4	1264.1	1289.9	1315.1	1340.0	1364.7	1414.0	1463.2
		n 1.5244	1.5416	1.5631	1.5926	1.6186	1.6430	1.6658	1.6874	1.7280	1.7655
280	411.2	v 1.66	1.72	1.81	1.95	2.09	2.22	2.35	2.48	2.72	2.95
		h 1203.1	1218.7	1238.4	1266.2	1291.9	1317.2	1342.2	1367.0	1416.4	1465.7
		n 1.5185	1.5362	1.5580	1.5873	1.6133	1.6375	1.6603	1.6818	1.7223	1.7597
300	417.5	v 1.55	1.60	1.69	1.83	1.96	2.09	2.21	2.33	2.55	2.77
		h 1204.1	1220.2	1240.3	1268.2	1294.0	1319.3	1344.3	1369.2	1418.6	1468.0
		n 1.5129	1.5310	1.5530	1.5824	1.6082	1.6323	1.6550	1.6765	1.7168	1.7541
350	431.9	v 1.33	1.38	1.46	1.58	1.70	1.81	1.92	2.02	2.22	2.41
		h 1206.1	1223.9	1244.6	1272.7	1298.7	1324.1	1349.3	1374.3	1424.0	1473.7
		n 1.5002	1.5199	1.5423	1.5715	1.5971	1.6210	1.6436	1.6650	1.7052	1.7422
400	444.8	v 1.17	1.21	1.28	1.40	1.50	1.60	1.70	1.79	1.97	2.14
		h 1207.7	1227.2	1248.6	1276.9	1303.0	1328.6	1353.9	1379.1	1429.0	1478.9
		n 1.4894	1.5107	1.5336	1.5625	1.5880	1.6117	1.6342	1.6554	1.6955	1.7323
450	456.5	v 1.04	1.08	1.14	1.25	1.35	1.44	1.53	1.61	1.77	1.93
		h 1209	1231	1252	1281	1307	1333	1358	1383	1434	1484
		n 1.479	1.502	1.526	1.554	1.580	1.603	1.626	1.647	1.687	1.723
500	467.3	v 0.93	0.97	1.03	1.13	1.22	1.31	1.39	1.47	1.62	1.76
		h 1210	1233	1256	1285	1311	1337	1362	1388	1438	1489
		n 1.470	1.496	1.519	1.548	1.573	1.597	1.619	1.640	1.679	1.715

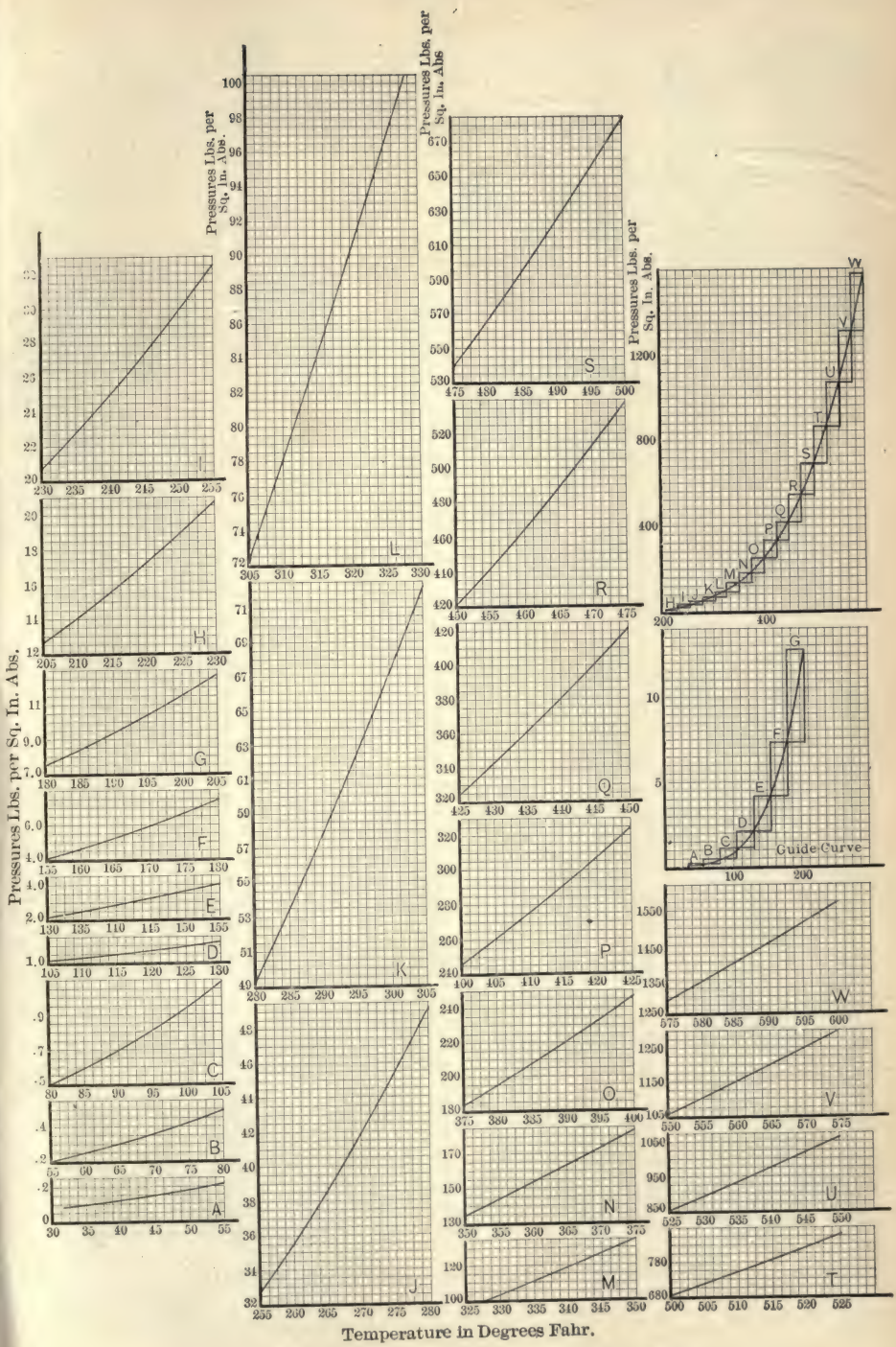


CHART A.—Steam, Pressure-temperature (Table XLVII).

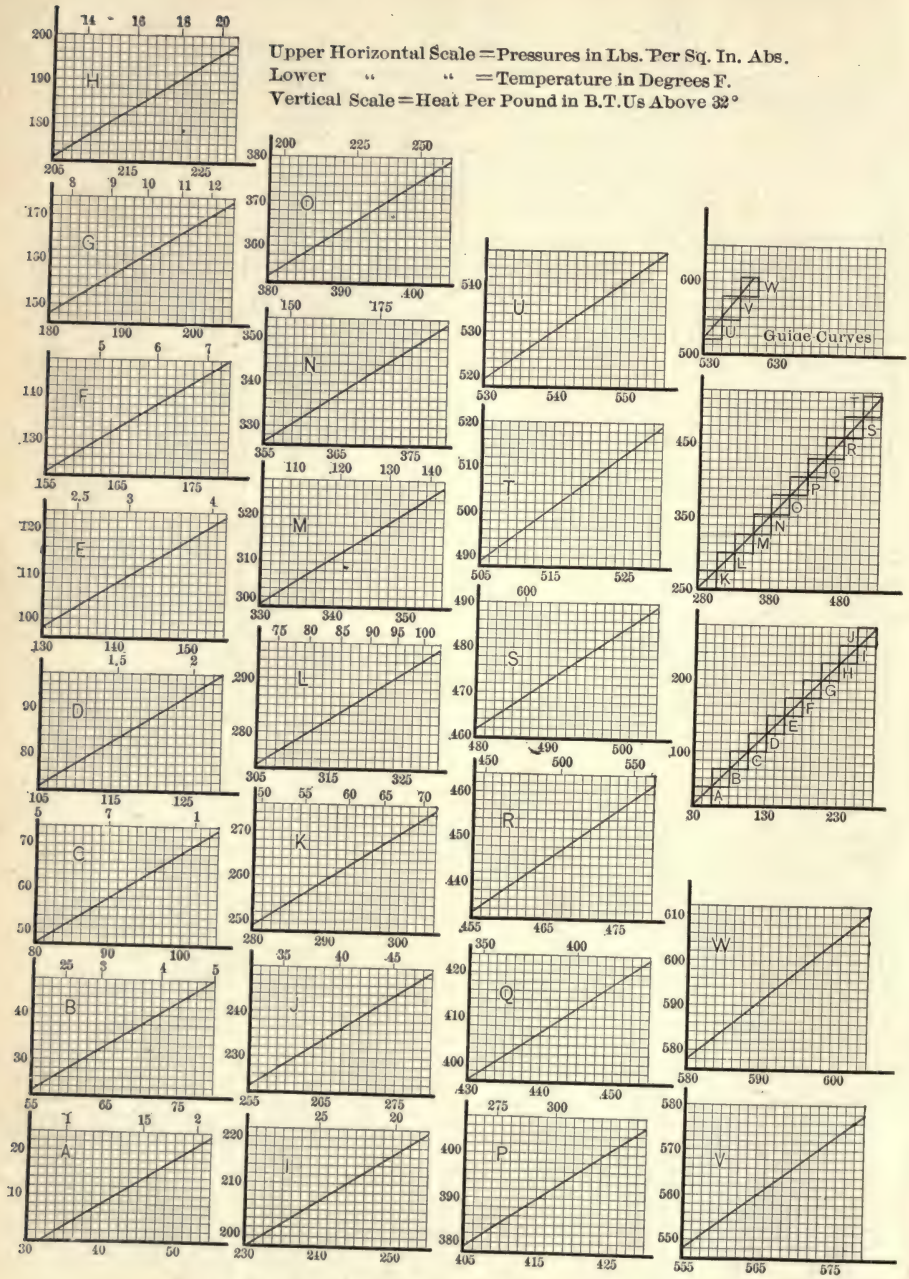


CHART B.—Steam, Heat of the Liquid (Table XLVII).

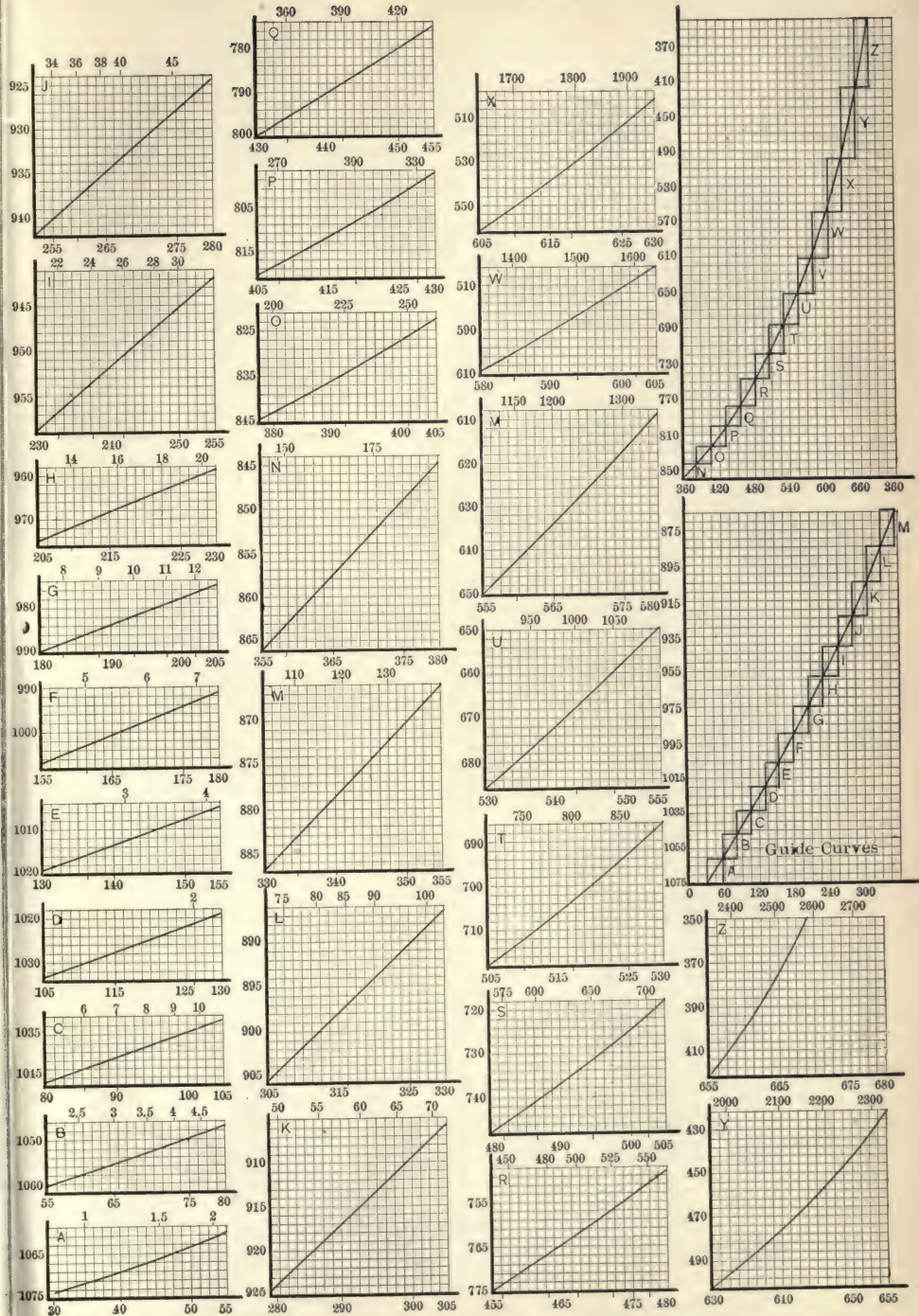


CHART C.—Steam, Latent Heat (Table XLVII).

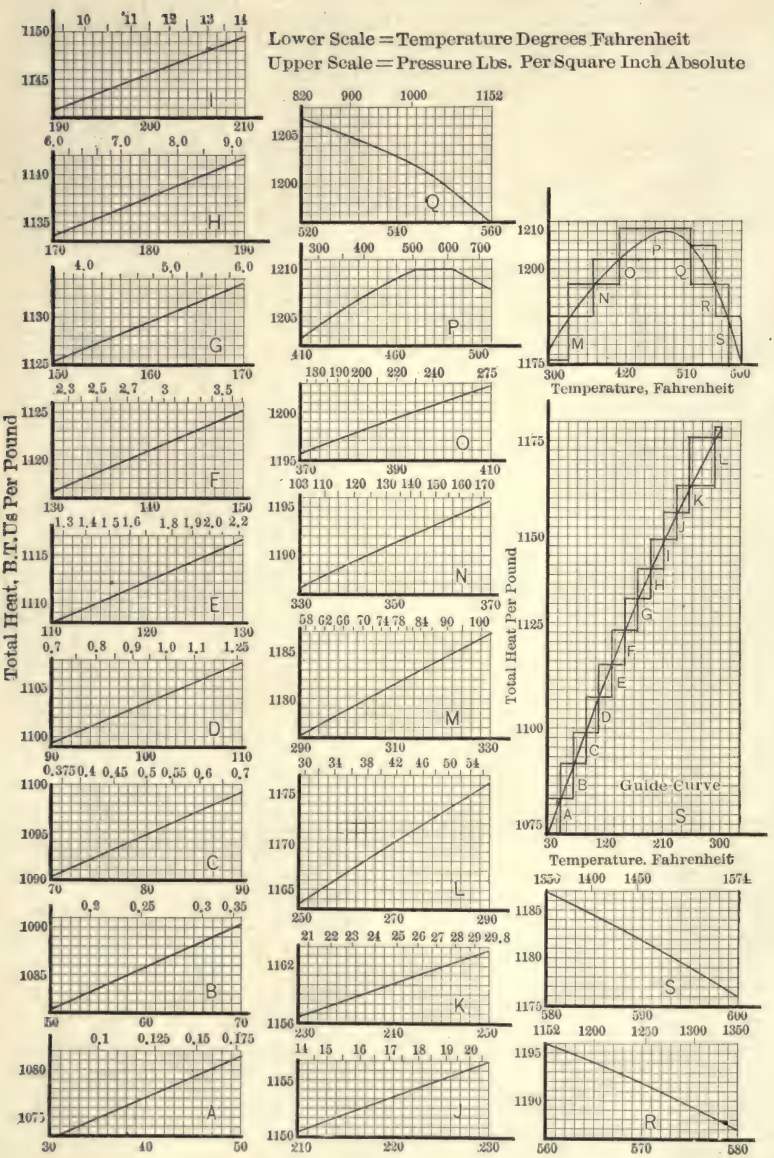


CHART D.—Steam, Total Heat (Table XLVII).

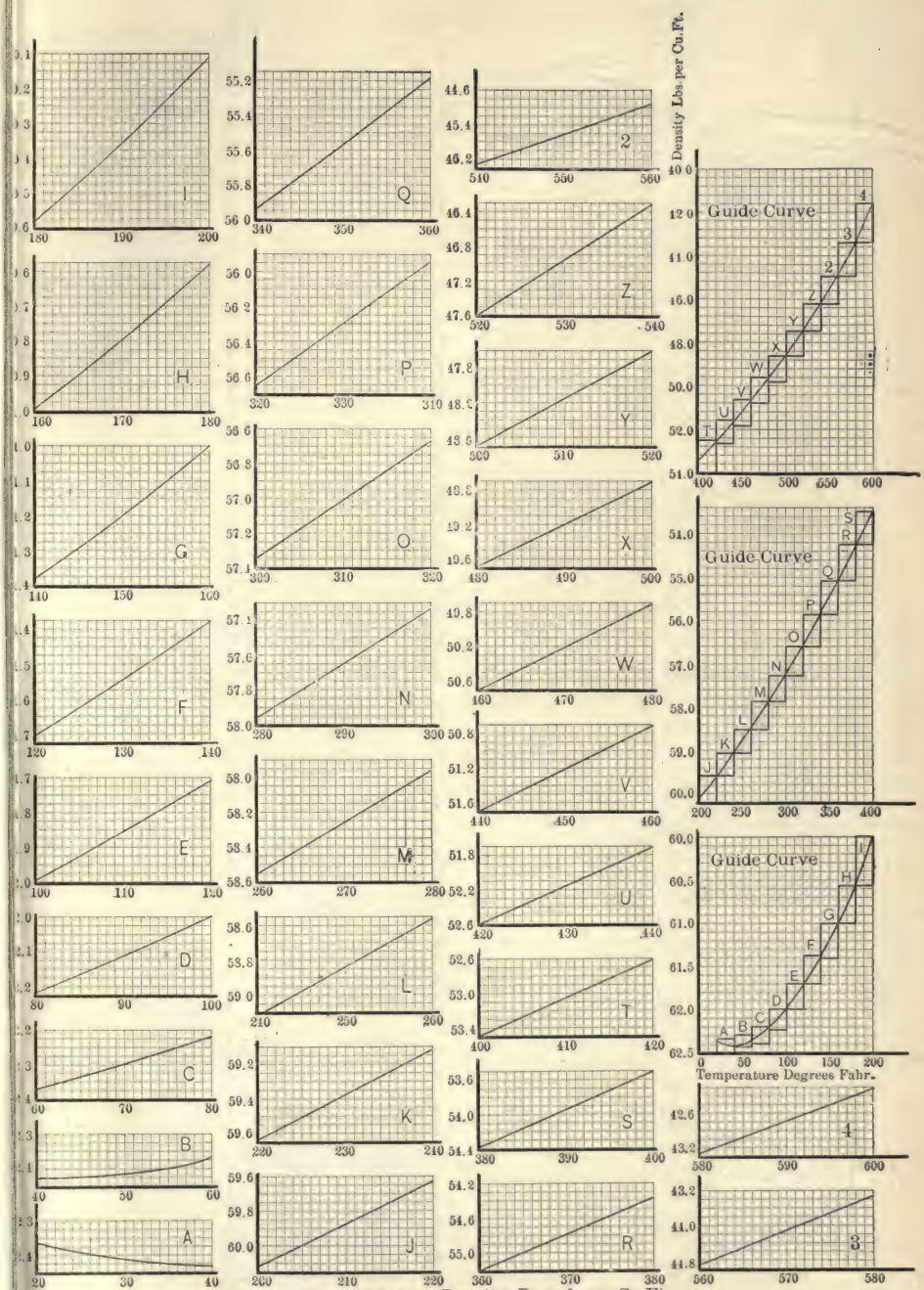


CHART E.—Steam Specific Volume and Density of the Liquid (Table XLVII)

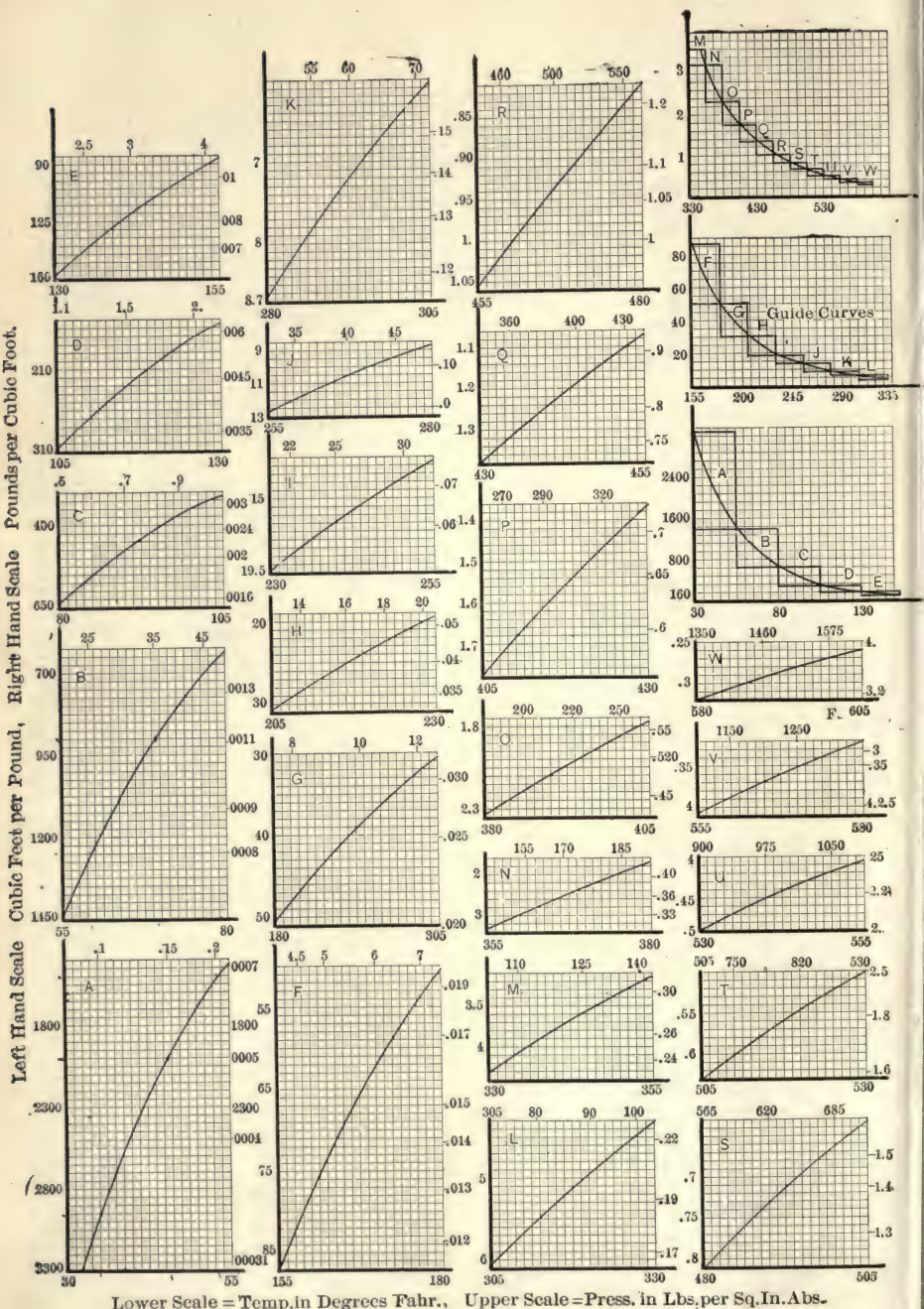


CHART F.—Steam Specific Volume and Density of the Vapor (Table XLVII).

TABLE XLIX

PROPERTIES OF SATURATED AMMONIA VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat. Above 32° F.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
420	−40	9.85	−4.85	−74	603.5	529.5	25.72	.0388	.02365	42.3	45.18	558.32	.1624	1.437
421	−39	10.1	−4.60	−73.2	602.75	529.55	24.80	.0403	.02368	42.27	45.34	557.41	.1600	1.432
422	−38	10.33	−4.37	−72.2	602.05	529.85	24.00	.0417	.0237	42.24	45.50	556.55	.1580	1.427
423	−37	10.6	−4.10	−71.2	601.4	530.2	23.30	.0429	.02371	42.21	45.68	555.72	.1556	1.422
424	−36	10.9	−3.80	−70.2	600.75	530.55	22.68	.0441	.02372	42.18	45.76	554.99	.1532	1.417
425	−35	11.2	−3.50	−69.2	600.05	530.85	22.10	.0452	.02373	42.15	46.02	554.03	.1508	1.412
426	−34	11.6	−3.10	−68.2	599.4	531.2	21.46	.0466	.02374	42.12	46.20	553.2	.1486	1.407
427	−33	12	−2.7	−67.2	598.7	531.5	20.88	.0480	.02375	42.09	46.36	552.34	.1462	1.402
428	−32	12.3	−2.4	−66.3	598	531.7	20.36	.0491	.02377	42.06	46.54	551.46	.1440	1.397
429	−31	12.7	−2.0	−65.3	597.3	532	19.84	.0504	.02378	42.03	46.70	550.6	.1414	1.392
430	−30	13.1	−1.6	−64.4	596.6	532.2	19.36	.0517	.02381	42.00	46.88	549.72	.1393	1.387
431	−29	13.5	−1.2	−63.4	595.9	532.5	18.86	.0530	.02383	41.96	47.04	548.86	.1368	1.382
432	−28	14	−0.7	−62.4	595.05	532.55	18.38	.0544	.02384	41.93	47.18	547.87	.1344	1.378
433	−27	14.4	−0.3	−61.4	594.4	533	17.92	.0559	.02386	41.90	47.34	547.06	.1320	1.373
434	−26	14.8	+0.1	−60.3	593.7	533.4	17.46	.0573	.02388	41.87	47.49	546.21	.1300	1.368
435	−25	15.25	0.55	−59.3	592.9	533.6	17.02	.0587	.02390	41.84	47.64	545.26	.1272	1.363
436	−24	15.7	1.00	−58.2	592.05	533.75	16.57	.0603	.02391	41.81	47.78	544.27	.1232	1.358
437	−23	16.2	1.5	−57.2	591.35	534.15	16.12	.0618	.02393	41.78	47.92	543.43	.1228	1.354
438	−22	16.68	2.0	−56.2	590.6	534.4	15.70	.0637	.02396	41.74	48.06	542.54	.1206	1.349
439	−21	17.15	2.35	−55.2	589.8	534.6	15.28	.0654	.02398	41.71	48.20	541.6	.1184	1.344
440	−20	17.60	2.90	−54.2	589	534.8	14.86	.0676	.02400	41.68	48.32	540.68	.1160	1.340
441	−19	18.1	3.4	−53.1	588.4	535.3	14.54	.0686	.02401	41.65	48.46	539.94	.1138	1.335
442	−18	18.6	3.9	−52.1	587.7	535.6	14.05	.0717	.02403	41.61	48.58	539.12	.1116	1.330
443	−17	19.1	4.4	−51.1	587	535.9	13.70	.0730	.02405	41.57	48.71	538.29	.1092	1.325
444	−16	19.6	4.9	−50.1	586.3	536.2	13.34	.0742	.02407	41.54	48.83	537.47	.1068	1.320
445	−15	20.2	5.5	−49.0	585.6	536.6	13.02	.0764	.02409	41.51	48.96	536.64	.1048	1.316
446	−14	20.8	6.1	−48.0	584.9	536.9	12.70	.0787	.02411	41.47	49.08	535.82	.1022	1.311

TABLE XLIX—*Continued*

PROPERTIES OF SATURATED AMMONIA VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat. Above 32° F.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
447	−13	21.4	6.7	−47.0	584.2	537.2	12.50	.0800	.02413	41.44	49.20	535.00	.1000	1.307
448	−12	22	7.3	−46.0	583.5	537.5	12.12	.0820	.02415	41.41	49.31	534.19	.0978	1.302
449	−11	22.6	7.9	−44.9	582.8	537.9	11.84	.0844	.02417	41.37	49.42	533.38	.0956	1.298
450	−10	23.2	8.5	−43.9	582.05	538.15	11.58	.0862	.02419	41.34	49.54	532.51	.0930	1.293
451	− 9	23.8	9.1	−43.0	581.3	538.3	11.32	.0880	.02421	41.30	49.64	531.66	.0910	1.289
452	− 8	24.5	9.8	−42.0	580.65	538.65	11.06	.0900	.02423	41.27	49.75	530.90	.0888	1.284
453	− 7	25.1	10.4	−41.0	579.9	538.9	10.82	.0920	.02426	41.23	49.86	530.04	.0864	1.280
454	− 6	25.7	11.0	−40.0	579.2	539.2	10.58	.0940	.02427	41.20	49.96	529.24	.0840	1.275
455	− 5	26.4	11.7	−38.9	578.4	539.5	10.34	.0970	.02430	41.16	50.08	528.32	.0832	1.271
456	− 4	27	12.3	−37.8	577.6	539.8	10.12	.0990	.02432	41.12	50.18	527.42	.0796	1.266
457	− 3	27.7	13.0	−36.8	576.9	540.1	9.9	.101	.02433	41.09	50.28	526.62	.0772	1.262
458	− 2	28.4	13.7	−35.8	576.05	540.25	9.66	.103	.02436	41.05	50.38	525.67	.0750	1.257
459	− 1	29.1	14.4	−34.8	575.3	540.5	9.44	.106	.02439	41.01	50.48	524.82	.0728	1.253
460	0	29.7	15.0	−33.8	574.6	540.8	9.24	.108	.02440	40.98	50.58	524.02	.0708	1.249
461	1	30.5	15.8	−32.8	574	541.2	9.00	.111	.02442	40.94	50.68	523.32	.0682	1.244
462	2	31.2	16.5	−31.8	573.2	541.4	8.80	.114	.02445	40.90	50.78	522.42	.0660	1.240
463	3	31.9	17.2	−30.7	572.4	541.7	8.60	.116	.02446	40.87	50.87	521.53	.0636	1.236
464	4	32.7	18.0	−29.6	571.6	541.95	8.40	.119	.02448	40.83	50.96	520.64	.0612	1.231
465	5	33.5	18.8	−28.6	570.8	542.2	8.22	.122	.02450	40.79	51.05	519.75	.0595	1.227
466	6	34.3	19.6	−27.6	570	542.4	8.04	.124	.02454	40.75	51.14	518.86	.0572	1.222
467	7	35.1	20.4	−26.6	569.2	542.6	7.86	.127	.02457	40.71	51.22	517.98	.0550	1.218
468	8	36	21.3	−25.6	568.4	542.8	7.70	.130	.02459	40.67	51.31	517.09	.0526	1.214
469	9	36.9	22.2	−24.5	567.6	543.1	7.54	.132	.02460	40.64	51.40	516.20	.0504	1.210
470	10	37.8	23.1	−23.5	566.8	543.3	7.38	.136	.02463	40.60	51.48	515.32	.0486	1.205
471	11	38.7	24.0	−22.4	566	543.6	7.21	.139	.02466	40.56	51.56	514.44	.0460	1.201
472	12	39.7	25	−21.4	565.2	543.8	7.05	.142	.02468	40.52	51.64	513.56	.0440	1.197
473	13	40.6	25.9	−20.4	564.3	543.9	6.90	.145	.02469	40.48	51.72	512.58	.0412	1.193

TABLE XLIX—*Continued*

PROPERTIES OF SATURATED AMMONIA VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	Sp. Vol. of Liquid. Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
474	14	41.6	26.9	−19.3	563.5	544.2	6.75	.148	.02472	40.44	51.80	511.70	.0396	1.188
475	15	42.6	27.9	−18.2	562.6	544.4	6.60	.152	.02475	40.40	51.88	510.72	.0372	1.184
476	16	43.6	28.9	−17.2	561.8	544.6	6.45	.155	.02478	40.36	51.96	509.84	.0350	1.180
477	17	44.6	29.9	−16.2	560.9	544.7	6.32	.158	.02480	40.32	52.02	508.88	.0328	1.176
478	18	45.6	30.9	−15.1	560.1	545	6.18	.162	.02483	40.28	52.10	508.00	.0306	1.172
479	19	46.6	31.9	−14.0	559.2	545.2	6.04	.166	.02485	40.24	52.18	507.02	.0284	1.167
480	20	47.7	33.0	−13.0	558.4	545.4	5.90	.169	.02487	40.20	52.25	506.15	.0262	1.163
481	21	48.7	34.0	−12.0	557.6	545.6	5.78	.173	.02489	40.16	52.32	505.28	.0240	1.159
482	22	49.7	35.0	−11.0	556.8	545.8	5.66	.177	.02493	40.11	52.39	504.41	.0212	1.155
483	23	50.8	36.1	− 9.9	556	546.1	5.54	.180	.02495	40.07	52.46	503.54	.0196	1.151
484	24	51.9	37.2	− 8.9	555.1	546.2	5.43	.184	.02498	40.03	52.53	502.57	.0173	1.147
485	25	53.	38.3	− 7.8	554.3	546.5	5.32	.188	.02500	39.99	52.60	501.70	.0152	1.143
486	26	54.1	39.4	− 6.7	553.5	546.8	5.22	.192	.02503	39.95	52.66	500.84	.0128	1.138
487	27	55.3	40.6	− 5.6	552.6	547	5.12	.195	.02506	39.90	52.73	499.87	.0106	1.134
488	28	56.5	41.8	− 4.6	551.8	547.2	5.02	.199	.02509	39.86	52.80	499.00	.0084	1.130
489	29	57.8	43.1	− 3.5	550.9	547.4	4.93	.203	.02511	39.82	52.86	498.04	.0062	1.126
490	30	59.1	44.4	− 2.5	550	547.5	4.83	.207	.02513	39.78	52.92	497.08	.0040	1.122
491	31	60.3	45.6	− 1.5	549.1	547.6	4.74	.211	.02516	39.73	52.98	496.12	.0020	1.118
492	32	61.6	46.9	− 0.4	548.2	547.8	4.66	.215	.02518	39.69	53.04	495.16	.0	1.114
493	33	62.9	48.2	+ 0.7	547.3	548	4.57	.219	.02522	39.65	53.10	494.20	.0024	1.110
494	34	64.2	49.5	+ 1.8	546.4	548.2	4.48	.223	.02525	39.60	53.16	493.24	.0046	1.106
495	35	65.6	50.9	2.8	545.5	548.3	4.40	.227	.02527	39.56	53.22	492.28	.0072	1.102
496	36	67.0	52.3	4.0	544.6	548.6	4.31	.232	.02530	39.52	53.28	491.32	.0090	1.098
497	37	68.4	53.7	+ 5.0	543.6	548.6	4.23	.236	.02533	39.47	53.34	490.26	.0112	1.094
498	38	69.8	55.1	6.1	542.6	548.7	4.14	.242	.02536	39.43	53.40	489.20	.0134	1.090
499	39	71.3	56.6	7.2	541.6	548.8	4.06	.246	.02539	39.38	53.46	488.14	.0156	1.086
500	40	72.8	58.1	8.3	540.6	548.9	3.98	.251	.02542	39.34	53.51	487.14	.0176	1.082

TABLE XLIX—*Continued*

PROPERTIES OF SATURATED AMMONIA VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
501	41	74.2	59.5	9.4	539.5	548.9	3.91	.256	.02544	39.29	53.56	485.94	.0200	1.078
502	42	75.7	61.0	10.4	538.5	548.9	3.84	.260	.02548	39.25	53.61	484.89	.0220	1.074
503	43	77.2	62.5	11.5	537.5	549.1	3.77	.265	.02551	39.20	53.66	483.89	.0242	1.070
504	44	78.6	63.9	12.6	536.6	549.2	3.70	.270	.02554	39.16	53.71	482.89	.0264	1.066
505	45	80.2	65.5	13.7	535.6	549.3	3.64	.274	.02557	39.11	53.76	481.84	.0284	1.062
506	46	81.9	67.2	14.8	534.6	549.4	3.57	.280	.02559	39.07	53.81	480.79	.0306	1.058
507	47	83.6	68.9	15.9	533.6	549.5	3.51	.285	.02562	39.02	53.85	479.75	.0328	1.054
508	48	85.2	70.5	17.0	532.6	549.6	3.44	.291	.02564	38.98	53.90	478.70	.0348	1.050
509	49	87.0	72.3	18.1	531.6	549.7	3.38	.296	.02567	38.93	53.94	477.66	.0368	1.045
510	50	88.7	74	19.2	530.6	549.8	3.32	.301	.02570	38.89	53.98	476.62	.0390	1.042
511	51	90.5	75.8	20.5	529.6	550.1	3.26	.307	.02574	38.84	54.01	475.59	.0407	1.038
512	52	92.3	77.6	21.6	528.6	550.2	3.21	.312	.02577	38.80	54.04	474.56	.0432	1.033
513	53	94.1	79.4	22.7	527.6	550.3	3.15	.317	.02580	38.75	54.08	473.52	.0452	1.029
514	54	96	81.3	23.8	526.6	550.4	3.08	.325	.02583	38.70	54.11	472.49	.0472	1.025
515	55	97.9	83.2	25.0	525.6	550.6	3.03	.331	.02586	38.65	54.14	471.46	.0496	1.021
516	56	99.9	85.2	26.0	524.6	550.6	2 97	.337	.02590	38.60	54.17	470.43	.0512	1.017
517	57	101.8	87.1	27.1	523.6	550.7	2.91	.344	.02594	38.55	54. 0	469.40	.0538	1.013
518	58	103.8	89.1	28.2	522.6	550.8	2.86	.350	.02597	38.50	54.23	468.37	.0560	1.009
519	59	105.8	91.1	29.3	521.6	550.9	2.80	.357	.02600	38.45	54.25	467.35	.0580	1.005
520	60	107.8	93.1	30.4	520.6	551.0	2.77	.361	.02604	38.40	54.28	466.32	.0601	1.001
521	61	109.6	94.9	31.5	519.5	551.1	2.72	.367	.02607	38.35	54.30	465.25	.0624	.998
522	62	111.5	96.8	32.6	518.5	551.1	2.67	.375	.02610	38.30	54.32	464.18	.0645	.994
523	63	113.5	98.8	33.7	517.5	551.2	2.63	.380	.02614	38.25	54.34	463.16	.0666	.990
524	64	115.5	100.8	34.8	516.4	551.2	2.58	.387	.02618	38.20	54.35	462.10	.0688	.986
525	65	117.5	102.8	35.9	515.4	551.3	2.53	.397	.02621	38.16	54.37	461.03	.0706	.982
526	66	119.6	104.9	37.0	514.3	551.3	2.49	.401	.02624	38.11	54.38	459.97	.0728	.978
527	67	121.8	107.1	33.1	513.3	551.4	2.44	.410	.02628	38.06	54.39	458.91	.0750	.974

TABLE XLIX—*Continued*

PROPERTIES OF SATURATED AMMONIA VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Liquid.
528	68	123.9	109.2	39.2	512.2	551.4	2.40	.417	.02631	38.01	54.40	457.80	.0770	.970
529	69	126.1	111.4	40.4	511.1	551.5	2.35	.426	.02635	37.96	54.41	456.74	.0792	.967
530	70	128.4	113.7	41.5	510.1	551.6	2.31	.432	.02638	37.91	54.42	455.68	.0809	.963
531	71	130.8	116.1	42.6	509.0	551.6	2.27	.440	.02642	37.86	54.42	454.63	.0832	.959
532	72	133.2	118.5	43.6	508	551.6	2.23	.448	.02645	37.81	54.43	453.57	.0852	.955
533	73	135.6	120.9	44.8	506.9	551.7	2.20	.455	.02649	37.76	54.43	452.52	.0872	.951
534	74	138	123.3	45.8	505.9	551.7	2.16	.463	.02652	37.72	54.44	451.46	.0892	.948
535	75	140.5	125.8	47.0	504.8	551.8	2.12	.472	.02654	37.67	54.44	450.36	.0913	.944
536	76	143	128.3	48.0	503.7	551.7	2.08	.481	.02658	37.62	54.44	449.26	.0933	.940
537	77	145.5	130.8	49.1	502.6	551.7	2.05	.488	.02661	37.57	54.44	448.16	.0954	.936
538	78	148.1	133.4	50.2	501.5	551.7	2.01	.497	.02665	37.52	54.43	447.07	.0976	.932
539	79	150.7	136.0	51.3	500.4	551.7	1.98	.505	.02668	37.47	54.43	445.97	.0996	.928
540	80	153.2	138.5	52.4	499.3	551.7	1.94	.515	.02672	37.42	54.42	444.88	.1016	.925
541	81	155.6	140.9	53.5	498.1	551.6	1.91	.524	.02675	37.37	54.41	443.69	.1038	.921
542	82	158.3	143.6	54.6	496.9	551.5	1.88	.532	.02679	37.32	54.41	442.49	.1058	.917
543	83	161.0	146.3	55.8	495.7	551.5	1.85	.541	.02683	37.26	54.40	441.30	.1080	.913
544	84	163.8	149.1	56.9	494.6	551.5	1.82	.550	.02688	37.21	54.39	440.21	.1100	.910
545	85	166.8	152.1	58.0	493.4	551.5	1.79	.559	.02690	37.15	54.38	439.02	.1122	.906
546	86	169.2	154.5	59.2	492.3	551.5	1.76	.568	.02695	37.10	54.36	437.94	.1142	.902
547	87	172.2	157.5	60.3	491.1	551.5	1.72	.580	.02699	37.05	54.35	436.75	.1164	.899
548	88	175.1	160.4	61.5	490	551.5	1.70	.588	.02703	37.00	54.33	435.67	.1184	.895
549	89	178	163.3	62.7	488.8	551.5	1.68	.595	.02706	36.94	54.32	434.48	.1206	.891
550	90	181	166.3	63.9	487.7	551.6	1.65	.606	.02710	36.89	54.30	433.40	.1227	.887
551	91	183.8	169.1	65.2	486.5	551.7	1.62	.615	.02715	36.83	54.28	432.22	.1248	.883
552	92	186.8	172.1	66.3	485.3	551.6	1.60	.625	.02719	36.78	54.26	431.04	.1268	.880
553	93	190	175.3	67.4	484.1	551.5	1.57	.637	.02724	36.72	54.24	429.86	.1288	.876
554	94	193.2	178.5	68.5	482.9	551.4	1.54	.647	.02727	36.67	54.22	428.68	.1308	.872

TABLE XLIX—*Continued*

PROPERTIES OF SATURATED AMMONIA VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pound per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
555	95	196	181.3	69.7	481.7	551.4	1.52	.658	.02732	36.61	54.20	427.50	.1328	.868
556	96	199	184.3	70.8	480.5	551.3	1.49	.668	.02735	36.55	54.17	426.33	.1348	.864
557	97	202.4	187.7	72.0	479.3	551.3	1.47	.680	.02739	36.50	54.15	425.15	.1369	.860
558	98	205.6	190.9	73.0	478.1	551.1	1.44	.691	.02743	36.44	54.12	424.03	.1390	.857
559	99	209	194.3	74.2	476.8	551.0	1.42	.701	.02747	36.39	54.09	422.71	.1410	.853
560	100	212.5	197.8	75.3	475.5	550.8	1.40	.714	.02753	36.33	54.07	421.43	.1432	.849
561	101	215	200.3	76.4	474.3	550.7	1.38	.724	.02756	36.27	54.04	420.26	.1452	.846
562	102	218.5	203.8	77.6	473.1	550.7	1.36	.735	.02761	36.22	54.01	419.09	.1472	.842
563	103	222	207.3	78.8	471.9	550.7	1.34	.746	.02765	36.16	53.98	417.92	.1492	.838
564	104	225.5	210.8	79.8	470.6	550.4	1.32	.758	.02770	36.10	53.95	416.65	.1512	.836
565	105	229	214.3	81.0	469.4	550.4	1.30	.769	.02775	36.04	53.92	415.48	.1532	.831
566	106	232.5	217.8	82.2	468.1	550.3	1.28	.781	.02779	35.99	53.89	414.26	.1552	.827
567	107	236	221.3	83.4	466.8	550.2	1.26	.794	.02783	35.93	53.86	412.94	.1572	.823
568	108	239	224.3	84.6	465.6	550.2	1.24	.806	.02787	35.87	53.83	411.77	.1592	.820
569	109	243	228.3	85.8	464.3	550.1	1.22	.820	.02791	35.82	53.80	410.50	.1612	.816
570	110	247	232.3	86.9	463	549.9	1.20	.833	.02796	35.76	53.76	409.24	.1632	.812
571	111	251	236.3	88.1	461.7	549.8	1.18	.847	.02801	35.70	53.72	407.98	.1653	.809
572	112	255	240.3	89.2	460.4	549.6	1.16	.862	.02806	35.64	53.69	406.71	.1673	.805
573	113	259	244.3	90.4	459.1	549.5	1.14	.873	.02810	35.59	53.65	405.45	.1693	.801
574	114	263	248.3	91.5	457.8	549.3	1.13	.885	.02815	35.53	53.61	404.19	.1714	.798
575	115	267	252.3	92.7	456.5	549.2	1.11	.900	.02819	35.47	53.57	402.93	.1734	.794
576	116	271	256.3	93.8	455.1	548.9	1.09	.917	.02824	35.41	53.53	401.62	.1753	.791
577	117	275.5	260.8	95.0	453.8	548.8	1.08	.926	.02828	35.35	53.49	400.31	.1773	.787
578	118	280	265.3	96.1	452.4	548.5	1.06	.943	.02833	35.29	53.45	398.95	.1793	.784
579	119	284	269.3	97.3	451	548.4	1.04	.957	.02839	35.23	53.40	397.60	.1813	.780
580	120	288	273.3	98.4	449.6	548.0	1.03	.970	.02843	35.17	53.36	396.24	.1832	.776
581	121	292	277.3	99.6	448.3	547.9	1.01	.985	.02848	35.11	53.31	394.99	.1852	.772

TABLE XLIX—*Continued*

PROPERTIES OF SATURATED AMMONIA VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
582	122	296	281.3	100.8	447	547.8	1.000	1.0	.02853	35.05	53.26	393.74	.1872	.769
583	123	299	284.3	101.9	445.6	547.5	.99	1.01	.02857	34.99	53.21	392.39	.1893	.765
584	124	303	288.3	103.0	444.3	547.3	.975	1.02	.02863	34.93	53.17	391.13	.1913	.761
585	125	307.5	292.8	104.2	443.0	547.2	.960	1.04	.02867	34.87	53.12	389.88	.1933	.758
586	126	312	297.3	105.4	441.7	547.1	.945	1.06	.02873	34.81	53.06	388.64	.1953	.754
587	127	316	301.3	106.6	440.3	546.9	.930	1.07	.02877	34.75	53.01	387.29	.1973	.750
588	128	320	305.3	107.8	439.0	546.8	.920	1.08	.02881	34.69	52.95	386.05	.1993	.746
589	129	325	310.3	109.0	437.5	546.5	.910	1.09	.02887	34.63	52.90	384.60	.2013	.743
590	130	330	315.3	110.2	436.2	546.4	.890	1.12	.02893	34.57	52.85	383.35	.2033	.739
591	131	335	320.3	111.3	434.8	546.1	.88	1.13	.02898	34.50	52.79	382.06	.2053	.736
592	132	340	325.3	112.4	433.5	545.9	.87	1.15	.02902	34.44	52.73	380.77	.2072	.732
593	133	345	330.3	113.6	432.1	545.7	.855	1.17	.02909	34.38	52.66	379.44	.2092	.728
594	134	350	335.3	114.8	430.8	545.6	.84	1.19	.02914	34.32	52.60	378.20	.2112	.725
595	135	355	340.3	116.0	429.4	545.4	.83	1.20	.02919	34.25	52.53	376.87	.2133	.721
596	136	360	345.3	117.1	428.0	545.1	.815	1.23	.02925	34.19	52.47	375.53	.2153	.718
597	137	365	350.3	118.2	426.6	544.8	.805	1.24	.02931	34.12	52.40	374.20	.2173	.714
598	138	370	355.3	119.4	425.3	544.7	.79	1.26	.02938	34.06	52.32	372.98	.2193	.711
599	139	375	360.3	120.6	423.9	544.5	.78	1.28	.02941	34.00	52.25	371.65	.2213	.707
600	140	380.5	365.8	121.8	422.5	544.3	.77	1.30	.02945	33.94	52.18	370.32	.2232	.703
601	141	386	371.3	123.0	421.0	544.0	.76	1.31	.02952	33.88	52.10	368.95	.2253	.700
602	142	391	376.3	124.2	419.6	543.8	.75	1.33	.02957	33.82	52.03	367.57	.2273	.697
603	143	396	381.3	125.4	418.1	543.5	.74	1.35	.02962	33.75	51.96	366.19	.2292	.693
604	144	402	387.3	126.6	416.6	543.2	.73	1.37	.02968	33.69	51.88	364.77	.2312	.690
605	145	407	392.3	127.9	415.2	543.1	.72	1.39	.02974	33.63	51.80	363.40	.2332	.686
606	146	412	397.3	129.0	413.7	542.7	.71	1.41	.02980	33.56	51.72	362.03	.2352	.683
607	147	418	403.3	130.3	412.3	542.6	.7	1.43	.02985	33.50	51.64	360.66	.2372	.679
608	148	424	409.3	131.5	410.8	542.3	.69	1.45	.02992	33.43	51.55	359.25	.2392	.675

TABLE XLIX—*Continued*

PROPERTIES OF SATURATED AMMONIA VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
609	149	429	414.3	132.8	409.3	542.1	.68	1.47	.02998	33.37	51.46	357.84	.2412	.672
610	150	435	420.3	134.0	407.8	541.8	.67	1.49	.03003	33.30	51.38	356.42	.2432	.668
611	151	440	425.3	135.2	406.3	541.5	.66	1.51	.03010	33.23	51.29	355.01	.2450	.665
612	152	448	433.3	136.3	404.9	541.2	.65	1.54	.03016	33.17	51.20	353.70	.2470	.661
613	153	452	437.3	137.6	403.4	541.0	.641	1.56	.03021	33.10	51.10	352.30	.2490	.658
614	154	457	442.3	138.8	401.9	540.7	.632	1.58	.03028	33.03	51.00	350.90	.2511	.654
615	155	463	448.3	140.0	400.3	540.3	.624	1.60	.03034	32.97	50.90	349.40	.2530	.651
616	156	469	454.3	141.2	398.8	540.0	.616	1.62	.03039	32.90	50.80	348.00	.2549	.648
617	157	475	460.3	142.4	397.3	539.7	.608	1.64	.03046	32.83	50.70	346.60	.2569	.644
618	158	481	466.3	143.6	395.7	539.3	.600	1.66	.03053	32.76	50.60	345.10	.2588	.640
619	159	486	471.3	144.9	394.0	538.9	.594	1.68	.03058	32.69	50.50	343.50	.2608	.637
620	160	492	477.3	146.2	392.5	538.7	.588	1.72	.03065	32.62	50.40	342.10	.2628	.633
621	161	498	483.3	147.4	391.0	538.4	.578	1.75	.03072	32.55	50.29	340.71	.2648	.630
622	162	504	489.3	148.6	389.4	538.0	.568	1.76	.03080	32.48	50.18	339.22	.2668	.626
623	163	510	495.3	149.9	387.7	537.6	.560	1.78	.03086	32.40	50.06	377.64	.2688	.623
624	164	518	503.3	151.1	386.1	537.2	.550	1.82	.03094	32.33	49.95	336.15	.2707	.619
625	165	523	508.3	152.4	384.4	536.8	.542	1.85	.03100	32.26	49.83	334.57	.2726	.616
626	166	529	514.3	153.6	382.8	536.4	.532	1.88	.03109	32.18	49.72	333.08	.2746	.612
627	167	536	521.3	154.9	381.1	536.0	.524	1.91	.03115	32.11	49.60	331.50	.2764	.608
628	168	542	527.3	156.1	379.5	535.6	.516	1.94	.03122	32.04	49.48	330.02	.2784	.605
629	169	549	534.3	157.4	377.8	535.2	.508	1.97	.03129	31.97	49.36	328.44	.2802	.601
630	170	556	541.3	158.6	376.1	534.7	.502	1.99	.03135	31.90	49.24	326.86	.2820	.597
631	171	563	548.3	159.7	374.4	534.1	.494	2.02	.03143	31.82	49.12	325.28	.2840	.594
632	172	570	555.3	160.8	372.7	533.5	.488	2.05	.03149	31.75	49.00	323.70	.2860	.590
633	173	575	560.3	162.0	371.0	533.0	.482	2.07	.03157	31.67	48.88	322.12	.2880	.586
634	174	584	569.3	163.2	369.3	532.5	.476	2.10	.03165	31.60	48.75	320.55	.2898	.583
635	175	590	575.3	164.5	367.6	532.1	.470	2.13	.03171	31.53	48.62	318.98	.2918	.579

TABLE XLIX—*Continued*

PROPERTIES OF SATURATED AMMONIA VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
636	176	598	583.3	165.7	366.0	531.7	.464	2.16	.03179	31.45	48.50	317.50	.2936	.575
637	177	605	590.3	166.9	364.1	531.0	.458	2.18	.03187	31.38	48.36	315.74	.2956	.572
638	178	612	597.3	168.1	362.4	530.5	.454	2.20	.03195	31.30	48.23	314.17	.2976	.568
639	179	620	605.3	169.4	360.6	530.0	.450	2.22	.03203	31.22	48.10	312.50	.2994	.564
640	180	626	611.3	170.6	358.8	529.4	.446	2.24	.03210	31.15	47.96	310.84	.3010	.560
641	181	634	619.3	171.8	356.9	528.7	.440	2.27	.03218	31.07	47.82	309.08	.3032	.557
642	182	640	625.3	173.0	355.0	528.0	.434	2.30	.03226	31.00	47.68	307.32	.3050	.553
643	183	648	633.3	174.3	352.9	527.2	.428	2.33	.03234	30.92	47.54	305.36	.3070	.549
644	184	656	641.3	175.5	351.0	526.5	.420	2.38	.03241	30.85	47.40	303.65	.3089	.545
645	185	664	649.3	176.8	349.0	525.8	.416	2.40	.03250	30.77	47.26	301.74	.3108	.542
646	186	670	655.3	178.0	347.0	525.0	.408	2.45	.03259	30.67	47.12	299.88	.3128	.538
647	187	679	664.3	179.3	345.0	524.3	.402	2.49	.03266	30.62	46.98	298.02	.3148	.536
648	188	686	671.3	180.5	343.0	523.5	.396	2.53	.03274	30.54	46.84	296.16	.3168	.530
649	189	694	679.3	181.8	341.0	522.8	.392	2.55	.03283	30.46	46.70	294.30	.3188	.526
650	190	702	687.3	183.0	339.0	522.0	.386	2.57	.03292	30.38	46.55	292.45	.3206	.522
651	191	710	695.3	184.3	336.9	521.2	.380	2.63	.03300	30.30	46.40	290.50	.3224	.518
652	192	719	704.3	185.5	334.8	520.3	.374	2.67	.03310	30.21	46.26	288.54	.3244	.514
653	193	728	713.3	186.8	332.8	519.6	.370	2.70	.03319	30.13	46.11	286.69	.3264	.510
654	194	736	721.3	188.0	330.7	518.7	.364	2.75	.03329	30.04	45.96	284.74	.3284	.506
655	195	744	729.3	189.2	328.6	517.8	.360	2.78	.03339	29.96	45.82	282.58	.3304	.502
656	196	752	737.3	190.4	326.5	516.9	.354	2.82	.03349	29.87	45.67	280.83	.3324	.498
657	197	762	747.3	191.7	324.4	516.1	.350	2.86	.03360	29.78	45.52	278.88	.3344	.494
658	198	770	755.3	192.9	322.3	515.2	.344	2.90	.03370	29.68	45.37	276.93	.3364	.490
659	199	780	765.3	194.2	320.1	514.3	.340	2.94	.03380	29.59	45.21	274.89	.3384	.486
660	200	788	773.3	195.5	318.0	513.5	.336	2.98	.03390	29.50	45.05	272.95	.3403	.482

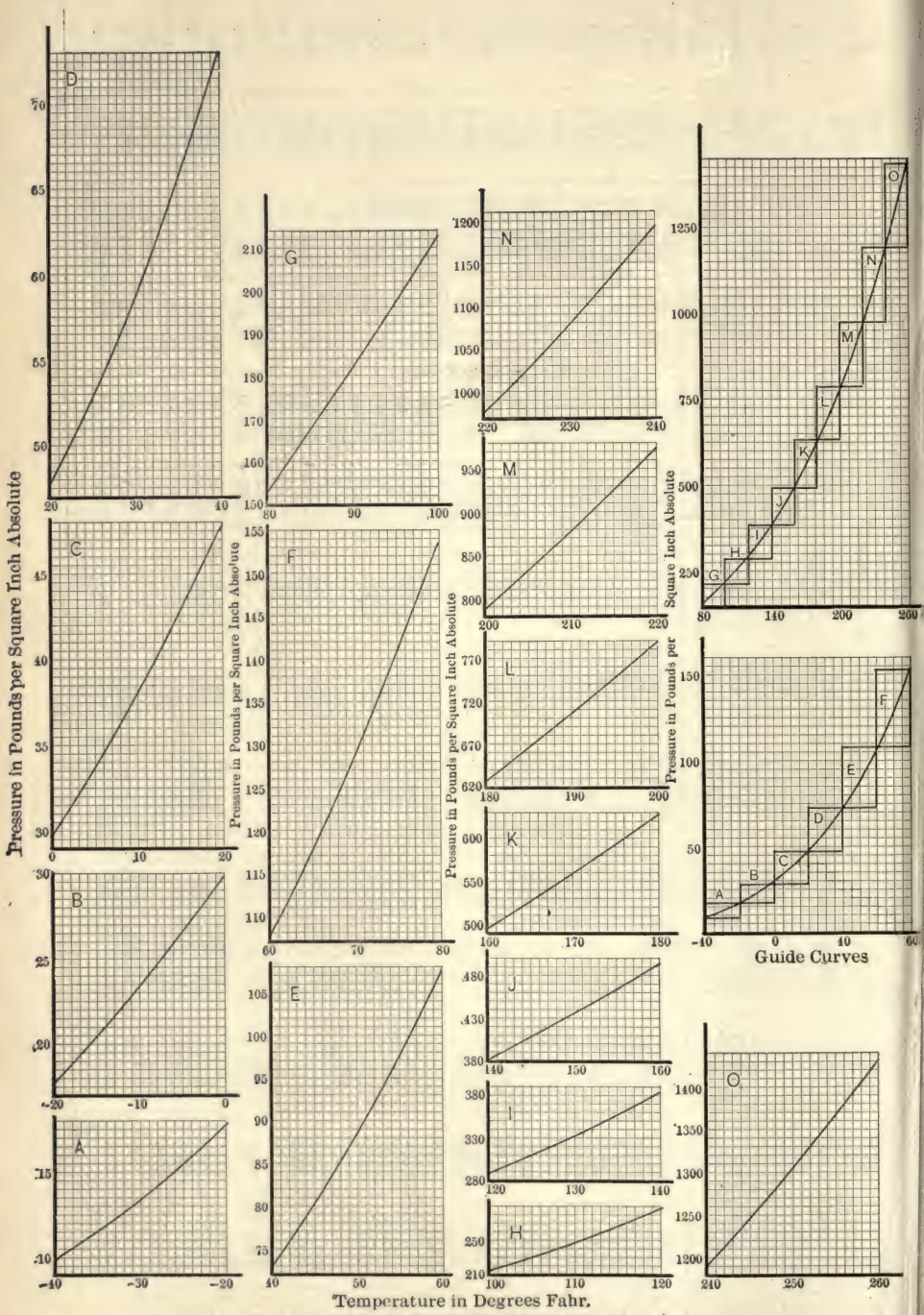


CHART G.—Ammonia, Pressure-temperature (Table XLIX).

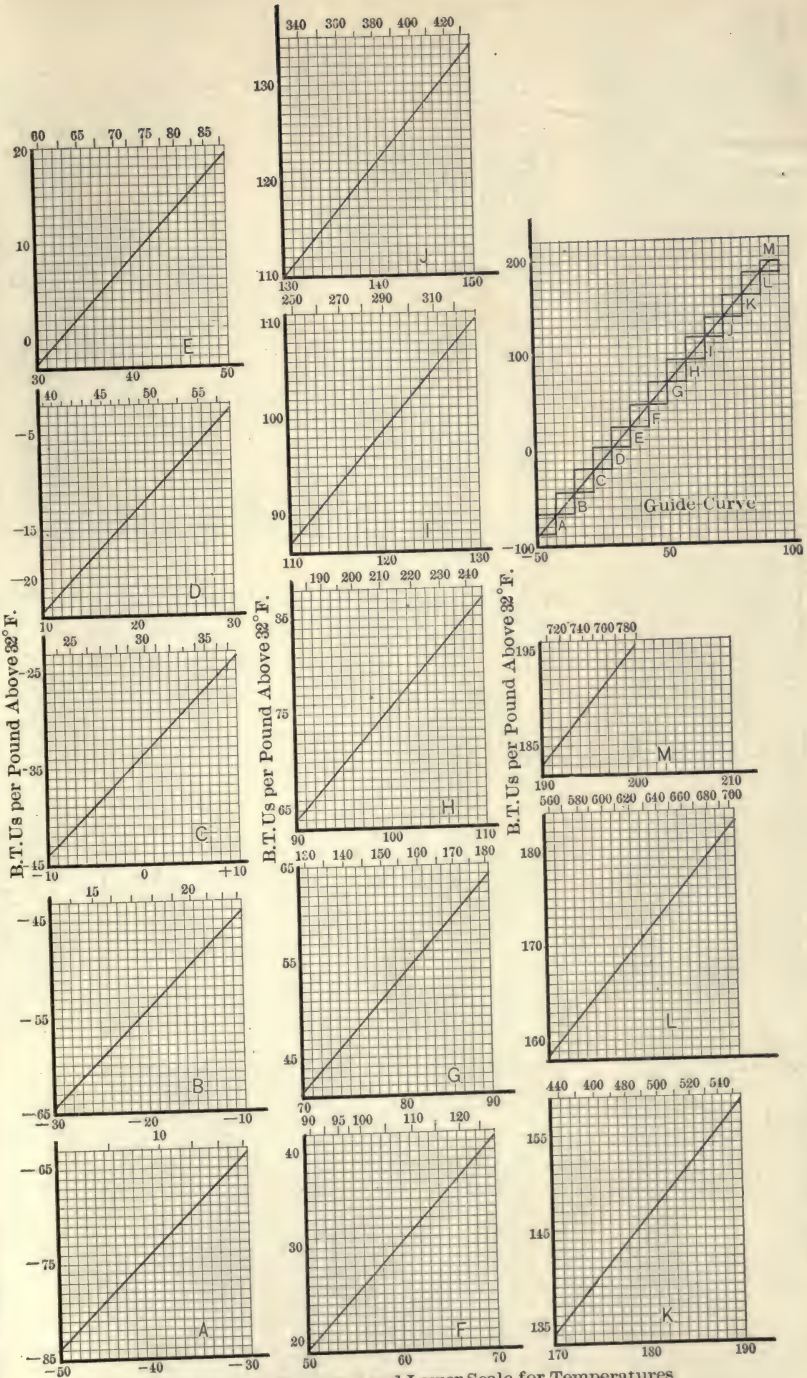


CHART H.—Ammonia, Heat of the Liquid (Table XLIX).

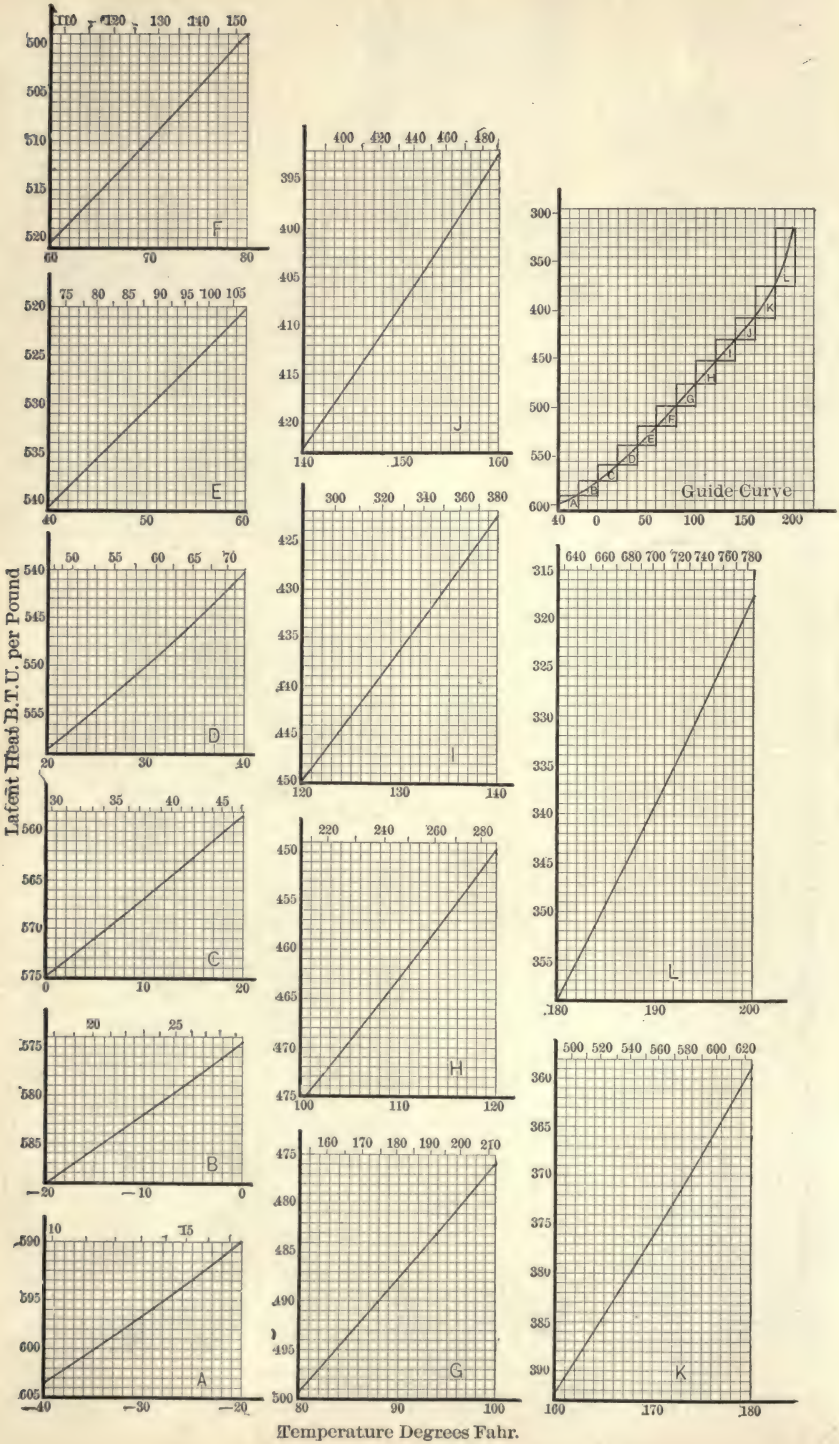


CHART I.—Approximate Latent Heat (Table XI-XV)

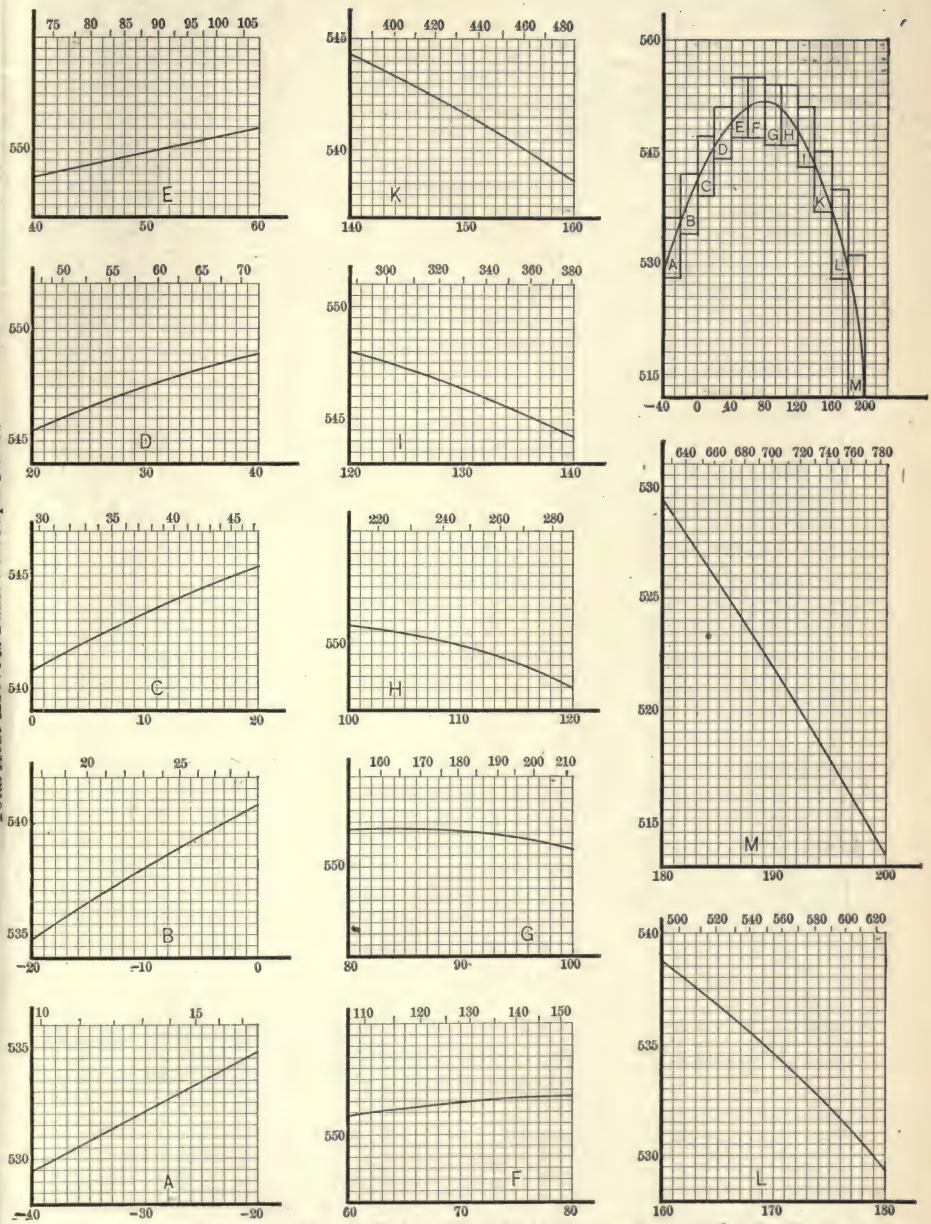
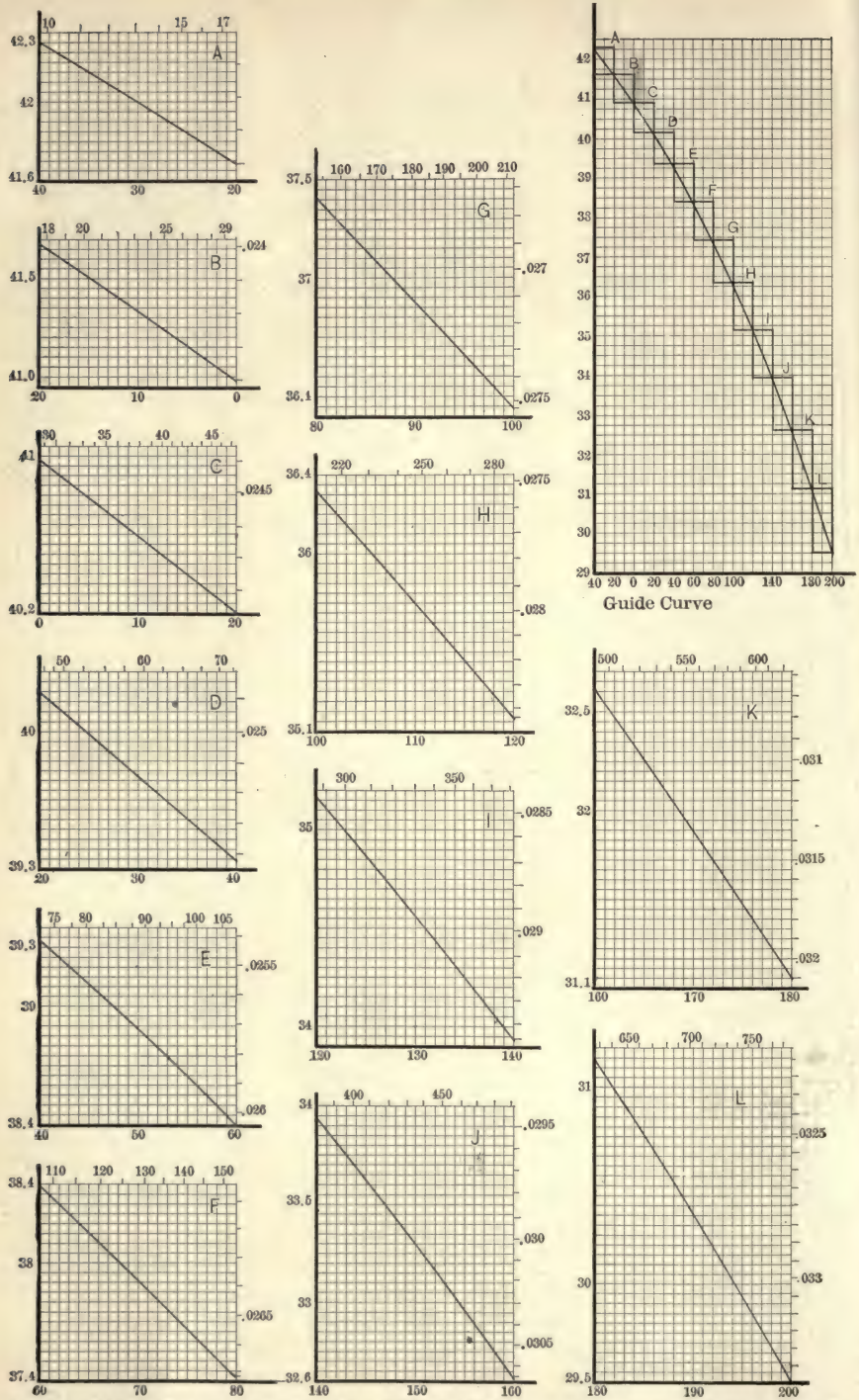
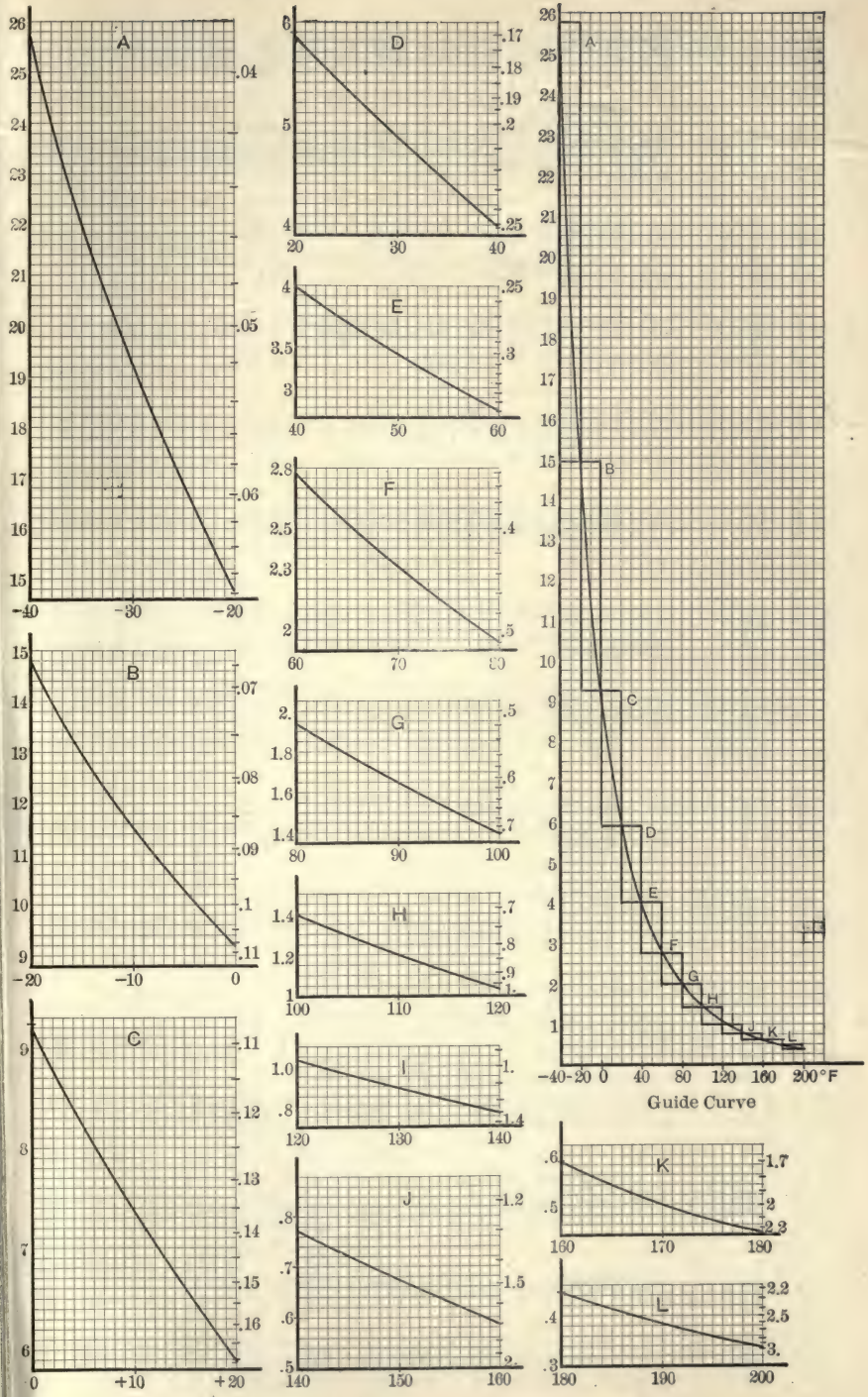


CHART J.—Ammonia, Total Heat (Table XLIX).



Left Hand Scale=lbs. per cu. ft. Density **Right Hand Scale**=cu. ft. per lb. Specific Volumes
Lower " =Degrees F. Temp. Upper " =lbs. per sq. in. Pressures

CHART K.—Ammonia, Specific Volume and Density of the Liquid (Table XLIX)



Curve Showing Effect of Temperature Veriation upon the Specific Volume and Density of Saturated Ammonia Vapor

TABLE L

PROPERTIES OF SATURATED CARBON DIOXIDE VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Liquid Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
440	−20	219.5	204.8	−24	122.6	98.6	.01576	63.45	.3782	2.644	14.80	107.8	.05129	.2786
441	−19	223	208.3	−23.6	122.2	98.6	.01578	63.37	.3708	2.697	14.79	107.41	.05068	.2771
442	−18	227.9	213.2	−23.2	121.8	98.6	.015804	63.27	.3636	2.747	14.78	107.02	.04952	.2755
443	−17	232.6	217.9	−22.8	121.4	98.6	.015830	63.17	.3564	2.808	14.76	106.64	.04882	.2740
444	−16	237.8	221.1	−22.4	121.0	98.6	.015856	63.07	.3494	2.860	14.74	106.26	.04769	.2725
445	−15	242.6	227.9	−22.0	120.6	98.6	.01588	62.97	.3425	2.919	14.73	105.87	.04697	.2710
446	−14	248	233.3	−21.6	120.2	98.6	.015908	62.86	.3360	2.976	14.72	105.48	.04583	.2695
447	−13	253.8	239.1	−21.2	119.8	98.6	.015936	62.75	.3300	3.030	14.70	105.1	.04506	.2680
448	−12	259	244.3	−20.8	119.4	98.6	.015961	62.66	.3240	3.086	14.68	104.72	.04391	.2665
449	−11	264.6	249.9	−20.3	118.95	98.6	.015988	62.54	.3180	3.143	14.66	104.29	.04317	.2649
450	−10	270	255.3	−19.9	118.5	98.6	.016016	62.43	.3121	3.204	14.64	103.86	.04196	.2633
451	− 9	274	259.3	−19.5	118.2	98.6	.016044	62.30	.3062	3.258	14.62	103.58	.04122	.2621
452	− 8	277.7	263.0	−19.0	117.65	98.6	.016072	62.22	.3016	3.311	14.61	103.04	.04002	.2603
453	− 7	281.8	267.1	−18.6	117.20	98.6	.01610	62.11	.2970	3.367	14.60	102.60	.03925	.2587
454	− 6	286	271.3	−18.2	116.85	98.65	.016128	62.00	.2922	3.422	14.59	102.26	.03848	.2573
455	− 5	290.2	275.5	−17.8	116.45	98.65	.016156	61.90	.2875	3.478	14.58	101.88	.03725	.2559
456	− 4	294.7	280	−17.3	116.05	98.7	.016184	61.78	.2830	3.534	14.55	101.5	.03647	.2545
457	− 3	299.2	284.5	−16.9	115.6	98.7	.016212	61.68	.2785	3.590	14.53	101.07	.03518	.2530
458	− 2	303.8	289.1	−16.4	115.20	98.75	.01624	61.58	.2743	3.642	14.51	100.69	.03441	.2514
459	− 1	308.7	294	−16.0	114.75	98.75	.016269	61.46	.2702	3.700	14.50	100.25	.03360	.2500
460	0	313.5	298.8	−15.5	114.3	98.8	.016298	61.35	.2661	3.756	14.49	99.81	.03278	.2484
461	1	318.7	304.0	−15.0	113.85	98.80	.016324	61.25	.2620	3.817	14.48	99.37	.03151	.2470
462	2	323.8	309.1	−14.6	113.4	98.8	.016353	61.15	.2580	3.876	14.47	98.93	.03068	.2454
463	3	329	314.3	−14.1	113.00	98.85	.016382	61.05	.2540	3.937	14.46	98.54	.02986	.2441
464	4	334	319.3	−13.7	112.55	98.85	.016410	60.94	.2500	4.000	14.45	98.1	.02853	.2426
465	5	339.5	324.8	−13.2	112.10	98.85	.016440	60.83	.2460	4.065	14.44	97.66	.0277	.2411

TABLE L—*Continued*

PROPERTIES OF SATURATED CARBON DIOXIDE VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
466	6	345	330.3	−12.8	111.65	98.85	.016472	60.71	.2424	4.125	14.43	97.22	.02683	.2396
467	7	350.6	335.9	−12.3	111.15	98.85	.016504	60.58	.2387	4.190	14.42	96.73	.02597	.2380
468	8	356	341.3	−11.8	110.7	98.9	.016532	60.48	.2351	4.253	14.42	96.28	.02463	.2365
469	9	362	347.3	−11.3	110.20	98.90	.016563	60.37	.2316	4.317	14.42	95.78	.02375	.2350
470	10	368	353.3	−10.9	109.8	98.9	.016594	60.26	.2282	4.382	14.42	95.38	.02288	.2336
471	11	373.8	359.1	−10.4	109.3	98.9	.016628	60.12	.2250	4.44	14.42	94.88	.02197	.2321
472	12	380	365.3	−10.05	108.90	98.85	.016664	60.00	.2217	4.514	14.42	94.48	.02061	.2307
473	13	386	371.3	− 9.5	108.35	98.85	.016700	59.88	.2184	4.581	14.42	93.93	.01970	.2291
474	14	392.5	377.8	− 9.05	107.9	98.85	.016736	59.75	.2152	4.649	14.42	93.48	.0189	.2276
475	15	399	384.3	− 8.6	107.4	98.8	.016772	59.63	.2121	4.717	14.42	92.98	.01786	.2261
476	16	405.5	390.8	− 8.15	106.95	98.8	.016808	59.49	.2090	4.785	14.42	92.53	.01695	.2247
477	17	412	397.3	− 7.65	106.40	98.75	.016844	59.36	.2060	4.854	14.42	91.98	.01553	.2231
478	18	418.7	404.0	− 7.15	105.90	98.75	.016884	59.23	.2029	4.933	14.41	91.49	.01458	.2215
479	19	425.5	410.8	− 6.7	105.4	98.7	.016924	59.08	.1997	5.010	14.39	91.01	.01363	.2200
480	20	432	417.3	− 6.2	104.9	98.7	.016964	58.92	.1965	5.092	14.37	90.53	.01268	.2185
481	21	439	424.3	− 5.65	104.35	98.7	.017004	58.80	.1933	5.173	14.34	90.01	.01171	.2170
482	22	446	431.3	− 5.1	103.75	98.65	.017044	58.66	.1905	5.249	14.32	89.43	.01074	.2152
483	23	453	438.3	− 4.6	103.15	98.55	.017084	58.52	.1876	5.330	14.30	88.85	.009248	.2136
484	24	460	445.3	− 4.1	102.6	98.5	.017128	58.37	.1848	5.411	14.28	88.32	.00826	.2120
485	25	467	452.3	− 3.6	102	98.4	.017172	58.23	.1820	5.495	14.26	87.74	.00726	.2104
486	26	474.5	459.8	− 3.05	101.35	98.3	.017216	58.09	.1793	5.577	14.24	87.11	.006247	.2085
487	27	482	467.3	− 2.55	100.75	98.2	.017260	57.94	.1765	5.665	14.20	86.55	.005226	.2069
488	28	489	474.3	− 2.05	100.15	98.1	.017314	57.75	.1738	5.754	14.16	85.99	.004202	.2052
489	29	496.5	481.8	− 1.55	99.55	98	.017358	57.61	.1711	5.844	14.12	85.43	.003164	.2036
490	30	503	488.3	− 1.0	98.95	97.95	.017404	57.45	.1685	5.935	14.08	84.87	.002119	.2019
491	31	510.5	495.8	− .5	98.35	97.85	.017450	57.30	.1658	6.031	14.04	84.29	.001065	.2003

TABLE L—*Continued*

PROPERTIES OF SATURATED CARBON DIOXIDE VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
492	32	518.2	503.5	.0	97.75	97.75	.017500	57.11	.1633	6.123	13.99	83.76	.0	.1987
493	33	526	511.3	.5	97.15	97.65	.017555	56.96	.1607	6.222	13.94	83.21	.001073	.1971
494	34	533.7	519.0	1.0	96.45	97.45	.017612	56.78	.1583	6.317	13.90	82.55	.002151	.1952
495	35	541	527.3	1.6	95.75	97.35	.017668	56.57	.1559	6.414	13.85	81.90	.003234	.1934
496	36	549	534.3	2.2	95.00	97.20	.017724	56.41	.1534	6.523	13.80	81.20	.004181	.1915
497	37	557	542.3	2.75	94.30	97.05	.017784	56.22	.1510	6.622	13.74	80.56	.005419	.1897
498	38	565	550.3	3.3	93.6	96.9	.017844	56.03	.1486	6.729	13.67	79.93	.006523	.1880
499	39	572.7	558.0	3.85	92.85	96.7	.017908	55.84	.1462	6.840	13.60	79.25	.007644	.1861
500	40	580.2	565.5	4.4	92.1	96.5	.017974	55.63	.1441	6.940	13.54	78.56	.008753	.1842
501	41	588	573.3	5.0	91.3	96.3	.018040	55.43	.1417	7.067	13.46	77.84	.009869	.1822
502	42	596	581.3	5.6	90.5	96.1	.018112	55.21	.1392	7.184	13.36	77.14	.0110	.1803
503	43	603.5	588.8	6.2	89.65	95.85	.018184	54.99	.1370	7.299	13.27	76.38	.01215	.1782
504	44	611.2	596.5	6.8	88.8	95.6	.018256	54.78	.1347	7.424	13.17	75.63	.01329	.1762
505	45	619	604.3	7.4	88.0	95.4	.018334	54.60	.1324	7.553	13.07	74.93	.01444	.1742
506	46	627.5	612.8	8.0	87.1	95.1	.018416	54.27	.1301	7.692	12.97	74.14	.01563	.1721
507	47	635.5	620.8	8.6	86.1	94.7	.018496	54.05	.1278	7.825	12.86	73.24	.01681	.1698
508	48	644	629.3	9.2	85.2	94.4	.018576	53.84	.1255	7.968	12.75	72.45	.01798	.1677
509	49	652.7	638.0	9.8	84.25	94.05	.018666	53.57	.1233	8.110	12.64	71.61	.01919	.1655
510	50	661	646.3	10.4	83.3	93.7	.018760	53.30	.1211	8.258	12.52	70.78	.02037	.1633
511	51	669	654.3	11.0	82.40	93.40	.018850	53.10	.1190	8.403	12.40	70.00	.02160	.1613
512	52	677	662.3	11.6	81.45	93.05	.018940	52.80	.1168	8.562	12.26	69.19	.02283	.1591
513	53	685	670.3	12.25	80.50	92.75	.019038	52.52	.1147	8.718	12.12	68.38	.02406	.1569
514	54	693.5	678.8	12.9	79.5	92.4	.019136	52.26	.1126	8.881	11.98	67.52	.02543	.1547
515	55	701.5	686.8	13.55	78.50	92.05	.019236	51.98	.1105	9.050	11.85	66.65	.02658	.1524
516	56	710	695.3	14.2	77.45	91.65	.019336	51.71	.1085	9.217	11.72	65.73	.02844	.1500
517	57	719.5	704.8	14.85	76.30	91.15	.019448	51.41	.1065	9.390	11.59	64.71	.02973	.1476

TABLE L—*Continued*

PROPERTIES OF SATURATED CARBON DIOXIDE VAPOR

Abs. Temp. ° F.	Scale, Temp. ° F.	Pressure, Pounds per Sq.in. Absolute.	Pressure, Pounds per Sq.in. Gage.	Heat of Liquid Above 32° F.	Latent Heat.	Total Heat Above 32° F.	Sp. Vol. of Liquid, Cu.ft. per Pound.	Density of Liquid, Pounds per Cu.ft.	Sp. Vol. of Vapor, Cu.ft. per Pound.	Density of Vapor, Pounds per Cu.ft.	External Latent Heat.	Internal Latent Heat.	Entropy of Liquid.	Entropy of Vapor.
518	58	729	714.3	15.5	75.15	90.65	.019560	51.12	.1045	9.570	11.46	63.69	.03105	.1451
519	59	738.5	723.8	16.15	74.00	90.15	.019672	50.83	.1026	9.747	11.33	62.67	.03237	.1425
520	60	748	733.3	16.8	72.85	89.65	.019792	50.52	.1006	9.940	11.19	61.66	.03371	.1400
521	61	758	743.3	17.55	71.60	89.15	.019920	50.20	.0986	10.142	11.04	60.56	.03507	.1374
522	62	768	753.3	18.3	70.3	88.6	.020044	49.87	.0966	10.352	10.89	59.41	.03643	.1348
523	63	778	763.3	19.1	69	88.1	.020168	49.67	.0946	10.571	10.74	58.26	.03783	.1319
524	64	788	773.3	19.9	67.65	87.55	.020298	49.28	.0926	10.799	10.56	57.09	.03926	.1291
525	65	798.5	783.8	20.7	66.3	87.00	.020440	48.86	.0906	11.038	10.37	55.93	.04067	.1263
526	66	809.5	794.8	21.5	64.9	86.4	.020592	48.57	.0886	11.287	10.18	54.72	.04218	.1234
527	67	820.5	805.8	22.3	63.5	85.8	.020744	48.18	.0866	11.547	9.98	53.52	.04363	.1205
528	68	830.5	815.8	23.1	62.1	85.2	.020896	47.85	.0846	11.820	9.78	52.32	.04517	.1176
529	69	841	826.3	23.9	60.6	84.5	.021050	47.45	.0826	12.107	9.58	51.02	.04683	.1145
530	70	851.5	836.8	24.75	59.05	83.8	.021240	47.08	.0806	12.407	9.36	49.69	.04837	.1114
531	71	860.5	845.8	25.6	57.45	83.05	.021420	46.68	.0786	12.722	9.12	48.33	.0500	.1082
532	72	871	856.3	26.5	55.8	82.3	.021600	46.30	.0766	13.055	8.87	46.93	.05169	.1048
533	73	882	867.3	27.4	54.2	81.6	.021800	45.87	.0746	13.405	8.62	45.58	.05343	.1017
534	74	892	877.3	28.4	52.5	80.9	.022000	45.45	.0727	13.757	8.37	44.13	.05454	.09831
535	75	902	887.3	29.4	50.8	80.2	.022232	44.98	.0708	14.124	8.11	42.69	.05718	.09495
536	76	912.5	897.8	30.4	49.05	79.45	.022472	44.50	.0690	14.493	7.86	41.19	.05901	.09151
537	77	923	908.3	31.4	47.25	78.65	.022720	44.00	.0672	14.881	7.60	39.65	.06098	.08798
538	78	934	919.3	32.5	45.35	77.85	.023000	43.48	.0653	15.314	7.33	38.12	.06298	.08429
539	79	944.5	929.8	33.6	43.45	77.05	.023280	42.94	.0634	15.773	7.03	36.44	.06506	.08061
540	80	955	940.3	34.7	41.6	76.3	.023600	42.37	.0616	16.234	6.72	34.88	.06714	.07704

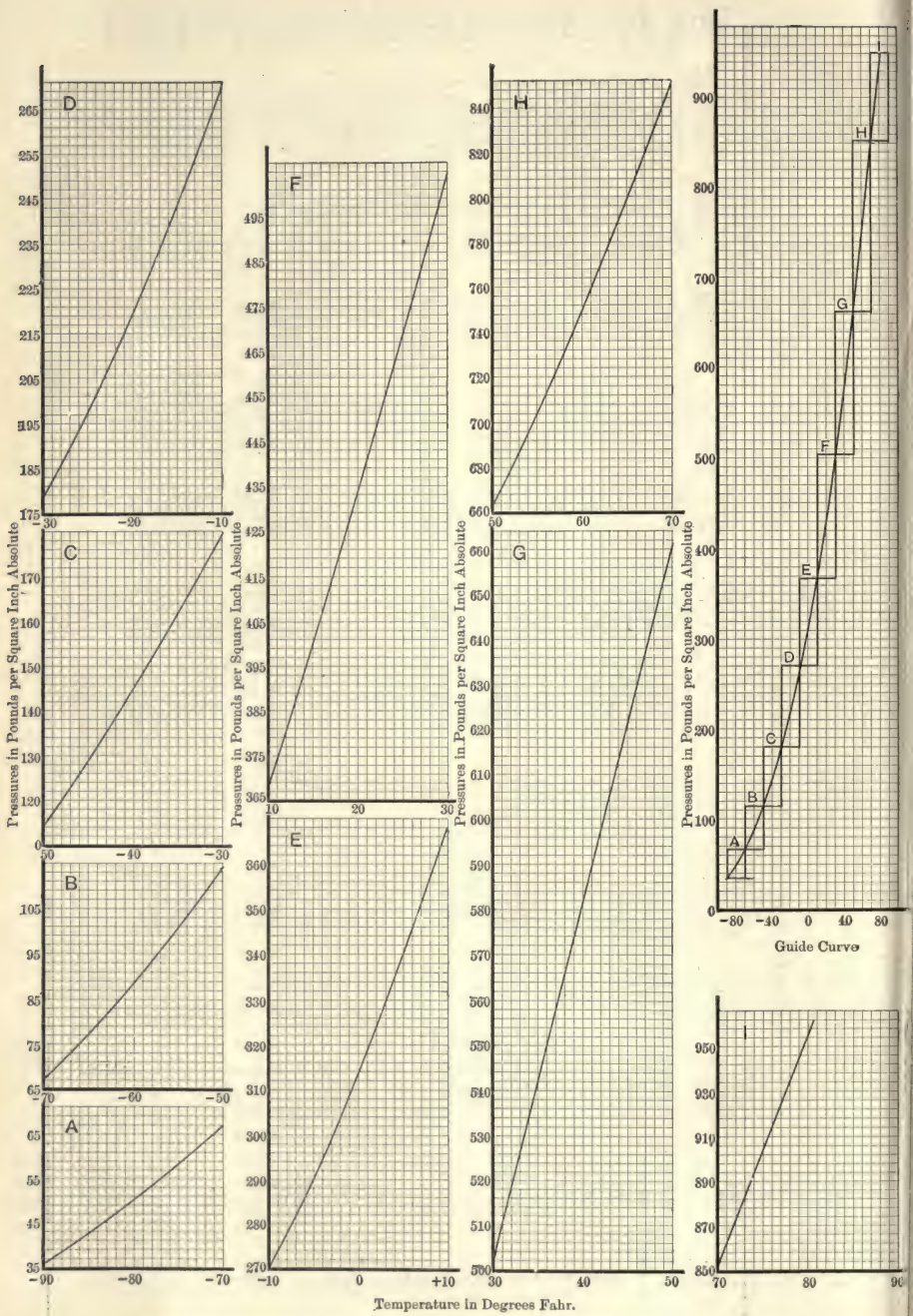


Chart to Show Pressure Temperature Relation of Carbon Dioxide Vapor

CHART M.—Carbon Dioxide, Pressure-temperature (Table L).

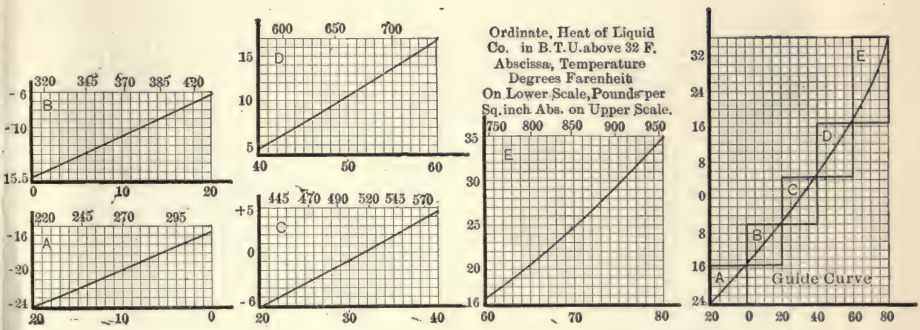
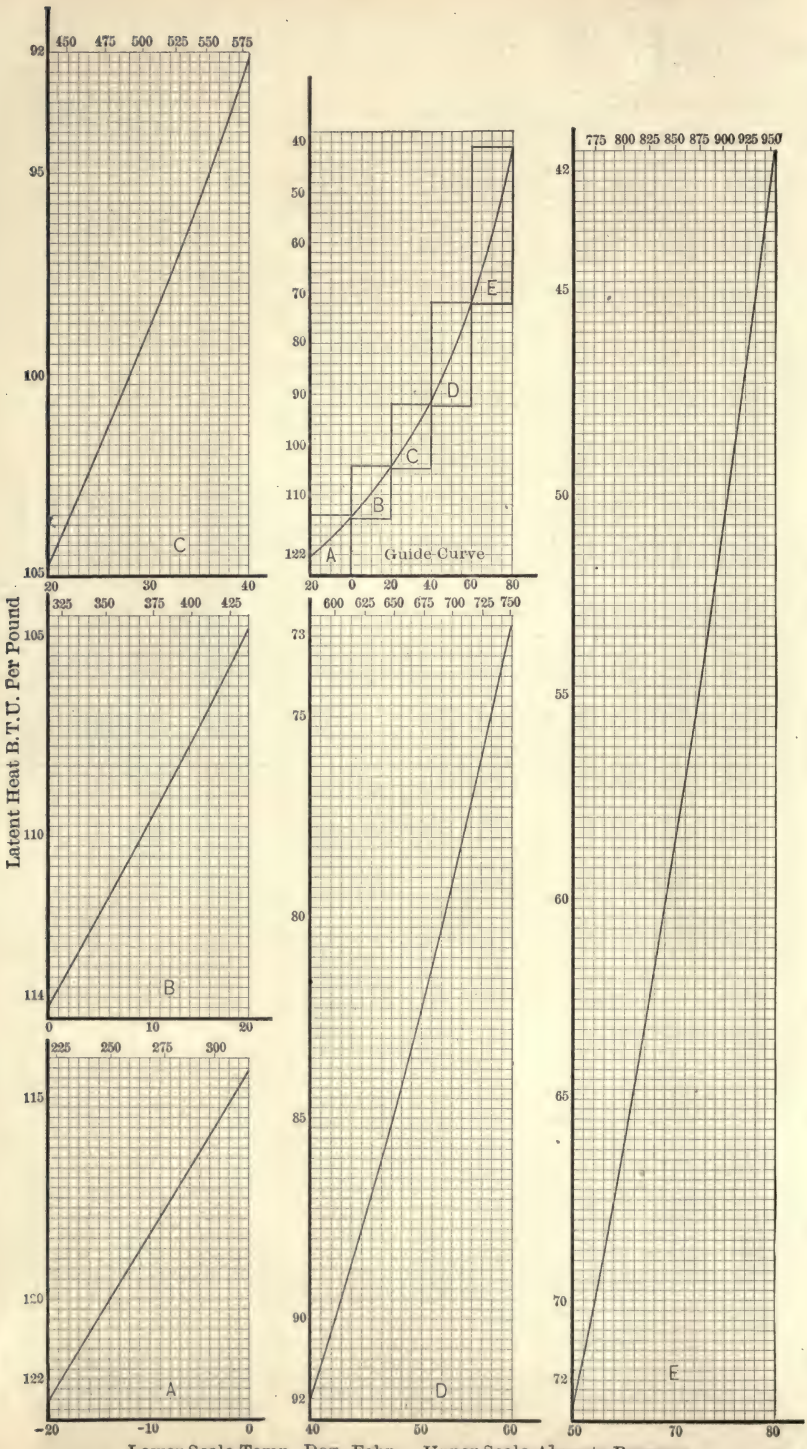


CHART N.—Carbon Dioxide, Heat of the Liquid (Table L).


Latent Heat B.T.U. Per Pound
Guide Curve
A
B
C
D
E
Lower Scale-Temp Deg. Fahr. Upper Scale-Absoute Pressure

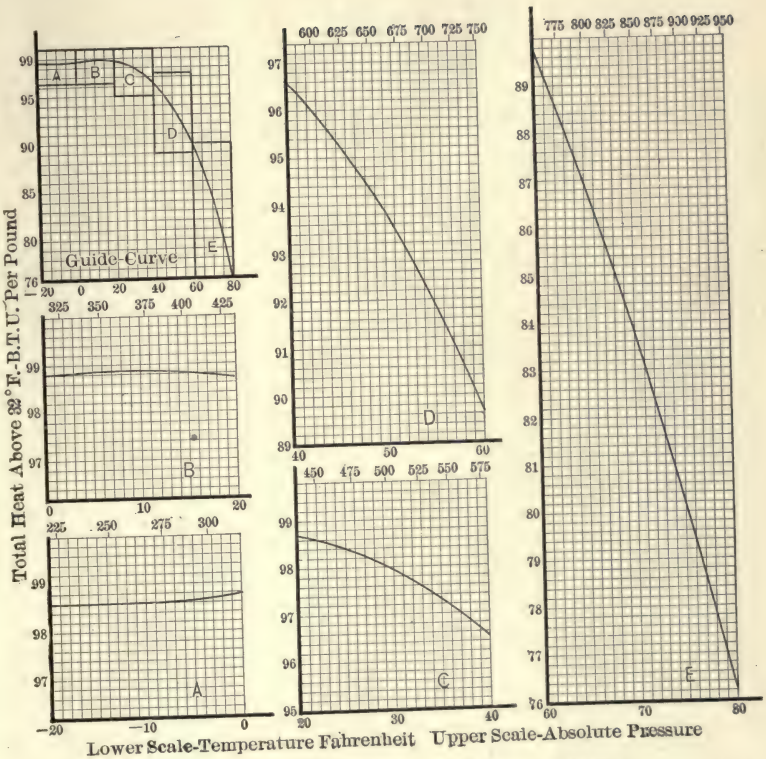


CHART P.—Carbon Dioxide, Total Heat (Table L).

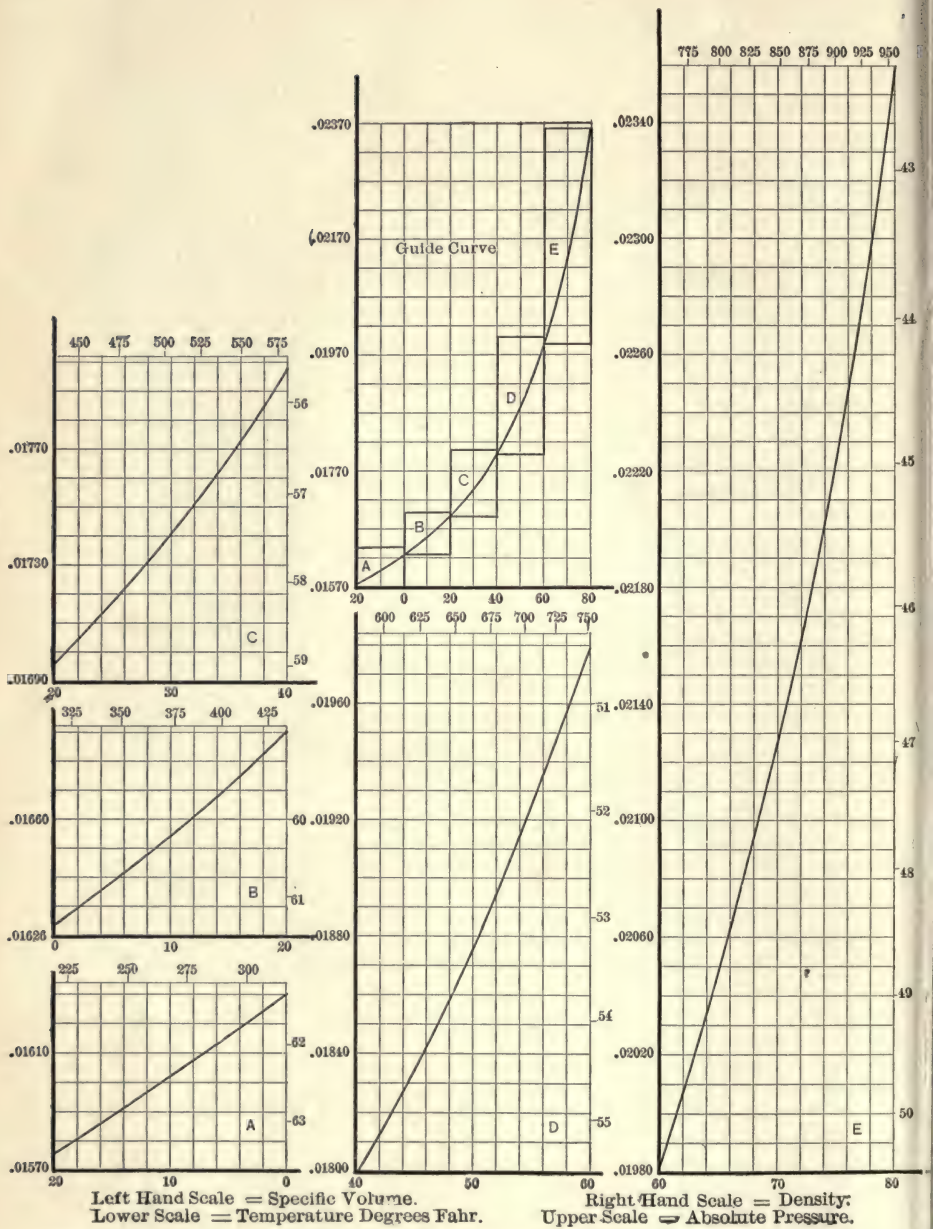


CHART Q.—Carbon Dioxide, Specific Volume and Density of the Liquid (Table L).

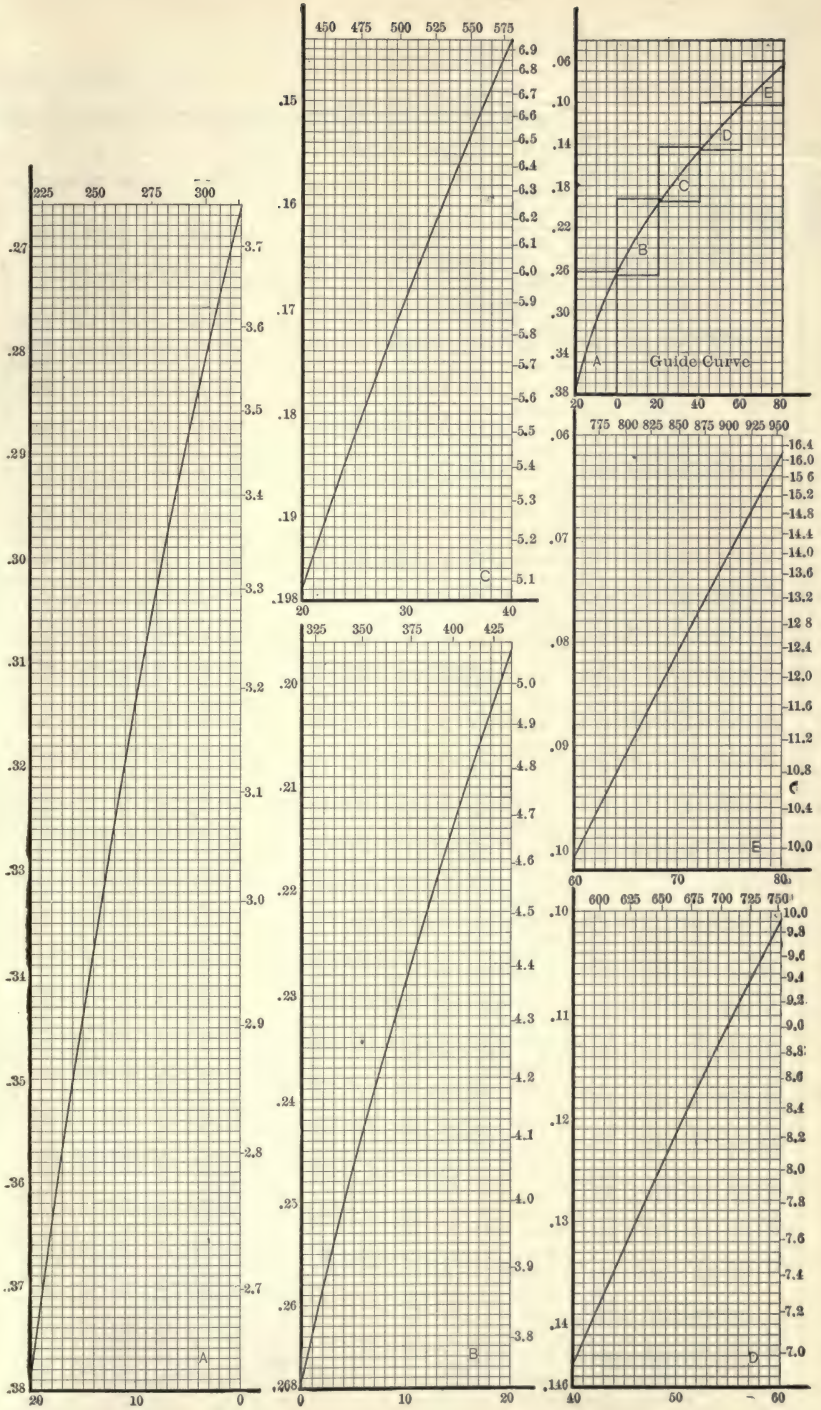


CHART R.—Carbon Dioxide, Specific Volume and Density of the Vapor (Table L).

TABLE

SOLUTIONS OF

RELATION BETWEEN PRESSURE, TEMPERATURE,

Per Cent NH_3 by Weight.	Degrees Baumé.	Specific Gravity.	Pounds per Square Inch Gage											
			0	5	10	15	20	25	30	35	40	45	50	55
			206.3	223.6	234.9	247.4	256.2	263.8	270.4	277.1	282.8	288.1	292.9	297.5
1	...	...	204	219	232	242	251	260	267	274	280	286	291.5	297
			201.4	219.3	231.5	243.3	251.7	259.4	266.4	272.7	278.4	283.7	288.5	293.1
1.84	11	.993	198.5	214	226	236.5	245.5	254	261.5	269.5	274.5	281	286.5	292
			201.1	218.5	230.8	242.1	250.9	258.6	265.5	271.9	277.6	282.8	287.7	292.2
2	...	...	194	212.5	225	235.5	244.5	253	260.5	267.5	273.5	280	285.5	291
			195.8	213.2	225.5	236.6	245.6	253.3	260.2	266.8	272.3	277.5	282.4	286.9
3	...	...	191	206	219	229	238	246.5	254	261.5	267	274.5	280	285
			191.5	208.8	221	232.3	241	248.7	255.7	262	267.7	272.9	277.8	282.4
3.80	12	.986	186.5	200.5	214	224.5	233	241.5	249.5	256	262.5	269.5	274.5	280.5
			190.5	207.7	220	231.2	240	247.6	254.7	260.9	266.7	271.8	276.1	281.4
4	...	...	185	200	213	223	232	240.5	248	255	261	268	273.5	279.5
			185.2	202.4	214.6	225.8	234.6	242.2	249.3	255.6	261.4	266.5	271.4	276.1
5	...	...	180	195	207.5	217.5	226.5	235	242	249	255	262.5	268	273.5
			183.5	200.7	212.8	224.1	232.8	240.5	247.5	253.8	259.6	264.8	270.2	274.1
5.30	13	.979	178	192.5	206	216	225	234	240.5	252.5	254	261	266	272
			180	197.1	209.2	220.5	229.2	237	243.9	250.2	256.1	261.2	266.7	271.2
6	...	...	175	189.5	202	212.5	221	229.5	237	248.5	249.5	257	262.5	268
			175.8	193	205	216.2	224.9	232.6	239.6	246.0	251.8	257	262.1	266.7
6.80	14	.972	171	185.5	198.5	208.5	217	225	232.5	239.5	245.5	252.5	258	263.5
			170	192.1	204	215.3	223.9	231.7	238.6	245.1	250.8	256.1	261.1	265.8
7	...	...	170	184.5	197.5	207.5	216	224	231.5	238.5	244.5	251.5	257	262.5
			168.8	187.2	199.1	210.3	218.9	226.9	233.7	240.1	245.9	251.2	266.2	260.8
8	...	...	165.5	180	193	203	211.5	219.5	227	233.5	239.5	246	252	257.5
			165.4	185.8	197.8	209	217.7	225.4	232.4	238.6	244.2	249.3	254.1	258.7
8.22	15	.966	164.5	179	191.5	202	210.5	218.5	226	232.5	239	245	250.5	256.5
			160.8	182.5	194.5	205	214.3	222	229	235.2	240.8	245.9	250.7	255.3
9	...	...	161	175.5	188.5	198.5	207	215	222.5	229	235	241.5	247	252.5
			156	177.7	189.6	200.6	209.2	216.9	223.9	230.1	235.5	240.6	245.4	250
10	16	.960	156.5	171.5	184.5	194.0	203	211	218	225	230.5	237	242.5	247.5
			156.4	173.2	185.1	196.1	204.7	212.4	219.4	225.6	231	236.1	240.9	244.5
11	...	...	152.5	167.5	179.5	190	198.5	206.5	213.5	220	226	232.5	237.5	242.5
			151.9	168.9	180.6	191.9	199.6	208.3	214.8	221	226.4	231.5	236.4	240.0
12	...	...	149	163	175.5	185.5	194.5	202.5	209.5	216	222	228	233	238
			151	168	179.9	191.0	199.6	207.3	213.6	219.6	225.0	230.3	234.4	239.0
12.17	17	.953	147.5	162	174.5	184.5	192.5	201.5	208.5	215	221	227	232.5	237
			147.5	164.4	176.4	187.4	196.1	203.7	210.1	216.1	221.4	226.8	230.8	235.5
13	...	...	144.5	159	171	181.5	190	198	205	211.5	217.5	223.5	228.5	233.5
			143.7	160.5	172.3	183.4	192	199.7	206	212.1	217.6	222.7	227.2	231.8
13.88	18	.946	141	155	167.5	178	186.5	194.5	201.5	207.5	214	219.5	224.5	230.0
			143.2	160	171.8	182.9	191.5	199.2	205.5	211.6	217.1	222.2	226.7	231.3
14	...	...	140.5	154.5	167	177.5	186	193.5	201	207	213.5	219	224	228.5
			139	155.8	167.6	178.7	187.3	195.0	201.3	207.4	212.9	218.0	222.5	227.1
15	...	...	137	151	163	173.5	182	190	197	203	209.5	215	220.0	225
			134.8	151.6	163.4	174.5	183.1	190.8	197.1	203.2	208.7	213.8	218.3	222.9
16	...	...	132.5	147	159	169.5	178	186	192.5	199	205	211	215.5	220.5
			133.8	150.6	162.3	173.3	181.4	189.5	196	201.8	207.1	212.3	217.1	221.7
16.22	19	.94	131.5	146	157.5	168.5	177	185	192	198	204.5	210	215.0	220.0
			130.6	147.4	159.1	170.1	178.2	186.3	192.8	198.6	203.9	209.1	213.9	218.5
17	...	...	129	143	155	165.5	174	182	188	195	201	207	211.5	216.5
			126.2	142.9	154.6	165.6	174.2	181.9	188.9	195.1	200.7	205.7	209.5	214.1
18.03	20	.935	125	139	151	161.5	170	177.5	184.5	191	197	202.5	207.5	212.5
			122.3	138.9	150.7	161.6	170.3	177.9	185.0	191.1	196.8	201.7	205.6	210.1
19	...	...	121.5	135.5	147.5	157.5	166.5	173.5	180.5	187	193	198.5	203.0	208.5

LI

AMMONIA IN WATER

AND PER CENT NH_3 IN SOLUTION

Above One Standard Atmosphere												Specific Gravity.	Degrees Baumé.	Per Cent NH_3 by Weight.
60	65	70	75	80	85	90	95	100	105	110	115			
301.9	306.3	310.4	314.4	318.2	321.8	325.2	328.5	331.7	334.8	337.8	340.7			
301.5	306	310	315	318.5	322	325.5	329	307.5	335.5	339	341.5	...	...	1
297.5	301.8	306	310	313.8	317.4	320.8	324.1	327.3	330.4	333.4	336.3			
296.5	301	305.5	310	313.5	317.5	321	324.5	330.5	331	334	337	.993	11	1.84
296.7	300.9	305.2	309.2	312.9	316.6	320	323.2	326.5	329.6	332.6	335.4			
295.5	300	304.5	309	312.5	316	320	323.5	327	330	333	336	...	...	2
291.4	295.6	300	303.9	307.6	311.3	314.7	317.9	321.2	324.3	327.3	330.1			
289.5	294.5	299	303	307	311	314.5	317.5	320.5	324	327.5	330	...	...	3
286.8	291.1	295.3	299.3	303.1	306.7	310.1	313.4	316.6	319.7	322.7	325.6			
284.5	290	294	298.5	302	306.5	310	313	316	320	323	325.5	.986	12	3.80
285.7	290.1	294.2	298.3	302.1	305.6	309.1	312.4	315.5	318.7	321.6	324.5			
284	289	293	297.5	301	305.5	309	312	315	318.5	326.5	324.5	...	...	4
280.4	284.8	288.9	293	296.3	300.3	303.8	307.1	310.2	313.4	316.3	319.2			
278.5	283	287.5	292	295.5	299.5	303	306	310	313	316.5	319	...	...	5
279.2	283.5	287.1	291.7	295.5	299.1	302.5	305.8	309	312.1	315.1	318			
276.5	281.5	285.5	290	294	298	301	304.5	307.5	311	315	317.5	.979	13	5.30
275.6	280	284.1	288.2	291.9	295.5	299	302.2	305.5	308.5	311.6	314.4			
273	277.5	281.5	286	290	294	302	300.5	304	307	310.5	313.5	...	...	6
271.1	275.4	279.6	283.6	287.4	291	294.4	297.1	300.9	304	307	309, 9			
269	278.5	277.5	281.5	285.5	289.5	303	296	300.5	303	306.5	309	.972	14	6.80
270.1	274.5	278.6	282.7	286.4	290.1	293.5	296.7	300	303	306.1	308.9			
267.5	277.5	276.5	281	284.5	288.5	302	295	299.5	302	305	308	...	...	7
265.2	269.6	273.7	281.7	281.5	285.2	288.6	291.7	295.1	298.1	301.2	303.9			
262	267	271.5	275.5	279.5	283.5	287	290	293	296.5	300	303	...	...	8
263.1	267.4	271.6	275.6	279.4	283	286.4	289.7	292.4	296	299	301.9			
261	266	270	274.5	278	282.5	286	289	292	295.5	296.5	301.5	.966	15	8.22
259.7	264	268.2	272.2	276	279.6	283	286.3	289.6	292.6	295.6	308.5			
257	262	266.5	270.5	274.5	278	282	285	282	291.5	294.5	297.5	...	...	9
254.4	258.7	262.9	266.9	270.7	274.3	277.7	281	284.2	287.3	230.3	293.2			
252.5	257.5	261.5	265.5	269.5	274	277	280	277	287	290	293	.960	16	10
249.9	254.2	258.4	262.4	266.2	268.8	273.2	276.5	279.7	282.8	285.8	288.7			
247.5	252.5	256.5	260.5	264.5	268.5	272.5	275	272	282	285	288	...	...	11
245.4	249.8	253.9	257.9	261.7	264.3	268.7	272	275.2	278.3	281.3	289.2			
242.5	247.5	251.5	256	259.5	264	267.5	270	267	277	280	283	...	...	12
243.4	247.7	251.9	255.4	259.7	263.3	266.7	270	273.2	276.3	279.3	282.2			
242	246.5	251	255	253.5	263	266.5	269	266.5	276	279	282	.953	17	12.17
239.9	244.2	248.4	251.8	256.2	259.8	263.1	266.5	269.6	272.8	275.7	278.6			
238	243	247	251	255	258	263	266	262.5	272.5	275.5	278.5	...	...	13
236.2	240.5	244	248.7	252.5	256.1	259.8	262.8	266	269.1	272.1	275			
234.5	239	243.5	247	250.5	255	259.0	261.5	258	268.5	271.5	274.5	.946	18	13.88
235.7	240	243.5	248.2	252	255.6	259	262.3	265.5	268.6	271.6	274.5			
234	238.5	242.5	246.5	250	254.5	258.5	261	257.5	268	271	274	...	...	14
231.5	235.8	239.4	244	247.8	251.4	254.8	258.1	261.3	264.4	267.4	270.3			
229.5	234	238.5	242.5	246	250	254	256.5	260	263.5	266.5	270	...	...	15
227.3	231.6	235.1	239.8	243.6	247.2	250.6	253.7	257.1	260.2	263.2	266.1			
225	230	234	237.5	241.5	246	249.5	252	255.5	259	262	265	...	...	16
226.1	230.4	234.6	238.6	242.4	246	249.4	252.7	255.9	259	262	264.9			
224.5	229	233.5	237	241	245	248.5	251.5	254.5	258	261	264	.94	19	16.22
222.9	227.2	231.4	235.4	239.2	242.8	246.2	249.5	252.7	255.8	258.8	261.7			
221	225.5	230	233	237.5	241.5	245	248	251	254.5	257.5	260.5	...	...	17
218.5	222.8	227	231	234.8	238.4	241.8	245.1	248.3	251.4	254.4	257.3			
217	221.5	225.5	229.5	233	237.5	241	243.5	247	250	253	256.5	.935	20	18.03
214.6	218.8	223.1	227	230.9	234.4	237.9	241.1	244.4	247.4	250.5	253.4			
213	217.5	221.5	225	229	233	237	239.5	243	246	249	252	...	...	19

TABLE

SOLUTIONS OF

RELATION BETWEEN PRESSURE, TEMPERATURE,

Per Cent NH_3 by Weight.	Degrees Baumé.	Specific Gravity.	Pounds per Square Inch Gage											
			0	5	10	15	20	25	30	35	40	45	50	55
19.87	21	.928	119.4	135.9	147.6	158.6	167.2	174.4	181.5	187.2	192.5	197.5	202.3	206.9
			118	132	144	154	163	170.5	177	184	189.5	195.5	200.5	205
20	...	...	118.9	135.5	147.1	158.2	166.7	174.4	181.1	186.7	192.1	197	201.9	206.4
			117.5	131.5	143.5	153.5	162.5	170	176.5	183.5	189	195	200	204.5
21	...	...	115.2	131.8	143.4	154.5	163.0	170.7	177.4	183.0	188.4	193.3	198.2	202.7
			114	128	140	150	158.5	166	173	179.5	185	191	195.5	200
21.75	22	.921	112.9	129.4	141	151.9	160.5	168.2	174.6	180.1	185.3	190.3	195.1	199.7
			111.5	125.5	137.5	147	155.5	163.5	170	176.5	182.5	188	193.0	197.5
22	...	...	112	128.5	140.1	151.0	159.6	167.3	173.7	179.2	184.4	189.4	194.2	198.8
			110.5	124	136.5	146	154.5	162.5	169	175.5	181.5	187	191.5	196
23.03	23	.915	108	124.5	136.1	147	155.6	163.3	170.0	175.4	180.2	185.2	190.0	194.6
			107	120.5	132.5	142.5	150.5	158.5	165	171.5	177.5	183	187.5	192.5
24	...	...	114.8	121.3	132.9	143.8	152.4	160.1	166.8	172.2	177.0	182	186.8	191.4
			103.5	117	129	138	147	154.5	161.5	168	174	179	184	188.5
24.99	24	.909	101.5	117.8	129.3	140.1	148.6	156.3	163	168.4	173.6	178.6	183.2	187.8
			99	113.5	125.5	135	143.5	151	158	164.5	170	175.5	180	185
26	...	...	98.3	114.6	126.2	136.9	145.5	153.1	159.8	165.3	170.4	175.5	179.9	184.7
			95.5	110.0	122.0	131.5	140	147	154	160.5	166.5	171.5	176.5	181
27	...	...	95.1	111.4	123.1	133.7	142.3	150.0	156.6	162.1	167.2	172.4	176.7	181.5
			92.5	106.5	118.5	128	136.5	143.5	150.5	157	162.5	168	172	177.5
27.66	25	.904	93.0	109.4	121.0	131.7	140.1	147.9	154.5	159.9	165.1	170.3	174.4	178.9
			90.0	104.0	116.5	126	134	141.5	148.5	154.5	160.5	165.5	171	175
28	...	...	92.0	108.3	120.0	130.6	139.1	146.8	153.4	158.9	164.0	169.3	173.3	177.9
			89.0	103	115	124.5	132.5	140	147	153.5	159	163	169.5	173.5
29	...	...	88.9	105.2	117.0	127.5	136	143.8	150.3	155.8	161	166.2	170.2	174.8
			86.0	99.5	111.5	121	129	136.5	143	149.5	155	160.5	165	170
29.60	26	.898	87	103.3	114.7	125.4	133.9	141.6	148.2	153.8	159	164.3	168.1	172.7
			83.5	97.5	109.5	119	127	134.5	141	147	152.5	158	163.5	167.5
30	...	...	85.8	102.1	113.5	124.2	132.7	140.4	147	152.6	157.8	163.1	166.9	171.6
			82.5	96.5	108	117.5	125.5	133	139.5	146	152	157	162	166
31.05	27	.891	82.6	98.8	110.2	120.9	129.4	137.1	143.5	149.2	154.5	159.8	163.6	168.3
			79.0	93.0	104.5	114	122	129.5	136	142	148	153	158.5	162.5
32	...	...	80.1	96.2	107.6	118.3	126.8	134.5	140.9	146.6	151.9	157.2	161.0	165.7
			76.0	89.5	101	110.5	118.5	126	132.5	138.5	144.5	149.5	154.5	159
33	...	...	77.4	93.5	104.9	115.6	124.1	131.8	138.7	143.9	149.2	154.5	158.3	163.0
			73.0	86.5	98	107	115.0	122.0	129	135	140.5	146	151.5	155.5
33.25	28	.886	76.5	92.6	103.9	114.6	123.1	130.8	137.8	143	148.3	153.6	157.4	162.1
			72.0	85.5	97	106.5	114.5	121.5	128	134	140	145	150.0	154.5
34	...	...	74.6	90.7	102	112.7	121.2	128.9	135.9	141.1	146.4	151.7	155.5	160.2
			69.5	83.0	94.5	104.0	111.5	119	125.5	131.5	137.5	142.5	147.5	152
35	...	...	72	88.1	99.4	110.1	118.6	126.3	133.3	138.5	143.8	149.1	152.9	157.6
			67.5	80.0	91.5	100.5	108.5	115.5	122	128	134.0	139	144	148.5
35.60	29	.881	70.4	86.5	97.8	108.5	117	124.7	131.7	137.9	142.2	147.5	151.3	156.0
			64.5	78.0	89	98.5	106	113.5	120	126	132	136.5	142	146
36	...	...	60.5	85.6	96.9	107.5	116.1	123.8	130.8	137.0	141.7	147.2	151.0	155.7
			63.5	77	88	97	105	112.5	118.5	124.5	130	135	140	145
37	...	...	67.2	83.3	94.6	105.2	113.8	121.5	128.5	134.7	140.7	146.8	150.2	154.9
			60.5	73.3	85.0	94	101.5	108.5	115.0	121.5	127	132	137	141
38	...	...	65.0	81.0	92.3	104.9	111.5	119.2	126.2	132.5	138.4	143.9	149.4	154.0
			57.5	70.5	81.5	90.5	98.5	105.5	112	117.5	123.5	138.5	133.5	137.5
38.20	30	.875	64.5	80.5	91.8	102.5	111.0	118.7	125.7	132	138.1	143.6	149.3	153.9
			56.5	70.0	81.0	90	97.5	105	111.5	117.0	123.0	127.5	133	137.0

LI—*Continued*

AMMONIA IN WATER

AND PER CENT NH_3 IN SOLUTION

Above One Standard Atmosphere												Specific Gravity.	Degrees Baumé.	Per Cent NH_3 by Weight.
60	65	70	75	80	85	90	95	100	105	110	115			
211.3	215.6	219.8	223.8	227.6	231.2	234.6	237.9	241.1	244.2	247.2	250.1			
209.5	214	218	221.5	225	229.5	233	236	239	242	245.5	248	.928	21	19.87
210.8	215.2	219.3	223.4	227.1	230.7	234.1	237.4	240.7	243.8	246.7	249.6			
209	213.5	217.5	221	224.5	229	232.5	235.5	238.5	241.5	245	247.5	...	...	20
207.1	211.5	215.6	219.7	223.3	227	230.4	233.7	237	240.1	243	245.9			
205	209.5	213.5	217.5	221	224.5	227.5	231	234.5	237.5	240.5	243.5	...	...	21
204.1	208.4	212.6	216.6	220.4	224	227.4	230.7	233.9	237	240	242.9			
202	206.5	210.5	214	218	221.5	225.5	228.5	232	234.5	237.5	240.5	.921	22	21.75
203.2	207.5	211.7	215.7	219.5	223.1	226.5	229.8	233	236.1	239.1	242			
201	205.5	209.5	213	215	220.5	224.5	227	230.5	233	236.5	239.5	...	...	22
199	203.3	207.5	211.5	215.3	218.9	222.3	225.6	228.8	231.9	234.9	237.8			
196.5	201.5	205	209	211	216.5	220	223	226.5	229	232.5	235	.915	23	23.03
195.8	200.1	204.2	208.3	212.1	215.7	219.1	222.4	225.6	228.7	231.7	234.6			
193	197.5	201.5	205	207	212.5	216	219	222.5	225	228.5	231	...	...	24
192.2	196.5	200.7	204.7	208.5	212.1	215.5	218.8	222	225.1	228.1	231			
188.5	193	197.5	201.5	205	208.5	212	215.0	218.5	221.5	224.5	227	.909	24	24.99
189.1	193.3	197.5	201.6	205.3	208.9	212.2	215.6	218.9	221.9	225	237.8			
185.5	190	194	197.5	201.5	205	208	211.5	214.5	271.5	220.5	223.5	...	...	26
185.9	190.2	194.3	198.4	202.2	205.7	209	212.5	215.8	218.7	221.8	234.7			
181.5	186	190	194	197.5	201	204.5	207.5	210.5	213.5	216.5	219.5	...	...	27
183.3	187.6	191.8	195.8	199.6	203.2	206.6	209.9	213.1	216.2	219.2	222.1			
179	183.5	187.5	191.5	195	198.5	202	205.5	208.5	211	214.5	217	.904	25	27.66
183.2	186.6	190.7	194.8	198.5	202.2	205.6	208.8	212.1	215.1	218.2	221.0			
177.5	182	186.5	190	193.5	197.5	200.5	204	207	210	212.5	215.5	...	...	28
180.2	183.5	187.6	191.8	195.4	199.1	202.6	205.7	209.0	212.1	215.1	217.9			
174	178	182.5	186	190	193.5	196.5	200	203	206	209	211.5	...	...	29
178.1	181.4	185.6	189.6	193.4	197.0	200.4	203.7	206.9	210	213.0	215.9	.898	26	29.60
171.5	176	180	184	187.5	191	194.5	198	201	203.5	207	209.5			
176.9	180.2	184.4	188.4	192.2	195.8	199.2	202.5	205.7	208.8	211.8	214.7	...	...	30
170	174.5	179	182.5	186	189.5	192.5	196.5	199.5	202.0	205	208			
173.5	177.0	181.2	185.2	189.0	192.6	196	199.3	202.5	206.6	209.6	212.5	.891	27	31.05
166.5	171	174.5	178.5	182.5	185.5	189	192.5	195.0	198.0	201	204.5			
170.9	174.4	178.6	182.6	186.4	190	193.4	196.7	199.9	204	207	209.9	...	...	32
163	167	167.5	175	178.5	182	185.5	188.5	192	194.5	197.5	200.5			
168.2	171.7	175.9	179.9	183.7	187.3	190.7	194.0	197.2	201.3	204.3	207.2	...	...	33
159.5	163.5	163.5	171.5	175	178.5	181.5	185	188	191.0	194	196.5			
157.3	170.8	175	179	182.8	186.4	189.8	193.1	196.3	200.4	203.4	206.3	.886	28	33.25
169.0	163	162.5	170.5	174.5	177.5	180.5	184	187.5	190	193	196.0			
165.4	168.9	173.1	177.1	180.9	184.5	187.9	191.2	195.4	198.5	201.5	204.4	...	...	34
156	160	160	168	171.5	175.5	178	181.5	184.5	187.5	190	193.0			
162.8	166.3	170.5	174.5	178.3	181.9	185.3	188.6	192.8	195.9	198.9	201.8	...	...	35
152.5	156.5	156.5	164	168	171.0	174	177.5	180.5	183.5	187	189.5			
161.2	164.7	168.9	172.9	176.7	180.3	183.7	187.0	191.2	194.3	197.3	200.2	.881	29	35.60
150.5	154.5	154.5	163	165.5	169	172	175.5	178.5	181.0	184.5	187			
160.8	164.5	168.7	172.7	176.5	180.1	183.5	186.8	191	193.9	196.9	199.8	...	...	36
149.0	153	153.0	160.5	160.5	167.5	170.5	174	177.0	179.5	182.5	185.5			
159.7	163.7	167.9	171.9	175.8	179.3	182.7	186.0	190.2	192.8	195.8	198.7	...	...	37
145.5	149.5	149.5	157	153	164.0	167	170.5	173	176.0	179.5	182.0			
158.6	162.9	167.1	171.1	175	178.5	181.9	185.2	189.4	191.7	194.7	197.6	...	...	38
142	146	146	153.5	150	160.5	163.5	166.5	170	172.5	175.5	178.5			
158.3	162.6	167	171.0	174.8	178.4	181.8	185.1	188.3	191.4	194.4	197.3	.875	30	38.20
141.5	145.5	145.5	153	149.5	160	163	166	169.5	172	175	178			

TABLE LII

AMMONIA—WATER SOLUTIONS

VALUES OF PARTIAL PRESSURES OF AMMONIA AND WATER VAPOR FOR VARIOUS TEMPERATURES AND PER CENTS OF AMMONIA IN SOLUTION

Per cent NH_3	2.5				5.0				7.5			
Temperature ° F.	Partial Pressure of Ammonia Vapor.	Partial Pressure of Water Vapor.	Total Pressure Sum of Partials.	Total Pressure from New Standards.	Partial Pressure of Ammonia Vapor.	Partial Pressure of Water Vapor.	Total Pressure Sum of Partials.	Total Pressure from New Standards.	Partial Pressure of Ammonia Vapor.	Partial Pressure of Water Vapor.	Total Pressure Sum of Partials.	Total Pressure from New Standards.
	Press. Inches Hg.				Press. Inches Hg.				Press. Inches Hg.			
32.	.236	.177	.413	..	.512	.158	.670	....	.788	.158	.946	..
35.6	.256	.197	.453	..	.571	.197	.768	....	.867	.197	1.064	..
39.2	.276	.236	.512	..	.591	.236	.827	....	.945	.216	1.161	..
42.8	.295	.276	.571	..	.650	.276	.926	....	1.041	.256	1.297	..
46.4	.315	.315	.630	..	.709	.315	1.024	....	1.16	.295	1.455	...
50.0	.354	.355	.709	..	.788	.355	1.343	....	1.28	.335	1.615	1.6
53.6	.394	.413	.807	..	.866	.394	1.260	....	1.415	.374	1.789	1.8
57.2	.434	.472	.906	..	.965	.452	1.417	....	1.575	.433	2.008	2.
60.8	.492	.532	1.024	..	1.062	.511	1.573	1.6	1.75	.473	2.223	2.1
64.4	.552	.590	1.142	..	1.18	.590	1.770	1.9	1.925	.552	2.477	2.5
68.	.611	.670	1.281	1.3	1.319	.649	1.958	2.	2.125	.611	2.736	2.9
71.6	.670	.748	1.318	1.5	1.455	.728	2.183	2.2	2.34	.689	3.029	3.1
75.2	.729	.847	1.576	1.6	1.592	.826	2.418	2.6	2.58	.788	3.368	3.6
78.8	.807	.945	1.752	1.8	1.75	.925	2.675	2.8	2.835	.866	3.701	3.9
82.4	.885	1.06	1.945	2.	1.925	1.043	2.968	3.	3.09	.985	4.075	4.1
86	.985	1.2	2.185	2.1	2.125	1.180	3.305	3.5	3.49	1.122	4.612	4.8
89.6	1.085	1.36	2.445	2.5	2.30	1.34	3.64	3.8	3.70	1.28	4.98	5.2
93.2	1.18	1.515	2.695	2.8	2.52	1.495	4.015	4.1	4.06	1.435	5.495	5.8
96.8	1.28	1.69	2.97	3	2.725	1.672	4.397	4.5	4.42	1.615	6.035	6
100.4	1.38	1.89	3.27	3.4	3.01	1.870	4.880	5	4.82	1.81	6.63	6.7
104.0	1.455	2.125	3.580	3.8	3.29	2.085	5.375	5.2	5.27	2.03	7.30	7.3
107.6	1.655	2.36	4.015	4	3.58	2.30	5.88	6	5.72	2.245	7.965	8
111.2	1.811	2.62	4.431	4.6	3.90	2.56	6.46	6.5	6.18	2.50	8.68	8.8
114.8	1.970	2.95	4.920	5	4.23	2.815	7.045	7	6.78	2.76	9.54	9.4
118.4	2.15	3.21	5.36	5.2	4.58	3.11	7.69	7.8	7.33	3.05	10.38	10.2
122.0	2.320	3.54	5.860	5.9	4.96	3.44	8.40	8.5	7.89	3.37	11.26	11.3
125.6	2.520	3.88	6.400	6.4	5.35	3.80	9.15	9	8.55	3.70	12.25	12.
129.2	2.740	4.29	7.030	7	5.80	4.22	10.02	10.	9.25	4.07	13.32	13.2
132.8	2.955	4.73	7.685	7.8	6.25	4.65	10.90	11	9.89	4.5	14.39	14.4
136.4	3.15	5.21	8.36	8.2	6.72	5.12	11.84	12	10.06	4.98	15.04	15.8
140	3.37	5.77	9.14	9	7.2	5.63	12.83	12.9	11.45	5.49	16.94	16.9

	10				12.5				15			
32	1.21	.158	1.368	1	1.58	.138	1.718	1.5	2.11	.138	2.248	2
35.6	1.24	.177	1.417	1.5	1.72	.157	1.877	1.8	2.3	.157	2.457	2.5
39.2	1.36	.197	1.557	1.5	1.89	.177	2.067	2.1	2.54	.177	2.717	2.8
42.8	1.495	.236	1.731	1.7	2.09	.217	2.307	2.5	2.79	.217	3.007	3
46.4	1.67	.276	1.946	1.9	2.31	.256	2.566	2.8	3.07	.256	3.326	3.2
50	1.87	.315	2.185	2	2.56	.295	2.855	3	3.41	.295	3.705	3.8
53.6	2.05	.355	2.405	2.4	2.82	.335	3.155	3.3	3.76	.335	4.095	4.1
57.2	2.28	.413	2.693	2.9	3.12	.394	3.514	3.7	4.14	.374	4.514	4.7
60.8	2.52	.472	2.992	3	3.45	.453	3.903	4	4.55	.433	4.983	5
64.4	2.79	.532	3.322	3.4	3.82	.512	4.332	4.5	5.02	.492	5.512	5.5

TABLE LII—*Continued*

Per cent NH_3	10				12.5				15			
Temperature ° F.	Partial Pressure of Ammonia Vapor.	Partial Pressure of Water Vapor.	Total Pressure Sum of Partials.	Total Pressure from New Standards.	Partial Pressure of Ammonia Vapor.	Partial Pressure of Water Vapor.	Total Pressure Sum of Partials.	Total Pressure from New Standards.	Partial Pressure of Ammonia Vapor.	Partial Pressure of Water Vapor.	Total Pressure Sum of Partials.	Total Pressure from New Standards.
	Press. Inches Hg.				Press. Inches Hg.				Press. Inches Hg.			
68	3.09	.590	3.680	3.8	4.22	.571	4.791	5	5.55	.552	6.102	6
71.6	3.4	.670	4.070	4	4.61	.65	5.26	5.4	6.1	.631	6.731	7.7
75.2	3.74	.767	4.507	4.6	5.04	.729	5.769	6	6.7	.71	7.41	7.6
78.8	4.09	.847	4.937	5	5.55	.827	6.377	6.6	7.33	.81	8.14	8
82.4	4.49	.965	5.455	5.4	6.08	.926	7.006	7	7.98	.906	8.886	8.9
86	4.9	1.1	6.0	6.1	6.66	1.04	7.70	7.8	8.66	1.005	9.665	9.9
89.6	5.35	1.24	6.59	6.8	7.26	1.18	8.44	8.5	9.5	1.12	10.62	10.7
93.2	5.86	1.4	7.26	7.4	7.92	1.32	9.24	9.3	10.35	1.26	11.61	11.9
96.8	6.37	1.555	7.925	7.9	8.63	1.47	10.10	10	11.28	1.42	12.70	12.8
00.4	6.94	1.75	8.69	8.8	9.38	1.67	11.05	11	12.25	1.59	13.84	13.9
04.0	7.5	1.95	9.45	9.5	10.18	1.87	12.05	12	13.22	1.77	14.99	15
07.6	8.19	2.165	10.355	10.4	11.02	2.07	13.09	13	14.30	1.98	16.28	16.3
11.2	8.88	2.42	11.30	11.4	11.9	2.32	14.22	14.4	15.45	2.2	17.65	17.8
14.8	9.6	2.68	12.28	12.2	12.88	2.56	15.44	15.7	16.62	2.44	19.06	19
18.4	10.38	2.97	13.35	13.3	13.85	2.83	16.68	17	17.9	2.69	20.59	20.6
22.0	11.22	3.25	14.47	14.5	14.95	3.13	18.08	18	19.3	2.97	22.27	22.2
25.6	12.05	3.58	15.63	15.5								
29.2	12.95	3.96	16.91	17								
32.8	13.95	4.37	18.32	18.2								
36.4	15.0	4.81	19.81	20								
40	16.5	5.29	21.79	21.2								

	17.5				20				22.5			
32	2.72	.138	2.858	2.8	3.46	.118	3.578	3.5	4.37	.118	4.488	4.6
35.6	3.0	.157	3.157	3.1	3.84	.138	3.978	4	4.85	.138	4.988	5
39.2	3.29	.177	3.467	3.5	4.22	.158	4.378	4.3	5.33	.158	5.488	5.9
42.8	3.62	.217	3.837	3.9	4.65	.177	4.827	4.9	5.86	.177	6.037	7
46.4	4.02	.256	4.276	4.2	5.12	.217	5.337	5.1	6.43	.197	6.627	6.7
50	4.41	.295	4.705	4.8	5.63	.256	5.886	5.9	7.07	.236	7.306	7.3
53.6	4.87	.335	5.205	5.2	6.2	.295	6.495	6.4	7.74	.275	8.015	8
57.2	5.36	.374	5.734	5.9	6.8	.335	7.135	7.1	8.48	.315	8.795	9
60.8	5.92	.433	6.353	6.5	7.49	.394	7.884	7.8	9.3	.354	9.654	9.7
64.4	6.5	.492	6.992	7	8.2	.453	8.653	8.6	10.18	.394	10.574	10.8
68	7.13	.552	7.682	7.8	9.0	.512	9.512	9.5	11.12	.453	11.573	12
71.6	7.8	.631	8.431	8.5	9.85	.571	10.421	10.3	12.15	.512	12.662	12.9
75.2	8.55	.71	9.26	9.3	10.75	.65	11.40	11.5	13.25	.571	13.821	14
78.8	9.33	.788	17.118	10.3	11.75	.73	12.48	12.4	14.45	.65	15.10	15.2
82.4	10.2	.866	11.066	11.4	12.75	.85	13.60	13.6	15.85	.729	16.579	17
86	11.1	.966	12.066	12	13.9	.905	14.805	15	17.40	.807	18.207	18
89.6	12.1	1.08	13.18	13.3	15.05	1.14	16.19	16.1				
93.2	13.2	1.22	14.24	14.5	16.30	1.26	17.56	17.9				
96.8	14.35	1.36	15.71	15.8	17.75	1.4	19.15	18.9				
100.4	15.6	1.5	17.1	17	19.35	1.55	20.90	20.6				
104.0	16.95	1.67	18.62	18.6	21.05	1.71	22.76	22.3				
107.6	18.45	1.85	20.30	19.9								

Table LIII

ABSORPTION OF GASES BY LIQUIDS

Selected from Smithsonian Physical Tables.

Values of x_t = volume of gases referred to 32° F. and 29.92 ins. Hg which one volume of water can absorb at atmospheric pressure and temperature of first column.

Temperature.		CO_2.	CO.	H.	N.	O.	Air.	NH_3.	H_2S.	Methane.	Ethylene.
° C.	° F.										
0	32	1.797	.0354	.02110	.02399	.04925	.02471	1174.6	4.371	.04573	.2563
5	41	1.450	.0315	.02022	.02134	.04335	.02179	971.5	3.965	.04889	.2153
10	50	1.185	.0282	.01944	.01918	.03852	.01953	840.2	3.586	.04367	.1837
15	59	1.002	.0254	.01875	.01742	.03456	01795	756.0	3.233	.03903	.1615
20	68	.901	.0232	.01809	.01599	.03137	.01704	683.1	2.905	.03499	.1488
25	77	.772	.0214	.01745	.01481	.02874	.....	610.8	2.604	.02542	....
30	86	....	.0200	.01690	.01370	.02646	.....	....	....	.....	....
40	104	.506	.0177	.01644	.01195	.02316	.....	.....	....	.....	....
50	122	....	.0161	.01608	.01074	.02080	.....	....	....	.....	....
100	212	.244	.0141	.01600	.01011	.01690	.....	....	....	.....	....

TABLE LIV

ABSORPTION OF AIR IN WATER (WINKLER, 1904)

Air free of CO_2 and NH_3 measured at 29.92 ins. and 32° F.

Temperature. ° C.	Cu.ft. Oxygen at 29.92 ins. Hg per 1000 cu.ft. water.	Cu.ft. Nitrogen, Oxygen, etc., per 1000 cu.ft. water.	Sum of Oxygen and Nitrogen.	Temperature. ° C.	Cu.ft. Oxygen at 29.92 ins. Hg per 1000 cu.ft. water.	Cu.ft. Nitrogen, Oxygen, etc., per 1000 cu.ft. water.	Sum of Oxygen and Nitrogen.
0	10.19	18.99	29.18	16	6.89	13.25	20.14
1	9.91	18.51	28.42	17	6.75	13.00	19.75
2	9.64	18.05	27.69	18	6.61	12.77	19.38
3	9.39	17.60	26.99	19	6.48	12.54	19.02
4	9.14	17.18	26.32	20	6.36	12.32	18.68
5	8.91	16.77	25.68	21	6.23	12.11	18.34
6	8.68	16.38	25.06	22	6.11	11.90	18.01
7	8.47	16.00	24.47	23	6.00	11.69	17.69
8	8.26	15.64	23.90	24	5.89	11.49	17.38
9	8.06	15.30	23.36	25	5.78	11.30	17.08
10	7.87	14.97	22.84	26	5.67	11.12	16.79
11	7.68	14.65	22.33	27	5.56	10.94	16.50
12	7.52	14.35	21.87	28	5.46	10.75	16.21
13	7.35	14.06	21.41	29	5.36	10.56	15.92
14	7.19	13.78	20.97	30	5.26	10.38	15.64
15	7.04	13.51	20.55				

TABLE LV

HEATS OF COMBUSTION OF FUEL ELEMENTS AND CHEMICAL COMPOUNDS, SELECTED FROM LANDOLT AND BÖRNSTEIN, MEYERHOFFER AND SMITHSONIAN TABLES AND THOMSEN'S THERMO-CHEMISTRY

Substance.	Products.	B.T.U. per Lb.	Authority for B.T.U. per Lb.	Cu.ft. per Lb. at 32° F. and 29.92 ins. Hg.	Authority for Volume.	B.T.U. per Cu.ft. at 32° F. and 29.92 ins. Hg.
Carbon, C	CO_2	**14544**	Favre and Silberman			
Carbon, C	"	14647	Berthelot			
Graphite, C	"	14222	Berthelot			
Graphite, C	"	14033	Favre and Silberman			
Diamond, C	"	14146	Berthelot			
Carbon, soft, C	CO	4451	Favre and Silberman			
	"	4480	Berthelot			
	"	**4351**	Calc. from Thomsen			
Hydrogen, H_2	H_2O liquid 64° F.	61200	Thomsen	177.9093	Rayleigh	344
	" 90° F.	60854	Andrews			
	" 64° F.	61477	Favre and Silberman			
	" 212° F.	**60626**	Calc. from Thomsen			**341**
	vapor 212° F.	**51892**	Calc. from Thomsen			**292**
	" 212° F.	51717	Favre and Silberman			
Carbon monoxide, CO:	CO_2	4369	Thomsen	12.8090	Ledux	341
	"	**4325**	Favre and Silberman			**338**
	"	4376	Andrews			341
Sulphur, S:	SO_2 gas	**3998**	Thomsen			
	" "	3897	Berthelot			
	SO_3 liquid	5810	Thomsen			
Methane, CH_4:	CO_3 and H_2O liquid 64° F.	23841	Thomsen	22.349	Thomsen	
	" " 64	24017	Berthelot			
	" " 212	**23646**	Calc. from Thomsen			**1066**
	" vapor 212	**21463**	Calc. from Thomsen			**959**

Ethane, C_2H_6:	CO_2 and H_2O	liquid 64	22226	Thomsen	11.9354	Kolbe	
	"	" 64	22338	Berthelot			
	"	" 212	**22027**	Calc. from Thomsen			**1845**
	"	vapor 212	20280	Calc. from Thomsen			**1699**
Ethylene, C_2H_4:	"	liquid 64	21429	Thomsen	12.578	Saussure	
	"	" 64	21927	Berthelot			
	"	" 212	**21300**	Calc. from Thomsen			**1700**
	"	vapor 212	**20053**	Calc. from Thomsen			**1595**
Acetylene, C_2H_2:	"	liquid 64	21465	Thomsen	13.135	Berthelot	
	"	" 64	21724	L. B. M.		"	
	"	" 212*	**21399**	Calc. from Thomsen		"	**1529**
	"	vapor 212	20727	Calc. from Thomsen			**1578**
Propane, C_3H_8:	"	liquid 64	21650	Thomsen	8.08	Avogadro's Law †	
	"	" 64	21616	Berthelot		"	
	"	" 212*	**21460**	Calc. from Thomsen			**2656**
	"	vapor 212	**19872**	Calc. from Thomsen		"	**2459**
Propylene, C_3H_6:	"	liquid 64	21120	Thomsen	8.47	"	
	"	" 64	21398	Berthelot			
	"	" 212*	**20772**	Calc. from Thomsen			**2476**
	"	vapor 212	**19724**	Calc. from Thomsen			**2329**
Allylene, C_3H_4:	"	liquid 64	21038	Thomsen	8.90	"	
	"	" 64	21285	Berthelot			
	"	" 212*	**20935**	Calc. from Thomsen			**2352**
	"	vapor 212	**20060**	Calc. from Thomsen			**2254**
Butane, C_4H_{10}:	"	liquid 64	21326	Thomsen	6.1751	Frankland	
	"	" 212*	**21141**	Calc. from Thomsen			**3424**
	"	vapor 212	**19634**	Calc. from Thomsen			**3180**
Butylene, C_4H_8:	"	liquid 64	20912	Thomsen	6.35	Avogadro's Law †	
	"	" 212*	**20758**	Calc. from Thomsen			**3268**
	"	vapor 212	**19510**	Calc. from Thomsen			**3072**

* The value at 212° F. based on specific heat calculated by means of atomic weight.
† Approximate molecular weights used.

TABLE LV—*Continued*

HEATS OF COMBUSTION OF FUEL ELEMENTS AND CHEMICAL COMPOUNDS, SELECTED FROM LANDOLT AND BÖRNSTEIN, MEYERHOFFER AND SMITHSONIAN TABLES AND THOMSEN'S THERMO-CHEMISTRY

Substance.	Products.	B.T.U. per Lb.	Authority for B.T.U. per Lb.	Cu.ft. per Lb. at 32° F. and 29.92 ins. Hg.	Authority for Volume.	B.P.U. per Cu.ft. at 32° F. and 29.92 ins. Hg.
Pentylene, C_5H_{10}:	CO_2 and H_2O liquid 64	20674	Favre and Silberman	5.08	Avogadro's Law †	
	" " 212 *	**20516**	Calc. from F. &. S			**4039**
	" vapor 212	**19268**	Calc. from F. & S.			**3793**
Hexane, C_6H_{14}:	" liquid 64	20745	Stoleman	4.14	"	
	" " 212	**20610**	Calc. from Stoleman			**4970**
	" vapor 212	**19195**	Calc. from Stoleman			**4630**
Benzole, C_6H_6:	" liquid 64	18094	Berthelot	4.56	"	
	" " 212	**17976**	Calc. from Berthelot			**3942**
	" vapor 212	**17305**	Calc. from Berthelot			**3795**
Heptane, C_7H_{16}:	" liquid 64	20741	Stoleman	3.56	"	
	" " 212	**20640**	Calc. from Stoleman			**5797**
	" vapor 212	**19230**	Calc. from Stoleman			**5400**
Methyl alcohol, CH_3OH:	" liquid 64	10250	Thomsen	11.12	"	
	" " 64	9596	Berthelot			
	" " 212	**10203**	Calc. from Thomsen			**917**
	" vapor 212	**9113**	Calc. from Thomsen			**819**
Ethyl alcohol, C_2H_5OH:	" liquid 64	13325	Thomsen	7.73	"	
	" " 64	12748	Berthelot			
	" " 212	**13246**	Calc. from Thomsen			**1720**
	" vapor 212	**12100**	Calc. from Thomsen			**1570**

* The value at 212° F. based on specific heat calculated by means of atomic weight.
† Approximate molecular weights used.

INTERNAL THERMAL CONDUCTIVITY

Adapted from Landolt, Börnstein, Meyerhoffer, and Smithsonian Physical Tables and Professional Papers.

Substance.	Small Calories per Second per Sq.cm. per Degree C. per Cm. Thick.	B.T.U. per Hour per Sq.ft. per Degree F. per Inch Thick = K.	Authority.
Iron	$.1665(1-.000228t)$	$483\ [1-.000127(t-32)]$	Lorenz
	$.209(1-.00147t)$	$606\ [1-.00082(t-32)]$	Forbes
	$.197(1-.00002t)$	$571\ [1-.0000111(t-32)]$	Tait
	$.175(1-.0015t)$ ←	$507\ [1-.00083(t-32)]$	Stewart
	$.199(1-.00287t)$	$577\ [1-.00159(t-32)]$	Augstrom
	.1528 at 28° C.	443 at 82° F.	Hall
	.1627 at 100° C.	472 at 212° F.	Lorenz
Copper	$.7189(1+.000051t)$ ←	$2080[1+.0000278(t-32)]$	Lorenz
	$.71\ \ (t+.0014t)$	$2060[1+.000788(t-32)]$	Tait
	$1.08\ \ (1+.0013t)$	$3130[1+.000722(t-32)]$	Tait
	$1.027\ \ (1-.00214t)$	$2980[1-.00119(t-32)]$	Augstrom
	$.983\ \ (1-.00152t)$	$2850[1-.000845(t-32)]$	Augstrom
	$1.12\ \ (1-.001t)$	$3250[1-.00055(t-32)]$	Stewart
Brass, Yellow	$.2041(1+.002445t)$	$592\ [1+.00136(t-32)]$	Lorenz
Brass, Red	$.2460(1+.001492t)$	$713\ [1+.000892(t-32)]$	Lorenz
Steel, Hard	.0620	180.0	Kohlrausch
Steel, Soft	.1110 ←	322.0	Kohlrausch
Steel, Bessemer	.0964 at 15° C.	279.5	Kirchhoff
Aluminum	$.3435(1+.0005356t)$	$966\ [1+.0002980(t-32)]$	Lorenz
Lead	$.0836(1-.000861t)$	$242.5[1-.000479(t-32)]$	Lorenz
Tin	$.1528(1-.000687t)$	$443\ [1-.000382(t-32)]$	Lorenz
Zinc	.1528 at 15° C. ←	443	Kirchhoff and Hansem
Zinc	.2653 at 18° C.	770	Jaeger and Diesselhorst
Silver (highest of all)	1.0960	3180	Weber

TABLE LVI—*Continued*

Substance.	Small Calories per Second per Sq.cm. per Degree C. per Cm. Thick.	B.T.U. per Hour per Sq.ft. per Degree F. per Inch Thick $=K$.	Authority.
Slate	.0036	9.58	Lees Chorlton
Granite and sandstone	.0054	15.65	Average
Marble, limestone, etc.	.0047 to .0056	13.6 − 16.2	Herschel, Lebour, Dunn
Portland cement	.00071	2.06	Lees Chorlton
Plaster of Paris	.0007	2.03	Lees Chorlton
Soil	.00033 dry: .0016 wet	.958 dry; 4.64 wet	Lees Chorlton
Sand, white dry	.00093	2.7	Herschel, Lebour, Dunn
Chalk	.0002	.58	Herschel, Lebour, Dunn
Firebrick	.00028	.812	Hutton, Bland
Carbon	.000405	1.32	Forbes
Glass	.0011 to .0023	3.19 to 6.68	Averages
Diatomic earth	.00013	.377	Hutton, Bland
Paraffine	.00023 at 0° C. and .00168 at 100° C.	.667 at 0° C. and 4.88 at 100° C.	Weber
Ice	.00223; .00568	6.47; 16.48	Forbes, Newman
Snow, packed	.00051	1.48	Hjeltström
Sawdust	.00012	.348	Forbes
Woods	.0003 with grain, .00009 across grain	.871 with grain, .0261 across grain	Forbes
Strawboard	.0003	.871	Forbes
Pasteboard	.00045	1.305	Forbes
Asbestos paper	.00043	1.245	Lees Chorlton
Blotting paper	.00015	.435	Lees Chorlton
Felt	.000087	.252	Forbes
Cotton wool	.000043	.125	Forbes
Cotton wool pressed	.000033	.0957	Forbes
Flannel	.00012	.348	Forbes
Haircloth	.000042	.122	Forbes
Cork	.000717	2.08	Forbes
Leather, cowhide	.00042	1.22	Lees Chorlton

TABLE LVI—*Continued*

Substance.	Small Calories per Second per Sq.cm. per Degree C. per Cm. Thick.	B.T.U. per Hour per Sq.ft. per Degree F. per Inch Thick = K.	Authority.
LIQUIDS			
Water	.00120 at 0° C. ←	3.48	Weber
	.00136 from 9° C. to 15	3.94	Weber
	.00129 at 4° C.	3.74	Wachsmuth
	.00124 at 18° C.	3.6	Chree
	.00157 at 30° C.	4.56	Graetz
	.00222 at 108° C.	6.45	Lundquist
Methyl alcohol	.000495 from 9° C. to 15	1.435	Weber
Ethyl alcohol	.000423 from 9° C. to 15	1.22	Weber
Ethyl alcohol and water 50%	.0008 at 25° C.	2.32	Lees
Benzole	.000333 from 0° C. to 15	.972	Weber
Petroleum	.000355 at 13° C.	1.03	Graetz
GASES			
Air	$.0000568(1+.0019t)$	$.165[1+.000106(t-32)]$	Winkelmann
	.0000484 ←	.1405	Graetz
	.0000569	.165	Schwarze
	.000072	.209	Schleiermacher
Ammonia	$.0000389(1+.0026t)$	$.113\ [1+.00144(t-32)]$	Schwarze
Ethylene	$.0000395(1+.00445t)$	$.1145[1+.00248(t-32)]$	Winkelmann
Hydrogen	$.000327\ (1+.00175t)$ ←	$.95\ \ [1+.000974(t-32)]$	Winkelmann
Hydrogen	.000319 at 0° C.	.926	Eckerlein
Nitrogen	.0000524 from 7° C. to 8	.152	Winkelman
Oxygen	.0000563 from 7° C. to 8	.163	Winkelmann
Methane	.0000647	.188	Winkelmann
Carbon monoxide	.0000499 at 0° C.	.145	Winkelmann
Carbon dioxide	.0000307 at 0° C.	.0891	Winkelmann

TABLE LVII

RELATIVE THERMAL CONDUCTIVITY

CONDUCTIVITIES CARBON DIOXIDE AND RESISTANCES SILVER } = 1 at 32° F.

Substance.	Conductivity Carbon Dioxide = 1.	Resistance = $\frac{1}{\text{Conductivity}}$ Silver = 1.
Iron	5700	5.23
Iron (Wiederman and Franz)	4165	8.60
Copper	23000	1.52
Copper (Wiederman and Franz)	25760	1.36
Steel	3600	9.74
Steel (Wiederman and Franz)	4165	8.60
Aluminum	11000	3.18
Lead	2700	12.95
Lead (Wiederman and Franz)	2975	11.75
Tin	5000	7
Tin (Wiederman and Franz)	5320	6.58
Zinc	5000	7
Zinc (Wiederman and Franz)	9835	3.56
Silver	35000	1
Slate	117	3
Granite and sandstone	176	199
Marble, limestone, etc	153–182	228–192
Portland cement	23.2	1511
Plaster of Paris	22.8	1531
Soil	10.7 dry; 52.2 wet	3270 dry; 6700 wet
Sand, white dry	30.4	1150
Chalk	6.52	5370
Firebrick	9.12	3840
Carbon	13.2	2650
Glass	35.8 to 75	978 to 467
Diatomic earth	4.24	8260
Paraffine	7.50 at 0° C. to 55.0 at 100° C.	4670 at 32° F. to 637 at 212°
Ice	72.7; 18.5	481; 189.0
Sawdust	3.92	8940
Snow, packed	16.6	2110
Woods	9.8 w.g.; 2.94 a.g.	3570 with grain; 11900 ag.g
Strawboard	9.8	3570
Pasteboard	14.7	2380
Asbestos paper	14.0	2500
Blotting paper	4.9	7150
Felt	2.84	12300
Cotton wool	1.4	25000

TABLE LVII—*Continued*

RELATIVE THERMAL CONDUCTIVITY.

Substance.	Conductivity Carbon Dioxide = 1.	Resistance = $\frac{1}{\text{Conductivity}}$ Silver = 1.
otton wool, pressed	1.08	32400
lannel	3.92	8930
aircloth	1.37	25600
ork	2.34	1495
eather, cowhide	13.7	2560
Vater	39.09	896
Iethyl alcohol	16.12	2170
Iethyl alcohol (De Heen)	10.70	3270
thyl alcohol	13.78	2540
thyl alcohol (Henneberg)	12.07	2900
thyl alcohol 90% (Henneberg)	12.53	2990
thyl alcohol (Henneberg)	21.22	1650
enzole	10.83	3240
enzole (Weber)	11.25	3100
etroleum	11.56	3030
ir	1.85	18900
mmonia	1.27	27600
mmonia (Plank)	1.7	20600
thylene	1.28	27400
thylene	1.37	2960
ydrogen	10.65	3280
ydrogen (Stefan)	12.97	2960
ydrogen (Kindt and Warberg)	13.14	7100
itrogen	1.71	20450
xygen	1.83	19100
xygen (Stefan)	1.89	25500
[ethane	2.30	15200
[ethane (Stefan)	2.57	18500
arbon monoxide	1.62	21600
arbon monoxide (Kindt and Warberg)	1.81	19300
arbon dioxide	1.00	35000
arbon dioxide (Stefan)	1.15	30400
arbon dioxide (Kindt and Warberg)	1.09	32100
luminating gas (Plank)	4.94	13600

CHAPTER V

HEATING BY COMBUSTION.

FUELS, FURNACES, GAS PRODUCERS AND STEAM BOILERS

1. Origin of Heat and Transformation to Useful Form. Complexity Fuels as Sources of Heat. General Classification. Solid, Liquid, Gaseou Natural and Artificial. All heat for power purposes is derived from th combustion of fuels of some sort, whether natural or artificial, as also is near all the heat for the warming of things for domestic and industrial purpose but some heat is derived directly from bodies in their natural state, as fo example, when foods are cooled for safe storage or when ice is made. Thu the source of heat with which engineering operations must deal is (*a*), th of an exothermic chemical reaction, or (*b*), that of bodies in their natur state, and this heat when taken into some system for the purpose of accon plishing some industrial end must be traced through the whole series processes necessary to that end and finally disposed of, as no energy can l destroyed. The ultimate disposition of the heat is a matter of just as muc importance as its generation or acquirement at a point of origin, and th almost infinite variety of intermediate exchanges and transformations betwee its reception into the system and its final disposition.

Heat received into a system is ultimately disposed of in one of three way first, it is most commonly discharged from the system in the form of h water, hot gases or vapors, and from them communicated to the surroundin bodies or radiated into space; or second, it may be bound up by endotherm chemical reactions or changes of state as when certain chemical compoun or solutions are formed by heat absorption; or finally it may have been tran formed into energy of another form, permanent or transient, as when a mi cage has been lifted or light generated in the electric lamp. Omitting such di positions as correspond to transformation of energy into another form as b temporary, and to be treated in the next chapter, the two final dispositio are the inverse of the two original sources, chemical or physical changes state of bodies or temperature changes of the general surroundings, earth a water and air.

When the heat is received into the system at high temperatures as is the ca when its origin is the combustion of fuel, its progress is generally downward to temperature, each exchange from substance to substance resulting in low temperature but the final disposition by transfer to water or atmospher air, in steam condensers, for example, must take place at a temperature the heat carrier, in excess of that of the final heat receiver, that is, the fin

mperature of the steam, the heat of which must be disposed of by heating ıter or air must be higher than that of the available water that is to receive it. tween the reception and final disposition all sorts of things may be caused happen; power may be generated and transformed into all the various ms of energy; metals be extracted from their ores or chemical compounds value made from those of no value.

When, however, the heat is received at low temperatures, as when ice to be made or a quicksand to be frozen to permit of shaft sinking, or food to be preserved, or natural water cooled for industrial or drinking purposes, air to be dried of its moisture, then heat can be received into the system ly by having in the system something colder than the body whose heat is be taken. In general, these operations result in heat reception at a lower mperature than the surrounding air and water to which it must ultimately returned and which can receive it only when it has been raised to a higher nperature, so *that at least one step in the series of processes must be a raising temperature by doing work on the working substance, usually by compression.*

The practical use of heat, therefore, is absolutely limited as to process, thods and quantities by the conditions of reception and those of ultimate position, but between these limits all operations or processes may be classed as

(*a*) Heat transformation into work or vice versa;

(*b*) Change of substance form, chemically or physically, with heat absorption liberation,

(*c*) Heat transfer from regions of high to regions of low temperature.

These may also be grouped into two broader divisions, somewhat more nvenient than those above.

I. Thermal processes in which no mechanical work is done and involving at exchange or substance change with heat addition or abstraction.

II. Thermal processes involving the doing of mechanical work on, or by, working medium or heat carrier. These are all work-heat transformations.

It is with the first class that this chapter deals in as much as the majority fuels must be changed in character before the heat in them is in a form table for conversion into work. All work is produced by a change in the ume of a substance with or without a change in pressure. As before stated, se substances giving the greatest volume changes are the gases and vapors. *this reason then the heat in the fuel must be transferred to a gas or vapor less the fuel be originally one or the other of these.*

The first type of heat-carrying substance, a gas, is generally prepared under ssure by the so-called explosive burning of a gas with air in a closed ssel. As few of the natural fuels are gases, the first step in the utilization other forms of fuels for such cases as these, is to gasify them by means of producers, retorts or similar apparatus at the expense of some of their n heat or heat from another source.

The second type of heat-carrying substance, used for securing work at the pense of heat, a vapor, is obtained by evaporating some volatile liquid,

usually water, in a boiler by means of the heat produced by the combustio of any natural or artificial fuel.

Therefore, the problem of the fuels, their transformation, one form int another, more especially their gasification and complete combustion, is of th utmost practical importance and of considerable difficulty. Although th only combustible chemical elements of all fuels are carbon, hydrogen an to a small extent sulphur, and the complete combustion reactions few an simple, it must not be understood that fuels are simple in nature, their suitabl and proper combustion easy, or that it is possible to predict their heat o combustion or its effects with precision, even when their chemical compo sition is known. *No problem of equal practical importance is more difficu or elusive than the prediction of fuel effects by calculation, or the determinatio of the most suitable fuel for a given purpose.*

Fuels may be regarded as mixtures of the chemical elements, carbo hydrogen and sulphur, so far as their combustible parts are concerned, i the proportions indicated by their ultimate analysis, but it is known that thes elements make a great variety of compounds, so that if these compounds a present in a fuel, its heat of combustion cannot be the sum of the heats of co bustion of its chemical elements, but this sum less the amount necessary t break the bonds of molecular union. It would be more proper from th heat liberating standpoint, to say that the heat of combustion of a fuel is th sum of the heats of combustion of all those chemical compounds that as mixture, make up the fuel. Unfortunately this precise statement is of litt help because no one knows that a fuel except the gaseous varieties, does co sist of a mixture of separate and distinct fuel compounds, or what they ar It is probably nearer the truth to say that solid fuels are themselves chemic compounds of great molecular complexity with other chemical compound in *solution* or some equivalent state, and it is known that all liquid fuels a solutions of a lot of liquids in each other with perhaps a little of the simp mixture condition. If all the compounds that together constitute a fuel we known, if the heats of solution or absorption of such substances so held b the others were known and their quantities, then the heat of combustio of a fuel could be expressed as an algebraic sum of all these heats exactl This, however, is quite impossible, and even if it were possible would be comparatively little use because of its complexity and the fact that no tw samples of the same fuel externally judged, give quite the same results, thou all will average up within certain reasonably close limits.

There is another possible assumption with regard to a solid fuel comp sition that is, while approximate, quite useful, which considers the combustib to consist of fixed carbon or carbon that burns directly from the solid cok mixed with a gaseous combustible, termed the volatile, and the heat of co bustion as the sum of the heat of so much fixed carbon as is present and t heat of the volatile. This assumption leads to an estimate of the fuel val of one coal, from calorimeter determinations of others of the same class terms of the heat of combustion per pound of the volatile which is differe

or each class of coals, and that of the fixed carbon which is the same for all. n a precisely similar approximate way, may the gaseous and liquid fuels be reated, which treatment emphasizes the tremendously variable and complex ature of fuels which must be handled for the accomplishment of engineering nds and the necessity for some study of their nature that will throw light ot only on their calorific power, but on the best way in which to burn them o develop it to the fullest extent and in the most efficient manner, that is, vith least loss.

The very chemical complexity that the study of calorific powers reveals, s also responsible for the remarkably different behaviors of the fuels even f the same class, in fires. Furthermore, each different use to which fuels re put, requires within certain limits special properties in the fuel to make adaptable to the purpose. For example, internal combustion engines require aseous fuel and where gaseous fuel is not available, naturally it must be made rom whatever solid or liquid fuel is most available, availability depending artly on cost. Blast furnaces for the extraction of iron from the ore are est served with coke, and coke must be made for the purpose from coal, or vhere this is not available, a hard, natural coal may be substituted in some ases. Certain metallurgical and chemical manufacturing operations require very steady, high temperature such as can be obtained only with gaseous nd in some cases liquid fuel. Boilers are fired with all sorts of fuel, that vhich produces the cheapest steam being selected, and this may be a gas, n oil, or any one of the great number of different solid fuels that may be had n a given district, each of different quality and price and usually each giving different boiler horse power and efficiency, so it may be, that either the ost expensive coal per ton, or the lowest price coal per ton, or some inter- ediate one, may make the most steam per dollar's worth of fuel. Additional omplexity arises from the fact that the cheapest steam-making coal may ield so low a boiler horse-power as to make the boiler installation investment harge per unit of steam so great, that greater horse-power with more expen- ive fuel might really be more economical. In cities the household use of as for illumination is now combined with the use of the same gas for domestic eating, and a suitable gas must be manufactured to meet the demand for his double service.

These examples will suffice to show first: that the use to which a fuel is ut more or less dictates the type of fuel, second, that in apparatus capable f using more than one fuel the problem of selection is partly one of properties nd partly one of price, factors that may or may not be related; third, that vhile natural fuels exist in great variety it is good engineering to manufacture rom them other fuels to secure desired results. These facts make it necessary o enquire a little more closely than has been done into the properties of the atural and artificial fuels and their differences and the sort of process by vhich one class or kind of fuel may be manufactured from another.

The first step in such a study is a comprehensive classification of all fuels s given in the following Table LVIII as a basis for the treatment:

TABLE LVIII

GENERAL CLASSIFICATION OF FUELS

Physical State.	Name of Natural Fuel.	Name of Manufactured Fuel.
Solid.....	Wood, peat, lignite, bituminous and anthracite coal	Charcoal, coke, briquettes.
Liquid.....	Crude mineral oil.....	Alcohol, mineral oil, distillates and residues, oil, gasolene, kerosene, fuel coal, and oil tar.
Gaseous....	Natural gas.....	Coal gas, coke-oven gas, water gas, carburetted water gas, producer gas, blast-furnace gas, oil gas, oil and alcohol vapors.

The general processes by which the artificial fuels are made from the natural ones are divisible into groups. The first is a simple heating or roasting of natural solid fuels, which leaves a coke and produces a volatile mixture which on cooling leaves some permanent gases more or less saturated with vapors and some liquid generally termed tar or tar liquor. The second is analogous but as applied to liquid fuels is termed distillation or fractionation, but the third is quite different. It is based on the reactions of fixed carbon with the oxygen of the air and with steam to make carbon monoxide and hydrogen. Often these three type processes are mixed and all three take place at once and are associated with all sorts of dissociations and inter-reactions. It is however, convenient to think of all processes being divisible into

(*a*) Mere heating of the natural fuel;

(*b*) Chemical reactions between the fuel constituents, air, and steam.

In the succeeding sections there will be taken up in order the physical chemical, and thermal characteristics of the natural fuels, their treatment for the creation of the common manufactured forms of fuel, the combustion of all sorts, solid, liquid, and gaseous, both explosively and non-explosively together with the conditions which surround and control each change. Finally the gas producer and steam boiler as characteristic apparatus having certain properties will be analyzed and the laws of performance derived, both from experimental data and from purely rational considerations.

Prob. 1. The simplest power-plant consists of a boiler and non-condensing engine. Trace the heat from its source in the fuel to its final disposition.

Prob. 2. The above power-plant has had added to it various auxiliaries for increasing the efficiency. They consist of a feed-water heater in the exhaust of all auxiliaries and an economizer in the flue, a superheater in the first pass of the boiler, an induced draft steam-driven fan, and a condenser. Trace the heat through this system from the coal to its ultimate disposition.

Prob. 3. Gasolene is burned in the cylinder of a water-jacketed engine, provided ith an air-cooled radiator. Show where the original heat of the fuel goes.

Prob. 4. Coke is burned in a blast furnace and some of the gaseous products, which are ch in CO, are washed and used in gas engines driving the blast compressors. Between ıe blower and furnace the air passes over brickwork previously heated by burning ›me of the gas. The gas-engine cylinder is, of course, water jacketed. Through hat steps does that portion of the heat of the fuel not used up by the reduction of ıe iron ore pass and what is its ultimate disposition?

Prob. 5. Heat is removed from a cold-storage room and delivered to the atmospheric ir. The process in an ammonia-compression system is to allow ammonia to evaporate ı coils at a low pressure, compress the vapor and condense it by allowing water and ir to pass over tubes receiving the high pressure vapor. Through what transfer and ransformation processes does the heat pass before reaching the atmosphere? Assume he compressor to be steam driven and trace the heat of the coal as well.

Prob. 6. Air is compressed by a water-jacketed steam-driven compressor and dis- harged to a large pipe-coil receiver, then through a long pipe line, through a coal- red preheater to an air engine. Show where the air receives heat and what becomes f it and also what becomes of the heat of the coal.

Prob. 7. A coal-fired boiler supplies a steam-driven electric generator, the exhaust rom which passes to radiators placed throughout a building lighted by the current, he excess steam passes to the roof exhaust. Trace the heat of coal to its final dis- ›osition.

Prob. 8. Air and steam are supplied to a gas producer. What becomes of the heat ›riginally in the coal?

2. Natural Solid Fuels, Wood, Peat, Lignite, Bituminous and Anthracite Coal. Chemical and Physical Properties. Classifications Based on Ultimate and Proximate Analysis and on Behavior on Heating. All solid fuels are of vegetable origin in spite of the variations in properties from the hard anthracites o the soft peats, and while the tracing of the process of transformation of ach from its origin or one to another is more a problem of chemical geology r physical and organic chemistry than of the engineering of fuel manufacture r combustion, yet such an analysis throws so much light on the nature of he fuels and the methods of treatment in combustion or gasification as to ender a brief review decidedly worth while. Such an analysis will not only how the relation between the different varieties of solid fuels and their hemical, physical, and calorific properties, but will also show the intimate elation between the solid, liquid, and gaseous form of the natural fuels and he corresponding artificial products made from the first two as raw materials.

There are three ways of defining a fuel, first, by a general name, indicating eologic condition, behavior on heating, or some other generally understood ut not very definite characteristic, and which is responsible for the names ignite, peat, bituminous and anthracite, long flame, short flame, cannel, coking, aking, and gas coal, together with many others, some indicating size like un-of-mine, slack, broken, furnace, egg, nut, pea, buckwheat, and rice, the atter names applying principally to sizes of anthracite. Also the name of he district or mine is used and any American engineer knows pretty well

what is meant by Pocahontas coal or Pennsylvania anthracite or Illinois coal.

The second way of defining or fixing a solid fuel is based on what is termed its *proximate analysis*, which has a meaning dependent solely on the methods employed, the following applying to the laboratory work of the U. S. Geological Survey. One gram of pulverized coal selected with great care so that it represents as nearly as possible the average of the lot, the properties of which must be found, is heated in an oven at 221° F. for one hour and quickly weighed. The loss in weight is termed the *moisture* and the residue after burning the dried sample in a crucible termed the *ash*, the difference or *combustible*, being divided by other operations into two parts, the *volatile* and the *fixed carbon*, the former being the loss of weight by seven minutes heating of the dry sample, in closed crucibles, excluding the air, in the flame of a Bunsen burner and the latter the residue after subtracting the ash.

By complete analytical methods the chemical elements of the coal may be found and reported, which define the coal by its *ultimate* analysis, the methods being those common to quantitative organic chemistry. Special constituents reported by analysts for fuels are each determined by special methods out of place here.

Wood is in all cases the primary organic substance from which solid fuels are derived and the term must be used to include not only the trunks of trees but branches, leaves, and roots as well as small plants and mosses. It is composed chemically of cellulose, $C_6H_{10}O_5$, as the fiber, and of sap, or sap deposits between the fiber. Cellulose consists according to its formula, of a definite weight proportion of carbon, hydrogen, and oxygen, but the proportions of these same elements will be different in real woods by reason of the sap properties or the materials deposited by sap, which are of both nitrogenous and non-nitrogenous character. Sap is really a very complex substance consisting of protein, tannin and several vegetable acids, starch, sugar, essential oils and resins among its organic constituents, together with all sorts of inorganic salts derived from the soil. These are of some importance in explaining the differences found in the coals derived from the woods and are mentioned here to call attention to the relation. In the following Table LIX is compared the ultimate composition of pure cellulose with the average of maple, oak

TABLE LIX

COMPARISON OF CELLULOSE AND AVERAGE WOOD (DRY AND ASH FREE)

Constituent.	Cellulose.	Wood, Average of Maple, Oak, Pine, Willow.	Spores of Club Moss.
Carbon	44.44%	49.2%	63.0%
Hydrogen	6.17%	6.1%	8.6%
Oxygen	49.39%	.....	.....
Oxygen and nitrogen	.....	44.7%	28.4%

pine and willow woods and with moss fiber, dried and free from ash and therefore, excluding much of the impregnating matter, as given by Chevandier, together with Percy's values for the spores of club moss, which forms peat.

The table illustrates the increase of carbon content of wood over cellulose and still greater is the carbon content of the moss, but it does not indicate whether the carbon is free or combined with other elements. Of course, actual wood contains a great deal of water, even air-dried wood carrying from 15 per cent to 25 per cent of moisture, while the ash content will vary from one-tenth of one per cent up to perhaps 4 per cent, depending on the soil, the part of the plant, its age and a variety of similar circumstances, the ash of cellulose being, of course, zero.

It is from such chemical origin that the solid fossil fuels were derived by a natural process, termed carbonification, because it is characterized by an increase of carbon content over the original vegetable substance just as it does itself show an increase over cellulose, its main constituent fiber. These fossil solid fuels are usually divisible by age into younger and older groups, though local conditions may disturb the division, the former including *peat and lignite*, and the latter *bituminous and anthracite* coals, which calls attention to the fact that time is the first element in increase of per cent carbon. The next factor is the action of oxygen. Wood will absorb oxygen with a resulting slow reaction to CO_2 even at low temperatures, and so also, will all the coals of whatever grade. There is also a further reaction of organic nature between the hydrogen, whether free or combined and the carbon, to form hydrocarbons, among which is methane CH_4 as the principal one. Mixing of the deposits with sand and clay increases the ash content of the coals over that of the woods, just as sap deposits give to the woods an ash content not existing in cellulose.

It is certain that the processes of carbonification produce certain products consisting of carbon dioxide and hydrocarbons and perhaps many other complex compounds and leave a residue which is the coal. Increase of both pressure and temperature promotes the process, the coal residue containing a higher per cent of carbon and ash than the original plants from mixture and reaction with earths, water and air, while oxygen and nitrogen decrease, with time and the favorable nature of surrounding conditions. One very active agent in the changes is heat, promoting chemical reaction and expelling gases and the vapors of liquid resultant substances, which are compounds of carbon, hydrogen, and nitrogen, largely hydrocarbons and carbon dioxide. Vapors may condense in other colder places into which they are driven by the pressure, forming oil deposits, while gases may impregnate rocks and sands and remain in place when overlaid by impervious rock. Thus is the origin of natural gas and crude oils traced with reference to coal formation as products of successive decomposition and reactions beginning with wood, the coal constituting the residue, but there is good reason to believe that hydrocarbons forming natural gases and oils may have been formed from carbides of iron, as the U. S. Geological Survey has recently shown that

the regions of great magnetic needle deflection are those of known oil deposits.

The various coals are divided into classes with names for convenience of discussion, according to their properties relating to the completeness and character of the changes they have suffered, but unfortunately there is a general lack of agreement as to the meaning of the names. Coal classification is generally based on chemical properties, Muck making the total carbon content of the ultimate analysis the basis, Frazer, the fixed carbon divided by the volatile combustible matter of the proximate analysis, while Campbell of the U. S. Geological Survey uses the ratio of the total carbon to the total hydrogen of the ultimate analysis. These three classifications with the corresponding names are given in the following Table LX.

TABLE LX

CLASSIFICATION OF COALS BY COMPOSITION

Class.	Name.	Campbell. Total C. / Total H.	Frazer. Fixed C. / Vol. Comb.	Muck. Per Cent Total C, Dry and Ash Free	General Per cent Total C, Dry and Ash Free.
A	Graphite and graphitic coal	∞ to ?	Anthracite		
B	Anthracite (1)	? to 30	100 to 12	Anthracite, 95	Anthracite 97 to 92.5
C	Anthracite (2)	30 to 26			
D	Semi-anthracite	26 (?) to 23	12 to 8		92.5 to 87.5
E	Semi-bituminous	23 (?) to 20	8 to 5		87.5 to 75.0
F	Bituminous (1)	20 to 17	Bituminous 5 to 0	Common coal 82	
G	Bituminous (2)	17.0 to 14.4			Bituminous eastern U. S. 75.0 to 60
H	Bituminous (3)	14.4 to 12.5			
I	Bituminous (4)	12.5 to 11.2			Bituminous western U. S. 65 to 50
J	Lignite	11.2 to 9.3		70	Under 50
K	Peat	9.3 (?) to ?		59	
L	Wood or cellulose	7.2		50	

The latest of these classifications is Campbell's, which was proposed by reason of the failure of previous classifications to suitably divide the fifty odd U. S. samples, thoroughly studied by the Geological Survey, and which were mainly poor coals, into the lignite and bituminous classes, to which they

obviously belonged. It is to be regretted that there was not enough data to positively fix the numbers marked (?), but it is a fact that will more clearly appear later that no sharp line of division exists between one class and the next, each merging into the other by almost imperceptible gradations, as might properly be expected, considering their common origin.

Another basis of classification of considerable value depends on the behavior of coals on heating with respect to (*a*) the amount of gas produced, (*b*) the character of the coke residue, and (*c*) the changes in the mass before the coke sets. The U. S. Geological Survey divisions were based on the sort of residue left in a rectangular platinum box after heating a powdered sample of coal with ten parts of ground silica, and permits of division into *coking or non-coking coals*. This is similar to the practice of Muck except that he heats powdered coal alone and gives other names to the coal, according to the character of the coke residue. Similarly Grüner, Sexton and Hilt give names relating partly to the formation of coke and partly to the quantity of gas yield as indicated by the size of the flame of the burning volatile. These various conditions are compared in the following Table LXI, which indicates, first, that this sort of classification, while common in practice, is still not reduced to any accepted form; second, that as an indication of the use to which the different coals are well adapted, a classification based on behavior under heating, as to gas yield and character of coke formed may be more valuable than others based on composition; third, gas and coke property classification is not definitely related to composition, though there is some sort of relation.

These classification properties are not of mere laboratory importance, but largely control the actual useful service of the coals. Coals that melt too completely cannot well be burnt under boilers without constantly breaking up the cake that forms, so as to let air pass through, but moderate caking tendencies are good as preventing fine material from passing through the grate, so anthracite rice or dust is often advantageously mixed with some caking bituminous coal.

Peat is the least removed in character from its vegetable origin of all the solid fossil fuels and two general varieties are recognized with reference to origin, sometimes described as (1), high-bog peats, formed from heath and swamp moss and usually located in high altitudes, and (2), low-bog peats, formed largely from grasses about the borders of low bodies of water.

In most peats the fibrous structure is still visible and they range in color from yellow through brown to black; some are soft and others hard, hard especially when the fibrous structure has almost or nearly disappeared, and in all cases there is a very large moisture content even in air-dried peat. In but few cases is peat strong enough to resist crushing when piled high during combustion as in gas producers. Probably its most distinguishing characteristic is the very large percentage of nitrogen and oxygen and the poorly combustible character of its volatile matter. There are given in Table LXII on page 655 a few analyses of peats, as this sort of fuel is widely distributed and of growing importance.

TABLE LXI

CLASSIFICATION OF COALS BY GAS AND COKE QUALITIES

Behavior of Powdered Sample on Heating in Crucible.	Muck.						Sexton's English Names.	Grüner's German Names.	Hilt.	
	Name.	Class.	Ash and Moisture Free.			Per Cent Coke.			Name.	Fixed C / Volatile, Ash and Moisture Free
			C	H_2	O_2					
Does not melt, residue powder, same as coal.	Sand coal.	Anthracite and semi-anthracite.	93 to 90	4 to 4.5	3 to 5.5	90 to 82	Anthracite	Lean coal	Anthracite	20 to 9
Partly melts, residue mainly powder, rest soft.	Molten sand coal.	Dry bituminous, long flame.	80 to 75	4.5 to 5.5	15 to 19.5	60 to 50	Non-caking coal, long flame	Dry coal, long flame	Semi-caking sinter coal, poor in gas	9 to 5.5
Melts, residue compact and hard but not puffed.	Sinter coal.	Caking bituminous coal, long flame gas coal.	85 to 80	5 to 5.8	10 to 14.2	68 to 60	Gas coal	Fat coal, long flame	Caking or coking coal	5.5 to 1.2
									Caking gas coal	1.2 to 1.5
Melts, residue compact and hard somewhat puffed.	Caking, sinter coal.	Caking coal, proper, or forge coal.	89 to 84	5.5 to 5.0	5.5 to 11	74 to 68	Furnace coal	Fat caking coal	Sinter coal, rich in gas	1.5 to 1.25
Melts thoroughly, residue very hard and very much puffed.	Caking coal.	Caking bituminous coal	91 to 88	4.5 to 5.5	5.5 to 6.5	82 to 74	Coking coal	Fat coal, short flame	Sand coal, rich in gas	1.25 to 1.1

Table LXII

COMPOSITION OF PEATS (JÜPTNER)

Origin.	Composition Per Cent by Weight, Dry					Moisture Per Cent Air Dried.	Physical Properties.	Authority.
	C	H_2	N_2	O_2	Ash.			
appoge, Ireland	51.05	6.58	39	55	2.55	.....	.....	Kane
ulbeggen, Ireland	61.04	6.87	.30	.46	1.83	10	.....	"
hilipstown, Ireland	58.69	6.97	1.45	32.88	1.99	to 25	Pale, red brown	"
Vood of Allen, Ireland	61.02	5.77	.81	32.4	7.9	.....	Dark brown dense	"
auclaire, France	57.03	5.63	2.09	29.67	5.58	.....	Dark brown	Regnault
ony, France	58.09	5.93	31	77	4.61	.....	" "	"
remont, France	57.79	6.11	30	.37	3.33	.....	Incompletely decomposed	"
ammstein, Rheinfalls, Ger.	61.15	6.29	1.66	27.20	2.70	16.7	Solid & dense	Walz
einweden, Rheinfalls, Ger.	57.50	6.90	1.75	31.81	2.04	16.0	Lighter	"
iedermoor, Rheinfalls, Ger	47.90	5.8	42	80	3.50	17.0	Light, felty mass	"
rünwald, Germany	49.88	6.5	1.16	42.42	3.72	.....	.....	Websky
arz, Germany	50.86	5.80	.77	42.70	.57	.....	.....	"
arz, Germany	62.54	6.81	1.41	29.24	1.09	.....	.....	"
imm, Germany	59.47	5.52	2.51	31.51	18.53	.....	.....	"
undsmühl, Germany	59.70	5.70	1.56	33.04	2.92	.....	.....	"
witzerland	40.10	4.53	2.84	21.51	7.87	23.17	Pressed peat	Goppelsröder

AVERAGE CALORIFIC POWER OF PEAT (SCHEERE) B.T.U. PER POUND

ro per cent water, zero per cent ash = 9090 | 25 per cent water and zero per cent ash = 6840
ro per cent water, 15 per cent ash = 7992 | 30 per cent water and 10 per cent ash = 1462

Lignite is only one step removed from peat, yet it is very different in haracter, having been formed largely, if not entirely, from plants rich in resin, cluding the coniferous trees, cypress and palms, but in some cases others e included. Its appearance is non-fibrous and is either brown or black in lor. It readily absorbs oxygen and gives off carbon dioxide at all tempera- res. The volatiles of lignites is so largely carbon dioxide that they e chemically in the same general class as the peats, and this distinguishes th as much as any property from all other coals, the volatiles of which are ainly hydrocarbons. This is a most interesting distinction but must not be erpreted as meaning that no hydrocarbons are found in lignites but rather at they are mixed with large amounts of non-combustible diluents. To e lignite analyses in the general table there are added the following Austrian nite analyses, Table LXIII, for comparison from Jüptner.

TABLE LXIII

COMPOSITION OF AUSTRIAN LIGNITES

Origin.	Composition Per Cent by Weight.							B.T.U. per Lb.
	C	H_2	O_2	N_2	H_2O	Ash.	S	
Trefail, Styria, Austria..	44.95	3.67	16.93	.97	20.15	8.43	1.64	7895
Teplitz, Bohemia, Austria.....	44.93	3.12	12.51	.64	34.28	4.43	.50	7065
Dax, Bohemia, Austria..	50.12	4.06	13.14	.65	25.50	6.53	.93	8334

He gives the following as a characteristic average analysis:

LIGNITE (BROWN COAL) GENERAL AVERAGE COMPOSITION

Carbon	50–65%
Combustible H	1– 2%
Chemically combined water	20–30%
Moisture water (hygroscopic)	10–25%
Ash	5–10%

Many lignites contain water, as water of crystallization, so that, on drying even in the sun, loss of water will sometimes cause a freshly mined, hard, shiny piece to fall into a heap of sand-like particles. This is important in the problem of firing this fuel because varieties that behave like this, may fall through a boiler grate or pack a gas producer bed, so that the air blast cannot be forced through. When they contain bituminous matter that melts on heating this will act as a binder and may correct the disintegration evil in boiler fires or perhaps increase the packing difficulty in gas producers. There are among lignites, as among peats, very great ranges of change and variety but all varieties contain much water and yield much non-combustible matter in the volatile gases. The latter fact is the reason why classifications based on the per cent fixed carbon or its relation to the volatile of coals, often fails in distinguishing lignites from bituminous coals and why also even the total carbon hydrogen ratio also fails, though, of course, it is better since the ratio is very different for two fuels with the same total carbon one with mainly hydrocarbon volatile and the other with much carbon dioxide. In America lignites are found in Dakota, Texas, Arkansas, Louisiana, Mississippi, and Alabama with other varieties more properly bituminous in Montana, Idaho, Washington, Oregon, California, Wyoming, Utah, Colorado New Mexico, and Texas.

Bituminous coal is next to lignite in order of age, but embraces in its own class so great a variety of coals as to lead to the different designations of coking, non-coking, long flame and short flame, and to the four sub-classes based on composition, proposed by Campbell, in addition to a semi-bituminous

lass. It requires only a brief inspection of the general coal table at the end f this chapter, Table CIV, where are recorded the analysis of nearly two undred coals of all classes and grades, to show how hopeless must be any ttempt to draw a sharp line between bituminous coals of low grade and the gnites. In general practice perhaps no single thing distinguishes a bitu- inous coal more than the considerable quantities of very rich hydrocarbon olatile that characterizes the best varieties, such as come from the center f the Appalachian Mountains, from Pennsylvania to Ohio on the west, to labama on the south or from separate deposits in many Western States. erhaps the best division of bituminous varieties for practical purposes, however ndefinite it may be, is that descriptive of the flame or the coke. According o this, the lowest coal in the bituminous series is the non-caking long-flame oal.

Non-caking long-flame bituminous coals, called by the Germans "sand oals," leave coal particles after heating, unchanged as to form, except as to he cracking of large masses, but give off combustible hydrocarbons gases very reely. They are black or brown in color, generally hard, and include the English and Scotch splint coals, which have been used for blast furnaces nstead of coke and also for reverberatory furnaces. They occur in America but are most common in England and on the Continent. According o Sexton the English varieties contain about 40 per cent volatile and from 5 per cent to 55 per cent fixed carbon and yield about 60 per cent coke, which indicates that in coking some of the hydrocarbon volatile is decom- posed and its carbon deposited with the fixed carbon. According to Jüptner he total carbon of these coals is between 75 per cent and 80 per cent, hydrogen 5.5 per cent to 4.5 per cent, oxygen and nitrogen 19.5 per cent to 15.5 per ent, indicating the presence of non-combustible in the volatile. The calorific ower is placed by him, between 14,400 and 15,300 B.T.U. per pound, which s very high compared to lignites.

Cannel coals belong more nearly to this class than to any other, but are uite different in many ways from the general run of coals, hardly fitting nto such a series as is here described in any really logical place. They are ull black, hard but easily broken, give a very large amount of gas on heating nd a long flame when burning, crack as they burn but do not melt or cake. hey appear to be related to the shales that yield oil on distillation more han to other coals, a fact that is brought out by the very small fixed carbon ontent given by Sexton as only 7 per cent for English boghead cannel and he correspondingly large per cent of ash given as 22 per cent. This same oal yields, however, about 29 per cent of coke on roasting, proving how freely he volatile hydrocarbon is decomposed and in so doing deposits its carbon vith the fixed carbon to make coke. Industrially cannel coals are used only or gas making by retort roasting processes.

Bituminous Gas Coals come next in order and are distinguished from the revious class by the melting, caking or coking tendency during heating, nd by the greater volatile content or gas yield, and its hydrocarbon character,

though they contain nitrogen diluents and yield ammonia. Lumps will stick together but not puff up, nor melt completely with loss of original form, during heating. They are especially adapted to coal-gas making by the retort roasting process, to reverberatory furnaces because of their long flames, but may also be used to advantage in boiler furnaces. According to Jüptner the total carbon content is greater than for the non-caking variety, being between 80 per cent and 85 per cent, the hydrogen between 5.8 and 5 per cent, so that this is also a little higher, and oxygen and nitrogen 14.2 per cent to 10 per cent, somewhat lower, indicating gases of more distinctly hydrocarbon character with less non-combustible diluents. This also is indicated by a higher calorific power placed between 15,300 and 15,840 B.T.U. per pound.

Bituminous Furnace Coals is the term applied by Sexton to the next in the series and they have more strongly developed caking tendencies, softening and swelling during combustion and yielding long, luminous gas flames by the combustion of the hydrocarbon volatile. According to Sexton the volatile matter is between 25 per cent and 35 per cent, materially less in total quantity than for the long flame non-coking coals. The coke yield is from 65 per cent to 75 per cent, and the fixed carbon from 1 to 7 per cent, less than the coke indicating once more the decomposition of volatile in coking and deposit of soot with the fixed carbon. Jüptner places the total carbon between 84 per cent and 89 per cent, total hydrogen 5 per cent to 5.5 per cent, oxygen and nitrogen 11 per cent to 5.5 per cent. The rise of the hydrocarbons is indicated also by the rise of calorific power, which is placed between 15,840 and 16,740 B.T.U. per pound. These coals are used in England for domestic purposes and everywhere for boiler fires, forges, reverberatory furnaces and coal-gas making.

Bituminous Coking Coals are those next in order, distinguished not only by a strong melting tendency on heating, and by the very hard, large masses of coke they yield from slack coal, but equally important by a lesser gas yield which is responsible for the designation *short flame* applied to them. This is the most important industrial class of coals as it is applied to the making of blast furnace coke and is good for boiler work. The coke yield varies from 65 per cent to 80 per cent, depending as much on the oven as on the coal itself and the content of oxygen and nitrogen is less than for the preceding varieties. Lewes gives the following compositions, Table LXIV, for some good English coking coals which may be compared with others in the general coal table at the end of the chapter.

TABLE LXIV

COMPOSITION OF ENGLISH COKING COALS (LEWES)

Source.	C	H_2	O_2	N_2	S	Ash.	Coke.
Durham	83.47	6.68	8.17	1.42	.60	.20	62.70
South Wales	83.78	4.79	4.15	.93	1.43	4.41	72.60
Derbyshire	79.69	4.94	10.28	1.41	1.01	2.65	59.32
Lancashire	77.90	5.32	9.43	1.30	1.44	4.88	60.22

Semi-bituminous, Semi-anthracite, and Anthracite coals are not distinguishable on the basis of coke or flame properties but rather on the basis of hardness and composition and they merge very gradually from the last group in the order named to graphite. The volatile becomes of a simple character, being almost entirely methane in the anthracites and small in quantity, usually less than 4 per cent, whereas the more distinctly bituminous coals contain very large quantities of volatile which are complex hydrocarbons, some of them of the liquid form, termed tar, and which may be over 5 per cent by weight while the total volatile may be quite high. The total carbon content of anthracites may reach 98 per cent and as they are very hard, are not so easy to burn. Some varieties termed graphitic coals cannot be burned at all except by mixing with other varieties and only in the hottest fires. The calorific power of the anthracites approaches very closely to that of pure carbon, the small excess heating value of the volatile balancing the ash, but there is a most important exception found in small sizes. Anthracite generally occurs with slate streaks, which, as the coal is broken into commercial sizes, is separated out by hand and as sizes become smaller the slate cannot be distinguished from the coal so that the smaller sizes often contain as much as 35 per cent of ash. This relation is shown approximately in the following Table LXV, applying to the Wilkes-Barre and Scranton, Pa., products.

TABLE LXV

WILKES-BARRE ANTHRACITE SIZES AND AVERAGE ASH CONTENTS

Name.	Passes through Hole, Inches.	Passes over Hole, Inches.	Ash.	
			General Average.	One Mine.
Broken	$3\frac{1}{2}$	$2\frac{3}{4}$	5.00	
Egg	(*a*) $2\frac{3}{4}$	2	5.5	5.66
	(*b*) $2\frac{1}{2}$	$1\frac{3}{4}$		
Stove	(*a*) 2	$1\frac{3}{8}$	8	10.17
	(*b*) $1\frac{3}{4}$	$1\frac{1}{4}$		
Chestnut	(*a*) $1\frac{3}{8}$	$\frac{3}{4}$	12	12.67
	(*b*) $1\frac{1}{4}$	$\frac{3}{4}$		
Pea	$\frac{3}{4}$	$\frac{1}{2}$	15	14.66
Buckwheat	(*a*) $\frac{1}{2}$	$\frac{3}{8}$	19	16.62
	(*b*) $\frac{1}{2}$	$\frac{1}{4}$		
Rice or buckwheat, No. 2	(*a*) $\frac{3}{8}$	$\frac{3}{16}$	25	
	(*b*) $\frac{1}{4}$	$\frac{1}{8}$		

Semi-anthracites are usually the border coals of the anthracite fields as the semi-bituminous are border coals of more distinctly bituminous fields, but the latter may more often separately occur. In America these semi-bituminous coals include those from Clearfield, Cambria, and Somerset counties, Pa., Cumberland, Md., Pocahontas, Va., and New River, West Va. They are perhaps most definitely fixed by the volatile content of 18 per cent to

22 per cent, low ash and sulphur, and are considered the best boiler coals in America.

More than any single thing the character of the volatile characterizes the different coals and serves to group them naturally into classes, the bituminous coals, more especially those of the caking or coking class, having complex hydrocarbon mixtures with much of the liquid variety yielding tar; the non-coking semi-bituminous varieties yield a simpler set of hydrocarbons with anthracite at one end characterized by almost pure methane volatile, and lignites and peats at the other, characterized by much non-combustible matter in their volatile diluting the hydrocarbon constituents. It is not possible to say that the volatile gases or vapors that finally appear really existed in the coal as such. It is more likely, as has been pointed out, that they did not, but appear as products of decomposition of other oxygenated compounds.

In Table CIV there is presented a collection of proximate and ultimate analyses selected from thousands available, by reason of the high authority and reputation for accuracy of the analysts reporting them and because also they fairly well represent, when grouped together, the whole range of solid fuels from the peats to the anthracites. For each fuel the calorific power, as determined by the bomb calorimeter, is given and will be used later as a basis for a general study of calorific power. The reports of Mahler are representative of French, those of Bunte, of German, and those of Lord, working first with Haas and later alone as chemist for the U. S. Geological Survey, of American coals and scientific authority, and it is remarkable how well their results fi together. These analyses together with a few others, perhaps less authoritative constituting about 200 different coals, have been classified according to Camp bell's carbon hydrogen ratio for convenience, but some little errors ar introduced by the lack of consistency in moisture which is different for all an not reported by Mahler.

The impurities in coal, notably the ash and sulphur, have an appreciabl influence on the uses to which coals may be put, often not inferior, to th properties of the volatile or the calorific power of the coal. Sulphur generall occurs as iron pyrites, FeS_2, in nuggets or veins and on combustion usuall gives off SO_2 as a gas, leaving iron oxide, Fe_2O_3, as part of the ash if ther is insufficient air, as is usually the case in the interior of the cinder, while i the region of high temperature there will be formed iron sulphide, FeS, an sulphur which escapes as vapor. Sulphur is also present in some coals a calcium sulphate or gypsum, which leaves calcium sulphide when heated i contact with carbon. Sulphur in almost any form will make compounds wit other things present, which fact bears on the availability of the fuel for certai metallurgical operations; for example, the making of iron. Sulphides, especiall of iron, are apparently important factors in the fusing of the ash and th making of clinker. Both in the solid combined form and as a gas, sulphu is actively corrosive and grate bars as well as boiler plates and the pipes an scrubbers of gas generators will suffer from its presence so that high sulphu

oal is considered generally undesirable though sometimes some sulphur may e removed by washing.

Fusion of ash, and clinker formation is a serious matter, limiting the air upply, sticking to grate bars and when it contains sulphur actually corroding he bar by chemical reaction, while in gas producers clinker collection is often o severe as to stop the operation entirely. In fact certain coals cannot be used n producers at all because of this tendency, though otherwise desirable. The formation of clinker starts with the fusion of some one of the constituents f the ash or the fluxing of one constituent by another, the liquefied material owing over and enclosing solid matter, even large pieces of unburned coal, nd tending to stick to other pieces, to firebrick linings or to grate bars. The linkering tendency of a given coal cannot be predicted even when the sh is completely analyzed and the melting-points of its constituents nown, because the possible compounds of those constituents are not nd cannot be known. The chemical compounds in ash are usually xides of aluminum Al_2O_3, silica, SiO_2, iron FeO and Fe_2O_3, calcium CaO, otassium K_2O, magnesium MgO, sodium Na_2O, sulphur SO_2, with many nore and all combined with each other in unknown ways. Those occurring n the largest quantities are usually the alumina and silica, the melting-points f which are 3400° F. and 2510° F. respectively, which are within range of he usual furnace temperature of 3000° F. or thereabouts. The melting-oint of either of these constituents or a mixture of them is lowered by fluxing vith the other compounds, making glasses and slags, some of which melt below 000° F., so that in practically every ash some fusion will occur and clinker orm if the fire is pushed at all. Just what is to be done in any case to avoid rouble, in view of the certainty of some clinker and the possibility of a pro-ibitive amount cannot at present be settled on any scientific grounds, but s a matter of trial and judgment, first in the treatment of the most available oal and second in the selection of a substitute.

Prob. 1. Into what class of coal, anthracite, bituminous, etc., would coals Nos. 1, 2, 16, 25, 49, 64, 148, 188, and 194 of Table CIV fall?

Prob. 2. From Table CIV, classify according to Muck, the following coals, Nos. 5, 5, 90, 116, 141, 189, 191, 197.

Prob. 3. A coal gave as an ultimate analysis of the combustible, H = 9.5; C = 91; $(S+O_2+N_2) = 5.5$; and as proximate analysis, fixed carbon, 83 per cent; volatile, 7 per cent. Classify it according to Frazer and Muck.

Prob. 4. The analysis of a coal gave, C = 87.7; $H_2 = 4.7$; $O_2 = 5$; ash = 2.6. From the as and coke qualities of Table LXI, what names could be applied to this coal?

Prob. 5. How do the classifications according to Muck, Frazer, and Campbell gree on the following coals? Nos. 9, 25, 50, 60, 184?

Prob. 6. Taking Jüptner's figures of, C = 75–80 per cent, $H_2 = 5.5$–4.5 per cent, and $(O_2+N_2) = 19.5$–15.5 per cent, as the limits for non-caking, long-flame coals, which of hose given in the general table would fall in this class?

Prob. 7. How do the analyses of the lignites given in the table compare with the verage as given by Jüptner?

Prob. 8. Sexton gives 7 per cent as a fair value for the fixed carbon for English cannel coal. What coals in the general table substantiate this value?

Prob. 9. To what coals in the general table could the name gas-coal be applied using Jüptner's values as a standard?

3. Calorific Power of Coals and the Combustible of Coals. Calculation of Calorific Power from Ultimate and Proximate Analyses. Calorific Power of the Volatile. If the carbon, hydrogen and sulphur in coal were just mixed together, the calorific power would be the sum of the products of each fractional weight into the calorific power of the respective element, but no such simple relation exists, so that calculation of calorific power from the ultimate analysis must be based on some sort of assumptions as to molecular relations expressed or implied.

Most of the ordinary fuels are mixtures of chemical compounds of unknown molecular constitution and usually great complexity of molecular structure not at all revealed by ultimate analysis and only partly so by proximate analysis, and in the case of the liquids by fractional distillation. They are all compounds and mixtures of the elements carbon, hydrogen, nitrogen, oxygen, sulphur, and with or without neutral ash. This fact lies at the basis of most of the empiric formulas for their calorific power, for if the fuel elements C, H_2, and S, shown to be present by an ultimate analysis be assumed to be just mixed together in the weights given, the calorific power is the sum of that of all the parts and can be expressed symbolically.

Let Q_H = B.T.U. per pound of hydrogen;
" Q_C = B.T.U. per pound of carbon;
" Q_S = B.T.U. per pound sulphur;
" Q = B.T.U. per pound fuel;
" w_H = weight of hydrogen per pound fuel;
" w_C = weight of carbon per pound fuel;
" w_S = weight of sulphur per pound fuel.

Then

$$Q = Q_C w_C + Q_H w_H + Q_S w_S. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (754$$

This, however, takes no account of the oxygen which may be assumed to be free and so ignored, or assumed to be combined as an incombustible. By weight, 1 lb. H + 8 lbs. O_2, make 9 lbs. water, so that each pound of O_2 that is combined with H_2 as water takes from the total H_2 present $\frac{1}{8}$ of the oxygen weight, leaving as burnable only what is left or $(w_H - \frac{1}{8}w_O)$. Introducing this expression in the formula above

$$Q = Q_C w_C + Q_H (w_H - \tfrac{1}{8} w_O) + Q_S w_S, \quad . \quad . \quad . \quad . \quad . \quad (75$$

or factoring,

$$Q = Q_C \left[w_C + \frac{Q_H}{Q_C} (w_H - \tfrac{1}{8} w_O) + \frac{Q_S}{Q_C} w_S \right]. \quad . \quad . \quad . \quad . \quad . \quad (75$$

This is the form of one of the first empiric formulas as given by Dulong, the sulphur effect was neglected and 14,500 and 62,100 in round numbers taken as the calorific power of carbon and hydrogen respectively, giving the form

$$Q=14{,}500w_C+62{,}100(w_H-\tfrac{1}{8}w_O). \quad . \quad . \quad . \quad . \quad . \quad (757)$$

In the American Society of Mechanical Engineers Code for Boiler Testing there is recommended a formula which retains the sulphur effect and assigns 4000 as the calorific value, changing the others for carbon and hydrogen to give

$$Q=14{,}600w_C+62{,}000(w_H-\tfrac{1}{8}w_O)+4000w_S \quad . \quad . \quad . \quad . \quad (758)$$

Another older formula given by Mahler neglects both sulphur and combined oxygen-hydrogen effects, but introduces an empiric constant as follows:

$$Q=20{,}100w_C+67{,}600w_H-5540. \quad . \quad . \quad . \quad . \quad . \quad . \quad (759)$$

None of these expressions or others to be noted will agree with experimental determinations due to the very wide complexity of molecular structure of the fuels they are supposed to represent. Most of them have been developed for coals of a single class, thus, for example, the Dulong expression Eq. (757) was intended to nearly represent a class of bituminous coals characterized by much volatile hydrocarbon while Dulong gave another for lignites, the peculiarity of which is a high water constant as follows:

$$Q=14{,}500w_C+5333(w_H-\tfrac{1}{8}w_O)-1147(1\tfrac{1}{8}w_Ow_W) \quad . \quad . \quad . \quad (760)$$

where w_W is hygroscopic water, $1\frac{1}{8}w_O$ combined and fixed water, and the negative term is supposed to represent the heat to evaporate all the water present.

As an example of an attempt to base the formula at least in part on the physical character of the coal that of Goutal, Eq. (761), is especially interesting and suggestive:

$$Q=14{,}670w_C'+Kw_V, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (761)$$

where w_V is the volatile hydrocarbon weight, while w_C' represents the fixed carbon or coke, K is a constant supposed to have values, according to Goutal, as follows:

When volatile by	weight is	2–15%	then K is	23,400
"	"	15–30%	" "	18,000
"	"	30–35%	" "	17,100
"	"	35–40%	" "	16,200

Jüptner quotes a formula of the International Union of Steam Boiler Inspection Societies based on ultimate analysis as given in Eq. (762).

$$Q = 14{,}400w_C + 52{,}200(w_H - \tfrac{1}{8}w_O) + 4500w_S - 1080w_W, \quad . \quad . \quad . \quad (762)$$

where w_W is the hygroscopic water. He also notes some differences between the results of this formula and calorimeter tests by L. C. Wolf as follows:

For bituminous coal	± 2%
For lignite	± 5%
For peat	± 8%
For cellulose	− 7.9%
For wood	±12%

It has been pointed out that the solid fuels may be regarded as the sum of a combustible and a non-combustible part; and the combustible as the sum of a fixed carbon and a volatile part. On this basis it is possible to make approximations of calorific power if the nature of the volatile is known or the average calorific power of the volatile, without an ultimate analysis, using instead the simpler and easier proximate analysis which does not require elaborate chemical laboratory equipment for its determination.

The principal combustible gases of the volatile are hydrocarbons and these are of fairly constant calorific power per pound over a very considerable range of character. The highest calorific power of all is that of methane, 23,640 B.T.U. per pound, and few of those hydrocarbons that enter into coal volatiles so far as is known, have less than 75 per cent of this. If, therefore it is found, as might be expected, that one group of coals is characterized by the same group of hydrocarbons in its volatile and in even approximately constant proportions the calorific power of that volatile would be almost exactly constant, at least as constant as the calorific power of all the combustible taken together. It is also reasonable to expect that each class of coal would have a different group of hydrocarbon combustibles in the volatiles, the calorific power of which would vary accordingly, being highest for anthracites with methane as the principal constituent, lower for the bituminous varieties and least for lignites, the volatile of which is much diluted with non-combustible gases and vapors, similar to those yielded by wood. Thus the character of heating power of the volatile of the coals furnishes a new basis of classification with direct reference to availability as fuels.

To illustrate the peculiarities of the calorific power of coals and the dependence of calorific power on class of coal, a new table, Table CV, has been computed and some curves plotted, all derived from the data on the two hundred coal analyses reported in Table CIV. As a first step, the calorific powers as given in the latter table are plotted in Fig. 173, to horizontal coordinates of total carbon to total hydrogen, which is the basis of Campbell's classification. Vertically, there are laid off for each coal two points, one the calorific power

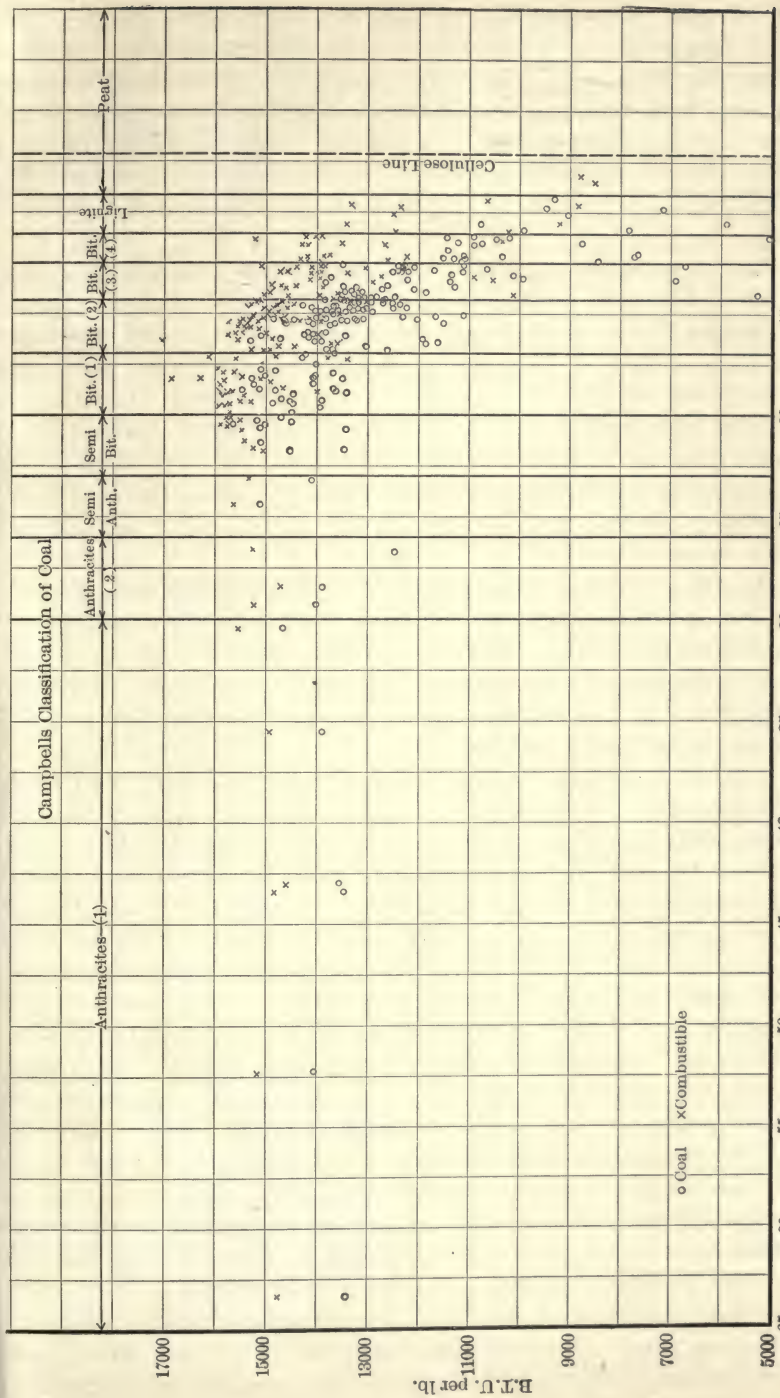


FIG. 173.—The Relation of Calorific Power of Coals, Lignites, and Peats to the Ratio of Carbon to Hydrogen.

per pound of coal as reported from the calorimeter test, and the other the heat of formation of the products. Vertical classification bands are drawn across the sheet to divide the classes. From this chart it appears that there is no very definite relation between calorific power and the Campbell class of the coal, though there is a distinct falling off toward the ratios 15 and over. Great confusion exists in the bituminous region and from there to the peats; so great as to make any representative curve an impossibility.

The failure to establish any definite relation between calorific power and the total carbon-hydrogen ratio, as illustrated in Fig. 173, naturally suggests a trial on another basis. Accordingly, the calorific powers, both calorimetric values and that by heat of formation of products, are replotted in Fig. 174 to a new horizontal coordinate, suggested by Frazer's classification, the ratio of fixed carbon to volatile. Here the points are differently distributed, but with no more hope of a smooth curve being drawn, through them. Again there is confusion in the bituminous region, especially where the fixed carbon to volatile ratio is less than 2, as here almost any calorific power may be found for the same value of the ratio.

The explanation of this situation must be found in the nature of the combustible and in its amount and kind, but especially in kind. As combustible is partly fixed carbon and partly volatile, and as fixed carbon has always the same calorific power, the real explanation must be sought in variations in the nature of the volatile itself, and, of course, its amount. In order that this possibility may be traced, the chemical and thermal properties of these coals are redetermined, as ash and moisture free, in Table CV at the end of the Chapter, which, therefore, gives the properties of the combustible matter only. In this table the calorific power of the combustible is reported, total and as divided between the fixed carbon and the volatile parts and finally the calorific power of the volatile itself per pound is found. To get this result, the calorific power of the coal as determined by the bomb calorimeter is divided by the sum of the partial weights of fixed carbon and volatile, the quotient being the calorific power per pound of total combustible. The product of the fractional weight of the fixed carbon and 14,544, its known calorific power, gives the heat due to the combustion of the fixed carbon part of the combustible, and this subtracted from the B.T.U. per pound of combustible gives the heat per pound of combustible derived from its volatile. The heat per pound of combustible derived from its volatile only, when divided by the fractional weight of volatile in the combustible gives the B.T.U. per pound of volatile itself. These values of the calorific power of the volatile matter of coals from Table CV are plotted in Fig. 175 and through the points a fair curve can be drawn at least as fair, as the determination of the values from the indirect experimental results of the calorimeter for the whole coal, are themselves accurate. It is reasonable to expect that direct determination of the calorific power of volatile alone, will lead to a much smoother curve, but even with the present data it is possible to make a really useful approximation

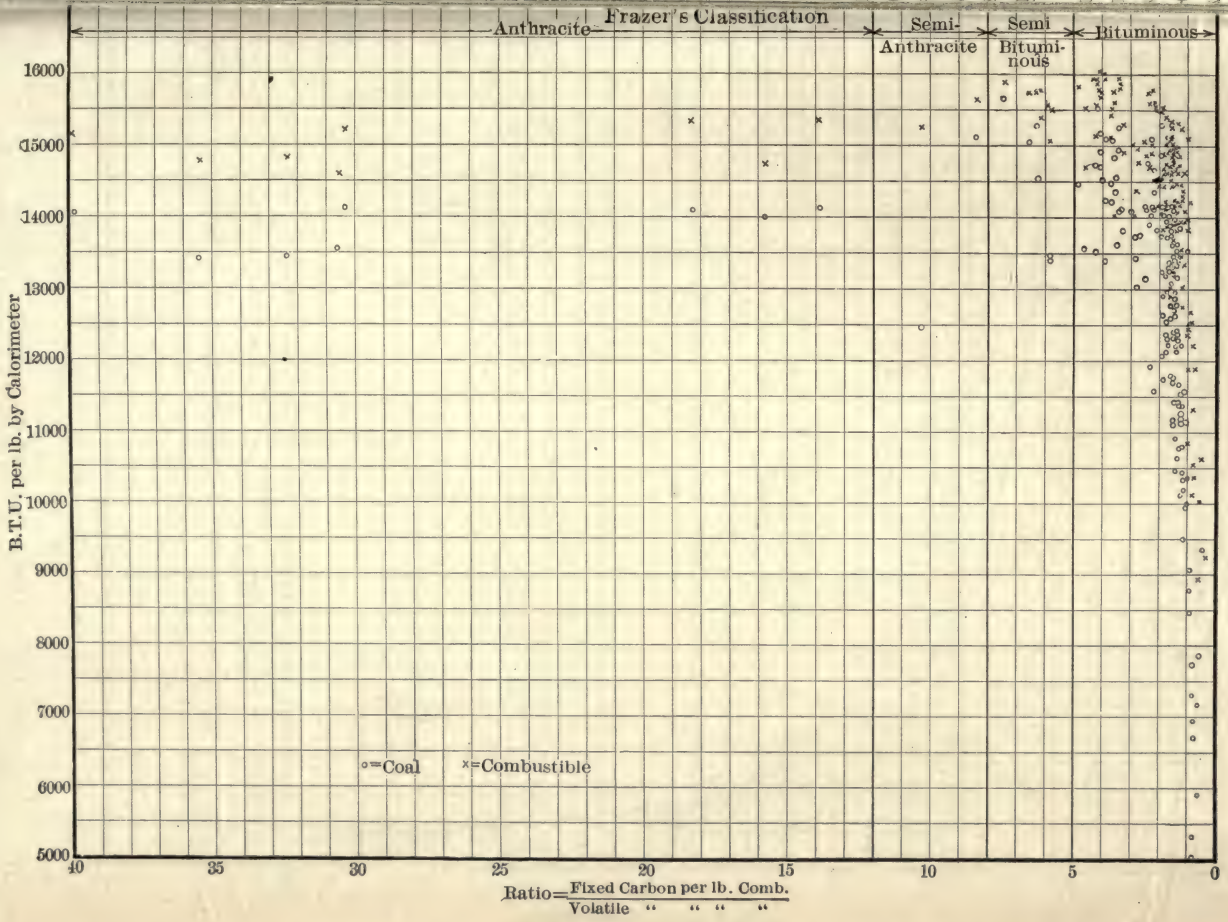


FIG. 174.—The Relation of the Calorific Power of Coals, Lignites, and Peats to the Ratio of Fixed Carbon to Volatile.

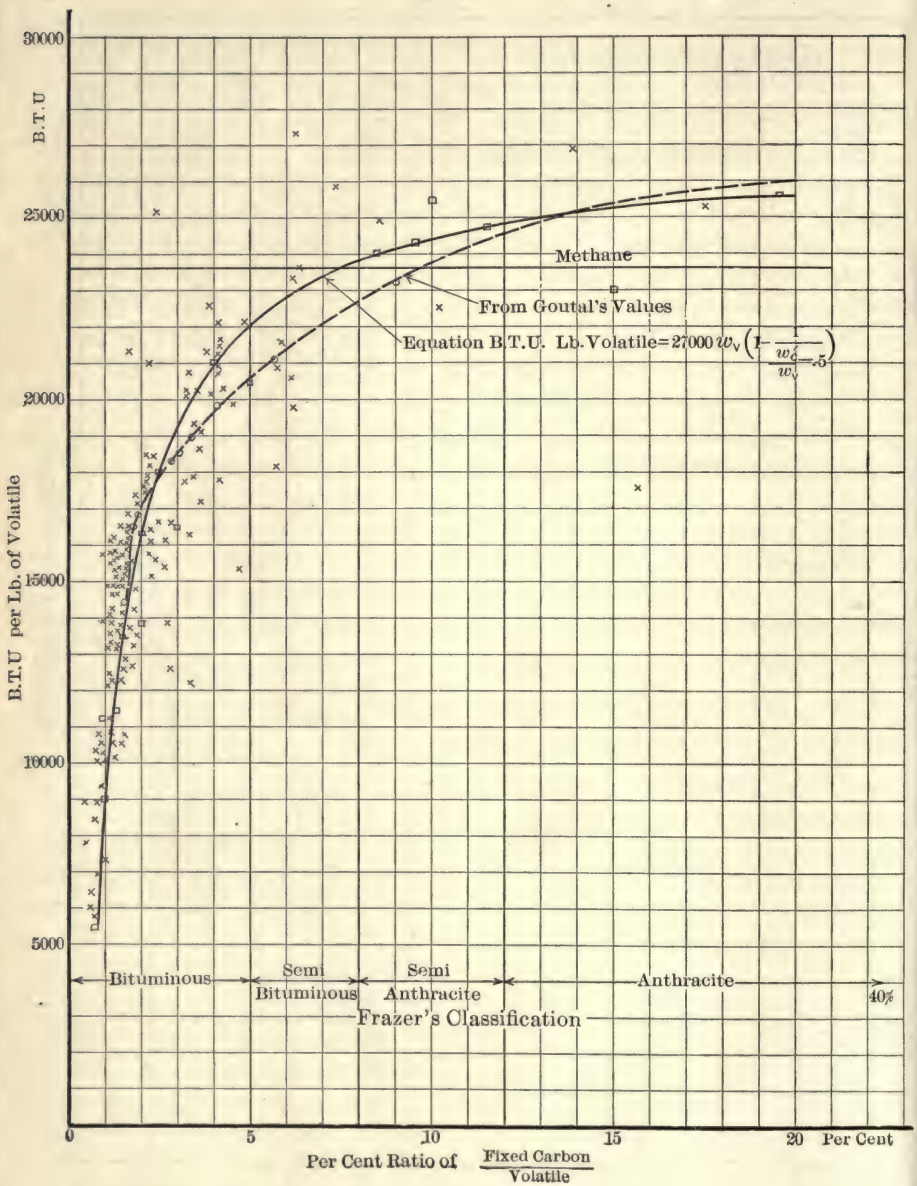


FIG. 175.—Calorific Power of the Volatile of Coals.

to the calorific power of a coal from its proximate analysis by the formula Eq. (763) derived from the smooth solid curve of Fig. 175.

Let w_C' = fractional weight of fixed carbon in coal;
" w_V = fractional weight of volatile in coal.

Then

$$Q = 14{,}544 w_C' + 27{,}000 w_V \left[1 - \frac{1}{\left(\frac{w_C'}{w_V}\right) + .5} \right] \quad . \quad . \quad . \quad . \quad (763)$$

This equation is of the same general form as Goutal's Eq. (761), but with his constant C evaluated as a function of the fixed carbon to volatile ratio, instead of assigning it separate values for each class of coal by the volatile content alone. The differences due to this are indicated in Fig. 175 by the relation of the dotted to the solid line.

These relations not only make it possible to calculate the calorific power of a coal with fair accuracy from its easily found proximate analysis, but more important in a broad sense it brings out the relation of the coals as fuels to the oils and natural gas fuels as will appear a little more clearly when these latter are examined.

Example. A coal which gives a heating value of 13,961 B.T.U. when tested in the calorimeter, was found to have an ultimate analysis as follows: C = 80.03, H_2 = 4.13, N_2 = 1.40, O_2 = 3.20, S = 1.90, ash = 9.34. How do the heating values derived by means of the Dulong Eq. (757) and A.S.M.E. Eq. (758) compare with the experimental?

Substituting the above values in Eq. (757),

$$Q = 14{,}544 \times 80.03 + 62{,}100 \left(.0413 - \frac{.0320}{8} \right) = 13{,}952 \text{ B.T.U. per lb.}$$

and

$$Q = 14{,}600 \times .8003 + 62{,}000 \left(.0413 - \frac{.0320}{8} \right) + 4000 \times .019 = 14{,}069. \text{ B.T.U. per lb.}$$

Prob. 1. Which of the following coals would be worth more per ton on the basis of the heating value found by the A.S.M.E. formula; by the others?

For the first C = 83.75; H_2 = 4.13; S = .57; O_2 = 2.65.
For the second C = 74.39; H_2 = 4.98; S = 3.44; O_2 = 6.42.
For the third C = 73.5; H_2 = 5.19; S = 2.54; O_2 = 8.05.

Prob. 2. A boiler horse-power is 33,486 B.T.U. received by the water in the boiler, per hour. If 70 per cent of the heat of the coal is available, what will be the horse-power of a boiler under which is being burned one ton per hour of the following coal, the correct calorific power of which is given by the Dulong expression? What is the weight of the coal per hour per boiler horse-power? C = 79.20; H_2 = 4.30; O_2 = 2.15.

Prob. 3. Natural gas of 1000 B.T.U. per cubic foot may be purchased in a certain locality for 10 cents a thousand cubic feet. What price could coal containing 75 per cent of C, 6 per cent of H_2, 1.5 per cent of O_2, and 1.5 per cent of S, bring in the same locality on the basis of heat content only?

Prob. 4. A gas engine is operated on producer gas. The producer is supplied with coal of the following composition: C = 80.15 per cent; H_2 = 3.35 per cent; N_2 = 1.32 per cent; O_2 = 4.28 per cent; S = .9 per cent; ash = 10 per cent. What is the efficiency for the system if the engine delivers one horse-power for each pound of coal fed to the producer?

Prob. 5. In a boiler plant it has been found that of the entire heat in the coal 60 per cent gets to the steam, 25 per cent escapes to the stack, 13 per cent to the ashpit, and the remainder is radiated from the boiler setting. For a coal of the following composition, what would be the number of B.T.U. per pound of coal in each loss and what would be the number actually used? C = 70; O_2 = 2.5; N_2 = 2; H_2 = 12_2; S = .5; ash = 13.

Prob. 6. What will be the per cent error in determining the calorific power of the following coals by Eqs. (754)–(763), the value by the calorimeter being considered correct? Nos. 12, 45, 98, 126, 147 of Table CV.

Prob. 7. Coal is bought by the calorific value as found by the formula of the A.S.M.E. code. What bonus or penalty would be involved if the true value is that of the calorimeter for the first five coals of the table?

Prob. 8. For coal No. 20 which of the formulas gives a result closest to the calorimeter value? Which for coal No. 2?

Prob. 9. What would be the relative error in assuming that Eq. (763) gave the true heating value of coal No. 9 compared to the value as found from the ultimate analysis, the calorimeter value being used as the true one?

4. Mineral Oil and Natural Gas Fuel. Chemical and Physical Properties. Calorific Power Direct and as Calculated for Oils from Ultimate Analysis or from Density, and for Gas from Sum of Constituent Gases. Mineral oils, no matter from what part of the world they come, are very much alike in ultimate composition, but most amazingly different in detailed properties such as boiling point. When clean, all are absolutely ash free, all yield vapors on heating and have calorific powers very close together, compared to the variations noted for coals. Analysis shows them to be hydrocarbon compounds or rather mixtures and solutions of a large number of hydrocarbons of the paraffine, ethylene and naphthalene series, the former predominating in American and the latter in Russian oils, these two countries furnishing the bulk of the world's supply having produced respectively 16½ million and 7 million tons in 1905.

From Pennsylvania crude oil the hydrocarbons in Table CVI at the end of the Chapter, of the paraffine series have been separated as reported partly by Lewes, and from Russian oils the corresponding ethylenes, and two naphthalenes by Redwood. In the paraffines, there are noted four isomers that is, compounds having the same ultimate analysis and fractional weight but different physical properties, while with the ethylenes, all of which have the same ultimate analysis but different molecular weights, are noted two naphthalenes which are isomeric modifications of the ethylenes, having the molecular arrangement indicated by the formula $C_nH_{2n-6}+H_6=C_nH_{2n}$

vhich have the same weight proportions but whose properties are more closely hose of the paraffines than the ethylenes. As pointed out in Chapter IV, he calorific power of all hydrocarbons varies with the value of n or their osition in the series and is somewhat different for the same value of n in differnt series. It is also true that both liquid and vapor densities vary with the alue of n so it might be expected that the calorific power would vary with the lensity of the oils, if they did not contain excessively variable proportions of lifferent series. This was foreshadowed by the Slaby formula for the hydrarbon gases and vapors which expresses calorific power as a linear function of lensity of those gases and is confirmed by the work of Sherman and Kropff at olumbia University, for the oils. In Table CVII at the end of this Chapter re given the results of their calorimeter determinations on 64 oils, ranging rom specific gravity of .7100 to .9644 or 67.7° Bé. to 15.2° Bé., together with he calculated value as a linear function of density by Eq. (764).

$$\text{B.T.U. per lb. oil} = 18{,}650 + 40(\text{Bé.} - 10) \quad . \quad . \quad . \quad . \quad (764)$$

n only $\frac{1}{9}$ of the cases is the error more than 1 per cent, in only $\frac{1}{30}$ over 2 per cent nd never exceeds 3 per cent, which is fairly satisfactory. This makes it posible to estimate calorific power of oils by simply taking the density, about as losely, in some cases more so, as could be done by an ultimate analysis and the pplication of the calorific powers of carbon and hydrogen to the respective ractions of each.

Inspection of the general table of properties of mineral oils, Table CVIII t the end of the Chapter shows that the carbon hydrogen ratio, which for the thylenes and naphthalenes is constant and equal to 6 and for the paraffines lways less than this, does sometimes exceed this value. In such cases it must e assumed that solid carbon is present, having been deposited by the decomosition of some of the oil under heat treatment before or after leaving the earth, nd such oils are invariably heavy and black in color. No natural or crude oil s heavy or dense because it consists only of heavy hydrocarbons, nor is any ne light because its hydrocarbons are exclusively light ones, but as all crudes ontain both heavy and light constituents the density is chiefly an indication f which class of hydrocarbons, the light or the heavy, predominate. These elations, which are of fundamental importance, in fixing the industrial value f an oil and in prescribing the treatment it shall receive in applying it to oil ngines, boiler furnaces or the making of oil gas, will be brought out more early in a succeeding section under heat treatment for fractional distillation. his fractionation of oils is the only known way of reducing the range of hydrarbon constituents and is a sort of separation process.

The calorific power of the oils cannot be as accurately predicted from their ltimate analysis by computation of the heat of formation of the products of ombustion, as it can fromt he density by Sherman and Kropff's formula, hough there seems to be in some cases, large differences between the experiental bomb values and those for the formula. In most cases if not all, these

can be traced to inaccurate experimental work, which, with oils, especially those that have light and easily volatile constituents, is a very difficult procedure. It is for this reason that two oil tables giving calorific powers are given, one containing the Sherman and Kropff values, known to be most accurate, and the other such values as are commonly reported in general engineering literature. To still more clearly bring out this point, there are plotted in Fig. 176 all the tabular values of calorific powers, exclusive of the heats of formation of the products, and through them the S. and K. line is drawn according to its equation. It will be noted how much better their experimental values lie with respect to this line than do others of more doubtful accuracy.

Natural gas like mineral oil is also chiefly a mixture of hydrocarbons though, of course, no such complexity of mixture can exist in the gases as in the liquids because so few of the hydrocarbons are gaseous at ordinary temperatures; most

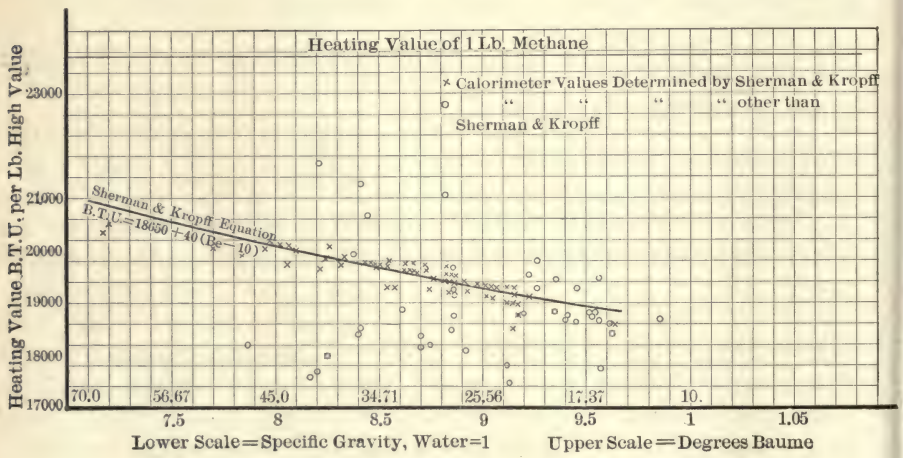


FIG. 176.—Calorific Power of Mineral Oils.

of them being liquids and a few solids. Accordingly, natural gas is a pretty definite thing and analysis shows it to be largely and in some cases almost entirely methane CH_4. One sample, No. 1, from the series of Table CIX at the end of the Chapter, that from West Virginia, is over 99 per cent methane, but another sample, No. 38, reported for the Pittsburgh district is only about half methane, and contains nearly 36 per cent by volume of free hydrogen. Practically all natural gases contain some CO, which is a product of oxygen reacting on carbon, probably in a coal and as might be expected there is correspondingly some nitrogen present, as would be the case when the oxygen that combined had come from the air, leaving the nitrogen free after combustion. Similarly in many cases some CO_2 is present, indicating a more complete oxidation of the carbon and accounting for more nitrogen on the assumption that the oxygen had been derived from air.

One important class of chemical constituents of some natural gas is the higher hydrocarbons such as ethylene, but more peculiarly the

iat at ordinary temperatures are liquid and are present in the gas as vapors those liquids together with a group that liquefy under pressure. This has led a process of compression of natural gas with subsequent cooling for the covery of liquid forms of hydrocarbons most of which are easily volatile and l of which are high in illuminating value. A recent paper of the Bureau of lines by Allen and Burrel discusses the recovery of liquid forms from natural as by compression, to which reference may be made, but which is not analyzed ere because of lack of space.

The calorific power of natural gas, like any other gas, is a thing very uch easier to predict than that for coals and oils with reasonable precision hen its constituent gases are known. Each of these has a definite calorific ower per cubic foot at standard conditions of 32° and 29.92″ Hg, so that the lorific power is to be found by multiplying the volumetric proportion of each nstituent by the proper calorific power. The sum will be the calorific power the gas in B.T.U. per cubic feet (standard) and, of course, there will be oth a high and a low value for every gas containing any hydrogen, free or com-ned and this includes all of the natural gases. For convenience, in calcula-ons on temperature rise, due to combustion, the B.T.U. per pound is usually lded. When some of the constituents are the heavier hydrocarbons, which e not separated by the absorption methods of gas analysis, they are generally ported together as heavy hydrocarbons, as illuminants, or in some equivalent rms. In all such cases, heat calculations require that a chemical constitu-on be assigned to these, usually ethylene and benzole in some proportion, judgment based on experience may indicate. This is the only element of ncertainty as to calorific power that is worth noting, and at present there is remedy available.

To illustrate the method of determination of density and calorific power er cubic foot and per pound one case is set down in convenient tabular form, able LXVI.

TABLE LXVI

ENSITY AND CALORIFIC POWER OF NATURAL GAS FROM CONSTITUENTS
(32° F. and 29.92″ Hg)

Kansas Natural Gas.	One Cubic Foot				Summary
	Contains		Yields B.T.U.		
	Cu.Ft.	Lbs.	High.	Low.	
					B.T.U./cu.ft. gas high = 1046.3
ethane, CH_4	.9820	.043935	1090.5	944.7	B.T.U./cu.ft. gas low = 950.1
eavy hydroc., C_6H_6	.0010	.000240	5.0	4.6	Cu.ft/lb. gas = 22.42
arbon monoxide, CO	.0025	.000195	.8	.8	Lbs./cu.ft. gas = .0446
xygen, O_2	.0025	.000223	....	...	B.T.U./lb. gas high =23458.
Total for gas	1.0000	.044593	1046.3	950.1	B.T.U./lb. gas low =21301.

Prob. 1. The following oil was used in an engine and one horse-power hour (2545 B.T.U.) was obtained for each pound of the oil supplied. What was the thermal efficiency based on the heating value, (*a*) as derived by calculation from the ultimate analyses, and (*b*) by the Sherman and Kropff formula?

Sp.gr. at 60° F. = .926. $C = 83.36$; $H_2 = 12.41$; $S = .5$; $(N_2 + O_2) = 3.83$.

Prob. 2. At one time a melting device was used to clear away snow from city streets. The fuel used was oil and the heat generated was used to melt the snow placed a tank above the fire, the water running off to the sewer as soon as formed. Assuming 90 per cent of the heat of the fuel to be available, that the snow is put in at a temperature of 20° F. and that the water runs away at a temperature of 40° F., how many pounds of snow will be disposed of per pound of the following oil?

$C = 85.5$; $H_2 = 14.2$; $(O_2 + N_2) = .3$.

Prob. 3. To prevent frost damage in orchards, crude oil is burned in smudge pots and the temperature of the air in the orchard kept above 32° F. Taking the specific heat of air as .243, how many cubic feet can be heated from 30° to 40° F. by a gallon of California crude oil, the density of which is 16.85° Bé. and the ultimate analysis by weight is $H_2 = 11.3$ $C = 85.75$; $S = .67$? Compute the results by S. and K. formula and by heat of formation of products.

Prob. 4. The analyses given are for a West Virginia and an Ohio natural gas respectively by volume. On the basis of the heat of formation of the products of each, what would be their relative heating values?

	CH_4	H_2	CO	C_2H_4	N_2	CO_2	O_2
West Virginia..	99.5	....	...	...	....	...	.5
Ohio.....	93.35	1.64	.41	.35	3.41	.25	.39

Prob. 5. The oil, the analysis of which is given below by weight, may be had for 5 cents per gallon. At what price will the gas, the analysis of which is also given but by volumes, be an equally economical fuel, the economy being based solely on the heat value?

Oil: $O_2 = 82$; $H_2 = 14.8$; $O_2 + N_2 = 3.2$.

Gas: $CH_4 = 75.99$; $H_2 = 6.1$; $C_2H_4 = 18.12$; $CO_2 = .34$.

Prob. 6. From the ultimate analysis of the two compare the heating value of 1 bbl. (50 gals.) of oil No. 11, Table CVIII and one ton (2000 lbs.) of coal No. 98 and 191, Table CIV.

Prob. 7. A ton of coal occupies roughly 40 cu.ft. Compare the space occupied by a million B.T.U. in the form of fuel oil (Table CVIII) and a bituminous coal (Table CIV).

Prob. 8. Assuming a boiler efficiency of 70 per cent for both fuels, what will be the consumption of a crude oil (Table CVIII) and an anthracite coal (Table CIV) per boiler horse-power?

Prob. 9. An oil-burning locomotive is developing 1000 boiler horse-power. What would be the necessary oil tank capacity for a two-hour run? The oil used is No. 37 Table CVIII and the boiler efficiency is 65 per cent.

Prob. 10. What would be the space required to carry an equivalent amount of coal No. 24, for the same conditions as in Prob. 9?

Prob. 11. Oil and gas may both be procured in a given locality. Assuming the gas to be equivalent to No. 19, Table CIV, and the oil to be equivalent to No. 69 Table CVIII, what would the oil be worth per gallon to equal the gas at 7 cents per 1000 cubic feet based on heating value alone?

5. Charcoal, Coke, Coke Oven and Retort Coal Gas as Products of Heating Wood and Coal. Chemical, Physical, and Calorific Properties per Pound. Calorific Power of Gases per Cubic Foot in Terms of Constituent Gases. Yield of Gas and Coke per Pound of Coal. When *wood* is heated, a distillation process begins at about 400° F. before which the discharge is mainly water vapor, but after which a series of complex gases and vapors are liberated, some of which may condense. Some of these distillates were present as such in the wood, and are liberated unchanged, but most of them are compounds formed by the heat action in the fiber, sap and ash constituents by mutual chemical reactions. The products of heating wood as given by Jüptner show the surprising complexity of the process and products which are divisible into five groups:

PRODUCTS OF WOOD DISTILLATION (JÜPTNER)

(1) Hygroscopic water,

(2) Gas, consisting mainly of

- (*a*) Acetylene, C_2H_2
- (*b*) Ethylene, C_2H_4
- (*c*) Benzole, C_6H_6
- (*d*) Napthalene, $C_{10}H_8$
- (*e*) Carbon monoxide, CO
- (*f*) Carbon dioxide, CO_2
- (*g*) Methane, CH_4
- (*h*) Hydrogen, H_2

(3) Liquid tar, consisting of

- (*a*) Benzol, C_6H_6
- (*b*) Naphthalene, $C_{10}H_8$
- (*c*) Paraffine, $C_{20}H_{42}$ to $C_{22}H_{46}$
- (*d*) Retene, $C_{18}H_{18}$
- (*e*) Phenol, C_6H_6O
- (*f*) Oxyphenic acid, C_6H_6O
- (*g*) Crysylic acid, C_7H_8O
- (*h*) Phlorylic acid, C_7H_8O
- (*i*) Creosote { $C_7H_8O_2$, $C_8H_{10}O_2$, $C_9H_{12}O_2$
- (*j*) Resins

(4) Pyroligneous acid, consisting of

- (*a*) Acetic acid, $C_2H_4O_2$
- (*b*) Propionic acid, $C_3H_6O_2$
- (*c*) Acetone, C_3H_6O
- (*d*) Wood alcohol, CH_3OH

(5) Wood charcoal,

When the heating of wood takes place in closed chambers, the wood melts just as do the coking coals, but at temperatures in the neighborhood of 600° F. and it yields a sort of hard coke quite different from charcoal. The whole process is summed up in the following Table LXVII from Violette which also indicates the changes in the result brought about by gradually rising temperatures of distillation, which are briefly, an increased gas yield continuously, and a liquid yield first increasing and then decreasing, showing that at high tem-

TABLE LXVII

(PRODUCTS OF WOOD DISTILLATION (CHERRY) (VIOLETTE)

(Wood previously dried at 302° F. with moisture loss 13 to 17 per cent)

Temp. F. of Charring.	Per cent by Weight.		Composition of Charcoal.				Remarks.	
	Volatile.	Charcoal.	C	H_2	O_2+N_2	Ash.		
320	2.00	98.00	47.605	6.064	46.271	.085		Open chamber
356	11.41	88.58	48.936	5.840	45.123	.117		
392	22.90	77.10	51.817	3.995	43.976	.226	Partly dry wood	
428	32.50	67.50	54.570	4.150	41.394	.217	Partly charred wood	
464	49.21	50.79	61.307	5.507	32.705	.516		
500	58.77	40.23	67.890	5.038	26.494	.559		
536	63.84	36.16	72.639	4.705	22.095	.568	Red charcoal begins	
572	66.39	33.61	73.236	4.254	21.962	.569	Red coal (for hunting gun powder) gradually getting darker	
608	67.77	32.23	73.573	4.831	21.086	.519		
644	68.47	31.53	75.202	4.406	19.926	.478		
809	81.13	18.87	81.643	1.961	15.246	1.162	Black charcoal for military powder	
1873	81.25	18.75	81.974	2.297	14.149	1.597		
2012	81.60	18.40	83.292	1.702	13.794	1.225		
2282	82.06	17.94	88.138	1.415	9.260	1.199	Very black dense solid charcoal	
2372	82.54	17.46	90.811	1.583	6.490	1.151		
2732	82.69	17.31	94.566	.7395	3.840	.665		
3000(?)	85.00	15.00	96.517	.621	.936	1.946		
	Gases.	Charcoal	Liquid.					
320	1.0	98.0	1.0	49.017	5.305	45.532	.154	Wood turned brown, tube brown	Closed chamber
356	5.0	93.2	1.8	56.523	6.188	37.094	.198	Red brown coal, tube red tar drops	
392	1.0	87.4	2.6	61.042	5.247	33.427	.294	Charcoal, heavy tar deposit in tube	
428	1.2	86.4	1.6	66.418	4.983	28.015	.589	Common black charcoal	
464	1.5	82.5	2.5	67.134	5.167	25.923	1.771	" " "	
500	1.5	82.8	2.2	67.621	5.010	25.258	2.031	Tar drops on charcoal	
536	1.5	82.7	2.3	64.601	5.424	26.768	3.201	Very hard coal	
572	1.8	78.3	3.7	67.576	4.565	27.327	.584	Black molten mass, no wood structure	
608	2.0	78.7	1.3	65.618	4.760	25.523	4.072	Molten coal like soft coal coke	
644	2.0	78.5	1.5	77.075	4.707	14.041	3.838	" " " " " "	

peratures some of the liquid is decomposed into gaseous constituents. In all cases the yield of charcoal decreases but its quality changes. The charcoal at higher temperatures contains more carbon, about the same hydrogen and regularly lessening amounts of oxygen and nitrogen, the sum of which is, however, always large and indicates why peats and lignites near the woods have volatiles so weak in combustible matter. Quick coking produces only half the charcoal that results from a slow process.

Peat on heating also gives off an equally large variety of volatile products, the nature of which is intimately related to the fuel value and its treatment in boiler fires and producers. The following is the result of heating a Swiss peat obtained by Vohl which it is interesting to compare with that for wood, already given and for the coals to follow;

PRODUCTS OF SWISS PEAT DISTILLATION (VOHL)

1. Water, associated with liquids called tar water, 25 per cent by weight, consisting of

Bases { *a*. Ammonia; *b*. Methylamin; *c*. Picolin; *d*. Lutidin; *e*. Anilin; *f*. Cæspidin

Acids { *g*. Carbon dioxide; *h*. Carbon disulphide; *i*. Hydrocyanic; *j*. Acetic; *k*. Propionic; *l*. Butyric; *m*. Valieranic; *n*. Phenol

2. Gas, 17.62 per cent by weight, consisting of
   (*a*) Heavy hydrocarbons, C_nH_{2n} (*c*) Hydrogen, H_2
   (*b*) Methane, CH_4 (*d*) Carbon monoxide, CO

3. Liquid, 5.37 per cent by weight condensed from the gas and consisting of
   (*a*) Tar, specific gravity, .820
   (*b*) Heavy mineral oil, specific gravity, .855
   (*c*) Paraffine

4. Peat coal or coke, 25 per cent by weight.

The most striking difference between the products of peat and wood distillation from the fuel standpoint is the appearance of heavy hydrocarbon in the former in the gas form and an increase of it in the liquid forms or a general rise of hydrocarbon combustible matter in the volatile, because the liquids appear first as vapors mixed with the permanent gases. A comparison of the products of different *coking* peats is given in the following Table LXVIII from Kane and Sullivan who roasted 100 pounds in retorts.

TABLE LXVIII

PRODUCTS OF PEAT DISTILLATION (KANE AND SULLIVAN)

Origin.	Per Cent by Weight of Products.				Composition of Tar Water. Per cent. by Weight					Composition of Tar Per cent. by Weight		
					Ammonia.		Acetic Acid.					
	Water.	Tar.	Coke.	Gas.	NH_3	$(NH_4)_2SO_4$	$C_2H_4O_2$	Acetate of Lime.	Wood Alcohol.	Paraffine.	Fuel Oil.	Lubricating Oil.
Light and dense mixed Mt. Lucas Philipstown.....	23.60	2.00	37.50	36.90	.302	1.171	.076	.111	.092	.024	.684	.469
Light peat Wood of Allen.	32.27	3.58	39.13	25.02	.187	.725	.202	.302	.171	.179	.721	.760
Dense peat Wood of Allen.	38.10	2.76	32.65	26.49	.393	1.524	.286	.419	.197	.075	.571	.565
Upper layer of Ticknevin.	38.63	2.92	31.11	32.34	.210	.814	.196	.287	.147	.170	.262	.617
Same distilled at red heat.	32.10	3.34	23.44	42.12	.195	.756	.208	.365	.161	.196	.816	.493
Upper layer of Shannon..	38.13	4.12	21.87	35.69	.404	1.576	.205	.299	.132	.181	.829	.680
Dense peat.....	21.19	1.46	18.97	57.74	.181	.702	.161	.236	.119	.112	.647	.266

Lignite yields on heating much the same sort of products as does peat but there are two groups, one in which the hydrocarbon or bituminous substances are larger than in the other, probably because of different origin. The coke and gas are not of much value and being little used there is but little data available. The coke from such of the lignites as do not disintegrate by reason of loss of water of crystallization will be a little less than half the coal by weight, the volatile from one-seventh to one-third and tar water from one-eighth to one-fifth. Thus, in the lignites that will coke, there is an increase in volatile combustible while those that disintegrate have combustible close to peat. The former might properly be called sub-bituminous coal by reason of the difference and the latter solidified peat or lignite proper.

Bituminous coals are the great raw materials for roasting treatment for the manufacture of coke, (*a*) in beehive ovens where the volatile is burned and wasted, (*b*) in by-product coke ovens which yield not only valuable coke but also illuminating and fuel gases besides chemical by-products, and also for the primary manufacture of illuminating gas in retorts where the coke is a by-product. The study of the distillation or roasting of the bituminous coals is, therefore, not only important as throwing light on their fuel properties but also because of its relation to the illuminating gas and coke industries. Much is known of these properties and their use in plants of the various sorts, so much, that what can be said here will be the merest sketch in which the fuel properties will be dwelt upon rather than the chemical by-products recovery, the production of illumination or the metallurgical properties of coke, each of which is itself a big subject.

While there is a very great difference between the products from the dif-

erent bituminous coals, the following by Wagner may be taken as a basis of comparison of the results with those of wood, peat and lignite.

Products of Bituminous Coal Distillation (Wagner)

Coal: 78 per cent C, 4 per cent disposable H_2, $1\frac{1}{2}$ per cent N_2, .8 per cent S, 5.7 per cent combined H_2O, 5 per cent hygroscopic H_2O, 5 per cent ash.

1. Tar water or ammonia water containing,
   - (*a*) Water, carbonate of ammonia and sulphide of ammonia.
   - (*b*) Chloride, cyanide, sulphocyanide of ammonia.

2. Gas containing,
   - (*a*) Illuminants { gases—acetylene, ethylene, propylene, butylene; vapors { benzole, styrole, naphthalene, propyl, butyl, acetylnaphthalene;
   - (*b*) Combustible non-illuminants—hydrogen, methane, carbon monoxide;
   - (*c*) Impurities—carbon dioxide, ammonia, cyanogin, rhoden, sulphuretted hydrogen, carbon disulphide, nitrogen.

3. Liquid tar containing,
   - (*a*) Liquid hydrocarbons—benzole, toluol, propyl, butyl, etc.;
   - (*b*) Solid hydrocarbons—naphthalene, etc.;
   - (*c*) Substances containing oxygen—phenol, creosote, aniline, etc.;
   - (*d*) Asphaltic substances and resins.

4. Coke, 70 to 75 per cent of the coal containing 90 to 95 per cent of combustible matter, and 10–5 per cent ash.

Comparison of these results with previous ones shows most clearly a rise of hydrocarbons in both gaseous and liquid form, which together constitute the volatile of the coal, so that the volatile has combustible properties very close to that of vapor of oils. The diluent materials in the volatile are small in amount but may increase on excessive heating. The manner of heating, as to time and temperature, largely control the nature of the products, gases coke or by-products. Moderate temperatures are necessary to secure high yields of ammonia and for highly illuminating gas, *high* temperatures increase the coke at the expense of the carbon of the hydrocarbons, decomposing them to methane, hydrogen and carbon. The products differ, moreover, with the period of coking, as the temperature does also as a rule, fresh cold coal being

charged and slowly heating up. During the first periods most of the richer hydrocarbons forming the tar and illuminants come off, constituting the high calorific power part of the volatile. Toward the end of the coking period and that of highest temperature the gases are free of tar and low in calorific and illuminating power. It has been suggested that the first period corresponds to the formation of coke and ends when the coke has *set* or taken its permanent size and shape, while the second is a period of coke decomposition or rather distillation of the freshly set coke and this is a useful distinction to keep in mind for boiler and gas producer application of fuels. Two sets of data will serve to illustrate the variation of products with coal, and the variation of gas with the time of coking, the former, Table LXIX, by Jüptner, and the second, reported by Blauvelt for both a high and a low volatile American coal in the Solvay by-product coke oven. These latter results relate entirely to the gas composition its quantity and rate of production, and are given in graphical form in Fig. 177.

TABLE LXIX

PRODUCTS OF BITUMINOUS GAS COAL DISTILLATION (JÜPTNER)

(Variation with coal composition)

Coal from		Pas De Calais.		England.	Commentry	Blanzy.
Coal composition, per cent by weight	Moisture	2.17	2.70	3.31	4.34	6.17
	Ash	9.04	7.06	7.21	8.80	10.73
	O_2	5.56	6.66	7.71	10.10	11.70
	H_2	5.06	5.36	5.40	5.53	5.64
	C	88.38	86.97	85.89	83.37	81.66
	N_2	1	1	1	1	1
Products of distillation, per cent by weight	Gas	13.70	15.08	15.81	16.95	17.00
	Tar	3.90	4.65	5.08	5.48	5.59
	Ammonia water	4.59	5.57	6.80	8.61	9.86
	Coke	71.48	57.63	64.90	60.88	58.00
	Coal dust	6.33	7.07	7.41	8.08	9.36
Gas produced per kg coal	Vol. cubic meter	30.13	31.01	30.64	29.73	27.44
Volumetric analysis of gas	CO_2	1.47	1.58	1.72	2.79	3.13
	CO	6.68	7.17	8.81	9.86	11.93
	H_2	54.21	52.79	50.10	45.45	42.26
	CH_4	34.37	34.43	35.03	36.42	37.14
	C_6H_6	.79	.99	.96	1.04	.88
	C_2H_4	2.48	3.02	3.98	4.44	4.76

Probably no coals have as high a gas yield as the English cannel coals and related varieties, so the following figures, Table LXX, by Robinson, are of interest as representative of high yields.

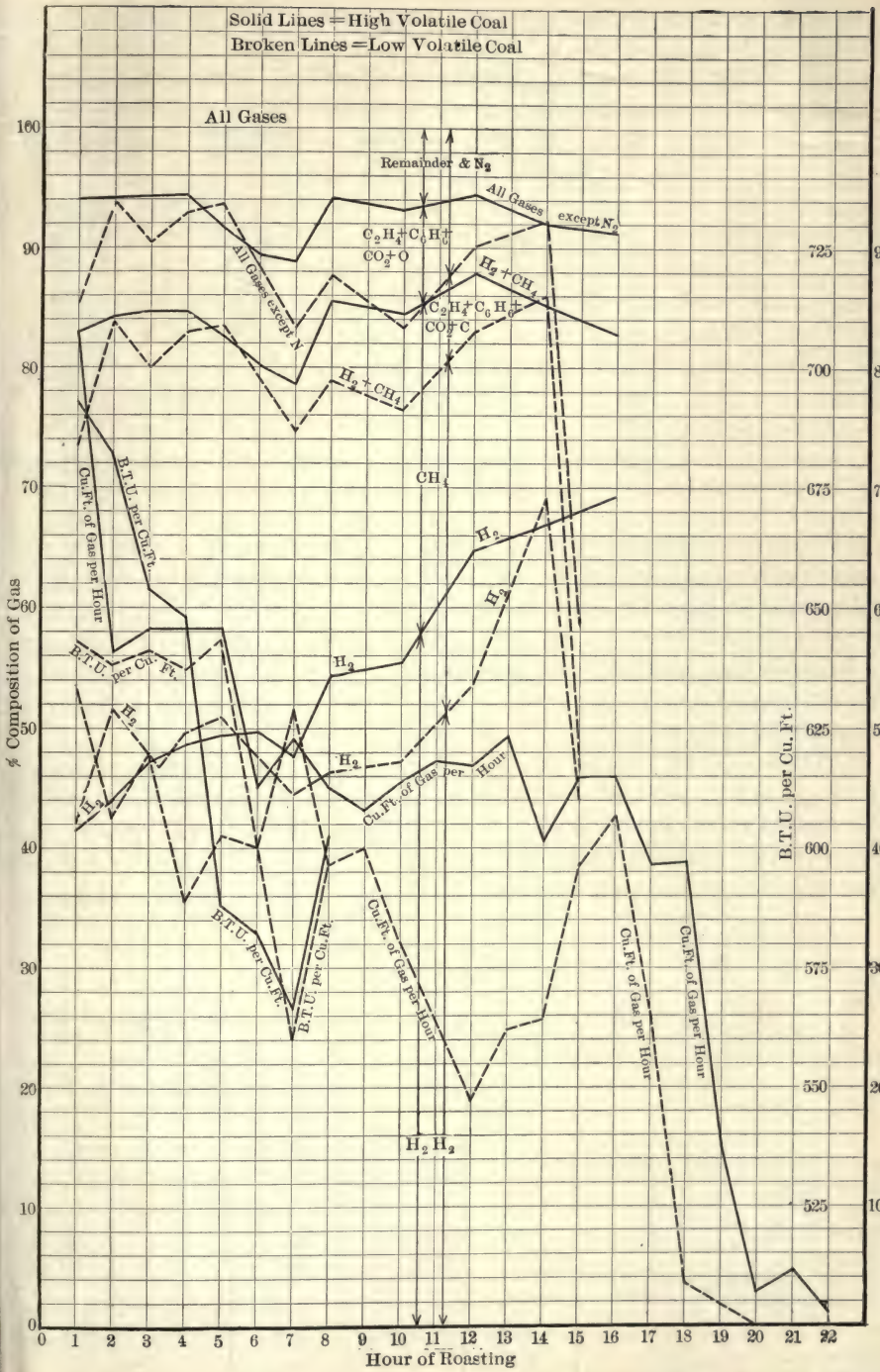


FIG 177.—Variation in the Composition of By-product Coke Oven Gas with Time of Roasting.

TABLE LXX

GAS YIELD OF ENGLISH CANNEL COAL (ROBINSON)

Coal.	Cubic Feet Retort Gas per Pound Coal	Pounds Gas. / Pounds Coal	Cubic Feet per Pound of Gas.
Newcastle	5.13	.1776	28.87
Scotch parrot	4.24	.2080	30.39
Boghead cannel	6.70	.3866	17.32
Lesmahagon cannel	5.05	.2848	17.73
Lucehall cannel	5.09	.2058	24.73
Neath, South Wales	5.00	.1790	27.93
Derbyshire	4.20	.1375	30.52
Derbyshire soft	3.20	.1143	27.99
Staffordshire	4.67	.1348	34.66

Average 4½ cu.ft. gas per pound of coal, weighing .036 lb. cubic foot and giving about 680 B.T.U. per cubic foot.

Coke ovens, so far as the fuel processes are concerned, are exactly the same as gas-house retorts except for size, shape and temperature. The former yield more coke than the latter where the temperatures are higher, time of gas contact with coal shorter, and with walls longer. The hydrogen is higher and hydrocarbons and tar yield lower in the retorts, probably because of the temperature and contact conditions but just how or why these variations occur no one seems to know.

In Table CX at the end of this Chapter are given a number of analyses of coal gas and by-product oven gases for comparison and taken from a variety of sources to show possible limits. The following, Table LXXI, gives a comparison of two analyses considered by Jüptner as typical of the two classes, with Blauvelt's typical coke oven gas and Güldner's typical retort coal gas.

TABLE LXXI

COMPARISON OF COKE OVEN AND RETORT COAL GASES

Constituents.	Coke Oven Gas Per Cent by Volume.			Retort Coal Gas Per Cent by Volume.	
	Blauvelt.	Güldner.	Jüptner.	Güldner.	Jüptner
Benzole, C_6H_6	1.2	.81	.61		1.54
Ethylene, C_2H_4	4.2	1.52	1.63	4.5	1.19
Methane, CH_4	35.5	32.40	36.11	35.0	36.00
Hydrogen, H_2	48.0	55.60	35.32	48.5	55.00
Hydrogen disulphide, H_2S	.....	.....	.43	.....	
Carbonic acid, CO_2	1.3	1.21	1.41	2.0	.87
Carbon monoxide, CO	5.1	7.08	6.49	7.0	5.40
Oxygen, O_2	.5	.....	.....	.25	
Nitrogen, N_2	4.2	1.38	.....	2.75	

The coke itself is a by-product in the gas retort and no attempt is made to et a good quality, the retort management and the selection of coal for it being oth guided by the desire for low coke and high gas yield with high illuminating alue. Beehive coke ovens are operated with regard to nothing but the largest roduction of good, hard coke of large pieces and good porosity, for use in oundry cupolas and in blast furnaces, while by-product coke ovens are designed ith several ends in view, first good coke, second large yield, third good gas r heating, power, and illumination, and fourth chemical by-product recovery. t has already been shown how the gases of the retort and coke oven compare; ney contain not the same quantities but nearly so and the same kinds of con-tituents, the differences being mainly in the hydrocarbons which for the retorts nd high heats are simpler in character with more hydrogen and less carbon ontent. The beehive coke is generally regarded as the best for iron making ut the difference compared to oven coke is not great. Retort coke is, however, ery different because (*a*) it is made from a non-coking or semi-coking coal nd (*b*) has much soot deposited from decomposed hydrocarbons and is very ense, close grained, hard, black and always in small pieces. In the beehive the ield from a given coal must always be less than from the oven because some is urnt with the gases and so is less than the fixed carbon, whereas in ovens or etorts the yield is always greater than the fixed carbon. Some coke analyses re given in Table CXI at the end of the Chapter from the U. S. Geological urvey reports of tests on American coals, given mainly to show its character-tic as a fuel; reference must be made to metallurgical works for analyses and roperties of the coke adapted for treatment of the metals or ores. The fol-owing three English cokes (Sexton) are given here for comparison:

ANALYSIS OF THREE ENGLISH COKES

Origin.	Composition Per Cent by Weight.				
	C	H_2	O_2	N_2	Ash.
unkenfield (Percy).....	85.84	.52	1.38	.86	11.40
est Durham (Kubale).....	93.15	.72	.90	1.28	3.95
verage Durham (Kubale).....	84.92	4.53	6.66	.65	2.28

A most interesting observation with regard to the relation between the coal, oke and gas, was made by Deulle and quoted with the following Table LXXII y Lewes. It is, that the best gas coals should contain from 7.5 to 9 per cent oxygen; less than this means a high coke yield but gas poor; more will yield ad coke, *indicating that the illuminating hydrocarbons are originally oxygenated ubstances like fatty acids or alcohols decomposing to form ethylene, methane, ydrogen and oxides of carbon, which are not primarily present as such.*

Table LXXII

RELATION BETWEEN OXYGEN IN COAL AND HYDROCARBONS IN GAS (RETORT)

Per Cent O_2 in Coal.		5 to 6	6.5 to 7.5	7.5 to 9	9 to 11	11 to 12
Volumetric composition of gas	CO_2	1.47	1.58	1.72	2.79	3.13
	CO	6.68	7.19	8.21	9.86	11.93
	H_2	54.21	52.79	50.10	45.45	42.26
	C_6H_6	.79	.99	.66	1.04	.88
	CH_4	34.37	34.43	35.03	36.42	37.14
	C_2H_4	2.48	3.02	3.98	4.44	4.76
Density of gas		.352	.376	.399	.441	.482

The calorific power and density of these coke oven and retort coal gases may be calculated from the constituent gases and their respective constants as was done for natural gas, the operations being most conveniently set down in tabular form, as follows in Table LXXIII.

Table LXXIII

DENSITY AND CALORIFIC POWER OF COKE OVEN GAS

From Constituents (32° F. and 29.92″ Hg)

Constituents of Average Coke Oven Gas.	One Cubic Foot				Summary	
	Contains		Yields B.T.U.			
	Cu.ft.	Pounds.	High.	Low.		
Hydrogen, H_2	.4200	.0023607	143.220	122.640	B.T.U. per cu.ft. gas high	642.16
Methane, CH_4	.3430	.0153470	365.638	328.937		
Carbon monoxide, CO	.0600	.0046840	20.460	20.460	B.T.U. per cu.ft. gas low	579.84
Heavy Hydrocarbons C_2H_4	.0200	.0015880	34.000	31.900		
Heavy Hydrocarbons C_6H_6	.0200	.0043800	78.840	75.900	Pounds per cu.ft. gas	.04031
Carbon dioxide, CO_2	.0250	.0030620	0.0	0.0		
Oxygen, O_2	.0110	.0009812	0.0	0.0	Cu.ft. per lb. gas.	24.807
Nitrogen, N_2	.1010	.0079070	0.0	0.0	B.T.U. per lb. gas high	15930
Total for gas	1.0000	.0403099	642.158	579.837	B.T.U. per lb. gas low	14384.

TABLE LXXIII, *Concluded*

DENSITY AND CALORIFIC POWER OF RETORT COAL GAS

FROM CONSTITUENTS (32° F. AND 29.92″ Hg)

Constituents of Average Retort Coal Gas.	One Cubic Foot				Summary
	Contains		Yields B.T.U.		
	Cu.ft.	Pounds.	High.	Low.	
Hydrogen, H_2	.5250	.0029509	179.025	153.3000	
Methane, CH_4	.3135	.0140310	334.191	300.6400	B.T.U. per cu.ft. gas high 604.6
Carbon monoxide, CO	.0860	.0067140	29.326	29.3260	
Heavy Hydrocarbons C_2H_4	.0110	.0008740	18.700	17.5450	B.T.U. per cu.ft. gas low 542.56
Heavy Hydrocarbons C_6H_6	.0110	.0024100	43.362	41.7450	
Carbon dioxide, CO_2	.0150	.0018400	0.0	0.0	Lbs. per cu.ft. gas .031872
Oxygen, O_2	.0035	.0003121	0.0	0.0	Cu.ft. per lb. gas 31.375
Nitrogen, N_2	.0350	.0027400	0.0	0.0	B.T.U. per lb. gas high 18969.5
Total for gas	1.0000	.0318720	604.604	542.5560	B.T.U. per lb. gas low 17022.7

Prob. 1. A natural gas which was practically 100 per cent CH_4 was procurable for cents per 1000 cu.ft. Compare the cost of coal gases Nos. 1 and 17 of Table CX at .00 per 1000 cu.ft., with this on a heat-unit basis.

Prob. 2. Five cubic feet of coal gas No. 10, Table CX, were made per pound coal, and the gas was sold at $1.10 per million B.T.U. What was the cost per ton (2000 lbs.) of coal if 50 per cent of the value of gas was charged to coal?

Prob. 3. What will be the cubic feet of gas used per horse-power per hour by an engine running on coal gas No. 71 with a thermal efficiency of 20 per cent?

Prob. 4. A hall is lighted by 50 lights each consuming 5 cu.ft. of gas per hour. If the hall be 100 ft. × 50 ft. × 20 ft. and the gas used be No. 43, how many times per hour must the air be changed to prevent a temperature rise of over 5° F.?

Prob. 5. A thousand cubic feet of coal gas No. 12, are being forced through a pipe per minute. How many of the following units are being transmitted per minute? (*a*) Foot-pounds; (*b*) B.T.U.; (*c*) Horse-power hours.

6. Distillate Oils, Kerosene, Gasolene, Residue Oils; and Oil Gas as Products of Heating Mineral Oils. Chemical, Physical, and Calorific Properties. Calorific Power of Fractionated Oils in Terms of (a) Carbon and Hydrogen; (b) Density per Pound, and Estimated Value per Cubic Foot of Vapor. Calorific Power of Oil Gas per Pound and per Cubic Foot in Terms of Constituent Gases. Yield of Distillates and Oil Gas. The discovery that crude mineral oil could be made to yield by simple heat treatment such valuable products as kerosene for lamps and engines, and gasolene, which latter has made possible the modern motor boat, automobile, and aeroplane, was one of great industrial importance and it is a little surprising to find so little scientific information available as to the precise nature, chemically and molecularly, of such important raw materials and products. It is, of course, known that all are hydrocarbons and that any sample contains many, and these usually of more

than one series, and it appears that each sample consists of a mixture of several, more or less soluble in each other. Each hydrocarbon having a different boiling-point from the others, it should be possible to separate them by distillation but not very well or completely because of mutual influences. In the first place, if one is dissolved in the other the boiling-point of the solvent will be raised by the substance dissolved and the effect will be different as the proportions vary and these will vary with continued boiling. Again, vapors of constituents having high boiling-points will escape with other vapors coming off at their own lower boiling-points in some proportion because each exerts its own vapor tension in a mixture. Therefore, while distillation will accomplish some sort of separation no distillate from a complex mixture of solutions can itself be simple but will be of the same general nature, except that the proportions of the constituents will be different. This explains why distillates obtained at low temperatures will leave residues when they are themselves heated even to a very much higher temperature than that at which they were obtained.

The separation and naming of the various oil products is a purely local procedure and though based on difference in boiling-points mainly, is not practiced in the different refineries in the same manner. Therefore, a product of a given name, like gasolene, may be quite different, one sample from another. The following Table LXXIV is given by Robinson as the average practice in separating and naming the products of American and Russian crude oil.

TABLE LXXIV

AVERAGE DISTILLATION PRODUCTS OF CRUDE MINERAL OILS (ROBINSON)

Class.	Name of Product.	Average Per Cent Yield.	Specific Gr. 60° F.	Bé.	Boiling-Point, F.
Petroleum ether	Cymogene	small	.590	107	32
	Rhigolene	.1	.625–.631	94–92	64
	Gasolene	1 –1.5	.635–.658	91–83	86–158
Petroleum spirit	*C* naphtha (benzene)	10	.680–.700	76–70	140–212
	B naphtha	2 – 2.5	.717–.72	65	175–250
	A naphtha (benzene)	2– 2.5	.742–.745	58	212–265
Lamp kerosene	Water white	12 –20	.780–.785	49	300–575
	Ordinary kerosene	40 –55	.800–.810	44	300–700
Intermediate	Gas oil	.....	.85	35	
Heavy oils	Lubricating oil	17.5	.885–.920	28–22	
	Paraffine	2	.980	13	
	Residue and loss	5 –10			
Petrol	Gasolene or benzene	5 –16	.725–.765	63–53	
Lamp oils	Kerosene	30 –40	.817–.828	41–39	
Intermediate	Solar oil	10 –12	.840–.860	37–33	
Lubricating oils	Spindle oil	12 –15	.870–.897	31–26	
	Engine oil	25 –40	.908–.912	24	
	Cylinder oil	3 –5	.915–.920	23–22	
Fuel oil	Residue, astatki or gondron	10 –15	.900–.950	25–17	

but must be accepted with caution as the differences mentioned above may be considerable,t hough for lamp oil kerosene, considerations of safety have led to legal restrictions as to boiling and flash points that keep it reasonably constant.

A somewhat more common distribution of products with the American names and densities as now understood in the oil business is given in the following Table LXXV.

TABLE LXXV

AMERICAN MINERAL OIL PRODUCTS AT 60° FAHRENHEIT

Kind of Oil.	Degrees Gravity, Baumé.	Weight of one gallon. 231 cu.in. in ounces.
Gasolene	93.5	83.5
Gasolene	91.	85.
Gasolene	90.	85.5
Gasolene	88.	86.
Gasolene	86.	86.5
Gasolene	78.5	90.
Gasolene	74.	91.5
Sumatra naphtha	74.	
Gasolene	72.	92.
Gasolene	62.0	97.5
Kerosene, 120°, water white	49.	104.5
Kerosene, 150°, water white	47.5	105.5
Kerosene	46.5	106.5
Celcius	42.	
Fuel oil, Pratt's	.....	108.
Fuel oil, Lima	.....	109.
Limpid residue, W. P	34.5	114.
Gas oil, E. W	34.5 to 35.0	
Paraffine, .865° sp.gr	32.5	115.5
Water, at 62° F		133.4

The percentages and gravities given in the preceding tables are supposed to be average practice but there is a question whether there is any such thing as average practice; certainly there is no such thing as an average crude oil. For example, California oils yield practically no burnable lamp oil, that is, clean, smokeless burning, but do yield much asphalt residue and no paraffine and practically all the lighter constituents are collected in one lot and called distillate, which has some of the properties of gasolene and some of kerosene. Also the quantity of any one product usually sold by its specific gravity, or Baumé, as a means of definition or specification can be varied greatly by mixing lighter and heavier constituents and this is a regular practice as can be shown by fractional distillation of the product. Densities are always given at 60° F. and are corrected for kerosene at other temperatures by adding or subtracting .0004 sp.gr. per degree F.

To illustrate, first, the variations in the products having a given trade name, and second, the complexity of product, the several fraction-

ation tests are given in Table CXII for crude petroleums and kerosenes and Table CXIII for gasolenes. These are plotted in curve form, Fig. 178 and Fig. 179 respectively, on which are indicated the boiling-points of known hydrocarbons and bands added for the class of distillate in accordance with the Robinson classification. Horizontal distances represent fractions distilled, a fraction being the per cent by volume that has been discharged between two given temperatures in a boiling mass, the temperature continually rising. Incidentally it may be noted that the temperature is different in the vapor than in the boiling liquid, though that of the liquid is usually taken. The rate of boiling or application of heat very seriously affects these curves, any one of which might easily be changed thereby.

The distillate between any two temperatures is by no means a simple substance nor is one sample the same as another, so that the gasolene collected in original manufacture between the same temperatures from different oils will not be the same. This is a fundamental characteristic of the boiling of solutions of different substances in each other, some of the vapor of every one will come off at any temperature and while at low temperatures the amount of heavy vapors is small they are always present and in varying amounts for different oils and especially for different ways and rates of boiling that cannot be fully explained here.

For the United States the following, Table LXXVI, gives an estimate of the oil characteristics from the different fields, divided into two groups. The first represents in output about 15 per cent of the total for the whole country and yields from 8 to 12 per cent of gasolene while the second represents about 85 per cent of the total production, and yields from 1 to 4 per cent of gasolene, the calorific power of all varying not over 12 per cent.

TABLE LXXVI

U. S. GASOLENE AND KEROSENE BEARING CRUDE OILS

Group I (8%–12% gasolene)		Group II (1%–4% gasolene)	
Pennsylvania	40–50 Bé.	Illinois	30–34 Bé.
Kentucky	40–44 "	Kansas	22–32 "
Ohio	37–40 "	Oklahoma	22–32 "
Indiana	37–40 "	Louisiana	22–32 "
		Texas	16–26 "
		California	12–28 "

The calorific power of distillates can, of course, be estimated from their carbon, hydrogen and sulphur contents by formulas of the Dulong type but very much closer results will be obtained from the density formula of the type derived by Sherman and Kropff. Samples of gasolene and kerosene, tested by the U. S. Geological Survey, yielded the following formulas (Eq. (765) and

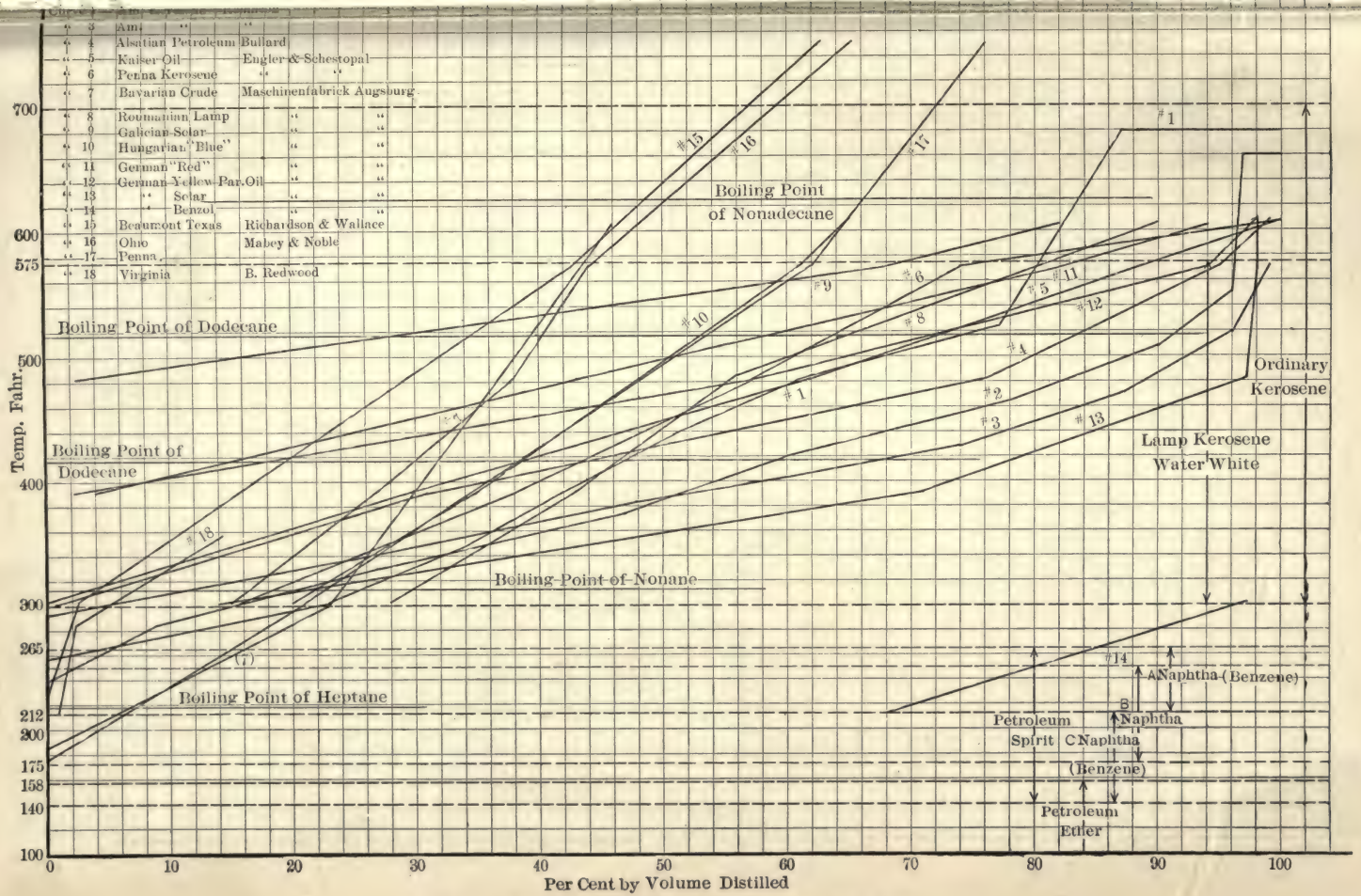


FIG. 178.—Fractional Distillation of Kerosene and Petroleums.

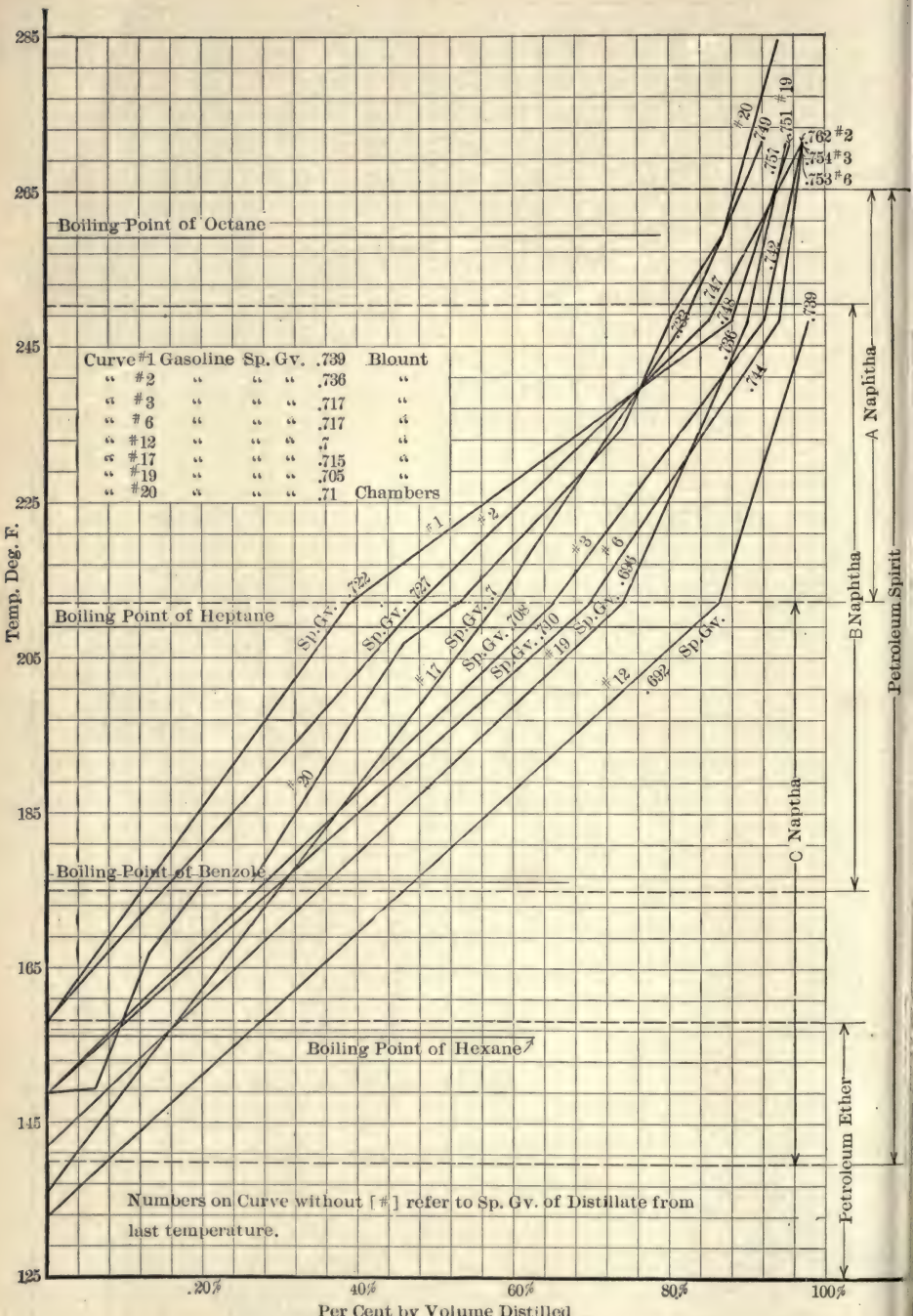


FIG. 179.—Fractional Distillation of Gasolenes.

Eq. (766)) under the manipulation of Stanton and Strong for the gasolenes and Allen and Strong for the kerosenes, both suggested by Eq. (764).

$$\text{B.T.U. per lb. gasolene} = 18{,}320 + 40(\text{Bé.} - 10), \quad . \; . \; . \quad (765)$$

$$\text{B.T.U. per lb. kerosene} = 18{,}440 + 40(\text{Bé.} - 10). \quad . \; . \; . \quad (766)$$

The details of these samples, with the measured and calculated calorific powers are given in the following Table LXXVII. The calorific power is calculated by the general Sherman and Kropff formula and by the Strong modification, showing how very close the new constant fits the case, the error being always less than 1 per cent.

TABLE LXXVII

CALORIFIC POWER OF GASOLENES AND KEROSENES

	Density, Sp. Gr. at 60° F.	Bé.	B.T.U. per lb.		Per Cent Error.	B.T.U. per lb.	
			By Calorimeter.	By S. and K. Formula.		By Strong Constants.	Error by Strong Constants.
Stanton and Strong—Gasolene	.7122	66.6	20581	20910	+1.61	20580	±.00
	.7165	65.3	20477	20862	+1.88	20532	+.27
	.7168	65.2	20527	20858	+1.61	20528	±.00
	.7175	65.1	20579	20854	+1.34	20524	−.27
	.7285	62.2	20401	20738	+1.65	20408	+.03
	.7285	62.2	20410	20738	+1.61	20408	−.01
	.7289	62.1	20407	20734	+1.60	20404	−.02
	.7289	62.1	20381	20734	+1.75	20404	+.11
	.7292	62.0	20390	20730	+1.67	20400	+.05
	.7292	62.0	20423	20730	+1.50	20400	−.11
	.7294	62.0	20389	20730	+1.67	20400	+.05
	.7301	61.8	20407	20722	+1.54	20392	−.07
Allen and Strong—Kerosene	.7912	46.9	19922	20126	+1.02	19916	+.03
	.7925	46.7	19894	20118	+1.13	19908	+.07
	.7930	46.5	19881	20110	+1.15	19900	+.10
	.7999	45.0	19872	20050	+ .95	19840	−.16

Residue oils always contain carbon in the free state as suspended solid matter and they are heavy and viscous in nature. However, the carbon may be removed by filtration, or the density reduced and fluidity increased by adding some light constituent, in which case they are very different from distillates or crudes of the same specific gravity, a fact that renders their use very difficult, as oils are generally bought on a specific gravity or Baumé specification. When the free carbon is not removed, the oils will burn very much more smoky than otherwise and such oils cannot be used in some oil engines at all, because of carbon

deposits, when the same engine can successfully use another oil of the same density and perhaps same calorific power. Specifications based on temperature fractions would avoid these difficulties.

Mineral oils when heated, not in a liquid mass but by slow admission to a hot chamber the temperature of which is above the boiling-point of some or all of their constituents, undergo a decomposition which may be called *destructive distillation* and is in many ways similar to the roasting of coal. It will yield a large quantity of permanent gases if the temperature be high enough and mixed with them a series of hydrocarbon vapors, mainly benzole if the temperature is that of good red heat, and there will be left behind some heavy liquid residue and coke. Such gas after washing and cooling is called oil gas and as made by the Pintsch process is the main illuminating medium of over 90 per cent of all American steam railroad cars. Usually the oil used for this purpose is the grade between the lamp oil or kerosene and the lubricating oil, but there is considerable variation in practice and price, the latter largely controlling practice in the choice of raw material. Too low a retort temperature gives poor gasification, and tar and condensible vapors, in excess; too high a temperature gives fixed carbon, reduced illuminants and increased hydrogen and each oil must be treated a little differently from the others. The gas consists mainly of hydrocarbons and hydrogen, the hydrocarbons being much the same as those from coal gas but in different proportions, and not usually separated from each other by analysts except into saturated and unsaturated groups beyond methane and ethylene, but even these as reported are often included with others by the methods of the analysis. The hydrocarbons called saturated are those absorbed by bromine in potassium bromide or concentrated sulphuric acid, and consist mainly of methane, CH_4, and ethane, C_2H_6. The rest called unsaturated, include ethylene, C_2H_4, butylene, C_4H_8, acetylene, C_2H_2, benzole, C_6H_6 and naphthalene, $C_{10}H_8$. Analysts frequently assume no ethane present and report only methane for the first absorption and call the rest heavy hydrocarbons or illuminants. Sometimes they are assumed to be ethylene or benzole and reported as such, but only the most careful chemists will verify results or reject assumptions. For this reason it is likely that the hydrocarbon analyses reported in the Table LXXVIII for oil gas are quite uncertain, but less so, than the hydrocarbons equivalent to kerosene and gasolene. A recent investigation of Pintsch oil gas at Columbia University showed it to consist of 63.1 per cent CH_4, 5.6 per cent H_2, 27.4 per cent illuminants, 0.8 per cent O_2, 0.4 per cent CO, and 27 per cent N_2. Blau gas is the name applied to the condensible constituents of the distillation rejected in the Pintsch process, and as obtained by very high compression they include only those that freely revaporize on release of pressure.

From calorimeter results the nature of the illuminating hydrocarbons in the above-mentioned Pintsch gas was found to be represented by carbon 89 per cent and hydrogen 11 per cent by weight which does not correspond to any one, but might be made up by mixtures of several hydrocarbons. In a somewhat similar way, the general conclusion has been reached that gasolene is most nearly represented by the simple hydrocarbon hexane, and kerosene by decane,

TABLE LXXVIII

PROPERTIES OF OIL GAS

No.	Description.	Volumetric Analysis.							At 32° F. and 29.92″ Hg Pressure.					
		CH_4	H_2	Heavy C_2H_4.	CO.	CO_2.	O_2	N_2	Lbs. Cu. Ft.	Cu. Ft. per Lb.	B.T.U. per Cu. Ft.		B.T.U. per Lb.	
											High.	Low.	High.	Low.
1	Thwaite oil gas	63.19	31.61	...	.4	..	..	5.06	.03427	29.18	893.5	818.0	26072	23869
2	Pintsch American oil	63.1	5.6	27.4	.4	..	.8	..	.05142	19.45	1173.	1074.	22815	20889
3	Pintsch American oil	61.2	6.4	28.3	.2	..	.7	..	.05109	19.6	1260.7	1064.	24710	20854
4	Oil gas	58.3	24.3	17.4	..	.9	..	..	.04313	23.2	995.9	803.9	23096	18650
5	Pintsch gas from petroleum residue	58.0	24.3	17.	..	..	..	..	.04081	24.5	990.	898.	24260	22000
6	Pintsch gas from paraffine oil	54.9	5.6	28.9	8.9	.9	..	..	.0591	16.92	1126.8	1034.8	19065	17509
7	American petroleum oil gas	53.7	4.8	41.2	.2	..	..	.1	.05726	17.46	1294.8	1192.0	22607	20812
8	Pintsch gas, Moore-head	52.5	18.5	23.5	1.0	.5	.5	3.5	.04777	17.32	1157.5	966.5	20060	16940
9	General	48.	32.	16.5	..	..	5.0	3.0	.04318	23.16	901.3	716.	20874	16583
10	Crude oil Retort gas, England	35.4	6.6	49.4	1.5	1.4	.3	..	.05972	16.750	1390.7	1107.	23282	18542
11	English shale oil gas, Young and Bell	19.	16.85	44.83	.63	..	.24	1.15	.04670	21.41	1043.1	966.0	22333	20682

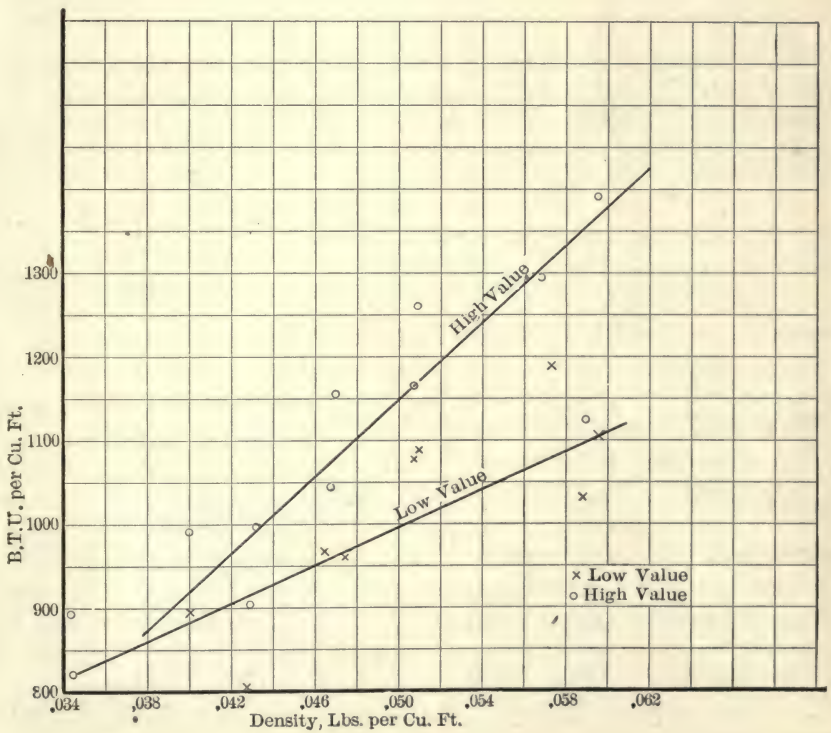


FIG. 180.—Calorific Power of Oil Gas.

but this is hardly better than the roughest sort of guess, only really needed when the volume of vapor per pound is to be estimated by the molecular weight method, the only available means.

The calorific power of these oil gases is plotted to a density base in Fig. 180, which shows a straight line relation with density somewhat similar to that found for liquid oils. There are very little data available on the production or yield of oil gas except that of a confidential character, because its manufacture is not very general, but the following figures (Table LXXIX) quoted by Güldner from the Gas Journal are useful as comparative if not as absolute data:

TABLE LXXIX

YIELD OF RETORT OIL GAS

Oil Used.	B-naphtha Sp.gr. .730.		Kerosene. Sp.gr. .807.		Heavy Oil, Sp.gr. .847.			Heavy Oil, Sp.gr. .884.		
Retort temp. F....	1112	1562	1112	1562	1112	1472	2012	932	1112	1562
Cu. inches gas per cubic inch oil...	451	625	469	582	401	513	594	213	368	657
Cu.ft. gas per lb...	10.4	13.8	9.3	11.0	7.6	9.7	11.3	3.7	6.7	12.3
Residue % weight.	11.4	5.1	21.4	7.5	28.5	12.2	18.0	62.3	41.5	9.4

The calculation of the calorific power of oil gas with the assumption of nature of heavy hydrocarbons as half ethylene and half benzene may be laid out conveniently in tabular form as for natural and coal gas, and for this a typical analysis is given in Table LXXX.

TABLE LXXX

DENSITY AND CALORIFIC POWER OF OIL GAS

FROM CONSTITUENT (32° F. AND 29.92″ Hg)

Constituents of Average Pintsch Oil Gas.	One Cubic Foot				Summary
	Contains		Yields B.T.U.		
	Cu.Ft.	Pounds.	High.	Low.	
Methane, CH_4.....	.525	.02347	559.6	504.5	B.T.U. per cu.ft. gas high..... 1290
Hydrogen, H_2.....	.185	.00104	63.0	54.2	B.T.U. per cu.ft. gas low..... 1196.5
Heavy Hydrocarbons C_2H_4	.117	.00930	198.9	186.6	Cu.ft. per lb. of gas 16.25
Heavy Hydrocarbons C_6H_6	.118	.02588	465.1	447.8	Lbs. per cu.ft. of gas .06153
Carbon monoxide, CO..	.010	.00078	3.4	3.4	B.T.U. per lb. high 20,962
Carbon dioxide, CO_2...	.005	.00061	.....	.....	B.T.U. per lb. low 19,443
Oxygen, O_2.....	.005	.00045	.....	.....	
Total for gas.....	.963	.06153	1290.0	1196.5	

The cooling of the distilling gases from either coal or oil in retorts produces a liquid generally termed tar, the relative character of which for the two cases is shown by the following ultimate analyses:

TAR ANALYSES

Tar from	Per Cent by Weight						
	C	H_2	N_2	O_2	Ash	S	B.T.U. per Lb.
Coal.....	89.21	4.95	1.05	4.20	.06	.53	15400
Oil.....	92.70	6.13	.11	.11	.05	.33	17300

7. Gasification of Fixed Carbon and Coke by Air Blast Reactions, Producing Air Gas, and Blast Furnace Gas. Comparative Yield per Pound of Coke and Air. Sensible Heat and Heat of Combustion of Gas. Relation of Constituents in Gas. Efficiency of Gasification. Carbon when hot enough may react with oxygen to form carbon monoxide or carbon dioxide and this is the basic principle for the gasification of carbon with the oxygen of air when the process is so controlled as to make a maximum of carbon monoxide, the product being called air gas. The principal source of air gas is the blast furnace in which the fuel is usually coke but may be a hard coal, though the process as there carried out is not controlled with a view to getting high carbon monoxide content primarily, but rather to best permit of the ore reduction to metal. However, the conditions are generally such as to yield an air gas almost as good as if its production were the end in view.

This gasification process is characterized by a greater precision of relation between conditions and results than any dealing with hydrocarbons whether derived from coal volatile or from oils, and, therefore, it yields better to prediction of results though not so well as might be expected at first glance. While from the fundamental chemical reactions the relative weights of carbon, oxygen and gas can be set down exactly for a complete reaction, there are certain gaps to be filled by estimation, in attempting to predict a gas made from a coal or a coke even when its composition is known. In the first place *the three substances*, CO, CO_2 *and* C *are known to be in equilibrium in all proportions at some temperature*, the ratio of CO to CO_2 in the presence of an excess of C depending on the temperature. Therefore, in any fire where carbon is a fuel there will always be some CO_2 and some CO but no one can say how much of each even though the equilibrium proportions are known for all temperatures, because the temperature is a *resultant* and not an *imposed* condition, and to attain equilibrium requires sufficient *time* of contact and this is always unknown. No real fuel is all fixed carbon and seldom is fixed carbon in the same state, so that dense carbons like retort coke require longer time or higher temperatures for reaction than charcoals, while coals containing other constituents than carbon,

such as nitrogen, oxygen, hydrogen, both free and combined in all sorts of ways, further complicates the problem. Cokes even when they yield no gases on high heating will do so when by partial combustion the cell walls are broken down and in such cases hydrocarbons will be added to the air gas even in the late stages of coke combustion.

Accordingly, air gas while consisting mainly of carbon monoxide will also contain some carbon dioxide, some hydrogen and possibly some methane, all besides the nitrogen derived from the air. Presence of oxygen in air gas is an indication of air addition after gas formation is complete and at a point where the gas has cooled below the ignition point of the combustible constituents. Air mixture with the gas in hot regions always results in the burning of some of the gas produced previously and is one cause of higher carbon dioxide content than the conditions of CO–CO_2–C equilibrium warrant at the temperature.

Air gas as usually made is merely the product of blasting air into a *thick* bed of coal, coke or charcoal, the depth of carbon being maintained at from 2 to 6 ft. to insure *sufficient time* of contact between gases and carbon to complete the reaction. The necessary depth depends partly on the blast pressure which determines the flow velocity in a bed of given porosity and partly on the sort or condition of the fixed carbon fuel, being least for charcoal. Air gas is seldom made for itself, but as a part of other processes, for example, the blast furnace yields air gas as a by-product of iron making, called in this case blast-furnace gas, also water-gas making, to be described presently, requires a heated bed usually so prepared by a preliminary making of air gas especially in the Lowe system of carburetted water-gas manufacture, where the air gas serves a double purpose, as it is burned to heat the carburetor after, by its formation, it has heated the carbon bed to a temperature suitable for water-gas making. Probably the nearest approach to straight air-gas making with a view to securing it for a gas engine fuel is that of the Tait producer, though in all gas producers some air gas is made to be mixed with water gas separately or simultaneously as made.

To show the characteristics of the composition of some air gases Table CXIV at the end of the Chapter has been compiled. These analyses show that the composition varies considerably for reasons just given, the carbon monoxide ranging from about 18 to 35 per cent, carbon dioxide from 4 to 16 per cent, hydrogen from less than 1 to over 12 per cent, methane, the only hydrocarbon, from zero to 5 per cent, being least for cokes and greatest for coals.

It is of value to examine some of these results in the light of such fundamental relations as are available, to indicate what a perfect air gas should be, the corresponding yield per pound of fuel and the *efficiency of the transformation*, this latter being *defined as the ratio of the heat of combustion of the gas formed, to that of the fuel from which it came, each burning to the same final products of* CO_2 *and* H_2O.

The first step is to establish such relations as may be possible between the CO and CO_2 existing together and two investigations are available for this purpose. First, that of Boudouard, who established the equilibrium ratio as a

function of temperature and later determined the time factors in attaining equilibrium with carbon in three different states, which latter relations were also studied by the second authority, Clement.

By leaving amorphous carbon in contact with CO_2 for very long times Boudouard found on analyzing the gases resulting, that, as might be expected, the relation of CO to CO_2 became finally constant for any one temperature and as temperature rose the per cent of CO increased. His numbers are tabulated below, Table LXXXI, as calculated from the derived formula by Dowson and Larter, together with the ratio $\frac{CO}{CO_2}$ which may be termed the *carbon oxide ratio* for want of a better name. These numbers are also plotted to coordinates of $\frac{CO}{CO_2}$ and temperatures, and the corresponding values of $\frac{CO}{CO+CO_2}$ by volume, are added as an extra scale in Fig. 181.

TABLE LXXXI

BOUDOUARD'S EQUILIBRIUM RELATIONS CO AND CO_2 WITH TEMPERATURE

Temp. F.	Per Cent by Vol.		$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$	Temp. F.	Per Cent by Vol.		$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$
	CO_2	CO				CO_2	CO		
835	98	2	.0204	.02	1407	20.0	80.0	4.00	.80
923	95	5	.052	.05	1447	15	85	5.68	.85
1000	90	10	.11	.10	1497	10	90	9.00	.90
1090	80	20	.25	.20	1533	7.5	92.5	12.34	.925
1153	70	30	.43	.30	1582	5	95	19.00	.95
1204	60	40	.67	.40	1610	4	96	24.00	.96
1252	50	50	1.00	.50	1646	3	97	32.33	.97
1297	40	60	1.50	.60	1697	2	98	49.00	.98
1348	30	70	2.33	.70	1790	1	99	99.00	.99
1376	25	75	3.00	.75	1886	0.5	99.5	199.00	.995

To attain these per cents of CO at such temperatures, which must exceed 1800° F. to exceed 99 per cent CO, requires a long time, and longer for some forms of carbon than others and longer than is available in air-gas producers or boiler fires to a still less degree. Moreover, the process in these cases is somewhat different for instead of making all CO_2 to be reduced at a fixed temperature to CO, air is blasted into a bed, *the temperatures are whatever result from the reaction* and the reaction such that both CO and CO_2 are produced at once, such CO_2 as is formed later reducing, and the CO_2 arising at first partly from direct burning of C with O and partly by burning of CO formed at one spot by O that escaped being used up at another. However, the time factor is a most important one, and operates in such a way as to prevent the attainment of as much CO as is indicated in the preceding table and curve at the given temperature, or if

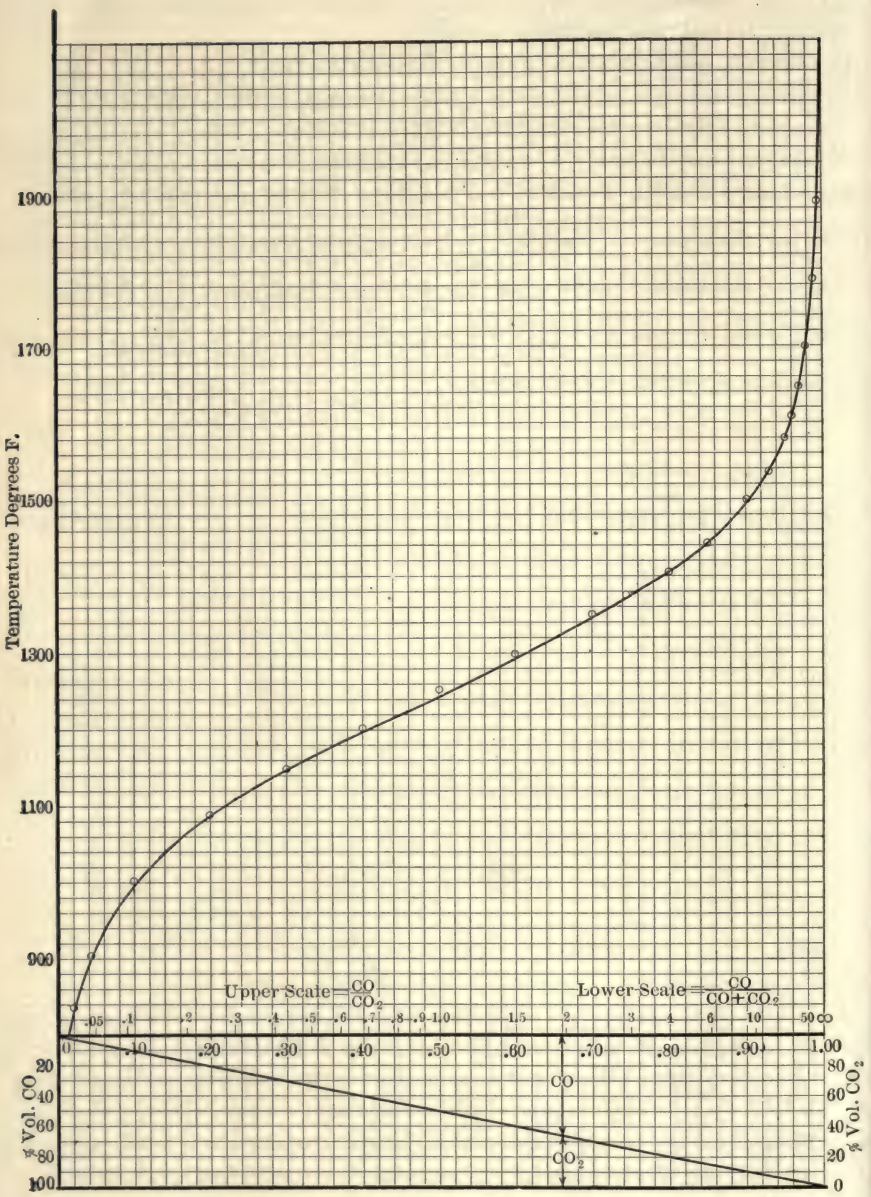


FIG. 181.—Boudouard's Equilibrium Relations for CO and CO_2 with Temperature.

high amount is to be attained in a limited time the temperature must be higher. herefore, if a curve be plotted for CO/CO_2 with temperature, for less time contact than is necessary for equilibrium it will be above the equilibrium rve and such data have been given by both Boudouard and Clement, repro-ced in Table CXV at the end of the Chapter. It must be observed that the me element depends on the mode of experiment, a fact that makes the problem ore complex especially when comparing results obtained from gases at *rest* a closed tube with another *through which a current passes.* This latter is the ore favorable as the gases sweep the carbon, and fresh CO_2 can reach it by echanical pushing away of the CO next to it, while in the former case displace-ent can only take place by diffusion. Boudouard used the closed tube and lement the open tube with current passing through; the former knew accu-tely the time of contact while the latter could only estimate it from the volume flow and temperature, a very uncertain method.

These results are plotted in two ways in Fig. 182 and Fig. 183, to bring it clearly the relations. In Fig. 182 the ratio $\frac{CO}{CO_2}$ is plotted as a function of me as derived, while in Fig. 183 as a function of temperature and in each case he equilibrium line is added as a reference.

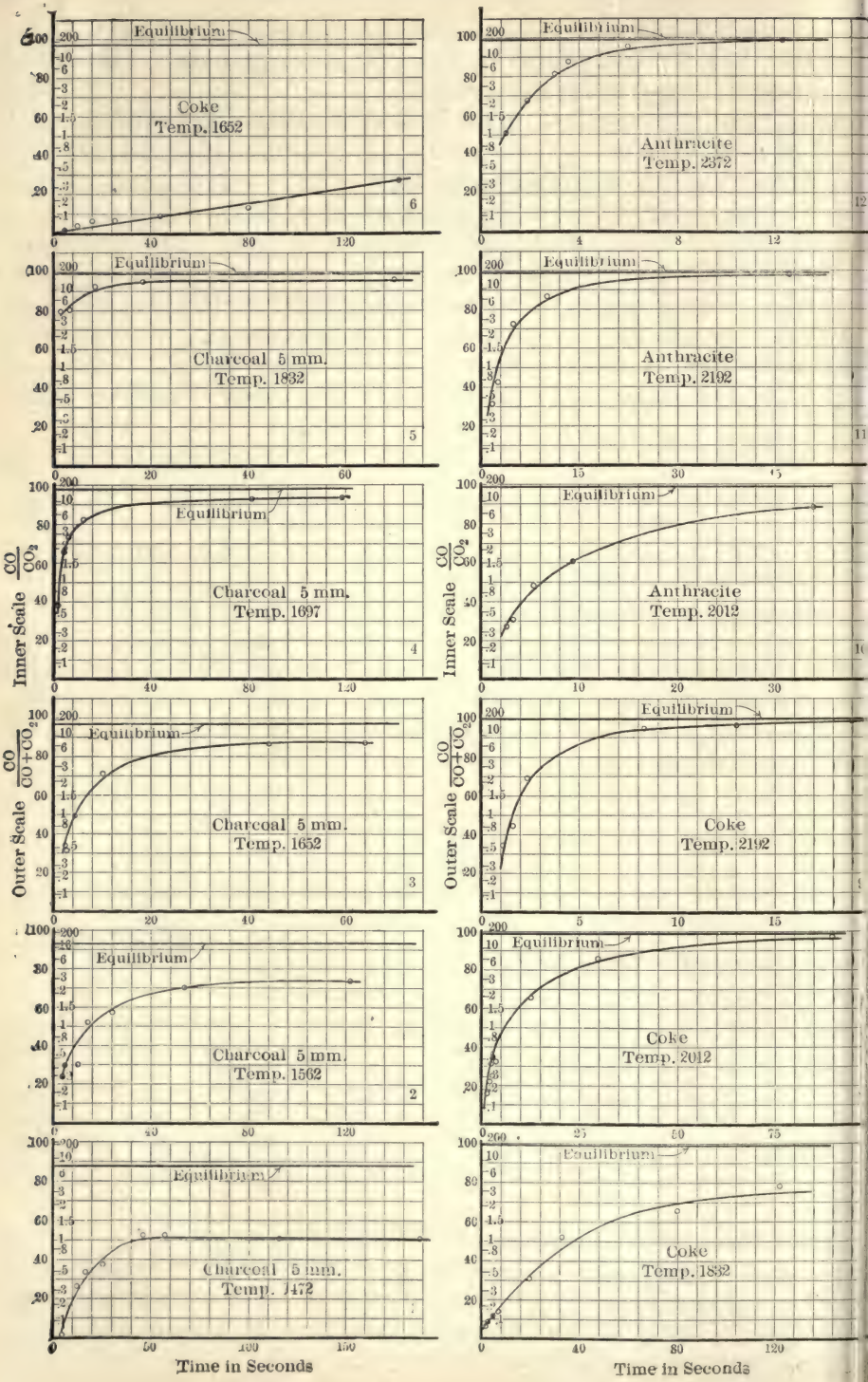
It is clear from these data how strong an influence the time and form of carbon xert, but in no case do the conditions correctly represent the *gas producer* as ell as another series by Boudouard, who passed *air* through a tube packed with oke in 5 mm. pieces, and held at a temperature of 1472° F. while the rate of ow was varied with the following results, Table LXXXII, the units of which ake them unfortunately not comparable with others.

TABLE LXXXII

CHANGE OF O_2 IN AIR TO CO AND CO_2 AT 1472° F. (BOUDOUARD)

Vel. air, in inches per Minute.	Per Cent by Volume.			$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$	Per Cent of Carbon Gasified in	
	CO_2	CO	CO_2+CO			CO_2	CO
6.10	18.20	5.20	23.40	.28	.222	77.8	22.2
16.47	18.43	3.80	22.23	.205	.171	82.9	17.1
79.31	18.92	1.88	20.80	.0995	.090	91.0	9.0
89.36	19.90	1.83	21.73	.0915	.084	91.6	8.4
195.20	19.40	.93	20.33	.0478	.046	95.4	4.6

These results show that decreasing the time of contact decreases the total arbon gasified and increases the CO_2 content of the gases, indicating that a eduction to CO of some CO_2 previously made is the order of reaction and that his is more complete the greater the time available but it is quite likely that igher temperatures would materially change the result.


Equilibrium
Coke
Temp. 1652
Anthracite
Temp. 2372
Charcoal 5 mm.
Temp. 1832
Anthracite
Temp. 2192
Charcoal 5 mm.
Temp. 1697
Anthracite
Temp. 2012
Charcoal 5 mm.
Temp. 1652
Coke
Temp. 2192
Charcoal 5 mm.
Temp. 1562
Coke
Temp. 2012
Charcoal 5 mm.
Temp. 1472
Coke
Temp. 1832
Inner Scale CO/CO2
Outer Scale CO/(CO+CO2)
Time in Seconds
Time in Seconds

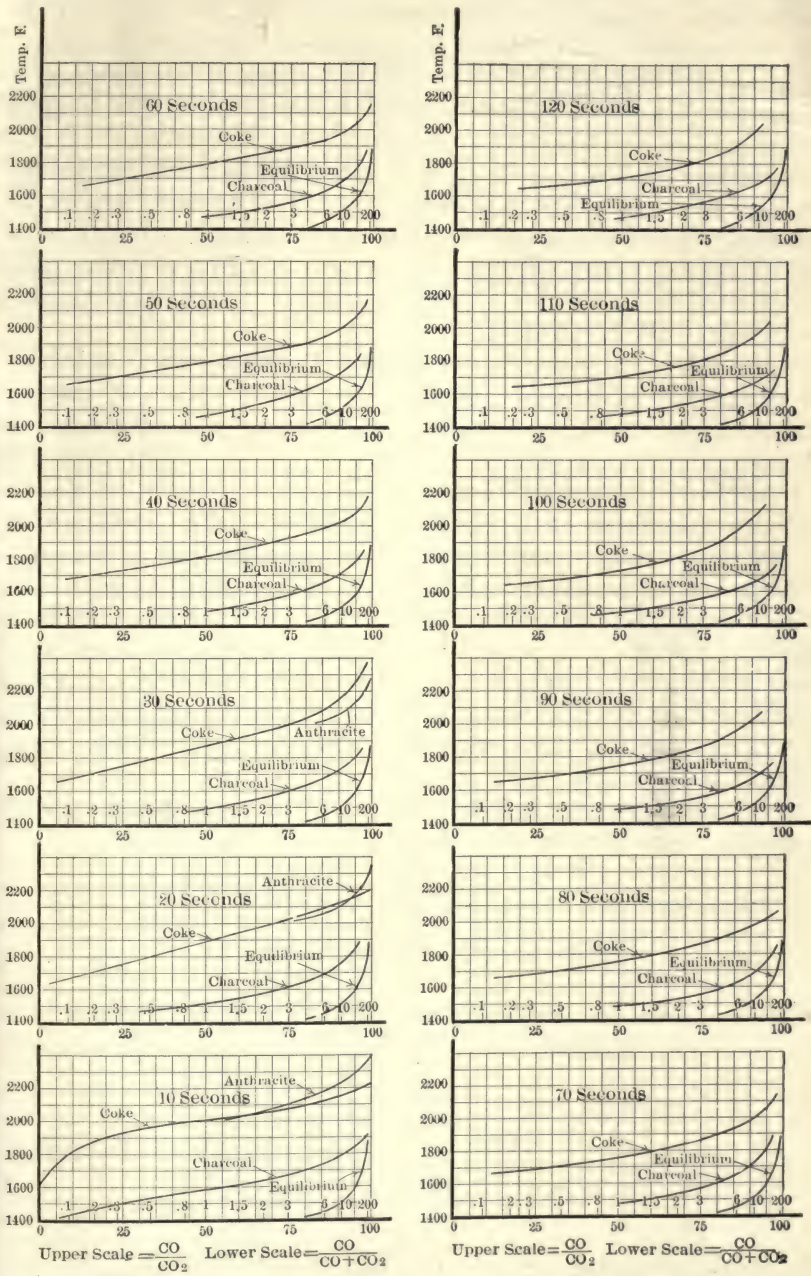


Fig. 183.—Rate of Reduction of CO_2 to CO in Contact with C with Temperature at Various Periods of Contact Time.

In using data on equilibrium it must be assumed that sufficient time has elapsed to allow of equilibrium being established and this will always be the case in producers except when blasted too vigorously, that is, overloaded. Thus in the case of the Brymbo blast furnace the gas of which is reported by Allen the attainment of a carbon oxide ratio of 6.0 indicates that the bed temperature must have reached an average value of 1460° F. from the curve if, (*a*) no other CO_2 was formed either by combustion or other chemical reaction beyond the highest temperature zone which is always the case with blast furnaces, (*b*) the flow through the bed and the bed itself were homogeneous, which is never the case, the bed being usually divisible into chimneys of high temperature and easy path of gases which always flow where the resistance is least, and if (*c*) the time were sufficient to attain equilibrium.

That there should be any difference at all in the temperatures of beds of fuel blasted with air and all excessively thick is due to cooling influences, external and internal as follows: (*a*) dry air is seldom used or available and frequently steam is added which on decomposition absorbs heat, (*b*) air and steam blasts are supplied at different temperatures and as the reactions fix only the *rise* of temperature, the temperature in the bed rises directly with initial mixture temperature, other things being equal; (*c*) walls absorb, transmit and discharge heat to water jackets or radiate it to the air; (*d*) in blast furnaces chemical reactions are absorbing heat at various points of the path, and giving off gases from ore and flux changes, otherwise disturbing the purely combustion reaction. Of these influences the strongest is the hydrogen decomposition and reaction with carbon, which produces what is termed a water gas, the characteristics of which will be examined after establishing the quantitative fundamental relations for air gas.

The two fundamental relations fixing the weight and volume proportions and heats for air-gas making are given by Eqs. (767) and (768).

$$C+O_2=CO_2+(14{,}544\times 12)\text{B.T.U.} \quad . \quad . \quad . \quad . \quad (767)$$

$$2C+O_2=2CO+2(4351\times 12)\text{B.T.U.} \quad . \quad . \quad . \quad . \quad (768)$$

Accordingly for oxygen reacting with carbon, and measuring volumes at 32° and 29.92 ins. Hg, the proportions being the same at any other pressure,

$$1\text{ lb. C}+\left\{\begin{array}{c}\frac{32}{12}=2.66\text{ lbs.}\\ \text{or}\\ \frac{358}{12}=29.8\text{ cu.ft.}\end{array}\right\}O_2=\left\{\begin{array}{c}\frac{44}{12}=3.66\text{ lbs.}\\ \text{or}\\ \frac{358}{12}=29.8\text{ cu.ft.}\end{array}\right\}CO_2+14{,}544\text{ B.T.U.} \quad (76$$

$$1\text{ lb. C}+\left\{\begin{array}{c}\frac{16}{12}=1.33\text{ lbs.}\\ \text{or}\\ \frac{358}{24}=14.9\text{ cu.ft.}\end{array}\right\}O_2=\left\{\begin{array}{c}\frac{28}{12}=2.33\text{ lbs.}\\ \text{or}\\ \frac{358}{24}=29.8\text{ cu.ft.}\end{array}\right\}CO+4351\text{ B.T.U.} \quad (77$$

For air reactions taking by $\left\{\begin{array}{l}\text{Wt., } O_2 = 23.1 \text{ per cent; } N_2 = 76.9 \text{ per cent}\\ \text{Vol., } O_2 = 20.9 \text{ per cent; } N_2 = 79.1 \text{ per cent}\end{array}\right\}$

the nitrogen to be added is $\left\{\begin{array}{l}3.33 \times \text{wt. of oxygen}\\ 3.78 \times \text{vol. of oxygen}\end{array}\right\}$

and the air involved is $\left\{\begin{array}{l}4.33 \times \text{wt. of oyxgen}\\ 4.78 \times \text{vol. of oxygen}\end{array}\right\}$

These values substituted in Eqs. (769) and (770) will give the weights, volumes and heats of reacting with carbon, Eqs. (771) and (772) applying when the product is CO_2, Eqs. (773) and (774) when it is CO.

$$1 \text{ lb. C} + \left\{\begin{array}{l}\left[\begin{array}{c}2.66 \text{ lbs.}\\ \text{or}\\ 29.8 \text{ cu.ft.}\end{array}\right] O_2\\ + \left[\begin{array}{c}8.857 \text{ lbs.}\\ \text{or}\\ 112.64 \text{ cu.ft.}\end{array}\right] N_2\end{array}\right\} = \left\{\begin{array}{l}\left[\begin{array}{c}3.66 \text{ lbs.}\\ \text{or}\\ 29.8 \text{ cu.ft.}\end{array}\right] CO_2\\ + \left[\begin{array}{c}8.857 \text{ lbs.}\\ \text{or}\\ 112.64 \text{ cu.ft.}\end{array}\right] N_2\end{array}\right\} + 14{,}544 \text{ B.T.U.} \quad (771)$$

$$1 \text{ lb. C} + \left\{\left[\begin{array}{c}11.517 \text{ lbs.}\\ \text{or}\\ 142.44 \text{ cu.ft.}\end{array}\right] \text{Air}\right\} = \left\{\begin{array}{l}\left[\begin{array}{c}3.66 \text{ lbs.}\\ \text{or}\\ 29.8 \text{ cu.ft.}\end{array}\right] CO_2\\ + \left[\begin{array}{c}8.857 \text{ lbs.}\\ \text{or}\\ 112.64 \text{ cu.ft.}\end{array}\right] N_2\end{array}\right\} + 14{,}544 \text{ B.T.U.} \quad (772)$$

$$1 \text{ lb. C} + \left\{\begin{array}{l}\left[\begin{array}{c}1.33 \text{ lbs.}\\ \text{or}\\ 14.9 \text{ cu.ft.}\end{array}\right] O_2\\ + \left[\begin{array}{c}4.429 \text{ lbs.}\\ \text{or}\\ 56.32 \text{ cu.ft.}\end{array}\right] N_2\end{array}\right\} = \left\{\begin{array}{l}\left[\begin{array}{c}2.331 \text{ lbs.}\\ \text{or}\\ 29.8 \text{ cu.ft.}\end{array}\right] CO\\ + \left[\begin{array}{c}4.429 \text{ lbs.}\\ \text{or}\\ 56.32 \text{ cu.ft.}\end{array}\right] N_2\end{array}\right\} + 4351 \text{ B.T.U.} \quad (773)$$

$$1 \text{ lb. C} + \left\{\left[\begin{array}{c}5.759 \text{ lbs.}\\ \text{or}\\ 71.22 \text{ cu.ft.}\end{array}\right] \text{Air}\right\} = \left\{\begin{array}{l}\left[\begin{array}{c}2.33 \text{ lbs.}\\ \text{or}\\ 29.8 \text{ cu.ft.}\end{array}\right] CO\\ + \left[\begin{array}{c}4.429 \text{ lbs.}\\ \text{or}\\ 56.32 \text{ cu.ft.}\end{array}\right] N_2\end{array}\right\} + 4351 \text{ B.T.U.} \quad (774)$$

Assuming that 1 lb. of carbon reacting with air produces both CO_2 and CO at the same time,

Let x = fraction of 1 lb. of carbon burning to CO_2,

" $(1-x)$ = fraction of 1 lb. of carbon burning to CO.

Then if the products of reaction of 1 lb. of carbon mix, the reaction is defined by Eq. (775)

$$1 \text{ lb. C} + \left\{ \begin{array}{r} x\left[\begin{array}{c}11.517 \text{ lbs.}\\ \text{or}\\ 142.44 \text{ cu.ft.}\end{array}\right]\text{Air} \\ +(1-x)\left[\begin{array}{c}5.759 \text{ lbs.}\\ \text{or}\\ 71.22 \text{ cu.ft.}\end{array}\right]\text{Air} \end{array} \right\} = \left\{ \begin{array}{r} x\left[\begin{array}{c}3.66 \text{ lbs.}\\ \text{or}\\ 29.8 \text{ cu.ft.}\end{array}\right]\text{CO}_2 \\ +(1-x)\left[\begin{array}{c}2.33 \text{ lbs.}\\ \text{or}\\ 29.8 \text{ cu.ft.}\end{array}\right]\text{CO} \\ +x\left[\begin{array}{c}8.857 \text{ lbs.}\\ \text{or}\\ 112.64 \text{ cu.ft.}\end{array}\right]\text{N}_2 \\ +(1-x)\left[\begin{array}{c}4.429 \text{ lbs.}\\ \text{or}\\ 56.32 \text{ cu.ft.}\end{array}\right]\text{N}_2 \\ +[14{,}544x+4351(1-x)] \text{ B.T.U.} \end{array} \right\} \quad (775)$$

This equation reduces to the two following forms, Eq. (776) giving the weight relations and Eq. (777) those for volumes each associated with the heats of reaction.

$$1 \text{ lb. C} + 5.759(1+x) \text{ lbs. air} = \left\{ \begin{array}{l} 3.66x \text{ lbs. } \text{CO}_2 \\ +\ 2.33(1-x) \text{lbs. CO} \\ +\ 4.428(1+x) \text{ lbs. } \text{N}_2 \\ +(10{,}193x+4351) \text{ B.T.U.} \end{array} \right\} \quad . \ . \ . \quad (776)$$

$$1 \text{ lb. C} + 71.22(1+x) \text{ cu.ft. air} = \left\{ \begin{array}{l} 29.8x \text{ cu.ft. } \text{CO}_2 \\ +29.8(1-x) \text{ cu.ft. CO} \\ +56.32(1+x) \text{cu.ft. } \text{N}_2 \\ +(10{,}193x+4351) \text{ B.T.U.} \end{array} \right\} \quad . \ . \ . \quad (777)$$

The final gas will have the following composition and carbon monoxid-dioxide ratio by volumes (Tables LXXXIII and LXXXIV). It should be noted with respect to the latter that the *ratio of carbon monoxide to dioxide by volumes in the gas is the same as the ratio of fractional weights of carbon burning to each as products.*

TABLE LXXXIII

COMPOSITION OF HYPOTHETICAL AIR GAS. GENERAL

Constituent.	By Weight.	By Volume.	$\frac{\text{CO}}{\text{CO}_2}$ by Vol.	$\frac{\text{CO}}{\text{CO}+\text{CO}_2}$ by Vol.
$\text{CO}_2 =$	$\frac{3.66x}{5.758x+6.758}$	$\frac{29.8x}{56.32x+86.12}$	$\frac{1-x}{x}$	$1-x$
$\text{CO} =$	$\frac{2.33(1-x)}{5.758x+6.758}$	$\frac{29.8(1-x)}{56.32x+86.12}$		
$\text{N}_2 =$	$\frac{4.428(1+x)}{5.758x+6.758}$	$\frac{56.32(1+x)}{56.32x+86.12}$		

If the carbon burns all to CO then $x=0$; if all to CO_2 then $x=1$; and the composition of the gas for the two cases will be given by Table LXXXIV.

TABLE LXXXIV

COMPOSITION OF HYPOTHETICAL AIR GAS. NO CO_2, AND NO CO.

Constituent.	By Weight Per Cent.		By Volume Per Cent.		$\frac{CO}{CO_2}$		$\frac{CO}{CO+CO^2}$	
	All CO	All CO_2	All CO	All CO_2	All CO	All CO_2	All CO	All CO_2
CO_2	0	$\frac{3.66}{12.516}=29.2$	0	$\frac{29.8}{142.44}=20.8$				
CO	$\frac{2.33}{6.758}=34.47$	0	$\frac{29.8}{86.12}=34.6$	0	∞	0	1	0
N_2	$\frac{4.428}{6.758}=65.53$	$\frac{8.856}{12.516}=70.8$	$\frac{56.32}{86.12}=65.4$	$\frac{112.64}{142.44}=79.2$				

The general relations between the carbon and air as raw materials and the gas as produced from them are given as follows, Eq. (778):

$$\left.\begin{array}{lll}
\text{1 lb. carbon makes } (5.758x+6.758) \text{ lbs. gas} & (a) \\
\text{1 lb. air makes } \left(\frac{5.758x+6.758}{5.759(1+x)}\right) \text{ lbs. gas} & (b) \\
\text{1 lb. gas requires } \left(\frac{1}{5.758x+6.758}\right) \text{ lbs. carbon} & (c) \\
\text{1 lb. gas requires } \left(\frac{5.759(1+x)}{5.758x+6.758}\right) \text{ lbs. air} & (d) \\
\text{1 lb. carbon makes } (56.32x+86.12) \text{ cu.ft. gas} & (e) \\
\text{1 lb. air makes } \left(\frac{56.32x+86.12}{5.759(1+x)}\right) \text{ cu.ft. gas} & (f) \\
\text{1 cu.ft. gas requires } \left(\frac{1}{56.32x+86.12}\right) \text{ lbs. carbon} & (g) \\
\text{1 cu.ft. gas requires } \left(\frac{5.759(1+x)}{56.32x+86.12}\right) \text{ lbs. air} & (h)
\end{array}\right\} \quad . \quad . \quad (778)$$

The heats of reaction can be expressed in terms of each of the volumes or weights entering, those for the raw materials or for the gas formed, and are set down in tabular equation form below, Eq. (779) first for any proportion of CO and CO_2, next for no CO and finally no CO_2.

HEATS OF REACTION FOR AIR GAS, B.T.U.

Heat of Reaction. General.	When CO = 0 or $x = 1$.	When $CO_2 = 0$ or $x = 0$.	Unit		
$10193x+4351$	14544	4351	Per Lb. carbon	(a)	(779)
$\frac{10193x+4351}{5.759(1+x)}$	$\frac{14544}{11.518}=1263$	$\frac{4351}{5.759}=755$	" Lb. air	(b)	
$\frac{10193x+4351}{5.758x+6.758}$	$\frac{14544}{12.517}=1162$	$\frac{4351}{6.759}=644$	" Lb. gas	(c)	
$\frac{10193x+4351}{56.32x+86.12}$	$\frac{14544}{142.44}=1021$	$\frac{4351}{86.12}=51$	" Cu.ft. gas	(d)	
$\frac{10193x+4351}{3.66x}$	$\frac{14544}{3.66}=3974$	.....	" Lb. CO_2	(e)	
$\frac{10193x+4351}{2.33(1-x)}$	.....	$\frac{4351}{2.33}=1867$	" Lb. CO	(f)	
$\frac{10193x+4351}{4.428(1+x)}$	$\frac{14544}{8.856}=1642$	$\frac{4351}{4.428}=983$	" Lb. N_2	(g)	
$\frac{10193x+4351}{29.8x}$	$\frac{14544}{29.8}=488$	.....	" Cu.ft. CO_2	(h)	
$\frac{10193x+4351}{29.8(1-x)}$	.....	$\frac{4351}{29.8}=146$	" Cu.ft. CO	(i)	
$\frac{10193x+4351}{56.32(1+x)}$	$\frac{14544}{112.64}=129$	$\frac{4351}{56.32}=77$	" Cu.ft. N_2	(j)	

These heats of reaction are all positive and act to raise the temperatur of the bed and the products of the reaction, and the temperature rise could b calculated if the specific heat of the products were known, or on any assumptio for its value. The heat of combustion of the gases formed, which is due to th CO, is the useful effect in gas producers and a loss in boiler fires to which th preceding relations also apply. The amount of this heat of combustion is, course, the difference between the heat of complete combustion per pound C ar its heat of reaction when combustible gases are formed by it. Therefore, th heat of combustion of the gas will be $14{,}544-(10{,}193x+4351)$ per pound carbon, from which the value per cubic foot and pound of gas can be found given below in tabular equation form, Eq. (780).

HEAT OF COMBUSTION OF HYPOTHETICAL AIR GAS, B.T.U.

General.	When CO = 0 or $x = 1$.	When $CO_2 = 0$ or $x = 0$.	Unit		
$10193(1-x)$	0	10193	Per lb. C	(a)	(78
$\frac{10193(1-x)}{5.758x+6.759}$	0	$\frac{10193}{6.759}=1508$	Per lb. gas	(b)	
$\frac{10193(1-x)}{56.32x+86.12}$	0	$\frac{10193}{86.12}=118$	Per cu.ft. gas	(c)	

Dividing the heat of combustion of the gas formed per pound carbon, the heat of combustion of the carbon, will give the efficiency of the produ as a fuel transformer from solid carbon to combustible gases.

$$\left\{\begin{array}{l}\text{Efficiency of gasification}\\ \text{of carbon}=\end{array}\right\}\left\{\begin{array}{ll}E=\dfrac{10{,}193(1-x)}{14{,}544} & (a)\\ =70\%\text{ if no } CO_2 \text{ is formed} & (b)\end{array}\right\}\quad\ldots\quad(781)$$

erefore, the efficiency of this process cannot exceed 70 per cent, the other 30 cent of the carbon heat appearing as sensible heat of the gas, some of which be used as will be shown later for the formation and dissociation of steam, ling hydrogen thereby to the gas, and raising the efficiency more and more ve 70 per cent as the hydrogen increases, except as such reactions cool the l and tend to make more CO_2 with its corresponding lessening air-gas efficiency ct. When the CO_2 content is not zero, the heat of combustion of the gas l efficiency of the process becomes less and the curves of Fig. 184 are plotted show graphically the relation. On the same sheet are given the gas mposition, calorific power of the gas and heats of reaction per pound of bon, and per pound of gas. The heat of reaction per pound of carbon is a asure of efficiency, and per pound of gas a measure of the temperature of gas when specific heats are known.

Air-gas making involves high temperatures, which cannot be calculated, l these are really so high as to make it an impracticable process for most ls because of the fusibility of the ash and formation of clinkers large enough stop the flow of blast. The blast furnace, fluxing and melting everything ontains to slag and iron does not suffer in this way and it is not impossible t fluxing of ash and removal as liquid slag may be practical for gas making in ducers some day. At the present time steam is almost universally introduced h the blast of air, and by its decomposition absorbs enough heat to prevent ous clinkering of most coals, though there are still some that give trouble.

The calorific power and density of an air gas can, like others, be calculated n the properties of its constituents and such a calculation is arranged in ular form below, Table LXXXV.

TABLE LXXXV

DENSITY AND CALORIFIC POWER OF BLAST FURNACE GAS

FROM CONSTITUENTS (32° F. AND 29.92″ Hg)

erage Composition of Blast Furnace Gas.	One Cubic Foot				Summary.
	Contains		Yeilds B.T.U.		
	Cu. Ft.	Pounds.	High.	Low.	
on monoxide, CO..	.2861	.022336	97.560	97.560	B.T.U. cu.ft. gas high..... 109.035
rogen, H_2.....	.0274	.000143	9.343	8.001	
hane, CH_4.....	.0020	.000089	2.132	1.918	B.T.U. cu.ft. gas low..... 107.479
on dioxide, CO_2...	.1139	.013974	.....	.....	Cu.ft. per lb. gas..12.3132
ogen, N_2.....	.5706	.044672	.....	.....	Lbs. per cu.ft. gas .081214
l for gas.....	1.0000	.081214	109.035	107.479	B.T.U. lb. gas high.....1342.57
					B.T.U. lb. gas low.....1323.41

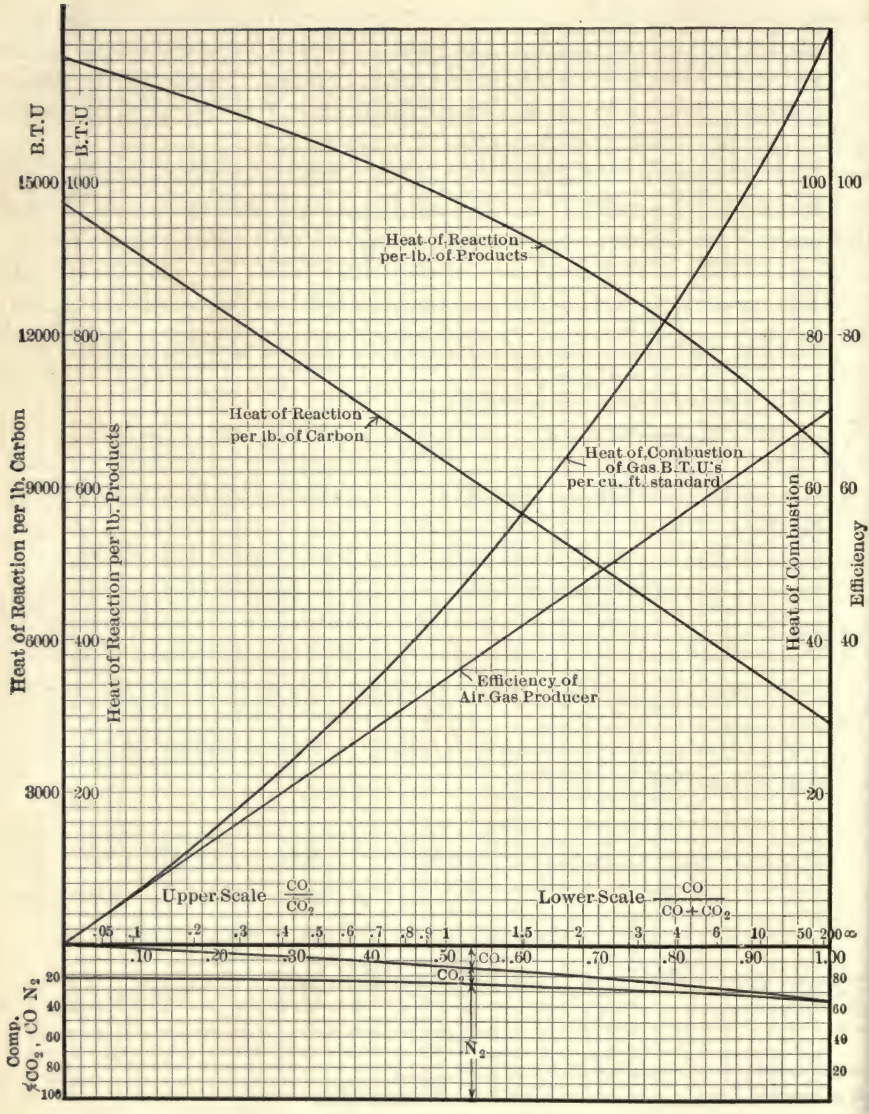


FIG. 184.—Characteristics of Air-gas.

Example 1. A sample of air gas showed a ratio of CO to CO_2 of 5.66. What was composition by weight and volume, pounds of carbon and air supplied per cubic foot gas made, and the heat of combustion per cubic foot of the gas?

$$\text{Since } \frac{CO}{CO_2} = \frac{1-x}{x} = 5.66,\ x = .15.$$

$$CO_2 \text{ by weight} = \frac{3.66 \times .15}{5.758 \times .15 + 6.758} = 7.2 \text{ per cent.}$$

$$CO \text{ by weight} = \frac{2.33 \times .85}{5.758 \times .15 + 6.758} = 26 \text{ per cent.}$$

$$N_2 \text{ by weight} = \frac{4.428 \times 1.15}{5.758 \times .15 + 6.758} = 66.8 \text{ per cent.}$$

$$CO_2 \text{ by volume} = \frac{29.8 \times .15}{56.32 \times .15 + 86.12} = 4.7 \text{ per cent.}$$

$$CO \text{ by volume} = \frac{29.8 \times .85}{56.32 \times .15 + 86.12} = 26.8 \text{ per cent.}$$

$$N_2 \text{ by volume} = \frac{56.32 \times 1.15}{56.32 \times .15 + 86.12} = 68.5 \text{ per cent.}$$

$$1 \text{ cu.ft. gas requires } \frac{1}{56.32 \times .15 + 86.12} = .0105 \text{ lb. C.}$$

$$1 \text{ cu.ft. gas requires } \frac{5.759(1.15)}{56.32 \times .15 + 86.12} = .0705 \text{ lb. air.}$$

The heat of combustion per cubic foot of gas is

$$\frac{10{,}193 \times .85}{56.32 \times .15 + 86.12} = 92 \text{ B.T.U.}$$

Prob. 1. If the gases Nos. 1 and 11 of Table CXIV were made with a bed perature of 1400° F., what was the ratio of actual $\frac{CO}{CO_2}$ per cent to the equilibrium cent? What would it have been if the temperature had been 1600° F.?

Prob. 2. In an air gas consisting only of CO and CO_2, what would be the largest ount of CO per cubic foot attainable with a bed temperature of 1200° F.?

Prob. 3. In a producer blasted with air alone 20 per cent of the carbon burns to $_2$ and 80 per cent to CO. How many cubic feet of CO, CO_2, and N_2 will be formed pound of carbon and how much air will be required? What heat will be developed the reaction per pound C, per pound gases, and by the gas per pound and per cubic t in burning?

Prob. 4. Should the carbon all burn to CO what would be the composition of the by weight and volume and how much air would be needed?

Prob. 5. How many cubic feet of air would be needed to make (*a*) 10 lbs. and (
10 cu.ft. of air gas containing 25 per cent CO and 5 per cent CO_2?

Prob. 6. A blast furnace yielded gas containing practically nothing but N_2, C
and CO_2, the percentages being 30 per cent for CO and 10 for CO_2. Assuming th
blast to have been preheated to a temperature of 300° F. and that the specific heat
the products was .25, what was the final temperature?

Prob. 7. What would be the heat of reaction in the above case per cubic foot ar
pound of gas made and per pound of carbon. What would be the heat of combustio
of the gas calculated from constituents and from the general equation for the prop
value of x.

Prob. 8. Neglecting the H_2 and CH_4 in the gases what was the efficiency of th
producer in the case of gases Nos. 1, 2, 3, and 9, Table CXIV?

Prob. 9. What is the calorific power and density of gas No. 14 of Table CXIV?

**8. Gasification of Fixed Carbon, Coke and Coal, Previously Heated, b
Steam Blast Reactions, Producing Water Gas. Composition and Relatio
of Constituents of Water Gas, Yield per Pound of Steam and Coal. He
of Combustion of Gas and Limitation of Yield by Negative Heat of Reactio**

Water gas is the term applied to the product obtained by blasting steam into
carbon, charcoal, coke or coal bed previously heated and *producer gas* generally,
Dowson gas in England and generator gas in Germany, where the heating of th
bed by air-gas making, proceeds at the same time as the steam reaction whic
of course, is endothermic. The fundamental relations of the temperature
the bed to the reaction characteristics were studied by Bunte with the resul
given in Table LXXXVI.

TABLE LXXXVI.

WATER GAS CHARACTERISTICS WITH BED TEMPERATURE (BUNTE)

Temp. F.	% Steam H_2O Decomposed.	Composition by Vol. of Water Gas.			$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$
		H_2	CO	CO_2		
1245 Dull red	8.8	65.2	4.9	29.8	.16	.141
1396	25.3	65.2	7.8	27.0	.29	.224
1540	41.0	61.9	15.1	22.9	.65	.397
1749	70.2	53.3	39.3	6.8	5.80	.853
1850	94.0	48.8	49.7	1.5	33.1	.972
1940	98.0	50.7	48.0	1.3	36.8	.975
2057 (White)	99.4	50.9	48.5	.6	80.8	.988

These results are plotted in Fig. 185 to a base of carbon oxide ratio, togeth
with the Boudouard equilibrium curve, and indicate that in no case was t
equilibrium even approximately attained, as for example, at 1750° F. Bunt
ratio was less than 6, while Boudouard's equilibrium value is about 75 in rou

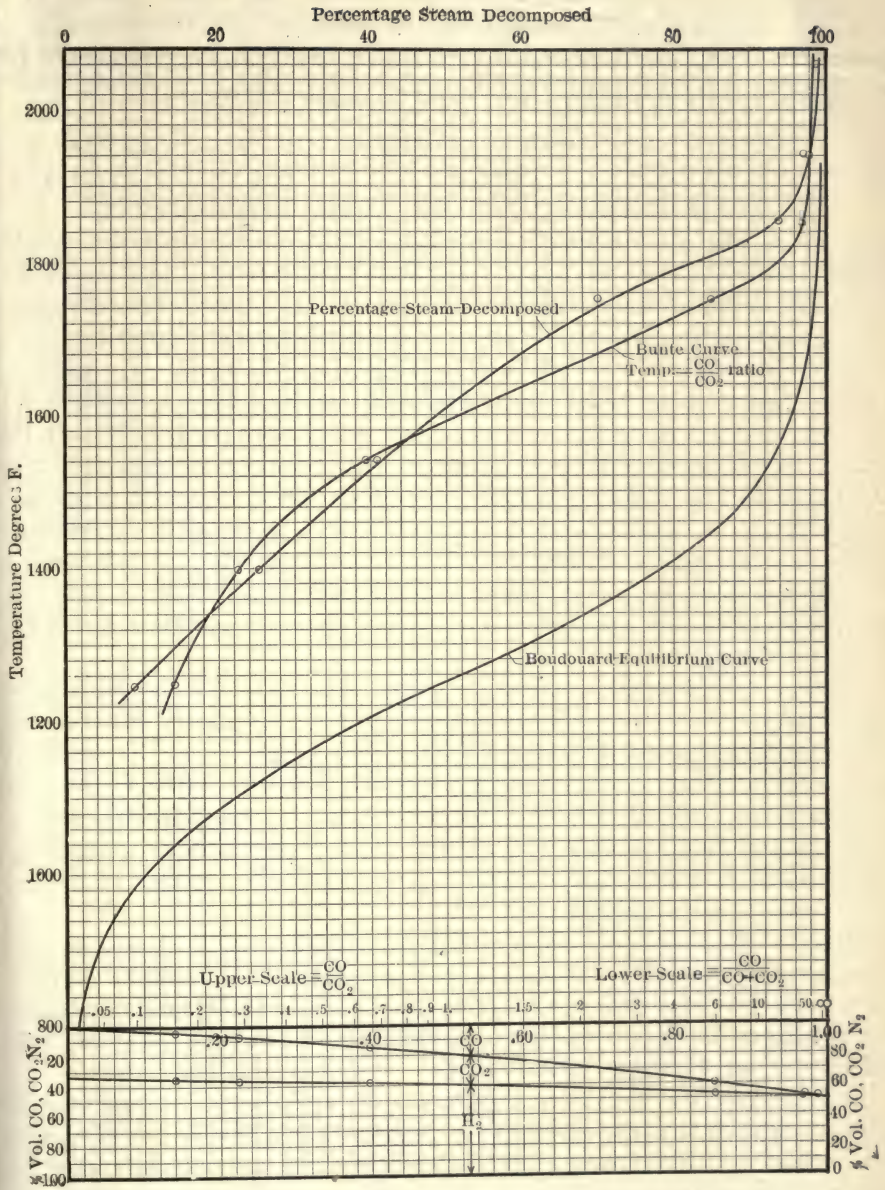


FIG. 185.—Characteristics of Water-gas at Various Bed Temperatures

numbers. Assuming both experimenters' results to be accurate, this would be a measure of the importance of the time element, but there is considerable doubt as to the accuracy especially of the temperatures of the bed reported by Bunte. There is, however, another explanation based on the reactions of CO with steam to form CO_2 and H_2 according to

$$CO + H_2O = CO_2 + H_2$$

which would tend to oxidize the CO when formed in the presence of steam, and it is reported by Bunte that at the temperature named, 1750, there was 30 per cent of steam free and still undecomposed. That this is the most probable explanation is indicated by the lesser discrepancy at low temperatures at which the above steam reaction takes place feebly or not at all. For example, at Bunte's lowest temperature 1245° F. his ratio is .16, while for equilibrium it would be about five times as much as against twelve times at 1750°F. In this work of Bunte the bed, previously heated to the temperature desired, must have begun to cool at once under the steam blast influence, which is precisely what happens in straight water-gas manufacture, an essentially intermittent process. Moreover, the cooling cannot be uniform because the resistance to flow is not uniform through a fuel bed, so that where the blast can pass most easily, there most of it will pass, and a non-homogeneous condition of temperature and extent of reactions result.

Another interesting fact shown by these data is the rapidity with which hydrogen forms in considerable volume even at low temperature and with only a small fraction of steam decomposed. There is a retardation of CO formation, until the temperatures have become considerable, at about 1400° F. the curve becomes more nearly horizontal and, for a very considerable temperature range the per cent hydrogen remains substantially constant between 65 per cent for 1250° F. where there is practically no CO, and 51 per cent at 2060° F. where there is nearly 49 per cent of CO.

Straight water-gas making, which these experiments illustrate, proceeds with decreasing temperature, but producer-gas making is a continuous process with steady temperatures, enough air oxygen combining with carbon exothermally to balance the endothermic heat of the water-gas making, to permit of the maintenance of a steady state.

In Table CXVI at the end of the Chapter are given some analyses of water gases which show carbon oxide ratios as high as 17.5 indicating the attainment of very high bed temperatures before steam blasting, a moderately slow flow through the bed to give the CO_2 a chance to reduce, and not an excessive amount of free H_2O to react with the CO formed. In some cases CH_4 is reported, indicating the use of a coal or an incomplete coke, some oxygen and nitrogen are also reported, indicating influx of air, but it is difficult to see how oxygen could exist unless its contact with combustible H_2, CO or CH_4 were delayed till cooling occurred. The hydrogen content of these water gases ranges a little above and

below 50 per cent which is about the same as found by Bunte for all temperatures above that at which the formation of CO was appreciable, 1800° F., in which case, the volumetric amount is substantially equal for the hydrogen and carbon monoxide in Bunte's tests and in some of the analyses reported. When the analyses depart from this equality the CO is always less than H_2 and as this was found by Bunte to be so for too low a temperature, it may be assumed that when such analyses are reported, the process had been carried on for too long a time, or started with too cold a bed. Usually five minutes is a fair average time for water-gas making in a previously heated coke or anthracite fuel bed.

The volumes, weights and heats involved in water gas making are given by two fundamental equations of chemical reactions, Eqs. (782) and (783), using the low calorific value for hydrogen, and derived from,

$$C+2H_2O=CO_2+2H_2+[(14{,}544\times12)-(51{,}892\times4)]\text{ B.T.U.},$$

$$C+\ H_2O=CO\ +\ H_2+[(4351\times12)-(51{,}892\times2)]\text{ B.T.U.}$$

Therefore,

$$C+2H_2O=CO_2+2H_2-33{,}040\text{ B.T.U.}, \quad . \ . \ . \ . \quad (782)$$

$$C+\ H_2O=CO\ +\ H_2-51{,}572\text{ B.T.U.} \quad . \ . \ . \ . \quad (783$$

These may be interpreted as follows:

$$1\text{ lb. }C+\left\{\left[\begin{array}{c}3\text{ lbs.}\\ \text{or}\\ 59.6\text{ cu.ft.}\end{array}\right]H_2O\right\}=\left\{\begin{array}{l}\left[\begin{array}{c}3.66\text{ lbs.}\\ \text{or}\\ 29.8\text{ cu.ft.}\end{array}\right]CO_2\\ +\left[\begin{array}{c}.333\text{ lb.}\\ \text{or}\\ 59.6\text{ cu.ft.}\end{array}\right]H_2\end{array}\right\}-2753\text{ B.T.U. . (784)}$$

$$1\text{ lb. }C+\left\{\left[\begin{array}{c}1.5\text{ lbs.}\\ \text{or}\\ 29.8\text{ cu.ft.}\end{array}\right]H_2O\right\}=\left\{\begin{array}{l}\left[\begin{array}{c}2.33\text{ lbs.}\\ \text{or}\\ 29.8\text{ cu.ft.}\end{array}\right]CO\\ +\left[\begin{array}{c}.167\text{ lb.}\\ \text{or}\\ 29.8\text{ cu.ft.}\end{array}\right]H_2\end{array}\right\}-4298\text{ B.T.U. . (785)}$$

Assuming that 1 lb. of carbon yields both reactions at the same time and that x represents the fractional part burning to CO_2, or $(1-x)$ the corresponding part that burns to CO, then the double reaction will be represented by Eq. (786).

$$1 \text{ lb. C} + \left\{ \begin{array}{r} x \left[\begin{array}{c} 3 \text{ lbs.} \\ \text{or} \\ 59.6 \text{ cu.ft.} \end{array} \right] \text{H}_2\text{O} \\ +(1-x) \left[\begin{array}{c} 1.5 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] \text{H}_2\text{O} \end{array} \right\} = \left\{ \begin{array}{r} x \left[\begin{array}{c} 3.66 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] \text{CO}_2 \\ +(1-x) \left[\begin{array}{c} 2.33 \text{ lbs.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] \text{CO} \\ +x \left[\begin{array}{c} .333 \text{ lb.} \\ \text{or} \\ 59.6 \text{ cu.ft.} \end{array} \right] \text{H}_2 \\ +(1-x) \left[\begin{array}{c} .167 \text{ lb.} \\ \text{or} \\ 29.8 \text{ cu.ft.} \end{array} \right] \text{H}_2 \\ -[(2753x+4298(1-x)] \\ \text{B.T.U.} \end{array} \right\} \quad (786$$

This reduces to the two following forms, Eqs. (787) and (788), the first fo weights and the second for volumes, all volumes being for standard gas con ditions.

$$1 \text{ lb. C} + 1.5(1+x) \text{lbs. } \text{H}_2\text{O} = \left[\begin{array}{c} 3.66x \text{ lbs. } \text{CO}_2 \\ +2.33(1-x) \text{ lbs. CO} \\ +.167(1+x) \text{ lbs. } \text{H}_2 \end{array} \right] + (1545x - 4298) \text{ B.T.U.} \quad (787$$

$$= \left[\begin{array}{c} 29.8\,x \text{ cu.ft. } \text{CO}_2 \\ +29.8(1-x) \text{ cu.ft. CO} \\ +29.8(1+x) \text{ cu.ft. } \text{H}_2 \end{array} \right] + (1545x - 4298) \text{ B.T.U} \quad (788$$

From these equations the weight, volume, heat and temperature changes ca be set down in each of the various units.

The composition of the gas will be for any proportion of CO to CO_2 as give below in Table LXXXVII.

TABLE LXXXVII

COMPOSITION OF HYPOTHETICAL WATER GAS. GENERAL

Constituents.	By Weight.	By Volume.	$\frac{\text{CO}}{\text{CO}_2}$ (Vols.)	$\frac{\text{CO}}{\text{CO}+\text{CO}_2}$
CO_2	$\frac{3.66x}{1.5x+2.5}$	$\frac{29.8x}{29.8x+59.6}$	$\frac{1-x}{x}$	$1-x$
CO	$\frac{2.33(1-x)}{1.5x+2.5}$	$\frac{29.8(1-x)}{29.8x+59.6}$		
H_2	$\frac{.167(1+x)}{1.5x+2.5}$	$\frac{29.8(1+x)}{29.8x+59.6}$		

It appears from this that, like the case of air-gas, the carbon monoxide-dioxide ratio by volume is the same as the fractional weight of carbon burning to these two constituents respectively.

When the carbon burns all to CO, $x=0$, or all to CO_2, $x=1$, the composition of the gas will be that given in Table LXXXVIII.

TABLE LXXXVIII

COMPOSITION OF HYPOTHETICAL WATER GAS. NO CO_2, AND NO CO

Constituents	By Weight, Per Cent.		By Volume, Per Cent.		Ratio $\frac{CO}{CO^2}$		Ratio $\frac{CO}{CO+CO_2}$	
	All CO	All CO_2	All CO	All CO_2	All CO	All CO_2	All CO	All CO_2
CO_2	0	$\frac{3.66}{4}=91.7$	0	$\frac{29.8}{89.4}=33$	∞	0	1	0
CO	$\frac{2.33}{2.50}=93.3$	0	$\frac{29.8}{59.6}=50$	0				
H_2	$\frac{.167}{2.50}=6.7$	$\frac{.333}{4}=8.3$	$\frac{29.8}{59.6}=50$	$\frac{59.6}{89.4}=67$				

The equality of volume of CO and H_2 when no CO_2 is formed is pretty well borne out by the test results quoted, at least as perfectly as the equilibrium ratio for the temperature permits, when the temperature is high and free steam is not in contact with CO in the hot zone, while the 67 per cent by volume of hydrogen for no CO formed, is pretty closely approached by Bunte's 65.2 per cent when there was 4.9 per cent CO at the lowest temperature of 1245° F.

The general relations between the carbon and steam as raw materials and the gas produced are given by Eq. (789).

$$\left.\begin{array}{lll}
\text{1 lb. carbon makes } (2.5+1.5x) \text{ lbs. of gas} & (a) \\
\text{1 lb. steam makes } \left(\frac{2.5+1.5x}{1.5(1+x)}\right) \text{ lbs. of gas} & (b) \\
\text{1 lb. gas requires } \left(\frac{1}{2.5+1.5x}\right) \text{ lbs. of carbon} & (c) \\
\text{1 lb. gas requires } \left(\frac{1.5(1+x)}{2.5+1.5x}\right) \text{ lbs. of steam} & (d) \\
\text{1 lb. carbon makes } (29.8x+59.6) \text{ cu.ft. gas} & (e) \\
\text{1 lb. steam makes } \left(\frac{29.8x+59.6}{1.5(1+x)}\right) \text{ cu.ft. gas} & (f) \\
\text{1 cu.ft. gas requires } \left(\frac{1}{29.8x+59.6}\right) \text{ lbs. of carbon} & (g) \\
\text{1 cu.ft. gas requires } \left(\frac{1.5(1+x)}{29.8x+59.6}\right) \text{ lbs. of steam} & (h)
\end{array}\right\} \quad (789)$$

The heats of reaction per unit of each quantity entering are obtainable by dividing the total heat of reaction by the volume or weight of each substance entering, and these are set down in the tabular equation below, Eq. (790).

HEATS OF REACTION FOR WATER GAS, B.T.U. LOW.

General Heat of Reaction.	When CO =0, $x=1$.	When CO_2 =0, $x=0$.	Unit		
$1545x-4298$	-2753	-4298	Per Lb. carbon	(a)	(790)
$\frac{1545-4298x}{1.5(1+x)}$	$-\frac{2753}{3}=-918$	$-\frac{4298}{1.5}=-2865$	" Lb. steam	(b)	
$\frac{1545x-4298}{1.5x+2.5}$	$-\frac{2753}{4}=-688$	$-\frac{4298}{2.5}=-1719$	" Lb. gas made	(c)	
$\frac{1545x-4298}{29.8x+59.6}$	$-\frac{2753}{89.4}=-31$	$-\frac{4298}{59.6}=-72$	" Cu.ft. gas made	(d)	
$\frac{1545x-4298}{3.66x}$	$-\frac{2753}{3.66}=-752$	.....	" Lb. CO_2	(e)	
$\frac{1545x-4298}{2.33(1-x)}$	.....	$-\frac{4298}{2.33}=-1842$	" Lb. CO	(f)	
$\frac{1545x-4298}{.167(1+x)}$	$-\frac{2753}{.33}=-8259$	$-\frac{4298}{.167}=-25788$	" Lb. H_2	(g)	
$\frac{1545x-4298}{29.8x}$	$-\frac{2753}{29.8}=-92$	.....	" Cu.ft. CO_2	(h)	
$\frac{1545x-4298}{29.8(1-x)}$	.....	$-\frac{4298}{29.8}=-144$	" Cu.ft. CO	(i)	
$\frac{1545x-4298}{29.8(1+x)}$	$-\frac{2753}{59.6}=-46$	$-\frac{4298}{29.8}=-144$	" Cu.ft. H_2	(j)	

These heats all go to the lowering of temperature, being negative, and the temperature drop can be determined only by assuming an average specific heat for the products which are 93 per cent CO and 7 per cent H_2 approximately by weight in one, and 92 per cent CO_2 and 8 per cent H_2 approximately by weight in the other limiting case, and at the same time assuming the sensible heat capacity and the weight of the large fuel bed, and the weight of gas flow through it per minute. This would give an approximation to the rate of temperature lowering, which would be of doubtful value. In practical operation the process is controlled by observation and usually the bed drops in temperature enough in five minutes, from the highest starting temperature practicable to avoid serious clinkering, to that at which the process is no longer feasible, and this last temperature may be put roughly at 2000° F. and 1700° F. if excessive CO_2 is to be avoided in the gas.

The pound of carbon for which the total heat of reaction is $(1545x-4298)$ B.T.U. could give on combustion 14,544 B.T.U. so that the gas which results must yield the difference $(14{,}544+4298-1545x)$ B.T.U. (low) per pound of carbon with corresponding values in other units, as in tabular Eq. (791).

HEAT OF COMBUSTION OF HYPOTHETICAL WATER GAS, B.T.U. LOW

General	When CO = 0, $x = 1$	When $CO_2 = 0$, $x = 0$	Unit		
$18842 - 1545x$	17297	18842	Per lb. C	(*a*)	. (791)
$\frac{18842 - 1545x}{2.5 + 1.5x}$	$\frac{17297}{4} = 4342$	$\frac{18842}{2.5} = 7537$	Per lb. gas	(*b*)	
$\frac{18842 - 1545x}{29.8x + 59.6}$	$\frac{17297}{89.4} = 174$	$\frac{18842}{59.6} = 316$	Per cu.ft. gas	(*c*)	

Dividing the heat of combustion of the gas per pound of carbon by the heat of its complete combustion per pound will give the *apparent* efficiency of the process by Eq. (792).

$$E = \left(\frac{18{,}842 - 1545x}{14{,}544}\right) \quad (a)$$

$$= \frac{18{,}842}{14{,}544} = 130 \text{ per cent if no } CO_2 \text{ is formed} \quad (b) \qquad . \; . \; (792)$$

This is, of course, impossible continuously, and results from ignoring the heat that must be put into the bed before admitting steam to make the steam reaction which is so strongly endothermic, a possibility. Neglecting the heat necessary for the making of steam from water as it is possible sometimes to secure it from waste sources, the previous making of air gas can be computed in terms of the quantity that must be made, or in terms of the amount of carbon used in doing it, to permit of a given amount of water-gas production subsequently.

Thus, for air-gas making, the heat of reaction per pound C is $(10{,}193x + 4351)$ while for water-gas making it is $(1545x - 4298)$ For these to equalize

(Heat of reaction per pound C) × (lbs. C) used for air gas = (heat of reaction per pound C) × (lbs. C) used for water gas.

Therefore

$$\frac{\text{Heat of air gas react. per lb. C}}{\text{Heat of water gas react. per lb. C}} = \frac{\text{Lbs. C gasified to make water gas}}{\text{Lbs. C gasified to make air gas}}$$

$$= \left(\frac{10{,}193x + 4351}{1545x - 4298}\right) \quad (a)$$

$$= \frac{4351}{4298} = 1.01 \text{ if } CO_2 = 0 \quad (b) \qquad . \; . \; (793)$$

Similarly,

(Heat of reaction per lb. of air gas) × (lbs. air gas)
= (heat of reaction per lb. water gas) × (lbs. water gas).

Therefore

$$\frac{\text{Heat of reaction per lb. air gas}}{\text{Heat of reaction per lb. water gas}} = \frac{\text{Lbs. water gas made}}{\text{Lbs. air gas made}} = \frac{644}{1719}$$

$$= .37 \text{ if } CO_2 = 0 . \quad (794)$$

Or in terms of volumes

$$\frac{\text{Heat of reaction per cu.ft. air gas}}{\text{Heat of reaction per cu.ft. water gas}} = \frac{\text{Cu.ft. water gas made}}{\text{Cu.ft. air gas made}} = \frac{51}{72}$$

$$= .70 \text{ if } CO_2 = 0 \quad (795)$$

Therefore, assuming no CO_2 to be formed, which is the limiting case, 2 cu.ft. of water gas can be made for each 3 cu.ft. of air gas previously made, in round numbers, or about a third of a pound of water gas for each pound of air gas, or about equal weights of carbon must be used for each. If, as in many cases, the process is intermittent and the air gas is thrown away, the efficiency would be about $\frac{130}{2} = 65$ per cent for water-gas making alone, and proportionately higher as some of the air gas or its heat was put to useful duty.

The density and calorific power of a water gas can, of course, be calculated from its constituents by the general method, and the results for a typical water gas are tabulated below, Table LXXXIX.

TABLE LXXXIX

DENSITY AND CALORIFIC POWER OF WATER GAS

FROM CONSTITUENTS (32° F. and 29.92″ Hg)

Constituents of Average Water Gas.	One Cubic Foot				Summary
	Contains		Yields B.T.U.		
	Cu. Ft.	Lbs.	High.	Low.	
Hydrogen, H_2	.4557	.002561	155.39	133.06	B.T.U. per cu.ft. gas high.....357.04
Carbon monoxide, CO	.4485	.035014	152.94	152.94	B.T.U. per cu.ft. gas low.....329.86
Methane, CH_4	.0441	.001971	47.01	42.29	
Heavy Hydrocarbons C_2H_4	.0010	.000080	1.7	1.57	Cu.ft. per lb. gas.21.917
Carbon dioxide, CO_2	.0445	.005460	.....	.....	Lbs. per cu.ft. gas...045623
Oxygen, O_2	.0050	.000446	.....	.....	B.T.U. per lb. high..7825
Nitrogen, N_2	.0012	.000094	.....	.....	B.T.U. per lb. low 7228
Totals	1.0000	.045626	357.04	329.86	

Example 1. A sample of water gas showed a ratio of CO to $(CO + CO_2)$ of .9. What was its composition by weight and volume; pounds of carbon and of steam supplied per cubic foot of gas made, and the heat of combustion per cubic foot of the gas? Since $CO \div (CO + CO_2) = 1 - x$, $(1 - x) = .9$, and $x = .1$.

$$CO_2 \text{ by weight } \frac{3.66 \times .1}{1.5 \times .1 + 2.5} = 13.8 \text{ per cent.}$$

$$\text{CO by weight} \quad \frac{2.33\times.9}{1.5\times.1+2.5} = 79.3 \text{ per cent.}$$

$$\text{H}_2 \text{ by weight} \quad \frac{.167\times1.1}{1.5\times.1+2.5} = 6.9 \text{ per cent.}$$

$$\text{CO}_2 \text{ by volume} = \frac{29.8\times.1}{29.8\times.1+59.6} = 4.7 \text{ per cent.}$$

$$\text{CO by volume} = \frac{29.8\times.9}{29.8\times.1+59.6} = 42.8 \text{ per cent.}$$

$$\text{H}_2 \text{ by volume} = \frac{29.8\times1.1}{29.8\times.1+59.6} = 52.5 \text{ per cent.}$$

$$\text{1 cu.ft. gas requires} \quad \frac{1}{29.8\times.1+59.6} = .16 \text{ lb. C.}$$

$$\text{1 cu.ft. gas requires} \quad \frac{1.5(1+.1)}{29.8\times.1+59.6} = .264 \text{ lb. steam.}$$

The heat of combustion per cubic foot of gas is

$$\frac{18{,}842-1545\times.1}{29.8\times.1+59.6} = 298 \text{ B.T.U. (low value)}$$

Prob. 1. In a water-gas producer the bed temperature was found to be 1800° F. According to the results of Bunte's experiments, what would be the composition of the gas and the per cent of steam decomposed? How would the CO to CO_2 ratio, compare with that for the air-gas producer?

Prob. 2. Lewes gives as an average analysis of water gas the following: $H_2=51.9$; $CO=40.08$; $CH_4=.1$; $CO_2=4.8$; $N_2=3.13$. How does the percentage of hydrogen as given compare with that found by the general equation in terms of x.

Prob. 3. How much carbon and how much steam will be required to make 1000 cu.ft. of gas which is 50 per cent H_2, 45 per cent CO, and 5 per cent CO_2 by volume, and what will be the heat of reaction per pound of carbon and per pound of gas?

Prob. 4. A producer gives a gas in which the ratio of CO to $(CO+CO_2)$ is .8. How many cubic feet of gas will be made per pound of C and per pound of steam?

Prob. 5. What will be the heat of combustion of the gas No. 10 in Table CXVI per pound and per cubic foot? Calculate this by the general equation in terms of x, and from the analysis.

Prob. 6. For the same gas what would be the efficiency of the producer?

9. Gasification of Coals by Steam and Air Blasts, Resulting in Producer Gas. Composition and Relation of Constituents of Producer Gas, Yield per Pound of Fixed Carbon, Air, and Steam. Modification of Composition by Addition of Volatile of Coal. Heat of Combustion of Gas, Sensible Heat, and Efficiency of Gasification. Horse-power of Gas Producers. The most important of these gasifying processes is the most complex, and as carried out in the power-gas producer is as important in gas-power systems using coal fuel as is steam generation in the boilers of steam-power systems. The gas producer is a continuously operated apparatus, having a brick-lined casing to hold its thick fuel bed, and provided with means for blasting the bed with a mixture

of steam and air in more or less closely regulated proportions. This gives rise to a condition of steady state, as to temperature and reactions, which are maintained as long as working conditions permit and the most important of these is the physical condition of the bed as to porosity, ash content, homogeneity and clinker. Neglecting these physical conditions, it may be assumed, that the blast enters all points of the supply part of the bed uniformly, and passes through all parts at an equal rate, all parts of the blast remaining in contact with the carbon for the same length of time, and that finally that all points in any cross-section of the bed at right angles to the path are at an equal temperature. These things are not really as assumed, but it is difficult enough to fix any relations even with these assumptions and impossible without them except in a qualitative manner.

In such producers there is made a combination of water gas and air gas, the ratio depending on the amounts of carbon reacting respectively with the oxygen of air and with the oxygen of steam. In all cases the exothermic heat of the air-gas process supports and more or less balances the endothermic heat of the water-gas process. The gas will thus contain as combustibles *from the fixed carbon and steam*, carbon monoxide, hydrogen, nitrogen, carbon dioxide, and some uncombined oxygen, usually very small in amount. Mixed with this gas will be the *products of the coal volatile*, which are most complex in character and amount as might be inferred from the discussion of retort-coal gas and coke-oven gas. The nature and amounts of these additional products of the volatile of the coal depends, of course, on the coal itself but also to a very considerable degree on the manner of treatment in producers and it is in this respect only, that different gas producer processes differ one from the other, however different they may look or vary in structure.

Producers may be divided into the following classes with respect to the treatment of the volatile of the coal.

1. *Up draft.* Coal is fed at top and blast at the bottom. Volatile and moisture roast off at the top layer in the presence of gases from the coke-bed reactions and in a temperature equal to that of the gas leaving.

2. *Down draft.* Coal and blast are supplied to the top. Volatile and moisture distilled at the point of supply are mixed with the air and steam of the blast and pass down through the entire bed. Volatile is partly burned and partly decomposed in the hot zone.

3. *Combined up and down draft.* Blast enters both top and bottom, coal is fed at the top and the gas leaves at the bottom. Volatile passes through hot zones, not so hot as (2) but hotter than (1) and in the presence of some air but less than (2). Volatile is partly burned, partly decomposed and partly unchanged.

4. *Combined up and down draft, blast entering only at bottom.* Volatile and moisture are roasted off at the top, and rising, are brought around the bed to its bottom, entering there with the blast gases, the finished gas leaving at a mid point. Usually, this is imperfectly executed, some volatile passing straight down to the gas outlet. Volatile is all burned as it enters the bed.

In the first class are to be found all standard forms of anthracite and coke producers, while the others are intended for bituminous coals, yielding much tar when supplied to the first class, and which cannot be completely separated from the gas, clogging mains and interfering with the operation of the engine. In no case can a caking or coking coal be properly handled in any standard producer because of draft interference due to melting of the coal without prohibitively costly attendance to keep the cake broken up and in some cases not even then. Even non-caking bituminous coals give some trouble, not only because of the tar they yield, but because also of the variability of the gas which is a mixture of water gas, air gas and more or less decomposed rich hydrocarbon volatile. Even when the ratio of air and water gas is kept constant, the volatile of the coal distilling off at a variable rate, will seriously impair the constancy of the gas quality, because the volatile of bituminous coals has about six times the heating power of the gasified carbon. With lignites the high moisture is mixed with the volatile, absorbing heat in its liberation and in the steam reactions with the other constituents like CO and so introducing a further cause of interference in a manner depending on its path.

In the tables at the end of this Chapter, Tables CXVII and CXVIII, are collected some producer gas analyses, the first from a wide range of sources to illustrate the ranges of composition, while the second are from Fernald, reporting for the U. S. Geological Survey the results of trials of bituminous coals and lignites, ranging from high to low grade in a common updraft anthracite or coke type of producer to which was added as an auxiliary a tar extractor of the centrifugal fan order. In these latter tests the volatile is all reported as methane and the calorific power calculated on that basis. For each case the carbon oxide ratio has been added, to indicate more clearly the relations of the CO, CO_2 and H_2, derived mainly from the coke bed reaction, the hydrocarbons being purely roasting products. Of course, in down draft and mixed draft producers some of the volatile has also reacted, part of it in a manner similar to the bed.

Inspection of the general table, Table CXVII, shows a pretty steady value of the carbon-oxide ratio between 4 and 6, except for certain special conditions indicating that the temperatures and rates of blast feed are about the same in all, or that where high rates of combustion are used and time of contact small, the temperatures are allowed to rise to compensate. When the ratio departs from this, a reason is always available, for example, increase of steam in the blast, produced a regular decrease in the $\frac{CO}{CO_2}$ ratio according to the Bone and Wheeler tests on air blasts saturated with water vapor at 60° F. to 80° F. in which the ratio changed from 5.2 to 1.2, and the hydrogen, as might be expected, rose from 16.60 to 22.65 per cent by volume. The increase of hydrogen with increase of steam and the corresponding reduction of temperature and $\frac{CO}{CO_2}$ ratio, is well

confirmed by all results on Mond gas plants, operated primarily for ammonia recovery which requires low temperatures. In these the ratio falls between 6 and 7, indicating according to Boudouard, temperatures above 1200° F. On the other hand reduction of steam permits the bed to attain high temperatures and the gas will contain little hydrogen which is indicated by the Thwaite test on charcoal where the hydrogen is .2 per cent by volume, which could easily have been derived from the moisture in even pretty dry air alone, and the $\frac{CO}{CO_2}$ ratio = 42.5, corresponding to a temperature somewhere above 1500° F. These results, therefore, show effectively how the steam supply controls the bed temperature and the gas quality as to CO and H_2 content and, therefore, why for the constant gas quality needed in power gas, accurate control of bed and blast conditions is so necessary.

In the second table, Table CXVIII, reporting the Geological Survey tests the first remarkable thing noticeable is the regularity with which the carbon oxide ratio $\frac{CO}{CO_2}$ decreases with decrease of CO_2 and the small value of the ratio compared to that obtained in other operating tests. This is an indication of very low temperature working, for the ratio is as small as .87 and never rises above 2.4, possibly due to high rates of blasting. It is too much to expect equilibrium to be established in any producer, yet departures from it give rise to interpretations as to what was happening in one case that did not take place in the other. The general average of the ratio of CO to CO_2 being so low in the U.S.G.S. tests leads to the conclusion that; (*a*) the producer was driven to higher capacity than the average of others leaving less time for establishment of equilibrium; (*b*) more steam was used, resulting in lower temperatures and leaving a residue of undecomposed steam to react on the CO that was formed reducing it to CO_2. There is no relat on, whatever, between the efficiencies reported for these tests and any of the fundamental factors that control efficiency so that these efficiencies must be considered as in error probably because of the shortness of the runs and the irregularity of working reported. Producer efficiencies cannot be obtained except by absolutely steady rate of output, which is necessary to a steady thermal state in the producers and for long periods of time because of the slowness of gasification compared to the body of coal in the producer. In these tests the producer could hold about 5000 lbs. of coal in all stages from green coal to ash and it was burnedt a rates ranging from 270 lbs. per hour for West Virginia No. 12 to 590 lbs. per hour for Texas No. 1, the lengths of run variyng from $4\frac{1}{2}$ hours for Missouri No. 2 to 43 hours for Alabama No. The rate of combustion per hour, therefore, varied from a little over 5 per cent to 12 per cent of the weight in the full producer, making possible a very large error in judging equality of conditions of the bed before and after the run, for such short periods of time. For with the 5 per cent rate a particle of coal would be roughly 20 hours in passing through, as it would take this time to consume what was under it when it was fired.

It is desirable to establish such standards of relation between the quantities involved as were found by analysis for air gas and water gas, to be applied to the producer gas case for the purpose of comparison of results, with possibilities, and this may be done by similar methods. From what has been said it is evident that when air gas and water gas are made simultaneously from the fixed carbon or coke, and mixed with the volatile more or less decomposed, that *the final gas will depend on (a) the proportion of the air gas to water gas; (b) the volume of combined air and water gas to the volatile of the coal; (c) the degree to which the volatile is decomposed or suffers reactions.* The first step, therefore, in a fundamental analysis is the establishment of formulas for the mixed gasified carbon gases in terms of the quantities of constituents, carbon, air and steam.

From Eq. (776) of Section 7 for air gas in terms of weights the following Eq. (796) is derived by dividing by $5.759(1+x)$ and rearranging.

$$\text{lb. air}+\left[\frac{1}{5.759(1+x)}\right]\text{lbs. C}=\left\{\begin{array}{l}\left[\dfrac{3.66x}{5.759(1+x)}\right]\text{lbs. CO}_2\\ +\left[\dfrac{2.33(1-x)}{5.759(1+x)}\right]\text{lbs. CO}\\ +\left[\dfrac{4.428}{5.759}\right]\text{lbs. N}_2\\ +\left[\dfrac{10{,}193x+4351}{5.759(1+x)}\right]\text{B.T.U.}\end{array}\right\}\quad\ldots\quad(796)$$

The corresponding volume relation follows from Eq. (777) and is given by Eq. (797).

$$\text{lb. air}+\left[\frac{1}{5.759(1+x)}\right]\text{lbs. C}=\left\{\begin{array}{l}\left[\dfrac{29.8x}{5.759(1+x)}\right]\text{cu.ft. CO}_2\\ +\left[\dfrac{29.8(1-x)}{5.759(1+x)}\right]\text{cu.ft. CO}\\ +\left[\dfrac{56.32}{5.759}\right]\text{cu.ft. N}_2\\ +\left[\dfrac{10{,}193x+4351}{5.759(1+x)}\right]\text{B.T.U.}\end{array}\right\}\quad\ldots\quad(797)$$

Two similar equations are given below, Eqs. (798) and (799) derived from the water-gas relations Eqs. (787) and (788) of Section 8, for S lbs. of steam acting with carbon.

$$\text{lbs. H}_2\text{O}+\left[\frac{S}{1.5(1+x)}\right]\text{lbs. C}=\left\{\begin{array}{l}\left[\dfrac{3.66xS}{1.5(1+x)}\right]\text{lbs. CO}_2\\ +\left[\dfrac{2.33(1-x)S}{1.5(1+x)}\right]\text{lbs. CO}\\ +\left[\dfrac{.167S}{1.5}\right]\text{lbs. H}_2\\ +S\left[\dfrac{1545x-4298}{1.5(1+x)}\right]\text{B.T.U.}\end{array}\right\}\quad\ldots\quad(798)$$

$$S \text{ lbs. } H_2O + \left[\frac{S}{1.5(1+x)}\right] \text{ lbs. C} = \left\{ \begin{array}{l} \left[\frac{29.8xS}{1.5(1+x)}\right] \text{ cu.ft. } CO_2 \\ + \left[\frac{29.8(1-x)S}{1.5(1+x)}\right] \text{ cu.ft. CO} \\ + \left[\frac{29.8S}{1.5}\right] \text{ cu.ft. } H_2 \\ + S\left[\frac{1545x-4298}{1.5(1+x)}\right] \text{ B.T.U.} \end{array} \right\} \quad . \quad . \quad . \quad (799$$

If 1 lb. of air and S lbs. of steam together react on carbon and the oxygen o the air and that of the steam act the same, or in symbols x is the same for bot reactions, then together they will give a result expressed by Eqs. (800) and (801) one in terms of gas volumes and the other weights, both giving the heats of reac tion and both obtained by addition.

$$\left\{ \begin{array}{l} \left[\begin{array}{l} 1 \text{ lb. air} \\ + S \text{ lbs. } H_2O \end{array} \right] \\ + \left[\left(\frac{1}{5.759} + \frac{S}{1.5}\right)\frac{1}{1+x}\right] \text{lbs. C} \end{array} \right\} = \left\{ \begin{array}{l} \left[\frac{3.66x}{(1+x)}\left(\frac{1}{5.759} + \frac{S}{1.5}\right)\right] \text{ lbs. } CO_2 \\ + \left[\frac{2.33(1-x)}{(1+x)}\left(\frac{1}{5.759} + \frac{S}{1.5}\right)\right] \text{ lbs. CO} \\ + \left[\frac{4.428}{5.759}\right] \text{ lbs. } N_2 \\ + \left[\frac{.167S}{1.5}\right] \text{ lbs. } H_2 \\ + \left[\frac{10{,}193x+4351}{5.759(1+x)} + \frac{S(1545x-4298)}{1.5(1+x)}\right] \text{B.T.U.} \end{array} \right\} \quad . \quad (800$$

$$\left\{ \begin{array}{l} \left[\begin{array}{l} 1 \text{ lb. air} \\ + S \text{ lbs. } H_2O \end{array} \right] \\ + \left[\left(\frac{1}{5.759} + \frac{S}{1.5}\right)\frac{1}{1+x}\right] \text{lbs. C} \end{array} \right\} = \left\{ \begin{array}{l} \left[\frac{29.8x}{(1+x)}\left(\frac{1}{5.759} + \frac{S}{1.5}\right)\right] \text{ cu.ft. } CO_2 \\ + \left[\frac{29.8(1-x)}{(1+x)}\left(\frac{1}{5.759} + \frac{S}{1.5}\right)\right] \text{cu.ft. CO} \\ + \left[\frac{56.32}{5.759}\right] \text{ cu.ft. } N_2 \\ + \left[\frac{29.8S}{1.5}\right] \text{ cu.ft. } H_2 \\ + \left[\frac{10{,}193x+4351}{5.759(1+x)} + \frac{S(1545x-4298)}{1.5(1+x)}\right] \text{ B.T.U.} \end{array} \right\} \quad . \quad (80$$

These two equations are fundamental to the gasification of the fixed carb part of a coal in a gas producer fed by steam and air together.

The composition of the producer gas is shown in Table XC, below for general case and for the two limiting cases when $CO = 0$ and when $CO_2 = 0$.

TABLE XC

COMPOSITION OF HYPOTHETICAL PRODUCER GAS FROM THE FIXED CARBON

Constituents	By Weight.	By Volume.	When $x=0$.		When $x=1$.	
			By Wt.	By Vol.	By Wt.	By Vol.
CO_2	$\dfrac{\left(\frac{3.66x}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}{1+S+\left[\frac{1}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]}$	$\dfrac{\left[\frac{29.8x}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]}{\left[\frac{29.8}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}$	0	0	$\dfrac{.318+1.222S}{1.087+1.333S}$	$\dfrac{2.59+9.93S}{12.37+29.8S}$
CO	$\dfrac{\left[\frac{2.33(1-x)}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]}{1+S+\left[\frac{1}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]}$	$\dfrac{\frac{29.8(1-x)}{1+x}\left[\frac{1}{5.759}+\frac{S}{1.5}\right]}{\left[\frac{29.8}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}$	$\dfrac{.405+1.556S}{1.174+1.667S}$	$\dfrac{5.17+19.87S}{14.95+39.73S}$	0	0
H_2	$\dfrac{\frac{.167S}{1.5}}{1+S+\left[\frac{1}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]}$	$\dfrac{\frac{29.8S}{1.5}}{\left[\frac{29.8}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}$	$\dfrac{.111S}{1.174+1.667S}$	$\dfrac{19.87S}{14.95+39.73S}$	$\dfrac{.111S}{1.087+1.333S}$	$\dfrac{19.87S}{12.37+29.8S}$
N_2	$\dfrac{\frac{4.428}{5.759}}{1+S+\left[\frac{1}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]}$	$\dfrac{\frac{56.32}{5.759}}{\left[\frac{29.8}{1+x}\right]\left[\frac{1}{5.759}+\frac{S}{1.5}\right]+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}$	$\dfrac{.769}{1.174+1.667S}$	$\dfrac{9.78}{14.95+39.73S}$	$\dfrac{.769}{1.087+1.333S}$	$\dfrac{9.78}{12.37+29.8S}$
Ratio $\dfrac{H_2}{N_2}$	$\dfrac{.167\times5.759}{1.5\times4.428}S=.144S$	$\dfrac{29.8\times5.759}{1.5\times56.32}S=2.031S$	$.144S$	$2.031S$	$.144S$	$2.031S$
Ratio $\dfrac{CO}{CO_2}$	$\dfrac{2.33(1-x)}{3.66x}=.636\left[\dfrac{1-x}{x}\right]$	$\dfrac{1-x}{x}$	∞	∞	0	0
Ratio $\dfrac{CO}{CO+CO_2}$	$\dfrac{1-x}{1+.572x}$	$1-x$	1	1	0	0

The general relations between the various quantities of raw materials and products are given by Eq. (802) below.

1 lb. carbon makes

$$\left[\frac{1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}{\left(\frac{1}{5.759}+\frac{S}{1.5}\right)\left(\frac{1}{1+x}\right)}\right]\text{lbs.}=\left[\frac{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}{\left(\frac{1}{5.759}+\frac{S}{1.5}\right)\left(\frac{1}{1+x}\right)}\right]\text{cu.ft gas.}\quad (a)$$

1 lb. steam makes

$$\left[\frac{1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}{S}\right]\text{lbs.}=\left[\frac{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}{S}\right]\text{cu.ft. gas.}\quad (b)$$

1 lb. air makes

$$\left[1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)\right]\text{lbs.}=\left[\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8S}{1.5}\right]\text{cu.ft. gas.}\quad (c)$$

1 lb. gas requires

$$\left[\frac{\left(\frac{1}{5.759}+\frac{S}{1.5}\right)\left(\frac{1}{1+x}\right)}{1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}\right]\text{lbs. carbon.}\quad (d)$$

1 lb. gas requires

$$\left[\frac{S}{1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}\right]\text{lbs. steam.}\quad (e)$$

1 lb. gas requires

$$\left[\frac{1}{1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}\right]\text{lbs. air.}\quad (f)$$

1 cu.ft. gas requires

$$\left[\frac{\left(\frac{1}{5.759}+\frac{S}{1.5}\right)\left(\frac{1}{1+x}\right)}{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}\right]\text{lbs. carbon.}\quad (g)$$

1 cu.ft. gas requires

$$\left[\frac{S}{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}\right]\text{lbs. steam.}\quad (h)$$

1 cu.ft. gas requires

$$\left[\frac{1}{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}\right]\text{lbs. air.}\quad (i)$$

(802)

In order to show the composition of the gas by volume for various proportions of steam to air in the blast assuming all the steam to react, the four curves of Fig. 186 have been plotted. The left-hand curve is for a ratio of $\frac{CO}{CO_2}=$ the other three for ratios of 6 15, and infinity. The first three cover the range obtained in practice, extending from a value that is lower, to one that is higher than usually obtained, while the last is the limiting case. The composition by volume is plotted vertically and the value of S, which is the weight of steam

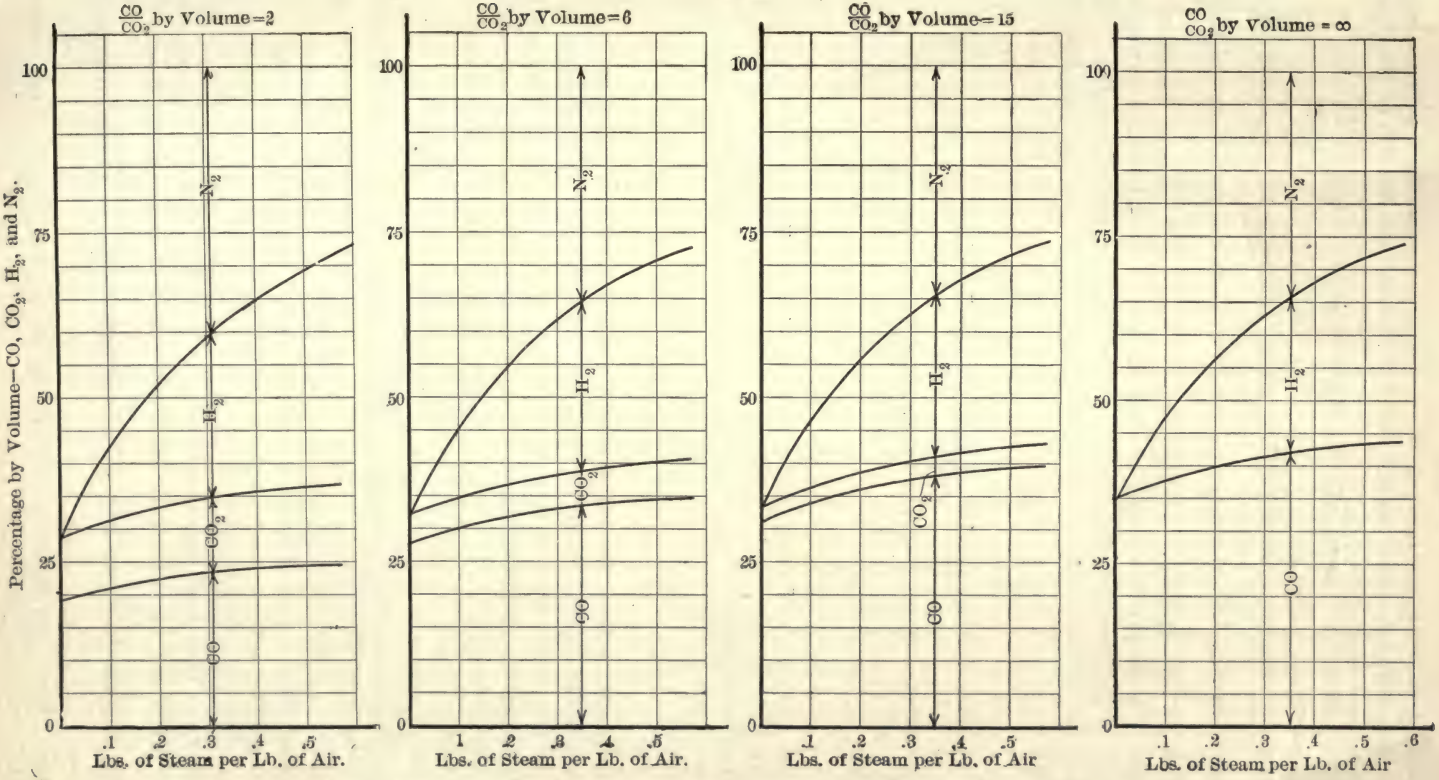


FIG. 186.—Composition of Hypothetical Producer Gas from Fixed Carbon.

per pound of air reacting, horizontally. These are separately plotted because it is not convenient to show on one sheet the variations in each of the four gas constituents as a function of the two prime variables S and $\frac{CO}{CO_2}$.

Heats of reaction are plotted as a function of S alone, laid off horizontally in Fig. 187, and a separate curve drawn for each value of the $\frac{CO}{CO_2}$ ratio, 2, 6, 15 and infinity. The vertical distances are heats of reaction, first, per pound of gases produced and second, per pound of carbon, the former being a measure of temperature rise, and the latter of efficiency of reaction. These two heats are given below as derived from Eq. (800), in the two Eqs. (803) and (804).

$$\left\{\begin{array}{l}\text{Heat of reaction B. T. U. per pound of gas produced.}\end{array}\right\} = \left\{\frac{\left[\left(\frac{10{,}193x+4351}{5.759}+S\left(\frac{1545x-4298}{1.5}\right)\right]\left(\frac{1}{1+x}\right)}{\left[1+S+\left(\frac{1}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)\right]}\right\} \quad . \quad (803)$$

$$\left\{\begin{array}{l}\text{Heat of reaction B. T. U. per pound of carbon gasified.}\end{array}\right\} = \left\{\frac{\left[\left(\frac{10{,}193x+4351}{5.759}\right)+S\left(\frac{1545x-4298}{1.5}\right)\right]}{\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}\right\} \quad . \quad . \quad . \quad . \quad (804)$$

Air saturated with water vapor carries a definite weight of vapor per pound of air and as the air blast is to carry water vapor in some proportion it is convenient to define the amount in terms of the temperature of moisture-saturated air that carries the definite quantity in question. This is most conveniently done in curve form, Fig. 188, where the temperature of water-saturated air is plotted vertically to horizontals of pounds of steam per pound of air.

As already pointed out, the greater the quantity of steam in the blast the more endothermic heat will be taken up from the exothermic heat of the air-gas reaction and when these two are equal there will be *no temperature* rise of the products of gasification over that of the blast. This is a limiting case, for which of course no reactions could take place. In order that it may, some temperature rise must be permitted or, in other terms, the net heat of reaction must be exothermic and sufficient in quantity to allow the final gas to be as hot as necessary. This final gas should not be less than 1100° F., corresponding roughly to 250 B.T.U. exothermic heat, per pound of gases, which fixes a limiting value of S on the chart, Fig. 187, depending on the $\frac{CO}{CO_2}$ ratio to be allowed in the gas. For $\frac{CO}{CO_2}=6$, the corresponding value of S is .215 which would be

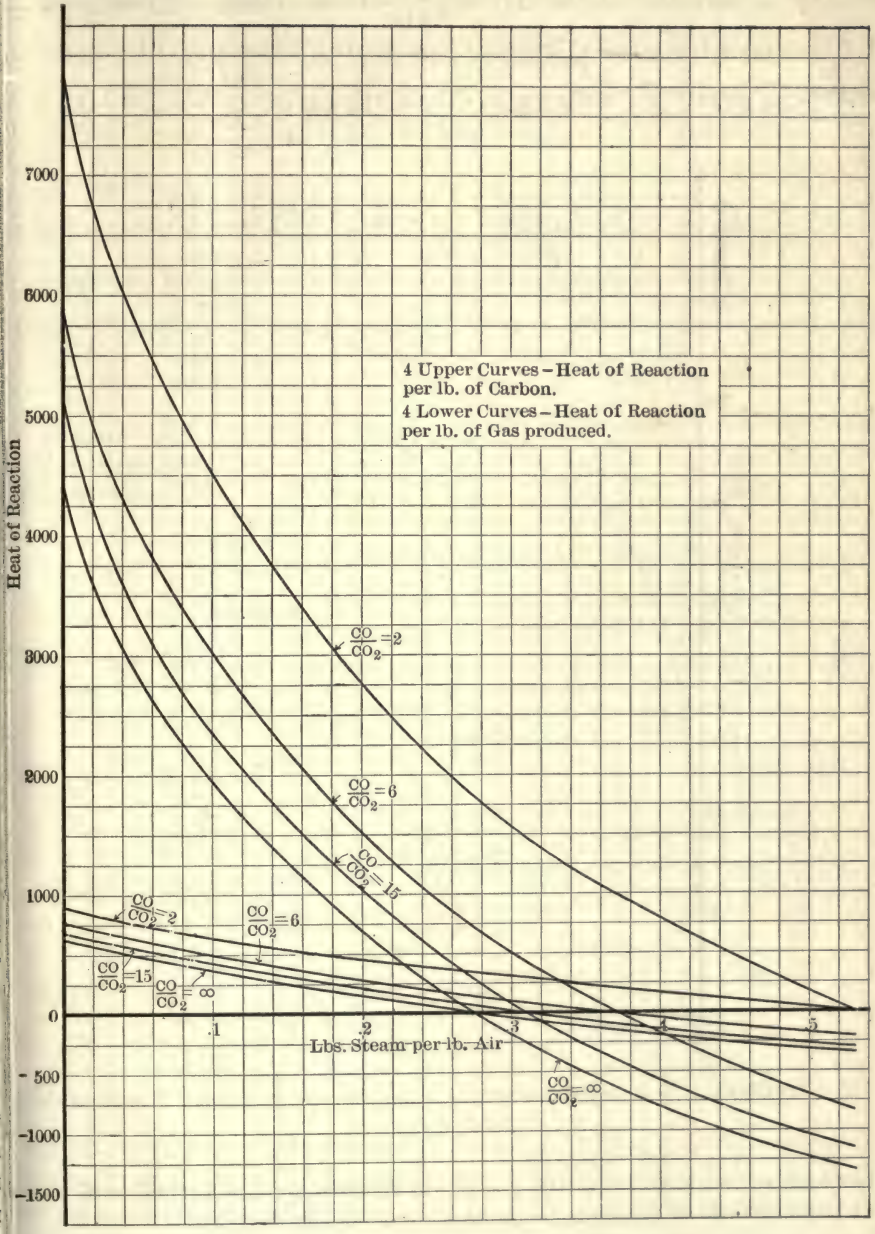


FIG. 187.—Heats of Reaction for Hypothetical Producer Gas from Fixed Carbon, B. T. U.

carried by air saturated at 150° F., for $\frac{CO}{CO_2}=15$ the value of S that gives 250 B.T.U. per pound of gases exothermic heat, is .164, which would be carried by air saturated at 142° F., while for no CO_2 in the gas or $\frac{CO}{CO_2}=\infty$, $S=.15$.

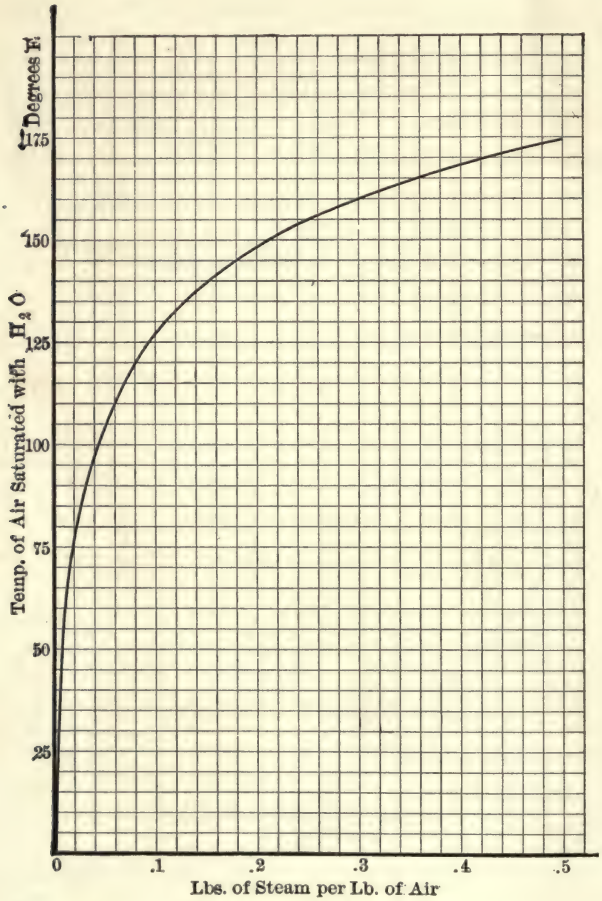


FIG. 188.—Temperature of Air Blast at which Saturated Air Carries a Given Weight of Steam per Pound of Air.

The heat of combustion of the gas produced from 1 lb. of carbon is of course 14,544 less the heat of reaction per pound of carbon or, Eq. (805)

$$\left\{\begin{array}{l}\text{Heat of combustion of gas from one pound of carbon B. T. U, low.}\end{array}\right\}=14{,}544-\left[\frac{\left(\frac{10{,}193x+4351}{5.759}\right)+S\left(\frac{1545x-4298}{1.5}\right)}{\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}\right] \quad . \quad 80$$

Therefore the efficiency of the reaction is Eq. (806),

$$E=1-\left[\frac{\left(\frac{10{,}193x+4351}{5.759}\right)+S\left(\frac{1545x-4298}{1.5}\right)}{14{,}544\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}\right] \quad . \quad . \quad . \quad . \quad (806)$$

From this equation it appears that the efficiency of the reaction is primarily a question of the quantity of steam used for any given value of $\frac{CO}{CO_2}$, so the efficiency as given by this equation is plotted vertically to horizontal values of S, one curve each, for values of $\frac{CO}{CO_2}$, 2, 6, 15 and ∞ in Fig. 189.

The heat of combustion of the gas itself can be found by dividing the heat of combustion of the gas produced from 1 lb. of carbon by its volume and is given by Eq. (807),

$$\left.\begin{aligned}\text{B.T.U. cu.ft. gas (standard, low)} &= \left[\frac{14{,}544-\frac{\left(\frac{10{,}193x+4351}{5.759}\right)+S\left(\frac{1545x-4298}{1.5}\right)}{\left(\frac{1}{5.759}+\frac{S}{1.5}\right)}}{\frac{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}{\left(\frac{1}{5.759}+\frac{S}{1.5}\right)\left(\frac{1}{1+x}\right)}}\right] \quad (a)\\ &= \frac{1}{1+x}\left[\frac{14{,}544\left(\frac{1}{5.759}+\frac{S}{1.5}\right)-\left(\frac{10{,}193x+4351}{5.759}\right)-\left(\frac{1545x-4298}{1.5}\right)S}{\left(\frac{29.8}{1+x}\right)\left(\frac{1}{5.759}+\frac{S}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8S}{1.5}}\right] \quad (b)\end{aligned}\right\} \quad (807)$$

If CO_2 is absent $x=0$ and this becomes

$$\text{B.T.U. cu.ft. gas (standard, low)}=\frac{1770+6831S}{14.95+39.73S} \quad . \quad . \quad . \quad . \quad (808)$$

If no steam is supplied S = 0 and B.T.U. per cubic foot gas = 118, which checks the value found for air gas with 70 per cent efficiency. With .10 lb. steam per pound air, B.T.U. per cubic foot gas = 130, for which condition the reaction efficiency is 86.6 per cent. These figures show that steam in the blast increases both calorific power of gas and the thermal efficiency of its production, the steam-efficiency relations have been shown in Fig. 189, so, to correspond the calorific power of gas is plotted in Fig. 190 to the same horizontals, pounds of steam per pound of air.

To complete the series of graphic results, the gas production in cubic feet standard, per pound of fixed carbon is plotted in Fig. 191, to horizontals of S, one curve each for $\frac{CO}{CO_2}$ values of $\propto$, 2, 6, and 15, Eq. (802).

So far the results all relate to the pure process of gasifying carbon with steam and air, but in real producers there are additional factors to be accounted for, but this cannot be done quantitatively. There must be considered the heat losses of radiation, steam making, distillation of volatile, escape of unchanged

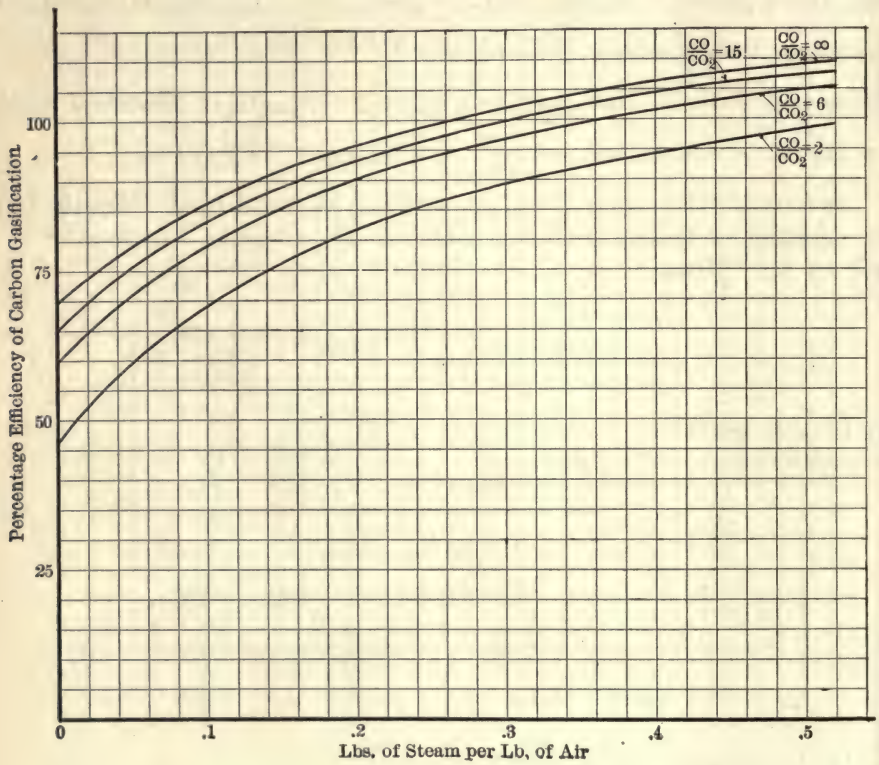


FIG. 189.—Efficiency of Hypothetical Gas Producer Gasifying Fixed Carbon.

steam and heats of reaction of volatile parts that suffer chemical change after being liberated. The production of the volatile always consumes less heat per B.T.U. in the gas formed, than the gasification of carbon, so that the coals which yield much volatile in the producer should give a higher efficiency than those that do not, unless some of the volatile is burned or its heat used up in other endothermic reactions. In fact, this gain by the volatile generally serves to pretty well balance the other heat losses just mentioned so that the real producer can have efficiencies pretty close to those calculated for the gasification of carbon if the volatile is 10 per cent and over. The addition of the volatile or its products to the gasified carbon will not change the relation of the CO, CO_2, H_2 and

N_2 constituents very much, if at all, but will reduce the per cent of each. The following general relation must hold.

$$\left\{\begin{matrix}\text{Cubic feet gas}\\ \text{per pound coal}\end{matrix}\right\} = \left\{\begin{matrix}\left[\begin{matrix}\text{cu.ft. volatile}\\ \text{per lb. volatile}\end{matrix}\right]\times\left[\begin{matrix}\text{Weight volatile}\\ \text{per lb.coal}\end{matrix}\right]\\ +\left[\begin{matrix}\text{cu.ft. gasified}\\ \text{C per lb. C}\end{matrix}\right]\times\left[\begin{matrix}\text{Wt. fixed carbon}\\ \text{per lb. coal}\end{matrix}\right]\end{matrix}\right\} \quad . \quad . \quad (809)$$

This Eq. (809) must be more or less exactly evaluated before the effects of volatile addition on each constituent can be estimated and to do this it is necessary

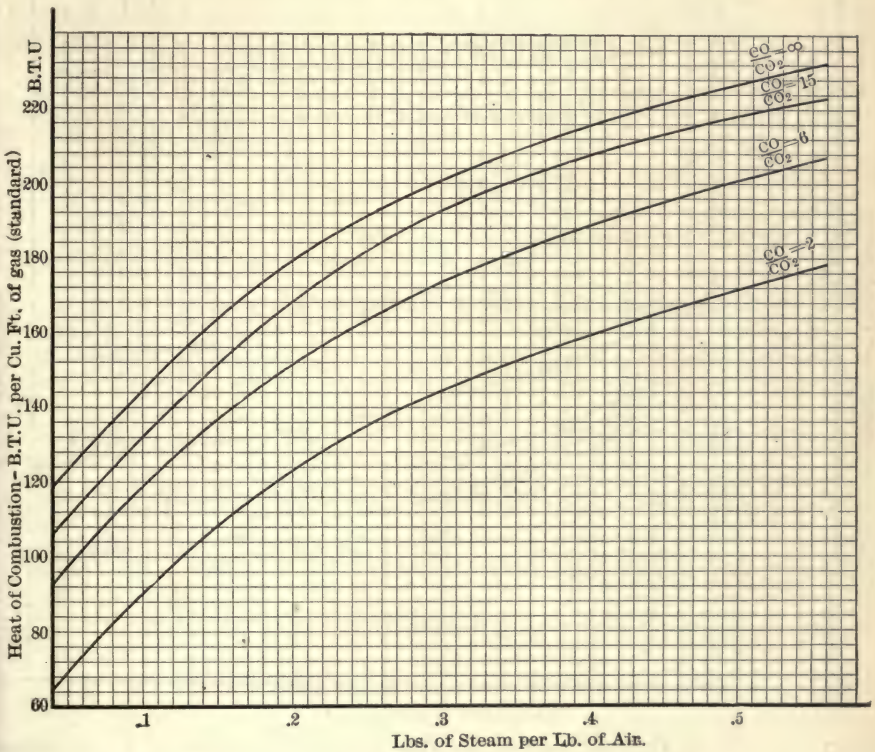


FIG. 190.—Calorific Power of Hypothetical Producer Gas from Fixed Carbon. Low Value.

to have some data on the cubic feet of gas per pound of volatile and the proportions of constituents making it up. This can be done by considering it to behave as coal gas from retorts or as coke oven gas. It was shown for the English cannel coals which yield the largest amount of volatile that they give on the average about 4.5 cu.ft. gas per pound coal, and are about 60 per cent by weight of volatile, so that the cubic foot of gas per pound of volatile = 7 app. Gas coals in retorts or coke ovens will yield about 8 or 9 cu.ft. gas per pound volatile, and about the same for quite a range of volatile per cent. The gasified

fixed carbon yield is somewhere near 90 cu.ft. per pound. Therefore, a coal with 30 per cent volatile and 60 per cent fixed carbon would give in round numbers 56 cu.ft. of mixed volatile and gasified carbon, of which the volatile makes up 4 per cent by volume, which has small influence on the proportion. The real influence may, however, be less, as in an up-draft producer, some of the volatile will condense to tar, several of the U.S.G.S. tests showing about .2 lb. tar per pound volatile of the proximate analysis, so that one-fifth of the volatile condensed. With down-draft producers the volatile may completely change and the gas show no trace except a small per cent of CH_4.

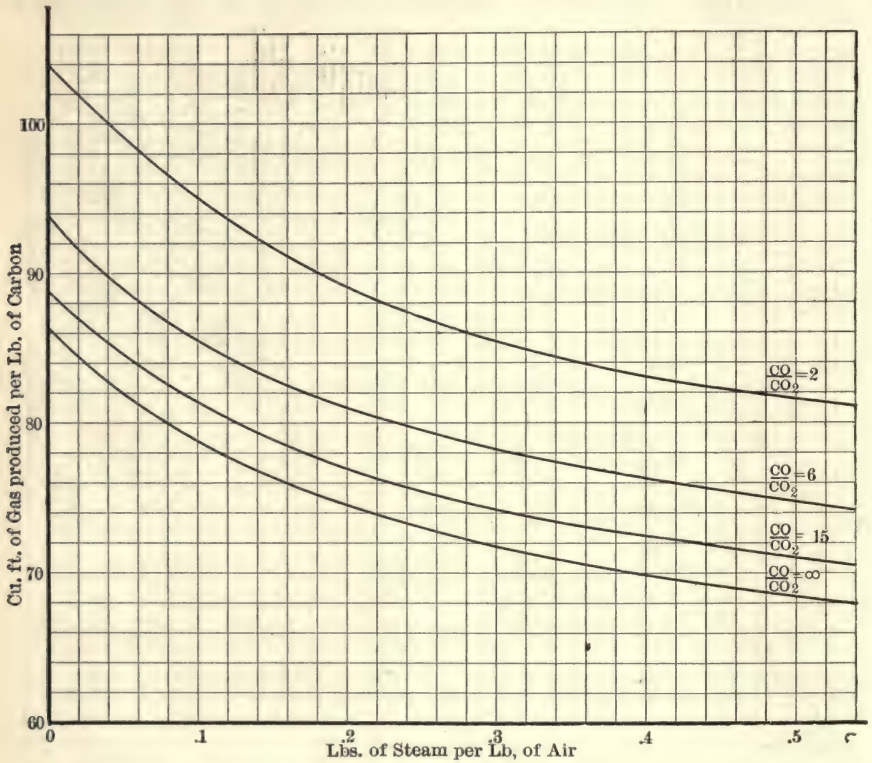


FIG. 191.—Quantity of Hypothetical Producer Gas (Standard) per Pound of Fixed Carbon.

Lignite coal will yield a volatile quite different in character and in up-draft producers will add something like the following gas, reported by Güldner for the yield by roasting brown lignite in retorts, weighing .0633 lb. cu.ft. or of specific volume 15.8 cu.ft. per pound.

COMPOSITION OF LIGNITE VOLATILE (GÜLDNER)

H_2.	CH_4.	CnH_{2n}.	CO.	H_2S.	CO_2.	O_2.	N_2.
24.3	16.5	1.4	8.1	1.1	17.0	3.1	28.5

Attention has already been called to two most important factors in the gasifying process which have only recently come within the scope of experimental measurement and these are the physical condition of the bed as to porosity, homogeneity, uniformity of gas flow, and the temperature of the bed with its homogenity of distribution. As part of the general investigations on fuels in producers, Clement and Grine of the U.S.G.S. measured the temperature

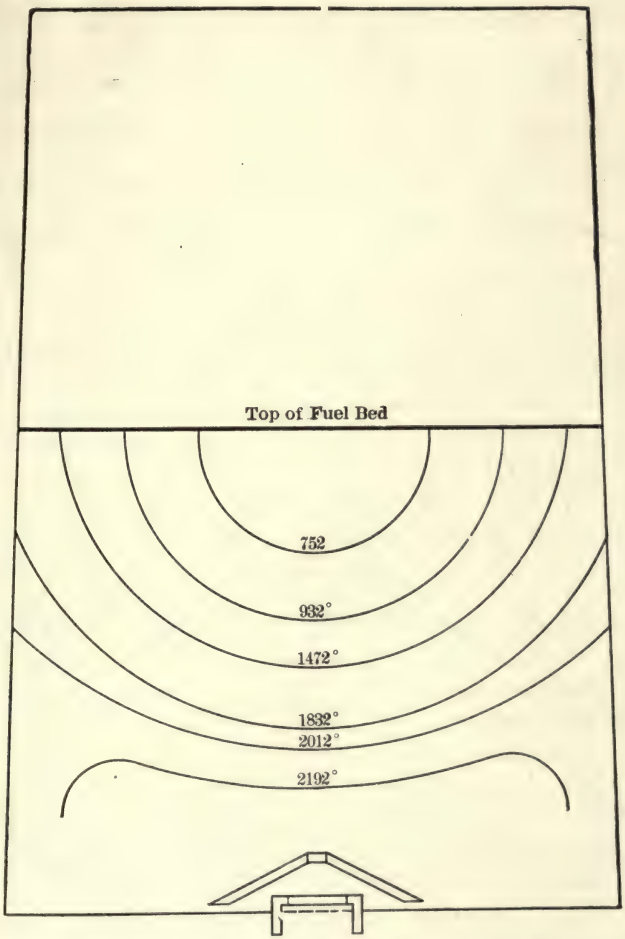


FIG. 192.—Section of Producer showing Temperature Variation.

in the bed of a producer 7 ft. internal diameter, up-draft, steam jet blower, delivering the blast under a hood in the center of the producer bottom at a point normally covered with ash. The results are plotted in the diagram, Fig. 192, in the form of constant temperature lines. The highest temperature is found in the bottom and the same as would be found in a boiler fire when supplied with steam in the air. The temperature falls slowly and then more rapidly toward

the top of the fuel bed, kept at $5\frac{1}{2}$ ft. above the blast entrance. The curvature of these lines shows that the blast rises toward the sides more than in the center and the difference is very marked indeed. The explanations and discussion of gas analyses which were made at the same time from the bed interior cannot be taken up here, the main facts being, first, that a non-homogeneous condition exists, partly because the blast is impelled outward by the hood but also because the resistance at the walls is less than anywhere else, second, that the temperatures actually attained are very much higher than correspond to equilibrium of the $\frac{CO}{CO_2}$ ratio reported. This ratio for the gases leaving, which were, of course, the mixture of all sorts of different gases from different parts of the bed, was $\frac{CO}{CO_2}=3$, for which the equilibrium temperature is 1376° F., while in this bed the temperature was 2200° F. at which value the equilibrium value of the ratio $\frac{CO}{CO_2}$ is over 200.

It has been found possible to generate a usable power gas from a jet of coal dust or oil blasted into a combustion chamber with air insufficient for complete combustion, if the walls and blast be so arranged as to keep the hot products in contact with glowing free carbon liberated from the fuel by its decomposition. Some analyses of such gas are given in Table CXIX at the end of the Chapter for Western heavy oils, which show a comparatively low CO content, though not so high a per cent CO_2 as might be expected from the bed type of solid carbon producer. As the temperatures in these chambers are quite high, it must be concluded that equilibrium between CO and CO_2 has not been established and the reason may be either too high a velocity of the gases, which is the same as insufficient time, or what is more likely, *not enough free carbon present at the right time and place for* CO_2 *reduction.* These gases if passed through a coke bed would be very much improved, in fact would become almost perfect as to combustibles. Another characteristic of these analyses is the large CH_4 content and with it an also large amount of heavy hydrocarbons. These two things are characteristic of the oil fuel as raw material, the methane having come from decomposition of higher hydrocarbons, yielding free carbon, some of which went into CO and some was left behind as coke in the producer.

An almost identical operation with pulverized coal that has recently been tried gave gas analyses (Latta) as in Table CXX at the end of the chapter. The powder and air are blown into a large combustion chamber without steam, all hydrogen coming from coal volatile, or from moisture in air and coal. As the coal was bituminous, ranging from 60 to 77 per cent fixed carbon and 18 to 33 per cent volatile, the small quantity of illuminants or heavy hydrocarbons shows that these were attacked by simple decomposition or by oxygen, more likely the latter, as the methane is also low while in coal gas it would be much higher and even higher also in up-draft producer gas with the same coal. The carbon monoxide-dioxide ratios are very much the same as for the oil-gas pro-

lucer and probably for the same reason, insufficient amounts of free carbon or the more complete reduction of CO_2 to CO, because the temperatures leveloped are quite high enough.

TABLE XCI

DENSITY AND CALORIFIC POWER OF PRODUCER GAS

	Constituent.	One Cubic Foot				Summary	
		Contains		B.T.U.			
		Cu.Ft.	Lbs.	High.	Low.		
Low volatile Anthracite	Carbon monoxide, CO..	.261	.020376	89.0	89.0	B.T.U. per cu.ft. gas high	142.3
	Hydrogen, H_2.....	.1500	.000843	51.2	43.8	B.T.U. per cu.ft. gas low	134.7
	Methane, CH_4.....	.0020	.000089	2.1	1.9		
	Carbon dioxide, CO_2...	.053	.000052	...	...	Cu.ft. per lb. gas.....	14.36
	Oxygen, O_2.....	.002	.000178	...	...	Lbs. per cu.ft. gas.....	.0696
	Nitrogen, N_2.....	.532	.041650	...	...		
	Total.....	1.00	.069638	143.3	134.7	B.T.U. per lb. gas high.	2043
						B.T.U. per lb. gas low..	1934
Peat	Carbon monoxide, CO..	.272	.021235	92.8	92.8	B.T.U. per cu. ft. gas high	128.9
	Hydrogen, H_2.....	.009	.000051	3.1	2.6	B.T.U. cu.ft. gas low...	125.1
	Methane, CH_4.....	.031	.001385	33.0	29.7		
	Carbon dioxide, CO_2...	.121	.014845	...	...	Cu.ft. per lb. gas.....	12.21
	Nitrogen, N_2.....	.567	.044390	...	...	Lbs. per cu.ft. gas.....	.0819
	Total.....	1.000	.081906	128.9	125.1	B T.U. lb. gas high	1574
						B.T.U. lb. gas low	1627
Bituminous, Updraft	Carbon monoxide, CO..	.253	.019751	86.3	86.3	B.T.U. per cu.ft. gas high	164.3
	Hydrogen, H_2.....	.092	.000517	31.4	26.9	B.T.U. per cu.ft. gas low	155.5
	Methane, CH_4.....	.031	.001385	33.0	29.7		
	Heavy Hydro., C_2H_4...	.008	.000636	13.6	12.6	Cu.ft. per lbs. gas.....	13.88
	Carbon dioxide, CO_2...	.034	.004171	...	...	Lbs. per cu. ft. gas.....	.0720
	Nitrogen, N_2.....	.582	.045565	...	...		
	Total.....	....	.072025	164.3	155.5	B.T.U. per lb. gas high..	2280
						B.T.U. per lb. gas low..	2158

Horse-power, as a term applied to a gas producer, is just as much out of place but just as necessary as when applied to a steam boiler. It is out of place because no mechanical work is done by either piece of apparatus, both being concerned with, and used for supplying the means for doing work in steam and gas engines. This gives the reason for the necessity of a name as engines are rated at, and develop a given horse-power, so it is natural to associate the horse-power name and number with the producer or boiler that makes it possible, however indirect or scientifically unsound it may be. There is, therefore necessary, some standard of producer as of boiler horse-power and it should be based on the capacity of either to supply respectively the requisite amount of gas or steam per hour that an engine may need to develop and maintain one horse-power of output. No such standard has, however, been generally accepted, but one can be easily created.

It will be not far wrong to say the average good gas engine of almost any size above 100 H.P. can develope a horse-power on 10,000 B.T.U. per hour in the gas, low value, and many can do better, as low as 8500 having been proved possible, though a few two-cycle engines may use as high as 12,500. *The figure 10,000 B.T.U. per hour being an easily remembered round number and well within the range of practical performance with a fair margin of capacity for very good engines, may well be adopted as the equivalent of a gas-producer horse-power.* Therefore,

$$\left.\begin{aligned}\text{H.P. of gas producer} &= \frac{\text{B.T.U. per hr. (low) in gas produced}}{10{,}000} \quad . \quad . \quad . \quad (a)\\ &= \frac{(\text{Cu.ft. gas per hr.}) \times (\text{B.T.U. cu.ft. gas})(\text{low})}{10{,}000} \quad . \quad (b)\end{aligned}\right\} \quad (810)$$

In the following tabular form, Table XCI, are given some average density and calorific properties for the producer gases, illustrating their calculation from the volumetric constituents in the usual way.

Example 1. A producer is blasted with air and steam in the ratio of 5 to 1 by weight. The ratio of CO to (CO_2+CO) is .8. What will be the composition of the gas by volume and weight, the pounds of carbon, air, and steam required per cubic foot of gas and the heat of combustion per cubic foot of gas?

For an air steam ratio of 5, S will be $\frac{1}{6}=.1667$. Since $CO \div (CO+CO_2)=1-x$, $=.8$ or $x=.2$.

The CO_2 by weight is $\left[\dfrac{\dfrac{3.66\times.2}{1.2}\left(\dfrac{1}{5.759}+\dfrac{.1667}{1.5}\right)}{1+.1667+\dfrac{1}{1.2}\left(\dfrac{1}{5.759}+\dfrac{.1667}{1.5}\right)}\right]=12.4$ per cent.

The CO by weight is $\left[\dfrac{\dfrac{2.33\times.8}{1.2}\left(\dfrac{1}{5.579}+\dfrac{.1667}{1.5}\right)}{1+.1667+\dfrac{1}{1.2}\left(\dfrac{1}{5.759}+\dfrac{.1667}{1.5}\right)}\right]=31.6$ per cent.

The H_2 by weight is $\left[\dfrac{\dfrac{.1667\times.1677}{1.5}}{1+.1667+\dfrac{1}{1.2}\left(\dfrac{1}{5.759}+\dfrac{.1667}{1.5}\right)}\right]=1.3$ per cent.

The N_2 by weight is $\left[\dfrac{\dfrac{4.428}{5.759}}{1+.1667+\dfrac{1}{1.2}\left(\dfrac{1}{5.759}+\dfrac{.1667}{1.5}\right)}\right]=54.8$ per cent.

The CO_2 by volume is $\left[\dfrac{\dfrac{29.8\times.2}{1.2}\left(\dfrac{1}{5.759}+\dfrac{.1667}{1.5}\right)}{\dfrac{29.8}{1.2}\left(\dfrac{1}{5.759}+\dfrac{.1667}{1.5}\right)+\dfrac{56.32}{5.759}+\dfrac{29.8\times.1667}{1.5}}\right]=7$ per cent.

he CO by volume is $\left[\frac{\frac{29.8\times.8}{1.2}\left(\frac{1}{5.759}+\frac{.1667}{1.5}\right)}{\frac{29.8}{1.2}\left(\frac{1}{5.759}+\frac{.1667}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8\times.1667}{1.5}}\right]=28.1$ per cent.

he H_2 by volume is $\left[\frac{\frac{29.8\times.1667}{1.5}}{\frac{29.8}{1.2}\left(\frac{1}{5.759}+\frac{.1667}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8\times.1667}{1.5}}\right]=16.4$ per cent.

he N_2 by volume is $\left[\frac{\frac{56.32}{5.759}}{\frac{29.8}{1.2}\left(\frac{1}{5.759}+\frac{.1667}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8\times.1667}{1.5}}\right]=48.5$ per cent.

cu.ft. gas requires $\left[\frac{\left(\frac{1}{5.759}+\frac{.1667}{1.5}\right)\frac{1}{1.2}}{\frac{29.8}{1.2}\left(\frac{1}{5.759}+\frac{.1667}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8\times.1667}{1.5}}\right]=.0116$ lb. C.

cu.ft. gas requires $\left[\frac{1}{\frac{1}{.1667}\left[\frac{29.8}{1.2}\left(\frac{1}{5.579}+\frac{.1667}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8\times.1667}{1.5}\right]}\right]=.0083$ lb. steam.

cu.ft. gas requires $\left[\frac{1}{\frac{29.8}{1.2}\left(\frac{1}{5.759}+\frac{.1667}{1.5}\right)+\frac{56.32}{5.759}+\frac{29.8\times.1667}{1.5}}\right]=.0495$ lb. air

he heat of combustion of the gas per pound of carbon is, low value,

$$14{,}544-\left[\frac{\frac{10{,}193\times.2+4351}{5.759}+.1667\left(\frac{1545\times.2-4298}{1.5}\right)}{\left(\frac{1}{5.759}+\frac{.1667}{1.5}\right)}\right]=13471 \text{ B.T.U.}$$

nd from the above, 1 cu.ft. of gas requires .0116 lb. of carbon. Hence the heat f combustion of the gas is 13,476×.0116 or 156 B.T.U. per cubic foot.

Prob. 1. The following anlysis of coke producer gas was taken when the producer vas receiving little steam. CO = 28; H = 4; $CO_2 = CH_4 = 4$; $N_2 = 60$. What was the atio of steam to air?

Prob. 2. If the air-steam ratio had been raised to 4, what would have been the omposition of the gas for the same CO to CO_2 ratio?

Prob. 3. For an air-steam ratio of 6 and a CO to (CO_2 + CO) ratio of .9, what vill be the carbon, steam, and air required for 1000 cu.ft. of gas and what will be he heating value of the gas?

Prob. 4. A coal used in a producer consists of 85 per cent fixed carbon, 5 per cent olatile, and 10 per cent ash. The CO to CO_2 ratio and the air-steam ratio are both 5. f the volatile has a heating value of 20,000 B.T.U. per pound and it is assumed that t comes off uniformly, what will be the heat value of the gas per pound and per cu.ft.?

Prob. 5. A coke producer blast is saturated at 140° F. What is the maximum CO to CO_2 ratio which could be had for a continuous run and the efficiency?

Prob. 6. For a ratio of CO to CO_2 of 10, what would be the minimum steam ratio for a continuous process and the corresponding efficiency of reaction for a fixed carbon bed?

Prob. 7. Coal No. 6 of Table CIV is used in a producer. The volatile is the same as gas No. 5 of Table CX and CO to CO_2 ratio is 6 with an air blast saturated at 160° F. What will be the composition of the gas and the efficiency of the producer? 500 B.T.U. per pound of coal is required to liberate the volatile.

Prob. 8. Coal No. 86 of Table CIV is used in a producer. What will be the volume of gas per ton of coal and what will be the composition for the following assumptions: CO to $CO_2 = 5$; $S = .2$; volatile of coal is the same as No. 12 of Table CX.

Prob. 9. In Table CXVIII are given some producer tests. Compare the composition of the gas as found with that as calculated and the actual efficiencies. Assume S to be .2.

10. Combustion Effects. Final Temperature, Volume and Pressure for Explosive and Non-Explosive Combustion. Estimation of Air Weights and Heat Suppression Due to CO in Products from Volumetric Analysis. When used for industrial purposes fuels are burned in one of two radically different characteristic ways, designated as, (*a*) explosive, and, (*b*) non-explosive combustion, and while in every case there is a rise of temperature it may be accompanied by a volume or by a pressure change or both, rise of pressure being especially characteristic of explosive combustion. In any case the fundamental effect of combustion is a temperature rise which can be *only estimated* as to amount because of the great uncertainty of specific heats coupled with the dissociation phenomenon which retards further oxidation of fuel elements after the temperature has reached a certain value. Accordingly, all calculations of combustion effects must be regarded as more or less approximate and to the results, factors must be applied to bring them into accord with direct observation of effects, which factors measure by their magnitude the errors of the method and the uncertainty of fundamental physical constants. The application of such factors, which is necessary even with the most precise methods, opens the way for quick estimation of results by shorter methods, which are almost universally used in engineering work concerned with explosions, especially those in the cylinders of internal combustion engines.

Usually *non-explosive combustion* takes place at constant pressure and explosive at constant volume but these are not essential relations as either kind may proceed under any conditions of pressure or volume relation. The essential distinction between *explosive* and non-explosive combustion is to be found in the *self-propagation* or progress of the flame or reaction causing it, for the former. For combustion to be explosive there must be a *mixture* of fuel and oxygen which may be diluted with neutral gases like nitrogen, carbon dioxide or water vapor within certain limits. Moreover, the mixture must be of the most intimate and homogeneous sort though the fuel may be in the solid liquid or gaseous

state. Each particle of oxygen must be in contact with the fuel particle the combustion of which it is to support, so that if the fuel is solid it must be in powdered form suspended in the oxygen; if liquid, it must be sprayed in, in as fine a mist form as is possible. The ideal condition of the fuel for explosive combustion is, however, the gaseous or vapor form, mixed homogeneously with the supporting oxygen. Mixtures that comply with these specifications will explode, that is, combustion once started by an ignition of any part of the mass at a single point, will propagate itself throughout the entire mass if it is at rest. The flame will at any instant be on a surface ideally of no thickness, but actually, if there be some inactive gases present, of measurable thickness, which surface separates the burnt from the unburnt. This surface advances through the mixture at a speed entirely independent of any motion of the mixture mass, and the combustion proceeds at a rate determined by this speed of propagation peculiar to the mixture itself and not by any surrounding conditions. If the mixture be in a closed chamber the whole mass will burn in a short time; if it be supplied to a combustion chamber continuously as is done sometimes in gas and oil furnaces and those consuming powdered coal, the combustion will proceed at the same rate against the current of supply, the velocity of which must be greater than the rate of propagation to prevent *back flash*, and be subsequantly reduced in the furnace below the rate of propagation to prevent *blow off*.

If the mixture contain not enough oxygen to support combustion of all the fuel present, then the combustion may still be explosive within certain limits, to be discussed later, but the combustion cannot be complete when the explosion takes place in a closed chamber and can only be complete when supplied in a stream, as in Bunsen burners or from furnace nozzles or burners, provided a supporting atmosphere of extra air be supplied externally to the jet or flame. In such cases the rate of combustion will be less because some of the fuel must wait until it can find oxygen; the flame will accordingly be longer and fill greater volumes of the combustion chamber, the rate will be least and the flame volume greatest for no previous mixture, all air for combustion in this case being derived from the external supporting atmosphere. This latter condition is found in common illuminating gas flames and in the fires of coal with much volatile, which rises from the bed practically without air mixture, all air necessary for combustion being admitted above the fire, meeting the gas when it can and the last particles burning in an atmosphere much diluted with the products of previously burnt fuel. Such conditions promote escape of unburnt gases, hydrocarbons, carbon monoxide or hydrogen, even when some oxygen is still unused and appears as free oxygen in the products of combustion.

Similarly, for solid fuels the combustion proceeds as fast as the air can reach the fuel, the rate of combustion being fixed by the rate of air supply but not directly proportional thereto, because of the stream form of the air between the particles of fuel and the diluting, separating influence of previously burnt products tending to keep air and fuel apart. Thus, solid fuel combustion always must take place with excess air or be incomplete, for if just the right amount were supplied some oxygen would slip between the coal particles without

touching and the quantity of combustible gases formed above would always be greater than this slippage air could burn. More air supplied with a view to burning this gas is ineffective because of lack of means of pre-mixing, so that while the fuel might all be burnt partly in the solid and partly in gaseous form it can be done only with excess air, absence of free oxygen in the flue gas being according to universal experience accompanied by much unburnt fuel.

While it is possible to calculate the weight of air necessary to complete the combustion of a pound of fuel from the reaction equation or similarly the volume of air per cubic foot of fuel, the above discussion indicates that such calculations are not sufficient to permit a judgment of the effects of combustion to be expected in practical working with either explosive or non-explosive, continuous or intermittent fires. In all cases, however, the temperature rise should be given by the ratio of heat liberated by combustion to the product of the specific heat of the products, into the weight of products. The weight of the products will always be the weight of air per pound of fuel chemically necessary for combustion, plus the excess air per pound of fuel, plus one, and the heat liberated is the calorific power per pound of fuel if it is all burnt. When, however, the air is insufficient to burn the fuel and in the case of solid fuel, sometimes also when it is sufficient, the full calorific power cannot be developed and heat will be lost.

Furthermore, the full heat of combustion will not be available for temperature rise if in the course of that rise the dissociation temperature for the products is reached, for after that time further union, heat liberation and temperature rise will cease. Abstraction of some heat will permit combustion to proceed but without any more rise of temperature. . These considerations show that the calculation of temperature rise on combustion or the consequent increase of volume at constant pressure, or increase of pressure at constant volume, when based on purely physical constants, is hardly more than an academic exercise even when the constants are positively known, but decidedly so at the present time, when the dissociation constants and specific heats at high temperature are practically unknown in spite of much work to find them.

Accordingly, to calculate *probable* effects of combustion experimentally determined factors must be relied upon.

Let Q = B.T.U. per pound fuel;

" t_2 and t_1 = final and initial temperatures F.;

" T_2 and T_1 = final and initial temperatures absolute;

" V_2 and V_1 = final and initial volumes cu.ft.

" P_2 and P_1 = final and initial pressures lbs. sq.ft.;

" p_2 and p_1 = final and initial pressures lbs. sq.in.;

" w_P = lbs. products per lb. fuel = wt. air chemically necessary + wt. of excess air per lb. fuel + 1;

" β = fraction of B.T.U. per lb. of fuel actually causing rise of temperature;

C_p, C_p', C_v, C_v' = specific heats *at* and *mean specific* heats, at constant pressure and volume for the products of combustion,

Then

$$\left.\begin{aligned} T_2-T_1=t_2-t_1&=\frac{\beta Q}{w_P C_p'}, \quad \text{when the pressure is constant} \quad (a)\\ &=\frac{\beta Q}{w_P C_v'}, \quad \text{when the volume is constant} \quad (b)\end{aligned}\right\} \quad (811)$$

For explosions at constant volume,

$$\frac{P_2}{P_1}=\frac{p_2}{p_1}=\frac{T_2}{T_1}=1+\left(\frac{\beta Q}{w_P C_v' T_1}\right), \quad (812)$$

$$P_2-P_1=\frac{P_1}{T_1}\left(\frac{\beta Q}{w_P C_v'}\right), \quad (813)$$

Also for constant pressure combustion,

$$\frac{V_2}{V_1}=\frac{T_2}{T_1}=1+\left(\frac{\beta Q}{w_P C_p' T_1}\right), \quad (814)$$

$$V_2-V_1=\frac{V_1}{T_1}\left(\frac{\beta Q}{w_P C_p'}\right) \quad (815)$$

These equations as set down are perfectly correct but of no practical use, without evaluating β, C_p' and C_v'. To illustrate the point: consider the combustion of carbon monoxide in oxygen, the temperature of which as measured by Mallard and Le Chatelier, was found to be 5800° F. The heat of combustion of CO burning to CO_2 is 4369 B.T.U. per pound CO and the weight of products is $\frac{44}{28}=1.57$ lbs. Taking the specific heat of carbon dioxide at constant volume at .2025 the value at 32° and constant, and assuming $\beta=1$ that is, all heat developed,

$$T_2-T_1=t_2-t_1=\frac{4369}{1.57\times.2025}=13{,}740° \text{ F.}$$

This is pretty far away from the measured value of 5800, which latter is really only 40 per cent of that derived by the calculation based on constant specific heat. Introducing the variable specific heat the result is a little better but still fails to agree with the measurement by a pretty wide margin, as will appear from what follows.

The most commonly accepted, but still uncertain form for the specific heat of gases at high temperatures in terms of the value at 32°, is given by a first or second degree equation, from which the mean value between 32°

and any temperature can be derived, and this multiplied by the temperature range will give the corresponding quantity of heat. To solve this equation for the final temperature in terms of the quantity of heat causing it is troublesome by algebraic methods but easy by a chart such as Fig. 193. Here one set of the Mallard and Le Chatelier values for the mean specific heatof various gases given in Eq. (816) has been used to calculate the temperature rise above 32° for various quantities of heat and plotted. For any heat increment per pound of gases there is a corresponding temperature increment that can be read off directly. Thus, for CO_2, consider 1 lb. to receive 1000 B.T.U., starting at 32° F., the temperature rise would be 3290° F.3−2° F.=3258°, whereas from 1000° F. as a starting point this same 1000 B.T.U.would yield a temperature of 3690° F. or a rise of 2690°.

$$\left\{\begin{matrix}\text{Mean specific heat}\\ \text{Mallard \& Le Chatelier}\\ \text{Pressure constant.}\end{matrix}\right\} = \left\{\begin{matrix} .149+.000048t & \text{for} & CO_2\ldots(a)\\ .433+.000060t & \text{“} & H_2\ldots(b)\\ .242+.000024t & \text{“} & N_2\ldots(c)\\ .212+.000021t & \text{“} & O_2\ldots(d)\\ 3.3+.000015t & \text{“} & H_2\ldots(e)\end{matrix}\right\} \quad \ldots (816)$$

Applying this to the combustion of CO to CO_2, yielding $\frac{4369}{1.57}=2783$ B.T.U. per pound of CO_2, the temperature rise above 32° is 6790 and the final temperature 6822° F., a little better than for constant specific heats but not good enough.

Other values for the variable specific of gases are given by different experimenters but are not quoted here as the results obtained by their use are no closer to the observed temperatures.

In place of the mean specific heats of the products taken by the variability law for the several constituents, the specific heat of air may be substituted at its value for low temperatures $C_v=.170$ and $C_p=.243$ approximately. Then introducing a new constant C' by the relation, ($C'\div$constant specific heat of air)=($\beta\div$specific heat of products), the temperature rise will be given by

$$\left.\begin{matrix} T_2-T_1=t_2-t_1=C'\dfrac{Q}{.170w_P}, & \text{for constant volume} & (a)\\ =C'\dfrac{Q}{.243w_P}, & \text{for constant pressure} & (b)\end{matrix}\right\} \quad . \quad (817)$$

Estimation of temperature rise by this second method, Eq. (817), is possible whenever a direct observation on a similar case permits of the determination of the value of C' for such cases. This is always possible in furnaces into which pyrometers may be inserted, but it is not possible for explosions in closed chambers because of the shortness of the time, but in such cases the pressure of explosion can be quite accurately determined, much closer than there is any

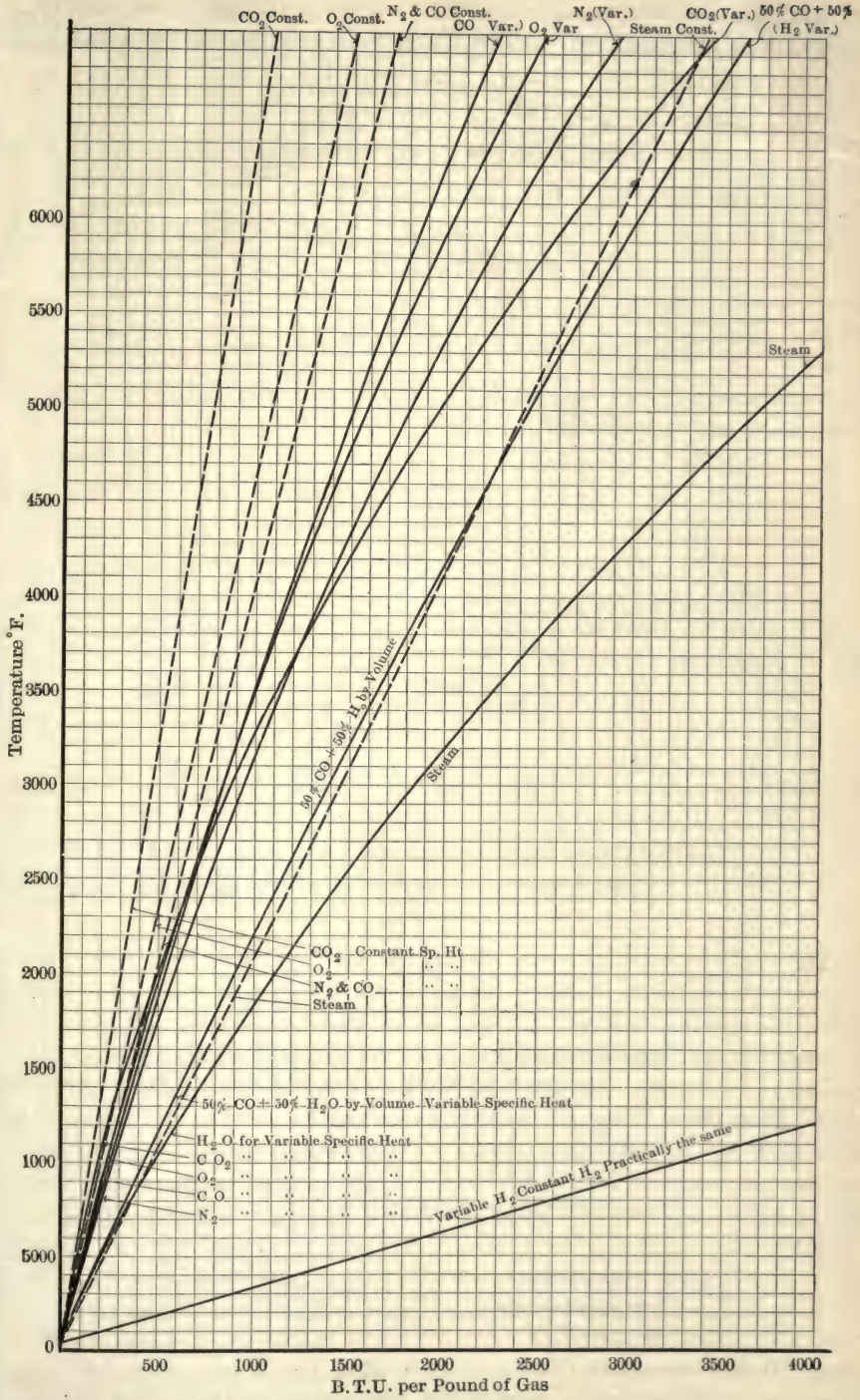


FIG. 193.—Relation Between Temperatures and Heat for Gases According to the Constant and Variable Specific Heat.

necessity for and closer many times than the temperature. From such pressure observations the coefficient can be found from relations of the following kind.

Let H = B.T.U. per cu.ft. gas at 32° and 14.7 lbs. sq.in. low value.
" a = cu.ft. air per cu.ft. gas.
" n = cu.ft. neutral added to air-gas mixture containing one cu.ft. gas.

Then from Eq. (813) if the specific heat is assumed constant

$$P_2 - P_1 = \frac{P_1}{T_1}\left(\frac{\beta Q}{w_p C}\right) = \frac{R}{V_1}\frac{\beta Q}{C_v} = \frac{778(C_p - C_v)}{C_v}\frac{\beta Q}{V_1} \quad . \quad . \quad . \quad . \quad (818)$$

But

$\frac{Q}{V_1}$ = B.T.U. per lb. fuel, divided by cubic feet of gases containing or formed from one pound of fuel, which is equal to B.T.U. per cubic foot mixture at pressure P_1 and temperature T_1. Hence

$$\frac{Q}{V_1} = \left(\frac{H}{1+a+n}\right) \times \frac{p_1}{14.7} \times \frac{492}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (819)$$

$$p_2 - p_1 = \frac{778}{144}(\gamma - 1)\ \beta\left[\frac{H}{1+a+n} \times \frac{p_1}{14.7} \times \frac{492}{T_1}\right] . \quad . \quad . \quad . \quad (820)$$

Introduce the constant C and the value of γ for air according to the equation of condition, Eq. (807),

$$[\gamma(\text{for mixture}) - 1]\beta = [\gamma(\text{for air}) - 1]C = .4C \quad . \quad . \quad . \quad (821)$$

$$p_2 - p_1 = \frac{778 \times .4 \times 492}{144 \times 14.7} C \frac{p_1}{T_1}\left(\frac{H}{1+a+n}\right) = 72.33C \frac{p_1}{T_1}\left(\frac{H}{1+a+n}\right)$$

$$= 72.33C\left(\frac{\text{initial press. lbs. per sq.in. abs.}}{\text{initial temp. F. abs.}}\right) \times \left\{\begin{matrix}\text{B.T.U. per cu.ft. mixture at 32° F. and} \\ \text{14.7 lbs. per sq.in abs.}\end{matrix}\right\} \quad (822)$$

For explosions the pressure rise given by Eqs. (820) and (822) have been determined hundreds of times for all the fuels used in gas-engine cylinders which include practically every one of the gases previously examined, as well as some vapors like kerosene, gasolene and alcohol, separately and mixed in various proportions. The pressure rise has been measured with initial pressure of one atmosphere in special experimental chambers and in the engine cylinder themselves by indicators, the mixture being compressed previous to explosion and for all ranges of fuel and initial pressure, a most extraordinary constancy on the value of C has been observed, which may be summarized by Eq. (823)

$$\left\{\begin{matrix}\text{Heat suppression factor} \\ \text{for pressure rise}\end{matrix}\right\} = C = \left\{\begin{matrix}.45 \text{ minimum} \\ .55 \text{ maximum}\end{matrix}\right\} \quad . \quad . \quad (823)$$

To illustrate the procedure in determining the heat suppression factor the following calculation and experimental determinations for an oil gas, recently

carried out with some care, will serve the purpose. In all cases the lower calorific power is taken.

Gas analysis per cent by volume—

$CO_2 = 0$
$C_2H_4 = 20.93$
$C_6H_6 = 7.42$
$H_2 = 6.$
$C_2H_4 = 62.15$
$O_2 = 0.75$
$CO = 0.3$
$N_2 = 2.65$
Total, 100.00

Of this $CO_2 = 0.00$
$N_2 = 2.65$
Total, Neutral, $= 2.65$

Air required $= 11.68$ volumes per vol. gas, therefore, cu.ft. best mixture per cu.ft. gas $= 12.68$, of which

Neutral in gas $= .029$
Neutral in nitrogen in air $= 9.24$
Neutral in mixture $= 9.269$ vols. in 12.68 vols. mixture $= 73.2\%$ by vol. Therefore the detonating or neutral free, gas mixture in the best mixture $= 26.8\%$ by vol. The low heating value of the gas was 1060 B.T.U. per cu.ft., whence that of the best mixture is $\frac{1060}{12.68} = 89$ B.T.U. per cu.ft.

With these data the following mixture characteristics for various proportions of air and gas will follow for the explosion ranges, as given in Table XCII. These limiting proportions for explosive combustion, constituting the first and last columns of the table, were determined by direct experiment.

TABLE XCII

CHARACTERISTICS OF EXPLOSIVE MIXTURES OF PINTSCH OIL GAS AND AIR

Volume of gas	1	1	1	1	1	1	1	1	1
Volume of air	8	9	10	11	12	13	14	15	16
Percentage of gas in mixture	11.1	10.	9.08	8.34	7.7	7.14	6.67	6.25	5.88
Excess or neutral air	.....	.....	.....	.....	.32	1.32	2.32	3.32	4.32
Active air	8	9	11	11	11.68	11.68	11.68	11.68	11.68
Excess or inactive gas	.315	.229	.144	.057					
Active gas	.685	.771	.856	.943	1	1	1	1	1
Neutral in active air	6.33	7.12	7.91	8.69	9.24	9.24	9.24	9.24	9.24
Neutral in active gas	.0202	.0227	.0252	.0278	.0295	.0295	.0295	.0295	.0295
Total inactive	6.665	7.372	8.079	8.777	9.59	10.59	11.59	12.59	13.59
% inactive or excess	74.1	73.72	73.5	73.0	73.8	75.6	77.3	78.7	79.9
% detonating gas	25.9	26.28	26.5	27.00	26.2	24.4	22.7	21.3	20.1
B.T.U. per cu.ft. of mixture (avail.)	80.6	81.7	82.5	83	81.5	75.7	71.6	66.3	62.5
Calculated pres. rise	163	165	166.6	167.6	164.6	153	144 6	134	126
Observed pres. rise	73.5	80	81	79.5	77.5	70.0	64.5	59	50.5
Ratio $\frac{\text{observed pres.}}{\text{calculated pres.}} = C$	.45	.485	.486	.476	.472	.46	.446	.44	.40

It is quite remarkable how small is the variation in the quantity of active or detonating mixture for all the ranges of explosive proportions and this explains why the pressures vary to so small a degree.

The relation of pressure rise to air-gas ratio in the mixture is clearly shown in Fig. 194 for this gas without the addition of neutral. On the same figure is added a set of curves on the results of a set of experiments conducted with carburetted water gas which had for its purpose the determi-

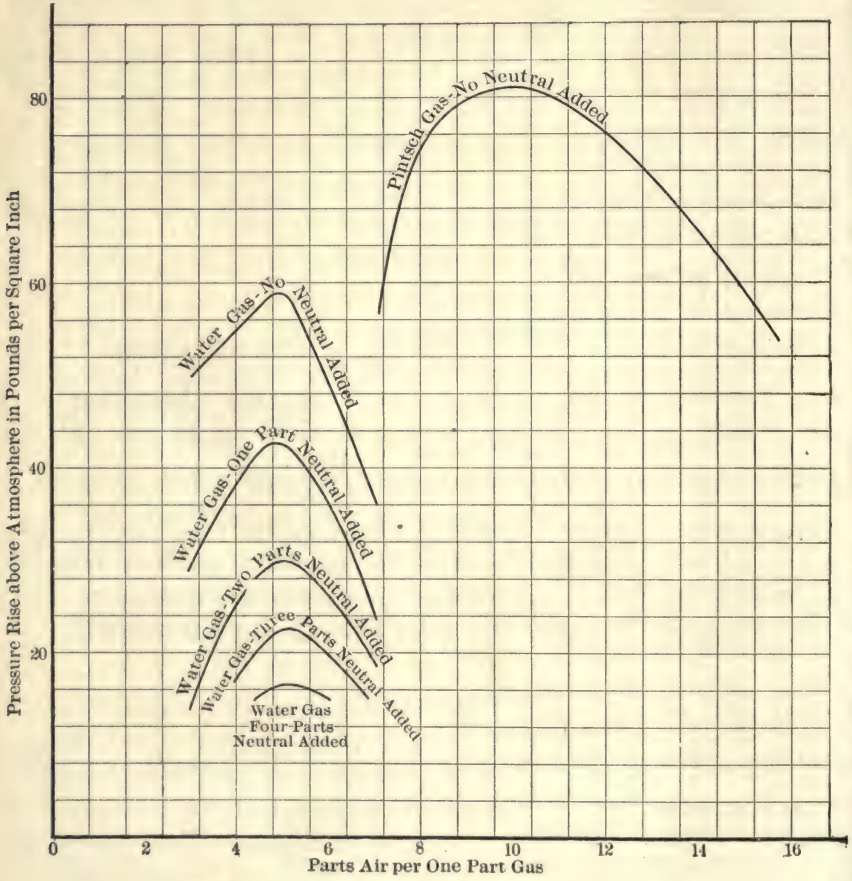


FIG. 194.—Air-gas Mixture Explosion Pressures.

nation of the explosion pressures for various amounts of neutral added t air-gas mixtures of different proportions for the whole range of mixture whic would explode. These curves show the same general form as that for th oil gas and air and indicate clearly that the maximum explosion pressure c pressure rise above atmosphere is always obtained for the combining propo tions of air to gas and that the pressure rise decreases as the amount c neutral or inactive gases increases.

To facilitate calculations with characteristic analyses of common gases the ata of Table CXXI at the end of the Chapter have been prepared and illustrates nother remarkable thing regarding explosive mixtures of these gases with air, iz., the small variations in the B.T.U. per cu.ft. of standard air gas mixtures n spite of a tenfold or more variation in the B.T.U. per cu.ft. of gas, which xplains why explosions of so called rich gases give rise to pressures but little reater than those for the so-called lean, poor or weak gases. The lowest alue is about 60, that for blast furnace gas, the highest value is 107, that for erosene, not twice as much, though the calorific of the fuel vapor per cu.ft. s for kerosene nearly ten times that for blast-furnace gas.

In this table gasolene is assumed to be heptane, C_7H_{16}; kerosene to be lodecane, $C_{12}H_{26}$; alcohol to be pure methyl, C_2H_5OH and the hydrocarbon lluminants of other gases to be distributed as indicated in the table. *It is ery uncertain just what these hydrocarbons are, but it seems to make very little ifference so far as the mixture heats are concerned, as the following calculations ill show:*

For the paraffine series, C_nH_{2n+2}.

$$\left.\begin{aligned} \text{Per cent C} &= \frac{12n}{12n+2n+2} = \frac{6n}{7n+1}. \quad (a) \\ \text{Per cent } H_2 &= \frac{2n+2}{12n+2n+2} = \frac{n+1}{7n+1}. \quad (b) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (824)$$

Neglecting the heats of formation

.T.U. per lb. = per cent C × 14,544 + per cent H_2 × 51,892 (low value)

$$= \frac{14{,}544 \times 6n + 51{,}892(n+1)}{7n+1} = \frac{139{,}156n + 51{,}892}{7n+1} \quad . \quad . \quad . \quad . \quad (825)$$

ubic feet vapor per pound (standard) $= \frac{358}{12n+2n+2} = \frac{358}{14n+2}$ . . . . (826)

$$\left.\begin{matrix} \text{B.T.U. per cubic foot} \\ \text{vapor (standard) low} \end{matrix}\right\} = \frac{\frac{139{,}156n+51{,}892}{7n+1}}{\frac{358}{14n+2}} = 777.4n + 289.9 \quad . \quad . \quad . \quad (827)$$

By the reaction formula, $C_nH_{2n+2} + \frac{3n+1}{2} O_2 = nCO_2 + (n+1)H_2O$, it follows hat

Cubic feet air per cubic foot vapor $= \frac{3n+1}{2.21} = 7.15nx + 2.38$ . . . (828)

Cubic feet air-vapor mixture per cubic foot vapor $= 7.15n + 3.38$ . . (829)

Therefore

$$\text{B.T.U. per cubic foot air-vapor mixture} = \frac{777.4n+290}{7.15n+3.38} \quad . \quad . \quad . \quad (830)$$

$$= 101 \text{ app. for } n = 1$$
$$= 105 \text{ app. for } n = 20$$
$$= 108 \text{ app. for } n = 100.$$

This indicates that from the lightest, methane, to the heaviest of the series, the heat per cubic foot of combining or best mixture with air will be between 102 and 110 approximately and the same will be true for any group of hydrocarbons such as make a real oil.

For the ethylene series C_nH_{2n}

$$\left.\begin{array}{l}\text{B.T.U. per lb.} = 20053 \\ \text{Cubic feet vapor per lb.} = \dfrac{358}{14n}\end{array}\right\} \therefore \text{B.T.U. per cu.ft. vapor} = 785n \text{ app.} \quad (831)$$

$$\text{Cubic feet air per cubic foot vapor} = 7.15n \quad . \quad . \quad . \quad . \quad (832)$$

$$\text{Cubic feet mixture per cubic foot vapor} = 71.5n+1. \quad . \quad . \quad (833)$$

Therefore

$$\text{B.T.U. cubic feet of mixture} = \frac{758n}{7.15n+1} \quad . \quad . \quad . \quad . \quad . \quad (834)$$

$$= 99 \text{ for } n = 2$$
$$= 106 \text{ for } n = 20$$
$$= 106 \text{ for } n = 100$$

This shows the same is true with even narrower limits for this series, *hence for any hydrocarbon oil, B.T.U. per cubic foot best mixture with air measured at standard conditions, will be between* 100 *and* 108, *no matter what it is.*

The weight characteristics of the mixture are added to the volumetric in Table CXXI for convenience in the use of the relations indicated by the preceding equations and summarized as:

Temperature rise is proportional to B.T.U. per lb. of products or of mixture. (835)

Pressure rise for explosions is proportional to B.T.U. per cu.ft of initial mixture . . . . . (836)

In furnace work the temperature rise determination, as well as the final temperature, is made along similar lines to that used for the pressure rise due to explosions in closed chambers, whether the fuel be solid coal or coke, liquid oil, or gas, unmixed, partially mixed with air or completely mixed in explosive proportions. High temperature attainment is always facilitated by initial heating of the blast or mixture supplied, as the combustion determines the tem-

perature *rise* and final temperature is the sum of *initial temperature* and temperature rise. The final temperature does not rise the same amount, however, as the initial temperature, except for low temperatures, indicating the operation of the dissociation limitation, but it does rise appreciably, showing that all heat suppression is not due to dissociation and may be charged to higher specific heats than are believed to hold. This, however, is at present a controversial point among physicists the full discussion of which is not worth while here in view of the prime purpose of getting numerical results that are reasonably close to the truth by whatever means are available.

In furnaces, especially hand-fired boiler furnaces, the temperature is constantly fluctuating with the air supply, draft, condition of bed and all sorts of minor influences and the diagram, Fig. 195, by the U. S. Geological Survey, taken from one of their boiler tests illustrates this admirably. According to this diagram the furnace temperature rose to a maximum of about 3000° F. and fell to about 2000° F., as limits, due to operating conditions, the rise and fall being cyclic with firing. Opening the door, of course, chills the fire unless previously the gases had insufficient air. Depositing coal results in rapid distillation, as the coal was bituminous, in one case 36 per cent volatile and in the other 21 per cent, and the gases in burning raise the temperature in the combustion chamber beyond the bridge wall, while that of the bed drops due to less air passage and larger per cent of CO formed in it, as well as to the heat absorption of volatile distillation. The explanation of variations in simultaneous temperatures in the bed, over it, and in the gas combustion space beyond would take too much space here, but are related to the combustion of gas compared with coke and the air supply above and below the fire. All temperatures in the furnace would be lower with anthracite coal especially in its smaller sizes.

As carbon yields 14,544 B.T.U. per pound when completely burned to CO_2 and in so burning, requires 11.52 lbs. air per pound yielding 12.52 lbs. of products, the value of β in equation (811), assuming the temperature rise to be at the high value of 2400° F., will be

$$\beta = \left[\frac{t_2 - t_1}{\frac{Q}{.243 w_P}}\right] = \frac{2400 \times .243 \times 12.52}{14{,}544} = .5. \quad . \quad . \quad . \quad . \quad (837)$$

It thus appears that for ordinarily high coal furnace temperatures the heat suppression coefficient is of the same order of magnitude as the corresponding factor for explosion pressures, but it must be noted that for lower temperatures this value will be higher as for example, when a little carbon, oil or gas is burned with a large excess of air, limiting the temperature rise. Thus, if there were 100 per cent excess air in the furnace, the temperature rise would be only 1200° for the same heat suppression factor whereas experience shows that the temperature will be closer to 2000°, which must be explained somehow, and the most rational explanation is the dissociation one, according to which the temperature attainable cannot rise much after dissociation sets in.

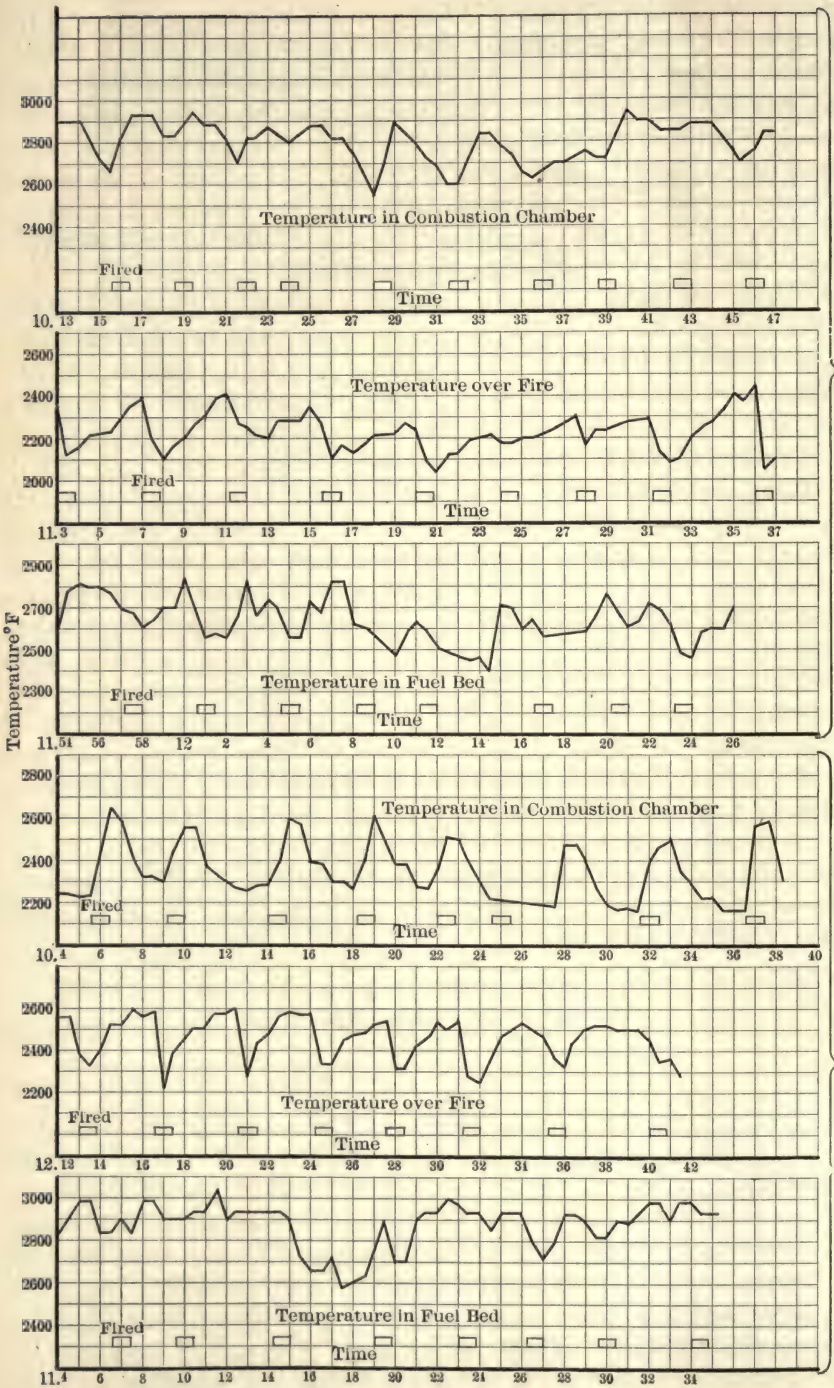


FIG. 195.—Temperatures in a Boiler Furnace.

This discussion shows the essentially approximate nature of temperature rise calculations for furnaces and explosions and that the search for correct specific heats of mixtures of various constituents in the products at assumed temperatures from supposedly correct constants is at the present time a purely academic procedure, which in practical work is to be discouraged until such time as *constants for all factors* are determined, including dissociations, which when used will give results that at least nearly agree with direct measurements.

According to the operation of the dissociation limit in furnaces, combustion cannot proceed after the attainment of a certain maximum temperature unless heat be abstracted, in which case it will proceed to keep up the temperature when it would otherwise fall, and there is evidence that this happens in both furnaces and gas-engine cylinders, though it is not quite absolute. It has been many times observed that the first part of the expansion line in gas engines does not follow the same expansion law as the last part, having a slope that indicates continued heat addition in spite of the certainty of simultaneous heat abstraction of the jacket. This, generally termed the *after burning*, is evidence of heat suppression, followed by liberation when temperature falls enough, but as some indicator cards do not show the action it is hardly conclusive.

It might be expected in furnaces that if at the time dissociation is limiting combustion, the gases be *suddenly* cooled below their ignition temperature, so suddenly as to leave insufficient time for union which would take place by gradual heat abstraction, that they would show unburned fuel elements. This is always found to be the case in boiler fires and is one of the explanations of smoke and of the presence of CO and some hydrogen and hydrocarbons in the gases when smoke is much in evidence, even though there is free oxygen present at the same time, the sudden cooling being done by contact of flame with the cold boiler tubes and plates. In this connection the following analyiss of soft coal soots by Sexton is of interest, the free carbon having separated out from the hydrocarbon gases at the high temperatures, perhaps by a preliminary reduction to ethane and acetylene, and not having time to burn either from deficiency of oxygen or dissociation limitations while in the high temperature and being prevented later by quick chilling.

COMPOSITION OF BITUMINOUS COAL SOOT (SEXTON) PER CENT BY WEIGHT

	I	II	III	IV
Carbon	39.0	86.94	68.5	75.3
Hydrocarbons	14.3	3.3–5.2	4.4	3.9
Sulphur	.....	.....	4.8	3.2
Sulphuric acid	4.0			
Ash	36.67	8–9.7	22.7	16.3

In no boiler furnace is combustion ever complete without excess oxygen indicated by free O_2 in the analysis of flue gas, and seldom so, even with appreciable amounts of it, so that analysis of flue gases of boiler furnaces or even other furnace fires is a most important practical method of indicating the attainment of good or bad fire conditions, leading to correction of faults in adjustment of

air supply above and below the grates, and at the same time giving a measure of the amount of unburnt fuel escaping, the amount of excess air being used and the weight of products per pound of coal. Flue-gas analysis is, therefore, the indicator of furnace efficiency and it, together with the temperature of escaping gases, measures the heat discharged to the chimney and unavailable for evaporating water.

The weight of combining air, excess air and total products, can be computed from the volumetric relations of the flue-gas constituents approximately for the fixed carbon of the fuel but not so easily for its volatile. Considering the fixed carbon alone, the fundamental relation for no excess air is given by Eq. (777) Section (7) which is,

1 lb. C+5.759(1+x) lbs. of air give a gas consisting by volume of

$$\left(\frac{1}{56.32x+86.12}\right)\begin{bmatrix}29.8x & \text{of } CO_2\\ 29.8\ (1-x) & \text{of CO}\\ 56.32(1+x) & \text{of } N_2\end{bmatrix} \text{ or } \left(\frac{1}{56.32x+86.12}\right)\begin{pmatrix}29.8(CO+CO_2)\\ 56.32(1+x)N_2\end{pmatrix}.$$

The weight of air per pound of carbon is 5.759(1+x) lbs. which can be evaluated numerically in terms of flue-gas volumetric relations when (1+x) can be found in terms of their CO and CO_2 contents. From these volumetric relations

$$\frac{\%\text{ by volume } N_2}{\%\text{ by volume CO}+\%\text{ by volume } CO_2}=\frac{56.32(1+x)}{29.8}=1.89(1+x). \quad . \quad (838$$

Hence,
$$(1+x)=\frac{1}{1.89}\times\frac{\%N_2}{\%\text{ CO}+\%CO_2}. \quad . \quad . \quad (839$$

Denoting by N_2, CO, and CO_2 the volumetric per cent of each,

$$\therefore \text{ Pounds air per pound C}=\frac{5.759}{1.89}\left(\frac{N_2}{CO+CO_2}\right)=3.1\left(\frac{N_2}{CO+CO_2}\right). \quad . \quad . \quad (840$$

If there is excess air there should be no free CO in the flue gases, but as the gases flow in streams the oxygen may not have come in contact with the unburned CO, its presence indicating unused air though not necessarily in excess chemically. It is, however, customary to consider all unused air as excess air whether chemically in excess or not, although uncombined air might better describe it.

If A = weight of the excess or uncombined air in pounds per pound of carbon, then if N_2', CO', and CO_2' are volumetric per cents exclusive of the excess air of the flue gas,

$$\text{Pounds air per pound C}=3.1\left(\frac{N_2'}{CO'+CO_2'}\right)+A. \quad . \quad . \quad . \quad (84$$

But the presence of the O_2 and N_2 of the extra air will change all the ratios. Each pound excess air per pound C will add 12.387 cu.ft. (std.) to the flue gas

of which 21 per cent or 2.701 cu.ft. is oxygen and 79 per cent or 9.686 cu.ft. is nitrogen. The total nitrogen will then be $[56.32(1+x)+9.686A]$ for A pounds excess air, and $2.701A$ cu.ft. oxygen will be with it. The total weight of air per pound C is $=5.759(1+x)+A$, the evaluation of which requires an expression for A and $(1+x)$ in terms of volumetric relations, which is not possible exactly. By trial the following is found to express the relation fairly well:

$$\frac{N_2+.055O_2}{CO+CO_2}=\frac{56.32(1+x)+9.686A+.055\times 2.7A}{29.8}$$

$$=1.89(1+x)+\left(\frac{9.834}{29.8}\right)A=1.89(1+x)+.33A$$

$$=.33[5.7(1+x)+A],$$

or

$$\frac{3N+.165O_2}{CO+CO_2}=5.7(1+x)+A. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (842)$$

Similar relations can be found for fuels consisting partly of hydrogen, but they are too complex to be of much help, so that the estimation of weight of air and excess air from the volumetric flue-gas analysis is possible *by a simple formula, only for fuels that are all or mainly fixed carbon.* The hydrogen relations can be worked out by expressions similar to those for producer gas and as flue-gas analysis is always made with cold gases none of the H_2O formed will show in the analysis. There will be added 337 cu.ft. of nitrogen (std.) for each pound of hydrogen and it will be derived from 34.64 lbs. of air required. This will have the effect of increasing the nitrogen per cent and decreasing the CO_2, CO and O_2 per cents in the flue gases from high volatile coal, oil, or hydrocarbon gases, for the same disposition and perfection of carbon combustion.

The heat loss due to unburned CO in the flue gas can be expressed in terms of the volumetric relations of CO to CO_2. If the carbon burns partly to CO_2 and partly to CO and x represents the fraction burning to CO_2, then $(1-x)$ represents the fraction burning to CO and 10,193 $(1-x)$ is the heat lost due to CO per pound of carbon.

B.T.U. per pound carbon, heat loss, due to unburnt CO $=10{,}193(1-x)$ (843)

But in the gases there will be

$29.8x$cu.ft. of CO_2 per lb. C,
$29.8(1-x)$ cu.ft. of CO per lb. C.

Hence designating by CO and CO_2 the respective per cents of each by volume

$$\frac{CO}{CO+CO_2}=\frac{29.8(1-x)}{29.8x+29.8(1-x)}=\frac{29.8(1-x)}{29.8}=1-x.$$

Therefore

B.T.U. per pound C, heat loss, due to unburned CO $=10{,}193\left(\frac{CO}{CO+CO_2}\right)$ (844)

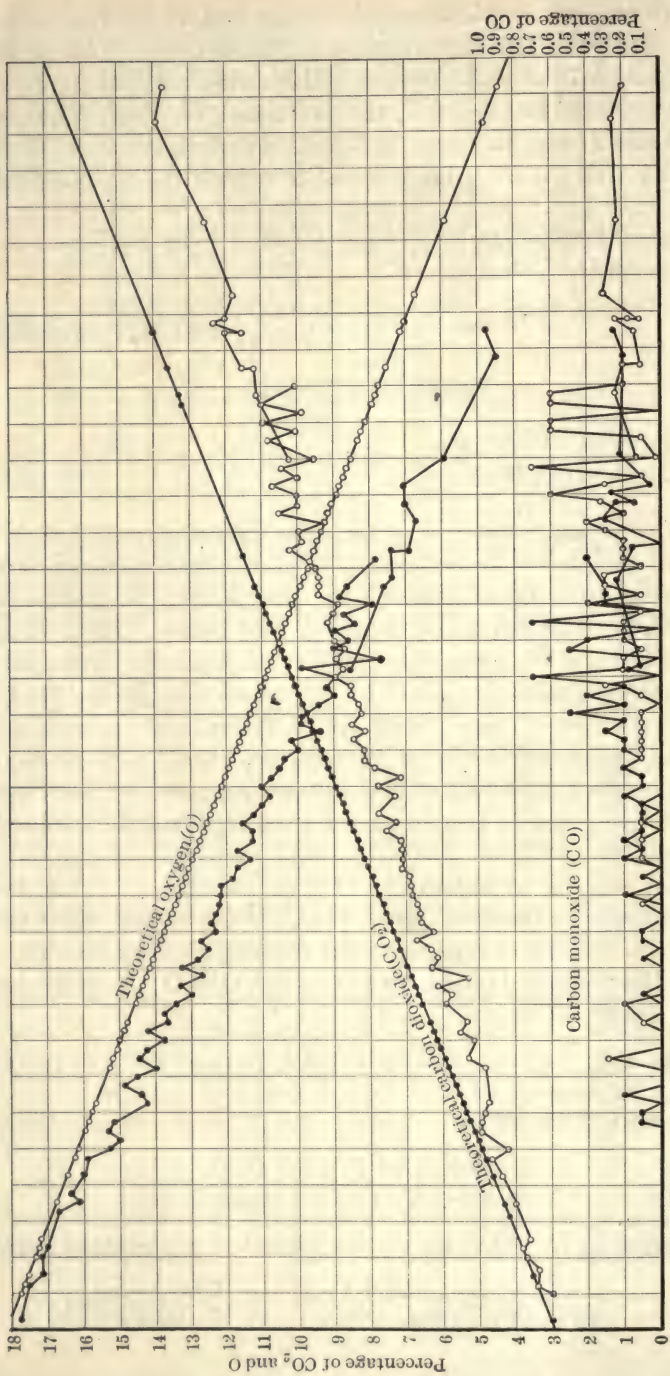


FIG. 196.—Diagrammatic Representation of Boiler Flue Gas Analysis.

At this point it is of interest to examine the sort of constituents that are ordinarily found in boiler flue gases, and in Table CXXII at the end of the Chapter are some careful determinations as compiled by Uehling, that may be regarded as representative of the resultant of all the complex conditions and reactions in a boiler fire. His conclusions are most reliable and are quoted: "There is a definite relation between the per cent of CO_2 and O_2 contained in the flue gas. The percentage of O_2 falls almost exactly in the same ratio as CO_2 goes up and vice versa. The analysis also shows conclusively that there is no relation between either O_2 or CO_2 and CO. High O_2 or low CO_2 is no evidence that CO will not be present or that the combustion is more complete with high CO_2 or that low O_2 means incomplete combustion. There is a tendency toward larger CO contents with larger CO_2 contents."

The total per cent of CO, CO_2 and O_2 together in the U. S. Geological Survey tests varied from 16.7 to 20 per cent, and of 1130 analyses only one total fell below 17 per cent, four below 18 per cent, 20 between 18 and 20 per cent, 287 between 19 and 20 per cent, 277 between 20 and 20.8 per cent. The best per cent of CO_2 varies with the carbon hydrogen ratio of the fuel, being smaller for large carbon hydrogen ratios. As a matter of interest in comparison with gas producer work the ratios of CO to CO_2 and of CO to $(CO+CO_2)$ is added to the tables and the curve Fig. 196, by Uehling is also added as it is of great assistance in getting a correct idea of the relations.

Nothing could be more clearly shown than the utter impossibility of completely burning without excess air, which carries off heat otherwise usefully available, than these curves and analyses, which represent more than usually good adjustment of fires, though expert adjustment with no end in view but perfect combustion, would improve results to some extent. In actual working, boiler capacity is just as important as efficient combustion and actual conditions represent attainable compromises.

Prob. 1. The flue-gas analysis taken on a boiler trial gave $O_2 = 10$ per cent; $CO_2 = 10$ per cent; $H_2 = 80$ per cent. If the coal used consisted of 80 per cent C and 20 per cent ash and the stack temperature was 400° F. above the room, what was the stack loss per pound of coal?

Prob. 2. 1000 heat-units are added to a pound of gas at 32° F. at constant pressure. The gas consists of 30 per cent CO, 40 per cent N_2, 5 per cent CO_2, and 25 per cent H_2. Calculate the final temperature on basis of constant specific heat and give what you consider a likely value for the actual temperature.

Prob. 3. A pound of air is compressed from 1 to 5 atmospheres above atmosphere and then 1000 heat-units are added at constant volume. What will be the final temperature?

Prob. 4. A gas requires 5 cu.ft. of air to completely burn it. Its heating value is 900 B.T.U. per cubic foot under standard conditions. A mixture of one part of this gas with 5, 6 and 7 cu.ft. of air at a temperature of 100° F. and at a pressure of 25 lbs. per square inch gage is ignited in a closed vessel. What is the probable pressure after combustion?

Prob. 5. A pound of coal gave the following analysis: $C = 70$ per cent; $H_2 = 5$ per cent; $O_2 = 5$ per cent; ash $= 20$ per cent. If twice the air chemically needed were

supplied, what would be the maximum temperature as calculated on basis of constant specific heats? What would be the flue-gas analysis?

Prob. 6. In a boiler test it was found that 20 per cent of the heat in the coal went up the flue as sensible heat and in unburned carbon. The coal contained 13,500 B.T.U. per pound. The flue gas analysis showed $CO_2 = 12$ per cent, $CO = 6$ per cent, $O_2 = 2$ per cent, $N_2 = 80$ per cent. If the coal is assumed to be 80 per cent C and the boiler-room temperature is 80° F., what was the flue temperature?

Prob. 7. The pressure in a gas-engine cylinder before ignition was 70 lbs. gage and after ignition 350 lbs. gage. The temperature before ignition was 300° F. For a mixture containing 110 B.T.U. per cubic foot under standard conditions, what was the ratio of observed to calculated pressure rise?

Prob. 8. Take an analysis of coal gas from the Table CX and on the basis of there being .5 of the calculated pressure rise, find the pressure exerted on the side of a room $50' \times 10'$ if there should be an explosion of the gas in it with atmospheric pressure and temperature before the explosion and with best with mixture.

11. Temperature of Ignition and Its Variation with Conditions. Limits of Proportion Air-Gas-Neutral, or Detonating Gas and Neutral, for Explosive Combustion of Mixtures. Limits of Adiabatic Compression for Self-Ignition of Mixtures. It is only with gaseous fuel previously mixed with measured air that best combustion results can be obtained, perfectly complete combustion without excess air, and this is easily managed in engine cylinders. Gas and oil and powdered coal fired boilers may approximate this, but with the burners in common use explosive mixtures cannot be used without endangering the settings should the flow be momentarily interrupted and then continue, and just such serious accidents have happened. With specially designed burners and furnaces, explosive combustion is perfectly feasible in boiler and other fires but as yet these are not widely used. Mixtures of fuel and air are explosive at the chemical proportions and for a considerable range beyond, usually more on the excess fuel side than that of excess air. Two of the most authoritative tables of limits, those of Eitner and Bunte, given below, are relatively but not absolutely correct. This is true because the limit of explosive proportions is said to be reached when a mixture will not explode and this is as much a question of how the ignition is attempted, as it is one of proportion. Flames will ignite mixtures that will not explode with electric sparks, some sparks will ignite mixtures that others will not, and mixtures will ignite when a flame is plunged into the mass when they cannot be ignited at a touch hole or at one corner of their chamber.

When, however, the following experimental observations are considered the complexity of the ignition becomes more apparent. With carbon it is known that the state determines the temperature at which combustion proper sets in, it being lower for porous varieties of charcoal, higher for denser varieties and very high for graphitic forms, but it is also known that all forms will react with oxygen, yielding carbon dioxide at some temperature. This leads to a suspicion that there is no such thing as *the ignition temperature* of carbon and probably also for other fuel elements and compounds. Consider the

ydrocarbons which in flames burning without previous air mixture become ecomposed; ethylene, for example, according to Sexton splitting up into cetylene and methane $3C_2H_4=2C_2H_2+2CH_4$ between 1500° F. and 1800° F. r ordinary furnace temperatures, and methanere acting at 2000° F., $2CH_4=$ $_2H_2+3H_2$ and finally acetylene $C_2H_2=C_2+H_2$ at 2430°F. Who can say n view of this possibility just what takes fire when ethylene is ignited, is it thylene as a whole or is it methane or acetylene or hydrogen separated out

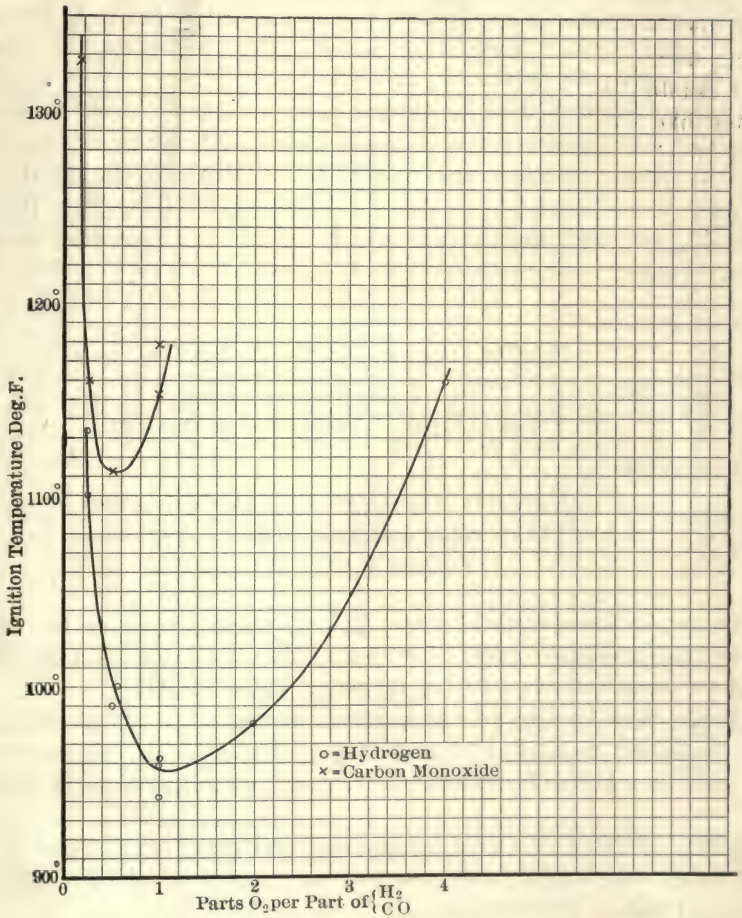


FIG. 197.—Temperature of Ignition for H_2, and CO, mixed with O_2.

by the attempt to ignite? There is no answer to the question, but a little ight can be thrown on it by some recent experiments prompted by previous work of Davy followed by Mallard and LeChatelier, who observed that for gaseous mixtures the *ignition temperature true or apparent, was different for different proportions of air and fuel and likewise still different when neutrals were present.* Results, however, were quite variable with the method of procedure and quite

inconsistent and it was not till Falk, by rapidly compressing a previously determined mixture, heating it by compression uniformly throughout to ignition, the temperature of which was calculated by the adiabatic law, that any progress could be said to have been made. His results are shown graphically in Fig. 197 for carbon monoxide-oxygen and hydrogen-oxygen mixtures, which are the principal fuel elements in power producer gas.

From the curve it is clear that there is a regular variation of ignition temperature with proportion and a minimum point for each fuel which corresponds to the H_2+O_2, proportions for hydrogen, indicating that it reacts in two stages first to H_2O_2, and later to $2H_2O$, whereas, for carbon monoxide the minimum point corresponds to the detonating mixture.

When the combustible and its oxygen must be ignited in the presence of neutral matter, like excess oxygen or excess combustible which when not combining are in effect neutral, or real neutrals like nitrogen, the ignition temperature may be calculated from the two following formulas Eqs. (845) and (846), working out each independently of the other and accepting whichever is the lower value.

$$\left\{\begin{matrix}\text{Ignition temperature}\\ \text{of } H_2 \text{ with } O_2\\ \text{and } N_2\end{matrix}\right\} = \left\{\begin{matrix}\text{[Ignition temperature for } H_2 \text{ with } O_2 \text{ alone]}\\ +\left[\dfrac{54\times\text{volume of inert gases}}{\text{Vol. of } H_2 \text{ or } O_2 \text{ (whichever is smaller)}}\right]\end{matrix}\right\} . \quad (845)$$

$$\left\{\begin{matrix}\text{Ignition temperature}\\ \text{of CO with}\\ O_2 \text{ and } N_2\end{matrix}\right\} = \left\{\begin{matrix}\text{[Ignition temperature for CO with } O_2 \text{ alone]}\\ +\left[\dfrac{144\times\text{volume of inert gases}}{\text{Volume of CO}}\right]\end{matrix}\right\} . \quad (846)$$

By the use of these equations the ignition temperature for five samples of hypothetical producer gase have been calculated as an example, the various steps being set down in the following Table XCIII.

The results show that in all but the first case with $H_2 = 12.6$ per cent, which increases to 26 per cent, it is the hydrogen that controls the ignition temperature and that the difference for wide range of proportions is really not great.

The same apparatus was used for gasolene and ethyl alcohol and gave the following values which were fairly constant for explosive mixtures through a 100 per cent range of proportions.

$$\left.\begin{matrix}\text{(Ignition temperature of gasolene mixtures with air)} = 986^\circ \text{ F. } (a)\\ \text{(Ignition temperature of alcohol mixtures with air)} = 1292^\circ \text{ F. } (b)\end{matrix}\right\} . \quad (847)$$

These temperatures of ignition would never be reached by compression of cold charges adiabatically to the pressures used in engine cylinders, yet for even small compressions the charges ignite themselves or preignite, proving that the compression temperatures are very high in such cylinders and must become so *by reason of heat added to the charge before or during compression*

TABLE XCIII

CALCULATED IGNITION TEMPERATURES FOR PRODUCER GAS

No.	Producer Gases, Composition by Volume.				Volumes O_2 to Burn 100 Volumes Gas.	Volumes Air to Burn 100 Volumes Gas.	Total Volume Mixture.
	CO_2	CO	H_2	N_2			
1	.0	38.9	12.6	48.5	25.75	123.08	223.08
2	3.7	34.2	16.1	46.0	25.15	120.22	220.22
3	7.2	29.9	19.4	43.5	24.65	117.82	217.82
4	10.7	25.6	22.7	41.0	24.15	115.44	215.44
5	13.6	21.5	25.8	39.1	23.65	113.05	213.05

No.	Volume of Inert Gas. When		n'	n	Temperature for Zero Inert Gas from Fig. 197.		Calculated Ignition Temperature for the Mixture.	
	CO Burns.	H_2 Burns.			For CO.	For H_2.	Based on CO.	Based on H_2.
1	164.73	204.18	4.13	16.2	1112	982	1708	1857
2	168.42	196.07	4.93	12.18	1118	963	1824	1620
3	172.97	188.72	5.93	9.73	1128	979	1984	1503
4	177.04	181.39	6.92	7.99	1148	954	2146	1385
5	180.80	174.35	8.4	6.76	1175	957	2385	1322

As the temperature rise before compression is multiplied by compression, it may be assumed that this is the controlling factor and the following calculation shows to what extent this may be expected to take place.

Let T_1, t_1, and T_2, t_2 be absolute and scale temperatures before and after compression to ignition temperature from one atmosphere to P_2 atmospheres absolute, adiabatically.

Then assuming the exponent, $s = 1.4$.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{.29} \quad \text{and} \quad T_1 = \frac{T_2}{\left(\frac{P_2}{P_1}\right)^{.29}}$$

From this the initial temperature for mixtures that really preignite at a known compression in engine cylinders can be calculated and three such cases are given below.

CALCULATION OF INITIAL TEMPERATURE FROM PREIGNITION PRESSURES IN GAS ENGINES

Fuel.	Compression Ratio in Cylinder when Preigniting, $\frac{P_2}{P_1}$	Temperature of Ignition, Falk.	Corresponding Initial Temperature.	
		T_2 Absolute F.	T_1 Absolute F.	t_1 Scale F.
Producer gas { CO 29.9, H_2 19.4, CO_2 7.2 }	9	1080+460=1540	816	356
Alcohol	13	1292+460=1752	833	373
Gasolene	7	986+460=1446	824	364

This shows that in using these fuels properly mixed with air in cylinders the amount of compression allowable without self-ignition *varies with the fuel mixture ignition temperature and with the initial temperature,* which cannot be allowed for the three fuels above to rise over 350° F., and does reach pretty close to this value, probably 300°, as some heat will be derived from the walls during compression. This temperature, when charges are taken in at about 60° F. on the average, is a measure of the heating effect of the hot walls during suction, coupled with the heat added by the residue hot gases from a previous explosion, and left in the clearance.

In the following Table XCIV are given some of the usually practiced limits of compression in gas engines, which, of course, are subject to considerable variations with type of engine and its management, and which are controlled as much by cylinder heating as by the natural temperatures of ignition of the mixtures themselves.

TABLE XCIV

COMPRESSIONS COMMONLY USED IN GAS ENGINES

Fuel.	Type of Engine.	Compression, Pounds per Square Inch Absolute.	Average Compression, Pounds per Square Inch Abs.
Gasolene	Automobile	60–95	75
Gasolene	Stationary	70–100	90
Kerosene	Hot bulb, 250–500 r.p.m.	30–75	60
Kerosene	Vaporized before entering cylinder	45–85	65
Natural gas	Medium and large engines	75–130	115
City gas	Small engines	60–110	85
Producer gas	Medium engines	100–160	135
Blast gas	Large engines	120–190	155

The setting on fire of a combustible, or more scientifically, its ignition, is, therefore, not the simple operation that it might seem, especially when the fuel is in the form of explosive gaseous mixture. It has been the custom to say that the fuel, when raised to *its* ignition temperature in the presence of oxygen, will at once begin to burn and ignition temperature tables like the following Table XCV are common, though probably wrong, surely so for gases.

Not only do the Falk data prove the controlling influence of proportions on ignition temperatures for gases but the following Table XCVI from Violette proves that for solid charcoal that the ignition temperature is dependent on its origin, which may be assumed to control its state. In this case the prime variable is the temperature at which the charcoal was formed by wood distillation, which it has been pointed out controls the density and character of the residue and probably similar conditions maintain for other fuels.

Temperatures of ignition for explosive gaseous mixtures are intimately related to the limiting proportions for explosive combustion, because inflammation will propagate or pass of itself, from particle to particle in the mass, *only*

when the heat liberated by combustion at one point is able to raise the next one to its temperature of ignition. It is impossible yet, to state algebraically just what this relation is, but in connection with it, an examination of some data on gas and air mixtures showing what proportions will and what will not explode is worth while.

TABLE XCV

IGNITION TEMPERATURES, °F

Substance.	Ignition Temperature.	Substance.	Ignition Temperature.
Carbon, C	752 (Sexton)	Methane, CH_4	1201 (Meyer)
Soft coal	600	Methane, CH_4	1213 (LeChatelier)
Anthracite	750	Ethane, C_2H_6	1141 (Allen)
Peat	430	Ethylene, C_2H_4	1124 (Allen)
Lignite dust	300 (Strohmeyer)	Ethylene, C_2H_4	1124 (Meyer)
Hydrogen, H_2	1077 (Olsen)	Propylene, C_3H_6	940 (Allen)
Hydrogen, H_2	1124 (Meyer)	Acetylene, C_2H_2	1038 (Allen)
Hydrogen, H_2	1031 (Le Chatelier)	Acetylene, C_2H_2	896 (Robinson)
Carbon monoxide, CO	1253 (Allen)	Propane, C_3H_8	1017
Carbon monoxide, CO	1347 (Meyer)	Alcohol, C_2H_5OH	1292
Carbon monoxide, CO	1211 (Le Chatelier)	Coal gas	1100 (Robinson)
Methane, CH_4	1212 (Allen)		

TABLE XCVI

VARIATION OF IGNITION TEMPERATURE OF CHARCOAL—(VIOLETTE)

Temperature of Distillation at which Charcoal was made, ° F.	Temperature of Ignition of the Charcoal in Air, ° F.
520	650–680
570	680–715
600	680–700
800	750
1800–3000	1110–1500
Above 3000	2300

In the previous section some curves were given to show the explosion pressures for various mixtures of air, gas and neutral or inert matter, which curves, stopped at points where the mixture was non-explosive, and the corresponding proportions can be read off directly. A series of values for the limiting proportions of air-gas mixtures is given in Table CXXIII, at the end of the Chapter, showing wide differences for different fuels with no apparent relation between them, nor are any controlling influences indicated, though some are really known to exist.

Pressure and temperature variations act irregularly, but in general tend to widen the limits; rise of temperature and pressure, more particularly the

latter, will render explosive, proportions otherwise not so, while neutral dilution always narrows the limits. A series of experiments by the writer indicated a tendency toward a limit of explosibility when the active or detonating mixture was equal to, or greater than, about 17 per cent of the total, or the neutral inactive gases not in excess of 83 per cent. This active mixture consists of the combustible constituents of the gas with just the right amount of oxygen to burn them, and is generally called the *detonating gas or detonating mixture.* All excess oxygen or excess fuel is classed as neutral and the following table shows the relations in various mixtures of (air-gas-neutral) of the detonating mixture to the whole mixture.

This limitation of explosibility by limiting quantities of detonating mixture or total inactive constituents noted above for all ranges of one gas, was observed by Bunte to air-gas mixtures alone for all kinds of gases with a comparatively small range of variation, as in Table XCVII.

TABLE XCVII

PER CENT DETONATING MIXTURE IN TOTAL AIR-GAS MIXTURE AT EXPLOSIVE LIMITS OF PROPORTION (BUNTE)

Gas.	When Air is in Excess.	When Gas is in Excess.
Carbon monoxide	24.75	15.78
Hydrogen	14.17	21.16
Water gas	18.80	20.94
Acetylene	11.72	14.03
Coal gas	17.40	31.15
Ethylene	16.40	23.91
Alcohol	15.00	24.17
Marsh gas	18.30	27.47
Ether	19.25	22.61
Benzene	19.87	22.25
Pentane	21.60	22.47

These figures taken in conjunction with those for large neutral additions suggest the possibility of a *universal law for all explosive gaseous mixtures whatever the fuel or the nature of constituents associated with it, that explosion is no longer possible when there are sufficient inactive particles of gas present to prevent a heat propagation that will allow one group of burning fuel and oxygen particles to set fire to the next separated from it by an insulating layer of dead gas or even allow the oxygen to reach the fuel, and that this per cent of inactive is nearly constant for all.*

One remarkable thing brought out by the table illustrating this general law is the narrowness of the range for rich hydrocarbons, especially on the excess air side. Practically no excess air is permissible, a fact that indicates the necessity in gasolene and kerosene engines for most accurate adjustment of proportions, without which there is almost a certainty that excess fuel will be used by an engine operator, all excess being direct waste.

Prob. 1. Acetylene gas is escaping into a room which is $15' \times 20' \times 10'$. How many cubic feet of gas must escape into the room and mix with the air before there is a possibility of an explosion? Should there be nothing to ignite the mixture, how many cubic feet of the gas must there be in the room before the upper limit of explosibility is reached? Should the gas have been coal gas, what would have been the quantities?

Prob. 2. A sample of gas gave the following analysis by volume: CO = 35 per cent; CO_2 = 6 per cent; H_2 = 14 per cent; N_2 = 35 per cent. If this gas be mixed with the combining proportion of air to what temperature must it be heated to cause it to ignite?

Prob. 3. A gas engine is running on a natural gas which may be considered as all methane. The combining proportions of this gas with air are in the ratio of 9.5 to 1. Taking the limits of proportion for explosive mixture from the general law what would be the cubic feet of air per cubic foot of gas for the leanest and richest mixture which would still ignite?

Prob. 4. Gasolene is stated to have an ignition temperature of 986° F. If the temperature of the mixture in the cylinder of an engine using this fuel is 400° F. at the beginning of compression, what will be the maximum compression which can be carried without preignition? The value of s may be taken as 1.4.

Prob. 5. If instead of gasolene there be used in the engine of Prob. 4, the following gas, by how much could the compression be raised without preignition occurring? CO = 37.6 per cent; H_2 = 11.3 per cent; CO_2 = 3.7 per cent; N_2 = 47.4 per cent.

12. Rate of Combustion of Solid Fuels with Draft. Propagation Rates, Normal, and Detonating for Explosive Gaseous Mixtures. The last combustion characteristic to be examined is the rate of combustion or speed with which the combination, once started, proceeds. In boiler fires the coal rests on grates and the rate with which it may burn determines the amount of steam that the boiler can make per hour, assuming a constancy of heat losses to flues and setting. When gases or oils are burned under boilers or in other furnaces the rate of combustion determines the size of furnace necessary for the combustion of the requisite fuel for the duty expected of the heat, while in gas engine cylinders the time of combustion is almost vanishingly small if efficiency is to be secured. For the latter case the explosion should always be as near complete as possible before expansion begins, and should not start too soon toward the end of compression or the work of compression will be too great. To get an idea of the time available, assume an automobile engine running at 1200 R.P.M. or 20 revolutions per second, completing one revolution in .05 second. Then ignition and combustion must take place during 18° of crank rotation near the dead center or $\frac{18}{360} = .05$ of the revolution the time for the whole process to be completed is $.05 \times .05 = .0025$ or about $\frac{1}{400}$ second, for a path not over 6 ins., corresponding to a flame speed of .005 second per foot or 200 ft. per second. The very largest engine with a cylinder of 4 ft. diameter, turning at 60 R.P.M. or 1 revolution per second, for the same crank angle requires .05 second, whence

for 1 ft. of mixture the speed must be $\frac{.05}{4} = .012$ second approximately and the flame speed about 85 ft. per second.

By elaborate apparatus Mallard and LeChatelier photographed the progress through a tube, of a flame passing through an explosive mixture of known characteristics. The photographic plate was moved across the tube during the flame passage giving a picture shown in Fig. 198 which is really a diagram of speed of propagation. Horizontal distances represent time, while verticals represent length of tube traversed or linear travel of flame through the mixture. The slope of the line, or distance (vertical), divided by time (horizontal), is the speed of propagation and as the line varies in slope the speed of flame propagation is a variable quantity. It appears, however, that at the beginning of combustion in the open tube the speed is first uniform then undulating or wavelike and sometimes at the end, very great, as indicated by the nearly vertical record line. *This rapid mode of propagation is termed the explosive, or better, the detonating wave* and is always reached in good mixtures if there is distance enough to be traversed, or if there is agitation of the mixture or pressure waves developed equivalent to agitation, and is quite common in engine cylinders. It was at first believed to follow the speed law of sound or other impact waves in gases but Dixon has shown that this is not true, the speed being much greater than that of sound but it is constant.

Therefore, in explosive mixtures the rate of combustion is at first uniform and slow, and rapidly accelerates to a high constant value. The following Table XCVIII, gives Berthelot's and Dixon's experimental determination in feet per second for the detonating waves, compared with sound waves.

TABLE XCVIII

VELOCITY OF EXPLOSIVE OR DETONATING WAVES

Mixture.	Velocity in Feet per Second.		Authority.
	Detonating Wave.	Sound Wave.	
$2H_2+O_2$	9220	1696	Berthelot
$2CO+O_2$	3583	1076	"
CH_4+2O_2	7504	1142	"
$C_2H_4+3O_2$	7248	1050	"
$2C_2H_2+5O_2$	8143	1060	"
$C_2H_4+2O_2$	7202	938	"
$C_2H_2+O_2$	8961	.....	Dixon
H_2+Cl_2	5693	.....	"
$2H_2+O_2+6H_2$	11588	.....	"
$2H_2+O_2+5O_2$	5591	.....	"

Various attempts have been made to derive a fundamental formula for the speed of the wave in terms of physical constants of the mixture, and whil

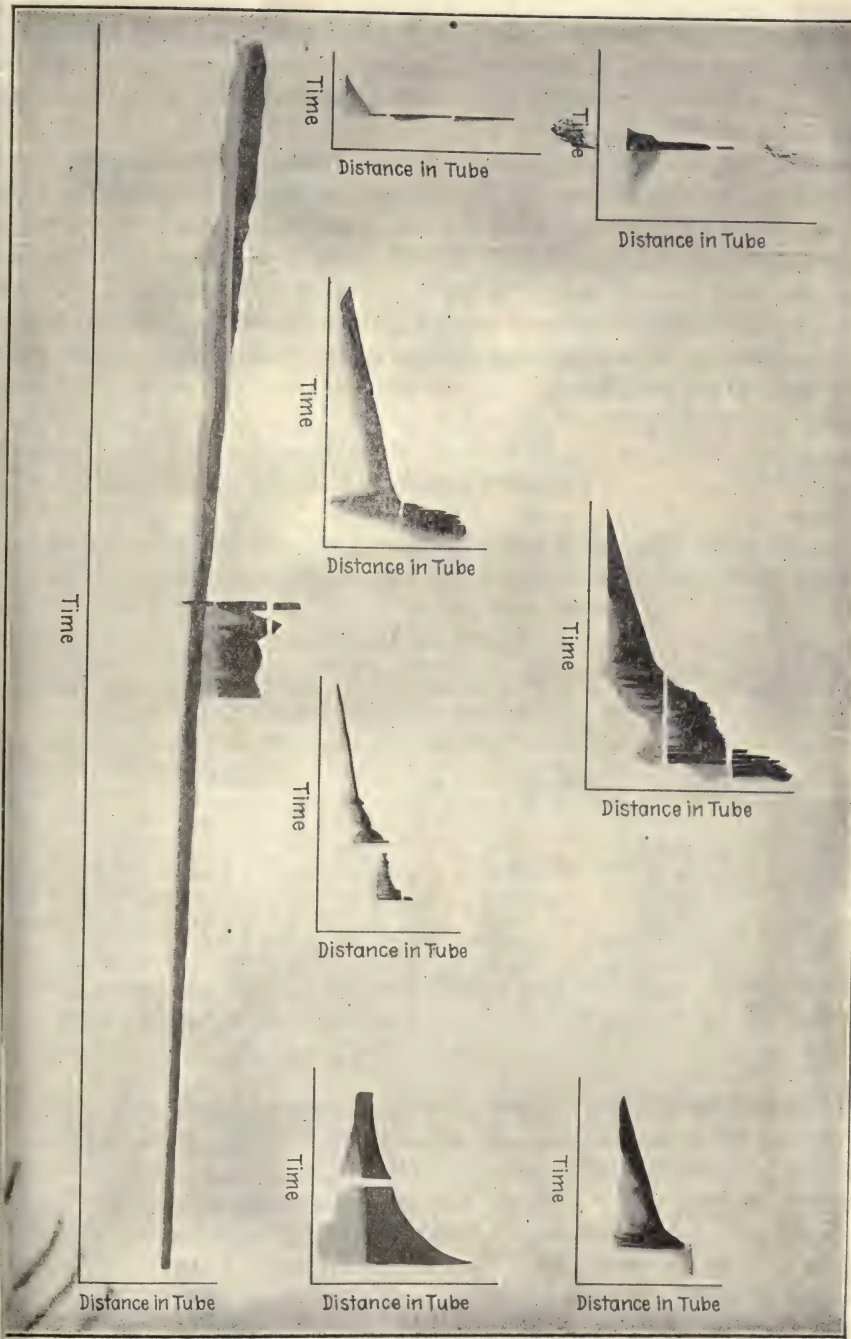


FIG. 198.—Propagation of Flame Through Explosive Mixtures. Photographically Recorded. For Combustion in Glass Tubes.

some are reasonably satisfactory no one of them seems to fit all cases or give results agreeing with measurements. If in cylinders, as often happens, the combustion seems to be too slow, then it is certain that the wave has not been set up and it is as a matter of fact undesirable to have it develop because of the accompanying shocks.

For the slow, uniform propagation the best data on the conditions and rates are given by Mallard and LeChatelier, whose results show that the walls have an influence, the nature of which is a cooling one, abstracting heat at various rates compared to that of generation. *When cooling is strong enough to take heat away as fast as it is generated, propagation of explosion is impossible* and this always takes place in narrow slits between plates, in very small tubes and in cold wire gauze screens. Rise of temperature always increases the rate, thus for H_2 and air with 30 per cent H_2,

$$\left\{\begin{array}{l} r = 10.75 \text{ ft.sec. at } 60^\circ \text{ F.} \\ r = 14.26 \text{ ft.sec. at } 212^\circ \text{ F.} \end{array}\right\}$$

The greatest single influence in fixing the rate of propagation is the nature and proportion of the mixture as shown by the following figures of Table XCIX.

TABLE XCIX

RATE OF PROPAGATION (UNIFORM) FEET PER SECOND FOR DIFFERENT PROPORTIONS IN MIXTURES AT ATMOSPHERIC PRESSURE

H_2 per 100 Vols.	r	C_2H_4 per 100 Vols.	r	$11.65(C_2H_4)+x$	r
6	0	5.6	0	$x = .5N_2$	1.38
10	1.97	6.0	.098	" $.0N_2$	.984
20	6.40	10.0	1.37	" $1.4N_2$	.623
30	10.82	12.0	2.01	" $.5CO_2$	1.02
40	14.34	14.0	1.18	" $1.0CO_2$	23.48
50	11.32	16.0	.33		
60	7.54	16.2	0		
70	3.61				
80	0				

In all cases neutral dilution has a strong influence on reduction of rate of propagation, reducing it for all air-gas mixtures to zero, when the limit of proportion has been reached at somewhere about 83 per cent of total neutral. In general the uniform or slow normal rate of propagation r never exceeds 14.1 ft. per second and occurs with 40 per cent of gas or 30 per cent excess H_2; for C_2H_4 and air the maximum is 2 ft. per second and occurs in the mixture containing 12.2 per cent gas or an excess of 42 per cent; for CO and air the maximum 6.56 ft. per second, and for illuminating gas and air the maximum is 4.1 ft. per second, occurring when the mixture contains 15 per cent excess gas, all at atmospheric pressure.

These figures show that in engine cylinders, pressure and temperature rise by compression, are necessary to make the mixtures burn in the time available, as the rate of combustion at atmospheric pressure is not high enough. Of course, if the detonating wave is set up, there will be more than enough time.

Rate of combustion of coal on grates is measured in pounds per square foot per hour, and if the coal burned as fast as the air flowed through the bed, the rate should increase directly with the square root of the draft, but be different for every different condition of porosity of bed, thickness of bed, or in general, resistance to flow. As a matter of fact coals with much volatile are burned as much above the grate, in the gasified form of hydrocarbon and carbon monoxide as on it. The gas thus made may easily equal in weight that of the fixed carbon left on the grate. In this case the rate of combustion will not vary with the square root of the draft, and especially if the coal cakes some the rate will be constantly varying from minute to minute. The following figures, Table C, are commonly quoted, but judged from present-day practice are considerably wrong, as later figures will show.

TABLE C

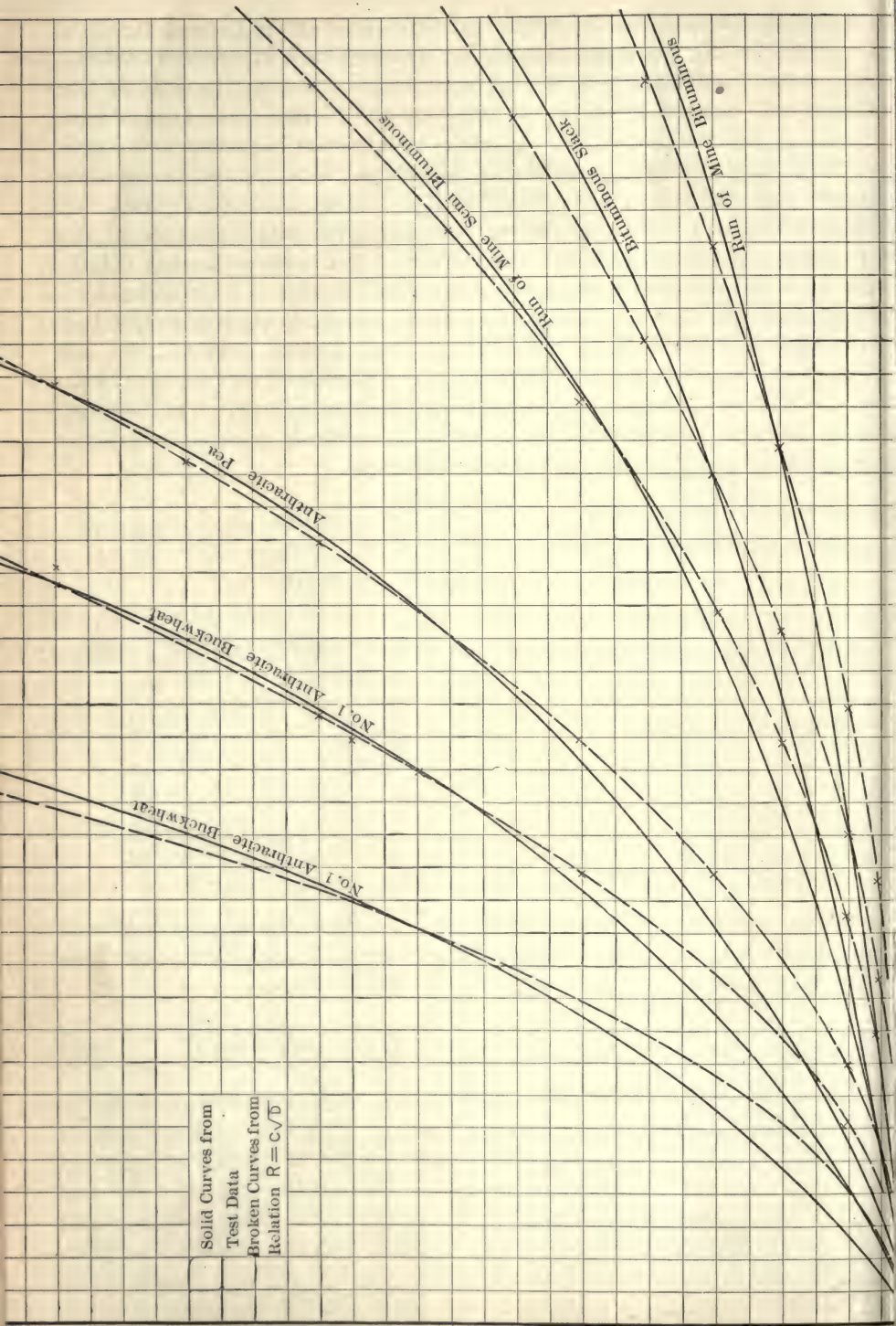
RATES OF COMBUSTION OF COAL

Type of Furnace.	Pounds Coal per Hour per Square Feet Grate.	
	Rankine.	Grüner.
Stationary boilers	4– 16	8– 20
Marine boilers	16– 24	
Locomotive	40–120	81–102
Lead smelting	.....	12– 16
Copper smelting	.....	15– 30
Puddling smelting	.....	20– 30
Steel smelting	.....	41– 81

On the assumption that the coal will burn as fast as the air passes through the bed, and that air will pass with a velocity proportional to the square root of the draft measured between ash pit and furnace, then

$$(\text{Lbs. coal sq.ft. grate per hr.}) = C\sqrt{(\text{draft in inches of water})}. \quad . \quad . \quad (848)$$

The constant C may be determined from experiment approximately, as it is not a true constant for a number of reasons. Consider a given coal on a given grate, then C will vary with thickness of fire, age of the fire, amount and condition of ash and clinker and amount of cake on the surface if the coal is of the caking variety. A change of size of coal with no other change will affect the air flow and the speed of carbon union with the air that does pass, and a change in proportion of surface and bottom air will also modify C, as will a change in grate. A grate that breaks up a bed like the step stoker will


Run of Mine Bituminous
Bituminous Slack
Run of Mine Semi Bituminous
Anthracite Pea
No. 1 Anthracite Buckwheat
No. 1 Anthracite Buckwheat
Solid Curves from Test Data
Broken Curves from Relation $R = c\sqrt{D}$

pass much more air than one that does not like a chain grate, and a fireman that hand slices often, lets more air pass than one that does not. A coal that has much volatile may burn as much above as on the grate, and its rate of combustion be very different at the same draft from another with less volatile, everything else being the same.

In spite of all these influences the rate does follow fairly well the relation of Eq. (848), though the constant must be selected with care. One general statement of the relations for a variety of coals is given by the Stirling Boiler Company, compiled from tests on their boiler and is reproduced in Fig. 199 with some additions. Assuming the middle rate reported for each coal to be correct, the square root curve is drawn through this point for higher and lower

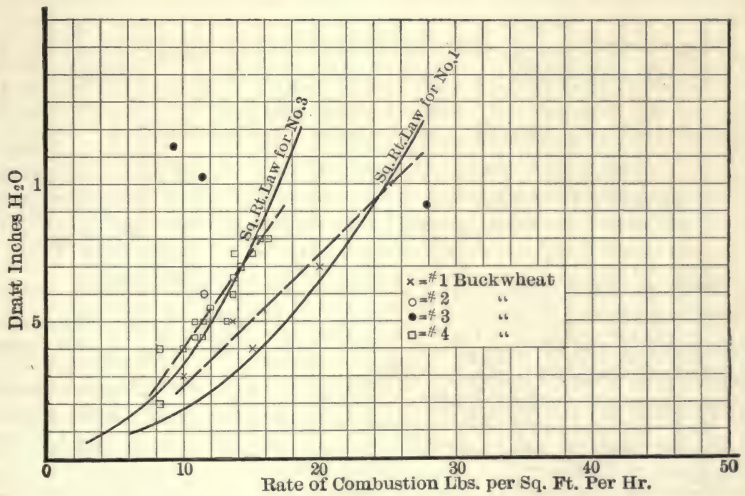


FIG. 200.—Rate of Combustion of Anthracite Coal.

drafts and from it the following constants, Table CI, are found for hand-fired furnaces:

TABLE CI

CONSTANTS OF PROPORTION ["C" in Eq. (834)] FOR RATE OF COAL COMBUSTION]

Values of C in Rate of Combustion, Eq. (848).	Coal name.
17.3	Anthracite rice
24.5	No. 1, Anthracite buckwheat
31.5	Pea anthracite
81.6	Run-of-mine bituminous
50.2	Run-of-mine semi-bituminous
59.3	Bituminous slack

To show how far actual cases depart from these averages, the results o some special determinations are separately plotted in Fig. 200 for anthracites and in Fig. 201 for bituminous coals, from Tables CXXIV and CXXV at the en of the Chapter. The highest rates and drafts are used on locomotives, nex

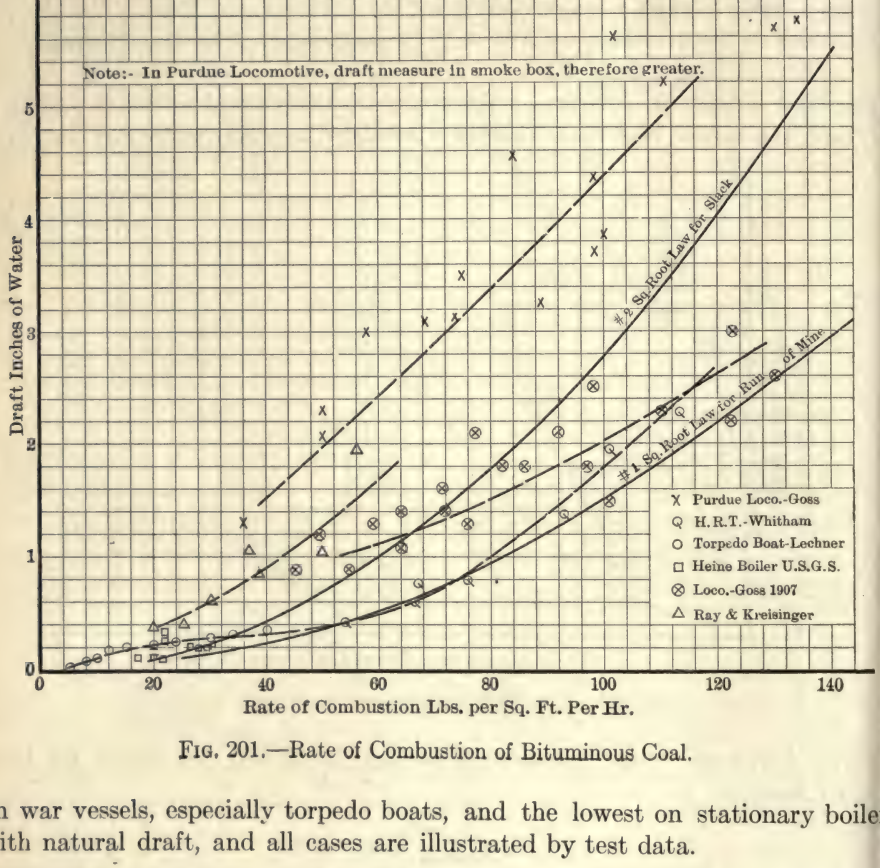


FIG. 201.—Rate of Combustion of Bituminous Coal.

on war vessels, especially torpedo boats, and the lowest on stationary boiler with natural draft, and all cases are illustrated by test data.

Prob. 1. A boiler rated at 180 H.P. has 60 sq.ft. of grate area. If 70 per cent o the heat of the coal is absorbed by the water, what must be the rate of combustio for No. 1 buckwheat containing 11,000 B.T.U. per pound. What draught will b required for the rate? When the capacity was raised to 270 H.P. the efficiency fe to 60 per cent. What draught would then be required to burn the necessary coal?

Prob. 2. A gasolene engine has a cylinder 5×5 ins. and runs at 1000 R.P.M Ignition occurs 12 degrees ahead of dead center. What must be the velocity of flam travel to give complete inflammation at dead center with cylindrical clearance 20% o the stroke in length?

Prob. 3. A stationary boiler had a grate area of 60 sq.ft. and was rated at 180 H.I A locomotive boiler having a grate area of the same amount was rated at 500 H.I Assuming that both boilers were fired with run-of-mine bituminous and that th respective efficiencies were 65 and 50 per cent, what draughts would be needed in each

Prob. 4. The coal No. 16 in the general coal table at the end of this chapter is being red at the rate of 50 lbs. per square foot of grate surface. What must be the rate f firing for coal No. 121 of the same table to produce the same heat?

Prob. 5. 100,000 ft. of natural gas are burned per hour under a boiler. Assume a atural gas from the general table at the end of the chapter and an anthracite coal rom the coal table and find what rate of combustion must be maintained with the oal on a grate area of 100 sq.ft. to develop the same heat?

Prob. 6. Coal No. 100 of the general table is being burned at the rate of 75 lbs. er hour per square foot of grate surface on a grate containing 120 sq.ft. What draught ill be needed? What grate area will be needed to produce the same heat with anthra- ite pea coal of 12,000 B.T.U. per pound burned at the same rate?.

Prob. 7. Ten barrels of fuel oil are burned per hour under a boiler. At what rate ust bituminous coal be burned under another boiler having 60 sq.ft. of heating surface o produce the same heat? Take any fuel oil from oil table and bituminous coal from oal table.

Prob. 8. No. 2 buckwheat anthracite, having a heating value of 11,600 B.T.U., s being burned with a draught of .7 in. on a grate with 80 sq.ft. of surface. On another rate run-of-mine bituminous is being burned with .5 in. draught. If the calorific ower of the latter is 14,000 and the rate of combustion follows the square root law, hat is the necessary area of the second grate for equal heat generation?

Prob. 9. What would be the relative rates of combustion for equal generation for oals Nos. 10, 25, 61, 113, 162, 185, 201 of the general table?

13. Steam Boiler Evaporative Capacity and Horse-power. Horse-power nits, Equivalent Rates of Evaporation and of Heat Absorption. Factors of vaporation. Relation between Absorption Rates and Rates of Heat Genera- on. Influence of Heating and Grate Surface, Calorific Power of Fuels, Rates f Combustion and Furnace Losses. It seems a little curious to have to confess hat the steam boiler, which has been built in one form or another for nearly wo hundred years, is the least understood to-day of all the important structural lements of plants dealing with heat for power or other purposes, yet such is he fact. Burning coal under and passing the furnace gases over surfaces olding boiling water looks like the simplest sort of process, and if it made no ifference how much steam could be made from a square foot of surface or om a pound of coal it would be simple.

The difficulties in dealing with boiler processes are all concerned with the stablishment of some fundamentally sound relations for the capacity of eating surface to absorb heat in terms of the amount of heat developed by he burning of coal, oil or gas in the furnace, when the dimensions are known. fter all these years of experience, and in spite of much theorizing and thou- nds of recorded tests it is not possible to-day to calculate from any fundamental lations either the amount of steam that will be developed per hour with a ven fuel, or the weight of steam that will be produced per pound of fuel. Of ourse, from empiric relations this can be done, because nothing is simpler han a comparison of the boiler in question and its fuel, with exactly or nearly milar ones for which tests have been made, yet even here it is possible to ake blunders by forgetting differences in operating conditions such as avail-

able draft or methods of firing. It must be said, therefore, that there is no generally accepted fundamental theory of either steam boiler capacity or efficiency, but there are some relations between certain important factors, that are established, so a discussion of steam boilers must be almost entirely confined to these relations which cannot yet be grouped together to constitute a general theory of the subject. To illustrate, it is possible to explain why a given boiler gives more steam with one coal than with another, or more steam at .4 in. draft than with .3 in. draft with the same coal; why, when twice the coal is burned, less than twice the steam is made and even to account for and measure the losses that are responsible for the difference; also in some cases, why with the same coal and draft one boiler makes more steam than another, either in pounds per hour or in pounds of steam per pound of coal, but it is not possible to compare two different kinds of boilers as to type and size with different methods of firing different coals with different drafts, in such a way as will explain or permit of prediction of difference in results due to all influences acting together. To state it a little differently there is no *absolute* measure of boiler performance as to capacity or efficiency as a basis of comparison to measure the goodness of a boiler as a boiler; comparisons must, therefore, be between one and another boiler, or one and another service condition; one boiler may be said to be better than another, or one condition more favorable and another worse, for the results desired, but hardly more than this is possible.

In this section will be discussed the various factors that seem to be determining influences in fixing the capacity or as it is generally measured in terms of horse-power, the horse-power of boilers and what determines it. Boiler horse-power like gas-producer horse-power is scientifically a bad term, but custom has sanctioned it and it will always be used, probably with more and more precision as to definition in the future than has been the case in the past. The term appears in the literature of the subject first during the period when steam pressures were low, about 70 lbs. gage and when steam engines were not as economical as they are now nor as different in type and in steam consumption. At that time it was easier to discuss *the average steam consumption of engines* than now, and the number that departed from the average was not great; this average being about 30 lbs. per hour per horse-power. Accordingly a boiler was said to have 100 horse-power capacity when it could make steadily 3000 lbs. of steam per hour and thus was boiler horse-power defined. In time all sorts of variations appeared, better and worse engines were built, each best adapted to some sort of service, higher steam pressures and boiler-feed temperatures were also used, so that no longer did the making of 30 lbs. of steam per hour, take from the fire the same amount of heat as when boiler pressures were uniformly lower and feed temperatures more constant. Also engines were built that operated with a lower water rate, less than half the old rate, while others, chiefly small or cheap ones and direct-acting pumps, often used three or four times the old average water rate. In short, variations in both engine and boiler conditions made it absurd to talk of average water rates of steam engines, and introduced correspondingly large differences in the amount of heat necessary

or the production of a pound of steam, so that an adjustment of *boiler horse-power* definition became necessary. This was done in America by the American Society of Mechanical Engineers, adopting a double definition which was

(*a*) *The evaporation of* 34.5 *lbs. of water per hour from* and *at* 212° *F.*;
(*b*) *The absorption by the water between feed condition and that of the steam leaving the boiler, of* 33,305 *B.T.U. per hour per pound.*

This last heat definition was most fortunate as it is an absolute unit, and whether it has any relation to engine requirements or not, is a matter of no importance whatever. It was believed to be the equivalent of the weight definition and would be, if the latent heat of evaporation at 212° were $\frac{33{,}305}{34.5}=965.36$, and also equivalent to the evaporation of 30 lbs. from 100° F. feed water to steam at 70 lbs. gage. The language of the committee of the Society adopting the standard in 1885 is worth quoting: "Your committee, after due consideration, has determined to accept the Centennial Standard, and to recommend that in all standard trials (boiler tests) the commercial horse-power be taken as an evaporation of 30 lbs. of water per hour from a feed-water temperature of 100° F. into steam (dry saturated) at 70 lbs. gage pressure (above standard atmosphere) which shall be considered to be equal to 34½ units of evaporation; that is, to 34½ lbs. of water evaporated from a feed-water temperature of 212° F. into steam (dry saturated) at the same temperature. This standard is equal to 33,305 thermal units per hour." A later committee, 1899, retained the 34½ lbs. from and at, but changed the heat equivalent to 33,317 B.T.U. per hour, based on a latent heat of 965.7 so that $965.7\times34.5=33{,}317$ B.T.U.

Now that the latent heat at 212° F. is by recent research known to be higher than 965.7, and the use of superheat is quite general though not contemplated by that committee, it is necessary to once more reconcile the double definition, one part with the other or to adopt a new standard, though the old ones must be understood by anyone trying to interpret old published test data, based on them. Considering the history of the term, it is believed that the retention of the 34½ lbs. evaporation from and at 212° is desirable, and according to the generally accepted Marks and Davis Steam Tables the latent heat corresponding is 970.4, therefore, *the heat equivalent of a boiler horse-power is* $970.4\times34.5=33{,}478.8$ *B.T.U. per hour.* This unit is adopted in this work and is applicable as well to superheated, as to wet or dry saturated steam. Therefore,

$$\text{boiler horse-power}=\left\{\begin{array}{c}\text{Absorption by water and steam in the boiler}\\ \text{of 33,478.8 B.T.U. per hour}\end{array}\right\}\ldots\quad(849)$$

With the heat basis as a standard, the weight of water evaporated per hour per boiler horse-power will, of course, vary regularly with the initial water temperature and final steam condition, and to facilitate practical work a *factor*

of evaporation is calculated for reduction of weights according to the relation of Eq. (851) derived by the following relations illustrating its use.

$$\begin{pmatrix}\text{Lbs. water evaporated}\\ \text{per hour per B.H.P.}\end{pmatrix}\times\begin{pmatrix}\text{B.T.U. per lb. steam above feed}\\ \text{water temperature}\end{pmatrix}=34.5\times 970.4.$$

or

$$\left.\begin{aligned}\begin{pmatrix}\text{Lbs. water evaporated}\\ \text{per hour per B.H.P.}\end{pmatrix}&=\frac{34.5\times 970.4}{\text{B.T.U. per lb. of steam above feed temp.}} && (a)\\ &=\frac{34.5}{\dfrac{\text{B.T.U. per lb. of steam above feed temp.}}{970.4}} && (b)\\ &=\frac{34.5}{\text{Factor of evaporation}} && (c)\end{aligned}\right\}\quad(850)$$

Hence

$$\left.\begin{aligned}(\text{Factor of evaporation})&=\frac{\text{B.T.U. per lb. of steam above feed temp.}}{970.4} && (a)\\ &=\frac{\left\{\begin{matrix}\text{Total heat per lb. of}\\ \text{steam above 32° F.}\end{matrix}\right\}-\left\{\begin{matrix}\text{Heat of liquid at feed}\\ \text{temp. above 32° F.}\end{matrix}\right\}}{970.4} && (b)\end{aligned}\right\}\quad(851)$$

The values of the factor of evaporation and equivalent pounds of water per hour per boiler horse-power are to be found from the steam tables or directly from the curves, Fig. 202, which also give the heat per pound for dry saturated, wet or superheated steam above any feed-water temperature by the following simple operations. Each of the upper curves gives directly the total heat per pound of steam above 32° and the distance between them and the lower curve intercept, that for any feed-water temperature, by a vertical distance. If, therefore, AB be the total heat for the steam above 32° at 100 lbs. per sq.in. absolute and 20° superheat and DE the heat of liquid at 200° F. feed temperature above 32°, then AC the vertical distance between these two points is the heat per pound of steam above the feed temperature 200° F. for 100 lbs. steam with 20° superheat. This can be marked on a slip of paper and read off on the extra scale to the right in terms of, heat in B.T.U., or factor of evaporation, or actual weight of water that must be evaporated per hour to give a boiler horse-power.

A sort of commercial rating of boilers has grown up as a convenience to purchase and sale, based solely on the surface of tubes and plates exposed to the heat, or so-called *heating surface*, and the figure used is either 10 or 12 sq.ft. for common forms, but varies from 8 to 16 for others. This is the result also of old average evaporations per square foot of surface for stationary boilers and suggests the dependence of capacity on the heat-absorbing power of the surface *which in such ratings is assumed to be constant.* To show what is the evaporation in pounds per hour per square foot of surface when 10 or 12 sq.ft. are allowed

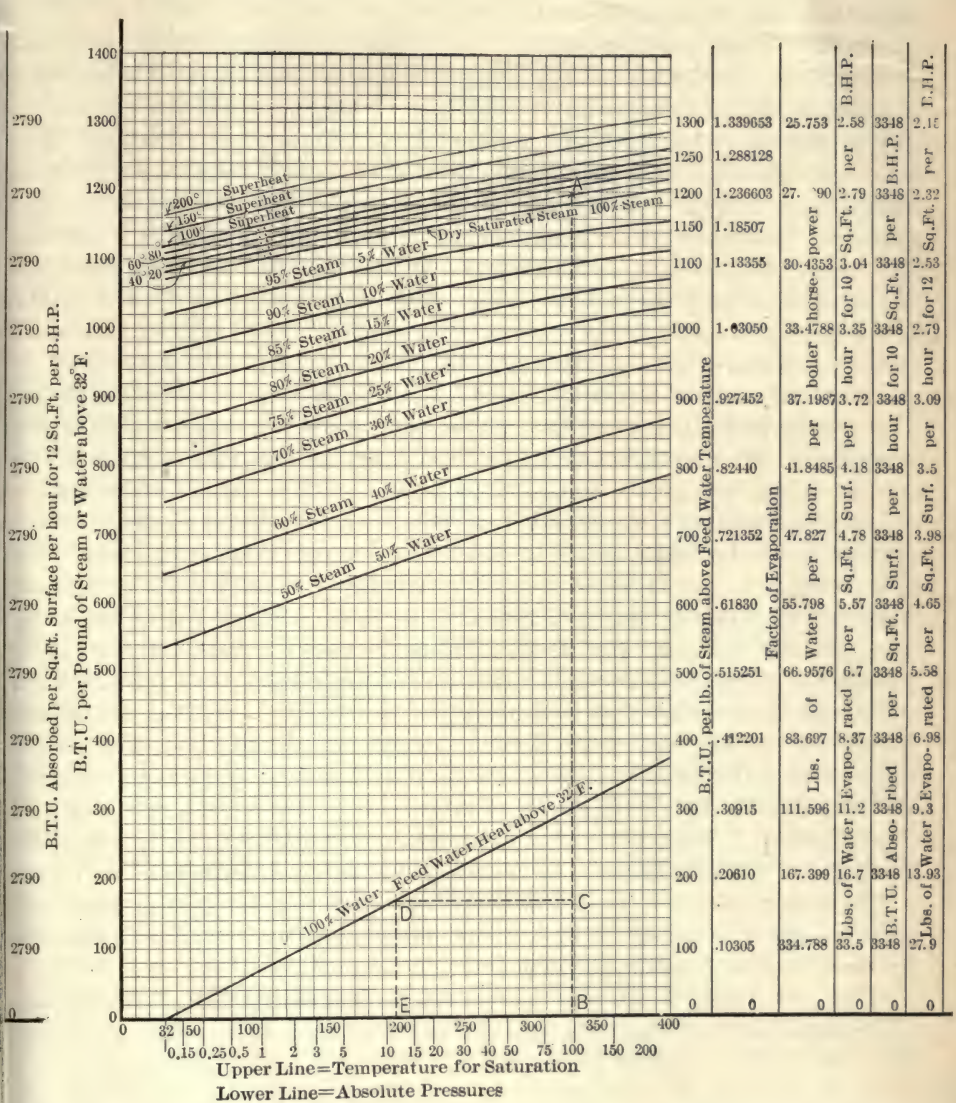


FIG. 202.—Heat per Pound of Steam above Feed Temperature. Evaporation per Hour per Boiler Horse-power. Factor of Evaporation.

per horse-power, with the corresponding rate of heat absorption in B.T.U. per hour per square foot, some extra scales are added to the right of Fig. 202 showing all relations at a glance.

The figures on evaporation per square foot of heating surface per hour, or better still, the heat absorbed per square foot of heating surface per hour, is a proper basis for comparing two sets of data for different boilers at equivalent capacities. Thus different boilers may be said to be operating at the same capacity when their heating surface rates of heat absorption are the same though one may be developing 50 and the other 1000 boiler horse-power actually. The rate of heat absorption by boiler heating surface is a quantity that must be dependent on two factors or terms, one an absolute figure and the other expressing the rate of change with generation. It is clear that as heat is generated faster it must certainly be absorbed faster unless the capacity of the water side of the heating surface for heat is lessened at the same time and this is not the case with ordinary boilers, but not beyond the bounds of possibility.

The experimental data on the transfer of heat showed conclusively that the capacity of water in tubes or tanks to absorb heat very much exceeded the capacity of gases to give it to the surface under a wide range of conditions, so it is not unreasonable to suppose that as gases bring more and more heat to water surfaces, it will be taken up by them at a rate which is some function of the rate of supply. If, therefore, by faster fuel combustion there is a regularly increasing quantity of heat available for the water in the form of hot gases, then more will be absorbed per hour, more steam made per hour, and more boiler horse-power developed per square foot of heating surface, and for the whole boiler, and these things must be functions of the rates of combustion and heat generation. In two different boilers the rate of increase of absorption may or may not bear the same relation to the rate of increase in generation, but it is quite possible for them to be similar in this respect,but at a given rate of generation in each it is more likely that the actual rates of absorption should differ by reason of the differences in structure. This latter difference should require a different surface per horse-power for the same amount of heat generated, even though a doubling of the rate of heat development produced the same *fractional increase* in horse-power of both.

Heat generation is, of course, essential to absorption, but it is necessary to distinguish between apparent and real generation in seeking relations between absorption and generation in accordance with the following definitions.

$$\begin{pmatrix}\text{Apparent heat generation in}\\ \text{boiler fires, B.T.U. per hr.}\end{pmatrix} = \begin{pmatrix}\text{Lbs. fuel supplied}\\ \text{per hr.}\end{pmatrix} \times \begin{pmatrix}\text{B.T.U. per lb. of}\\ \text{fuel as fired}\end{pmatrix} \quad (852)$$

$$\begin{pmatrix}\text{Real heat generation in boil-}\\ \text{er fires B.T.U. per hr.}\end{pmatrix} = \begin{pmatrix}\text{Lbs. fuel supplied}\\ \text{per hr.}\end{pmatrix} \times \begin{pmatrix}\text{B.T.U. per lb. of}\\ \text{fuel as fired}\end{pmatrix} \times \left[1 - \begin{pmatrix}\text{Fraction lost}\\ \text{in furnace}\end{pmatrix}\right]. \quad (853)$$

The fraction of heat lost in furnaces is that part of the calorific power represented by unburnt fuel in ashes, in cinders, in soot discharged through the flues, by unburned gases, carbon monoxide, hydrogen and hydrocarbons, by evaporation of moisture in the fuel and that conducted away from the *hot gases* by the furnace walls and settings between the point of firing and the place where absorption begins. This fraction is always appreciable and often large though very difficult to exactly measure, 20 per cent being not uncommon, in which case only 80 per cent of the calorific possibilities of the coal are available at all for absorption. These losses are mentioned here because of their importance but will be more fully discussed in the next section with others which together make the whole boiler efficiency less than 100 per cent.

With some fraction of the fuel heat available for absorption in the form of hot gases, it may be absorbed in one of two characteristic ways, first as radiant heat from the glowing coal, brickwork, or incandescent floating carbon particles in hydrocarbon flames, and second by actual contact of gases with surface. It is convenient and usual to characterize these as fire-box and tube absorption, respectively, although some tubes may get radiant heat as well as conducted heat, and all internal fire-boxes get some gas contact heat. Now, the rate at which fire-boxes, actual or equivalent, absorb heat, being governed by the laws of radiation, is proportional to the fourth power of the temperature difference, according to the Stefan and Bolzmann law, so it would seem to be quite independent of the rate of generation or rate of fuel combustion except as this may affect the temperature and extent of the radiant matter. This independence of radiant heat absorption with respect to rate of combustion or the constancy of radiant heat for all good fires seems to be really established. On the other hand the absorption by tubes or plates, of heat from hot gases passing through or around them, is governed by the laws of heat transfer from hot gases to water whatever they may be. It is certain that higher mean temperatures of the gases should increase the rate of absorption per hour, and it is also certain that any influences that make the dead gas film thinner, or promote contact between *fresh* hot gases and the tube itself, will increase the rate of absorption. One such influence on reduction of dead the film is the velocity of the gases, which is roughly proportional to the amount of air supply to the furnace, or the rate of combustion. *Whatever these laws are, they should be the same in kind for all boilers, but comparing any two, certain factors affecting the rate of absorption are fixed by the construction and may be very different in the two cases.* For example, the disposition of the heating surface may be such as to offer a long, narrow passage-way for the gases between the surfaces, or a short, wide one, or a narrow short one, or a wide long one. If the gases passed the water surface in infinitely thin streams, it seems likely that they would give up their heat almost instantly, so the length of such a passage would not make much difference. On the other hand if the passage were wide, the gases at the center of the stream might have to wait a long time to come into surface contact and might never do so, in which case they must give up their heat through surrounding gas layers which would take a long time and require long tubes.

It appears, therefore, as the disposition of the heating surface, the proportions of the gas passages between the absorbing surface, the relations of fire-box to flue surface and such matters as are fixed by construction, may vary through almost infinite ranges, that, however well known might be the fundamental laws of radiant heat flow and of transfer by conduction and convection, the rate of heat absorption in boilers would resist any definite generalization though offering a most attractive field for theorizing. This makes it seem all the more remarkable that experimental results should show such a consistency as is demonstrated below, even though the constants in the relation differ because of the *structural* influences noted above.

In selecting boiler data for discussion and analysis, one cannot be too careful in avoiding a most natural but improper and unscientific tendency, notable in boiler literature, which is, to pick out tests that seem to prove a preconceived theory, selecting the evidence to prove the case. The tests used here are believed to be free from improper manipulation, and to be fairly representative, though to be quite sure it would be necessary to analyze every authentic test in existence, which is beyond the scope of such a work as this.

The locomotive boiler is typical of internal fire-box construction and of operation with greatest "forcing," that is, highest rates of combustion and highest rates of evaporation per square foot of heating surface, while the ordinary stationary water-tube boiler is typical of much slower rates of operating conditions and of practically all tube construction. Accordingly, the first two sets of data selected for study are the report of Goss (Bulletin 402, U.S. G.S.) on 18 tests of an American locomotive boiler at the Purdue testing plant, with experienced and skillful experimenters, using two different, but both good quality coals, under four different boiler pressures and with different rates of combustion for each, all high. The boiler was of the extended wagon-top type, fire-box $72\frac{1}{16}$ ins. long, $34\frac{1}{2}$ ins. wide, 79 ins. deep, with fire-tubes 2 ins. in diameter and 16 ft. 5 ins. long, making the total heating surface of the boiler proper 1023 sq.ft. It was also equipped with a Cole return-tube superheater, $1\frac{1}{4}$ ins. outside diameter, 32 loops each 17.27 ft. long, adding 193 sq.ft. of superheating surface to 1023 sq.ft. of boiler surface, making the total heat absorbing surface 1216 sq.ft. The nature of the two different coals used is indicated by the proximate analysis and calorific power, one with twice the volatile of the other.

COALS USED IN GOSS LOCOMOTIVE BOILER TEST

Coal	Moisture, Per Cent.	Volatile, Per Cent.	Fixed C, Per Cent.	Ash, Per Cent.	B.T.U. per Pound.	
					Coal Dry.	Combustible
A.....	1.89	31.94	57.71	8.46	14047	15372
B.....	3.10	15.23	72.75	8.92	14347	15802

The first characteristic to be examined is the heat-absorbing capacity of the heating surface with respect to the heat developed in the furnace, which is plotted in Fig. 203, to coordinates representing horizontally, the rate heat developed and available for absorption per square foot of heating surface, and vertically the rate of heat absorption by heating surface both in B.T.U. per hour.

The experimental points all lie on straight lines whether the water surface be considered alone, the superheater surface alone, or the entire heating surface as one, within what may be regarded as the limit of experimental error. Taking the line representing the whole surface as characteristic of the whole heat-absorbing process or the boiler itself as a heat absorber, it is represented algebraically by Eq. (854) below.

$$\left.\begin{aligned} \left\{\begin{array}{l}\text{B.T.U. absorbed}\\ \text{per hour per sq.ft.}\\ \text{H.S.}\end{array}\right\} &= 1450 + .66 \left\{\begin{array}{l}\text{B.T.U. developed in fire and}\\ \text{available for absorption}\\ \text{per hour per sq.ft. H.S.}\end{array}\right\} \quad (a)\\ \left\{\begin{array}{l}\text{Boiler horse-power}\\ \text{per sq.ft. H.S.}\end{array}\right\} &= .0433 + .0000197 \left\{\begin{array}{l}\text{B.T.U. developed in}\\ \text{fire and available for}\\ \text{absorption per hour}\\ \text{per sq.ft. H.S.}\end{array}\right\} \quad (b) \end{aligned}\right\} \quad (854)$$

Similar results are found for a very different type of boiler, the Heine water tube, by the tests of the U. S. Geological Survey, though it was operated at very much lower rates, and on a great variety of coals ranging from low grade lignite to good bituminous. Over two hundred tests are represented in the data of Fig. 204, which is an almost perfect straight line within the limits of experimental procedure, and it is worth noting that the different coals are well distributed over the curve, showing how independent is the heating surface performance, of the sort of fuel used. The boiler had 2031 sq.ft. of heating and 40.55 sq.ft. of grate surface.

This line is represented by Eq. (855).

$$\left.\begin{aligned} \left\{\begin{array}{l}\text{B.T.U. absorbed}\\ \text{per hour per sq.ft.}\\ \text{H.S.}\end{array}\right\} &= 615 + .511 \left\{\begin{array}{l}\text{B.T.U. developed in fire and}\\ \text{available for absorption per}\\ \text{hour per sq.ft. H.S.}\end{array}\right\} \quad (a)\\ \left\{\begin{array}{l}\text{Boiler horse-power}\\ \text{per sq.ft. H.S.}\end{array}\right\} &= .0185 + .0000152 \left\{\begin{array}{l}\text{B.T.U. developed in}\\ \text{fire and available for}\\ \text{absorption per hour}\\ \text{per sq.ft. H.S.}\end{array}\right\} \quad (b) \end{aligned}\right\} \quad (855)$$

It thus appears, that from two totally different boilers, driven at very different rates, 4000 B.T.U. per sq.ft. heating surface maximum, absorbed in the Heine water-tube and nearly 15,000 maximum in the locomotive fire-tube boiler or about 4 to 1, and with all sorts of coal in the former, some about the same as in the latter, that the *heat absorbed varies by a straight-line law with the amount*

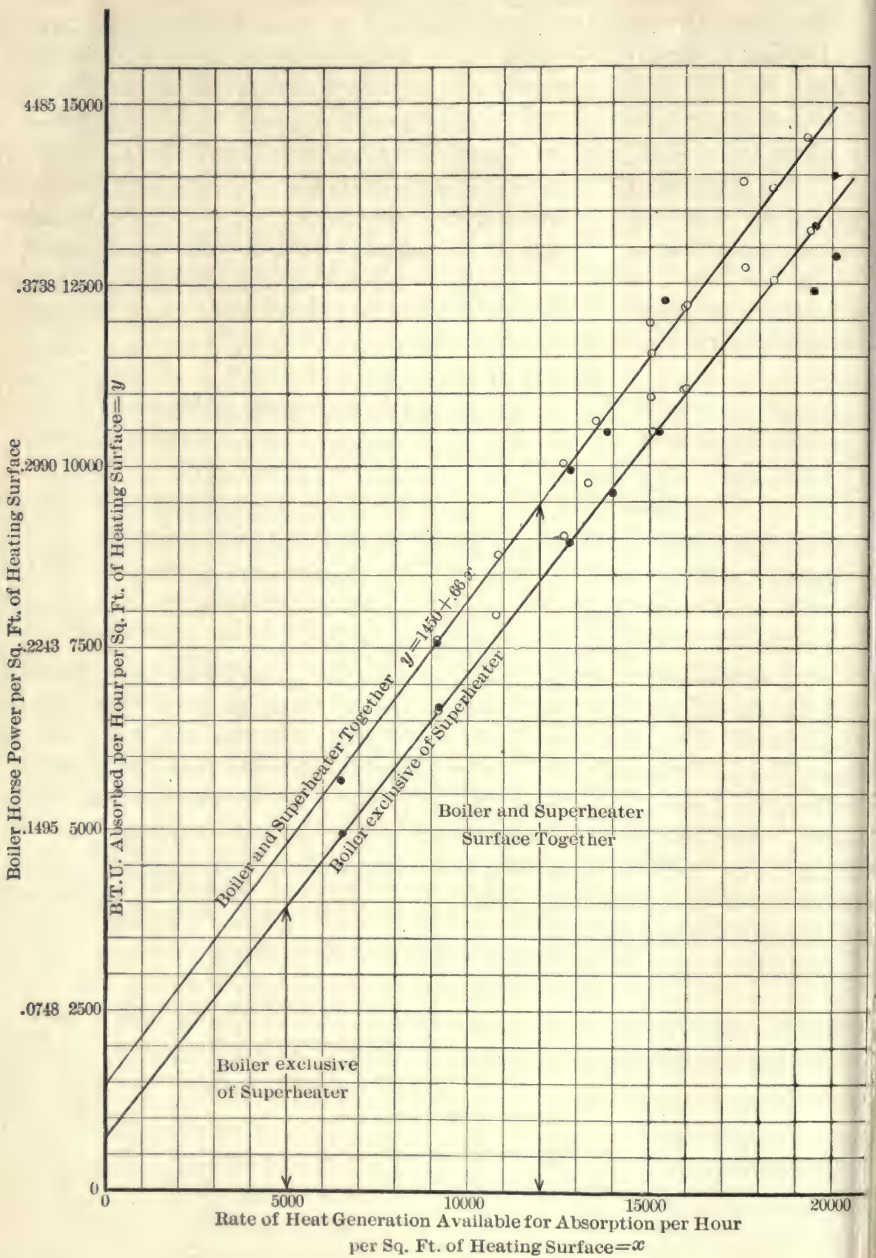


FIG. 203.—Relation between Rate of Heat Absorption oo B.H.P. and Rate of Heat Genera n Available for Absorption in a Locomotive Boiler.

developed, for both. Furthermore, the slope is nearly the same for both, though the constant for the water tube is about half that of the locomotive fire tube, which means that the *rate* of increase of absorption with increase of development or evolution of heat is nearly the same for both.

Extending the investigation a little further, more confirmatory results are obtained and for this three boilers are selected, the first representing the largest and the second the smallest for which authentic tests are available, while the third gives data for oil fuel to further check the influence of kind of fuel.

The largest boiler is that of the Detroit Edison Co., tests for which are reported by Jacobus, having a heating surface of 23,650 sq.ft. and of the curved tube Stirling type, fired with coal mechanically, on both Roney and Taylor

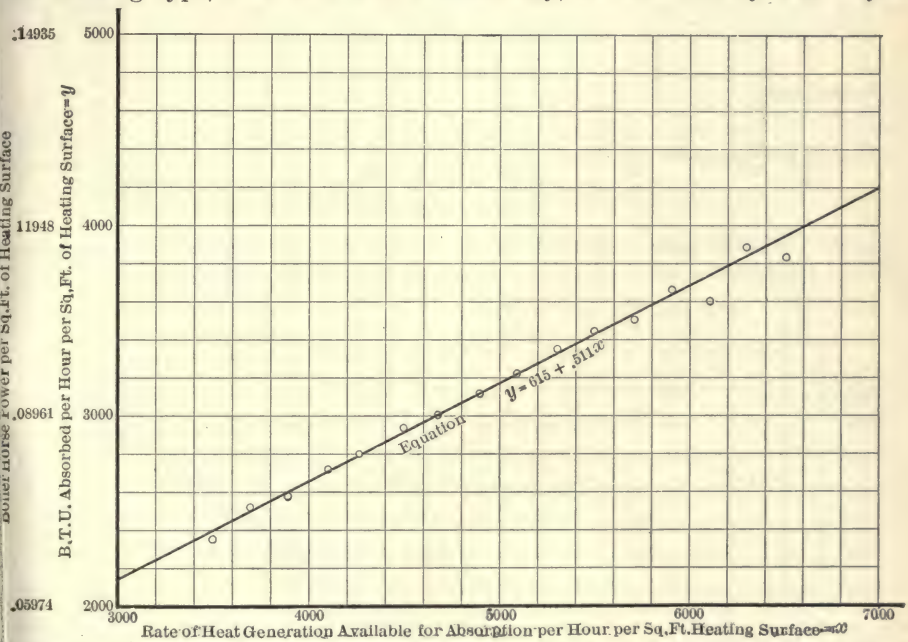


FIG. 204.—Relation between Rate of Heat Absorption or B.H.P and Rate of Heat Generation Available for Absorption for Heine Water Tube Boiler.

tokers. To the same coordinates as were used for the other cases the test ata are plotted in Fig. 205 and no better straight-line relation is possible. This ne is represented by the linear relation Eq. (856).

$$\left.\begin{aligned} &\left.\begin{matrix}\text{B.T.U. absorbed}\\ \text{per hour per sq.ft.}\\ \text{H.S.}\end{matrix}\right\} = 235.3 + .8235 \left\{\begin{matrix}\text{B.T.U. developed in fire}\\ \text{and available for absorp-}\\ \text{tion per hour per sq.ft.}\\ \text{H.S.}\end{matrix}\right\} \quad (a)\\ &\left.\begin{matrix}\text{Boiler horse-power}\\ \text{per sq.ft. H.S.}\end{matrix}\right\} = .0703 + .0000245 \left\{\begin{matrix}\text{B.T.U. developed in}\\ \text{fire and available for}\\ \text{absorption per hour}\\ \text{per sq.ft. H.S.}\end{matrix}\right\} \quad (b)\end{aligned}\right\} \quad (856)$$

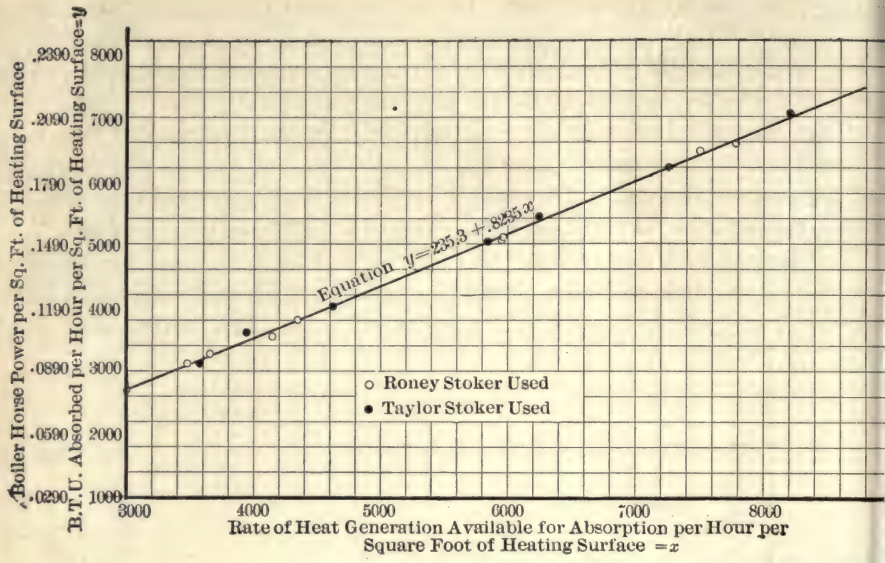


FIG. 205.—Relation between Rate of Heat Absorption or B.H.P. and Rate of Heat Generati Available for Absorption for 23650 sq.ft. Stirling Boiler. Detroit Edison Co.

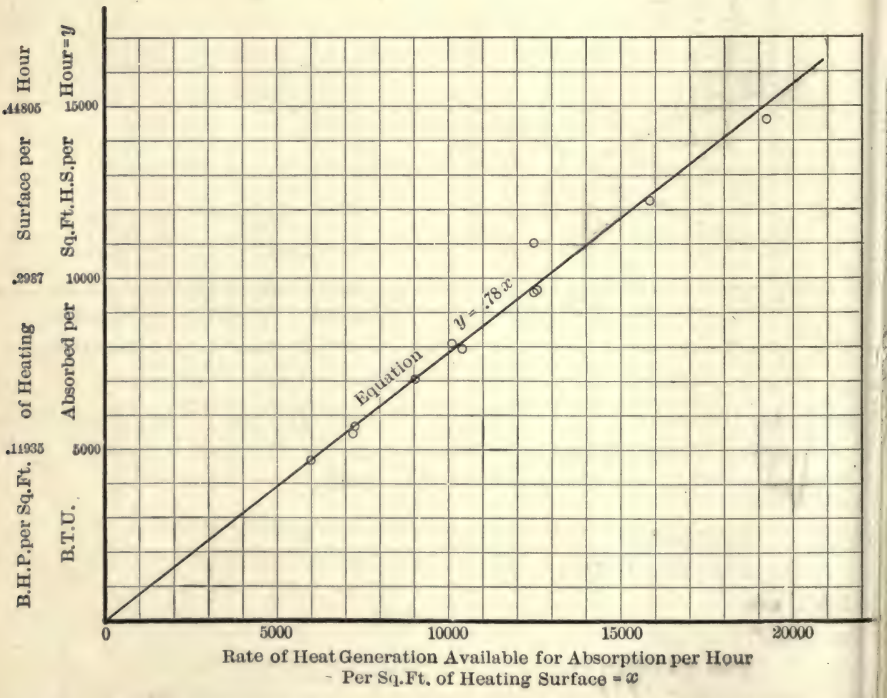


FIG. 206.—Relation between Rate of Heat Absorption or B.H.P. and Rate of Heat Genera Available for Absorption for White Steam Car Boiler.

The smallest boiler is represented by one used on the White steam automobile and for which tests are reported by Carpenter. This is of the "flash" type, gasolene fuel and carrying 45.8 sq.ft. of heating surface, the steam being of high pressure 200–500 lbs. gage and high superheat 400° F. These results are plotted in Fig. 206 and the resulting straight line is represented by Eq. (857).

$$\left.\begin{array}{l} \left\{\begin{array}{l}\text{B.T.U absorbed}\\ \text{per hour per sq.ft.}\\ \text{H.S.}\end{array}\right\} = .78 \left\{\begin{array}{l}\text{B.T.U. developed in fire and avail-}\\ \text{able for absorption per hour per}\\ \text{sq.ft. H.S.}\end{array}\right\} \quad (a) \\ \left\{\begin{array}{l}\text{Boiler horse-power}\\ \text{per sq.ft. H.S.}\end{array}\right\} = .000026 \left\{\begin{array}{l}\text{B.T.U. developed in fire and}\\ \text{available for absorption per}\\ \text{hour per sq.ft. H.S.}\end{array}\right\} \quad (b) \end{array}\right\} \quad (857)$$

To show how independent of the fuel are these relations, the tests of a Hohenstein water-tube boiler fired with liquid fuel under the direction of a special

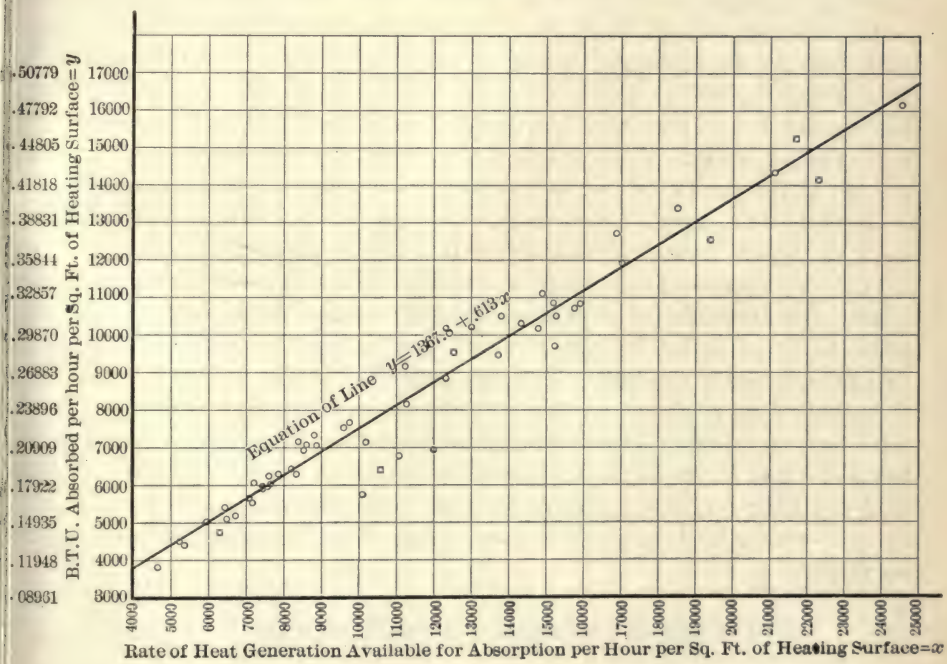


FIG. 207.—Relation between Rate of Heat Absorption or B.H.P. and Rate of Heat Generation Available for Absorption for Hohenstein Oil Fired Boiler.

board representing the U. S. Navy, are plotted in Fig. 207. This boiler had 2130 sq.ft. of heating surface and was worked under quite a wide range of conditions, nevertheless a straight-line relation maintains as given by Eq. (858).

$$\left\{\begin{array}{l}\text{B.T.U. absorbed}\\ \text{per hour per sq.ft.}\\ \text{H.S.}\end{array}\right\} = 1367.8 + .613 \left\{\begin{array}{l}\text{B.T.U. developed in}\\ \text{fire and available for}\\ \text{absorption per hour}\\ \text{per sq.ft. H.S.}\end{array}\right\} \quad (a)$$

$$\left\{\begin{array}{l}\text{Boiler horse-power}\\ \text{per sq.ft. H.S.}\end{array}\right\} = .0408 + .0000183 \left\{\begin{array}{l}\text{B.T.U. developed in}\\ \text{fire and available for}\\ \text{absorption per hour}\\ \text{per sq.ft. H.S.}\end{array}\right\} \quad (b) \qquad (85$$

There appears from the above examination of good data, to be no doul of the existance of a linear relation between heat absorbed and heat availab for absorption in boilers, regardless of boiler structure, of kind of fuel or ra of forcing. The only difference is to be found in the two constants and f this there is adequate explanation. When rate of heat development or evoluti becomes zero there appears to be some absorption in all but the White aut mobile boiler, because the line does not pass through zero, but cuts the coo dinate of absorption rate at a finite value. This value then must be a measu of the constant rate of absorption due to *radiant heat*, whether any hot gas carry heat to the absorbing surface or not. It is natural that this should diff for different boilers as it is a structural feature, depending on the exposure heating surface to heat rays from both incandescent fuel and *brickwork*. T rate at which the rate of absorption increases with the rate of evolution of heat a result of the proportions of the gas passages through the heating surface tub or around them, again a structural factor for which existing data are insuf cient to permit a numerical valuation in terms of dimensions alone.

It is often impossible to secure knowledge of furnace losses giving the amou of heat not available for absorption but still supplied in the fuel, so it is wor while to examine the relations between heat absorption and heat supplied fuel or *apparently available*. This heat supplied in the fuel is the product the weight fired per hour, and the calorific power of the fuel as fired. The fu nace loss, so called, is the difference between this product and that availal for absorption in the form of hot gases approaching the heating surface. Alg braically these relations are given by Eqs. (852) and (853).

Accordingly if the furnace loss as a fraction of the heat supplied be represent by L, then,

$$\text{Fraction of combustion heat really generated} = 1 - L, \quad . \quad . \quad (8$$

Whence

$$\left\{\begin{array}{l}\text{B.T.U. developed in}\\ \text{in fire and avail-}\\ \text{able for absorption}\\ \text{per hr. per sq.ft.}\\ \text{H.S.}\end{array}\right\} = (1-L) \times \left\{\begin{array}{l}\text{Lbs. of fuel sup-}\\ \text{plied per hour}\\ \text{per sq.ft. H.S.}\end{array}\right\} \times \left\{\begin{array}{l}\text{B.T.U. per}\\ \text{lb. of fuel}\\ \text{as fired}\end{array}\right\} \quad (8$$

Substituting this in the general linear equation between rate of absorption and rate of true generation, Eq. (861), gives the following relation between rate of absorption and rate of apparent generation, Eq. (862).

$$\left\{\begin{array}{l}\text{B.T.U. absorbed per}\\ \text{hour per sq.ft. H.S.}\end{array}\right\} = a + b \left\{\begin{array}{l}\text{B.T.U. developed in fire and avail-}\\ \text{able for absorption per hour per}\\ \text{sq.ft. H.S.}\end{array}\right\} \quad (861)$$

$$= a + b(1-L) \left\{\begin{array}{l}\text{Lbs. of fuel sup-}\\ \text{plied per hour}\\ \text{per sq.ft. H.S.}\end{array}\right\} \times \left\{\begin{array}{l}\text{B.T.U.}\\ \text{per lb.}\\ \text{of fuel}\\ \text{as fired}\end{array}\right\} \quad (862)$$

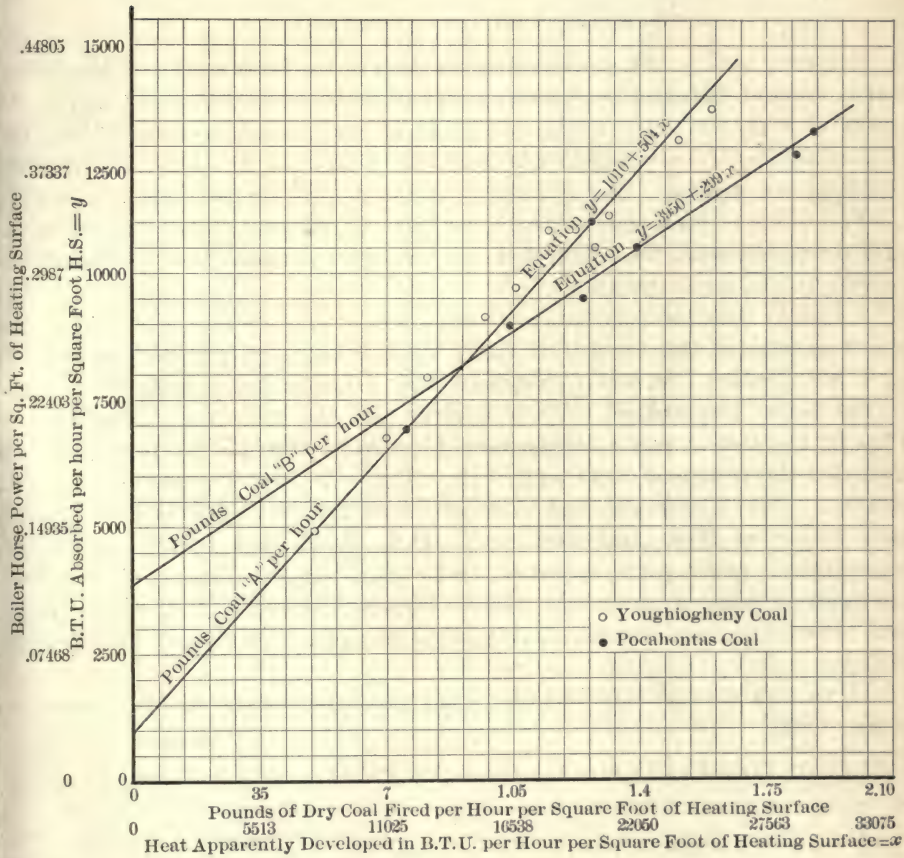


FIG. 208.—Relation between Rate of Heat Absorption or B.H.P., and Rate of Combustion or Apparent Heat Generation for a Locomotive Boiler.

If now L is a constant, then b being also a constant, $b(1-L)$ is likewise a constant, so that, this relation would also be linear and experimental determinations should be on a straight line. Should it happen in any case that a curve results, then *this is in itself a proof of a variable fractional furnace heat loss.* As a

general rule a straight line does result, so that constant fractional furnace heat losses are rather the rule than the exception. Figs. 208 and 209 demonstrate this clearly. It should be noted that as the ratio of heating surface to grate surface is fixed in any given boiler, the weight of coal per hour per square foot of H.S. is directly proportional to the rate of combustion. For a fuel of constant calorific power the last term of Eq. (862) will be constant, and a linear relation is established between rate of absorption or boiler horse-power and rate of combustion, a most valuable relation and true if,

(*a*) The furnace losses are a constant fraction of the heat in the fuel as supplied;

(*b*) The calorific power of the fuel is constant.

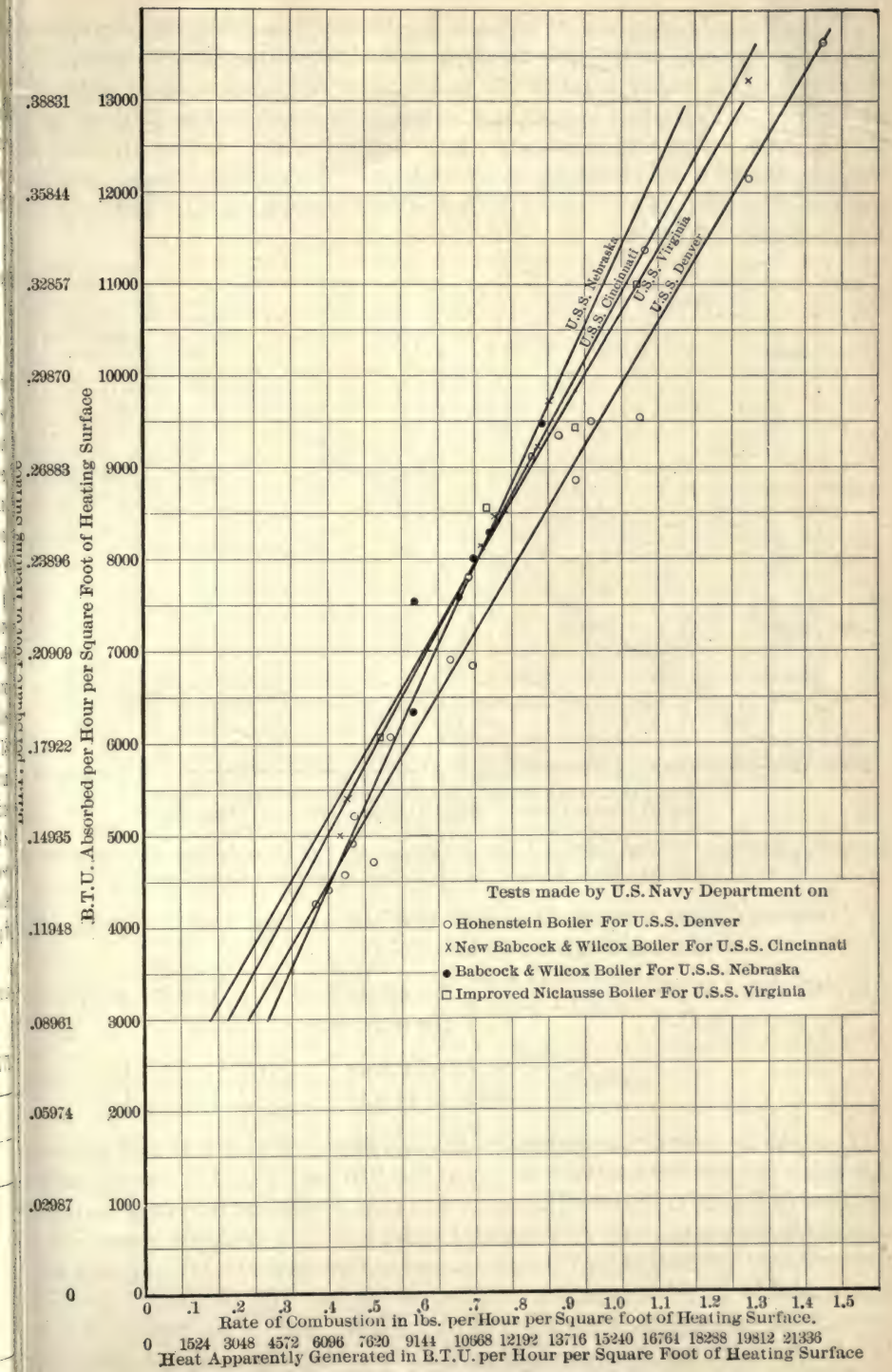
Referring to Fig. 208 representing the data of the Goss tests of a locomotive boiler, vertical distances representing rate of heat absorption and boiler horse-power to a double scale, horizontals, rate of combustion and equivalent rate of heat apparently developed, it is clear that a straight line is a fair representation of the results. The difference in the slopes for the lines representing the two different fuels shows clearly the effect of different furnace losses in one case compared with the other, but in each the fractional loss is constant as proved by the linear relation.

A similar conclusion is indicated by the four lines of Fig. 209, each representing a series of tests on a U. S. Navy water-tube boiler intended respectively for U.S.S. *Denver*, *Cincinnati*, *Nebraska* and *Virginia* taken from the record of the Navy Department. These boilers had the following characteristics: That of the *Denver* was a Hohenstein, 2174 sq.ft. H.S. for six tests and 213 sq.ft. for the rest, and 50.14 sq.ft. G.S. The coal used was Pocahontas run-of-mine. The *Cincinnati* and *Nebraska* had Babcock and Wilcox marine form boilers of 2640 and 4682 sq.ft. H.S., 63.25 and 111.72 sq.ft. G.S. respectively, and were operated under both closed ash-pit and close fire-room systems of draft with picked Pocahontas coal. A Niclausse boiler was tested for the *Virginia*, having 1852.56 sq.ft. of H.S. and 45.225 sq.ft. of G.S.

These straight-line relations give the following values for the constants of Eq. (862).

VALUES OF CONSTANTS FOR LOCOMOTIVE AND MARINE BOILER CAPACITY

Boiler.	Value of Constants, in Eq. (862).	
	a.	$b(1-L)$.
B. & W., U.S.S., *Cincinnati*	1350	.606
B. & W., U.S.S. *Nebraska*	50	.722
Niclausse, U.S.S. *Virginia*	1760	.566
Hohenstein, U.S.S. *Denver*	1090	.56
Locomotive, coal A	1010	.504
Locomotive, coal B	3950	.299


.38831
.35844
.32857
.29870
.26883
.23896
.20909
.17922
.14935
.11948
.08961
.05974
.02987
0
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
B.T.U. Absorbed per Hour per Square Foot of Heating Surface
U.S.S. Nebraska
U.S.S. Cincinnati
U.S.S. Virginia
U.S.S. Denver
Tests made by U.S. Navy Department on
○ Hohenstein Boiler For U.S.S. Denver
x New Babcock & Wilcox Boiler For U.S.S. Cincinnati
● Babcock & Wilcox Boiler For U.S.S. Nebraska
□ Improved Niclausse Boiler For U.S.S. Virginia
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5
Rate of Combustion in lbs. per Hour per Square foot of Heating Surface.
0 1524 3048 4572 6096 7620 9144 10668 12192 13716 15240 16764 18288 19812 21336
Heat Apparently Generated in B.T.U. per Hour per Square Foot of Heating Surface

It has been pointed out that when the furnace losses are not a constant fraction of the heat supplied in the form of fuel, then the curve between rate of absorption or boiler horse-power and rate of combustion or of apparent heat generation is really a curved, not a straight line, and in Fig. 210 one such case is given. This case represents a large number of tests on the Hohenstein boiler by the U. S. Navy testing board, with oil fuel for which a linear relation was found to hold between absorption and true generation rates allowing for furnace losses (Fig. 207).

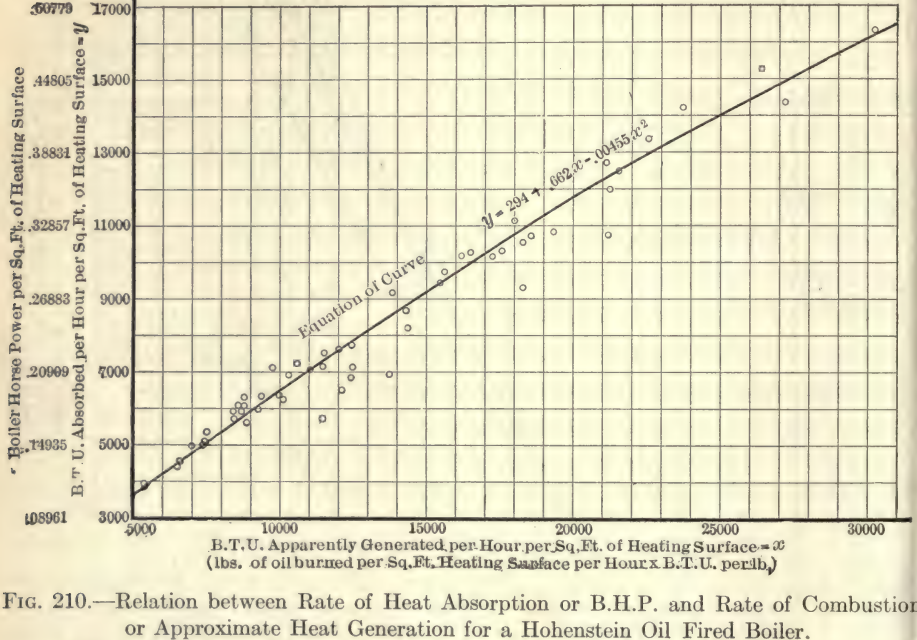


FIG. 210.—Relation between Rate of Heat Absorption or B.H.P. and Rate of Combustion or Approximate Heat Generation for a Hohenstein Oil Fired Boiler.

A second-degree equation fits fairly well the average curve through the experimental results and is given by Eq. (863):

$$\left\{\begin{matrix}\text{B.T.U. absorbed per}\\ \text{hour per sq.ft. H.S.}\end{matrix}\right\} = 294 + .662\left[\left\{\begin{matrix}\text{Lbs. oil per hour}\\ \text{per sq.ft. H.S.}\end{matrix}\right\} \times \left\{\begin{matrix}\text{B.T.U. per}\\ \text{lb. oil}\end{matrix}\right\}\right]$$

$$-.00455\left[\left\{\begin{matrix}\text{Lbs. oil per hour}\\ \text{per sq.ft. H.S.}\end{matrix}\right\} \times \left\{\begin{matrix}\text{B.T.U. per}\\ \text{lb. oil}\end{matrix}\right\}\right]^2. \quad (863)$$

It is the last term that represents the increasing furnace loss and to show this more clearly the relations of both Fig. 210 and Fig. 207 are replotted together in Fig. 211. Vertical distances as before, represent rate of absorption or boiler horse-power, but two horizontal scales are used, the lower representing apparent heat generation for the lower curve, and the upper scale real generation and available for absorption, for the upper curve. The horizontal distance between the two curves when referred to the lower scale is the furnace loss in B.T.U. per hour per sq.ft. of H.S. at the time when the real and apparent

ates of generation have the values represented by the points of intersection. Thus, when the real rate of generation of heat available for absorption is 15,000 B.T.U. per hour, point D, the furnace loss is represented by AB, and the apparent generation rate is 17,650 B.T.U. per hour, point C.

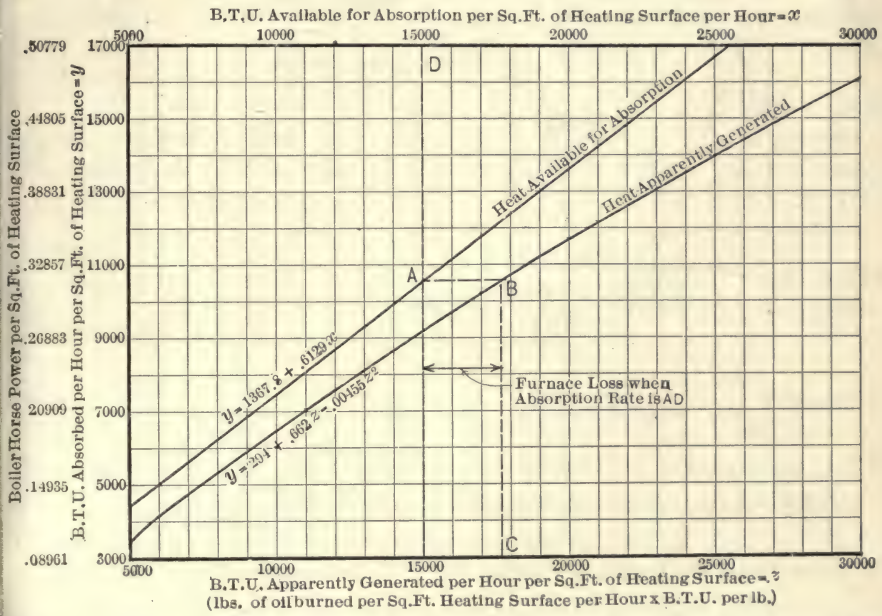


FIG. 211.—Relation between Heat Absorbed, Apparently Generated, Available and Heat Lost in Furnace.

The general fundamental equations (864 a,b,c,) may be changed as to form to others following containing different but related variables, and in some one these different forms they apply to existing test data.

$$\left.\begin{array}{l}\left.\begin{array}{l}\text{B.T.U. absorbed per hour}\\ \text{per square foot of heat-}\\ \text{ing surface}\end{array}\right\} = a + b\left\{\begin{array}{l}\text{B.T.U. developed in fur-}\\ \text{nace and available for ab-}\\ \text{sorption per hour per sq.ft.}\\ \text{of heating surface}\end{array}\right\} \quad (a)\\[2ex] \left.\begin{array}{l}\text{Lbs. evaporation from}\\ \text{and at 212° F. per}\\ \text{square foot of H.S. per}\\ \text{hour}\end{array}\right\} = \frac{a}{970.4} + \frac{b}{970.4}\left\{\begin{array}{l}\text{B.T.U. developed}\\ \text{in furnace and}\\ \text{available for ab-}\\ \text{sorption per hr.}\\ \text{per sq.ft. heating}\\ \text{surface}\end{array}\right\} \quad (b)\\[2ex] \left.\begin{array}{l}\text{Boiler horse-power per}\\ \text{square foot of heating}\\ \text{surface}\end{array}\right\} = \frac{a}{33479} + \frac{b}{33479}\left\{\begin{array}{l}\text{B.T.U. developed}\\ \text{in furnace and}\\ \text{available for ab-}\\ \text{sorption per hr.}\\ \text{per sq.ft. of heat-}\\ \text{ing surface}\end{array}\right\} \quad (c)\end{array}\right\} \quad (864)$$

Multiplying throughout by the heating surface in square feet gives thre more formulas Eq. (865):

$$\left.\begin{aligned}
&\left\{\begin{matrix}\text{B.T.U. absorbed per hour}\end{matrix}\right\} = a\left\{\begin{matrix}\text{Sq.ft. of heating surface}\end{matrix}\right\} + b\left\{\begin{matrix}\text{B.T.U. developed in furnace and available for absorption per hr.}\end{matrix}\right\} && (a)\\
&\left\{\begin{matrix}\text{Lbs. evaporated per hr. from and at 212° F.}\end{matrix}\right\} = \frac{a}{970.4}\left\{\begin{matrix}\text{Sq.ft. of heating surface}\end{matrix}\right\} + \frac{b}{970.4}\left\{\begin{matrix}\text{B.T.U. developed in furnace and available for absorption per hr.}\end{matrix}\right\} && (b)\\
&\left\{\begin{matrix}\text{Boiler horsepower}\end{matrix}\right\} = \frac{a}{33479}\left\{\begin{matrix}\text{Sq.ft. of heating surface}\end{matrix}\right\} + \frac{b}{33479}\left\{\begin{matrix}\text{B.T.U. developed in furnace and available for absorption per hr.}\end{matrix}\right\} && (c)
\end{aligned}\right\} \quad (865$$

Putting

$$\left\{\begin{matrix}\text{B.T.U. generated per hour}\end{matrix}\right\} = \left\{\begin{matrix}\text{Lbs. fuel per hour}\end{matrix}\right\} \times \left[\left(\begin{matrix}\text{B.T.U. per lb. fuel}\end{matrix}\right) - \left(\begin{matrix}\text{B.T.U. furnace loss per lb. fuel}\end{matrix}\right)\right]$$

three more formulas result, involving the fuel consumption rate, Eq. (866):

$$\left.\begin{aligned}
&\left\{\begin{matrix}\text{B.T.U. absorbed per hour}\end{matrix}\right\} = a\left\{\begin{matrix}\text{Sq.ft. of heating surface}\end{matrix}\right\} + b\left\{\begin{matrix}\text{Lbs. of fuel per hr.}\end{matrix}\right\}\left\{\left(\begin{matrix}\text{B.T.U. per lb.of fuel}\end{matrix}\right) - \left(\begin{matrix}\text{B.T.U. furnace loss per lb. fuel}\end{matrix}\right)\right\} && (a)\\
&\left\{\begin{matrix}\text{Lbs. evaporated per hr. from and at 212° F.}\end{matrix}\right\} = \frac{a}{970.4}\left\{\begin{matrix}\text{Sq.ft. of heating surface}\end{matrix}\right\} + \frac{b}{970.4}\left\{\begin{matrix}\text{Lbs. of fuel per hr.}\end{matrix}\right\}\left\{\left(\begin{matrix}\text{B.T.U. per lb. of fuel}\end{matrix}\right) - \left(\begin{matrix}\text{B.T.U. furnace loss per lb.}\end{matrix}\right)\right\} && (b)\\
&\left\{\begin{matrix}\text{Boiler horsepower}\end{matrix}\right\} = \frac{a}{33,479}\left\{\begin{matrix}\text{Sq. ft. of heating surface}\end{matrix}\right\} + \frac{b}{33,479}\left\{\begin{matrix}\text{Lbs. of fuel per hr.}\end{matrix}\right\}\left\{\left(\begin{matrix}\text{B.T.U. per lb. of fuel}\end{matrix}\right) - \left(\begin{matrix}\text{B.T.U. furnace loss per lb. fuel}\end{matrix}\right)\right\} && (c)
\end{aligned}\right\} \quad (86$$

Into these equations the grate surface may enter through the rate of fu consumption by Eq. (867):

$$\begin{aligned}(\text{Lbs. fuel per hr.}) &= (\text{lbs. fuel per sq.ft. of grate surface per hr.}) \times (\text{sq.ft. gra surface})\\ &= (\text{rate of combustion}) \times (\text{sq.ft. of grate surface.}) \quad . \quad . \quad (86\end{aligned}$$

Whence

$$\left.\begin{aligned}
&\left\{\begin{matrix}\text{B.T.U. absorbed per}\\ \text{hour}\end{matrix}\right\} = a\left\{\begin{matrix}\text{Sq.ft. of heat-}\\ \text{ing surface}\end{matrix}\right\}\\
&+b\left\{\begin{matrix}\text{Sq.ft. of}\\ \text{grate}\\ \text{surface}\end{matrix}\right\}\left\{\begin{matrix}\text{Rate of}\\ \text{com-}\\ \text{bustion}\end{matrix}\right\}\left\{\left(\begin{matrix}\text{B.T.U per}\\ \text{lb. of fuel}\end{matrix}\right)-\left(\begin{matrix}\text{B.T.U. fur-}\\ \text{nace loss}\\ \text{per lb. of}\\ \text{fuel}\end{matrix}\right)\right\} \qquad (a)\\
&\left\{\begin{matrix}\text{Lbs. evaporated per hr.}\\ \text{from and at 212° F.}\end{matrix}\right\} = \frac{a}{970.4}\left\{\begin{matrix}\text{Sq.ft. of heat-}\\ \text{ing surface}\end{matrix}\right\}\\
&+\frac{b}{970.4}\left\{\begin{matrix}\text{Sq.ft. of}\\ \text{grate}\\ \text{surface}\end{matrix}\right\}\left\{\begin{matrix}\text{Rate of}\\ \text{com-}\\ \text{bustion}\end{matrix}\right\}\left\{\left(\begin{matrix}\text{B.T.U. per}\\ \text{lb. of fuel}\end{matrix}\right)-\left(\begin{matrix}\text{B.T.U. fur-}\\ \text{nace loss}\\ \text{per lb. fuel}\end{matrix}\right)\right\} \qquad (b)\\
&\left\{\begin{matrix}\text{Boiler horse-power}\end{matrix}\right\} = \frac{a}{33{,}479}\left\{\begin{matrix}\text{Sq.ft. of heat-}\\ \text{ing surface}\end{matrix}\right\}\\
&+\frac{b}{33{,}479}\left\{\begin{matrix}\text{Sq.ft. of}\\ \text{grate}\\ \text{surface}\end{matrix}\right\}\left\{\begin{matrix}\text{Rate of}\\ \text{com-}\\ \text{bustion}\end{matrix}\right\}\left\{\left(\begin{matrix}\text{B.T.U. per}\\ \text{lb. of fuel}\end{matrix}\right)-\left(\begin{matrix}\text{B.T.U. fur-}\\ \text{nace loss}\\ \text{per lb. fuel}\end{matrix}\right)\right\} \qquad (c)
\end{aligned}\right\} \quad (868)$$

As the *ratio* of heating surface to grate surface is frequently used as a variable it may be introduced conveniently by Eq. (869).

$$\text{Ratio of heating surface to grate surface} = R = \frac{\text{Sq.ft. heat. sur.}}{\text{Sq.ft. grate sur.}} = \frac{A_h}{A_g}. \quad . \quad . \quad (869)$$

Whence

$$\left.\begin{aligned}
&\text{(B.T.U. absorbed per hour)}\\
&\quad = A_h\left[a+\frac{b}{R}\left\{\begin{matrix}\text{Rate of}\\ \text{com-}\\ \text{bustion}\end{matrix}\right\}\left\{\left(\begin{matrix}\text{B.T.U.}\\ \text{per lb.}\\ \text{of fuel}\end{matrix}\right)-\left(\begin{matrix}\text{B.T.U. furnace}\\ \text{loss per lb. of}\\ \text{fuel}\end{matrix}\right)\right\}\right] \quad . \quad . \quad (a)\\
&\text{(Lbs. evaporation per hr. from and at 212° F.)}\\
&\quad = \frac{A_h}{970.4}\left[a+\frac{b}{R}\left\{\begin{matrix}\text{Rate of}\\ \text{com-}\\ \text{bustion}\end{matrix}\right\}\left\{\left(\begin{matrix}\text{B.T.U.}\\ \text{per lb.}\\ \text{of fuel}\end{matrix}\right)-\left(\begin{matrix}\text{B.T.U. furnace}\\ \text{loss per lb. of}\\ \text{fuel}\end{matrix}\right)\right\}\right] \quad . \quad . \quad (b)\\
&\text{(Boiler horse-power)}\\
&\quad = \frac{A_h}{33{,}479}\left[a+\frac{b}{R}\left\{\begin{matrix}\text{Rate of}\\ \text{com-}\\ \text{bustion}\end{matrix}\right\}\left\{\left(\begin{matrix}\text{B.T.U.}\\ \text{per lb.}\\ \text{of fuel}\end{matrix}\right)-\left(\begin{matrix}\text{B.T.U. furnace}\\ \text{loss per lb. of}\\ \text{fuel}\end{matrix}\right)\right\}\right] \quad . \quad . \quad (c)
\end{aligned}\right\} \quad (870)$$

One of the best analytical investigations of the laws of boiler-surface heat absorption is that of Professor John Perry, who arrives at conclusions very

similar to the above. According to his mathematical analysis, based on the kinetic theory of gases and using the fundamental idea suggested by Osborn Reynolds, *the flue part of a boiler, that is, all surface getting its heat from hot gas contact, will absorb always the same fraction of the heat that gets to it.* The fraction depends on the relation of surface to cross-section of gas passage, that is, its mean hydraulic depth and on nothing else, except a constant of proportionality, the nature of which is somewhat hazy.

$$\frac{\text{Heat absorbed by flue}}{\text{Heat supplied to flue}} = \begin{pmatrix}\text{Constant depending on dimensions of gas}\\ \text{passage between heating surface}\end{pmatrix} = C. \quad (871)$$

From an analysis of the data of an old French locomotive test made on a boiler made up of sections, the evaporation in each of which could be measured, Perry concludes that the absorption or the evaporation in the fire-box alone is given by

$$\left\{\begin{matrix}\text{B. T. U. absorbed}\\ \text{per hour by fire-box}\end{matrix}\right\} = A \times (\text{grate surface}) + B \times (\text{pounds coal per hour}). \quad (872)$$

Assuming that the flues can absorb a constant fraction of what is left after the fire-box has taken out some of the heat of combustion from the hot gases, then neglecting furnace losses,

$$\left.\begin{aligned}\left\{\begin{matrix}\text{B.T.U. absorbed per hour by flues}\end{matrix}\right\} &= C \times \left\{\begin{pmatrix}\text{B.T.U. developed in furnace and available for absorption per hour}\end{pmatrix} - \begin{pmatrix}\text{B.T.U. absorbed by fire-box per hour}\end{pmatrix}\right. \quad (a)\\ &= C \times \left\{\begin{pmatrix}\text{B.T.U. developed in furnace and available for absorption per hour}\end{pmatrix} - A\begin{pmatrix}\text{Grate surface}\end{pmatrix} - B\begin{pmatrix}\text{Lbs. coal per hour}\end{pmatrix}\right\} \quad (b)\end{aligned}\right\} \quad (873)$$

$$\left\{\begin{matrix}\text{Total B.T.U. absorbed}\\ \text{per hour by boiler}\end{matrix}\right\} = \left\{\begin{matrix}\text{B.T.U. absorbed per hour}\\ \text{by (fire-box+flues).}\end{matrix}\right\}$$

$$\left.\begin{aligned}&= \left\{\begin{matrix}A(\text{Grate surface}) + B(\text{Lbs. coal per hour})\\ + C\left\{\begin{pmatrix}\text{B.T.U. developed in furnace and available for absorption per hour}\end{pmatrix} - \begin{pmatrix}A(\text{Grate surface}) + B(\text{Lbs. coal per hour})\end{pmatrix}\right\}\end{matrix}\right. \quad (a)\\ &= C\begin{pmatrix}\text{B.T.U. developed in furnace and available for absorption per hour}\end{pmatrix} + \left\{\begin{matrix}A(1-C)\,(\text{Grate surface})\\ + B\left(1 - \begin{pmatrix}C\end{pmatrix}\text{Lbs. coal per hour}\right)\end{matrix}\right\} \quad (b)\end{aligned}\right\} \quad (874)$$

This Eq. (874), reached by Perry partly by mathematical analyses starting with the kinetic theory of gases and partly from fire-box evaporation data for one boiler, is substantially the same as Eq. (865*a*), which is purely empiric, and based on many tests of different boilers, conducted with far more than usual accuracy. To show this similarity it is only necessary to assume that

$$M \times \left\{ \begin{array}{l} \text{B.T.U. developed in furnace} \\ \text{and available for absorption} \\ \text{per hour} \end{array} \right\} = \text{Lbs. coal per hour.}$$

Also that

$$\text{Grate surface} = \frac{\text{Heating surface}}{\text{Ratio of heating surface to grate surface}}$$

which on substitution in (Eq. (874*b*) gives

$$\begin{aligned} (\text{B.T.U. absorbed per hour}) &= C\left(\begin{array}{c} \text{B.T.U. developed in furnace and} \\ \text{available for absorption per hour} \end{array} \right) \\ &+ \frac{A(1-C)}{R}\left(\begin{array}{c} \text{Sq.ft. of} \\ \text{heating} \\ \text{surface} \end{array} \right) + B(1-C)M\left(\begin{array}{c} \text{B.T.U. developed in fur-} \\ \text{nace and available for} \\ \text{absorption per hour} \end{array} \right) \\ &= \frac{A(1-C)}{R}\left(\begin{array}{c} \text{Sq.ft. of} \\ \text{heating} \\ \text{surface} \end{array} \right) + \{B(1-C)M + C\}\left(\begin{array}{c} \text{B.T.U. developed in fur-} \\ \text{nace and available for} \\ \text{absorption per hour} \end{array} \right) \end{aligned} \qquad (875)$$

As the ratio of heating surface to grate surface R is a constant for one boiler, the two coefficients above become the constants (*a*) and (*b*) of Eq. (865*a*). To state it otherwise, his conclusion is identical with the new facts in the case if the ratio R is constant and if the furnace losses are a constant fraction of the apparent generation. If the approximate identity above noted is real then his most interesting conclusion, the most promising of all boiler transfer theories, is supported and *flues always will take out a constant fraction of the heat carried to them by the hot gases*, the fraction being larger, as they are longer and the cross-section of gas passage smaller compared to the heating surface at its perimeter or edge.

Prob. 1. A boiler receives 10,500 lbs. of water per hour at a temperature of 50° F. and turns it into steam at 150 lbs. per square inch gage, with 100° of superheat. What boiler horse-power is being developed and what is the factor of evaporation for this case?

Prob. 2. A boiler has a heating surface of 1050 sq.ft. On the basis of 10 sq.ft. per horse-power, how many pounds of steam will it make per hour from feed-water at 150° F. and steam 95 per cent dry at 125 lbs. per square inch gage?

Prob. 3. A boiler with 1000 sq.ft. of heating surface makes 4000 lbs. of steam per hour from feed-water at 200° F. and steam at 200 lbs. gage with 3 per cent moisture. What is the number of square feet of heating surface per horse-power?

Prob. 4. What would be the factors of evaporation for the following cases:

Feed Water Temperature.	Steam Pressure Gage.	Moisture in Per Cent or Superheat in ° F.
50	110	5%
150	50	100°
212	0	0
250	200	150°
300	400	3%
125	100	50°
212	0	10%
100	150	5%
70	125	200°
212.	0	500°

Prob. 5. 3000 lbs. of coal having a heating value of 12,500 B.T.U. per lb. are fired per hour. If the cinder loss is 5 per cent, CO loss 3 per cent, moisture loss $\frac{1}{2}$ per cent, and radiation and other furnace losses are 5 per cent, what is the rate of real generation per hour in B.T.U.?

Prob. 6. If 15 per cent of the heat really generated is lost to the stack and the efficiency of the heating surface is 80 per cent, what will be the horse-power of boiler, efficiency of boiler as a whole, and pounds of steam made per hour from feed water at 100° F., and steam at 100 lbs. gage dry, and saturated?

Prob. 7. For a boiler, the constants of Eq. (856*a*) were, $a = 1000$ and, $b = .55$. The ratio of heating surface to grate surface was 60 and grate was 60 sq.ft. in area. The coal fired per hour was 1500 lbs. and 90 per cent of the heat in the coal was actually generated. If the B.T.U. per pound of coal were 13,450, what was the horse-power of boiler?

Prob. 8. A boiler has 3500 sq.ft. of heating surface and a grate surface of 50 ft. The constants in the straight-line equation are $a = 1000$ and $b = .5$. Show how the horsepower will vary with the rate of combustion for coal containing 12,000 B.T.U. per pound and a constant furnace loss of 15 per cent.

Prob. 9. A boiler follows the same law as that of the Heine boiler, but with a constant (a) equal to 60 per cent of that of the Heine. If the furnace efficiency is 90 per cent and constant what must be the rate of combustion per boiler horse-power for $\frac{1}{2}$, $\frac{3}{4}$, 1, and $1\frac{1}{2}$ times the rated capacity? The boiler has 15,000 sq.ft. of heating surface, a ratio of heating surface to grate surface of 45, and is rated on 10 sq.ft. of heating surface. The coal used is No. 20 of the general table.

Prob. 10. The rate of combustion in a boiler varies from 20 to 80 lbs. per hour per square foot of grate surface. The grate area is 150 sq.ft. and ratio of heating surface to grate surface is 60. For a constant furnace loss of 15 per cent and a coal having a heating value of 14,000 heat-units, what will be the variation in horse-power, the relation being the same as for the Goss tests with Coal A?

14. Steam-boiler Efficiency, Furnace and Heating-surface Efficiency. Heat Balances and Variation in Heat Distribution. Evaporation and Losses per Pound of Fuel. Defining the efficiency of a boiler, inclusive of grate, furnace, setting and heating surface, as the ratio of the heat that is actually absorbed and retained by the steam per pound of coal, to the calorific power of the coal

s fired, it may be said that there is no fundamental theory or absolute standard, s was the case for evaporative capacity, but like it there are known and estab- shed relations between the factors that together make it less than 100 per cent. tudy of boiler efficiency must be based, therefore, not on what might be expected f the heat absorption surface but negatively, as it were, on the nature and extent f the losses or the several parts of the original heats that do not get into the water r steam, whether operating, to reduce the amount available for absorption efore the gases reach the surface or, on the other hand, to leave a residue f sensible heat in the gases after they have swept the surface.

Boiler heat losses can be divided, grouped and classified in many different ays of course, but the following is a very useful one.

Boiler-heat Loss Classification

1. *Furnace and Setting Loss.* This includes all those amounts of heat er pound of coal that tend to reduce the sensible heat of the gases available or heating surface absorption except radiant heat taken up directly by the eating surface, such as:

(*a*) That necessary to evaporate the moisture in the coal;

(*b*) That due to burning hydrogen to vapor instead of to water, the difference between high and low calorific power of hydrogen burnt per pound of coal;

(*c*) That radiated (1) from furnace and setting before absorption by water or steam;

(*d*) That due to unburned gases CO, H_2 and hydrocarbons in the flue gases;

(*e*) That due to unburned fixed carbon, in ash dropping through grate, or in soot and cinders in the flues, or discharged from the stack.

2. *Flue Loss.* This includes the amount of sensible heat still carried by ie gases that have swept over the heating surface, and is the product the total weight of gases per pound of coal, including water vapor and xcess air, into the mean specific heat and excess of temperature over that the air supply. Obviously, this is more or less governed by the steam pres- ire and temperature as the leaving gases can never be made cooler than the st boiler surface they touch and if this is superheater surface the temperature ay be high, but if it is feed-water supply surface it may be low. Otherwise is loss is governed by the control of air supply, so that too much excess is not be used, but also and more fundamentally by the absorbing laws of heating rface, as it is the heat residue after absorption.

On the assumption that all the rest of the heat gets into the steam and water, l the heat is thus accounted for. *That which gets into the water and steam ay be regarded as the useful effect,* but this may not be warranted if some steam id water leakages exist or radiation occurs from the steam or water surfaces iemselves. In these cases there may be a difference of opinion as to whether ie losses should be credited or debited because from the standpoint of boiler

goodness as a heat absorber all heat carried off by leakages is credit heat, for i has been taken from gases, but on the other hand from the steam user's stand point this heat is of no value as it cannot run an engine. The steam and wate surface radiation is, of course, also a steam user's loss, but the boiler abs rbe it from the gases and considering the boiler as an absorber rather than a con server of heat, it is a credit. No confusion is likely to result in practical wor if these items are clearly understood, circumstances in each case will indicat clearly where the items belong.

The preceding classification indicates that *there may be more than one standar of efficiency of boilers* or rather that boiler efficiency is divisible into parts, whic is a valuable way of keeping the processes clearly divided and of establishin a basis of analysis.

Boiler-efficiency Definitions

1. *Furnace Efficiency* E_f, as a term is properly applied to the ratio of th sensible heat of the gases before heating surface absorbtion per pound of fue to the calorific power of the fuel, or

$$\left.\begin{aligned} E_f &= \frac{(\text{B.T.U. per lb. fuel}) - (\text{Furnace and setting loss per lb. fuel})}{\text{B.T.U. per lb. fuel}} \quad (a) \\ &= 1 - \frac{\text{Furnace and setting losses per lb. fuel}}{\text{B.T.U. per lb. fuel}} \quad (b) \end{aligned}\right\} . \quad (876$$

2. *Heating-surface Efficiency* E_s as a term is properly applied to the ratio o the heat absorbed by the boiler water and steam to the sensible heat brough to the heating surface by the hot gases, radiated to it per pound of fuel. A the heat usefully absorbed is the difference between the sensible heat of gase as developed in the furnace and the flue loss this may be set down in two way,

$$\left.\begin{aligned} E_s &= \frac{(\text{B.T.U. absorbed by water or steam per lb. fuel})}{(\text{B.T.U. per lb. fuel}) - (\text{furnace and setting losses per lb. fuel})} \quad (a) \\ &= \frac{(\text{B.T.U. per lb.fuel}) - (\text{furnace and setting losses per lb.fuel}) - (\text{flue losses per lb.fuel})}{(\text{B.T.U. per lb. fuel}) - (\text{furnace and setting losses per lb. fuel})} \quad (b) \\ &= 1 - \frac{\text{Flue loss per lb. fuel}}{(\text{B.T.U. per lb. fuel}) - (\text{furnace and setting losses per lb. fuel})} \quad (c) \end{aligned}\right\} 87$$

3. *Boiler Efficiency* E_b is most commonly applied to the ratio of heat absorb by water and steam per pound of fuel to its calorific power per pound or it is t product of heating surface and furnace efficiencies.

$$\left.\begin{aligned} E_b &= \frac{\text{B.T.U. absorbed by water per lb. fuel}}{\text{B.T.U. per lb. fuel}} \quad (a) \\ &= 1 - \left[\frac{\text{furnace and setting losses per lb. fuel}}{\text{B.T.U. per lb. fuel}}\right] \\ &\qquad - \left[\frac{\text{Flue loss per lb. fuel}}{\text{B.T.U. per lb. fuel}}\right] \quad (b) \\ &= E_f \times E_s. \quad (c) \end{aligned}\right\} (87$$

The experimental determination of all these losses and efficiencies is continually going on, as it has for many years, and from the data, boiler designers and makers with the assistance of inventors—who, however, generally ignore the test data available—all have continuously sought to improve results and are still trying. As a consequence one might expect to find modern boilers turned out by the best engineers ever so much more efficient than those built at the country cross-roads plate shop, but this is not the case, nor can it be said that one type is any more efficient than another, if small portable and other special forms be excluded, nor are recent designs more efficient than old ones. This is a most striking situation and is very nicely illustrated by the summaries made by Donkin after studying some four hundred tests of all kinds of boilers located everywhere and which are quoted from his book in Table CII.

TABLE CII

BOILER EFFICIENCY SUMMARIES (DONKIN)

Type of Boiler.	No. of Tests.	Mean of Best Two Efficiencies Per Cent.	Lowest of One Test Efficiency, Per Cent.	Mean of All Tests, Efficiency Per Cent.
Water tube, 1½-inch tubes	6	81.4	66.6	77.4
Locomotive	37	83.3	53.7	72.5
Lancashire	10	74.4	65.6	72.0
Two-story	9	76.1	57.6	70.3
Two-story	29	79.8	55.9	69.2
Dry back	24	75.7	64.7	69.2
Return-smoke tube	11	81.2	56.6	68.7
Cornish	25	81.7	53.0	68.0
Cornish	9	81.0	55.0	67.0
Wet back	6	69.6	62.0	66.0
Elephant	7	70.8	58.9	65.3
Water tube, 4-inch tubes	49	77.5	50.0	64.9
Lancashire	40	73.0	51.9	64.2
Cornish	3	65.9	60.0	62.7
Lancashire	107	79.5	42.1	62.4
Dry back	6	73.4	54.8	61.0
Lancashire 3-flue	6	66.7	52.0	59.4
Elephant	8	65.5	54.9	58.5
Lancashire	8	74.3	45.9	57.3
Vertical	5	76.5	44.2	56.2

These data certainly demonstrate that boiler efficiency, however many things may determine it, cannot be associated with boiler type, nor, it might be added, with size either. Admitting that in some cases the efficiencies may be high, that is, somewhere near 80 per cent, and that in other cases thay may be low, 50 per cent or even less, the causes of such a possible range from good to bad when the average good performance of all types of boilers may be and is about the same, are worth investigating even if prediction of efficiency for specific conditions is impossible. The charge of bad management will not account for all the facts even though a poor fireman may make the performance of the best boiler as poor as the worst one over designed, and there can be absolutely

no question as to very considerable differences in efficiency with the most skillful management, that can be traced partly to form and proportions of the boiler structure and partly to operating conditions more or less beyond the control of the operator. In a great many cases gains or losses in efficiency are found chargeable to almost absurdly irrelevant things and the literature of the subject, especially that circulated by makers of boiler attachments such as grates, firedoors, stokers, furnaces, baffles, draft apparatus, air preheaters, tube retarders and many other minor items, is full of such false conclusions, due no doubt to unskillful or over skillful testing that failed to discover which one of a possible dozen or so variables that may effect boiler efficiency was responsible for an improvement obtained, when the particular device was in use and confidently credited without qualification to it. Such unwarranted conclusions would be impossible if boiler tests were made with sufficient thoroughness to absolutely fix *all* conditions and permit of tracing the heat losses, then, and then only, could a change in efficiency be connected with the change in that one condition either of service or structure that really caused it.

A really complete statement of results of a boiler performance accounting for supply and distribution of heat, termed the heat balance of the boiler, is almost impossible, yet may be pretty closely approximated by careful experimental work. The degree to which the approximation fails to represent the true distribution is indicated by an item containing *unaccounted for losses* often grouped with leakage of steam and water, and radiation. To illustrate, three different heat balances are reported in Table CIII below for, first—the U. S. Geological Survey Heine water tube boiler of 210 HP. and 2031 sq.ft of heating surface as reported by Breckenridge; second—the fire tube locomotive boiler of 400 H.P. and 1216 sq.ft. of heating surface as reported by Goss, and, third—a Stirling water tube boiler.

TABLE CIII

THREE EXAMPLES OF HEAT BALANCE FOR BOILERS

Distribution of Heat of Coal, Per Cent.	510 H.P. Stirling.	210 H.P. Heine.	400 H.P. Locomotive
1. Absorbed by water and steam	74.87	60.30	57.00
2. Loss due to vaporization of coal moisture	.24	.26	5.00
3. Loss due to sensible heat of stack gases	10.73	15.19	14.00
4. Radiation leakage and unaccounted for	8.24	3.97	7.00
Total per cent of fuel heat developed	94.08	79.72	83.00
5. Loss due to unburned CO in stack gases	.42	2.17	1.00
6. Loss due to other forms of incomplete combustion, H_2, C_mH_n in gases and fuel in ash cinder, in flues setting and discharged from stack	5.50	18.11	16.00
Total per cent of fuel heat not developed	5.92	20.28	17.00
	100.00	100.00	100.00

About some of these items there may be a difference of possible interpretation, especially as to whether a given loss represents heat developed or undeveloped and more particularly as to whether heat developed, as indicated, is really available for boiler heating-surface absorption. For example, when the fuel contains hydrogen, the steam from which does not get a chance to cool below 212° F., the low value of it only is developed whereas the high value is reported by the calorimeter as the calorific power of the fuel. Again, the heat absorbed by vaporizing moisture in coal is developed heat but it is unavailable for heating surface absorption because it is taken up by coal water.

To avoid conflicts in commercial transactions in which the efficiencies and losses in boilers are sometimes the subject of legal guarantees, standards must be available as subjects of agreement whether correct scientifically or not, and such standards are established by the American Society of Mechanical Engineers, which should be consulted by everyone interested. Assuming that the loss due to vaporization of coal moisture is unavailable for absorption, and that the items of radiation, leakage and "unaccounted for," in which nearly all errors are concentrated, represent available heat, the furnace and heating-surface efficiencies can be estimated and it must be understood that it is never possible to do more than estimate these. Charging all losses representing heat not available for absorption against furnace efficiency, these efficiencies decrease.

	Stirling	Heine	Locomotive
Efficiency of furnace per cent.....	94	79	78
" " heating surface per cent..	81	76	73
" " boiler complete per cent .	75	60	57

These figures are given not as typical of these types of boilers or of any particular service conditions, though they are as fair as any, but rather to show how division of performance may be made. In actual service and even during the conduct of tests, conditions are continually changing, so that a whole series of tests under conditions that seem identical will produce different results and show that the conditions were not identical, and the causes of variation are due almost entirely to the impossibility of fire control or the maintenance of steady conditions therein. The ash is constantly accumulating and more in one spot than in another because air passes most freely where fuel resistance is least, and burns the coal faster at that point, the thickness of fire is varying, coal cannot be supplied with absolute uniformity and the air supply is not only varying all the time but the ratio of top to bottom air and side leakage vary more; all these things change furnace and fire conditions so as to disturb the heat balance.

Even if the furnace efficiency were kept constant through a more perfect fire and air control than are possible except in tests, the conditions for heat absorption by the heating surface may vary considerably. Probably the greatest of the disturbing influences that tend to baffle attempts at analysis and often ignored even by first-class investigators, *is the delayed and long-con-*

tinued combustion of the gases beyond the fire-bed or the relation of flame to completely burned hot gases.

All theories of heating-surface absorption that have ever been proposed are based on the assumption that the gases passing over the heating surface are immediately cooled thereby and continuously and regularly more cooled according to some assumed law. Pyrometric investigations in flues show that so long as there is a clear, visible flame there is practically no cooling and even where the flame becomes of the flickering, irregular sort, the cooling of the gases is very much slower than beyond the limit of flame, or, in other words, generation of heat continuing during part of, sometimes all of, the absorption period, is a fact that makes it practically impossible to theorize on absorption laws.

It must not be understood, however, that this theorizing is of no value because it is, and attention has already been called to Perry's idea of absorption as most useful, and reasonably in agreement with observation on rate of heat absorption related to rate of heat generation. The efficiency of heating surface being the ratio of the amount absorbed to that generated and available for absorption, can be derived from the relations between rates of generation and absorption whether these relations are derived from a theory or represent average experimental data. According to the experimental data analyzed in the last section the relation between absorption and generation rates could be expressed in either of the following ways:

$$\left\{\begin{array}{l}\text{B.T.U. absorbed per hr. per}\\ \text{sq.ft. of heating surface}\end{array}\right\} = a + b\left\{\begin{array}{l}\text{B.T.U. developed in fire and}\\ \text{available for absorption}\\ \text{per hr. per sq.ft. of heat-}\\ \text{ing surface}\end{array}\right\}$$

$$\left\{\begin{array}{l}\text{B.T.U. absorbed per hr. per}\\ \text{sq.ft. of grate surface}\end{array}\right\} = aR + b\left\{\begin{array}{l}\text{B.T.U. developed in fire}\\ \text{and available for absorp-}\\ \text{tion per hr. per sq.ft. of}\\ \text{grate surface}\end{array}\right\}$$

from which two possible expressions for efficiency of the heating surface follow both of the same general form Eq. (865) since efficiency is

$$E_s = \frac{\text{B.T.U absorbed}}{\text{B.T.U. developed in fire and available for absorption}}$$

$$\left.\begin{array}{l} = b + \dfrac{a}{(\text{B.T.U. developed in fire and available for absorption per hr. per sq.ft. H.S.})} \quad (a)\\ = b + \dfrac{aR}{(\text{B.T.U. developed in fire and available for absorption per hr. per sq.ft. G.S.})} \quad (b)\end{array}\right\} \quad (87$$

These are equations of an equilateral hyperbola asymptotic to $100b$ per cent efficiency as indicated by AB in Fig. 212. They seem to be at fault because they show that efficiency may exceed 100 per cent with small amounts o

generation, which means, however, only that the relation is not true for very small generations. For the working ranges of generation rates the heating surface efficiency line is substantially straight and falling with increase of generation. According to Perry's division of absorption between fire-box and flue or their equivalents for the particular structure in question, the efficiency of the flue part of the heating surface is independent of the rate of heat supply to it, so that all variations in the efficiency of the whole heating surface must be chargeable to the part receiving radiant heat. Just how true this is, or just how the whole efficiency of heating surface should, and really does vary with rate of generation cannot be settled with any available data, though there is enough to prove almost anything with a little skillful stretching.

The curve AB of Fig. 212 does, however, within the limits of the experimental points from which it was plotted, those for the Hohenstein oil-fuel boiler, represent the facts of these experiments, and comparison with Eq. (879) for heating surface efficiency in terms of losses indicates a most interesting meaning for the vertical distance from the curve AB to the 100 per cent line. This distance must stand for the flue loss per pound of fuel divided by the generation per pound fuel or the calorific power less the furnace and setting loss and shows that for these experiments, the ratio is increasing with increase of total generation or rate of combustion.

If on the same diagram, the line CD represents the overall boiler efficiency then the vertical distance from it to the 100 per cent line must represent the sum of the losses in the flue as sensible heat, together with the furnace and setting losses. Similarly, the vertical distance from CD to AB must represent the furnace and setting losses. If the two curves are parallel these latter losses are constant in amount; if they converge toward the right they must decrease with increase in heat generation, and if they diverge the losses are increasing with increase of production. In this particular case the losses first increase and then decrease and are separately plotted below, line EF.

In all cases the flue losses increase in quantity with increase in heat generation according to these data and it is worth while to investigate what meaning may be attached to this.

Setting down the expression for flue loss which is represented by the distance from the heating-surface efficiency line AB to the 100 per cent line, in symbols

$$\frac{\text{Flue loss per lb. fuel in sensible heat}}{\text{B.T.U. per lb. fuel} - \text{furnace and setting losses per lb. fuel}} = 1 - b - \frac{aR}{\text{B.T.U. generated per square foot grate}}. \quad . \quad . \quad . \quad (880)$$

For the above to be true either the numerator must increase or denominator decrease, that is, with increase of combustion rate the sensible heat per pound of fuel must increase or the furnace and setting losses per pound of fuel must increase, or both. This shows clearly how variations in the furnace conditions affect the heating-surface efficiency in spite of any supposed constancy of the efficiency

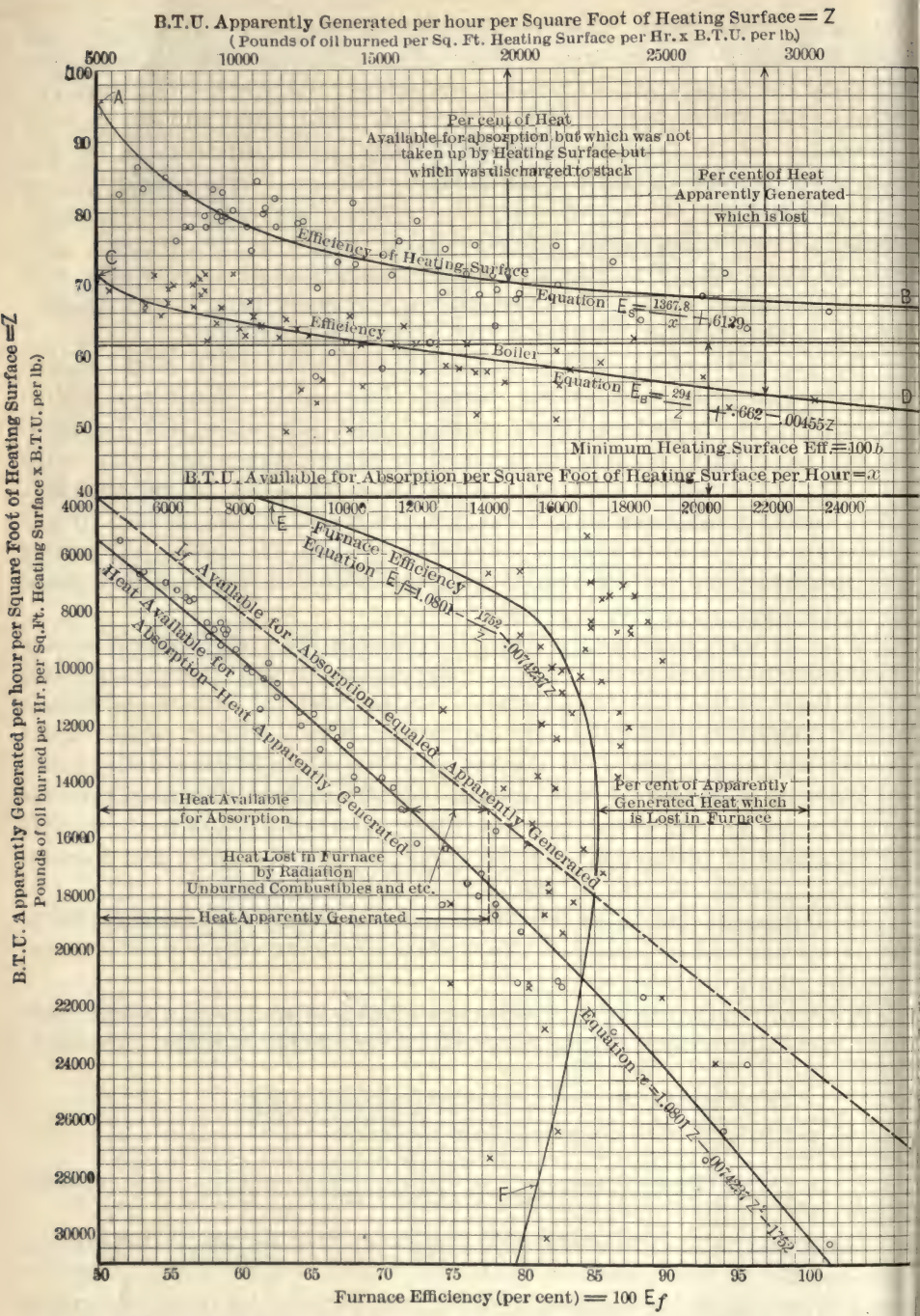


FIG. 212.—Curves showing the Relations which Exist between Quantities Entering into Boiler Efficiency.

f the flue part. As the experiments from which the conclusion is derived nclude all possible ranges in rate of combustion and kind of coal and several ery different types of boiler it must be accepted in spite of the apparent failure f the law of relation for low rates of generation where the curve passes the 00 per cent line.

The flue loss in sensible heat is the product of specific heat of gases, their eight per pound of fuel and the excess of temperature over the air supply and the flue part of the heating surface takes out always the same part C then

$$\text{Heat absorbed by flues} = C \times (\text{heat supplied to flue}). \quad . \quad . \quad . \quad (881)$$

$$\text{Flue loss} = (1-C) \times \text{heat supplied to flue}, \quad . \quad . \quad (882)$$

$$C_p w(t_{\text{stack}} - t_{\text{air}}) = (1-C) C_p w(t_{\text{furnace}} - t_{\text{air}}). \quad . \quad . \quad . \quad . \quad . \quad (883)$$

$$[(\text{stack temp.}) - (\text{air temp.})] = \text{constant} \times [(\text{furnace temp.}) - (\text{air temp.})] \quad (884)$$

rovided the weight of gases per pound of coal, w, is constant. Experimental bservation proves that the stack temperature always rises with increase in rate f combustion unless excess air be supplied at the same time, and as a rule the xcess air and weight of gases per pound of fuel decreases with increased rate f combustion or increased draft *if* the fire is thick enough and free from holes, herwise, and this is most common with moving-bar stokers and with anthracite nall sizes, whether hand or mechanically fired, the weights of gases per pound f coal decreases fast and the stack temperature increases not as fast as it should not at all.

In the U. S. Geological Survey series on the Heine boiler the combustion hamber temperature increased somewhat irregularly with rate of combustion nd rate of heat evolution as shown in Fig. 213, for a considerable variety of bals and at the same time the flue temperature bore the relation to combustion hamber temperature indicated in Fig. 214. Comparing individual readings, e ratio of excess temperature of combustion chamber over the boiler tem- rature, to the excess of flue temperature, the ratio was found to range from to 14, the best average being 7.3. This could hardly be called a constant tio and the reason may be due to the fact that the flame extended between e tubes more in some cases than in others, or it may be that the expectation constancy is based on wrong hypotheses, no one can say which, and yet this the best data on the subject in existence.

Examples of specific relations like the above could be cited almost without nit for other variables, and at the end, the problem would be little if any nearer solution, that is, the problem of finding and expressing these losses or the iciency of a boiler in terms of such variables of construction, or operation, evidently have some influence in making it what it is.

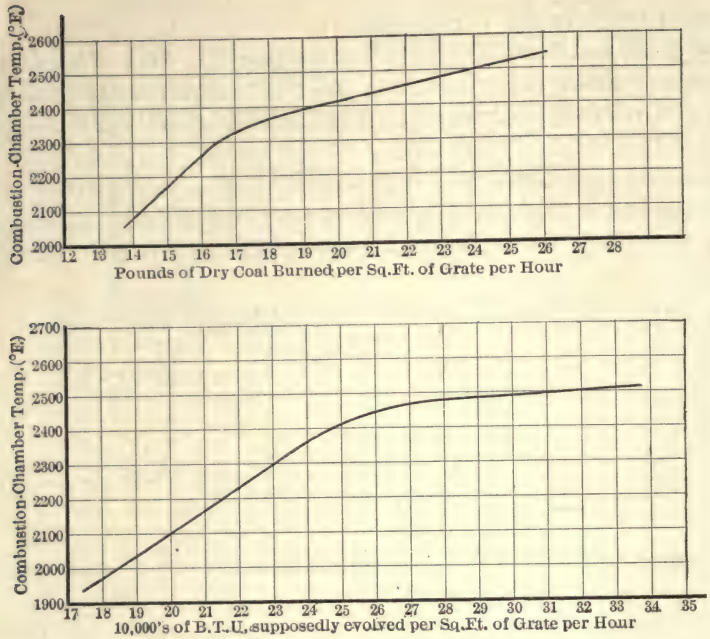


FIG. 213.—Relation of Combustion Chamber Temperature and Rate of Combustion or Rat of Heat Evolution.

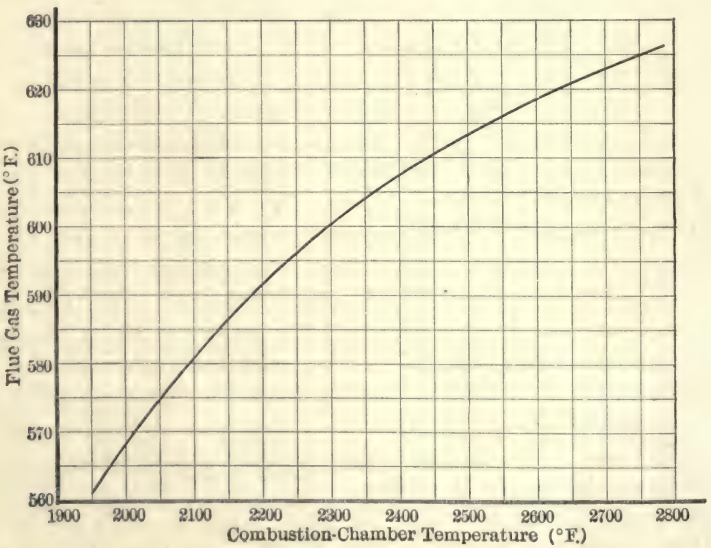


FIG. 214.—Relation between Flue Gas Temperature and Combustion Chamber Temperatu

However difficult division or analyses of losses may be, there is little doubt as to the sort of effect an increase or decrease of any one factor may have and it is, furthermore, usually possible to explain an overall efficiency curve when obtained, which will be clear from a few examples.

These overall efficiency curves are not always given in terms of per cent efficiency, nor is capacity, in terms of B.T.U. absorbed per hour, but for efficiency the evaporation actual or equivalent per pound of fuel is more common, and for capacity boiler horse-power or per cent boiler rating or rate of combustion. One such curve of considerable significance is that of Fig. 215, representing the results from the Interborough Railway boilers equipped with two Roney stoker furnaces, one 70 per cent of the grate area of the other and burning good semi-bituminous coal, first on one grate alone and then on both together. Increased capacity causes a decrease in efficiency regularly along a line

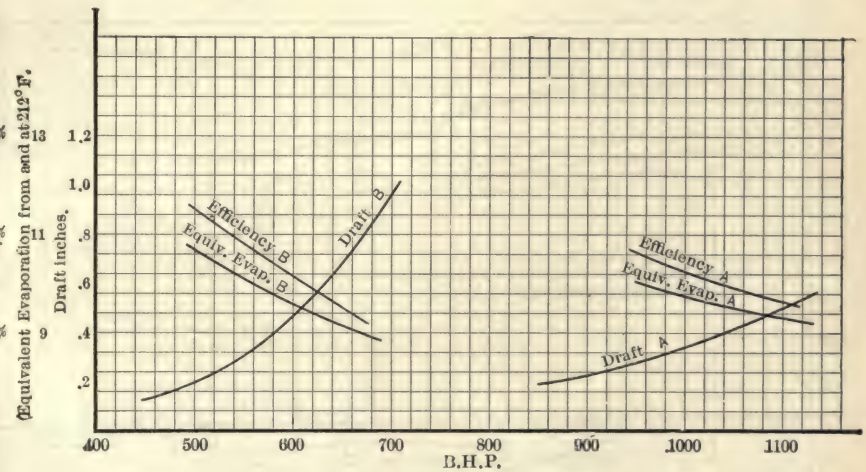


FIG. 215.—Relation between Horse-power and Draft or Efficiency.

almost straight with one grate, but when for a given capacity the second grate is used, the efficiency becomes higher after which it again decreases along a nearly parallel line, that is, in accord with a similar law. The only way this could happen is through a change in furnace conditions, probably traceable to excess air variations, as the draft suddenly decreases for a given amount of generation, when passing from the single to the double grate as is indicated by the draft curves. If there are holes or air leaks, as is the case with such stokers, this would cause a decrease of excess air and an increase of efficiency exactly as occurred when the draft changed from a high to low value for the *same or even greater* total rate of heat evolution. In the Goss locomotive tests the evaporation is reported per pound of dry coal as a function of equivalent evaporation per square foot of heating surface, and shows a similar nearly straight line relation as in Fig. 208, and different for the two coals used. The reason for this appears at once from Fig. 216, which shows the whole heat balance plotted

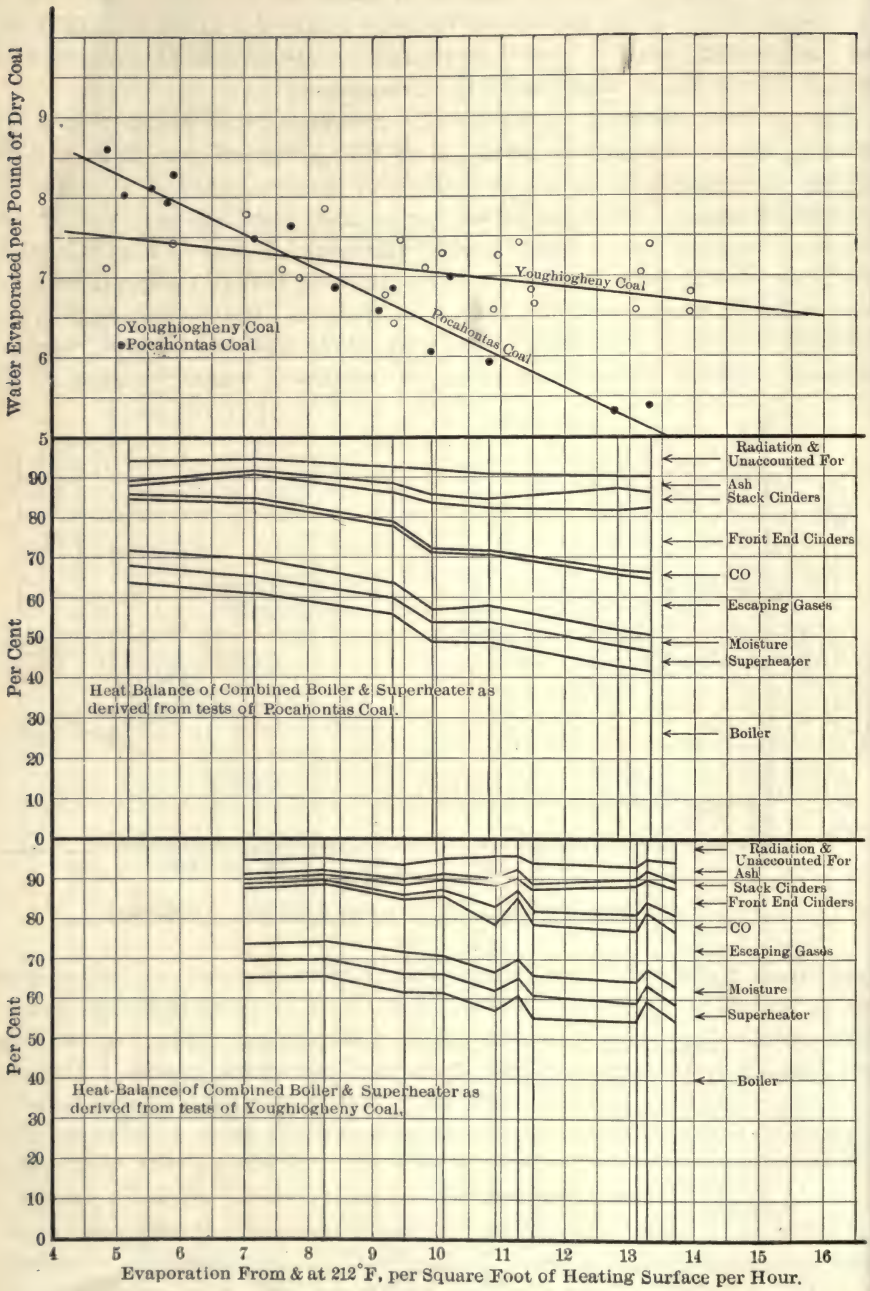


FIG. 216.—Heat Balance for Locomotive Boiler Working Under Various Rates of Evaporation

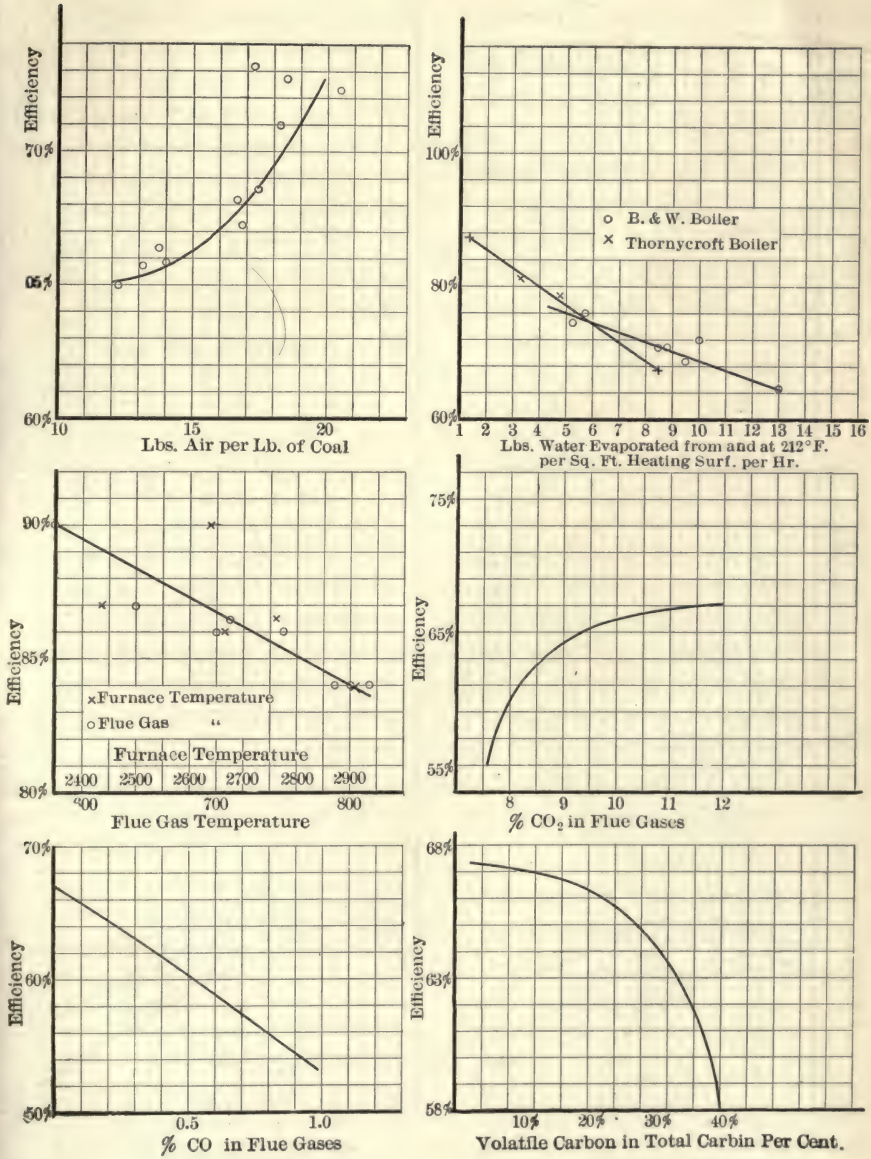


FIG. 217.—Influence of Various Factors on Boiler Efficiency

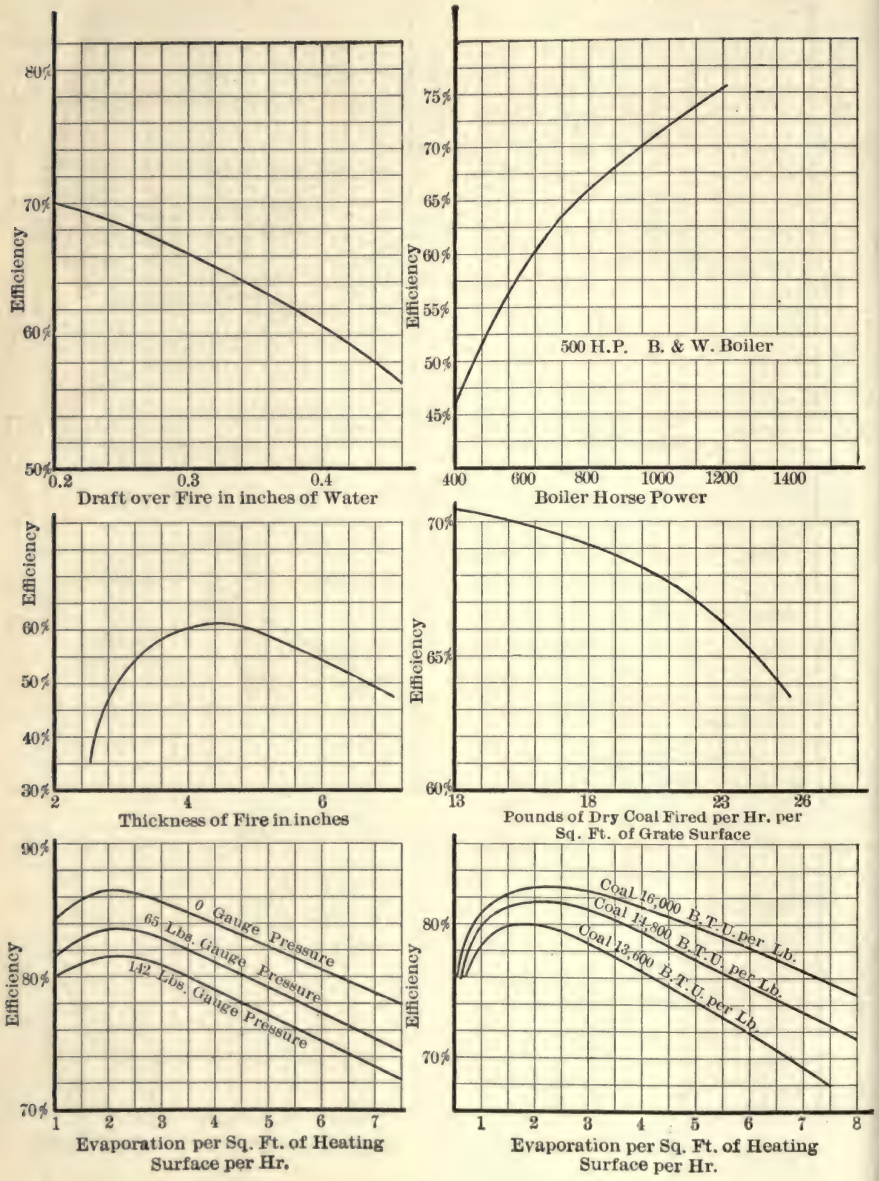


FIG. 218.—Influence of Various Factors on Boiler Efficiency.

to the same base and is mainly due to a difference in cinder loss which in both cases increases with boiler capacity and draft that produces it. The essential difference between this fire and that last discussed is in its very superior thickness, lack of agitation by stoker bars, and lack of air holes at sides or end.

Another good illustration of the controlling nature of some one condition is given by Myers, reporting results of boiler performance with tan bark as a fuel in leather-making works, the chief characteristic of the fuel being the large amount of moisture in proportion to the calorific power, which makes it unburnable in an ordinary furnace. The fuel showed about 9500 B.T.U. per pound when dry and ten samples contained 65½ per cent moisture average, leaving only 2665 B.T.U. per pound of fuel as fired, and which gave a boiler efficiency over all of 71 per cent when referred to dry tan bark and 54 per cent when referred to the bark as fired. Use of this wet fuel absolutely prevents the attainment of high efficiencies even in the best apparatus because of the moisture heat loss which is the controlling condition.

Another illustration of this fact is furnished by a test of the well-known B. and W. water tube boiler by the Pittsburgh Testing Laboratory, using blast-furnace gas fuel and which yielded a little over 54 per cent efficiency because of insufficient air and premixture with the gas, the flue gas analysis showing as high as 20 per cent escaping unburnt. With exactly the same boiler, except as to furnace and combustion chamber, an efficiency in excess of 80 per cent is not difficult with an oil fuel if care be exercised to control air supply in proportion to oil and means be provided to mix the oil and air; failure to do this may yield just as bad results as noted for the blast-furnace gas. These little incidents could be multiplied without end but will suffice to show that boiler efficiency is controlled not so much by the boiler, as by the furnace, not so much by boiler designer as by fireman, but the analysis shows that there is a problem of design, more concerned, however, with high capacity of surface, and therefore, cost and size of boiler, than with efficiency improvements.

A number of boiler tests are given in Figs. 217 and 218 with some one item of importance, selected to show the effect of various conditions of service and fuels in the same and different boilers, all of which are self explanatory in view of this discussion.

Prob. 1. A furnace is fired with coal containing 12,500 B.T.U. per pound and needing 14 lbs. of air per pound of coal. In the ash there is 5 per cent of the weight of the coal in unburned carbon; the coal has 5 per cent moisture with it; there is 3 per cent of CO in the flue gas; the stack temperature is 500° above that of the room, which is 80° F., and twice the combining proportion of air is supplied. For the above data and allowing for 5 per cent radiation, of which 4 per cent is from the setting and 1 per cent from steam surface, what is the boiler efficiency?

Prob. 2. A boiler makes 6 lbs. of steam per hour at 100 lbs. gage pressure from feed-water at 60° F. per pound of coal fired. The coal is No. 66 of the general table and the ash loss is 8 per cent, radiation 5 per cent, and the CO loss 2 per cent. What is the heating surface efficiency, the furnace efficiency, the boiler efficiency.

Prob. 3. The heating surface efficiency of a boiler is 80 per cent when making 7 lbs. of steam per hour, 97 per cent dry from feed at 100° F. per lb. coal. The coal used contains 12,500 B.T.U. per pound. What is the furnace loss and boiler efficiency?

Prob. 4. The efficiency of a certain boiler is 72 per cent. The steam pressure is 125 lbs. gage, feed-water 180° F. and there is 125° of superheat. The coal used is No. 87 of the table and 100 per cent excess air is supplied. Flue temperature is 100° above steam temperature and the room is 70° F. The evaporation per pound of coal is 6.5. What is the heating surface and furnace efficiency?

Prob. 5. If in the Hohenstein boiler test, for which curves are plotted in Fig. 212, the oil used had a heating value of 18,000 B.T.U. per pound and the heating surface was 2130, what was the efficiency of the boiler when 1.5 tons of oil were burned per hour, and what the furnace efficiency and the heating-surface efficiency?

GENERAL PROBLEMS ON CHAPTER V

Prob. 1. Natural gas consisting practically of nothing but methane is used in a gas engine. What would be the greatest amount of air which could be mixed with the gas and still have the engine run? The least? For the best mixture? To what pressure would the gas have to be compressed to cause self ignition, the compression being adiabatic and the temperature at the beginning being 300° F.?

Prob. 2. In a boiler test it was found that the room temperature was 70° F. and the stack temperature 450° F. The coal was of the following composition: H = .10, O = .04, C = .70, ash = .16. The flue-gas analysis was: CO_2 = 8 per cent; CO = 4 per cent; O_2 = 8 per cent; N_2 = 80 per cent. What was the B.T.U. loss per pound of coal due to sensible heat, to unburned carbon in flue gases and to moisture in coal?

Prob. 3. A boiler efficiency of over 80 per cent is unusual. What would be the highest evaporation from and at 212° F. that might reasonably be expected from a pound of any one oil?

Prob. 4. The average flue gas analysis during a boiler test showed CO_2 = .12; O_2 = .04; CO = .01; N_2 = .80. The coal was 80 per cent carbon and 20 per cent ash. What was the amount of air used per pound of coal and what was the stack loss if the flue temperature was 450° F. and boiler room 90° F.?

Prob. 5. For a bed temperature of 1700° F., what would be the CO to CO_2 ratio for air gas and water gas according to Boudouard's and Bunte's experiments? How much steam should be decomposed?

Prob. 6. A boiler has an efficiency of 70 per cent and is steaming at the rate of 150 boiler H.P. with feed-water at 60° F., steam pressure 125 lbs. gage, and 100° of superheat. The grate area is 215 sq.ft. For pea coal of 12,000 B.T.U. per pound, what must be the draft for this condition? Should the rating be forced up to 200 H.P., how would the draft have to be changed?

Prob. 7. A certain coal is said to evaporate 7 lbs. of water from a feed temperature 60° and at a pressure of 150 lbs. gage to 96 per cent steam. What would be the equivalent evaporation from and at 212° F. for this coal?

Prob. 8. Coal costing $1.00 per ton made 5 cu.ft. of coal gas, similar to No. 8, Table X, per pound of coal, and the gas sold for $1.50 per million B.T.U. The other expenses of manufacture were such as to just balance the profit on the gas. At what price must the coke be sold per ton to net 10 per cent of the receipts for the gas?

Prob. 9. In a certain boiler it is found that 68 per cent of the heat of the fuel gets into the steam. How many pounds of steam could be made per pound of coal No. 62, if the heating value were that as found from the ultimate analysis? How would the results differ if the heating value was that as determined from Eq. (757), from Eq. (759), from Eq. (773)?

Prob. 10. Which of the following boilers is developing the greatest boiler horsepower if the actual pounds of water evaporated per hour is the same for each?

Feed Temp.	Press Ga.	Quality or Superheat
40	100	.95
60	150	... 100°
60	100	... 100°
225	100	... 100°
225	100	.95
40	200	.95
40	200	... 100°

Prob. 11. Taking an analysis of one of each of the following gases from the tables and assuming the actual pressure rise to be .5 of the calculated, find which would give the greatest pressure when burned with the correct amount of air at constant volume, starting with a pressure of 20 lbs. per square inch absolute and a temperature of 200° F. Natural gas, coal gas, air gas, carburetted water gas, producer gas.

Prob. 12. In a given boiler the heat absorbed per hour bears a straight line relation to that available, as follows:

$$(\text{Absorbed}) = 1000 + .6\ (\text{available}).$$

When 1000 lbs. of bituminous coal yielding 14,000 B.T.U. per pound are being burned per hour with an unburned fuel loss of 8 per cent, what will be the boiler horse-power? What should the grate surface be for a draft of .5 in. of water?

Prob. 13. Coal No. 20 of Table CIV is used in a gas producer having a blast saturated with steam at 130° F. What will be the weight of coal required per million B.T.U. in the gas if the CO to CO_2 ratio is 6, and the volatile is the same as gas No. 4 Table CX, and requires 500 B.T.U. per pound of coal for its liberation? What will be the producer efficiency and the heating power of the gas?

Prob. 14. A boiler is being fired with coal No. 14 of the general coal table. The heating value of the coal is based upon formula (763). How many pounds of steam at 100 lbs. per square inch gage with 100° superheat may be made per pound of coal from feed-water at 60° F., with a boiler efficiency of 72 per cent?

Prob. 15. A gas engine running on producer gas showed a thermal efficiency 20 per cent. The producer used no steam and the ratio of CO to CO_2 was 5.5. What was the economy in cubic feet of gas per hour per horse-power and in pounds of coal per hour per horse-power if the fuel were all fixed carbon?

Prob. 16. Air and steam are supplied to a gas producer, the gas from which is used in a water-cooled gas engine, direct-connected to an electric generator, the current from which supplies motors and lamps. How is the heat originally in the coal distributed? Estimate quantities.

Prob. 17. When gasolene explodes what is the per cent of gasolene present in the air mixture? What for alcohol?

Prob. 18. An automobile engine is delivering 30 net horse-power and has a thermal efficiency of 15 per cent. What will be its radius of operation at this load on a 10 gallon tank of gasolene for a one-ton car with an average resistance of 20 per cent of its weight?

Prob. 19. A gas engine is running on carburetted water gas. The pressure before ignition is 80 lbs. gage and after ignition is 400 lbs. gage. The temperature before ignition is 800° F. Assuming the gas to be one in the table find the ratio of observed to calculated pressure rise.

Prob. 20. A water gas was found to have this analysis: $H_2 = 50$ per cent: $CO = 40$ per cent; $CH_4 = .5$ per cent; $CO_2 = 5$ per cent; $N_2 = 4.5$ per cent. How does this value of H_2 compare with that as found from the general equation in terms of the carbon monoxide, dioxide ratio? Explain the difference found.

Prob. 21. How much run-of-mine bituminous coal would be burned on a 100-ft. grate with 1-in., $1\frac{1}{2}$-in., and 3-in. draft? What boiler horse-power would be developed in a Heine type with a ratio of heating to grate surface of 50?

Prob. 22. What would be the factor of evaporation for the following cases? What weight of water must be evaporated per hour per boiler horse-power?

Feed Temp.	Pressure	Quality or Superheat	
40	100	.90	
150	100	1.00	
200	100	....	150°
50	200	.95	
50	200	....	100°

Prob. 23. In beehive coke ovens all the gas made is wasted. Considering that the gas made is No. 4 of Table CX, and that 4 cu.ft. are made per pound of coal, calculate the heat-units which are being thrown away per ton of coal? How many horse-power hours could be developed in a gas engine of 20 per cent thermal efficiency, by the gas?

Prob. 24. What are the thermal efficiencies of the two following systems: A steam engine is delivering one horse-power hour for 1.6 lbs. of coal while a gas-engine producer set delivers the same power on 1.1 lbs. of coal. The coal has the following composition: C = .7; O = .025; N = .02; H = .12; S = .05; ash = .13.

Prob. 25. A boiler plant consisting of 12 boilers is supplied by one feed pump. The combined capacity of the boilers is 2000 H.P., the feed temperature is 180° F., pressure 120 lbs. gage, quality 98 per cent. What must be the displacement of the pump, allowing 25 per cent extra for slippage and leakage?

Prob. 26. It is found that for a given boiler test the loss due to moisture in the coal is 2 per cent, that due to CO, etc., in flue gas is 5 per cent, cinder and soot loss per cent, and radiation from fire 2 per cent. What is the furnace efficiency? If the flue loss is 10 per cent, what is the heating surface efficiency and boiler efficiency?

Prob. 27. Coke is used in a producer and may be regarded as 88 per cent C and 12 per cent ash. The steam air ratio in the blast is .3 and the CO to CO_2 ratio in the gas is 5.5. What will be the analysis of the gas by weight and volume and its heating value? What will be the efficiency of the producer?

Prob. 28. It is desired to sell coal No. 23 of the coal table, gas No. 5, of the natural gas table, and oil No. 12 of the oil table, at one dollar per million B.T.U. What must be the price per ton of coal, 1000 cu.ft. of gas and per barrel (50 gals.) of oil?

Prob. 29. A plant requires 1,000,000 B.T.U. per hour in the form of air gas. A test of samples taken from the gas line show an average value for the ratio of CO to CO_2 of 4. How many pounds of coal will be needed per hour by the producer? How many cubic feet of air per hour must be supplied as blast?

Prob. 30. Oil is burned under a battery of boilers. The plant is equipped with the usual heat-saving appliances such as economizer, feed-water heater, condenser, and superheater. Trace the heat in the oil to its ultimate disposition Estimate quantities.

Prob. 31. In the case of a leak of the following gases into a room which would earliest create a condition favorable for an explosion—Acetylene, benzene vapor, Blaugas,

ether, gasolene, methane? Which would pass the danger zone first? Assume mixing means to be present.

Prob. 32. For car lighting Pintsch gas is carried in tanks under a pressure of 10 atmospheres. How does a cubic foot of it at this pressure compare in heating value to an equal volume of kerosene under normal pressure?

Prob. 33. A producer is blasted with an air blast saturated at 150° F., and the CO CO_2 ratio is 5. What would be the composition of the gas by weight and volume and its calorific power? What the thermal efficiency from coke and gas yield per ton of coke?

Prob. 34. A locomotive has a fire-box of 65 sq.ft. area and when running at full speed has an available draft of 6 ins. of water. The coal used is run-of-mine bituminous of 13,500 B.T.U. per pound. With an efficiency of 60 per cent what boiler horse-power could be developed? For a feed-water temperature of 40° F., a boiler pressure of 190 lbs. gage and a quality of 98 per cent how much water would be required for a two-hour run? How much coal would be needed for the same time? Check the horse-power for a 50 to 1 ratio of H.S. to G.S. by the absorption law of the Goss test.

Prob. 35. A barrel of crude oil (50 gals.) is worth \$1.10 at a certain point, while bituminous coal is worth \$6.00 per ton at the same point. If the former gives a boiler efficiency of 85 per cent and the latter 75 per cent, see if it is cheaper to burn oil than coal.

Prob. 36. A ton of coal of which 85 per cent was carbon is fed per hour to a producer. If there were 5 times as much CO in the gas as CO_2 and no H_2, how many pounds and cubic feet of gas were made per hour? How much sensible heat must be carried away in the scrubber water?

Prob. 37. Coal No. 15 of Table CIV is used in a producer. The heat required to drive off the volatile is 600 B.T.U. per pound of coal and the volatile may be taken as having the composition of gas No. 5 of Table CX. For a CO to CO_2 ratio of 5 and a value $S = .3$, what will be the composition and heating value of the gas?

Prob. 38. Air is compressed to as high a pressure as 2000 lbs. per square inch. Should it be possible to compress natural gas No. 2 of the gas table to the same pressure without liquefaction what would be the B.T.U. per 1000 cubic feet?

Prob. 39. What will be the cost per 1000 lbs. of dry steam made at 100 lbs. gage pressure from water at 60° F., when the boiler efficiency is 70 per cent, with (*a*) coal No. 29 of the coal table at \$4.00 per ton, (*b*) oil No. 15 of the oil table at 75 cents per barrel, and (*c*) gas No. 3 of the natural gas table at 20 cents 1000 cubic feet?

Prob. 40. What will be the efficiency of fixed carbon gasification in the case of the following producer gases: Air gas when CO to $CO_2 = 5$, and CO to $CO_2 = 6$, water gas when CO to CO_2 is 8.

Prob. 41. Classify coal Nos. 11, 21, 30, 36, 47, 62, 81, 91, 110, and 121, according to the classification of Frazer, Muck, and Campbell.

Prob. 42. A gas engine is running on producer gas No. 7 of the table. Assuming that the best mixture of gas and air is used, what will be the pressure after explosion for a compression pressure of 125 lbs. gage? What would be the effect of supplying 25 per cent excess air, 25 per cent deficiency of air?

Prob. 43. A large gas engine is running on producer gas No. 5 of the producer gas table. Taking the proper value for the usual compression of such an engine and assuming the compression to be adiabatic with $\gamma = 1.4$, how hot would the gases have to be at the beginning of compression to cause self-ignition?

Prob. 44. A coke producer is blasted with air saturated at 120° F. What is the maximum amount of hydrogen which could exist in the gas formed? Would this be when the CO or CO_2 was high? For the case of CO to $CO_2 = 7$ and the same value of S, what would be the per cent of hydrogen by volume and the heating value of the gas?

Prob. 45. For the combining proportions of air and oil gas No. 1, from the table, what temperature would be required to cause ignition?

Prob. 46. The ratio of heat absorbed by a boiler to that available is represented by equation of the first degree and for this case $(a) = 700$ and $(b) = .62$. What will be the boiler horse-power when 90 per cent of the heat of coal No. 126 of the general table is available for absorption and 1500 lbs. are burned per hour? What the thermal efficiency of heating surface and boiler?

Prob. 47. If the furnace losses are 10 per cent and the flue losses 15 per cent what will be the boiler efficiency and horse-power for a boiler under which coal No. 22 of the general table is burned at the rate of 400 lbs. every 10 minutes? What will be the amount of feed-water required for this boiler if its temperature is 200° F. and the steam pressure and temperature are 150 lbs. gage and 410° F. respectively ?

Prob. 48. Coal No. 7, of the general coal table, is used in a producer. The volatile is the same as gas No. 5 of Table CX and 450 B.T.U. per pound of coal are needed to liberate it. The CO to CO_2 ratio is 6 and the blast is saturated at 140° F. The gas is used in an engine which requires 100 ft. of it per horse-power per hour. The same coal is burned in a steam plant and a horse-power hour secured for 1.4 lbs. Which plant shows the best efficiency?

TABLE CIV

COMPOSITION AND CALORIFIC POWER OF CHARACTERISTIC COALS

No.	Name, Source, Size, Authority.	Total C / Total H	Proximate.				Ultimate.						B.T.U. per Lb.	
			% Moisture.	% Volatile.	% Fixed C.	% Ash.	% H_2.	% C.	% N_2.	% O_2.	% S.	% Ash.	By Calorimeter.	By Calculation.
1	Anth. de la Mare, Grand Couche, France, Mahler	63.3	4.4	2.5	88.4	4.7	1.37	86.56	...	2.97	...	4.7	13442	13420
2	Pa. anth., Treverton, Isherwood	52.31	.84	6.67	85.66	6.83	1.73	90.66	...	.78	...	6.83	14025	14235
3	Anth. Pennsylvania, Mahler	43.35	3.45	2.72	87.92	5.9	2.00	86.46	...	2.2	...	5.9	13471	13787
4	Anth. Hay-Daong (Tonkin), France, Mahler	43.00	3.26	2.24	89.8	4.00	2.00	86.11	...	4.47	...	4.00	13559	13736
5	Bit. Pa., Ormsby, U. S. A., Isherwood	35.5	1.25	39.03	53.72	6.00	2.47	87.57	1.04	2.67	...	6.00	13901	14034
6	Anth. Keban, France, Mahler	30.44	2.8	4.78	86.79	5.45	2.73	85.75	...	2.73	...	5.45	14090	14124
7	Anth. Commentry, France, Mahler	29.32	1.78	2.97	89.87	5.4	2.89	84.93	...	5.00	...	5.4	14130	14104
8	Anth. Blanzy, Ste. Barbe, France, Mahler	28.40	1.76	5.52	56.42	6.3	2.92	82.75	...	6.28	...	6.3	13991	13805
9	Pa. anth., culm, Scranton, U.S.G.S., No. 3	26.7	2.08	2.27	74.32	16.33	2.81	75.21	.80	4.08	.77	16.33	12472	12673
10	Anth. Creusot, France, Mahler	24.4	1.8	10.10	86.65	1.45	3.66	89.39	...	3.7	...	1.45	15127	15220
11	Anth., Grande Combe, Purts Petassus, Fr., Mahler	23.15	.83	11.16	85.74	7.25	3.63	84.07	...	4.22	...	7.25	14130	14428
12	Ruhr coal, Hörde, Germany, Bunte	21.75	.8	13.04	76.32	9.84	3.68	80.08	4.	11*	1.49	9.84	13468	12937
13	Ruhr coal, Bickefeld, Germany, Bunte	21.68	.8	13.23	81.99	3.98	4.04	85.63	3.	56*	1.99	3.98	14539	14983
14	Semi-fat d'Anzin, Fosse St. Marc, France, Mahler	21.37	1.35	13.79	84.16	1.7	4.14	88.47	4.	34*	...	1.7	15106	15377
15	Semi-anth.,Coalhill, Ark., Spadra Bed,U.S.G.S.,No.5	20.7	1.28	12.82	73.69	12.21	3.74	77.29	1.39	3.36	2.01	12.21	13406	13589
16	Semi-fat, Roche-la-Moliere, France, Mahler	20.57	.9	13.39	82.26	4	4.17	85.69	...	5.24	...	4	15151	14991
17	Semi-fat, Aniche, France, Mahler	20.48	.63	11.38	83.99	4	4.2	85.93	...	5.24	...	4	15668	15044
18	Semi-fat, Grande Combe, France, Mahler	20.45	6.1	12.79	82.80	3.8	4.27	87.16	...	4.16	...	3.8	15058	15265
19	Ruhr coal, Fröhlich Morgensonne, Ger., Bunte	20.27	.7	14.12	83.55	1.63	4.41	89.27	2.	74*	1.25	1.63	15194	15707
20	Semi-bit. Pocahontas r. of m., W. Va., Lord & Haas	20.27	.8	18.30	73.65	7.25	4.13	83.75	.85	2.65	.57	7.25	14512	14707
21	Semi-bit. r. of m., Pochontas, Va., Lord & Haas	20.11	.63	18.62	75.12	5.63	4.25	85.46	.85	3.24	.57	5.63	14733	15029
22	Same	19.84	.8	18.30	73.65	7.25	4.22	83.75	.85	3.36	.57	7.25	14512	14762
23	Same coal bed, Zenith, W. Va., U.S.G.S., No. 11	19.60	.8	16.90	70.80	11.50	4.03	79.12	1.04	3.78	.53	11.50	13970	13972
24	Bituminous r. of m., Windber, Pa., U.S.G.S., No. 1.	19.50	1.10	15.80	75.69	7.41	4.20	81.98	1.36	3.56	1.49	7.41	14499	14529
25	Pocahontas run-of-mine, Lord & Haas	19.45	.85	18.60	75.75	4.80	4.39	85.40	.85	3.94	.62	4.80	14906	15112
26	Ruhr coal, Dannenbaum, Germany, Bunte	19.32	1.84	21.04	73.97	3.15	4.38	85.18	4.	39*	1.06	3.15	14544	14968

* O + N

27	Bit. lump, Huntington Bed, Bonanza, Ark., U.S.G.S. No. 2	19.30	.74	16.26	73.66	9.34	4.13	80.03	1.40	3.20	1.90	9.34	13961	14220
28	Semi-bit. r. of m. Pocahontas, Big Sand, W. Va., U.S.G.S. No. 12	19.20	.62	18.05	74.38	6.95	4.36	83.63	1.34	3.03	.69	6.95	14733	14834
29	Fat, Anzin, France, Mahler	19.10	1.1	20.19	73.70	5	4.39	83.75	...	5.76	...	5	14492	14842
30	Bit. lump and nut, Huntington, Ark., U.S.G.S., No. 1	18.90	1.17	17.38	68.12	12.88	4.00	75.68	1.47	4.70	1.27	12.88	13410	13483
31	Bit. lump and slack, Huntington, Jenny Lind, Ark., U.S.G.S. No. 3	18.80	.8	19.75	67.65	11.80	4.07	76.37	1.55	4.91	1.30	11.80	13655	13626
32	Semi-bit., r. of m., New River Bed, Sun, W. Va., U.S.G.S. No. 7	18.80	.76	20.54	73.61	5.09	4.38	82.41	1.05	5.87	1.20	5.09	14857	14689
33	Ruhr coal, Bonifacius, Germany, Bunte	18.78	1.09	16.64	75.67	6.60	4.23	79.60	6.77*		1.71	6.60	13567	14191
34	Fat, Lens, France, Mahler	18.75	1	19	78.45	1.55	4.68	87.74	...	5.03	...	1.55	15505	15600
35	Semi-bit., r. of m., Pocahontas field, Mora, W. Va., U.S.G.S. No. 10	18.70	.65	18.80	75.92	4.63	4.58	85.91	4.07	3.24	.57	4.63	15190	15297
36	Fat, Roche-la-Moliere, France, Mahler	18.47	1.35	21.86	73.79	3	4.63	85.64	...	5.39	...	3	15268	15263
37	Fat, Rouchamp, France, Mahler	18.36	1.2	20.70	68.39	9.7	4.31	79.2	...	5.59	...	9.7	14102	14132
38	Bituminous, Cumberland, Md., Isherwood	18.21	1.25	13	80.75	5	4.75	86.5	...	2.5	...	5	15300	15460
39	Ruhr coal, Oberhausen, Germany, Bunte	18.20	.57	17.32	71.96	10.15	4.36	79.30	4.66*		.96	10.15	13536	14176
40	Ruhr coal, Mathias Stinnes, Germany, Bunte	18.17	1.28	19.99	71.70	7.03	4.49	81.65	4.02*		1.53	7.03	14116	14597
41	Ruhr coal, Lothringen, Germany, Bunte	18.15	1.49	22.23	72.19	4.09	4.55	82.63	6.22*		1.02	4.09	14112	14775
42	Fat, Carmaux, France, Mahler	18.04	1.5	21.10	75.90	1.5	4.72	85.2	7.07*		...	1.5	15084	15252
43	Fat, Mines des Ports (Gard), France, Mahler	17.92	.77	17.42	72.89	8.91	4.37	78.24	4.7*		...	8.91	13763	14028
44	Semi-bit. r. of m., New River Bed, Rush Run, W. Va., U.S.G.S. No. 6	17.80	.64	21.74	72.53	5.09	4.70	83.62	1.70	4.23	.66	5.09	14942	15079
45	Fat, Ste. Etienne, France, Mahler	17.73	1.25	19.75	75	4	4.77	81.55	...	5.43	...	4	15106	14753
46	Ruhr coal, Holland, Bunte	17.46	.99	20.19	73.96	4.86	4.77	83.37	5.00*		1.00	4.86	14220	15017
47	Ruhr coal, Shamrock, Bunte	17.19	1.10	20.44	71.53	6.93	4.79	82.36	3.63*		1.19	6.93	14360	14882
48	Ruhr coal, Westende, Germany, Bunte	17.10	1.18	18.55	72.43	7.84	4.76	81.36	3.33*		1.53	7.84	14233	14720
49	Ruhr coal, general, Bunte	17.02	1.42	28.04	66.92	3.62	4.81	81.96	6.62*		1.57	3.62	14112	14836
50	Ruhr coal, Consolidaten, Germany, Bunte	16.88	1.14	23.71	70.04	5.11	4.85	81.82	6.12*		.96	5.11	14089	14841
51	Ruhr coal, Victoria Mathias, Germany, Bunte	16.83	.98	25.28	70.46	3.28	4.80	80.72	8.66*		1.66	3.28	13746	14650
52	Bituminous, Clover Hill, W. Va., Johnson	16.81	1.34	31.70	56.83	10.13	4.96	83.39	...	1.17	...	10.13	12625	15135
53	Ruhr coal, Zollverein, Germany, Bunte	16.79	1.64	25.03	67.03	6.30	4.75	79.77	5.68*		1.86	6.30	13763	14482
54	Ruhr coal, Vollmond, Germany, Bunte	16.71	.92	21.52	69.75	7.81	4.77	79.76	5.44*		1.30	7.81	13822	14472
55	Ruhr coal, Graf Moltke, Germany, Bunte	16.60	1.51	24.59	63.50	10.40	4.54	75.25	6.72*		1.58	10.40	13025	13759
56	Saar coal, von der Heydt, Germany, Bunte	16.42	3.9	34.40	50.93	10.77	4.21	69.07	10.93*		1.12	10.77	11660	12642
57	Ruhr coal, Dahlbusch, Germany, Bunte	16.41	2.07	24.83	56.11	16.99	4.23	69.49	6.37*		.85	16.79	11912	12704

* $O+N$

TABLE CIV—*Continued*

COMPOSITION AND CALORIFIC POWER OF CHARACTERISTIC COALS

No.	Name, Source, Size, Authority.	Total C / Total H	Proximate.				Ultimate.						B.T.U. per Lb.	
			% Moisture.	% Volatile.	% Fixed C.	% Ash.	% H_2.	% C.	% N_2.	% O_2.	% S.	% Ash.	By Calorimeter.	By Calculation.
58	Ruhr coal, Friedrichs Ernestine, Germany, Bunte	16.33	1.54	28.38	65.12	4.96	4.94	80.59	6.	85	1.12	4.96	13925	14761
59	Saar coal, St. Ingbert, Germany, Bunte	16.32	1.73	29.81	65.63	2.83	4.99	81.49	8.	31	.65	2.83	14036	14903
60	Bituminous, Midlothian, W. Va., Johnson	16.31	.67	33.49	56.4	9.44	5.74	83.62	...	.64	...	9.44	15361	15643
61	Bit., Blue Creek, r. of m., Ala., W. B. Phillips	16.25	...	...	...	...	4.45	72.34	.89	12.25	1.06	10.16	11925	13254
62	Gas coal, Bethune, France, Mahler	16.21	1.2	28.80	65.90	4.1	5.09	82.42	...	7.19	...	4.1	14778	14983
63	Bit. r. of m., Upper Freeport Bed, Bretz, W. Va., U.S.G.S. No. 4	16.10	.98	28.72	61.87	8.43	4.85	78.21	1.5	6.11	.90	8.43	14139	14376
64	Gas coal, Lens, France, Mahler	16.04	1.05	29.55	66.40	3	5.22	83.73	...	7.01	...	3	15111	15343
65	Ruhr coal, Pluto, Germany, Bunte	16.04	1.52	...	...	2.78	5.05	80.97	9.	27*	.41	2.78	13935	14854
66	Ruhr coal, Graf Beust, Germany, Bunte	16.03	.59	24.98	71.14	3.29	5.13	82.24	10.	95*	1.68	3.29	13475	14071
67	Lignitic flaming coal, Blanzy, Ste. Marie, France, Mahler	15.98	3.9	...	...	1.9	1.97	79.38	...	9.86	...	1.9	14158	12740
68	Bit. r. of m., Upper Freeport Bed, Coalton, W. Va., U.S.G.S. No. 5	15.90	.65	20.20	59.97	10.18	4.78	76.36	1.48	6.21	.99	10.18	13828	14063
69	Ruhr coal, Recklinghausen, Germany, Bunte	15.90	1.44	27.18	66.70	4.48	5.11	81.22	6.	32*	1.43	4.48	14168	14967
70	Bit. r. of m., Kanawha Bed, Powelton, W. Va., U.S.G.S. No. 9	15.70	1.01	29.53	62.67	6.79	5.04	79.35	1.63	6.39	.80	6.79	14371	14630
71	Bit. r. of m., Upper Freeport Bed, Richard, W. Va., U.S.G.S. No. 3	15.50	1.00	30.25	58.38	10.37	4.71	76.12	1.44	6.09	1.07	10.37	13736	14091
72	Gas coal, Wigan, Lancashire, Eng., Mahler	15.48	.6	...	...	10.9	5.06	78.38	...	5.06	...	10.9	13970	14467
73	Ruhr coal, Ewald, Germany, Bunte	15.45	2.18	...	...	2.43	5.13	77.27	10.	36*	.63	2.43	13662	14664
74	Lignitic flaming coal, Montoic, France, Mahler	15.42	4.3	...	...	4.8	5.12	76.31	...	9.47	...	4.8	14022	14205
75	Ruhr coal, Mont Cenis, Germany, Bunte	15.40	2.5	25.67	53.96	17.87	4.30	66.20	7.	43*	1.70	17.87	11563	12303
76	Saar coal, Frankenholz, Germany, Bunte	15.37	1.99	37.21	54.38	6.42	5.03	77.40	7.	90*	1.26	6.42	13433	14356
77	Bit. r. of m., Thacker Coal, W. Va., Lord & Haas	15.35	1.40	35.00	51.10	6.50	5.14	78.90	1.42	6.88	1.16	6.50	13982	14637
78	Saar coal, Dodweiler, Germany, Bunte	15.31	1.32	33.19	59.72	5.77	5.11	78.26	8.	57*	.97	5.77	13508	14520

79	Bit. r. of m., Kanawha field, Ansted, W. Va., U.S.G.S. No. 3	15.30	1.60	32.12	58.92	7.36	5.16	78.75	1.38	6.43	.92	7.36	14153	14618
80	Gas coal, Commentry, France, Mahler	15.30	3	...	...	3.4	5.25	80.18	...	8.17	...	3.4	14166	14844
81	Gas coal, Firminy, France, Mahler	15.30	1.23	...	...	3.6	5.30	81.27	...	8.59	...	3.6	14690	15033
82	Saar coal, Friedrichsthal, Germany, Bunte	15.27	2.03	37.14	54.43	6.40	4.98	76.20	9.	28*	1.11	6.40	13217	13146
83	Gas coal, Montrambert, France, Mahler	15.26	.84	...	...	3	5.33	81.27	...	9.55	...	3	14884	15051
84	Bituminous, Beaver Creek, Pa., Lord & Haas	15.25	1.5	31.33	55.42	8.75	4.89	74.6	1.40	6.90	1.96	8.75	13248	13905
85	Saar coal, Heinitz, Germany, Bunte	15.19	2.24	29.17	56.13	12.46	4.64	70.29	9.	61*	.78	12.46	12076	13064
86	Pa. bit., Clinton, Lord & Haas	15.14	2.55	35.6	53.8	8.05	4.86	73.57	1.24	7.87	1.86	8.05	13140	13730
87	Pa. bit., Carnegie, Lord & Haas	15.13	1.45	36.42	56.2	5.93	5.10	77.2	1.68	7.22	1.42	5.93	13844	14388
88	Bit. nut, Thacker coal, W. Va., Lord & Haas	15.10	1.35	36.35	56.25	6.05	5.19	78.40	1.40	7.56	1.40	6.05	14161	14606
89	Lump., bit., Hocking Valley, O., Lord & Haas	15.09	6.72	37.13	50.32	5.83	4.6	69.42	1.46	10.3	1.67	5.83	12388	12952
90	Darlington coal, Middle Kittatinny, Wampum, Pa., Lord & Haas	15.07	.75	38.53	55.77	4.95	5.17	77.93	1.65	7.95	2.35	4.95	14085	14574
91	Hocking Valley, r. of m., Ohio, Lord & Haas	15.04	6.65	34.14	49.54	9.67	4.42	66.5	1.43	10.66	1.67	9.67	11693	12419
92	Saar coal, Püttlingen, Germany, Bunte	15.02	3.93	31.12	53.72	11.23	4.57	68.67	10.	80*	.80	11.23	11759	12770
93	Pa. bit., Turtle Creek, Lord & Haas	15.00	1.08	34.38	56.59	7.95	5.10	76.56	1.67	6.04	1.6	7.95	13734	14292
94	Bit., Pittsburgh coal, Carnegie, Pa., Lord & Haas	14.93	1.07	37.79	55.06	6.08	5.13	76.57	1.64	8.82	1.76	6.08	13977	14317
95	Bit., Wakeford, Ohio, Lord & Haas	14.93	1.55	37.29	53.34	7.82	4.98	74.39	1.4	6.42	3.44	7.82	13426	13982
96	Saar coal, Itzenpletz, Germany, Bunte	14.92	3.61	32.67	59.02	4.70	4.85	75.11	10.	68*	1.05	4.70	13906	13806
97	Darlington coal, Middle Kitatinny, Hoytdale, Pa., Lord & Haas	14.91	1.60	36.40	57.65	4.35	5.22	77.83	1.65	9.38	1.57	4.35	14013	14548
98	Indiana, bit., Noyes, McTaggart & Craven	14.80	8.98	34.49	50.3	6.28	4.76	70.5	1.36	16.29	1.39	6.28	13084	13095
99	Same as 97	14.76	2.70	35.10	53.50	8.70	4.93	73.78	1.34	10.57	1.68	8.70	13041	13580
100	Bit. Pittsburgh coal, Turtle Creek, Pa., Lord & Haas	14.75	1.75	36.20	53.00	9.05	5.05	74.48	1.37	8.39	1.66	9.05	13309	13941
101	Darlington coal, Middle Kitatinny, Beaver Creek, Pa., Lord & Haas	14.74	1.50	34.33	55.42	8.75	5.06	74.60	1.40	8.23	1.96	8.75	13248	13998
102	Saar coal, König, Germany, Bunte	14.74	1.21	37.72	54.32	6.75	5.20	76.69	8.	05*	2.10	6.75	13628	14391
103	Bit. Pittsburgh coal, Carnegie, Pa., Lord & Haas	14.70	1.45	36.42	56.20	5.93	5.26	77.20	1.68	8.51	1.42	5.93	13844	14473
104	Bit. r. of m., Pittsburgh Bed, Kingmont, W. Va., U.S.G.S. No. 1	14.70	1.35	36.92	55.36	6.37	5.26	78.31	1.55	7.61	.90	6.37	14164	14645
105	Darlington coal, Middle Kitatinny, Wampum, Pa., Lord & Haas	14.70	.70	36.80	55.85	6.65	5.22	76.81	1.62	8.52	1.18	6.65	13748	14413
106	Upper Freeport coal, Yellow Creek, O., Lord & Haas	14.69	1.23	38.72	50.88	9.17	4.98	73.15	1.40	7.41	3.89	9.17	13615	13845
107	Bit. Pittsburgh coal, Turtle Creek, Pa., Lord & Haas	14.67	1.08	34.48	56.59	7.95	5.22	76.56	1.67	7.00	1.60	7.95	13734	14364
108	Lignitic flaming coal, Decazeville, Bourran, France, Mahler	14.64	3.5	...	...	4.85	5.17	74.73	...	11.76	...	4.85	13344	14013

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TABLE CIV—*Continued*

COMPOSITION AND CALORIFIC POWER OF CHARACTERISTIC COALS

No.	Name, Source, Size, Authority.	Total C / Total H	Proximate.				Ultimate.						B.T.U. per l.b.	
			% Moisture.	% Volatile.	% Fixed C.	% Ash.	% H_2.	% C.	% N_2.	% O_2.	% S.	% Ash.	By Calorimeter.	By Calculation.
109	Saar coal, Kohlwald, Germany, Bunte	14.61	4.05	35.74	54.56	5.65	5.03	73.48	10.	86	.93	5.65	12580	13773
110	Bit. r. of m., Straight Creek, East Field, Kent, U.S.G.S. No. 1	14.60	1.92	36.56	57.08	4.44	5.36	78.31	1.85	8.80	1.24	4.44	14319	14690
111	Bit. r. of m., Wheatcroft, Western Field, Ken., U.S.G.S. No. 4	14.60	2.54	36.08	46.79	14.59	4.53	66.5	1.28	8.43	4.67	14.59	12294	12804
112	Upper Freeport coal, Palestine, Ohio, Lord & Haas	14.55	2.45	36.60	52.70	8.25	5.06	73.64	1.24	9.47	2.34	8.25	13212	13712
113	Bit., lump and nut, Warrior Field, Horse Creek, Ala., U.S.G.S. No. 1	14.50	1.55	33.10	53.71	12.64	4.96	72.16	1.66	7.85	.73	12.64	12958	13572
114	Bit. lump and nut, Warrior Field, Carbon Hill, Ala., U.S.G.S. No. 2	14.50	2.58	33.15	51.74	12.53	4.79	69.24	1.55	10.87	1.02	12.53	12449	13025
115	Bit., lump and nut, Pittsburgh Bed, West Mineral, Kan., U.S.G.S. No. 5	14.50	1.84	37.40	54.97	10.79	4.96	71.90	1.09	7.40	3.86	10.79	13199	13629
116	Upper Freeport coal, East Palestine, Ohio, Lord & Haas	14.46	.82	34.98	52.65	11.89	4.88	70.58	1.24	7.70	3.65	11.89	12796	13381
117	Upper Freeport coal, Waterford, Ohio, Lord & Haas	14.45	1.55	37.29	53.34	7.82	5.15	74.39	1.40	7.80	3.44	7.82	13426	14108
118	Lignitic flaming coal, Blanzy, Ste. Eugenie, France, Mahler	14.45	1.7	...	...	7.8	5.14	75.27	...	10.08	...	7.8	13474	14063
119	Pa. bit., Creedmoor, Lord & Haas	14.45	1.09	38.91	51.14	8.86	5.15	74.45	1.60	7.05	1.80	8.86	13493	14032
120	Saar coal, Reden, Germany, Bunte	14.43	3.45	34.25	56.08	6.62	5.06	72.98	11.	30*	.99	6.62	12548	13722
121	Bit. r. of m., Weir-Pittsburgh Bed, Clarksburg, W. Va., U.S.G.S. No. 2	14.40	1.46	40.14	50.50	7.90	5.09	74.44	1.37	7.70	3.50	7.70	13860	14053
122	Mahoney coal, Salemville, Ohio, Lord & Haas	14.37	3.15	35.00	50.95	10.90	4.95	71.13	1.23	9.93	1.86	10.90	12722	13419
123	Ohio bit., Cambridge, Lord & Haas	14.35	2.43	37.79	50.36	9.42	4.92	70.61	1.44	8.17	3.01	9.42	12758	13372
124	Darlington coal, Middle Kitatinny, Clinton, Pa., Lord & Haas	14.31	2.55	35.50	53.80	8.05	5.14	73.57	1.24	10.14	1.86	8.05	13140	13901
125	Bit., Hartshorne, Hartshorne, Ind. Terr., U.S.G.S., No. 2	14.30	1.70	37.19	49.79	11.32	5.00	71.49	1.72	8.91	1.56	11.32	12969	12511

126	Upper Freeport coal, Palestine Ohio, Lord & Haas.	14.26	2.15	36.70	50.70	10.45	5.00	71.29	1.34	9.28	2.64	10.45	13845	12524
127	Same	14.22	1.65	37.45	51.32	9.58	5.15	73.23	1.47	8.82	1.75	9.58	13194	13853
128	Upper Freeport coal, Steubenville, O., Lord & Haas.	14.21	1.47	39.23	51.54	7.66	5.26	74.73	1.44	8.06	2.85	7.66	13507	14181
129	Pittsburgh coal, Carnegie, Pa., Lord & Haas	14.16	1.08	37.67	52.00	9.25	5.19	73.50	1.44	8.08	2.54	9.25	13313	13966
130	Pittsburgh coal, North Mansfield, Pa., Lord & Haas	14.16	2.10	36.20	52.65	9.05	5.15	73.91	1.23	8.89	1.77	9.05	13237	13962
131	Upper Freeport coal, Salemville, Ohio, Lord & Haas	14.15	2.80	36.30	52.80	8.10	5.13	72.62	1.23	9.92	3.00	8.10	13097	13793
132	Same, New Galilee, Ohio, Lord & Haas	14.15	2.30	36.70	52.30	8.70	5.20	73.57	1.35	8.94	2.24	8.30	13322	13943
133	Pittsburgh coal, Creedmore, Pa., Lord & Haas	14.11	1.09	38.91	51.14	8.86	5.27	74.45	1.60	8.02	1.80	8.86	13493	14115
134	Bit. r. of m., McAlester Bed, Edwards, Ind. Ter., U.S.G.S. No. 3	14.10	3.45	37.45	47.82	11.28	4.85	68.18	1.50	10.57	3.63	11.28	12469	13004
135	Saxon brown coal, Bachbei, Ziebingen, Ger., Bunte.	14.03	45.33	28.33	24.45	1.99	2.56	35.93	13.	20*	.99	1.99	5339	6818
136	Bit., lump and nut and slack, Weir-Pittsburgh Bed, Yale, Kan., U.S.G.S. No. 2	13.90	2.23	31.87	47.63	18.27	4.56	63.14	.94	6.69	6.40	18.27	11880	12203
137	Lignitic flaming coal, Decazeville, Tramont, France, Mahler	13.89	1.58	...	...	2.8	5.43	75.27	...	14.92	...	2.8	13489	14240
138	Darlington coal, Middle Kitaninny, Wampum, Pa., Lord & Haas	13.85	2.85	37.50	50.85	8.80	5.25	72.82	1.33	8.55	3.25	8.80	13147	13903
139	Ala. bit., Mary Lee (top), W. B. Phillips	13.77	...	...	...	...	5.54	76.18	.55	8.93	1.15	8.9	13223	14484
140	Bit. r. of m., Marion, Ill., U.S.G.S., No. 3	13.70	5.96	30.29	52.16	11.59	4.92	67.30	1.43	12.99	1.77	11.59	12103	12850
141	Lignite, Trifail, Styria, Mahler	13.68	.71	...	...	4.75	4.78	65.45	...	24.30	...	4.75	11311	12427
142	Upper Freeport coal, Cambridge, O., Lord & Haas.	13.61	2.43	37.79	50.36	9.42	5.19	70.61	1.44	10.33	3.01	9.42	12758	13545
143	Lump and slack, Henryetta Bed, Ind. Ter., U.S.G.S. No. 1	13.60	3.87	35.73	50.05	10.35	5.14	69.85	1.29	11.38	1.99	10.35	12620	13355
144	Cannel gas coal, Middrie, France, Mahler	13.51	3.95	...	...	4.7	6	76.55	...	8.8	...	4.7	13865	14781
145	Lump and fine, Laddsville, Wapello Co., Iowa, U.S.G.S. No. 1	13.40	5.21	31.76	46.51	16.52	4.61	61.80	.97	10.90	5.20	16.52	11392	12012
146	Saxon brown coal, Greppen, Germany, Bunte	13.33	22.85	38.64	27.45	11.06	3.25	43.77	17.	54*	1.93	11.06	6966	8355
147	Jackson Co., Ohio, West, Lord & Haas	13.30	8.65	34.30	55.40	11.65	5.37	71.42	1.43	19.49	.64	11.65	12231	13669
148	Lignite, Vaurigard, France, Mahler	13.24	3.14	...	...	6.75	4.51	59.80	25.	8*	...	6.75	9965	11433
149	Mix. bit., Osage River, Johnson	13.20	1.67	41.83	51.16	5.34	6.20	81.85	...	1.98	...	5.34	13744	15663
150	Upper Bavarian coal, Haushmner, Grobkohle, Germany, Bunte	13.13	7.37	36.13	43.19	13.31	4.4	58.01	12.	02*	4.87	13.31	10121	11331
151	Lump, McAlester Bed, Lehigh, Ind. Ter., U.S.G.S. No. 4	13.10	4.91	37.79	43.90	13.40	4.84	63.21	1.38	13.15	4.02	13.40	11389	12287
152	Hocking Valley coal, lump, Middle Kitatinny, Ohio, Lord & Haas	12.98	6.72	37.13	50.32	5.83	5.35	69.42	1.46	16.27	1.67	15.83	12220	12953
153	Jackson Co., Ohio, North, Lord & Haas	12.95	8.45	34.09	54.09	3.38	5.50	71.20	1.45	17.71	.76	3.38	12402	13729

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TABLE CIV—*Concluded*

COMPOSITION AND CALORIFIC POWER OF CHARACTERISTIC COALS

No.	Name, Source, Size, Authority.	Total C / Total H	Proximate.				Ultimate.						B.T.U. pr Lb.	
			% Moisture.	% Volatile.	% Fixed C.	% Ash.	% H_2.	% C.	% N_2.	% O_2.	% S.	% Ash.	By Calorimeter.	By Calculation.
154	Lump, Atchison Field, Atchison, Kan., U.S.G.S. No. 4	12.90	3.57	37.00	46.80	12.63	5.04	65.02	1.07	7.91	8.33	12.63	12337	12845
155	Jackson Co., Ohio, Center, Lord & Haas	12.90	8.26	35.15	53.49	4.10	5.43	70.05	1.49	17.09	1.84	4.10	12303	13553
156	R. of m., Rich Hill Field, Sprague, Mo., U.S.G.S. No. 1	12.90	3.50	35.35	40.77	20.38	4.64	60.00	.99	8.64	5.53	20.38	11144	11760
157	Hocking Valley coal, r. of m., Middle Kitaninny, Ohio, Lord and Haas	12.88	6.65	34.14	49.54	9.67	5.16	66.50	1.43	15.57	1.67	9.67	11736	12868
158	Indiana bit., Lancaster, Noyes, McTaggart & C.	12.84	12.66	37.44	47.22	2.68	5.56	71.41	1.54	18.42	.62	2.68	10645	13783
159	Jackson Co., Ohio, South, Lord & Haas	12.77	7.02	37.66	50.82	4.48	5.49	70.12	1.50	16.96	1.45	4.48	12348	13684
160	Same, Eastern District, Lord & Haas	12.75	8.50	37.75	57.10	2.65	5.55	70.79	1.46	18.60	1.95	2.65	12337	14035
161	Hocking Valley coal, lump, Middle Kitaninny, Ohio, Lord & Haas	12.72	6.40	36.05	49.05	8.50	5.36	68.18	1.44	15.09	1.43	8.50	12132	13224
162	Lump and nut, Earlington, Ky., Western Field, U.S.G.S. No. 2	12.70	5.36	38.99	46.27	9.38	5.33	67.64	1.25	12.68	3.72	9.38	12312	13219
163	Indiana bit., New Pittsburgh, Noyes, McTaggart & Craven	12.62	6.83	39.92	39.93	13.3	5.07	62.88	1.01	13.06	7.46	13.3	11134	12518
164	R. of m., Earlington, Ky., U.S.G.S. No. 3	12.60	5.85	36.90	46.96	10.29	5.27	66.75	1.43	12.66	3.60	10.27	12292	12048
165	R. of m., Barnett, Morgan Co., Mo., U.S.G.S. No. 4	12.60	5.39	44.91	44.47	5.23	5.77	72.45	.75	10.25	5.55	5.23	13528	14258
166	Saxon brown coal, Alfred, Germany, Bunte	12.58	36.26	33.39	23.27	7.08	3.29	41.41	9.	84	3.12	7.08	6734	8143
167	Upper Bavarian coal, Pernsberger, Fördeköhle, Germany, Bunte	12.48	10.18	34.69	33.08	22.05	3.83	47.78	10.	92	5.24	22.05	8478	10379
168	R. of m., Hamilton, Marion Co., Iowa, U.S.G.S. No. 2	12.40	4.25	37.02	41.47	16.99	4.84	60.36	1.46	11.65	5.20	16.99	11182	11538
169	R. of m., Coffeen, Montgomery Co., Ill., U.S.G.S. No. 6	12.30	5.13	32.68	47.46	14.73	4.88	60.51	1.23	14.20	4.45	14.73	11158	11921
170	R. of m., Booneville, Warwick Co., Ind., U.S.G.S. No. 2	12.30	6.24	37.49	42.76	13.51	5.11	62.97	1.25	12.56	4.60	13.51	11538	12442

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171	Lump, Altoona, Polk Co., Iowa, U.S.G.S. No. 3....	12.30	4.52	40.96	38.99	15.53	4.93	60.62	.93	11.16	6.83	15.53	11356	12079
172	Lump and nut, Belleville Field, O'Fallen, Ill., U.S.G.S. No. 1.....	12.20	6.28	38.92	41.06	13.72	5.09	62.01	1.07	13.86	4.25	13.72	11144	12295
173	R. of m., Cambria Field, Cambria, Wyo., U.S.G.S. No. 2.....	12.20	2.73	37.61	37.40	22.26	4.54	55.29	.80	12.94	4.17	22.26	10364	10962
174	Saxon brown coal, Marie Louise, Germany, Bunte..	12.17	29.27	35.83	27.61	7.29	3.73	45.40	10.	72	3.59	7.29	7742	9007
175	Saxon brown coal, Menzelwitz, Fortschritt, Germany, Bunte.....	12.10	27.13	37.28	27.27	8.32	3.67	44.47	14.	69	1.72	8.32	7306	6763
176	R. of m., Mildred, Sull. Co., Ind., U.S.G.S. No. 1..	11.90	8.66	34.86	42.67	13.81	5.20	62.20	1.22	14.99	2.58	13.81	11405	12302
177	Lump, Belleville Field, Troy, Ill., U.S.G.S. No. 4...	11.60	11.40	32.45	44.30	11.85	5.33	61.79	1.17	18.52	1.34	11.85	10911	12271
178	Lignite, Terre de Feu, France, Mahler.....	11.59	16.5	...	...	18.5	3.86	46.16	...	14.98	...	18.5	8795	9054
179	Lump, Centerville, Appanoose Co., Iowa, U.S.G.S. No. 4.....	11.50	10.03	37.27	41.22	11.48	5.31	61.25	.94	16.56	4.46	11.48	11227	12304
180	Black lignite and washed nut, Red Lodge, Mont., U.S.G.S. No. 1.....	11.50	9.05	36.70	43.03	11.22	5.25	60.41	1.36	20.00	1.76	11.22	10777	12039
181	Lignite, Josefsziche in Schwanenkirchen, Ger., Bunte	11.34	40.35	26.65	18.11	15.89	2.54	28.80	9.	55	2.87	15.89	4640	5843
182	R. of m., Bevier Field, Mo., U.S.G.S. No. 2.....	11.30	9.14	34.53	39.02	17.31	4.96	56.25	.99	15.19	5.30	17.31	10451	12400
183	R. of m., Charlton, Lucas Co., Ia., U.S.G.S. No. 5.	11.20	9.22	32.71	44.52	13.55	5.35	59.89	1.22	16.57	3.42	13.55	10989	13191
184	Black lignite, lump and slack, Gallop Field, New Mexico, U.S.G.S. No. 1.....	11.20	10.86	35.14	46.90	7.10	5.73	64.34	1.05	21.14	6.64	7.10	11435	13097
185	Same, slack, U.S.G.S. No. 2.....	11.20	8.13	34.82	37.63	19.22	5.65	56.71	.98	16.74	1.30	19.22	10202	11725
186	Lignite, Pressdorf von Hofmark Steinfels, Germany, Bunte.....	11.00	16.47	52.28	25.97	5.28	4.48	49.31	4.	07	.39	5.28	7855	9903
187	Brown lignite, Hoyt, Wood Co., Tex., U.S.G.S. No. 2	10.90	10.66	39.42	40.11	9.81	5.28	57.31	1.06	25.83	.71	9.81	9904	11563
188	Black lignite, Boulder Field, Lafayette, Colo., U.S.G.S. No. 1.....	10.60	13.49	37.11	43.03	6.37	5.75	61.13	1.22	24.95	.58	6.37	10791	11401
189	Peat of Pschorrschwige, Germany, Bunte.....	10.59	29.14	41.26	22.69	6.91	3.66	38.76	21.	27	.26	6.91	5909	7867
190	Brown lignite, Lehigh Field, N. Dak., U.S.G.S. No. 1	10.1	15.42	38.73	33.61	12.24	5.22	52.66	.71	27.15	2.02	12.24	9061	10904
191	Brown lignite, Williston Field, N. Dak., U.S.G.S. No. 2.....	9.8	16.70	37.10	39.49	6.71	5.61	55.16	.91	30.98	.63	6.71	9491	11449
192	Peat of Ostrach, Germany, Bunte.....	9.78	11.06	52.78	27.64	5.52	4.70	45.93	29.	18	.61	5.52	7187	9553
193	Black lignite, Sheridan Field, Wyo., U.S.G.S. No. 1.	9.6	17.69	37.96	39.56	4.79	6.09	58.41	1.09	29.99	.63	4.79	10355	12212
194	Brown lignite, Houston Co., Tex., U.S.G.S. No. 1..	9.4	13.40	42.75	29.00	14.85	5.51	52.06	.95	25.33	1.04	14.85	9358	10991
195	Cannel coal, Albertite, Nova Scotia, Kent.....	9.04	...	...	...	...	9.14	82.67	8.	19	...	...	....	17564
196	Cannel coal, Tasmanite, Tasmania, Kent.....	7.62	...	...	...	...	10.41	79.34	4.	93	5.32	...	....	17872

*O+N

TABLE CV

COMBUSTIBLE AND VOLATILE OF COALS, LIGNITES, AND PEAT

No.	Name, Source, Size, Authority.	Combustible = Coal Less (Moisture + Ash).									B.T.U. Lb. Vol. = B.T.U. Lb. Comb., Less C in Comb. × 14544 divided by Vol. in Comb.	Ratio of Fixed C. / Vol.
		Ratio of Total C. / H.	Proximate.		Ultimate.			B.T.U. per Pound.				
			Volatile, %	Fixed C, %	H_2 %	C %	O_2 N_2 S %	Total	From Fixed C.	From Volatile.		
1	Anth. de la Mure, Grande Couche, France, Mahler	63.5	2.75	97.25	1.50	95.24	3.26	14789	14144	645	23455	35.36
2	Anthracite, Treverton, Penn., U. S. A., Isherwood	52.31	7.23	92.77	1.87	98.18	.91	15190	13492	1698	23480	12.82
3	Anthracite, Pennsylvania, U. S. A., Mahler	43.35	3.00	97.00	2.20	95.37	2.43	14816	14107	709	23600	32.33
4	Anthracite, Hoy Daong (Tonkin), France, Mahler	43.00	3.17	96.83	2.16	92.86	4.99	14618	14083	535	16877	30.54
5	Bit., Pa., Ormsby, U. S. A., Isherwood	35.50	42.20	57.8	2.66	94.50	2.84	14971	8406	6565	15557	1.37
6	Anthracite, Keban, France, Mahler	30.44	5.20	94.80	3.07	93.40	5.39	15357	13788	1569	30173	18.23
7	Anthracite, Commentry, France, Mahler	29.32	3.19	96.81	3.12	91.49	5.39	15221	14080	1141	35768	30.34
8	Anth., Blanzy, Ste. Barbe, France, Mahler	28.40	6.00	94.00	3.17	90.60	6.83	14725	13671	1054	17567	15.66
9	Anthracite culm, Scranton, U.S.G.S. No. 3	26.7	8.91	91.09	3.37	89.88	6.75	15256	13248	2008	22536	10.22
10	Anthracite Creusot, France, Mahler	24.41	10.44	89.56	3.78	92.93	3.83	15635	13026	2609	24990	8.57
11	Anth., Grande Combe Purts Petassus, Fr., Mahler	23.15	6.71	93.29	3.95	91.46	4.59	15372	13568	1804	26885	13.90
12	Ruhr coal, Hörde, Germany, Bunte	21.75	14.6	85.4	4.12	89.62	6.26	15072	12421	2651	18157	5.74
13	Ruhr coal, Bickefeld, Germany, Bunte	21.68	13.9	86.1	4.24	89.93	5.83	15269	12522	2747	19763	6.19
14	Semi-fat d'Anzin, Fosse St. Marc, France, Mahler	21.37	14.08	85.92	4.27	91.26	4.48	15401	12496	2905	20632	6.10
15	Semi-anth., Spadra Bed, Coalhill, Ark., U.S.G.S. No. 5	20.7	14.82	85.18	4.26	88.04	7.70	15497	12388	3109	20978	5.75
16	Semi-fat Roche-la-Moliere, France, Mahler	20.57	14.00	86.00	4.38	90.11	5.51	15781	12508	3273	23378	6.14
17	Semi-fat, Auiche, France, Mahler	20.48	11.93	88.07	4.40	90.10	5.49	15901	12809	3092	25918	7.38
18	Semi-fat, Grande Combe, France, Mahler	20.45	13.38	86.62	4.46	91.19	4.35	15761	12598	3163	23639	6.47
19	Ruhr coal, Fröhliche Morgensonne, Ger., Bunte.	20.27	14.5	85.5	4.51	91.40	4.09	15556	12435	3121	21524	5.89
20	Semi-bit., Pocahontas, W. Va., Lord & Haas	20.27	19.9	80.1	4.50	91.00	4.50	15782	11650	4132	20763	4.03
21	Semi-bit., r. of m., Pocahontas, Va., Lord & Haas	20.11	19.86	80.14	4.94	90.56	4.50	15717	11655	4062	20413	4.04
22	Same	19.84	19.9	80.10	4.55	90.3	5.15	15782	11650	4132	20763	4.03
23	Same bed, Zenith, W. Va., U.S.G.S. No. 11	19.60	19.27	80.73	4.55	89.4	6.05	15923	11741	4182	21702	4.19
24	Bit., r. of m., Windber, Pa., U.S.G.S. No. 1	19.50	17.27	82.73	4.54	88.54	6.92	15847	12032	3815	22090	4.79
25	Run of mine, Pocahontas, Virginia, Lord & Haas	19.45	19.71	80.29	4.61	89.71	5.68	15799	11674	4122	20913	4.07

26	Ruhr coal, Dannenbaum, Germany, Bunte	19.32	22.2	77.80	4.62	89.65	5.73	15309	11315	3994	17991	3.50
27	Bit., lump, Huntington Bed, Bonanza, Ark., U.S.G.S No. 2	19.30	18.08	81.92	4.55	88.28	7.17	15520	11914	3606	19945	4.53
28	Semi-bit., r. of m., Pocahontas, Field, Big Sandy, W. Va., U.S.G.S. No. 12	19.20	19.53	80.47	4.68	89.88	5.44	15930	11704	4226	21638	4.12
29	Fat, Anzin, France, Mahler	19.10	21.51	78.49	4.67	89.20	6.14	15433	11416	4017	18675	3.65
30	Bit., lump and nut, Huntington Bed, Ark., U.S.G.S. No. 1	18.90	20.33	79.67	4.59	86.87	8.54	15684	11587	4097	20152	3.92
31	Bit., lump and slack, Huntington Bed, Jenny Lind, Ark., U.S.G.S. No. 3	18.80	22.60	77.40	4.61	86.59	8.80	14010	11257	2753	12181	3.42
32	Semi-bit., r. of m., New River Bed, Sun, W. Va., U.S.G.S. No. 7	18.80	21.81	78.19	4.61	86.83	8.56	15780	11372	4408	20211	3.58
33	Ruhr coal, Bonifacius, Germany, Bunte	18.78	18.00	82.00	4.59	86.23	8.57	14697	11926	2771	15394	4.55
34	Fat, Lens, France, Mahler	18.75	19.50	80.50	4.80	90.03	5.17	15910	11707	4203	21554	4.13
35	Semi-bit., r. of m., Pocahontas Field, Mora, W. Va., U.S.G.S. No. 10	18.70	19.85	80.15	4.80	90.08	5.12	16030	11657	4373	22030	4.04
36	Fat, Roche-la-Moliere, France, Mahler	18.49	22.85	77.15	4.84	89.53	5.63	15961	11221	4740	20744	3.38
37	Fat, Rouchamp, France, Mahler	18.36	23.23	76.77	4.84	88.89	6.27	15835	11165	4670	20103	3.30
38	Bituminous, Cumberland, Md., U. S. A., Isherwood	18.21	13.86	86.14	5.08	92.36	2.56	16320	12528	3792	27359	6.22
39	Ruhr coal, Oberhausen, Germany, Bunte	18.20	19.40	80.6	4.88	88.82	6.30	15161	11722	3439	17727	4.15
40	Ruhr coal, Mathias Stinnes, Germany, Bunte	18.17	21.8	78.2	4.90	89.05	6.05	16865	11374	5491	25193	5.58
41	Ruhr coal, Lothringen, Germany, Bunte	18.15	23.5	76.5	4.82	87.52	7.66	14944	11126	3818	16247	3.26
42	Fat, Carmaux, France, Mahler	18.04	21.75	78.25	4.87	87.84	7.30	15550	11381	4169	19168	3.60
43	Fat, Mines-des-Ports (Gard), France, Mahler	17.92	19.29	80.71	4.84	86.52	8.64	15601	11738	3863	20080	4.19
44	Semi-bit., r. of m., New River Field, Rush Run, W. Va., U.S.G.S. No. 6	17.80	23.06	76.94	4.95	88.11	6.94	15850	11190	4660	20208	3.34
45	Fat, Ste. Etienne, France, Mahler	17.73	20.84	79.16	5.03	89.23	5.47	15943	11513	4430	21257	3.80
46	Ruhr coal, Holland, Germany, Bunte	17.46	21.40	78.60	5.07	88.53	6.38	15103	11432	3671	17154	3.67
47	Ruhr coal, Shamrock, Germany, Bunte	17.19	22.20	77.80	5.21	89.55	5.24	15613	11315	4298	19360	3.50
48	Ruhr coal, Westende, Germany, Bunte	17.10	20.4	79.6	5.23	89.43	4.74	16179	11576	4603	22559	3.90
49	Ruhr coal, general, Germany, Bunte	17.02	29.5	70.5	5.07	86.31	8.62	14861	10254	4607	15620	2.39
50	Ruhr coal, Consolidaten, Germany, Bunte	16.88	25.3	74.7	5.17	87.27	6.53	15028	10864	4164	16458	2.95
51	Ruhr coal, Victoria Mathias, Germany, Bunte	16.83	26.4	73.6	5.01	84.31	10.68	14357	10704	3653	13837	2.79
52	Bituminous, Clover Hill, W. Va., U. S. A., Johnson	16.81	35.81	64.19	5.65	84.20	13.57	14423	9336	5087	14205	1.79
53	Ruhr coal, Zollverein, Germany, Bunte	16.79	27.2	72.8	5.16	86.65	8.19	14950	10588	4362	16037	2.68
54	Ruhr coal, Vollmond, Germany, Bunte	16.71	23.6	76.4	5.23	87.39	7.38	15311	11111	4200	17792	3.24
55	Ruhr coal, Graf Moltke, Germany, Bunte	16.60	27.1	72.9	5.15	85.43	9.42	14786	10603	4183	15435	2.69

TABLE CV—*Continued*

COMBUSTIBLE AND VOLATILES OF COALS, LIGNITES, AND PEATS

No.	Name, Source, Size, Authority.	Combustible = Coal Less (Moisture + Ash.)									B.T.U. Lb. Vol. = B.T.U. Lb. Comb. Less C in Comb. × 14544 divided by Vol. in Comb.	Ratio of Fixed C. / Vol.
		Ratio of Total. C. / H.	Proximate.		Ultimate.			B.T.U. per Pound.				
			Volatile, %	Fixed C, %	H_2 %	C %	O_2, N_2, S. %	Total	From Fixed C.	From Volatile.		
56	Saar coal, von der Heydt, Germany, Bunte	16.42	40.3	59.7	4.93	80.95	14.12	13664	8683	4981	12360	1.49
57	Ruhr coal, Dahlbusch, Germany, Bunte	16.41	30.7	69.3	5.23	85.85	8.92	14717	10079	4638	15107	2.26
58	Ruhr coal, Friedrich Ernestine, Germany, Bunte	16.33	30.5	69.5	5.28	86.19	8.53	14893	10108	4785	15688	2.28
59	Saar coal, St. Ingbert, Germany, Bunte	16.32	31.2	68.8	5.23	85.38	9.39	14707	10006	4701	15067	2.21
60	Bit., Midlothian, W. Va., U. S. A., Johnson	16.31	37.25	62.75	6.30	93.02	0.72	17088	9126	7962	21374	1.68
61	Bit., r. of m., Blue Creek, Ala., U. S. A., Phillips	16.25	....	....	4.95	80.52	14.53					
62	Gas coal, Bethune, France, Mahler	16.21	30.41	69.59	5.37	87.03	7.60	15602	10121	5481	18024	2.29
63	Bit., r. of m., Upper Freeport Bed, Bretz, W. Va., U.S.G.S. No. 4	16.10	31.70	68.30	5.29	85.42	9.29	15607	9934	5673	17893	2.15
64	Gas coal, Lens, France, Mahler	16.04	30.80	69.20	5.44	87.26	7.30	15748	10064	5684	18451	2.25
65	Ruhr coal, Pluto, Germany, Bunte	16.04	....	....	5.28	84.60	10.12					
66	Ruhr coal, Graf Beust, Germany, Bunte	16.03	26.0	74.0	5.13	82.24	12.63	14018	10762	3256	12519	2.84
67	Lignitic flaming coal, Blanzy Purts, Ste. Marie, France, Mahler	15.98	31.35	68.65	5.27	84.26	10.46	15031	9984	5047	16099	2.19
68	Bit., r. of m., Upper Freeport Bed, Coalton, W. Va., U.S.G.S. No. 5	15.90	32.75	67.25	5.32	85.02	9.66	15514	9780	5734	17505	2.05
69	Ruhr coal, Recklinghausen, Germany, Bunte	15.90	28.9	71.1	5.43	86.33	8.24	15075	10341	4734	16380	2.46
70	Bit., r. of m., Kanawha Bed, Powelton, W. Va., U.S.G.S. No. 9	15.70	32.03	67.97	5.41	85.13	9.46	15586	9886	5700	17793	2.12
71	Bit., r. of m., Upper Freeport Bed, Richard, W. Va., U.S.G.S. No. 3	15.50	34.13	65.87	5.47	84.93	9.60	15498	9580	5918	17336	1.93
72	Gas coal, Wigan, Lancashire, Eng., Mahler	15.48	31.64	68.36	5.72	88.57	5.72	15782	9942	5840	18454	2.16
73	Ruhr coal, Ewald, Germany, Bunte	15.45	....	....	5.38	83.10	11.52					
74	Lignitic flaming coal, Montoic, France, Mahler	15.42	37.07	62.93	5.64	86.95	10.42	15426	9153	6273	16919	1.70
75	Ruhr coal, Mont Cenis, Germany, Bunte	15.40	32.2	67.8	5.40	83.14	11.46	14521	9861	4660	14470	2.10
76	Saar coal, Frankenholz, Germany, Bunte	15.37	40.6	59.4	5.50	84.51	9.99	14666	8639	6027	14842	1.46
77	Bit., r. of m., Thacker coal, W. Va., Lord & Haas	15.35	38.0	62.00	5.49	84.39	10.12	15161	9017	6144	16168	1.63

78	Saar coal, Dudweiler, Germany, Bunte	15.31	35.7	64.3	5.50	84.23	10.27	14603	9352	5251	14709	1.80
79	Bit., r. of m., Kanawha Bed, Ansted, W. Va., U.S.G.S. No. 8	15.30	35.28	64.72	5.57	85.01	9.42	15545	9413	6132	17381	1.83
80	Gas coal, Commentry, France, Mahler	15.30	39.96	60.04	5.60	85.66	8.73	15134	8732	6402	16021	1.50
81	Gas coal, Firminy, France, Mahler	15.30	32.02	67.98	5.58	85.39	91.13	15431	9887	5544	17314	2.12
82	Saar coal, Friedrichsthal, Germany, Bunte	15.27	40.6	59.4	5.45	82.21	11.34	14438	8638	5800	14283	1.46
83	Gas coal, Montrambert, France, Mahler	15.26	34.27	65.73	5.54	84.52	9.94	15440	9560	5880	17158	1.92
84	Bit., Beaver Creek, Pa., U. S. A., Lord & Haas	15.25	38.25	61.75	5.45	85.12	9.43	14761	8981	5780	15111	1.61
85	Saar coal, Heinitz, Germany, Bunte	15.14	34.20	65.80	5.44	82.38	12.18	14157	9570	4587	13410	1.92
86	Bit., Clinton, Pa., U. S. A., Lord & Haas	15.14	39.83	60.18	5.44	82.29	12.27	14697	8752	5945	14923	1.51
87	Bit., Carnegie, Pa., U. S. A., Lord & Haas	15.13	39.32	60.68	5.51	83.35	11.14	14947	8824	6123	15569	1.54
88	Bit., nut, Thacker coal, W. Va., Lord & Haas	15.10	39.25	60.75	5.52	83.45	11.03	15293	8834	6459	16453	1.55
89	Bit., lump, Hocking Valley, Ohio, Lord & Haas	15.09	42.46	57.54	5.26	79.38	15.36	14165	8369	5796	13650	1.36
90	Darlington coal, Middle Kitaninny, Wampum, Pa., Lord & Haas	15.07	40.86	59.14	5.44	81.99	12.57	14861	8601	6260	15320	1.45
91	Bit., r. of m., Hocking Valley, Ohio, Lord & Haas	15.04	40.80	59.20	5.28	79.47	16.25	13973	8610	5363	13140	1.45
92	Saar coal, Püttlingen, Germany, Bunte	15.02	36.7	63.3	5.39	80.94	13.67	13860	9206	4654	12681	1.72
93	Bit., Turtle Creek, Pa., U. S. A., Lord & Haas	15.00	37.79	62.21	5.61	84.16	10.23	15097	9048	6049	16007	1.65
94	Bit., Pittsburgh coal, Carnegie, Pa., Lord & Haas	14.93	40.70	59.30	5.46	81.53	13.01	15034	8625	6409	15747	1.46
95	Bit., Waterford, Ohio, U. S. A., Lord & Haas	14.93	41.15	58.85	5.50	82.08	12.42	14814	8559	6255	15200	1.43
96	Saar coal, Itzenplitz, Germany, Bunte	14.92	35.6	64.4	5.29	81.92	12.79	14075	9366	4709	13227	1.81
97	Darlington coal, Middle Kitaninny, Hoytdale, Pa., Lord & Haas	14.91	38.70	61.30	5.46	81.37	13.17	14900	8915	5985	15465	1.58
98	Bit., Indiana, U. S. A., Keyes, McTaggart & Craven	14.80										
99	Same as 97	14.76	39.62	60.38	5.34	78.92	14.74	14719	8780	5939	14985	1.52
100	Bit., Pittsburgh coal, Turtle Creek, Pa., Lord & Haas	14.75	40.58	59.42	5.55	81.89	12.56	14938	8642	6296	15515	1.46
101	Darlington coal, Middle Kitaninny, Beaver Creek, Pa., Lord & Haas	14.74	38.25	61.75	5.55	81.75	12.70	14722	8981	5741	15009	1.61
102	Saar coal, König, Germany, Bunte	14.74	41.0	59.0	5.65	83.32	11.03	14806	8581	6225	15183	1.44
103	Bit., Pittsburgh coal, Carnegie, Pa., Lord & Haas	14.70	39.32	60.68	5.59	82.07	12.34	14947	8824	6123	15570	1.54
104	Bit., r. of m., Pittsburgh Bed, Kingmont, W. Va., U.S.G.S. No. 1	14.70	40.01	59.99	5.61	85.65	10.74	15348	8725	6623	16553	1.50
105	Darlington coal, Middle Kitaninny, near Wampum, Pa., Lord & Haas	14.70	39.72	60.28	5.6	82.28	12.12	14839	8767	6072	15287	1.52
106	Upper Freeport coal, Yellow Creek, Ohio, Lord & Haas	14.69	43.21	56.79	5.48	80.54	13.98	14994	8260	6734	15584	1.31

TABLE CV—*Continued*

COMBUSTIBLE AND VOLATILES OF COALS, LIGNITES, AND PEATS

No.	Name, Source, Size, Authority.	Combustible = Coal Less (Moisture + Ash.)									B.T.U. Lb. Vol. = B.T.U. Lb. Comb., Less C in Comb. × 14544 divided by Vol. in Comb.	Ratio of Fixed C. / Vol.
		Ratio of Total, C/H.	Proximate.		Ultimate.			B.T.U. per Pound.				
			Volatile, %	Fixed C, %	H_2 %	C %	O_2, N_2, S %	Total.	From Fixed C.	From Volatile.		
107	Bit., Pittsburgh coal, Turtle Creek, Pa., Lord & Haas	14.67	37.86	62.14	5.67	83.17	11.16	15080	9037	6043	15958	1.64
108	Lignitic flaming coal, Decazeville (Bourran), France, Mahler	14.64	39.4	60.6	5.64	81.53	12.83	14549	8813	5736	14556	1.54
109	Saar coal, Köhlwald, Germany, Bunte	14.61	39.6	60.4	5.57	81.37	13.06	13931	8783	5148	12995	1.53
110	Bit., r. of m., Straight Creek, Eastern Field, Ky., U.S.G.S. No. 1	14.6	39.04	60.96	5.62	81.94	12.44	15292	8865	6427	16460	1.56
111	Bit., r. of m., Wheatcroft, Western Field, Ky., U.S.G.S. No. 4	14.6	43.54	56.46	5.30	77.86	16.84	14835	8212	6623	15215	1.30
112	Upper Freeport coal, Palestine, O., Lord & Haas	14.55	40.99	59.01	5.52	80.26	14.22	14803	8582	6221	15177	1.44
113	Bit., lump and nut, Warrior Field, Horse Creek, Ala., U.S.G.S. No. 1	14.5	37.41	62.59	5.68	82.60	11.72	15100	9103	5997	16030	1.67
114	Bit., lump, nut, and pea, Warrior Field, Carbon Hill, Ala., U.S.G.S. No. 2	14.5	39.05	60.95	5.48	79.16	15.36	13911	8865	5046	12922	1.56
115	Bit., lump and nut, Pittsburgh, Bed W. Mineral, Kan., U.S.G.S. No. 5	14.50	37.08	62.92	5.56	80.6	13.84	14602	9151	5451	14701	1.70
116	Upper Freeport coal, East Palestine, Ohio, Lord & Haas	14.46	39.92	60.08	5.54	80.1	14.36	14603	8738	5865	14692	1.51
117	Same coal, Waterford, Ohio, Lord & Haas	14.45	41.18	58.85	5.59	80.7	13.71	14814	8559	6255	15189	1.43
118	Lignitic flaming coal, Blanzy Purts, Ste. Eugenie, France, Mahler	14.45	35.8	64.2	5.60	83.17	11.14	14886	9337	5549	15500	1.79
119	Bit., Creedmoor, Pa., Lord & Haas	14.45	43.21	56.79	5.72	82.68	11.60	14983	8260	6723	15559	1.31
120	Saar coal, Reden, Germany, Bunte	14.43	37.9	62.1	5.60	80.79	13.61	13891	9032	4859	12820	1.64
121	Bit., r. of m., Weir-Pittsburgh Bed, Clarksburg, W. Va., U.S.G.S. No. 2	14.40	44.28	55.72	5.52	80.83	13.65	15291	8104	7187	16231	1.26

[illegible]	[illegible]	11.37	40.72	59.28	5.50	79.85	14.61	14727	8622	6105	14993	1.46
123	Bit., Cambridge, Ohio, Lord & Haas	14.35	42.87	57.13	5.58	80.11	14.31	14473	8309	6164	14388	1.33
124	Darlington coal, Middle Kitaninny, Clinton, Pa., Lord & Haas	14.31	39.82	60.18	5.59	80.01	14.40	14719	8752	5967	14982	1.51
125	Bit., Hartshorne Bed, Hartshorne, Ind. Ter., U.S.G.S. No. 2	14.30	42.76	57.24	5.63	80.62	13.75	14910	8325	6585	15400	1.34
126	Upper Freeport coal, East Palestine, Ohio, Lord & Haas	14.26	41.99	58.01	5.58	79.62	14.81	15010	8437	6573	15654	1.38
127	Same	14.22	42.19	57.81	5.7	80.99	13.31	14863	8408	6455	15300	1.37
128	Upper Freeport coal, Steubenville, O., Lord & Haas	14.21	43.22	56.78	5.7	80.93	13.37	14881	8258	6623	15324	1.31
129	Pittsburgh coal, Carnegie, Pa., Lord & Haas	14.16	42.01	57.99	5.71	81.0	13.29	14846	8434	6412	15263	1.38
130	Pittsburgh coal, North Mansfield, Pa., Lord & Haas	14.16	40.74	59.26	5.66	81.27	13.17	14898	8619	6279	15412	1.45
131	Upper Freeport coal, Salineville, Ol., Lord & Haas	14.15	40.74	59.26	5.59	79.02	15.39	14688	8619	6069	14897	1.45
132	Upper Freeport coal, New Galilee, O., Lord & Haas	14.15	41.24	58.76	5.7	80.58	13.72	14697	8547	6150	14915	1.42
133	Pittsburgh coal, Creedmoor, Pa., Lord & Haas	14.11	43.21	56.79	5.78	81.69	12.53	14983	8260	6723	15561	1.31
134	Bit., r. of m., McAlester Bed, Edwards, Ind. Ter., U.S.G.S. No. 3	14.10	43.92	56.08	5.47	76.87	17.65	14623	8156	6467	14724	1.28
135	Saxon brown coal, Bach bei Ziebingen, Ger., Bunte	14.03	57.8	42.2	5.69	68.89	25.42	10134	6138	3996	6913	.73
136	Bit., lump, nut, slack, Weir, Pittsburgh Bed, Yale, Kan., U.S.G.S. No. 2	13.90	40.09	59.91	5.58	77.26	17.16	14943	8713	6230	15540	1.49
137	Lignitic flaming coal, Decazeville (Tramont), France, Mahler	13.89	41.23	58.77	5.67	78.72	15.61	14110	8548	5562	13490	1.42
138	Darlington coal, Middle Kitaninny, Wampum, Pa., Lord & Haas	13.85	42.44	57.56	5.76	79.85	14.39	14881	8372	6509	15337	1.36
139	Bit., Mary Lee (top), Ala., W. B. Phillips	13.77										
140	Bit., r. of m., Marion, Ill., U.S.G.S. No. 3	13.70	36.74	63.26	5.57	76.12	18.31	14679	9200	5479	14910	1.72
141	Lignite, Trifail (Styria), Mahler	13.68	50.34	49.66	5.06	69.24	25.71	11909	7223	4686	9311	.99
142	Upper Freeport coal, Cambridge, O., Lord & Haas	13.61	42.87	57.13	5.75	77.95	16.32	14474	8300	6165	14381	1.33
143	Lump and slack, Henryetta, Bed Ind. Terr., U.S.G.S. No. 1	13.60	41.65	58.35	5.73	77.92	16.35	14712	8486	6226	14948	1.40
144	Cannel gas coal, Niddrie, France, Mahler	13.51	....	....	6.57	83.80	9.63	.....	.....	.....	.....	.....
145	Lump and fine, Laddsvale, Wapello Co., Iowa, U.S.G.S. No. 1	13.40	40.58	59.42	5.52	74.03	20.45	12905	8642	4263	10505	1.46
146	Saxon brown coal, Greppen, Germany, Bunte	13.33	58.5	41.5	4.92	66.62	29.46	10540	6036	4504	7610	.71
147	Jackson Co., Ohio, West, Lord & Haas	13.30	38.24	61.76	6.08	80.84	13.08	14189	8982	5207	13617	1.62
148	Lignite, Vaurigard, France, Mahler	13.24	49.95	50.05	5.01	66.36	28.63	10937	7279	3658	7323	1.002
149	Bit., Osage River, Miss., Johnson	13.20										

TABLE CV—*Continued*

COMBUSTIBLE AND VOLATILES OF COALS, LIGNITES, AND PEATS

No.	Name, Source, Size, Authority.	[Combustible = Coal Less (Moisture + Ash.) Ratio of Total, $\frac{C}{H}$.	Proximate. Volatile, %	Proximate. Fixed C, %	Ultimate. H_2 %	Ultimate. C %	Ultimate. O_2 N_2 S %	B.T.U. per Pound. Total.	B.T.U. per Pound. From Fixed C.	B.T.U. per Pound. From Volatile.	B.T.U. Lb. Vol. = B.T.U. Lb. Comb., Less C in Comb. × 14544 divided by Vol. in Comb.	Ratio of Fixed C. / Vol.
150	Upper Bavarian coal, Haushamer Grobkohle, Germany, Bunte	13.13	45.5	54.5	5.57	73.14	21.29	12759	7926	4833	10622	1.20
151	Lump, McAlester Bed, Lehigh, Ind. Ter., U.S.G.S. No. 4	13.10	46.26	53.74	5.59	72.99	21.42	13941	7816	6125	13240	1.16
152	Hocking Valley coal, lump, Middle Kitaninny, O., Lord & Haas	12.98	42.48	57.54	6.34	82.48	11.16	14166	8369	5797	13646	1.35
153	Jackson Co., Ohio, North, Lord & Haas	12.95	38.65	61.35	6.35	82.2	11.45	14162	8923	5239	13555	1.59
154	Lump, Atchison Field, Atchison, Kan., U.S.G.S. No. 4	12.90	44.2	55.8	5.77	74.42	19.81	14721	8116	6605	14943	1.26
155	Jackson Co., Ohio, Center, Lord & Haas	12.90	40.1	59.89	6.32	81.55	12.13	14078	8710	5368	13386	1.49
156	R. of m., Rich Hill Field, Sprague, Mo., U.S.G.S. No. 1	12.90	46.44	53.56	5.83	75.36	19.04	14640	7790	6850	14750	1.15
157	Hocking Valley coal, r. of m., Middle Kitaninny, O., Lord & Haas	12.88	40.80	59.20	6.43	82.78	10.79	13972	8610	5362	13142	1.45
158	Bit., Lancaster, Ind., Noyes, McTaggart & Craven	12.84	44.22	55.78	5.71	73.28	21.15	12573	8113	4460	10088	1.26
159	Jackson Co., Ohio, South, Lord & Haas	12.77	42.58	57.42	6.42	81.99	11.59	14148	8351	5797	13615	1.35
160	Same, Eastern District	12.75	39.8	60.2	6.35	81.05	12.60	13006	8755	4251	10681	1.51
161	Hocking Valley coal, lump, Middle Kitaninny, O., Lord & Haas	12.72	42.36	57.64	6.58	83.66	9.76	13905	8383	5522	13036	1.36
162	Lump and nut, Earlington, Ky., Western Field, U.S.G.S. No. 2	12.70	45.73	54.27	5.88	71.65	19.47	14441	7893	6548	14319	1.18
163	Bit., New Pittsburgh, Ind., Noyes, McTaggart & Craven	12.62	49.99	50.01	5.85	72.53	24.83	13943	7273	6670	13343	1.002
164	Same as 162, r. of m., U.S.G.S. No. 3	12.60	44.00	56.00	5.87	74.41	19.72	14657	8145	6512	14800	1.27
165	R. of m., Barnett, Morgan Co., Mo., U.S.G.S. No. 4	12.60	50.25	49.75	6.09	76.45	17.46	15134	7236	7898	15717	.99
166	Saxon brown coal, Alfred, Germany, Bunte	12.58	58.9	41.1	5.81	73.08	21.11	11884	5978	5906	10027	.697

167	Upper Bavarian coal, Penzberger Förderkohle, Germany, Bunte	12.48	51.2	48.8	5.65	70.50	23.85	12509	7097	5412	10570	.953
168	R. of m., Hamilton, Marion Co., Ia., U.S.G.S. No. 2	12.40	47.17	52.83	5.83	72.72	21.45	14246	7683	6563	13911	1.12
169	R. of m., Coffeen, Montgomery Co., Ill., U.S.G.S. No. 6	12.30	40.78	59.22	5.72	70.96	23.32	13923	8613	5310	13021	1.45
170	R. of m., Booneville, Warwick Co., Ind., U.S.G.S. No. 2	12.30	46.72	53.28	5.91	72.81	21.28	14377	7749	6628	14186	1.14
171	Lump, Altoona, Polk Co., Iowa, U.S.G.S. No. 3	12.30	51.23	48.77	5.84	81.76	22.40	14203	7093	7110	13878	.952
172	Lump and nut, Belleville Field, O'Fallen, Ill., U.S.G.S. No. 1	12.20	48.66	51.34	5.90	71.87	22.23	13933	7476	6466	13288	1.055
173	R. of m., Cambria, Field, Cambria, Wyo., U.S.G.S. No. 2	12.20	50.14	49.86	5.84	71.12	23.04	13817	7252	6565	13093	.994
174	Saxon brown coal, Marie Louise, Germany, Bunte	12.17	56.5	43.5	5.88	71.57	22.55	12203	6327	5876	10400	.769
175	Saxon brown coal, Menselwitz Fortschritt, Germany, Bunte	12.10	57.80	42.2	5.69	68.89	25.42	11318	6137	5181	8963	.73
176	R. of m., Mildred, Sullivan Co., Ind., U.S.G.S. No. 1	11.90	44.96	55.04	6.03	72.17	21.80	13030	8005	5025	11176	1.22
177	Lump, Belleville Field, Troy, Ill., U.S.G.S. No. 4	11.60	42.28	57.72	6.05	70.10	23.85	14216	8395	5821	13768	1.365
178	Lignite, Terre de Feu, France, Mahler	11.59	52.77	47.23	5.94	71.01	23.05	12670	6869	5801	10993	.895
179	Lump, Centerville, Appanoose Co., Iowa, U.S.G.S. No. 4	11.50	47.50	52.50	6.00	69.19	24.81	14303	7635	6668	14036	1.105
180	Black lignite, washed nut, Red Lodge, Mont., U.S.G.S. No. 1	11.50	46.03	53.97	5.92	68.04	26.04	13516	7849	5667	13212	1.17
181	Lignite, Josefszeche in Schwanenkirchen, Germany, Bunte	11.39	58.6	41.4	5.81	65.81	28.38	10366	6021	4345	7415	.706
182	R. of m., Bevier Field, Mo., U.S.G.S. No. 2	11.30	46.95	53.05	6.00	68.03	25.99	15245	7716	7529	16036	1.15
183	R. of m., Charlton, Lucas Co., Iowa, U.S.G.S. No. 5	11.20	42.35	57.55	6.19	69.28	24.53	14228	8384	5844	13797	1.36
184	Black lignite, lump and slack, Gallop Field, New Mexico, U.S.G.S. No. 1	11.20	42.83	57.17	6.17	69.33	24.60	13938	8315	5623	13129	1.334
185	Same, slack, U.S.G.S. No. 2	11.20	47.93	51.80	7.03	70.29	22.33	14081	7534	6547	13659	1.08
186	Lignite, Presstorf von Hofmark, Steinfels, Germany, Bunte	11.00	66.80	33.2	5.73	63.02	31.25	10038	4829	5209	7798	.497
187	Brown lignite, Hoyt, Wood Co., Tex., U.S.G.S. No. 2	10.90	49.56	50.44	5.85	63.55	30.60	12453	7336	5117	10325	1.017
188	Black lignite, Boulder Field, Lafayette, Colo., U.S.G.S. No. 1	10.60	46.31	53.69	6.14	65.29	28.57	13465	7808	5657	12213	1.16
189	Peat of Pschorrschwaige, Germany, Bunte	10.59	64.50	35.50	5.72	60.61	33.67	9242	5163	4081	6324	.55
190	Brown lignite, Lehigh Field, N. Dak., U.S.G.S. No. 1	10.10	53.54	46.46	5.96	59.99	34.05	12525	6757	5768	10773	.867

TABLE CV—*Concluded*

COMBUSTIBLE AND VOLATILES OF COALS, LIGNITES, AND PEATS

No.	Name, Source, Size, Authority.	Combustible = Coal Less (Moisture + Ash.)									B.T.U. Lb. Vol. = B.T.U. Lb. Comb., Less C in Comb. × 14544 divided by Vol. in Comb.	Ratio of Fixed C. / Vol.
		Ratio of Total. C. / H.	Proximate.		Ultimate.			B.T.U. per Pound.				
			Volatile, %	Fixed C, %	H_2 %	C %	O_2, N_2, S %	Total.	From Fixed C.	From Volatile.		
191	Brown lignite, Williston Field, N. Dak., U.S.G.S. No. 2	9.80	48.44	51.56	6.01	59.13	34.86	12391	7499	4892	10099	1.064
192	Peat of Ostrach, Germany, Bunte	9.78	65.6	34.4	5.84	57.11	37.05	8937	5003	3934	5997	.524
193	Black lignite, Sheridan Field, Wyo., U.S.G.S. No. 1	9.60	48.97	51.03	6.39	61.35	33.31	13357	7422	5935	12120	1.04
194	Bohemian peat, Mahler	9.60	68.93	31.07	5.96	57.21	36.82	10625	4519	6106	8858	.45
195	Brown lignite, Houston Co., Tex., U.S.G.S. No. 1	9.40	59.58	40.42	6.54	61.14	32.32	....	5879	....	.....	.678
196	Cannel coal, Obertite, Nova Scotia, Kent	9.04										
197	Oak wood, Mahler	8.57	....	....	5.88	50.44	43.69	8440	....	....	.....	.....
198	Norway pine, Mahler	8.48	....	....	6.02	51.08	42.90	8690	....	....	.....	
199	Cannel coal, Tasmanite, Tasmania, Kent	7.62										.....
200	Cellulose, Mahler	7.20	....	....	6.17	44.44	49.39	7560	....	....	.....	.....

TABLE CVI

PARAFFINES (C_nH_{2n+2}) FROM PENNSYLVANIA PETROLEUM

	Name.	Formula.	Boiling-point.		Specific Gravity at 32° F.	Molecular Weight Approx.	Composition by Weight.	
			° C.	° F.			% C.	% H.
Gas	Methane	CH_4	....	.....	.....	16	75	25
Gas	Ethane	C_2H_6	....	.....	.446	30	80.12	19.98
Gas	Propane	C_3H_8	−25	−13	.536	44	81.84	18.16
Gas	Butane	C_4H_{10}	0	32	.60	58	82.76	17.24
Gas	Pentane normal	C_5H_{12}	38	100.4	.627 at 57 .628	72	83.33	16.67
Liquid	Pentane iso	C_5H_{12}	30	86	.658 at 68	72	83.33	16.67
Liquid	Hexane normal	C_6H_{14}	69	156.2	.664	86	83.76	16.24
Liquid	Hexane iso	C_6H_{14}	61	141.8	.683 at 68	86	83.76	16.24
Liquid	Heptane normal	C_7H_{16}	97.5	207.5	.699	100	84.00	16.00
Liquid	Heptane iso	C_7H_{16}	91	195.8	.702 at 68	100	84.00	16.00
Liquid	Octane normal	C_8H_{18}	125	257	.703	114	84.21	15.79
Liquid	Octane iso	C_8H_{18}	118	224.4	.718 at 68	114	84.21	15.79
Liquid	Nonane	C_9H_{20}	136	276.8	.741	128	84.38	15.62
Liquid	Decane	$C_{10}H_{22}$	173	343.4	.73 at 68 .757	142	84.51	15.49
Liquid	Endecane	$C_{11}H_{24}$	182	359.6	.774 at −15 .765	156	84.62	15.38
Liquid	Dodecane	$C_{12}H_{26}$	198	388.4	.773 at −10 .776	170	84.71	15.29
Liquid	Tridecane	$C_{13}H_{28}$	216	420.8	.792	184	84.78	15.22
Liquid	Tetradecane	$C_{14}H_{30}$	238	460.4	.775 at 39	198	84.85	15.15
Liquid	Pentadecane	$C_{15}H_{32}$	258	496.4	.....	212	84.92	15.08
Liquid	Hexadecane	$C_{16}H_{34}$	280	536.	.775 at 64	226	84.96	15.04
Liquid	Octodecane	$C_{18}H_{38}$	....	.....	.....	254	85.02	14.98
Liquid	Eicosane	$C_{20}H_{42}$	205	401.	.778 at 99	282	85.10	14.90
Liquid	Tricosane	$C_{23}H_{48}$	234	453.	.779 at 118	324	85.18	14.82
Liquid		$C_{25}H_{52}$	....	.....	.....	352	85.23	14.77
Solid	Paraffine (myricle)	$C_{27}H_{56}$	....	.....	.....	380	85.26	14.74
Solid	Paraffine (ceryl)	$C_{30}H_{62}$	370	698	.....	422	85.31	14.69

ETHYLENES (C_nH_{2n}) AND NAPHTHALENES ($C_nH_{2n-6}+H_6$) FROM RUSSIAN PETROLEUM

Ethylenes Naphthalenes.							
Ethylene	C_2H_4	gas	.....	.....	28	85.7	14.3
Propylene	C_3H_6	gas	.....	.....	42	85.7	14.3
Butylene	C_4H_8	1	33.8	.635	56	85.7	14.3
Amylene	C_5H_{10}	36	96.8	.....	70	85.7	14.3
Hexylene	C_6H_{12}	70	158	.76	84	85.7	14.3
Heptylene	C_7H_{14}	84	183.2	.714	98	85.7	14.3
Octylene	C_8H_{16}	119	246.2	.733	112	85.7	14.3
Oct. Naphthalene	$C_8H_{10}+H_6$	136	276.5	.771	106+6	85.7	14.3
Nonylene	C_9H_{18}	....	.....	.....	126	85.7	14.3
Diamylene	$C_{10}H_{20}$	161	321.8	.777	140	85.7	14.3
	$C_{11}H_{22}$	180	356	.....	154	85.7	14.3
	$C_{12}H_{24}$	....	.....	.....	168	85.7	14.3
Dodeca Naphthalene	$C_{12}H_{18}+H_6$	196	384.8	.803	162+6	85.7	14.3
	$C_{14}H_{28}$	240	464.	.....	196	85.7	14.3
Triamylene	$C_{15}H_{30}$	248	478.4	.....	210	85.7	14.3
Tetraamylene	$C_{20}H_{40}$	over 390	over 734	.....	280	85.7	14.3

TABLE CVII

CALORIFIC POWER OF MINERAL OILS BY CALORIMETER AND CALCULATION BY DENSITY FORMULA OF SHERMAN AND KROPFF

No.	Class of Oil.	Sp.gr. at 15° C.	Degree Bé.	B.T.U. per Pound.		Error. %
				Calo-rimeter.	Calcul. S.&K.Form.	
1	Gasolene	.71	67.2	21120	20938	− .91
2	Gasolene	.7175	65.1	20389	20854	+2.33
3	Gasolene	.72	64.4	20527	20726	+ .99
4	Gasolene	.7709	51.6	20038	20314	+1.38
5	Kerosene	.7830	48.8	20018	20206	+ .92
6	California, refined	.7850	48.35	20014	20194	+ .89
7	West Virginia, crude	.7945	46.2	20030	20098	+ .33
8	Kerosene	.795	46.1	20135	20094	− .20
9		.7964	45.8	20236	20082	− .76
10	Ohio, crude	.8048	44.0	20068	20010	− .29
11	Pennsylvania, crude	.8059	43.7	20057	19998	− .29
12	California, refined	.8080	43.2	19802	19979	+ .88
13	Kansas, refined	.8103	42.8	19963	19962	± .00
14	West Virginia, crude	.8237	40.0	19766	19850	+ .42
15	California, refined	.8248	39.7	19827	19838	+ .05
16	West Virginia, crude	.8261	39.5	20021	19830	− .05
17		.8321	38.2	19757	19778	+ .11
18	Pennsylvania, crude	.8324	38.2	19782	19778	− .02
19	Ohio	.8418	36.3	19710	19702	− .04
20	Indian Territory	.8421	36.25	19795	19698	− .48
21		.8436	36.0	19924	19690	−1.17
22	Indian Territory	.8466	35.4	19685	19666	− .09
23	California, refined	.8500	34.7	19715	19638	− .38
24	Kansas, crude	.8510	34.5	19724	19630	− .47
25		.8514	34.45	19701	19630	− .35
26		.8534	34.05	19784	19610	− .86
27	Kansas, crude	.8580	33.20	19389	19578	+ .95
28	Illinois, crude	.8597	32.8	19379	19562	+ .95
29		.8616	32.5	19741	19550	− .95
30	California, refined	.8640	32.05	19555	19530	− .12
31	Pennsylvania, fuel oil	.8648	31.9	19656	19526	− .65
32		.8660	31.65	19555	19516	− .19
33	Pennsylvania, fuel oil	.8670	31.5	19530	19510	− .10
34	Indian Territory	.8690	31.1	19534	19494	− .20
35		.8708	30.8	19654	19482	− .88
36		.8712	30.7	19614	19478	− .68
37	Kansas, crude	.8745	30.1	19354	19454	+ .50
38	Pennsylvania, fuel oil	.8773	29.6	19428	19434	+ .03
39	Kansas, crude	.8800	29.0	19447	19410	− .19
40		.8807	29.0	19435	19410	− .47
41		.8810	28.9	19435	19406	− .15

TABLE CVII—*Continued*

CALORIFIC POWER OF HYDROCARBON OILS BY CALORIMETER AND CALCULATION BY DENSITY FORMULA OF SHERMAN AND KROPFF

No.	Class of Oil.	Sp.gr. at 15 °C.	Degrees Bé.	B.T.U. per Pound.		Error, %
				Calorimeter.	Calcul. S.&K. Form	
42	.....	.8820	28.75	19643	19400	−1.22
43	Kansas, crude.....	.8828	28.7	19249	19396	+ .73
44	.....	.8833	28.5	19474	19390	− .42
45	Indian Territory.....	.8860	28.0	19454	19370	− .42
46	.....	.8862	28.0	19372	19370	− .01
47	Indian Territory.....	.8900	27.3	19418	19342	− .39
48	Texas, crude.....	.8914	27.1	19242	19332	+ .45
49	.....	.8970	26.1	19355	19294	− .31
50	.....	.9007	25.4	19359	19267	− .47
51	.....	.9050	24.7	19228	19238	+ .05
52	.....	.9065	24.45	19352	19228	− .63
53	Kansas, crude.....	.9066	24.4	19089	19226	+ .69
54	.....	.9087	24.1	19282	19213	− .35
55	Kansas, crude.....	.9114	23.6	19303	19194	− .55
56	Texas, crude.....	.9137	23.2	19028	19178	+ .76
57	Texas, crude.....	.9153	22.95	19246	19168	− .39
58	Texas, crude.....	.9155	22.9	19008	19166	+ .80
59	California, crude.....	.9158	22.9	18572	19166	+2.58
60	Fuel oil.....	.9170	22.7	19103	19157	+ .28
61	California, crude.....	.9179	22.5	18779	19150	+1.94
62	California, crude.....	.9182	22.5	18985	19149	+ .83
63	Texas, crude.....	.9336	20.0	19080	19048	− .16
64	California, crude.....	.9644	15.2	18589	18858	+1.42

TABLE CVIII

PROPERTIES OF MINERAL OILS

No.	Name and Source.	Density.			Ultimate Analysis.				Prox. H_2O.	B.T.U. per Pound.			
		Sp.Gr.	F.	Bé.	C.	H_2.	O_2+N_2.	S.		By Calorimeter.	By S. & K. Form	High Value.	Low Value.
1	Coal tar, Paris gas works	1.044	....	6.112	82	7.6	....	....	....	.....	18595	16533	15870
2	Ogaio, crude	.985	32	12.135	87.1	10.4	2.5	....	....	18146	18735	18983	18065
3	California, fuel	.966	60	14.93	81.52	11.61	6.92	.55	....	18667	18847	18926	17903
4	California, Whittier	.9637	60	15.28	....	....	....	.845	8.71	18518	18861		
5	California, Whittier	.9629	60	15.39	....	....	....	.84	8.82	18596	18866		
6	California and Bakersfield fuel	.962	60	15.53	84.43	10.99	3.99	.59	....	.....	18871	18976	18005
7	Barbadoes fuel	.958	....	16.114	....	....	....	....	....	17718	18894		
8	California crude	.9572	60	16.24	86.3	16.7	....	.8	....	18646	18900	12723	21254
9	Russian residue	.956	....	16.43	....	....	....	....	....	19440	18907		
10	Hanover	.955	32	16.505	86.2	11.4	2.4	....	....	.....	18910	19488	18493
11	California crude	.9533	60	16.85	85.75	11.3	....	.67	....	18797	18924	19356	18363
12	California, Whittier and Los Angeles	.953	60	16.9	....	....	....	.98	4.93	18714	18926		
13	California, Whittier and Los Angeles	.9529	60	16.915	....	....	....	.955	4.62	18754	18926		
14	Texas fuel	.945	....	18.155	....	....	....	....	....	19242	18976		
15	California	.943	60	18.47	....	....	....	.735	1.06	18677	18989		
16	California, Whittier	.9417	60	18.67	....	....	....	.975	1.06	18626	18997		
17	California	.9410	60	18.783	....	....	....	1.010	.74	18705	19001		
18	California	.9407	60	18.829	....	....	....	.96	.42	18657	19003		
19	Baku Russia heavy	.938	60	19.26	86.6	12.3	1.1	....	....	19440	19021	20052	18978
20	Borneo	.936	....	19.58	....	....	....	....	....	18831	19033		
21	Baku, Russia residue	.928	60	20.95	87.1	11.7	1.2	....	....	22628	19088	19761	18739
22	Petroleum residue, Baku	.928	32	20.95	87.1	11.7	1.2	....	....	19832	19088	19761	18739
23	Petroleum residue, Baku	.928	....	20.95	87.1	11.7	1.2	....	....	19260	19088	19761	18739
24	Texas, Beaumont fuel	.926	60	21.25	83.26	12.41	3.83	.5	....	.....	19100	19654	18570
25	Texas, Beaumont crude	.924	60	21.56	84.60	10.90	2.87	1.63	....	.....	19112	18977	18025
26	Java	.923	60	21.71	87.1	12	.9	....	....	19496	19119	19943	18095

28	California	.920	60	22.17	84.0	12.7	1.2	....	....	.....	19136	19917	18807
29	Bumah fuel	.920	....	22.17	....	....	....	....	....	18864	19136		
30	Pennsylvania	.914	70	23.18	86.1	13.9	.....	.06	.....	.....	19176	20949	19735
31	Texas, Beaumont, Richardson & Wallace.	.912	70	23.514	85.03	12.3	.92	1.75	....	.....	19191	19894	18810
32	Pechelbronn	.912	32	23.514	86.9	11.8	1.3	....	....	17474	19191	19792	18761
33	Pechelbronn	.912	....	23.514	86.9	11.8	1.3	....	....	17816	19191	19792	18761
34	Shale oil, Ardeche	.911	....	23.682	80.3	11.5	8.2	....	....	16283	19196	18452	17647
35	Hanover crude	.892	32	26.95	80.4	12.7	6.9	....	....	.....	19328	19393	18283
36	Pechelbronn	.892	....	26.95	85.7	12	2.3	....	....	18036	19328	19739	18691
37	Ohio distillate	.887	60	27.84	84.2	13.1	2.7	....	....	18718	19364	20188	19044
38	Pennsylvania, crude, heavy	.886	32	28.01	84.9	13.7	1.4	....	....	19210	19370	20654	19457
39	Pennsylvania, heavy	.886	60	28.01	84.9	13.7	1.04	....	....	19224	19370	20654	19457
40	American residue	.886	32	28.01	....	....	....	....	....	19627	19370		
41	West Galicia	.885	32	28.095	85.3	12.6	2.1	....	....	18416	19374	20045	18944
42	Russian, crude, light	.884	32	28.38	86.3	13.6	.1	....	....	22628	19385	20796	19608
43	Balachany	.882	32	28.73	87.4	12.5	.1	....	....	21060	19400	20289	19198
44	Shale oil	.875	....	30.005	....	....	....	....	....	18217	19450		
45	West Virginia, heavy	.873	60	30.37	83.5	13.3	3.2	....	....	.....	19464	20207	19046
46	Calicia, Austria	.870	60	30.92	82.2	12.1	5.7	....	....	18416	19487	19291	18235
47	East Galicia	.870	32	30.92	82.2	12.7	5.7	....	....	18153	19487	19675	18545
48	Kansas, crude	.866	60	31.67	85.4	13.07	....	....	....	.....	19517	20345	19203
49	Schwabweiler	.861	32	32.603	86.2	13.3	.5	....	....	18844	19555	21200	19439
50	Baku, light	.844	....	35.87	86.3	13.6	....	....	....	20628	19685	20796	19608
51	West Virginia, light	.8412	....	36.435	84.3	14.1	1.6	....	....	18502	19705	20809	19578
52	West Virginia, crude	.841	60	36.47	84.3	14.1	1.6	....	....	18400	19709	20809	19578
53	West Virginia, light	.841	60	36.47	84.3	14.1	1.6	....	....	21240	19709	20809	19578
54	Ohio distillate	.838	60	37.07	....	....	....	....	....	19880	19733		
55	Schwabweiler	.829	32	38.89	79.5	13.6	6.9	....	....	.....	19806	19808	18620
56	Ohio, Mabery Noble	.829	70	38.89	85.0	13.8	.6	.6	....	.....	19806	20752	19547
57	Pennsylvania, crude	.826	60	39.50	82	14.8	3.2	....	....	17930	19830	20699	19606
58	Balakhany	.822	....	40.32	87.4	12.5	.1	....	....	21600	19863	19289	19198
59	American petroleum	.82	....	40.73	83.4	14.7	1.9	....	....	17588	19879	20042	19758
60	Pennsylvania, light	.816	....	41.57	82.0	14.8	3.2	....	....	17533	19913	19899	19606
61	Parma, Italy	.786	60	48.13	84	13.4	1.8	....	....	18215	20175	20341	19171
62	Parma, crude	.786	32	48.13	84	13.4	1.8	....	....	18218	20175	20341	19171
63	American, crude	....	....	....	83	14.4	3	....	....	19980	.....	20801	19544

TABLE CVIII—*Concluded*

PROPERTIES OF MINERAL OILS

No.	Name and Source.	Density.			Ultimate Analysis.				Prox. H_2O.	B.T.U. per Pound.			
		Sp.Gr.	F.	Bé.	C.	H_2.	O_2+N_2	S.		By Calorimeter.	By S. & K. Form	High Value.	Low Value.
64	Russian, crude	....	....	.....	85	11.5	3.5	....	....	18540	.....	19335	18332
65	American, refined	....	....	.....	85.5	14.2	.3	....	....	19883	.....	21045	19806
66	American, heavy residue	....	....	.....	87	13	....	....	....	19620	.....	20536	19401
67	Russian, heavy residue	....	....	.....	86.7	13	....	....	....	19449	.....	20433	19358
68	Canada, crude, Bothwell	....	60	.....	84.3	13.4	2.3	....	....	20410	.....	20385	19215
69	Ohio, crude	....	60	.....	80.2	17.1	2.7	....	....	21600	.....	22031	20538
70	Texas	....	60	.....	85.66	11.03	3.31	....	....	19240	.....	19145	18172
71	Borneo	....	60	.....	87.8	10.78	1.24	....	....	18830	.....	19305	18364
72	Burmah, India	....	60	.....	86.4	12.1	1.5	....	....	18865	.....	19902	18845
73	Caucasus, Russia	....	60	.....	84.94	13.96	1.25	....	....	18610	.....	20817	19649

TABLE CIX

COMPOSITION OF NATURAL GASES

No.	Source.	Authority.	Volumetric Analysis.							
			O_2.	CH_4.	C_2H_6.	H_2.	CO.	C_2H_4.	N_2.	CO_2.
1	West Virginia.....	Report Gas Eng. Com. N. E. L. A...	.4	99.5	.1					
2	Kansas.....	Report Gas Eng. Com. N. E. L. A...	.25	98.3	...	...	.25	...	1.2	
3	Caucasus.....	Bunsen.....	..	97.57	...	...	2.69			
4	Caucasus.....	Bunsen.....	..	95.56	...	...	4.4			
5	Kokomo, Ind.....	Levin.....	.3	94.16	...	1.7	.55	.3	2.8	.29
6	Kokomo, Ind.....	Eng. & M. J.....	.3	94.16	...	1.42	.55	.3	2.8	.29
7	St. Mary's, Ohio...	Levin.....	.35	93.85	...	2.74	.44	.2	2.98	
8	.....	Lucke.....	.35	93.85	...	2.14	.44	.2	2.98	
9	Marion, Ind.....	Eng. & M. J.....	.55	93.57	...	1.2	.6	.15	3.42	.3
0	Marion, Ind.....	Levin.....	.55	93.57	...	1.4	.6	.15	3.42	.3
1	Findlay, Ohio.....	Eng. & M. J.....	.39	93.35	...	1.64	.41	.35	3.41	.25
2	Findlay, Ohio.....	Levin.....	.39	93.35	...	1.84	.41	.35	3.41	
3	English.....	Lewes.....	..	93.16	2.94	...	1.0	...	2.9	
4	Russian.....	Lewes.....	..	93.1	3.26	.98	..	...	1.9	2.18
5	Caucasus.....	Bunsen.....	..	93.09	3.26	.98	..	...	.49	2.18
6	Anderson, Ind.....	Eng. & M. J.....	.42	93.07	...	1.86	.73	.47	3.02	.26
7	Anderson, Ind.....	Levin.....	.42	93.07	...	2.01	.73	.47	3.02	.26
8	Ohio.....	Lewes.....	.35	92.84	.35	1.89	..	...	3.82	.75
9	Fostoria, Ohio.....	Eng. & M. J.....	.35	92.84	...	1.89	.55	.20	3.82	.20
0	Muncie, Ind.....	Levin.....	.35	92.67	...	2.5	.4	.25	3.53	
1	Muncie, Ind.....	Eng. & M. J.....	.35	92.67	...	2.35	.45	.25	3.53	.25
2	Findlay, Ohio.....	Gill.....	.3	92.6	...	2.3	.5	.3	3.5	.3
3	.....	Lucke.....	.34	92.6	...	2.18	.5	.31	3.61	2.6
4	Caucasus.....	Bunsen.....	..	92.49	4.11	.94	.93	...	2.13	
5	Caucasus.....	Bunsen.....	..	92.24	4.26	...	3.50			
6	Leechburg, Pa.....	Hoyle.....	..	89.65	...	4.79	.26	4.39	...	.35
7	Penna. & W. Va....	Allen & Burrell.....	..	83.	16.4	...	..	...	.6	
8	West Virginia.....	Report Gas Eng. Com. N. E. L. A...	.15	81.5	17.6	.2	..	...	.55	
9	Butler County, Pa..	Hoyle.....	..	80.11	...	13.5	..	5.72	...	.66
0	Butler County, Pa..	Hoyle.....	..	75.44	...	6.1	..	18.12	...	.34
1	U. S.....	Ford.....	1.1	72.18	.7	20.6	1.	...	...	.8
2	Pittsburgh, Pa.....	Levin.....	.8	72.18	...	20.	1.	3.0	...	.8
3	Penna.....	Jüptner.....	..	67.0	5.0	22.	.6	1.0	3.0	.6
4	Pittsburgh, Pa.....	Hoyle.....	.8	67.0	...	22.	.6	5.0	3.0	.6
5	U. S.....	Ford.....	.8	65.75	1.	26.12	.8	...	...	.6
6	U. S.....	Ford.....	.78	60.7	1.	29.03	.58			
7	U. S.....	Ford.....	2.1	57.85	.8	9.64	1.0	...	23.41	
8	U. S.....	Ford	.8	49.58	.6	35.92	.4	12.3	...	.4

TABLE CX

COMPOSITION OF COKE OVEN AND RETORT COAL GAS

† Robinson

No.	Description.	Volumetric Analysis.								
		H	CH_4.	CO.	C_2H_4.	C_6H_6.	Heavy Hydrocarbons.	CO_2.	O_2	Remainder and N_2
1	Retort gas, Wright, 5½ hrs	67.12	22.58	6.12	.....	.....	1.79	1.50	.....	0.89
2	Wigan cannel coal, retort gas, Henry, 13th hr	60.0	20.0	10.00	.....	.....	0	.....	.....	10.0
3	Solvay coke oven	56.9	22.6	8.7	.....	.....	3.0	3.0	.....	5.8
4	Coke oven gas, average German, 1% water vapor	55.0	32.0	7.0	1.5	.8	.....	1.2	.....	2.5
5	Magdeburg retort gas	54.9	30.1	7.7	.....	.....	3.3	1.4	.2	2.4
6	Retort coal gas, Lewes, 5–6.5 O in coal	54.21	34.37	6.68	2.48	.79	.....	1.47		
7	Aachen retort gas	54.0	34.2	5.2	.....	.....	3.3	1.1	.....	2.2
8	† Norwich retort gas, bit. coal	53.79	36.11	3.40	.....	.....	3.26	.27	.14	3.03
9	† Southampton retort gas, bit. coal	53.59	36.74	3.59	.....	.....	3.09	.07	.39	2.53
10	† London retort gas, bit. coal	53.14	36.55	4.11	.....	.....	2.92	.09	.....	3.19
11	Coke-oven gas	53.0	35.0	6.00	2.0	.....	.....	2.0	.....	2.0
12	Common coal gas	52.9	31.8	7.18	5.0	.....	.....	.....	.....	3.12
13	Retort coal gas, Lewes, 6.5–7.5 O in coal	52.79	34.43	7.19	3.02	.99	.....	1.58		
14	Manchester Canal coal retort gas, Wright	52.71	31.05	4.47	.....	.....	11.19	.58		
15	Retort gas, Wright, 3½ hrs	52.68	33.54	6.21	.....	.....	3.04	1.49	.....	3.04
16	Average retort coal, Klumpp	52.5	31.35	8.6	.....	.....	1.3–2.2	1.5	.35	.35–1.05
17	Common coal gas	52.5	34.0	5.0	4.0	.....	.....	.....	.05	4.45
18	Retort coal gas, Sexton	51.88	31.8	9.1	.....	.....	5.2	.....	.....	2.02
19	† Brighton retort gas, bit, coal	51.62	38.15	4.14	.....	.....	3.76	.03	.23	2.07
20	London coal retort gas, Pryce	50.7	37.8	4.1	.....	.....	4.4	.....	.....	3.0
21	Retort coal gas, Newton, Mass.	50.59	34.80	6.16	.....	.....	5.23	1.16	.....	2.06
22	† Newcastle, Tyne, retort gas, bit. coal	50.5	36.71	3.37	.....	.....	3.62	.28	.23	5.29
23	Retort coal gas, Lewes, 7.5–9 O in coal	50.1	35.03	8.21	3.98	.66	.....	1.72	.....	.30
24	Paris retort gas	50.1	33.1	6.3	.....	.....	5.8	1.5	.50	2.7
25	Common coal gas	50.1	38.0	6.0	4.0	.....	.....	.....	.....	1.9
26	Coke oven gas, Wyer	50.0	36.0	6.0	4.0	.....	.....	1.5	.50	2.0

28	Frankfurt retort gas	49.8	32.6	8.8	.....	.....	4.0	2.3	.....	2.5
29	Berlin retort gas	49.7	32.7	9.5	.....	.....	4.6	2.5	.....	1.0
30	Königsberg retort gas	49.0	36.5	5.6	.....	.....	6.8	1.1	.....	1.0
31	† Gloucester retort gas, bit. coal	48.88	38.25	4.64	.....	.....	4.95	.03	.51	2.74
32	Dresden retort gas	48.7	33.4	8.0	.....	.....	3.0	1.5	1.4	4.0
33	Retort coal gas, average	48.49	35.9	6.61	.....	.....	3.83	.12	.....	5.05
34	Retort coal gas, Sexton	48.32	39.55	4.63	.....	.....	5.18	.....	.....	2.32
35	† Redhill retort gas, bit. coal	48.18	39.41	3.41	.....	.....	4.40	.74	.49	3.37
36	Coal gas, Bates	48.1	36.5	.60	4.3	.....	.....	.30	.40	9.8
37	‡ Good Solvay average coke oven gas	48.0	35.5	5.1	4.2	1.2	.....	1.3	.5	4.2
38	Retort coal gas	48.0	39.5	7.5	.....	.....	3.8	.....	.....	1.2
39	London retort gas	48.0	37.6	3.7	.....	.....	4.4	.....	.3	6.0
40	† London retort gas, bit. coal	47.99	37.64	3.75	.....	.....	4.41	.....	.26	5.95
41	Common coal gas	47.9	33.3	6.0	12.3	.....	.....	.....	.5	
42	Common coal gas	47.73	35.6	6.15	4.88	.....	.....	1.41	.31	3.92
43	Boston, Mass., retort coal gas	47.29	38.67	1.04	.....	.....	5.21	1.04	.....	6.75
44	Coke oven, Milwaukee	47.1	34.7	6.2	3.8	.....	.....	3.1	.3	4.8
45	Average retort coal, Klumpp	47.0	36.0	8.0	.....	.....	.5–4.3	1.6	.4	2.7–6.5
46	Newcastle, Staffordshire	46.31	39.01	3.74	.....	.....	4.53	.08	.11	6.22
47	Hannover retort gas	46.3	37.5	11.2	.....	.....	3.2	.8	.....	1.0
48	Hanrover, Ger., retort coal gas	46.27	37.55	.81	.....	.....	3.17	.81	.....	11.39
49	Lean coke oven, Klumpp	46.2	27.1	6.2	.....	.....	2.5	3.0	.6	14.4
50	Heidelberg retort coal gas	46.2	34.02	8.88	.....	.....	5.09	3.01	.65	2.15
51	Retort coal gas, Wyer	46.0	40.0	6.0	5.0	.....	.....	.5	.5	2.0
52	London coal retort gas, Chandler	46.0	39.5	7.5	.....	.....	3.8	.....	.....	3.2
53	Common coal gas	46.0	39.5	7.5	3.8	.....	.....	.6	.1	2.5
54	Cincinnati, Ohio, retort coal gas	45.85	39.26	.82	.....	.....	5.17	.82	.....	8.08
55	Manchester retort gas	45.6	34.9	6.6	.....	.....	6.5	3.7	.....	2.7
56	Coal retort gas— Bunsen & Roscoe	45.58	34.9	6.64	.....	.....	6.46	3.67	.....	2.75
57	† Nottingham retort gas, bit. coal	45.52	39.66	5.63	.....	.....	5.63	.81	.24	2.51
58	Retort coal gas, Lewes, 9–11 O in coal	45.45	36.42	9.86	4.44	1.04	.....	2.79		
59	† Bristol retort gas, bit. coal	44.57	40.7	4.77	.....	.....	4.58	.....	.27	5.11
60	Common coal gas	44.4	37.1	5.2	2.3	.....	.....	1.3	1.1	8.6
61	† Preston retort gas, cannel	43.95	39.33	4.64	.....	.....	6.22	.84	.25	4.77
62	Coal gas (16 c.p.)	43.5	35.0	11.5	.....	.....	5.5	2.0	.5	2.0
63	† Ipswitch retort gas, bit. coal	43.26	38.73	2.46	.....	.....	4.53	.06	.12	10.84

TABLE CX—*Concluded*

COMPOSITION OF COKE OVEN AND RETORT COAL GAS

No.	Description.	Volumetric Analysis.								
		H_2.	CH_4.	CO.	C_2H_2.	C_6H_6.	Heavy Hydrocarbons.	CO_2.	O_2.	Remainder and N_2.
64	† Sheffield retort gas, cannel	43.05	43.05	4.72	.....	.....	6.28	.24	.10	2.56
65	Retort coal gas, Lewes, 11–12 O in coal	42.26	37.14	11.93	4.76	.88	.....	3.13		
66	Average coke oven, Klumpp	42.0	34.3	6.0	.....	.....	4.0	2.5	1.1	10.1
67	Coal retort gas, Humpedge	41.72	41.88	4.98	.....	.....	8.72	.....	.....	2.70
68	Otto coke oven, poor part of gas	41.6	29.6	6.3	.....	.....	2.8	3.2	.4	16.1
69	† Birmingham retort gas, bit. coal	40.23	39.	4.05	.....	.....	4.76	1.50	.36	10.1
70	† Leeds retort gas, cannel	40.23	42.74	5.02	.....	.....	7.28	.34	.07	4.32
71	Birmingham, Eng., retort coal gas	40.23	39.0	1.50	.....	.....	4.76	1.50	.....	13.01
72	Bonn retort gas	39.8	43.1	4.7	.....	.....	4.7	3.0	.....	4.7
73	Common coal gas	39.78	45.16	7.04	6.38	.....	.....	1.08	.06	.50
74	Glasgow, Scot., retort coal gas	39.18	40.26	.29	.....	.....	10.0	.29	.....	9.98
75	† Glasgow retort gas, cannel	39.18	40.26	7.14	.....	.....	10.0	.29	.06	3.07
76	Retort gas, Wright, 1½ hrs	38.33	44.03	5.68	.....	.....	5.98	2.09	.....	3.89
77	Otto coke oven, good part of gas	37.6	40.8	5.6	.....	.....	5.8	3.7	.4	6.1
78	Rich coke oven, Klumpp	37.4	40.4	7.1	.....	.....	5.8	2.1	1.6	5.6
79	† St. Andrews retort gas, cannel	36.63	42.13	5.16	.....	.....	10.04	2.73	.48	2.83
80	† Liverpool retort gas, cannel	36.44	44.28	3.39	.....	.....	7.90	1.70	.19	6.10
81	Liverpool, Eng., retort coal gas	36.44	44.28	1.70	.....	.....	7.90	1.70	.....	7.98
82	Retort gas cannel coal, Sexton	36.1	37.8	6.8	.....	.....	16.4	.....	.....	2.9
83	Cleveland, Ohio, retort coal gas	34.8	28.8	.20	.....	.....	11.2	.20	.....	24.8
84	Hoffman coke oven, Bates	33.32	36.31	6.49	2.04	.....	.....	1.41	.43	20.0
85	† Manchester retort gas, cannel	33.24	42.93	6.61	.....	.....	12.23	.35	1.0	3.64
86	Cannel-coal gas	27.7	50.0	6.8	13.0	.....	.....	.10	.....	2.4
87	Wigan cannel coal retort gas, Henry, 5th hour	21.3	56.0	11.0	.....	.....	7.0	.....	.....	4.7
88	Newcastle coal, 10 minutes	20.1	57.38	6.19	.....	.....	10.62	2.21	.....	3.50
89	Wigan cannel coal, retort gas, Henry, 1st hour	16.0	58.0	12.3	.....	.....	12.0	.....	.....	1.7

90	Wigan cannel coal retort gas, Henry, 1st hour		8.8	72.0	1.9	.....	.....	12.0	.....	.....	5.3
91	Wigan cannel coal retort gas, Henry, 1st hour		0	82.5	3.2	.....	.....	13.0	.....	.....	1.3
	Low volatile coal:										
92	Solvay oven, Blauvelt,	1st hr	42.1	31.6	4.6	6.0	.50	.....	.10	.80	14.3
93	" "	2d hr	51.6	32.8	4.3	3.6	.70	.....	.10	.80	6.1
94	" "	3d hr	46.8	33.2	4.9	3.8	1.10	.....	.10	.70	9.4
95	" "	4th hr	49.6	33.5	4.6	3.3	.90	.....	.10	1.10	6.9
96	" "	5th hr	50.8	33.1	4.6	3.7	.80	.....	.10	.80	6.1
97	" "	7th hr	44.4	30.1	4.4	2.5	.70	.....	.20	1.0	16.7
98	" "	8th hr	46.2	32.6	4.5	2.9	.50	.....	.20	.90	12.2
99	" "	10th hr	47.4	29.1	4.1	1.6	.40	.....	0	.90	16.5
100	" "	12th hr	53.6	29.5	4.6	1.2	.50	.....	.10	.60	9.9
101	" "	14th hr	69.2	17.0	4.4	.6	.20	.....	.10	.90	7.6
102	" "	15th hr	44.0	3.0	2.9	3.5	.30	.....	0	4.9	41.4
	High volatile coal:										
103	Solvay oven, Blauvelt,	1st hr	41.4	41.5	5.8	3.2	.90	.....	.90	.50	5.8
104	" "	2d hr	43.8	40.4	5.1	2.6	1.0	.....	.90	.40	5.8
105	" "	3d hr	47.2	37.6	4.9	2.1	1.0	.....	.90	.70	5.6
106	" "	4th hr	48.6	36.2	5.0	2.1	1.1	.....	1.1	.40	5.5
107	" "	5th hr	49.5	33.3	4.6	1.7	1.0	.....	.80	1.0	8.1
108	" "	6th hr	49.8	31.4	4.6	1.6	.90	.....	1.1	1.1	9.5
109	" "	7th hr	47.6	31.0	4.4	1.3	.90	.....	2.2	1.6	11.0
110	" "	8th hr	54.2	31.5	4.8	1.5	1.0	.....	.90	.50	5.6
111	" "	10th hr	55.3	29.1	4.9	2.0	.50	.....	1.0	.60	6.6
112	" "	12th hr	64.8	23.1	5.3	.3	.10	.....	.60	.40	5.4
113	" "	14th hr	67.0	18.2	5.3	.4	0	.....	.50	.60	8.0
114	" "	16th hr	69.4	13.6	6.2	0	0	.....	.2	.4	10.2

TABLE CXI

COMPOSITION OF UNITED STATES COKE

(Mainly from U. S. Geological Survey Reports)

Origin.	Moisture.	Volatile.	Fixed Carbon.	Ash.	Sulphur
From Connelsville bituminous coal, 72 hours roasting.	.23	1.32	88.18	10.27	.81
From Connelsville bituminous coal, 48 hours roasting.	.19	.51	89.6	9.7	.63
Foundry Ganley Mountain, U.S.Geological Survey. . .	.75	.35	86.38	12.52	.70
Foundry Milwaukee Solvay, U.S.G.S.	.27	.48	89.63	9.62	.79
From Connelsville, U.S.G.S.	.18	.32	88.75	10.75	.87
From Alabama coal, U.S.G.S. No. 1	.33	.72	82.63	16.32	.69
From Arkansas coal, U.S.G.S. No. 6	1.30	2.85	78.84	17.01	1.46
From Illinois coal, U.S.G.S. No. 2	1.57	2.83	75.42	20.18	2.73
From Illinois coal, U.S.G.S. No. 3	.96	.44	87.08	11.52	1.19
From Indiana coal, U.S.G.S. No. 1	1.16	1.24	84.81	13.19	1.77
From Indian Territory, U.S.G.S. No. 2	2.60	1.85	80.25	15.30	1.59
From Iowa, U.S.G.S. No. 1	2.11	1.79	77.01	19.09	4.23
From Iowa, U.S.G.S. No. 3	1.80	1.95	78.64	17.61	4.70
From Kentucky, U.S.G.S. No. 1	.51	.84	93.25	5.40	.8
From Kentucky, U.S.G.S. No. 4	.52	.73	86.40	12.35	2.3
From Missouri, U.S.G.S. No. 2	2.18	1.82	81.34	14.66	2.8
From West Virginia, U.S.G.S. No. 1	.40	1.95	87.47	.18	.7
From West Virginia, U.S.G.S. No. 2	.59	1.31	86.70	11.40	2.2
From West Virginia, U.S.G.S. No. 3	.38	.87	84.48	14.27	1.1
From West Virginia, U.S.G.S. No. 4	.20	1.15	85.42	13.23	.6
From West Virginia, U.S.G.S. No. 5	.42	.43	84.34	14.81	.8
From West Virginia, U.S.G.S. No. 6	1.00	1.85	89.60	7.55	.7
From West Virginia, U.S.G.S. No. 10	.60	.55	90.34	8.51	.8
From West Virginia, U.S.G.S. No. 12	1.00	.75	90.37	7.88	1.0
Connelsville average of 3, J. B. Proctor	....	....	88.96	9.74	.8
Chattanooga, Tenn., average of 4, J. B. Proctor	....	....	89.51	16.34	1.4
Birmingham, Ala., average of 4, J. B. Proctor	....	....	87.29	10.54	1.
Pocahontas, Va., average of 3, J. B. Proctor	....	....	92.53	5.74	.
New River, W. Va., average of 8, J. B. Proctor	....	....	92.38	7.21	.
Big Stone Gap, Ky., average of 7, J. B. Proctor	....	....	93.23	5.69	.
Alabama, run-of-mine, foundry, Moldenke	1.34	1.03	83.35	14.28	1.
Alabama washed slack, foundry, Moldenke	.75	.75	86.00	11.50	.
Colorado washed slack, foundry, Moldenke	.44	1.31	82.18	16.07	.
Illinois washed slack, foundry, Moldenke	2.78	.74	83.35	13.13	2.
Pennsylvania washed slack, foundry, Moldenke.	.23	.29	92.53	6.95	.
Pennsylvania washed slack, foundry, Moldenke.	.91	2.26	80.84	15.99	1.
Tennessee, foundry, Moldenke	.22	.11	92.44	7.23	.
Tennessee, foundry, Moldenke	1.67	1.6	76.87	19.86	2.
Virginia, foundry, Moldenke	.16	.80	93.24	5.80	.
Virginia, foundry, Moldenke	1.52	1.67	88.52	8.29	1.
West Virginia, foundry, Moldenke	.67	.46	95.47	4.00	.
West Virginia, foundry, Moldneke	.60	2.35	84.09	12.96	2.
Proposed standard foundry coke specification	.5	.75	89.75	9.0	

TABLE CXII

FRACTIONATION TESTS OF KEROSENES AND PETROLEUMS

No.	Class and Density of Original.	Volumetric Per Cent Distilled.	Temperature of Distillation.		Density of Distillate, 60° F.	Density, Baumé.
			Deg. F. at Beginning.	Deg. F. at End.		
1	American kerosene Robinson Sp.gr. .797 Bé. 45.67	23	257	302	.748	57.21
		11	302	347	.767	52.5
		8	347	392	.783	49.0
		9	392	437	.794	46.5
		10	437	482	.807	43.5
		16	482	527	.821	40.8
		7	527	572	.831	38.8
		3	572	680	.836	37.5
		Left as res.				
		13	680	.....	.843	36.5
2	Russian kerosene Robinson Sp.gr. .825 Bé. 39.9	9	239	284	.786	48.2
		18	284	329	.799	45.4
		20	329	374	.816	41.6
		13	374	419	.829	38.9
		18	419	464	.831	38.5
		12	464	509	.845	36.8
		6	509	554	.857	33.5
		1	554	680	.864	32.2
		Left as res.				
		3	680	.....	.877	29.8
3	American kerosene Robinson Sp.gr.	25	293	338		
		23	338	383		
		28	383	428		
		13	428	473		
		7	473	518		
		3	518	572		
4	Alsatian petroleum Engler & Schestopal Sp.gr. .801 Bé. 44.8	.08	.....	302		
		30.35	302	392		
		44.7	392	482		
		20.2	482	572		
		3.8	572	608		
5	"Kaiser" oil Engler & Schestopal Sp.gr. .795 Bé. 46.1	29.7	302	392		
		32.3	392	482		
		26.3	482	572		
		11.7	572	608		
6	Pennsylvania kerosene Maschinenfabrik, Augsburg Sp.gr. .800 Bé. 45	15.8	.....	302		
		22	302	392		
		19.25	392	482		
		16.8	482	572		
		26.15	572	608		

TABLE CXII—*Continued*

FRACTIONATION TESTS OF KEROSENES AND PETROLEUMS

No.	Class and Density of Original.	Volumetric Per Cent Distilled.	Temperature of Distillation.		Density of Distillate, 60° F.	Density, Baumé.
			Deg. F. at Beginning.	Deg. F. at End.		
7	Bavarian, crude Maschinenfabrik, Augsburg Sp.gr. .827 Bé. 39.5	14.7 12.1 10.7 9.1 7.8	..... 302 392 482 572	302 392 482 572 608		
8	Roumanian, lamp Maschinenfabrik, Augsburg Sp.gr. .815 Bé. 41.9	28.2 15.2 12.8 24.3 7.7 5.4	..... 302 392 482 572 608	302 392 482 572 608		
9	Galician, solar Maschinenfabrik, Augsburg Sp.gr. .874 Bé. 30.2	.... 2.7 65.1 14	..... 302 392 482 572 608	302 392 482 572 608		
10	Hungarian, blue Maschinenfabrik, Augsburg Sp.gr. .836 Bé. 37.5	20.1 15 12.6 12.2 4.3 2.4	..... 302 392 482 572 608	302 392 482 572 608		
11	German, red Maschinenfabrik, Augsburg Sp.gr. .870 Bé. 30.9	..... 3.7 38.9 39.3 12.3	..... 302 392 482 572 608	302 392 482 572 608		
12	German, yellow parif. oil Maschinenfabrik, Augsburg Sp.gr. .860 Bé. 32.8	..... 2.4 55.0 37.0 2.8	..... 302 392 482 572 608	302 392 482 572 608		
13	German, solar Maschinenfabrik, Augsburg Sp.gr. .860 Bé. 32.8	13.8 57.4 25.5 1.0 .1	..... 302 392 482 572 608	302 392 482 572 608		

TABLE CXII —*Continued*

FRACTIONATION TESTS OF KEROSENES AND PETROLEUMS

No.	Class and Density of Original.	Volumetric Per Cent Distilled.	Temperature of Distillation.		Density of Distillate, 60° F.	Density, Baumé.
			Deg. F. at Beginning.	Deg. F. at End.		
14	German, benzol	68	.....	212		
	Maschinenfabrik, Augsburg	28.7	212	302		
	Sp.gr. .873 Bé. 30.5	.....	302			
15	Beaumont, Texas	2.5	230	302		
	Richardson & Wallace	40.0	302	572	.8749	30.1
	Sp.gr. .912	20.0	572	752	.9089	24.2
	Bé. 23.5	25.0	752	.....	.9182	23.6
16	Ohio	23.0	185	302	.7297	62.3
	Mabey & Noble	21.0	302	572	.8014	45.1
	Sp.gr. .829	21.0	572	752	.8404	36.8
	Bé. 38.9	27.0	752	.....	.8643	32.2
17	Pennsylvania	21.0	176	302	.7188	65.2
	Sp.gr. .914	41.0	302	572	.7984	45.8
	Bé. 23.2	14.0	572	752	.8334	38.3
		23.0	752	.....	Paraffine	
18	Virginia, petroleum, heavy	1.0	.....	212		
	B. Redwood	1.3	212	284		
	Sp.gr. at 32° F. .873, Bé. 30.5	12.0	284	356		
19	Virginia, petroleum, light	1.3	.....	212		
	B. Redwood	4.3	212	248		
	Sp.gr. 32° F. .8412	11.0	248	284		
	Bé. 36.6	17.7	284	320		
		25.2	320	356		
		28.5	356	392		
20	Pennsylvania, light	4.3	.....	212		
	B. Redwood	10.7	212	248		
	Sp.gr. at 32° F. .816	16.0	248	284		
	Bé. 41.6	23.7	284	320		
		28.7	320	356		
		31.0	356	392		
21	Penn., heavy, B. Redwood	.....	.....	500		
	Sp.gr. at 32° F. .886. Bé.	12.0	500	536		
22	Java, petroleum	1.0	.....	212		
	B. Redwood	1.0	212	248		
	Sp.gr. at 32° F. .923	.....	248	320		
	Bé. 21.8	7.7	320	356		
		15.0	356	392		
		22.3	392	428		
		24.3	428	464		

TABLE CXII—*Concluded*

FRACTIONATION TESTS OF KEROSENES AND PETROLEUMS

No.	Class and Density of Original.	Volumetric Per Cent Distilled.	Temperature of Distillation.		Density of Distillate, 60° F.	Density, Baumé
			Deg. F. at Beginning.	Deg. F. at End.		
		.8	.....	212		
	Java, petroleum	3	212	248		
23	B. Redwood	9.3	248	284		
	Sp.gr. at 32° F. .827	16.3	284	320		
	Bé. 39.2	22.0	320	356		
		27.8	356	392		
	Java, petroleum	.....	.....	392		
24	B. Redwood	2.3	392	428		
	Sp.gr. at 32° F. .972	4	428	464		
	Bé. 14	9	464	500		
		2.1	.....	212		
		4.6	212	248		
	East Galicia, petroleum	8.7	248	284		
25	B. Redwood	13.7	284	320		
	Sp.gr. at 32° F. .870	14.3	320	356		
	Bé. 30.9	21.7	356	392		
		25.3	392	428		
		32.3	428	464		
		4	212	248		
		9.8	248	284		
	West Galicia, petroleum	14.3	284	320		
26	B. Redwood	23.3	320	356		
	Sp.gr. at 32° F. .885	27	356	392		
	Bé. 28.1	30.7	392	428		
		36.7	428	464		

TABLE CXIII

FRACTIONATION TESTS OF GASOLENES

No.	Class and Density of Original.	Volumetric Per Cent Distilled.	Temp. of Distillation.		Density of Distillate, 60° F.	Density, Baumé.
			Deg. F. at Beginning.	Deg. F. at End.		
1	Gasolene [Blount] Sp.gr. .739 Bé. 59.5	39 49 7.5 3.5	158 212 248 271	212 248 271	.722 .748 .757 .767	63.9 57.2 55.0 52.6
2	Gasolene [Blount] Sp.gr. .736 Bé. 60.2	48 37 11.5 2.5	158 212 248 271	212 248 271	.727 .747 .762 .767	62.5 57.5 53.9 52.6
3	Gasolene [Blount] Sp.gr. .717 Bé. 65.3	65.5 26.5 4.5 2.5	149 212 248 271	212 248 271	.708 .742 .754 .769	67.9 58.8 55.8 52.2
4	Gasolene [Blount] Sp.gr. .716 Bé. 65.5	69.0 22.0 4.5 3	149 212 248 271	212 248 271	.707 .743 .751 .770	68 58.5 56.5 51.9
5	Gasolene [Blount] Sp.gr. .716 Bé. 65.5	65.0 26.0 5.0 2.5	145 212 248 271	212 248 271	.704 .742 .753 .772	68.9 58.9 56 51.5
6	Gasolene [Blount] Sp.gr. .717 Bé. 65.3	70.0 24.0 3.0 1.5	149 212 248 271	212 248 271	.71 .744 .753 .769	67.2 58.2 55.9 52
7	Gasolene [Blount] Sp.gr. .719 Bé. 64.7	67.0 21.0 6.0 4.5	140 212 248 271	212 248 271	.706 .742 .750 .770	68.2 58.9 56.8 51.9
8	Gasolene [Blount] Sp.gr. .711 Bé. 66.9	66 24 6.5 2.5	140 212 248 271	212 248 271	.700 .731 .741 .762	70 61.6 58.9 53.8
9	Gasolene [Blount] Sp.gr. .715 Bé. 65.8	59 28.5 7.0 4.0	145 212 248 271	212 248 271	.701 .736 .750 .765	69.8 60.2 56.6 53.0
10	Gasolene [Blount] Sp.gr. .712 Bé. 66.7	62.0 25.0 7.0 5.0	145 212 248 271	212 248 271	.699 .730 .742 .758	70.1 61.8 58.8 54.8
11	Gasolene [Blount] Sp.gr. .710 Bé. 67.2	68 22.5 6.5 2.0	136 212 248 271	212 248 271	.699 .736 .750 .736	70.1 60.2 56.6 60.2

TABLE CXIII—*Continued*

FRACTIONATION TESTS OF GASOLENES

No.	Class and Density of Original.	Volumetric Per Cent Distilled.	Temp. of Distillation. Deg. F. at Beginning.	Temp. of Distillation. Deg. F. at End.	Density of Distillate, 60° F.	Density, Baumé.
12	Gasolene [Blount] Sp.gr. .700 Bé. 70	86.5 11.5 5	133 212 248 271	212 248 271	.692 .739	72.3 59.5
13	Gasolene [Blount] Sp.gr. .718 Bé. 65	59 29 8 3	145 212 248 271	212 248 271	.704 .742 .755 .768	69 58.8 55.5 52.5
14	Gasolene [Blount] Sp.gr. .717 Bé. 65.3	64 26 6.5 2.5	149 212 248 271	212 248 271	.705 .740 .754 .770	68.8 59.4 55.8 51.7
15	Gasolene [Blount] Sp.gr. .717 Bé. 65.3	68 23 5.5 2.5	149 212 248 271	212 248 271	.705 .743 .755 .773	68.8 58.6 55.5 51.2
16	Gasolene [Blount] Sp.gr. .717 Bé. 65.3	67.5 22 5.5 3.5	143 212 248 271	212 248 271	.706 .742 .758 .770	68 58.8 54.9 51.8
17	Gasolene [Blount] Sp.gr. .715 Bé. 65.8	58 24 9.5 6.5	136 212 248 271	212 248 271	.700 .733 .749 .770	70 61 57 51.8
18	Gasolene [Blount] Sp.gr. .705 Bé. 68.6	73 17.5 5 3	131 212 248 271	212 248 271	.697 .736 .751 .768	71 60.2 56.5 52.5
19	Gasolene [Blount] Sp.gr. .705 Bé. 68.6	74 15.5 5.0 4.0	140 212 248 271	212 248 271	.696 .736 .745 .764	71.1 60.3 57.9 53.2
20	Gasolene [Chambers] Sp.gr. .71 Bé. 67.18	6.67 6.66 6.67 6.67 6.66 6.67 6.67 6.66 6.67 6.67 6.66 6.67 7.67 5.66 4.37	148.8 149.2 167.0 176 176 186.8 197.6 206.6 212.0 219.2 226.4 233.6 248.0 258.8 284.0	149.2 167.0 176.0 176 186.8 197.6 206.6 212.0 219.2 226.4 233.6 248.0 258.8 284.0 311		

TABLE CXIV

COMPOSITION OF BLAST-FURNACE GAS AND AIR GAS

No.	Description.	Volumetric Analysis.						
		CO	H_2	CH_4	CO_2	N_2	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$
1	Brymbo-Derby	34.0	1.3	.....	5.7	59.0	5.97	.86
2	Coke in small Dowson producer, Dowson and Larter	32.6	1.0	.....	1.4	65.0	23.2	.96
3	Westphalia, Allen	30.7	3.3	.5	7.8	57.7	3.94	.8
4	Blast-furnace, splint coal, Sexton No. 3	30.1	6.2	3.2	5.4	55.1	5.58	.85
5	Durham coke, blast furnace, max. CO_2 content in one month	30.1	2.9	.....	7.1	59.9	4.24	.81
6	Durham coke, blast furnace, min. CO_2 content in one month	29.7	.9	.....	10.3	59.1	2.88	.74
7	Blast furnace, Upper Silesia, Germany	29.7	6.3	.....	7.8	56.2	3.81	.79
8	Blast furnace, charcoal, Ebelmann	29.6	.....	.....	5.9	63.4	5.01	.83
9	Metallurgical air gas, Lewes	29.0	2.5	.....	4.0	64.5	7.25	.88
10	Blast furnace, Westphalia, Germany, dry	29.0	4.0	.....	10.0	57.0	2.9	.74
11	Blast furnace, coke, Ebelmann	28.61	2.74	.20	11.39	57.06	2.52	.72
12	Coke, Lackawanna Steel Co	28.4	1.7	.....	11.8	.....	2.40	.71
13	Blast furnace, unwashed, Sexton	28.19	10.24	1.78	6.23	53.56	4.53	.82
14	Blast furnace, unwashed, Sexton	28.09	7.11	2.77	7.92	54.14	3.55	.78
15	Blast furnace, splint coal, Sexton No. 2	28.06	5.45	4.39	8.61	53.38	3.26	.76
16	Blast furnace, English	27.71	1.34	.....	8.62	62.33	3.21	.76
17	Blast furnace, Minette District, Germany	27.5	3.0	.54	10.0	54.5 +5.0	2.75	.73
18	Blast-furnace gas	27.5	3.0	.....	10.0	59.4	2.75	.73
19	Blast-furnace coal, Archibald	27.5	3.68	3.62	6.15	59.27	44.7	.82
20	Bituminous coke air gas, Loomis Pettibone	27.3	5.1	.8	2.7	.....	10.1	.91
21	Blast furnace, splint coal, Sexton No. 1	27.15	5.48	4.29	8.57	54.29	3.16	.76
22	Blast furnace, Cleveland, Eng., Robinson	27.1	1.3	.....	12.4	59.2	21.8	.69
23	Blast furnace, Glengarmock, washed, Robinson	27.0	2.0	5.0	6.0	60.0	4.5	.82
24	Blast furnace, Frodingham coke, Allen	26.9	2.4	.....	6.3	64.4	4.27	.80
25	Coke, Lackawanna Steel Co	26.9	3.0	.....	12.0	.....	2.24	.69
26	Coke, Lackawanna Steel Co	26.8	1.4	.....	8.6	.....	3.12	.76
27	Coke, Lackawanna Steel Co	26.6	2.4	.....	12.0	.....	2.22	.69
28	Blast-furnace, splint coal, Sexton No. 4	26.4	12.23	1.71	6.79	58.81	3.89	.80
29	Blast furnace, Westphalia, Germany, 10 per cent H_2O	26.1	3.6	.....	9.0	51.3 +10.0	2.90	.74
30	Coke, Lackawanna Steel Co	26.1	2.0	.....	12.1	.....	2.15	.68
31	Scotch blast furnace, Wishan	25.83	4.55	3.45	7.21	58.96	3.58	.78

TABLE CXIV—*Continued*

COMPOSITION OF BLAST-FURNACE GAS AND AIR GAS

No.	Description.	Volumetric Analysis.						
		CO	H_2	CH_4	CO_2	N_2	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$
32	Isabella Furnace, U. S. Steel Co., Gayley	25.8	.....	.....	12.6	4.25	2.04	.67
33	Producer gas, little steam	25.3	9.2	3.1	3.4	58.2	7.44	.88
34	Dowson gas, average	25.0	18.0	3.0	7.0	47	3.57	.78
35	Blast furnace, Glengarmock, unwashed, Robinson	25.0	7.0	4.0	7.0	57.0	3.57	.78
36	Producer gas, little steam	24.8	8.5	5.2	5.6	55.1	4.44	.82
37	Blast furnace, Wishan, Pellew	24.75	2.33	.75	5.75	66.42	4.30	.81
38	Blast furnace charcoal, Ebelmann	24.65	5.19	.93	12.01	57.22	2.05	.67
39	Producer gas, little steam	24.0	9.8	3.4	6.0	55.6	4.0	.80
40	Blast furnace, Lediebas, Germany, (coke) dry	24.0	2.0	2.0	12.0	6.0	2.0	.67
41	Blast-furnace coke	24.0	2.0	2.0	12.0	60.0	2.0	.67
42	Durham coke, Allen	23.84	2.34	.....	10.94	water 5.55		
						57.3	21.8	.69
43	Isabella Furnace, U. S. Steel Co., Gayley	23.2	.....	.....	16	.....	1.45	.59
44	Blast furnace, Upper Silesia, Germany	23.0	.....	.....	6.0	59.0 +12.0	3.83	.79
45	Producer gas, little steam	22.74	8.37	2.56	5.3	60.13	4.29	.80
46	Producer gas, little steam	22.1	6.8	3.74	4.84	61.68	4.57	.82
47	Blast furnace, Lediebas, Germany, coke, 10 per cent H_2O	21.6	1.8	1.8	10.8	54.0 +10.0	2.0	.67
48	Producer gas, little steam	20.8	6.9	2.2	4.6	64.9	4.53	.82
49	Isabella Furnace, U. S. Steel Co., Gayley	20.6	.....	.....	13.0	.....	1.58	.61
50	Producer gas, little steam	20.0	5.3	3.0	3.6	67.5	5.56	.85
51	Loomis Pettibone coal	20.0	14.0	2.0	8.2	55.5	2.44	.71
52	Loomis Pettibone wood	20.0	14.0	2.0	16.0	47.7	1.25	.55
53	Anthracite before making water gas	17.8	.....	.....	9.7	72.5	1.84	.65
54	Taylor gas, average	12.0	21.0	2.0	5	57.0	2.4	.71
55	Mond gas	12.0	29.0	2.0	14.5	42.5	.83	.45
56	Mond gas	11.5	28.5	2.1	15	42.9	.77	.43

TABLE CXV

RATE OF FORMATION OF CO FROM CO_2 AND CARBON

Form of Carbon.	Temp. Deg. F.	Time, Seconds.	Volumetric Analysis.				Authority.
			CO_2	CO	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$	
ne, amorphous	1472	480	13.6	86.4	6.43	.864	Boudouard
harcoal, 2–5 mm	1472	480	39.9	60.1	1.51	.601	
harcoal, hazel nut	1472	480	17.1	82.9	4.88	.829	
oke, 2–5 mm	1472	480	79.1	20.9	.26	.209	
oke, hazel nut	1472	480	83.6	16.4	.20	.164	
as carbon, 2–5 mm	1472	480	80.1	19.9	.25	.199	
as coke, hazel nut	1472	480	86.7	13.3	.15	.133	
Charcoal, 5 mm	1472	189	49.7	50.3	1.01	.503	Clement
	1472	116	49.6	50.4	1.01	.504	
	1472	57	48.2	51.8	1.07	.518	
	1472	46	47.8	52.2	1.09	.522	
	1472	24	62.5	37.5	.60	.375	
	1472	16	71.7	28.3	.40	.283	
	1472	12	75.5	24.5	.32	.245	
	1472	2.7	93.7	6.3	.067	.063	
	1472	1.6	96.1	3.9	.041	.039	
Charcoal, 5 mm	1562	123	25.7	74.3	2.88	.743	Clement
	1562	54	29.8	70.2	2.36	.702	
	1562	24	42.8	57.2	1.34	.572	
	1562	13	47.4	52.6	1.11	.526	
	1562	9.3	70.3	29.7	.42	.297	
	1562	4.6	70.3	29.7	.42	.297	
	1562	3.7	77.6	22.4	.29	.224	
	1562	3.3	77.5	22.5	.29	.225	
Charcoal, 5 mm	1652	64	12.7	87.3	6.87	.873	Clement
	1652	44	13.3	86.7	6.52	.867	
	1652	10	29.2	70.8	2.42	.708	
	1652	4.3	50.2	49.8	.99	.498	
	1652	2.8	68.9	31.1	.45	.311	
	1652	2.2	65.6	34.4	.52	.344	
Charcoal, 5 mm	1697	119	5.3	94.7	17.9	.947	Clement
	1697	81	6.7	93.3	13.9	.933	
	1697	12	15.2	84.8	5.57	.848	
	1697	5.8	28.2	71.8	2.54	.718	
	1697	4.3	35.8	64.2	1.79	.642	
	1697	2.3	62.5	37.5	.60	.375	
Charcoal, 5 mm	1832	70	5.1	94.9	18.6	.949	Clement
	1832	18.6	5.7	94.3	16.5	.943	
	1832	8.2	9.7	90.3	9.3	.903	
	1832	3.7	20.3	79.7	3.92	.797	
	1832	2.3	20.5	79.5	3.88	.795	
harcoal, 5 mm	2012	36.5	1.3	98.7	75.9	.987	Clement
	2012	10.4	1.7	98.3	57.8	.983	
	2012	4.97	1.9	98.1	51.6	.981	
	2012	3.6	2.7	97.3	36.0	.973	
	2012	1.9	5.4	94.6	17.5	.946	
Coke	1652	142	72.4	27.6	.382	.276	Clement
	1652	80	86.9	13.1	.151	.131	
	1652	44	90.6	9.4	.104	.094	
	1652	25	94.3	5.7	.061	.057	

TABLE CXV—*Continued*

RATE OF FORMATION OF CO FROM CO_2 AND CARBON

Form of Carbon.	Temp. Deg. F.	Time, Seconds.	Volumetric Analysis.				Authority.
			CO_2	CO	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$	
6. Coke	1652	16	95.1	4.9	.051	.049	Clement
	1652	9.6	97.4	2.6	.027	.026	
	1652	3.7	99.2	.8	.008	.008	
7. Coke	1832	123	21.6	78.4	3.62	.784	Clement
	1832	80	35.6	64.4	1.81	.644	
	1832	33	47.1	52.9	1.12	.529	
	1832	19	68.0	32.0	.47	.320	
	1832	6.4	86.1	13.9	.16	.139	
	1832	4.1	88.5	11.5	.13	.115	
	1832	3.1	90.8	9.2	.101	.092	
	1832	2.0	93.7	6.3	.067	.063	
8. Coke	2012	90	2.9	97.1	33.6	.971	Clement
	2012	30	14.6	85.4	5.85	.854	
	2012	13	33.9	66.1	1.95	.661	
	2012	6.7	44.4	55.6	1.25	.556	
	2012	3.2	68.3	31.7	.46	.317	
	2012	1.8	69.6	30.4	.437	.304	
	2012	1.7	76.0	24.0	.316	.240	
	2012	1.6	77.9	22.1	.284	.221	
	2012	1.5	78.6	21.4	.272	.214	
	2012	.96	86.7	13.3	.154	.133	
9. Coke	2192	19	1.1	98.9	89.7	.989	Clement
	2192	13	2.2	97.8	44.4	.978	
	2192	8.3	4.7	95.3	20.2	.953	
	2192	2.4	31.5	68.5	2.18	.685	
	2192	1.6	56.1	43.9	.78	.439	
	2192	1.1	66.5	33.5	.504	.335	
Coke	2372	8.9	.1	99.9	999	.999	Clement
	2372	4.1	2.1	97.9	46.5	.979	
	2372	2.1	6.8	93.2	13.7	.932	
	2372	1.1	16.6	83.4	5.02	.834	
10. Anthracite	2012	34	12.2	87.8	7.2	.878	Clement
	2012	9.4	39.9	60.1	1.5	.601	
	2012	5.4	52.3	47.7	.91	.477	
	2012	3.3	69.8	30.2	.43	.302	
	2012	2.4	73.5	26.5	.36	.265	
11. Anthracite	2192	47	.3	99.7	332.3	.997	Clement
	2192	10	14.4	85.6	5.95	.856	
	2192	5.1	28.5	71.5	2.5	.715	
	2192	2.8	57.7	42.3	.73	.423	
	2192	1.6	69.0	31.0	.45	.310	
12. Anthracite	2372	12.4	.1	99.9	999	.999	Clement
	2372	6.0	3.5	96.5	27.6	.965	
	2372	3.6	17.6	82.4	4.68	.824	
	2372	3.0	19.1	80.9	4.23	.809	
	2372	1.91	33.7	66.3	1.97	.663	
	2372	1.07	49.7	50.3	1.01	.503	

TABLE CXVI

COMPOSITION OF WATER GAS

No.	Description.	Volumetric Analysis.						Ratios.	
		H_2.	CO.	CH_4.	CO_2.	O_2.	N_2.	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$
1	Essen water gas, coke, Sexton	54.52	31.86	1.62	12.	00	0		
2	Dellurck process water gas, Lewes No. 3	52.76	37.50	..	4.08	.46	5.2	9.2	.90
3	Strong water gas, Moore	52.76	35.88	4.11	2.05	..	4.33	17.5	.95
4	Dellurck Process water gas, Lewes No. 1	52.43	38.30	..	4.73	.74	3.80	8.1	.89
5	Average water gas, Lewes	51.89	40.08	.10	4.80	..	3.13	8.35	.89
6	From anthracite before carburetting for illumination, O'Connor	51.8	43.4	..	3.5	..	1.3	12.4	.93
7	Dellurck Process water gas, Lewes No. 2	50.09	39.95	..	5.38	1.22	3.36	7.4	.88
8	Blue water gas, Morehead	50.0	43.25	.5	3.0	..	3.25	14.4	.935
9	Water gas. Allen	49.65	42.89	.75	2.97	..	3.74	14.4	.935
10	Uncarburetted water gas	49.55	45.89	..	3.87	..	.71	11.8	.92
11	Uncarburetted water gas	49.50	35.93	1.05	4.25	..	8.75	8.45	.89
12	Essen water gas, coke, Thorpe	49.17	43.75	.31	2.71	..	4.00	16.1	.94
13	Water gas before carburetting, Lowell	48.6	43.2	2.0	3.0	.4	2.8	14.4	.93
14	Average water gas, Lewes	48.31	35.93	1.05	4.25	.51	9.95	8.45	.89
15	Water gas before carburetting, average	47.97	42.75	4.23	2.80	.05	2.2	15.3	.94
16	Water gas before carburetting, average	45.57	44.85	4.41	4.45	.5	.77	10.1	.91
17	Lowe water gas, anthracite, Thorpe.	44.50	42.10	..	3.60	..	9.80	11.7	.92
18	Water gas, anthracite, Loomis Pettibone	44.3	42.4	2.7	3.5	.2	6.9	12.1	.92
19	Water gas, bituminous coke, Loomis Pettibone	42.1	32.6	2.9	5.3	.3	16.8	6.15	.86

TABLE CXVII

COMPOSITION OF PRODUCER GAS

No.	Description.	Volumetric Analysis.							Ratios	
		CO.	H_2.	CH_4.	Heavy Hydro carbons.	CO_2.	O_2.	Remainder and N_2.	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$
1	Bit., half water and half air gas, down draft	34.7	26.8	1.80	0.30	3.60	0.20	32.6	9.64	.907
2	Charcoal gas, Thwaite	34.1	0.20	...	...	0.80	...	64.9	42.6	.978
3	Charcoal gas, Thwaite	33.8	0.10	...	...	1.30	...	64.8	26.0	.964
4	Charcoal, Loomis Pettibone, down draft	31.4	9.30	1.90	...	1.20	0.20	56.0	26.2	.963
5	From sawdust in Switzerland	29.8	6.50	6.90	.30	6.00	...	50.5	4.97	.833
6	Producer gas, little steam	28.8	13.47	.10	...	6.20	.20	51.23	4.65	.824
7	From anthracite coal, little steam	28.0	4.00	4.00	...	4.00	...	60.00	7.00	.875
8	Producer gas, little steam	27.8	13.33	...	...	6.20	.40	52.27	4.49	.818
9	Anthracite, Koerting, up draft	27.8	13.33	...	...	6.20	.40	52.27	4.49	.818
10	Producer gas, Jones	27.6	15.30	1.4		3.90	...	51.80	7.08	.876
11	Producer gas, Monaco	27.5	16.67	...	...	8.40	.90	46.53	3.28	.766
12	Peat gas, Akermann	27.2	.90	3.10	...	12.10	...	56.7	2.25	.692
13	Ebelmann producer	27.2	14.0	...	...	5.50	...	53.3	4.95	.826
14	Ingham producer, Sexton	27.0	10.90	1.28	...	4.50	...	56.32	6.00	.857
15	Anthracite, up draft, less than $\frac{1}{4}$ load	27.00	13.90	...	...	4.00	.20	54.90	6.75	.873
16	From bituminous coal, American	26.97	13.00	.33	...	4.37	.21	55.12	6.17	.86
17	Bit., Loomis Pettibone, down draft	26.90	9.40	1.10	.30	3.60	.20	58.5	7.47	.882
18	Wilson producer, Sexton	26.89	11.55	1.45	...	4.00	...	56.11	6.72	.870
19	Bit. coal, up draft, Duff	26.80	13.40	4.40	...	4.40	...	51.00	6.10	.859
20	Wilson producer, bit. coal, England, Patterson and Stead	26.80	11.50	1.40	...	4.00	...	56.30	6.70	.870
21	Anthracite, Koerting, up draft	26.80	13.47	.10	...	6.20	.20	53.23	4.32	.812
22	Produesr gas, Jones	26.50	17.50	2.1		4.40	...	49.50	6.02	.854
23	Siemans closed hearth, steam blast, Sexton	26.40	12.13	2.00	...	9.16	...	50.31	2.88	.744
24	Producer gas	26.30	13.60	.40	...	4.80	.20	54.70	5 49	.846
25	Anthracite, Taylor, up draft	26.10	15.00	.20	...	5.30	.20	53.2	4.92	.831

26	Producer gas	26.10	15.00	.20	...	5.30	.20	53.2	4.92	.851
27	Siemans closed hearth	26.00	1.90	.71	...	4.20	...	67.19	6.19	.860
28	Bit. coal, up draft	26.00	13.00	4.00		4.00	...	53.00	6.50	.867
29	Anthracite, Koerting, up draft	26.00	14.40	.20	...	6.00	1.20	52.2	4.33	.811
30	Producer gas	26.00	14.40	.20	...	6.00	1.20	52.2	4.33	.811
31	Producer gas	25.70	15.30	.20	...	5.50	.40	52.9	4.67	.825
32	Anthracite, Taylor, up draft	25.70	13.30	.20	...	5.50	.40	54.9	4.67	.824
33	Bit. and Anth., up draft, Smith, load $\frac{1}{4}$ to $1\frac{1}{4}$	25.60	22.90	...	...	4.80	.20	46.5	5.35	.842
34	Siemans open hearth, Snelus	25.60	...	4.40	...	4.30	...	65.7	5.95	.856
35	Coke fuel, Adams, whole test	25.5	12.0	.40	...	5.30	.65	56.15	4.81	.828
36	Producer gas, Witz	25.4	16.5	1.00		4.80	1.20	51.10	5.29	.842
37	Producer gas, little steam	25.3	9.2	3.10	.80	3.40	...	58.20	7.43	.883
38	Bit. coal, up draft	25.3	9.2	3.10	.80	3.40	...	58.20	7.43	.883
39	Coke fuel, last six hours	25.3	13.2	.35	...	5.40	.60	55.15	4.70	.824
40	Anthracite, up draft, Dowson	25.25	18.50	2.00	...	5.25	...	49.00	4.85	.829
41	Anth., English, Dowson, Lewes	25.17	18.90	1.40	...	5.98	...	48.55	4.20	.808
42	Dowson producer gas, Thorpe	25.07	18.73	.62	...	6.57	...	49.01	3.82	.792
43	Dowson producer, Anthracite, Eng., Foster	25.07	18.73	.31	.31	6.57	.03	48.98	3.82	.792
44	From anthracite	25.07	18.73	.31	.31	6.57	...	49.01	3.82	.792
45	Anthracite pea, English, Lewes	25.00	14.00	1.00	...	6.00	...	54.00	4.17	.806
46	From anth., American, Wyer	25.00	20.00	...	...	5.00	.50	49.50	5.00	.833
47	Producer gas, little steam	24.80	8.50	5.20	.40	5.60	.40	55.10	4.43	.816
48	Bit., Loomis Pettibone, down draft	24.80	8.50	1.00	.10	4.10	.10	61.40	6.06	.859
49	From Illinois coal	24.50	17.80	3.60	3.20	3.70	.40	46.80	6.62	.869
50	From bit. coal, American	24.47	11.79	.30	...	3.96	.18	59.30	6.18	.860
51	From bit. coal	24.40	8.60	2.40	...	5.20	...	59.40	4.69	.824
52	Siemans closed hearth, steam blast, Snelus	24.40	8.60	2.40	...	5.20	...	59.40	4.69	.824
53	Bituminous and anthracite coal, down draft, Loomis Pettibone	24.40	14.43	3.01	.20	8.80	.50	48.66	2.77	.735
54	Anth., up draft, Deutz Mathot	24.30	18.90	.57	...	6.60	.30	49.33	3.68	.786
55	Bit. coal, reversible, Thwaite Allen	24.00	3.30	...	...	5.00	...	67.70	4.80	.828
56	Producer gas, little steam	24.00	9.80	3.40	.40	6.00	.80	55.60	4.00	.800
57	From German lignite	23.99	10.64	.25	...	5.21	.63	59.28	4.60	.821
58	Producer gas, Jones	23.80	19.80	1.30		6.30	...	48.80	3.77	.792
59	$\frac{3}{4}$ caking, $\frac{1}{4}$ non-caking French coal, Sexton	23.70	8.00	2.20	...	4.10	.40	61.60	5.79	.854
60	Wilson producer gas, Sexton	23.66	10.55	3.05	...	5.25	...	59.49	4.51	.820
61	Producer gas	23.60	12.14	.10	...	5.60	3.00	55.56	4.21	.808
62	Anth. coal, up draft, less than $\frac{1}{4}$ load, Smith	23.50	12.00	....	...	5.00	2.00	57.50	4.70	.825

TABLE CXVII—*Continued*

COMPOSITION OF PRODUCER GAS

No.	Description.	Volumetric Analysis.							Ratios	
		CO.	H_2.	CH_4.	Heavy Hydro carbons.	CO_2.	O_2.	Remainder and N_2.	$\frac{CO}{CO_2}$.	$\frac{CO}{CO+CO_2}$.
63	Ingham producer, Sexton	23.41	13.00	2.22	...	4.69	...	56.68	5.00	.834
64	Wilson producer, Sexton	23.41	13.82	2.22	...	4.69	...	55.86	5.00	.834
65	Wilson producer, Sexton	23.11	14.81	1.14	...	4.84	...	56.10	4.78	.827
66	From bit. coal, American, Wyer	23.00	10.00	3.00	.50	5.00	.50	58.00	4.60	.822
67	Producer gas, Uppenborn	23.00	17.00	2.00		6.00	...	52.00	3.83	.794
68	Anthracite coal, load $\frac{1}{4}$ to $1\frac{1}{4}$, Smith	23.00	22.60	...	...	4.00	1.00	49.40	5.75	.852
69	Anthracite coal, load $\frac{1}{4}$ to $1\frac{1}{4}$, Smith	23.00	23.50	...	...	7.00	2.00	44.50	3.28	.767
70	Producer gas	23.00	14.14	...	...	7.60	.20	55.06	3.02	.752
71	Bit., down draft, Loomis Pettibone	22.80	10.13	2.20	...	5.60	.40	58.87	4.07	.865
72	Producer gas	22.80	17.53	...	...	7.80	.20	51.67	2.92	.745
73	Producer gas, little steam	22.74	8.37	2.56	.36	5.30	.54	60.13	4.29	.809
74	Producer gas	22.70	16.57	...	...	7.80	...	52.93	2.91	.744
75	Producer gas	22.70	15.10	...	...	7.80	.20	54.20	2.91	.744
76	Bituminous coal	22.6	7.80	1.50	...	4.40	...	63.7	5.15	.838
77	Producer gas, Langer	22.5	24.00	...	...	7.50	...	46.0	3.00	.750
78	Anth., Loomis Pettibone, down draft	22.3	11.30	.70	...	5.50	.20	60.00	4.05	.802
79	Producer gas, little steam	22.10	6.80	3.74	.34	4.84	.40	61.78	4.57	.821
80	Producer gas	22.00	15.87	...	...	7.60	.20	54.33	2.89	.744
81	Anthracite, up draft, load $\frac{1}{4}$ to $1\frac{1}{4}$, Smith	22.00	21.30	...	...	5.00	1.00	50.70	4.40	.815
82	From bituminous coal	22.00	10.50	2.60	.60	5.70	.40	58.20	3.86	.794
83	Producer gas	21.90	16.80	...	...	8.40	.30	52.60	2.61	.724
84	Ebelmann producer	21.83	7.64	...	...	9.06	...	61.47	2.41	.709
85	Producer gas	21.70	16.04	...	...	8.80	.20	53.26	2.47	.711
86	Siemans closed hearth	21.60	9.60	3.60	...	5.00	...	60.20	4.32	.812
87	Gas from bituminous coal after heating in open-hearth furnace regenerator, Darby	21.60	17.70	2.00	.40	10.50	...	47.80	2.06	.673

88	Anthracite pea, English, Lewes	21.00	15.20	1.20	...	8.20	.20	54.20	2.56	.719
89	Anthracite, up draft, load ¼ to 1¼, Smith	21.00	21.50	...	...	9.00	1.00	47.50	2.33	.700
90	Bit., down draft, Loomis Pettibone	21.00	9.50	1.90	...	6.50	.60	60.50	3.23	.764
91	North Dakota, No. 2, U.S.G.S.	20.90	14.33	4.85	...	8.69	.23	51.00	2.402	.708
92	Producer gas, little steam	20.80	6.90	2.20	.20	4.60	.40	64.90	4.52	.819
93	Producer gas	20.50	13.00	...	...	9.80	.30	56.40	2.09	.678
94	Ingham producer, Sexton	20.40	12.60	3.50	...	5.50	...	58.00	3.70	.788
95	Anthracite coal, up draft, less than ¼ load, Smith	20.30	17.00	...	...	8.00	.70	54.00	2.54	.719
96	From lignite	20.20	25.00	...	...	6.60	2.20	46.00	3.06	.754
97	Dowson producer, anth. coal, last 6 hrs., Adams	20.13	15.64	1.16	...	6.09	.74	56.24	3.30	.766
98	Producer gas, little steam	20.00	5.30	3.00	.20	3.60	.40	67.50	5.55	.848
99	Indian Territory No. 1, U.S.G.S.	19.39	7.69	4.92	...	8.25	.11	59.64	2.35	.700
100	Dowson producer, anth. coal, average for whole test, Adams	19.05	15.59	1.31	...	5.93	.79	57.33	3.22	.764
101	Montana No. 1, U.S.G.S.	18.67	8.00	4.84	...	9.04	.36	59.09	2.065	.674
102	Bit. and anth., down draft, Sandberg	18.40	9.90	1.10	.10	5.10	...	65.40	3.61	.784
103	Lignite, Smith	18.30	25.60	...	...	8.40	1.80	45.90	2.18	.685
104	Texas No. 2, U.S.G.S.	18.22	9.63	4.81	...	9.60	.20	57.54	1.90	.654
105	Indian Territory, No. 4, U.S.G.S.	17.64	10.43	6.30	...	7.29	.23	58.11	2.42	.708
106	Colorado No. 1, U.S.G.S.	17.38	11.05	5.00	...	10.11	.55	55.91	1.715	.631
107	Bituminous coal, reversible, Thwaite, Lewes	17.16	7.33	4.	10	11.83	...	59.58	1.45	.593
108	Alabama No. 2, U.S.G.S.	16.65	7.20	5.64	...	8.16	.10	62.25	2.04	.670
109	From anthracite	16.60	24.20	2.00	...	11.30	...	45.90	1.47	.595
110	Siemans closed hearth, steam blast, Sexton	16.15	19.43	2.66	...	11.53	...	50.23	1.40	.584
111	West Virginia No. 4, U.S.G.S	15.82	11.16	3.74	...	10.16	.24	58.88	1.56	.610
112	Lignite	15.50	27.0	...	...	8.00	.35	46.00	1.94	.660
113	Wyoming No. 2, U.S.G.S.	15.46	10.79	5.52	...	10.21	.59	57.43	1.51	1.51
114	Illinois No. 3, U.S.G.S.	15.31	8.35	4.46	...	10.53	.15	61.20	1.455	.593
115	Illinois No. 4, U.S.G.S.	15.12	9.98	6.00	...	9.72	.12	59.06	1.558	.608
116	West Virginia No. 9, U.S.G.S.	14.77	9.51	6.65	...	8.90	.33	59.84	1.66	.623
117	Texas No. 1, U.S.G.S.	14.43	10.54	7.48	...	11.10	.22	56.23	1.302	.567
118	West Virginia No. 1, U.S.G.S.	14.34	2.81	5.56	...	10.50	.10	66.69	1.362	.577
119	West Virginia No. 12, U.S.G.S.	14.21	12.98	4.61	...	10.34	.12	57.74	1.372	.580
120	Indiana No. 1, U.S.G.S.	14.10	9.56	6.08	...	9.89	.25	60.12	1.427	.589
121	Lignite, Loomis Pettibone, down draft	14.10	13.80	2.60	.40	10.60	.30	58.20	1.33	.571
122	West Virginia No. 9, U.S.G.S.	13.70	9.55	6.60	...	10.40	.20	59.55	1.319	.569
123	Bituminous, Mond, Lewes	13.20	24.80	2.30	...	12.90	...	46.80	1.030	.506
124	From bituminous coal.	13.20	24.80	2.30	...	12.90	...	46.80	1.030	.506

TABLE CXVII—*Concluded*

COMPOSITION OF PRODUCER GAS

No.	Description.	Volumetric Analysis.							Ratios	
		CO.	H_2.	CH_4.	Heavy Hydrocarbons.	CO_2.	O_2.	Remainder and N_2	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$
125	West Virginia No. 7, U.S.G.S.	12.75	10.308	6.758	...	9.617	.084	60.483	1.323	.571
126	Iowa No. 2, U.S.G.S.	12.571	9.529	7.671	...	10.057	.171	60.001	1.25	.554
127	Kentucky No. 3, U.S.G.S.	12.45	10.92	6.52	...	10.87	.29	58.95	1.148	.535
128	Kansas No. 5, U.S.G.S.	12.40	9.05	7.417	...	10.267	.133	60.733	1.21	.547
129	West Virginia No. 8, U.S.G.S.	11.927	9.454	6.4	...	10.327	.218	61.674	1.153	.537
130	Indiana No. 2, U.S.G.S.	11.46	10.60	6.10	...	11.80	.070	59.97	.970	.493
131	Bituminous, up draft, excess steam, Mond, Humphrey	11.00	29.00	2.00	...	16.00	...	42.0	.687	.408
132	Bituminous, up draft, excess steam, Duff	11.00	28.00	2.50	...	15.50	...	43.0	.710	.415
133	Bit. coal, Mond gas, Sexton	11.00	27.20	1.80	.40	17.10	...	42.5	.644	.392
134	From bit. coal, much steam.	11.00	27.50	2.00	...	16.50	...	43.00	.666	.4
135	Coke and breeze, Mond, Allen	10.80	25.20	.40	...	16.80	...	46.80	.643	.391
136	Missouri No. 2, U.S.G.S.	10.53	7.63	6.33	...	12.07	.20	63.24	.873	.465
137	From bit. coal (much steam), Darby	10.50	24.80	2.60	.70	17.80	...	43.60	.590	.371
138	Siemans closed hearth, excess steam to make NH_3	10.00	23.00	3.00	...	15.00	...	49.00	.666	.400
139	German brown coal lignite	8.10	24.30	16.50	1.40	17.00	3.10	29.60	.475	.323
140	From coke, German, Meyer, 1st hr.	26.60	6.80	1.30	...	6.50	.10	58.70	4.09	.804
141	" " " 2d hr.	28.20	5.90	2.70	...	4.80	...	58.40	5.87	.854
142	" " " 3d hr.	27.80	6.40	2.60	...	4.20	...	59.00	6.62	.868
143	" " " 4th hr.	26.70	8.80	1.80	...	4.70	.10	57.90	5.68	.851
144	" " " 5th hr.	26.60	9.10	2.20	...	5.00	...	57.10	5.32	.843
145	" " " 6th hr.	29.00	8.00	1.70	...	4.00	...	57.30	7.25	.878
146	" " " 7th hr.	28.40	5.80	2.00	...	4.80	.20	58.80	5.92	.856
147	" " " 8th hr.	27.80	5.10	1.60	...	4.60	.10	60.80	6.05	.858
148	Mond producer, bit. coal, Bone & Wheeler, air blast, sat. H_2O at 60°	27.30	16.60	3.35	...	5.25	...	47.50	5.20	.838
149	" " " " " " 65°	25.40	18.30	3.40	...	6.95	...	45.95	3.66	.874

150	Mond producer, bit. coal, Bone & Wheeler, air blast, sat. H_2O at 70°.							21.70	19.65	3.40	...	9.15	...	46.10	2.37	.704
151	" " " " " " 75°.							18.35	21.80	3.35	...	11.65	...	44.85	1.57	.612
152	" " " " " " 80°.							16.05	22.65	3.50	...	13.25	...	44.55	1.21	.548
153	Coke Dowson producer, full load engine							27.65	9.85	....	...	3.80	.30	58.40	7.30	.879
154	" " no load engine							22.40	7.00	....	...	4.90	.50	65.20	4.56	.821
155	Anthracite Dowson producer, 1st hr., no load							10.20	11.05	2.75	...	11.20	.10	64.70	.91	.477
156	" " " 2d hr., no load							12.45	12.15	2.00	...	10.10	...	63.30	1.23	.553
157	" " " 3d hr., full load							22.25	15.90	1.95	...	5.26	.10	54.54	4.27	.810
158	" " " 4th hr., no load							18.30	13.60	1.20	...	9.05	.30	57.55	2.02	.670
159	" " " 5th hr., $\frac{1}{4}$ load							20.00	11.80	.75	...	8.35	.10	59.00	2.40	.706
160	" " " 6th hr., $\frac{1}{2}$ load							20.90	14.0	.70	...	7.70	...	56.7	2.72	.731
161	" " " 7th hr., full load							21.50	14.5	1.20	...	7.60	...	55.2	2.82	.738

TABLE CXVIII

GAS PRODUCER TESTS

U.S.G.S.—(FERNALD)

COAL AND GAS ANALYSIS

Coal.	Proximate Analysis of Coal.				B.T.U. per Pound Dry Coal.	B.T.U. in Gas per Pound Dry Coal.	Eff. of Producer Process.	Gas Analysis by Volumes.						Ratio, $\frac{CO}{CO_2}$	Ratio, $\frac{CO}{CO+CO_2}$	Tar.
	Moisture.	Volatile.	Fixed C.	Ash.				CO	H_2	CH_4 and Others	CO_2	O_2	Diff. and N			
Brown lignite, North Dakota, No. 2	39.56	27.78	26.30	6.36	11255	7830	63	20.90	14.33	4.85	8.69	.23	51.02	2.40	.7063	50 gals. 13,800 lbs. coal
Bit. semi-caking, Indian Territory, No. 1	5.00	36.51	49.98	8.51	13455	8620	64	19.39	7.69	4.69	8.25	.11	59.65	2.35	.7007	2½ bbls. 11,200 lbs. coal
Bit. clinker, Montana, No. 1	11.40	34.55	43.31	10.74	11934	6580	55	18.67	8.00	4.84	9.04	.36	59.10	2.05	.6730	10,200 lbs. coal
Brown lignite, Texas, No. 2	33.71	29.25	29.76	7.28	11086	8060	73	18.22	9.63	4.81	9.60	.20	57.53	1.90	.6549	60 gals. 9,050 lbs. coal
Bit. Indian Territory, No. 4	9.0	33.96	40.68	16.36	11392	9980	88	17.64	10.43	6.30	7.29	.23	58.11	2.30	.7075	50 gals. 6,300 lbs. coal
Black lignite, clinker slight, Colorado, No. 1	20.24	32.26	41.65	5.85	12245	7860	64	17.38	11.05	5.00	10.11	.55	55.90	1.72	.6322	50 gals. yellow 10,933 lbs. coal
Hard bit. non-caking, Alabama, No. 2	3.76	33.45	53.29	9.50	13365	9000	73	16.65	7.20	5.64	8.16	.10	62.24	2.05	.6711	Considerable, not measured
Semi-caking bit., West Virginia, No. 4	1.99	28.89	60.30	8.82	14202	11610	82	15.82	11.06	3.74	10.16	.24	58.88	1.56	.6080	2,100 lbs.coal
Bit. non-caking, Wyoming, No. 2	9.44	35.02	34.82	26.72	10656	6168	58	15.46	10.79	5.52	10.21	.59	57.43	1.52	.6022	60 gals. 12,100 lbs. coal
Bit. non-caking, no clinker, Illinois, No. 3	7.62	30.87	51.78	9.73	13041	8330	64	15.31	8.35	4.46	10.53	.15	61.19	1.46	.5924	60 gals. 10,500 lbs. coal
Bit. non-caking, no clinker, Illinois, No. 4	12.43	32.65	45.70	9.22	12834	8840	69	15.12	9.98	6.00	9.72	.12	59.06	1.56	.6086	75 gals. 10,500 lbs.coal

West Virginia, No. 9....	2.22	31.05	59.83	6.90	14548	11380	78	14.77	9.51	6.65	8.90	.33	59.85	1.66	.6239	50 gals. 6,000 lbs. coal
Brown lignite clinker, Texas, No. 1.....	33.50	32.34	23.80	10.36	10928	7260	66	14.33	10.59	7.48	11.10	.22	56.22	1.29	.5635	150 gals. 12,800 lbs. coal
West Virginia, No. 1....	1.61	36.85	55.40	6.14	14396	9260	64	14.34	2.81	5.56	10.50	.10	66.69	1.36	.5772..	6,900 lbs. coal
West Virginia, No. 12...	1.43	18.93	73.19	6.45	14825	10150	68	14.21	12.98	4.61	10.34	.12	57.75	1.38	.5788	50 gals. 8,100 lbs. coal
Bit. non-caking, Indiana, No. 1.....	11.51	36.04	42.37	10.08	13037	7730	60	14.10	9.56	6.08	9.89	.25	60.13	1.43	.5877	70 gals.black 11,700 lbs. coal
West Virginia, No. 9....	2.66	32.00	59.61	5.73	14580	8150	56	13.70	9.55	6.60	10.40	.20	59.55	1.32	.5684	120 gals. 1,300 lbs. coal
West Virginia, No. 7....	2.99	21.19	69.15	6.67	14720	13140	89	12.75	10.31	6.76	9.62	.08	60.48	1.32	.5699..	6,000 lbs. coal
Bit., Iowa, No. 2.....	16.69	31.42	31.19	20.70	10489	9300	88	12.57	9.53	7.67	10.06	.17	60.00	1.19	.5554	50 gals. 4,833 lbs. coal
Bit. semi-caking, Kentucky, No. 3.....	7.28	38.57	45.16	8.99	13226	8610	65	12.45	10.92	6.52	10.87	.29	58.95	1.15	.5338	100 gals. black 11,100 lbs. coal
Kansas, No. 5.....	4.35	31.97	52.43	11.25	13421	10500	79	12.40	9.05	7.42	10.27	.13	60.73	1.20	.5469..	4,000 lbs. coal
West Virginia, No. 8....	2.66	32.58	59.00	5.76	14558	9070	62	11.93	9.45	6.40	10.33	.22	60.67	1.16	.5359	75 gals. 8,900 lbs. coal
Bit. non-caking, Indiana, No. 2.....	8.72	39.60	41.95	9.73	12953	10140	79	11.46	10.60	6.10	11.80	.07	59.97	.98	.4926	60 gals. 6,900 lbs. coal
Bit., Missouri, No. 2....	11.60	35.28	38.28	14.84	11882	8820	74	10.53	7.63	6.33	12.07	.20	63.23	.87	.4659..	3,300 lbs. coal

Table CXIX

COMPOSITION OF OIL PRODUCER GAS

Name.	Volumetric Analysis, Per Cent.							Ratio.		B.T.U. per Cubic Foot.	
	CO	H_2	CH_4	C_nH_{2n}	O_2	CO_2	N_2	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$	High.	Low.
Process of International Amet. Co.	8.0	12.0	16.2	2.0	.2	4.2	57.4	1.9	.66	275	249
Do.	8.6	10.0	7.0	4.2	.3	5.4	64.4	1.6	.61	209	192
Do.	7.8	9.8	6.0	4.0	.4	6.5	65.5	1.2	.55	192	176
Lowe process.	7.3	47.4	28.6	10.0	.2	2.0	4.5	3.7	.78	661	605
" "	8.35	53.65	22.50	5.4	.4	2.25	7.45	3.7	.79	543	487
" "	6.0	46.0	26.0	10.30	.3	3.0	8.4	2.0	.67	630	566

Table CXX

COMPOSITION OF POWDERED COAL, PRODUCER GAS

Sample.	Volumetric Analysis, Per Cent.							Ratio.		B.T.U. per Cubic Foot.	
	CO	H_2	CH_4	C_nH_{2n}	O_2	CO_2	N_2	$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$	High.	Low.
1	15.85	6.17	4.09	....	1.4	9.2	63.29	1.7	.63	119	11
2	13.52	11.51	5.17	....	.3	8.1	61.40	1.7	.63	140	12
3	12.20	10.50	3.20	....	.0	7.6	66.50	1.6	.62	112	10
4	18.2	12.20	2.1	.1	.1	4.9	62.40	3.7	.79	128	11
5	13.8	10.4	2.5	.5	....	8.0	64.80	1.7	.63	118	10

CALORIFIC PROPERTIES OF BEST AIR-GAS MIXTURES—(Low Values)

(All at 32° F. and 29.92″ Hg.)

Name of Gas.	Volumetric Analysis, Per Cent.									Air.		Gas.		Gas B.T.U.		Mixture B.T.U.	
	CO	H_2	CH_4	C_2H_4	C_2H_6	C_6H_6	O_2	CO_2	N_2	Cu.ft. per Cu.ft. Gas.	Lbs. per Lb. of Gas.	Lbs. per per Cu.ft.	Cu.ft. per Pound.	Per Pound.	Per Cu.ft. 32° F. 29.92″ Hg.	Per Pound.	Per Cu.ft. 32° F 29.92″ Hg.
Carbon monoxide..	100	...	...	...	...	..	..	...	...	2.39	2.470	.07807	12.8090	4369	341	1258.6	100.59
Hydrogen.....	...	100	...	...	..	..	..	...	...	2.39	34.32	.005621	177.9093	51892	292	1469	86.1
Methane.....	...	...	100	...	..	..	..	...	...	9.56	17.244	.0447	22.349	21463	959	1176.4	90.81
Ethylene.....	...	...	...	100	..	..	..	...	...	14.34	14.557	.07951	12.578	20053	1595	1931.7	103.97
Ethane.....	...	...	...	...	100	..	..	...	...	16.73	16.115	.08379	11.9354	20481	1716	1196.6	96.78
Benzene.....	...	...	...	...	..	100	..	...	...	35.85	13.194	.2193	4.56	17305	3795	1219.17	102.98
Retort coal gas....	8.6	52.5	31.35	1.1	..	1.1	.03	1.5	3.5	5.0084	12.683	.031872	31.375	17022.7	542.6	1244.1	90.30
Coke-oven gas, rich	7.1	37.4	40.4	2.9	..	2.9	1.6	2.1	5.6	6.381	11.753	.04382	22.8206	15452.9	677.1	1211.7	91.74
Coke-oven gas, lean	6.2	46.2	27.1	1.3	..	1.2	.6	3	14.4	4.4597	9.300	.038705	25.8364	12458.4	482.2	1209.5	88.32
Coke-oven, gas, average.....	6.0	42	34.3	2.0	..	2.0	1.1	2.5	10.1	5.4301	10.872	.04031	24.807	14384	579.84	1211.6	90.1
Oil gas (retort)....	1.0	18.5	52.5	11.7	..	11.8	.5	.5	...	11.3936	14.943	.06153	16.25	19443	1196.5	1219.5	96.52
Carburetted water gas.....	28.1	21.8	30.7	12.9	..	..	:5	3.8	2.2	5.9774	8.9395	.05397	18.5288	12888	659.6	1296.6	94.53
Blast-furnace gas..	27.5	3.0	...	...	..	..	..	10.0	59.5	.7289	.7309	.08049	12.4239	1273.8	102.53	735.9	59.30
Water gas.....	35.93	49.5	1.05	...	..	..	..	4.25	9.27	2.1458	3.964	.04368	22.89377	6344	277.13	1278	88.09
Anthracite producer gas.....	25.7	15.3	.2	...	..	..	.4	5.5	52.9	.9989	1.1586	.069536	14.381	1928.9	134.13	893.6	67.102
Bituminous producer gas (up)...	14.34	2.81	5.56	...	..	..	.10	10.50	66.69	.9374	1.059	.079021	12.654	1397.4	110.43	680	59.15
Do. (down).....	20.0	14.0	2.0	.2	..	..	.1	8.2	55.5	1.023	1.162	.07105	14.074	1849.9	131.45	855.6	64.98
Do. mond.....	12	29	2.0	...	..	..	..	14.5	42.5	1.1711	1.4961	.063169	15.83022	2292	144.78	918.2	66.67
Lignite producer gas.....	20.90	14.33	4.85	...	..	..	.23	8.69	51.02	1.3056	1.5036	.070101	14.265	2276.9	159.62	909.4	69.23
Alcohol.....	...	...	...	...	..	..	..	...	...	14.424	9.06	.01285	7.782	12100.4	1554.9	1202.8	100.8
Gasolene.....	...	...	...	...	..	..	..	...	...	52.74	15.24	.2793	3.58	20519.7	5731.7	1263.5	106.66
Kerosene.....	...	...	...	...	..	..	..	...	...	88.71	15.079	.4748	2.106	20255	9617.7	1259.7	107.2
Natural gas, Kans.	.25	...	98.2	...	..	.1	.25	...	...	9.3969	17.004	.0446	22.42	21301	950.1	1183.1	91.38

Heating value of gases based on experimental value of constituents.

TABLE CXXII

COMPOSITION OF BOILER FLUE GASES—(VOLUMETRIC)

Average of.	Stat. Boiler, Illinois Coal, U. S. Geological Survey.					Locomotive Boiler, U. S. Geological Survey.				
	Analysis.			$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$	Analysis.			$\frac{CO}{CO_2}$	$\frac{CO}{CO+CO_2}$
	CO_2	O_2	CO			CO_2	O_2	CO		
4	3.4	17.5	0	0	0	10.16	8.49	.13	.0128	.0126
3	3.7	17.2	0	0	0	11.10	7.84	.23	.0207	.0203
5	4.4	16.3	0	0	0	11.15	7.52	.20	.0179	.0176
5	5.0	15.0	0	0	0	11.45	6.92	.00	0	0
5	5.3	14.7	.1	.0189	.0185	11.46	7.49	.10	.00875	.00865
5	5.9	14.4	.04	.0068	.00674	11.50	7.08	.17	.0148	.0147
6	6.2	14.1	.03	.00485	.00482	11.96	7.00	.23	.0193	.0189
9	6.4	13.7	.07	.0109	.0108	11.96	7.07	.14	.0117	.0155
16	6.6	13.0	.10	.0152	.0149	12.05	6.93	.15	.0125	.0123
9	6.8	12.6	.01	.00147	.0147	12.20	6.94	.05	.0041	.0407
14	7.0	12.8	.06	.0086	.0085	12.45	5.87	.22	.0177	.0174
20	7.2	12.6	.08	.0111	.011	13.57	4.49	.20	.0147	.0145
18	7.4	12.4	.00	0	0	13.87	4.75	.25	.018	.0177
20	7.6	12.9	.05	.0066	.00655					
14	7.8	12.1	.03	.00385	.00375					
30	8.0	11.7	.04	.005	.00498					
31	8.2	11.6	.10	.0122	.012					
27	8.4	11.3	.10	.0119	.01175					
16	8.6	11.1	.10	.0116	.0115					
17	8.8	10.8	.20	.0228	.0222					
19	9.0	10.7	.10	.0111	.011					
14	9.2	10.4	.10	.0109	.01075					
16	9.4	10.1	.20	.0213	.0208					
10	9.6	9.9	.20	.0208	.0204					
8	9.8	9.4	.20	.0204	.02					
8	10.0	9.2	.20	.020	.0196					
6	10.2	9.9	.20	.0196	.0192					
8	10.4	8.9	.5	.048	.046					
4	10.8	8.6	.02	.00185	.00185					
3	11.0	8.8	.36	.0327	.0317					
2	11.1	8.6	.30	.027	.0253					
1	11.4	7.9	.40	.035	.034					

TABLE CXXIII

LIMITS OF PROPORTION FOR EXPLOSIVE AIR-GAS MIXTURES

Gas.	Per Cent of Gas in the Mixture by Volume.			Authority.
	Combining Proportion.	When Air is in Excess.	When Gas is in Excess.	
Carbon monoxide	29.6	16.5	74.95	Eitner
" "	29.6	16.5	58.4	Bunte
" "	29.6	13.0	75	Clowes
Hydrogen	29.6	9.45	66.4	Eitner
"	29.6	9.45	54.4	Bunte
"	29.6	7.69	33.3	M.I.T.
"	29.6	5.00	72.0	Clowes
Water gas, theoretical	29.6	12.4	66.75	Eitner
" "	29.6	12.4	54.3	Bunte
" actual	.....	9.0	55.0	Clowes
" "	.....	3.8	16.7	M.I.T.
" "	.....	8.33	33.3	M.I.T.
Coal gas	14.9	7.9	19	Eitner
"	14.9	7.9	11.2	Bunte
"	14.9	5.3	16.7	Clerk
"	14.9	6.0	29	Clowes
"	.....	6.7	20	Clerk
"	.....	6.25	14.28	Grover
Boston illuminating gas	.....	6.67	25.0	M.I.T.
" "	.....	6.25	12.5	M.I.T.
" "	.....	6.67	20.0	M.I.T.
Acetylene	7.9	3.35	52.3	Eitner
"	7.9	3.35	49.0	Bunte
"	7.9	1.54	47.6	M.I.T.
"	7.9	2.96	66.7	
"	7.9	3.0	82.0	Clowes
Ethylene	6.5	4.1	14.6	Eitner
"	6.5	4.1	10.5	Bunte
Methane	9.5	6.1	12.8	Eitner
"	9.5	6.1	9.7	Bunte
"	9.5	5.0	13.0	Clowes
Ether	3.4	2.75	7.7	Eitner
"	3.4	2.75	5.0	Bunte
Benzene	2.7	2.65	6.5	Eitner
"	.....	2.65	3.9	Bunte
"	.....	2.4	4.9	Eitner
Pentane	2.6	2.4	2.5	Bunte
"	2.6	2.4	4.9	Eitner
Gasoline	.....	2.5	2.4	Bunte
" 86° Bé	.....	1.54	4.76	M.I.T.
" 71° Bé	.....	1.54	4.76	M.I.T.
" 65° Bé	.....	1.31	4.76	M.I.T.
Alcohol	6.5	3.95	13.65	Eitner
"	6.5	3.95	9.7	Bunte
Blau oil gas	.....	4.0	8.0	Hallock
Pintsch oil gas	.....	.....	.....	Lucke
Ethane	.....	4.0	22.0	Clowes

TABLE CXXIV
RATE OF COMBUSTION OF COAL WITH DRAFT.

[Conditions and Authority.		Rate of Combustion, Lbs. Coal per Hour per Square Foot	Draft in Inches of Water.
Locomotive, Purdue, reported by Goss for two coals: (a) Moisture 1.89, Volatile 31.94, Fixed carbon 57.71, Ash 8.46 (b) Moisture 3.10, Volatile 15.23, Fixed carbon 72.75, Ash 8.92 Draught taken in smoke box, furnace draft less by loss due to tube resistance, which is from one and one-half to three times the actual draft		114 108 94 69 101 99 91 91 53 82 75 50 134 131 89 74 58 36	5.18 5.15 4.28 3.09 5.60 3.85 3.69 4.37 2.04 4.56 3.50 2.25 5.74 5.65 3.22 3.10 3.00 1.25
Anthracite small coal	Ennis, buckwheat No. 1	10 15 20 25	.30 .45 .70 1.00
	Oneida, pea " buckwheat No. 1 " " No. 2 " " No. 3 Eckley No. 3	13.63 13.58 11.40 11.34 9.44	.38 .50 .60 1.04 1.13
	Coree, buckwheat Stoker, buckwheat " rice	19.80 32.9 28.0	.34 .58 .92
Torpedo boat reported by Lechner. Bit. coal. Locomotive type of boiler	Full grate	53.6 63.4 76.4 93.4	.40 .59 .79 1.37
	Half grate	66.7 101.4 113.0	.78 1.96 2.27
Whitham reporting Cambria run-of-mine bituminous coal under H.R.T. boiler. Ordinary conditions		5 8 10 12 14 15 16 18 20 22 25 28 30 34 36 40	.04 .11 .13 .17 .19 .20 .21 .23 .24 .26 .27 .29 .30 .32 .33 .36

RATE OF COMBUSTION OF COAL

U. S. Geological Survey Tests

Coal.	Proximate Coal Analysis.				Draft, Inches, H_2O.		Pounds per Square Foot per Hour.		
	Moisture.	Volatile.	Fixed C.	Ash.	Hood.	Furnace.	As Fired.	Dry.	Combustible.
Alabama No. 1	2.56	31.00	52.52	13.92	.36	.18	20.68	20.72	17.38
" No. 1, briquettes	2.63	33.00	50.96	13.41	.46	.18	19.48	18.97	16.40
" No. 2	4.83	32.98	48.65	13.54	.39	.17	22.60	21.54	18.60
Arkansas No. 1	1.99	18.61	66.36	13.04	.36	.12	17.30	16.90	14.80
" No. 1, briquettes	.94	21.21	67.65	10.20	.35	.15	18.93	18.74	16.63
" No. 2	1.07	16.86	73.65	8.42	.35	.15	16.00	15.70	14.38
" No. 2, briquettes	4.88	22.49	60.30	12.33	.37	.15	18.18	17.31	14.95
" No. 3	1.97	16.04	72.74	9.25	.40	.16	20.50	19.68	17.15
" No. 3, briquettes	2.60	17.35	62.04	18.01	.60	.22	21.00	20.49	16.80
" No. 4, briquettes	3.85	14.06	71.98	10.11	.45	.19	20.15	19.41	16.50
" No. 5	2.22	12.54	73.68	11.56	.53	.28	21.87	21.42	16.60
Colorado No. 1, lignite	19.78	35.85	39.00	5.37	.66	.33	22.15	17.80	15.22
Illinois No. 1	9.69	36.91	38.21	15.19	.40	.18	27.60	24.90	20.50
" No. 2, washed	10.45	37.77	41.72	10.06	.42	.15	24.95	22.36	20.05
" No. 3	8.51	31.19	48.75	11.55	.58	.21	23.20	21.23	17.90
" No. 4	13.47	33.48	41.59	11.46	.55	.16	23.00	19.84	17.00
" No. 4	12.58	32.44	43.63	11.35	.52	.21	26.50	23.13	19.90
" No. 6	13.19	32.31	39.62	14.88	.69	.23	25.80	22.34	18.88
Indiana No. 1, briquettes	11.74	38.79	43.23	6.24	.54	.144	24.55	21.70	20.05
" No. 1, washed	16.59	35.17	40.41	7.83	.485	.130	26.80	22.39	20.40
" No. 2	9.11	38.04	40.40	12.45	.516	.165	22.55	20.51	18.00
Indian Territory No. 1	7.65	33.96	46.30	12.09	.306	.122	20.75	19.17	16.40
" No. 2	3.71	36.21	50.31	9.77	.339	.124	22.25	21.50	18.90
" No. 3	4.79	37.30	47.58	10.33	.490	.180	22.46	21.43	18.05
" No. 4	6.24	35.44	45.33	12.99	.590	.210	22.41	21.04	18.50

TABLE CXXV—*Continued*

RATE OF COMBUSTION OF COAL

U. S. GEOLOGICAL SURVEY TESTS

Coal.	Proximate Coal Analysis.				Draft, Inches, H_2O.		Pounds per Square Foot per Hour.		
	Moisture.	Volatile.	Fixed C.	Ash.	Hood.	Furnace.	As Fired.	Dry.	Combustible.
Iowa No. 1	8.69	33.08	39.89	18.34	.460	.150	25.42	23.23	18.60
" No. 2	14.88	35.35	33.73	16.04	.620	.210	27.30	23.28	19.20
" No. 3	12.44	36.14	35.77	15.65	.580	.220	26.22	22.96	19.43
" No. 4	13.48	34.09	37.28	15.15	.500	.193	23.15	20.02	16.80
" No. 4, briquettes	13.24	36.50	37.85	12.41	.590	.220	24.30	21.11	18.15
" No. 5	16.01	31.76	38.83	13.04	.510	.220	27.61	23.23	20.00
Kansas No. 1	5.90	33.78	49.46	10.86	.430	.240	17.62	16.60	14.80
" No. 1	4.80	32.68	48.57	13.95	.390	.140	19.10	18.13	15.53
" No. 2	4.18	31.23	46.68	17.91	.470	.220	19.70	18.90	16.68
" No. 2, washed	5.82	34.32	51.22	8.64	.390	.160	21.30	20.08	17.85
" No. 3	2.03	33.52	50.99	13.46	.416	.142	19.20	18.77	16.17
" No. 3	2.25	34.30	51.05	12.40	.410	.250	16.82	16.45	14.30
" No. 4	5.51	36.32	43.59	14.58	.510	.190	19.21	18.18	15.65
" No. 5	4.31	32.42	51.36	11.91	.520	.170	21.53	20.62	18.10
Kentucky No. 1	2.89	35.61	55.59	5.91	.400	.150	20.50	19.95	18.40
" No. 2	7.76	37.91	45.75	8.58	.460	.160	23.58	21.75	19.38
" No. 2, briquettes	7.11	37.07	44.32	11.50	.490	.170	23.52	21.87	19.18
" No. 3	7.92	37.32	44.84	9.92	.470	.120	23.64	21.75	19.52
" No. 4	5.89	36.65	45.74	12.72	.550	.195	23.27	21.90	19.05
Missouri No. 1	7.28	34.88	40.64	17.20	.400	.170	24.00	22.30	18.00
" No. 1, briquettes	6.38	37.60	41.85	14.17	.440	.170	21.70	20.37	17.15
" No. 1, washed	7.93	36.81	44.21	11.05	.370	.160	22.45	20.71	18.23
" No. 2	13.09	32.88	37.33	16.70	.623	.198	28.74	25.00	19.88
" No. 2	11.57	31.77	39.76	16.90	.323	.130	22.82	20.20	16.95
" No. 3	18.63	26.18	29.98	25.51	.700	.280	26.75	21.85	15.70

Missouri No. 3, washed	20.78	31.18	39.61	8.43	.650	.260	27.42	21.72	19.10
" No. 4	12.24	40.10	42.11	5.55	.520	.160	23.45	20.64	19.21
New Mexico No. 1	11.90	37.85	41.57	8.68	.360	.140	26.80	23.70	21.60
" No. 2	9.92	37.30	36.11	16.67	.490	.230	29.22	26.37	19.71
" No. 2, briquettes	6.75	37.56	39.07	16.62	.540	.210	23.83	22.00	16.75
North Dakota No. 1, lignite	35.84	28.13	25.40	10.63	.575	.325	26.65	17.15	15.20
Pennsylvania No. 1	1.10	15.80	75.69	7.41	.480	.165	15.88	15.70	15.00
" No. 2	.59	16.61	76.76	6.04	.310	.153	16.72	16.60	16.10
" No. 3, briquettes	3.00	27.62	55.00	14.38	.610	.250	19.60	19.01	15.08
" No. 4	2.90	28.70	60.82	7.58	.450	.190	19.20	18.67	17.08
Texas No. 1, lignite	23.27	31.42	29.44	15.87	.520	.340	13.54	10.38	7.80
West Virginia No. 1	1.90	34.64	56.25	7.21	.310	.130	19.33	18.94	17.47
" No. 2	2.01	39.23	48.80	9.96	.460	.240	20.18	19.75	17.78
" No. 3	2.54	30.31	56.11	11.04	.500	.200	20.35	19.82	17.42
" No. 4	2.53	27.64	59.84	9.99	.480	.180	19.70	18.98	16.98
" No. 5	2.11	28.95	58.66	10.28	.510	.190	18.85	18.44	16.47
" No. 6	2.14	22.38	70.03	5.45	.450	.160	18.37	17.95	16.70
" No. 6	2.11	21.44	71.42	5.03	.463	.181	17.87	17.21	16.27
" No. 7	2.68	20.23	68.27	8.82	.530	.217	18.70	18.15	16.63
" No. 8	5.26	31.19	56.68	6.87	.370	.120	19.73	18.72	16.80
" No. 9	3.42	31.11	59.47	6.00	.500	.180	18.40	17.78	16.35
" No. 10	1.74	18.23	73.84	6.19	.380	.170	18.45	18.15	16.62
" No. 11	4.85	16.31	68.36	10.48	.520	.220	19.05	18.13	15.58
" No. 12	1.58	18.26	75.33	4.83	.435	.175	18.00	17.68	16.30
" No. 12, briquettes	2.32	24.02	67.46	6.20	.410	.140	18.15	17.66	15.90
Wyoming No. 1, lignite	21.81	40.56	31.61	6.02	.620	.230	28.95	22.69	20.68
" No. 2	11.10	35.55	34.58	18.77	.490	.140	29.80	26.51	19.80

CHAPTER VI

HEAT AND WORK. GENERAL RELATIONS BETWEEN HEAT AND WORK. THERMAL EFFICIENCY OF STEAM, GAS, AND COMPRESSED AIR ENGINES. FLOW OF EXPANSIVE FLUIDS. PERFORMANCE OF MECHANICAL REFRIGERATING SYSTEMS.

1. General Heat and Work Relations. Thermal Cycles. Work and Efficiency Determination by Heat Differences and Ratios. Graphic Method of Temperature Entropy Heat Diagram. Whatever is known of the existence of energy, its different manifestations or forms, the essential identity of the different forms, the possibility of converting one into another, and the laws of this transformation expressing quantitative relations between amounts of energy and changes of conditions of substances, has been all derived from very simple observations of the form and condition of substances, by a somewhat elaborate process of comparison and judgment, assisted by certain established mathematical principles. Experiments on substances suffering thermal changes are recorded directly in tables of observational data. The numerical values of any two series of readings are compared for the purpose of establishing, if possible, a general relation between them, a proceeding materially assisted by plotting one against the other on cross-section paper yielding a curve. The various points will seldom lie all on the curve because of experimental error whether human or instrumental. The smooth curve passing through or near the greatest number of points is considered as representing the true conditions of relation, especially when repetition tests of outlying points bring them closer to the curve. Such a curve is a graphic representation of a general relation between the two quantities involved and its equation is the mathematical expression for the *law* of relation. Two different laws so derived may, by similar methods, be combined with each other if any essential relation between the respective quantities really exists, and such a process will, if carried far enough, lead to more and more inclusive statements of the relation between various quantities. Each generalization is a law, properly so called, whether summarizing one set of observations, or combining two sets, or inclusive of all those ever made by anybody at any time. These laws may be stated in three ways:

(*a*) by equations;
(*b*) by curves, graphically;
(*c*) by words;

and of these, the statement in words is least valuable because of the inadequacy of language to express mathematical truths or to interpret physical mathematical facts. For solving numerical problems with actual substances, those laws which are least general and most concrete are most useful, because as a rule, they involve a lesser number of corrections and hypotheses than the broader generalizations which are reached only after the introduction of more and more assumptions, conditions and hypotheses. These various laws receive names when much used, the name sometimes being itself inclusive; all concerned with heat and work relations being thermodynamic laws, but the name may refer to a particular thing, such as the isothermal law for gases and the law of specific volume of saturated steam. In any case the law may be associated with an investigator's name, as in Boyle's or Avagadro's law, or they may have no name at all, appearing in books and papers simply as equation x, or figure y, especially when not easily expressed in words or not frequently used.

The relations of heat energy to work and the conditions for the transformation of one into the other, have been found by experience to yield quite readily to the process of generalization, into more and more broad statements, crystallizing finally into two, under the masterly manipulation of great physicists and there has grown up a tendency to call these two statements alone, *the laws of thermodynamics*. Thermodynamics is defined by Rankine, who helped to establish it, as " the reduction of the laws according to which such (heat and work) phenomena take place, to a physical theory or corrected system of principles."

Thermodynamics, as thus defined, is concerned with no numerical quantities, nor with any particular substance, nor for that matter with any actual substance whatever, but it is a physical theory of energy in relation to matter and expressible by two all-inclusive laws known as the first and second laws of thermodynamics, and as such, is a branch of natural philosophy. These two laws are inclusive of individual laws of particular thermal relations, only in the sense that the concrete cases *do not contradict them*, or have served to create them, but not in the sense that any particular concrete fact can be derived from the two laws of thermodynamics; in most cases it cannot.

Engineering is concerned with real substances and with numerical quantities, so that alone, the two laws of thermodynamic philosophy will not yield a solution of any practical numerical problem of design, or of analysis of test performance of actual heat machines or thermal apparatus. To guide such numerical thermal computations dealing with actual substances and apparatus, which is the province of Engineering Thermodynamics, two differences must be noted; first, numerical values for heat effects must be available for the various units involved, especially for that class known as physical constants, and second, their values must be known for actual substances or the materials of engineering. These physical constants include, for example, such quantities as the coefficients of expansion, the specific heats, latent heats of fusion and vaporization, the ratio of the pressure volume product to absolute temperature,

and the exponent " s " in adiabatic expansion of gases and vapors. These so-called thermal constants must be accepted as given in authoritative physical tables until such time as they are themselves expressible by laws with numerical coefficients, when the special law replaces the table either wholly or in part, as illustrated more or less completely in Chapter IV.

All the thermal constants are derivable by direct experiment, but in some cases, for example the specific heat of gases or superheated vapors at constant pressure and the latent heat of dry saturated vapors, the experimental difficulties are so great as to render indirect methods of determination preferable. From the general laws of heat and work a few important relations between these constants can be found, permitting of the determination of one of them more difficult to observe, from the values for others more easily measured, for example C_v can be found from C_p and R, or from C_p and γ, and the specific volume of dry saturated steam may be found by Clapeyron's equation. Not only are relations between the constants desirable for the determination of one from others, but also for purposes of checking observations made on different quantities, perhaps at different times and places, and by different experimenters. Even though some of these coefficients of heat effect may not prove to be really constant, nearly all of them being variables in fact, they are still called constants for want of an equally short and descriptive name. These general heat and work relations are extremely simple in conception and easy to use, but when mathematically stated by a series of differential equations may lead to a most complex series of derived equations to express which completely, would require perhaps a dozen volumes, but for engineering purposes this is not at all necessary or even desirable.

Experiment has shown that the state of a body is related to its heat content, and that a given body with an unknown heat content at any particular state or time may receive heat, may expand, may be compressed, or may lose heat in all sorts of ways, but if at the end of all these processes it finally is brought back to its original state, it must then have the same heat content as it had in the beginning, even though the numerical value of this heat content cannot be measured. This is a most significant fact because it provides a means of discussing heat taken in by the body, heat lost by it, and work done, each in its relation to the other. As energy is non-destructible, any work that has been done must be the exact equivalent of some heat that is not accounted for as heat, so that, assuming a body to pass through a series of thermal changes and to return to its original state, and therefore have the same heat content as before, it follows that the relations are as given in Eq. (885).

$$(\text{Heat added}) = (\text{Heat abstracted}) + (\text{Work done}). \quad . \quad . \quad . \quad (885)$$

To calculate the work done by a *complete* series of thermal changes it is only necessary to be able to calculate the heat added to the body or taken from it, during such a complete series thermal processes or algebraically as in Eq. (886)

$$(\text{Work done}) = (\text{Heat added}) - (\text{Heat abstracted}). \quad . \quad . \quad . \quad . \quad (886)$$

Defining *any single thermal process as a thermal phase*, such as evaporation at constant pressure, or expansion at constant temperature, and defining a *complete series of thermal processes bringing the substance back to its original physical state as a thermal cycle*, then a thermal cycle consists of a closed series of thermal phases, or on a diagram of changes the cycle is represented by a closed figure consisting of a number of separate connecting lines, each representing a phase. The work done by a thermal cycle is to be found directly, by subtracting from the algebraic sum of the heats added in each heat-adding phase, the corresponding algebraic sum of the heats abstracted in each heat-abstracting phase, so that the problem of finding the work done is resolved into

(*a*) The establishment of a thermal cycle, representative of the processes;
(*b*) The heat evaluation for each phase of that cycle.

This work is, of course, done at the expense of heat added to the substance and the ratio of the amount transformed into work, to the amount supplied, is the *thermal efficiency*, which is algebraically defined by Eq. (887).

$$\left.\begin{aligned}\left(\begin{matrix}\text{Thermal efficiency}\\ \text{of transformation}\\ \text{of heat into work}\end{matrix}\right) &= \left(\frac{\text{Work done}}{\text{Heat added}}\right) \quad (a)\\ &= \frac{(\text{Heat added}) - (\text{Heat abstracted})}{(\text{Heat added})} \quad (b)\\ &= 1 - \left(\frac{\text{Heat abstracted}}{\text{Heat added}}\right) \quad (c)\end{aligned}\right\} \quad (887)$$

To evaluate the various phase heats, positive and negative, graphic methods involving diagrams are of great assistance, but algebraic methods may also be used and for any particular problem, that method is used that saves most time, but judgment in the selection of method is acquired only by laborious experience in deriving solutions both ways. As algebraic methods are dependent directly on the relations of Chapter IV, it is only necessary to explain the graphic methods before proceeding to the detailed work of evaluating each phase by both methods.

It was found convenient in the study of work as a function of pressure-volume changes, to use diagrams the coordinates of which were the two factors, pressure and volume, the product of which, represented by an area, indicated to scale the quantity of work, the diagram showing how the work varied with either volume change or pressure change. A similar diagram can be developed, equally if not more useful, for studying quantities of heat as functions of temperature changes, by taking as one coordinate, absolute temperature, and for the other the heat per degree of absolute temperature, so that the area will indicate heat as the product of absolute temperature and heat per degree of temperature. As there is ordinarily little interest in the total

heat content of a substance counting from the state of no heat, and as the question under examination is usually concerned only with the heat received or lost between two thermal conditions or between two temperatures, it is common to take for the coordinates of the heat diagram absolute temperature for vertical distances and the heat gained or lost per degree of temperature starting anywhere, for horizontal distances. This horizontal distance, heat gained or lost per degree of absolute temperature was named by Rankine, the *thermodynamic function*, and by Clausius, the *entropy* and the latter and simpler name is now always used. Therefore, the diagram on which heat is represented by areas is known as the temperature-entropy diagram, just as the old diagram on which work was represented by areas is known as the pressure-volume diagram, the coordinate names, vertical and horizontal, being used for the name of the diagram. As the pressure-volume diagram was also called the work diagram, so may the temperature-entropy diagram be similarly called the heat diagram.

Horizontal distances on the temperature-entropy, or heat diagram are, from the above specifications of the diagram, *derived* quantities and not directly measurable physical quantities like volumes or temperatures. When a substance in condition A, receives or gives up heat to reach condition B, then the horizontal distance between the two corresponding diagram points is given by Eq. (888).

$$(\text{Entropy change from } A \text{ to } B) = \frac{\text{B.T.U. } (A \text{ to } B)}{\left(\begin{array}{c}\text{Absolute temperature } F \text{ at which the} \\ \text{heat exchange took place}\end{array}\right)} \qquad (888)$$

These horizontal distances or entropies can be laid off from an absolute zero of entropy or from any given value as a starting point, but as practical problems deal only with cyclic heat changes and not with total heat content, it is customary to plot only entropy differences in passing from one state to another, creating thus an artificial or *arbitrary zero* of entropy at the point where the heat content of the body is *least*. Entropies measured from this point are really differences in entropy between any point and that point where the body has least heat content. To avoid confusion the abscissa of any point will be set down as a difference. Introducing symbols,

Let ϕ_a, ϕ_b, etc. = the absolute and indeterminate entropy of the substance in the condition represented by points A, B, etc.;

" $(\phi_b - \phi_a)$ = change in entropy, in changing condition from A to B, equal to the horizontal distance between the diagram points

" T = absolute temperature F at which the heat is exchanged;

" Q_{ab} = the quantity of heat added to, or abstracted from the body in passing from state A to B.

Then

$$(\phi_b - \phi_a) = \frac{Q_{ab}}{T}. \quad \ldots \ldots \ldots \quad (889)$$

It is evident that as substances may change temperature during a change of heat content, T will vary with Q, and as T is the temperature *at* which the heat is received, it is unknown, so that solution of Eq. (889) is possible, first,

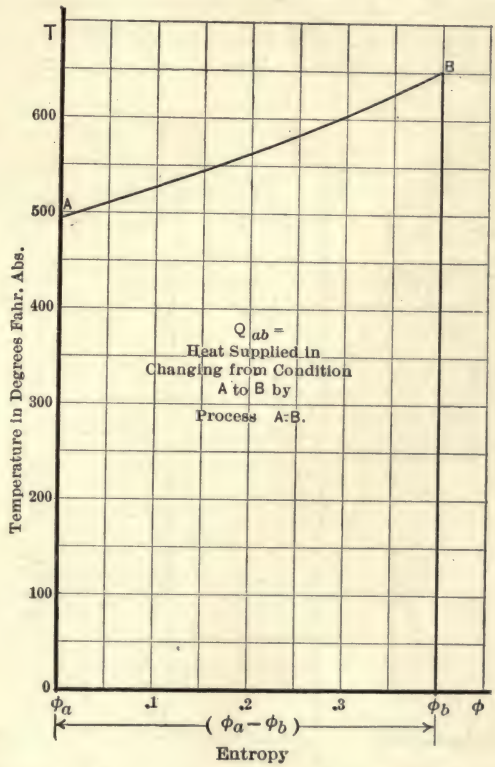


FIG. 219.—Heat Diagram to Temperature-entropy Coordinates Representing Quantity of Heat by the Area under the Thermal Process Curve.

by the use of an average temperature for the process, second, by what is preferable and equivalent, the substitution of a differential form, Eq. (890).

$$(\phi_b - \phi_a) = \int_a^b \frac{dQ}{T}. \quad \ldots \ldots \ldots \quad (890)$$

This formula Eq. (890) applies to all thermal processes in which the temperature changes with the heat content, such as heating solids, liquids, gases, and vapors, without change of state. When, however, the temperature does not change with heat reception or its loss, as for the boiling of liquids, condensing of vapor,

freezing of liquids, melting of solids, and for isothermal expansion of gases o superheated vapors, then Eq. (889), the simpler form, applies directly.

Referring to the diagram, Fig. 219, the point A, represents the origina state, B the final state after receiving the quantity of heat Q_{ab}, represented t scale by the area under the change of state curve, AB, down to the horizonta axis. It will be observed that heat added is laid off to the right and this is th usual way, accordingly heat abstracted is to be laid off to the left of the poin at which it begins. This arrangement makes it possible to easily deal wit cycles or complete thermal series of processes, as illustrated in Fig. 220, whic represents the following four-phase cycle.

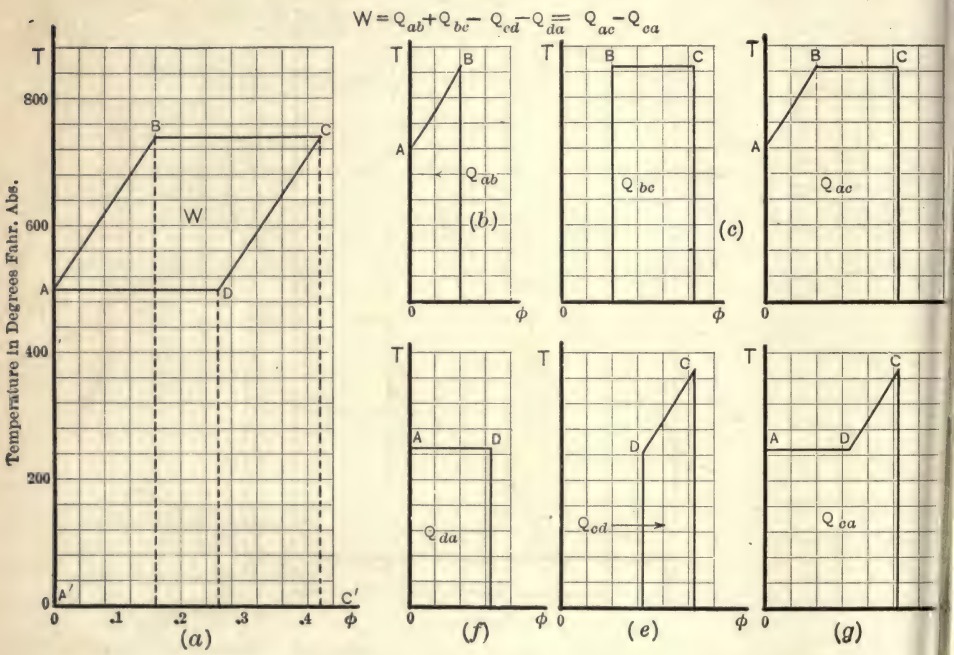


FIG. 220.—Heat Diagram of Complete Thermal Cycle to Temperature-entropy Coordinat showing Net Work done as the Algebraic Sum of Heat Areas under the Thermal Proc Curves.

(1) Heat addition with rising temperature from A to B;

(2) Heat addition at constant temperature from B to C;

(3) Heat abstraction with falling temperature from C to D;

(4) Heat abstraction at constant temperature from D to A.

Each of these heats is represented separately in diagrams (*b*), (*c*), (*e*), (*f*), the algebraic sum of all heats added, in diagram (*d*), of all heats abstra

in diagram (*g*), so that the heats added less the heats abstracted, is the difference between the areas under the curves *ABC* of (*d*), and *CDA* of (*g*), and this is of course, the area enclosed by the cycle *ABCD* on the main diagram (*a*).

It, therefore appears, that the evaluation of work done or the thermal efficiency of transformation becomes easy with such assistance as is rendered by the temperature-entropy diagram; the work is the area enclosed by the cycle diagram *ABCD* and the efficiency the ratio of enclosed area *ABCD* to all that area under the top boundary line *ABCC'AA*.

Practical application of this method of study of efficiency of heat transformation into work requires a preliminary establishment of the characteristics of the separate phases, which in turn involves the physical constants discussed in Chapter IV, where certain relations between them were assumed that must be proved on rational grounds.

Example 1. Evaluate the heat added to the substance in the change of condition represented by the line *AB* in Fig. 1, stating the results in heat units (B.T.U.) and in foot-pounds.

The area of the figure below the line *AB* down to absolute zero temperature measured by the planimeter or by adding narrow vertical strips, was found to be 5.67 sq.ins. as originally plotted. On the diagram, 1 inch represented 200° F. absolute in height and .2 entropy units horizontally, so that 1 square inch represents,

$$200 \times .2 = 40 \text{ B.T.U.}$$

The entire area represents, therefore,

$$40 \times 5.67 = 226.8 \text{ B.T.U.}$$

$$= 226.8 \times 778 = 176{,}500 \text{ ft.-lbs.}$$

Prob. 1. In Fig. 2 evaluate by the heat added from *A* to *B*, and from *B* to *C*, the heat removed from *C* to *D*, and from *D* to *A*, and find the net work done by the cycle, first in B.T.U. and then reduced to foot-pounds. What is the thermal efficiency of the cycle?

Prob. 2. A certain quantity of gas passes through a cycle in which it receives 1000 B.T.U. of heat. The mechanical work of the cycle is 90,000 ft.-lbs. How much heat is removed during the cycle, and what is the thermal efficiency?

Prob. 3. A steam engine has a thermal efficiency of 9.7 per cent. Each pound of steam entering the engine has 1050 B.T.U. more heat than the condensate leaving the condenser. How much heat must the condenser remove from each pound of steam?

Prob. 4. If an engine were of 100 per cent thermal efficiency, how many B.T.U. must be supplied per hour per H.P.? How many, if the efficiency were 20 per cent? If the efficiency were 5 per cent?

Prob. 5. A hypothetical engine receives 1000 B.T.U. at a constant temperature of 900° F. and rejects heat at a constant temperature of 90° F. No heat is added or removed while the temperature is changing. What is the thermal efficiency?

Prob. 6. A power plant using coal of 13,000 B.T.U. per lb., requires 2 lbs. of coal per hour per K.W. measured at the switchboard. What is the efficiency of the plant for converting heat of fuel into electrical energy?

Prob. 7. If a pound of water be evaporated at a constant temperature of 212° F. the heat to completely vaporize it will be 970.4 B.T.U. What is the increase in its entropy?

Prob. 8. To evaporate a pound of water at a temperature of 400° F. (247 lbs. per square inch absolute) requires 827.2 B.T.U. Find the increase of entropy due to evaporation.

Prob. 9. Anhydrous ammonia evaporating at a temperature of 0° F. absorbs 555 B.T.U. per pound. What is its increase in entropy due to evaporation?

Prob. 10. Plot to scale the $T\phi$ diagrams for each of these numerical problems.

Prob. 11. It has been proposed to apply the heat of condensation of steam after exhausting from a steam engine to the generation of ether vapor to work in an ether engine, the exhaust of which is to be condensed by water. Describe and sketch the steam and ether cycles showing what phases they have in common, what similar in kind, and what different, using $T\phi$ and PV coordinates.

2. General Energy Equation between Heat Change, Intrinsic Energy Change, and Work Done. Derived Relations between Physical Constants for Gases and for Changes of State, Solid to Liquid, and Liquid to Vapor. As already stated, work done by a substance is always done at the expense of its heat, and while doing work the substance may be receiving heat or it may not. If no heat is being added at the time the work is being done, that work is derived directly from the heat content of the substance and as a consequence the heat content is lessened or the energy reduced by an amount exactly equivalent to the work done. Should heat be added while work is being done, the change in the energy content of the substance is the difference between the heat added and work simultaneously done. It has been the custom to call the whole energy content of a body its intrinsic energy, so that the above expression becomes for a case of heat addition,

$$(\text{Change in intrinsic energy}) = (\text{Heat added}) - (\text{Work done}). \quad . \quad . \quad (89$$

For complete thermal cycles, the body returns to the original state has the same intrinsic energy at the end as at the beginning, and the change in intrinsic is zero. During the cycle, heat may have been received at one time and lost at another, so that for the " heat added " term in Eq. (891) there must be substituted a more general one representing the algebraic sum of all heats added, giving abstracted heats a negative sign, and similarly for the work term, there must be substituted one representing the algebraic sum of the works, work of compression being negative. Hence for a complete cycle,

$$\left.\begin{array}{l}\text{(Algebraic sum of heats added)} - \text{(Algebraic sum of works done)} = 0, \ (a) \\ \text{or} \\ \left(\begin{array}{c}\text{Algebraic sum of} \\ \text{heats added}\end{array}\right) = \text{(Heat added)} - \text{(Heat abstracted)} = \left(\begin{array}{c}\text{Net work} \\ \text{done.}\end{array}\right) (b)\end{array}\right\} \quad (892)$$

It was on this form of the expression, that the work and efficiency methods described in the last section, were based; it is, however, a special case of a more general expression for any kind of thermal process whether a complete cycle or not and consisting of any number of phases whether overlapping or separate, which is derivable as follows:

Let $\overline{U}$ = intrinsic energy of a body or its total energy of condition defined as the heat it has above a condition of absolute zero of heat, in B.T.U.

" Q = the heat gained by a body in passing from any state to any other, which is the sum of all heat directly added less all abstracted as such, in B.T.U.

" W = work done in passing from the first to the second state, which is the sum of all expansion less all compression work, or equal to the *net* work done, in ft.-lbs.;

" J = Joule's equivalent of one B.T.U. in ft.-lbs. = 778 approximately or 777.52, more exactly.

Then the more general expression for the relation between heat, work, and intrinsic energy, takes the following form, Eq. (893), which is one statement of the so-called first law of thermodynamics,

$$d\overline{U} = dQ - \frac{1}{J} dW. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (893)$$

This equation will be called for convenience the general energy equation for substances and is true for all sorts of thermal changes except those including as a part an irreversible process. In such cases the failure of the equilibrium of the state of the substance destroys the dependence of intrinsic energy on state alone which is otherwise true and assumed in the above equation.

Applying this equation directly to *perfect* gases gives some useful relations between the physical constants. For such hypothetical substances which all actual gases closely approximate at some time, $PV = RT$, for 1 lb., in foot-pound units. The differential form of this is given by Eq. (894).

$$d(PV) = PdV + VdP = RdT. \quad . \quad . \quad . \quad . \quad . \quad . \quad (894)$$

When gases receive a little heat dQ, it is conceivable that it may be divided into two parts, one part raising temperature without doing work, and therefore given by $C_v dT$ B.T.U. for 1 lb., and a part transformed directly into work and equal to PdV foot-pounds, hence

$$dQ = C_v dT + \frac{1}{J} PdV. \quad \ldots \quad (895)$$

Combining Eqs. (893) and (895),

$$d\overline{U} = C_v dT + \frac{1}{J} PdV - \frac{1}{J} dW. \quad \ldots \quad (896)$$

But as $dW = PdV$,

$$d\overline{U} = C_v dT \quad \ldots \quad (897)$$

Interpretation of Eq. (897) yields the conclusion, that *for perfect gases the intrinsic energy depends on the absolute temperature only.* Furthermore, the intrinsic energy of a perfect gas is equal to the product of its specific heat at constant volume into its absolute temperature plus a constant of integration, if this specific heat is not a function of temperature, which it probably is.

Combining Eqs. (894) and (895) by elimination of dV gives

$$dQ = C_v dT + \frac{R}{J} dT - \frac{1}{J} VdP$$

$$= \left(C_v + \frac{R}{J}\right) dT - \frac{1}{J} VdP. \quad \ldots \quad (898)$$

But the specific heat at constant pressure is given by Eq. (899),

$$C_p = \left(\frac{dQ}{dT}\right)_{P=c} \quad \ldots \quad (899)$$

Whence by Eq. (898),

$$C_p = C_v + \frac{R}{J},$$

or

$$R = J(C_p - C_v). \quad \ldots \quad (900)$$

This Eq. (900) proves the assumption made previously that the gas constant R is equal to the difference between the two specific heats expressed in foot-pounds.

Substitution of this value for R in Eq. (898) gives another form to the heat expression, in terms of dT and dP variables, Eq. (901),

$$dQ = C_p dT - \frac{1}{J} V dP. \quad \ldots \quad (901)$$

Giving to dT in Eq. (895) its value from Eq. (894),

$$dQ = \frac{C_v}{R} d(PV) + \frac{1}{J} P dV. \quad \ldots \quad (902)$$

This can be put into an integral form, Eq. (903), directly, and the factor simplified by using the relation,

$$\frac{C_v}{R} = \frac{C_v}{J(C_p - C_v)} = \frac{1}{J\left(\frac{C_p}{C_v} - 1\right)},$$

$$\left.\begin{aligned} \left(\begin{array}{l}\text{Heat added from state}\\ A \text{ to state } B \text{ in foot-}\\ \text{pounds}\end{array}\right) &= \left(\frac{1}{\frac{C_p}{C_v} - 1}\right)(P_b V_b - P_a V_a) + \int_a^b P dV \quad (a) \\ &= \left(\frac{1}{\frac{C_p}{C_v} - 1}\right)(P_b V_b - P_a V_a) + (\text{Work done}) \quad (b) \end{aligned}\right\} \quad (903)$$

When gases expand they follow, as has already been shown, the general law $PV^s = K$, and in so expanding the work done is given by Eq. (904).

$$W = \int_a^b P dV = K \int_a^b \frac{dV}{V^s} = \frac{P_a V_a{}^s}{1-s}(V_b{}^{1-s} - V_a{}^{1-s}) \quad \ldots \quad (904)$$

Substitution of this in Eq. (903) gives

$$JQ_{ba} = \left(\frac{1}{\frac{C_p}{C_v} - 1}\right)(P_b V_b - P_a V_a) + \left(\frac{P_a V_a{}^s}{1-s}\right)(V_b{}^{1-s} - V_a{}^{1-s})$$

$$= \left(\frac{P_a V_a{}^s}{\frac{C_p}{C_v} - 1}\right)(V_b{}^{1-s} - V_a{}^{1-s}) + \left(\frac{P_a V_a{}^s}{1-s}\right)(V_b{}^{1-s} - V_a{}^{1-s})$$

$$= \left[\frac{\left(\frac{C_p}{C_v} - s\right)}{\left(\frac{C_p}{C_v} - 1\right)}\right]\left(\frac{P_a V_a{}^s}{1-s}\right)(V_b{}^{1-s} - V_a{}^{1-s}) \quad \ldots \quad (905)$$

Hence

$$Q=\frac{1}{J}\left[\frac{\frac{C_p}{C_v}-s}{\frac{C_p}{C_v}-1}\right]\times(\text{Work of expansion}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (906)$$

To proceed further with the investigation of relations between physical constants requires a still more general energy equation involving more of them. One of the basic definitions of heat makes a quantity of heat proportional to the amount of its effect on pressure, on volume and on temperature, so that taking these quantities in pairs as independent variables, two fixing the third by the general property relations of matter, any heat increment may be equated to effect, by the three sets of algebraic relations of Eq. (907),

$$\left.\begin{aligned} dQ &= a\,dT+b\,dV, \quad (a)\\ &= c\,dT+d\,dP, \quad (b)\\ &= e\,dP+f\,dV. \quad (c) \end{aligned}\right\} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (907)$$

The change in intrinsic energy is given by the following set, Eq. (908), corresponding to the above, in which the factors (a), (b), (c), etc., are unit thermal effects or physical constants. Substituting Eq. (907) in Eq. (893).

$$\left.\begin{aligned} d\overline{U} &= a\,dT+b\,dV-\frac{1}{J}P\,dV=a\,dT+\left(b-\frac{P}{J}\right)dV \quad (a)\\ &= c\,dT+d\,dP-\frac{1}{J}P\,dV \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)\\ &= e\,dP+f\,dV-\frac{1}{J}P\,dV=e\,dP+\left(f-\frac{P}{J}\right)dV \quad (c) \end{aligned}\right\} . \quad . \quad (908)$$

These are, of course, partial differential equations and each must be a complete differential, that is, it must be possible to solve or integrate it. Therefore the differential coefficients of each coefficient with respect to the other variable, when its own variable is constant, must be equal to each other.

$$\frac{d}{dV}(a)_{T=C}=\left(\frac{da}{dV}\right)_{T=C}=\frac{d}{dT}\left(b-\frac{P}{J}\right)_{V=C} \quad . \quad . \quad . \quad . \quad . \quad (909)$$

This is expressed symbolically in Eq. (909) and there are two others, similar but not set down because they are not to be used.

The change of heat per degree of absolute temperature has been defined as the entropy change as in Eq. (910) for the differential increments.

$$d\phi = \frac{dQ}{T} \quad \ldots\ldots\ldots\ldots \quad (910)$$

Combining Eq. (910) with the original general equation, Eq. (907), there results some new relations between constants, Eq. (911), and which constitute one form of statement of the second law of thermodynamics:

$$\left.\begin{aligned} d\phi &= \frac{a}{T}\,dT + \frac{b}{T}\,dV \quad (a) \\ &= \frac{c}{T}\,dT + \frac{d}{T}\,dP \quad (b) \\ &= \frac{e}{T}\,dP + \frac{f}{T}\,dV \quad (c) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (911)$$

These are also complete differentials so that the differential coefficient relations yield the following, Eq. (912), and two others, similar, that will be omitted to save space.

$$\frac{d}{dV}\left(\frac{a}{T}\right)_{T=C} = \frac{d}{dT}\left(\frac{b}{T}\right)_{V=C} \quad \ldots\ldots\ldots \quad (912)$$

Performing the operations indicated in Eqs. (909) and (912), they may be combined.

From Eq. (909),

$$\left(\frac{da}{dV}\right)_{T=C} = \left(\frac{db}{dT}\right)_{V=C} - \frac{1}{J}\left(\frac{dP}{dT}\right)_{V=C} \quad \ldots\ldots\ldots \quad (913)$$

From Eq. (912),

$$\frac{1}{T}\left(\frac{da}{dV}\right)_{T=C} = \frac{1}{T}\left(\frac{db}{dT}\right)_{V=C} - \frac{b}{T^2},$$

or

$$\left(\frac{da}{dV}\right)_{T=C} = \left(\frac{db}{dT}\right)_{V=C} - \frac{b}{T} \quad \ldots\ldots\ldots \quad (914)$$

Equating Eq. (913) to Eq. (914) the result is expressed by Eq. (915).

$$\frac{b}{T}=\frac{1}{J}\left(\frac{dP}{dT}\right)_{V=C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (915)$$

This is a perfectly general expression, as no defining conditions or limiting hypotheses have yet been introduced except that of excluding non-reversible processes. Similar treatment will yield a long list of relations, omitted here because they may be found in works on the philosophy of thermodynamics and because they are not needed here. These particular ones are selected because they lead to values for specific heats which are defined mathematically by Eq. (916).

$$\left.\begin{aligned} C_v&=\left(\frac{dQ}{dT}\right)_{V=C} \qquad (a)\\ C_p&=\left(\frac{dQ}{dT}\right)_{P=C} \qquad (b)\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (916)$$

Referring back to Eq. (907) it is evident that,

$$C_v=a, \text{ and } C_p=c.$$

One step has been taken in evaluating the constant (a), by establishing a valu for the constant (b) in Eq. (915), and substituting this in Eq. (913), the com plete evaluation of the constant $a=C_v$, is given by Eq. (917),

$$\left(\frac{da}{dV}\right)_{T=C}=\frac{T}{J}\left(\frac{d^2P}{dT^2}\right)_{V=C},$$

whence

$$C_v=a=\frac{T}{J}\int\frac{d^2P}{dT^2}\,dV+K'. \quad . \quad . \quad . \quad . \quad . \quad (917$$

Therefore the specific heat at constant volume for *perfect* gases is a functio of absolute temperature only, or it is constant, because the constant of integra tion K' must be constant or a function of temperature only, and it may be zer By exactly similar reasoning, using other equations of the general group, can be shown that a similar conclusion is applicable to the constant, $c=C$ by Eq. (918),

$$C_p=c=\frac{T}{J}\int\frac{d^2V}{dT^2}\,dP+K''. \quad . \quad . \quad . \quad . \quad . \quad (91$$

These relations are derived here for two reasons: first, to show the general process of thermodynamic computation of relations between physical constants and second because this particular relation illustrates beautifully how utterly misleading some of the most laboriously reached of these conclusions may be. It was because of such a demonstration as this that for so long nobody even sought for a variation in the specific heat of superheated steam with pressure, which was ultimately found after engineering and not physical calculations showed that the old accepted values must be wrong. This does not indicate that conclusions based on such mathematical analyses will always be misleading because they are absolutely true for perfect gases, but they are to be adopted for real gases only with necessary restrictions based on how much departure there is by the real substance from the perfect gas.

When a substance changes state, a similar analysis starting with the general energy equation yields some useful relations between the new constants involved for these processes. Starting with the general statement Eq. (895) or Eq. (907*a*) that heat added is equal to, something (*a*) multiplied by the temperature change, added to the product of something else (*b*) and the volume change, it was found Eq. (915) by various transformations involving the two laws of thermodynamics that the coefficient (*b*) was proportional to the product of absolute temperature into the rate of change of pressure with temperature which when inserted in Eq. (907*a*) gives Eq. (919),

$$dQ = a dT + \frac{T}{J}\left(\frac{dP}{dT}\right) dV. \quad . \quad . \quad . \quad . \quad . \quad . \quad (919)$$

This equation may be applied to a change of state from solid to liquid and liquid to vapor at constant temperature, for which case it takes the form of Eq. (920), since $dT = 0$.

$$dQ = \frac{T}{J}\left(\frac{dP}{dT}\right) dV. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (920)$$

If the process be one of evaporation of 1 lb. and this be complete, then the heat added is the latent heat L, the final volume that of dry saturated vapor V_V, and the initial volume that of liquid at the temperature of boiling V_L, whence integration gives an expression for the physical constant, latent heat, Eq. (921),

$$L = \frac{T}{J}\left(\frac{dP}{dT}\right)(V_V - V_L). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (921)$$

This is the Clapeyron equation previously used and now established, by methods used by Clausius and Clapeyron, who derived it independently.

For fusion it is believed that latent heats are constant for all pressures, in which case the change in the freezing- or melting-point with pressure is given in degrees per unit of pressure by Eq. (922), where V_L = specific volume of liquid and V_S the volume of solid.

$$\frac{dT}{dP}=\frac{T(V_L-V_S)}{JL} \quad \ldots\ldots\ldots \quad (922)$$

It is interesting to note that for ice and water, ice being lighter than water, the difference above is negative so that increase of pressure lowers rather than raises the melting- or freezing-point. Rewriting the equation in the following form, Eq. (923), for vapor-liquid changes another interesting conclusion becomes possible:

$$\frac{JL}{V_V-V_L}=T\frac{dP}{dT} \quad \ldots\ldots\ldots \quad (923)$$

This indicates that latent heat in foot-pounds divided by the volume change in vaporization, which is in the nature of a pressure and may be regarded as the pressure resisting or opposing a change of state, is equal to the product of absolute temperature and the rate of change of vapor pressure with temperature. It has been remarked that this quantity given by Eq. (923) is nearly constant for several different vapors at the same pressure, so that each term is constant, Eq. (924):

$$\left.\begin{aligned}\frac{JL}{V_V-V_L}&=K\\ \text{or}\qquad T\frac{dP}{dT}&=K\end{aligned}\right\}\ \text{Approx. when } P \text{ is const.} \quad \ldots\ldots \quad (924)$$

It is noted, however, that the constant K rises with pressure so that at any pressure

$$T\frac{dP}{dT}=f(P),$$

whence on integration,

$$\log_e T=F(P)+K'. \quad \ldots\ldots\ldots \quad (925)$$

This Eq. (925) is the origin of form of some of the numerous equations for the relation between saturated vapor temperatures and pressures.

When two different substances A and B follow similar laws, according to

Eq. (924) then the boiling-points at equal pressures follow the law of constant ratio of Eq. (926),

$$\left(\frac{\text{Boiling-point of one substance}}{\text{Boiling-point of another substance}}\right) = (\text{Constant, at same pressure}). \quad . \quad (926)$$

It also follows that for substances having the same "K" or constant $\left(\frac{L}{V_V - V_L}\right)$ values, that latent heats are inversely proportional to the density of vapors when V_L is small compared to V_V and, therefore, that the generation of a cubic foot of any vapor in the group requires the same amount of heat.

These applications of general laws to vapors are subject to the same remarks as were referred to the derivation of physical laws from mathematical analysis, that they are no more correct than the hypotheses on which they are based and in the world of derivation and deduction it is easy to forget the hypotheses and make a serious blunder. As Mellor remarks in his Chemical Statics and Dynamics, "*No process of reasoning can establish a law of Nature. The elements of sameness—the law—must be actually discovered in the facts.*"

Example 1. Air expands under a piston from a volume of 1 cu.ft. and a pressure of 300 lbs. per square inch absolute to a volume of 5 cu.ft. at a pressure of 40 lbs. per square inch absolute. Assuming that expansion follows the law of constant PV^s, find the heat absorbed in B.T.U. Ratio of specific heats for air = 1.4.

The work done from Eq. (904) is

$$W = \frac{P_a V_a^s}{1-s}(V_b^{1-s} - V_a^{1-s}), \text{ and } s = \frac{\log\left(\frac{P_a}{P_b}\right)}{\log\left(\frac{V_b}{V_a}\right)}.$$

Substituting numerical values to find s,

$$s = \frac{\log \frac{300}{40}}{\log 5} = \frac{.875}{.699} = 1.25.$$

Therefore,

$$W = \frac{300 \times 1}{1-1.25} \times 144\left(\frac{1}{5^{1.251-1}} - 1\right) = \frac{43{,}200}{-.25} \times (.667 - 1) = 57{,}600 \text{ ft.-lbs.}$$

From Eq. (905)

$$(\text{Heat})_{ba} = \frac{1}{J}\left\{\frac{\frac{C_p}{C_v} - 1.25}{\frac{C_p}{C_v} - 1}\right\} \times W$$

$$= \frac{1}{778} \times \frac{1.4 - 1.25}{1.4 - 1} \times 57{,}600 = 27.7 \text{ B.T.U.}$$

Prob. 1. A pound of air at 60° F. and atmospheric pressure receives 500 heat-unit at constant pressure. How much of this heat will be expended in raising the tem perature of the air and how much in doing work?

Prob. 2. Show that when air expands isothermally the work done is equal to th heat added.

Prob. 3. A pound of air at 70° F. and a pressure of 12 lbs. per square inch absolut is compressed adiabatically until its volume is one-fourth of the original volume. On thousand heat-units are added at constant volume and the air then expands to th original pressure, whereupon it is cooled to the original temperature at constant pres sure. Show numerically that the algebraic sum of heats added is equal to the wor done.

Prob. 4. A pound of gas having the specific heat at constant volume of .2, change in volume from 100 cu.ft. to 150 cu.ft. while the pressure remains constant at 100 lbs per square inch absolute. How much heat was added to the gas?

Prob. 5. Prove that for a perfect gas $R = J(C_p - C_v)$ and show how much departur there is for air, methane, oxygen, nitrogen, carbon dioxide, steam.

Prob. 6. One thousand heat-units are supplied to a pound of air at 60° F. an 15 lbs. per square inch absolute pressure. What will be the temperature at the en of the heat reception and the work done, if (*a*) heat is received at constant volume, (*b* if it is received at constant pressure, and (*c*) if it is received during expansion accordin to $PV^{1.1} = \text{Const.}$

Prob. 7. Show that for adiabatic compression the work done is $C_v(T_2 - T_1)J$ foo pounds.

Prob. 8. It has been stated that $\dfrac{JL}{V_V - V_L}$ is approximately constant. Taking th values of L, V_V, and V_L from the tables of Chapter IV, compare the results obtain for steam, ammonia, and carbon dioxide at pressures of 10, 25, 50, 75, and 100 lb per square inch absolute.

Prob. 9. If air expands in a cylinder according constant to $PV^{1.3}$, find an expressi for the heat gained or lost.

Prob. 10. Determine whether heat is being gained or lost by ammonia vap during compression when it is found that s has the value 1.2 and derive a quantitati expression for it.

Prob. 11. Compare the change in intrinsic energy during vapor formation for o pound of steam, ammonia, and carbon dioxide at 32° F.

Prob. 12. Derive from the tables a formula for vapor pressure in terms of te perature, of the form of Eq. (925) for steam, ammonia, and carbon dioxide.

Prob. 13. From the equation derived in Problem 12, find $\dfrac{d^2P}{dT^2}$ and substituting Eq. (917), find C_v for steam, compare it with the values given in Marks and Dav and explain the differences found.

3. Quantitative Relations for Primary Thermal Phases, Algebraic, a Graphic, to PV and TΦ Coordinates. Constancy of P, V, and T for Gas and Vapors, Wet, Dry, and Superheated. Heating of solids, liquids, ga and vapors without change of state takes place according to the fundamen relation between temperature change and quantity of heat, Eq. (927).

$$(\text{Heat quantity}) = (\text{Specific heat}) \times (\text{Temperature change}) \times (\text{Weight}). \quad (92$$

the specific heat is constant this is a most simple expression, but it has been nown that specific heat is variable and given generally by an equation of the rm, Eq. (928), in which a, b, and c are constants, different for each substance.

$$\text{Specific heat} = C = a + bT + cT^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (928)$$

n this equation, if b and c are very small, the specific heat it reduces to ne form Eq. (929),

$$\text{Specific heat} = C = a, \text{ (substantially)} \quad . \quad . \quad . \quad . \quad . \quad (929)$$

rom these equations the entropy change can be determined for all substances ffering heat change without change of state, from Eq. (910), as follows:

$$\phi_2 - \phi_1 = \int_1^2 \frac{dQ}{T} = w \int_1^2 \frac{(\text{Specific heat})}{T}\, dT, \quad . \quad . \quad . \quad . \quad (930)$$

ut

$$\text{Specific heat} = \frac{dQ}{dT};$$

$$dQ = (\text{Specific heat})dT;$$

$$= (a + bT + cT^2)dT.$$

Hence

$$\phi_2 - \phi_1 = w \left\{ a \int_1^2 \frac{dT}{T} + b \int_1^2 dT + c \int_1^2 T dT \right\}$$

$$= w \left\{ a \log_e \frac{T_2}{T_1} + b(T_2 - T_1) + \frac{c}{2}(T_2^2 - T_1^2). \quad . \quad . \quad (931) \right.$$

his Eq. (931) is the entropy change for all heat gains or losses by solids, quids, gases and vapors not changing state when the specific heat is a function absolute temperature of the form given in Eq. (928), and reduces to the fol- wing special forms, Eq. (932, a, b, c, d), for the condition stated.

For constant specific heat C)	$(\phi_2 - \phi_1) = w\, C \log_e \frac{T_2}{T_1}$	(a)
For water if the specific heat be taken as unity	$(\phi_2 - \phi_1) = w \log_e \frac{T_2}{T_1}$	(b)
For gases and superheated vapors at constant volume with constant specific heat	$(\phi_2 - \phi_1) = wC_v \log_e \frac{T_2}{T_1}$	(c)
For gases and superheated vapors at constant pressure with constant specific heat	$(\phi_2 - \phi_1) = wC_p \log_e \frac{T_2}{T_1}$	(d)

. (932)

To illustrate the relation of these changes to each other there are plotted to PV and $T\Phi$ coordinates in Fig. 221 the following cases each for 1 lb. of substance:

(1) Heating of water from 32° F. to 400° F. with specific heat = 1;
(2) Heating of water from 32° F. to 400° F. with specific heat as given in the Marks & Davis Steam Tables using the columns called heat and entropy of liquid (above 32° F.);
(3) Heating of air from 32° F. and 20 lbs. sq.in. to 500° F. at constant volume with constant specific heat = (.17);
(4) Heating of air from 32° F. and 20 lbs. sq.in. to 500° F. at constant volume with variable specific heat, mean $C_v = .174 + .0000675(t - 32)$
(5) Heating of air at constant 20 lbs. per square inch pressure from 32° to 500° F. with constant specific heat $C_p = .239$;
(6) Heating of air at constant 20 lbs. per square inch pressure from 32° F. to 500° with variable specific heat $C_p = .239 + .0000675(t - 32)$

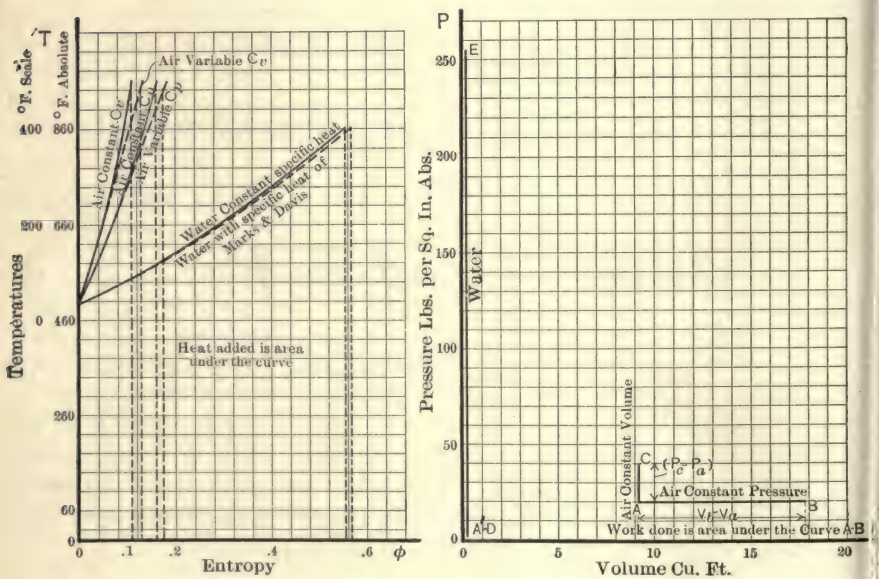


FIG. 221.—Changes in P, V, T, Φ, Heat, and Work, for *Constant Pressure* and *Constant Volu* Phases of Liquids, Gases and Vapors, without Change of State.

For constant pressure heating of a gas illustrated here by air, there w be an increase of volume depending on the temperature rise and work do which latter is measured by the volume increase, but as there are two possi final temperatures for the same amount of heat, one for the constant and one the larger variable specific heat, there will be two possible volumes increa and quantities of work done. In the example, the temperature has been fi so that the heat quantities differ by the differences between the areas on $T\Phi$ diagram under the solid curve for constant and under the dotted cu

for variable specific heat. These differences are so small and in the study of heat transformation into work so inconsequential that except when otherwise specifically stated, the constant value of specific heat will be used. This practice will save great labor as well and avoid the introduction of doubts which have real foundation, as to the accuracy of the determination of the variability laws so far announced. One special case of difference in practice is that for water and superheated steam, the entropy changes for which are calculated and form part of the steam tables under the heading " entropy of the liquid (above 32° F.)" and "entropy of the steam," but in approximate calculations no serious error will be introduced by assuming constancy of specific heat for water and unity as its value, but the variability for the steam cannot be so safely ignored.

Accordingly, with the above reservations the following relations of tabular Eq. (933) are found between the quantities for the gas heated at constant pressure from A to B., Fig. 221.

Related Quantities for Constant Pressure Heating	Formula	
Heat and Temp.	$Q_{ab} = C_p(T_b - T_a)$	(a)
Temp. and Volume	$V_b - V_a = V_a\left(\frac{T_b}{T_a} - 1\right)$	(b)
Heat and Volume	$V_b - V_a = V_a\frac{Q_{ab}}{C_p T_a} = \frac{R}{C_p}\left(\frac{Q_{ab}}{P_a}\right)$	(c)
Work and Temp.	$W = P_a(V_b - V_a) = P_a V_a\left(\frac{T_b}{T_a} - 1\right) = R(T_b - T_a)$	(d)
Work and Heat	$W = \frac{P_a V_a}{T_a}\frac{Q_{ab}}{C_p} = \frac{R}{C_p}Q_{ab}$	(e)
Entropy and Temp.	$\phi_b - \phi_a = C_p \log_e \frac{T_b}{T_a}$	(f)

(933)

A similar set may be written down, Eq. (934), almost by inspection for the case of constant volume heating from A to C, Fig. 221, which, of course, causes a pressure rise, during which no work can be done, or $W = 0$.

Related Quantities for Constant Volume Heating	Formula	
Heat and Temperature	$Q_{ac} = C_v(T_c - T_a)$	(a)
Temperature and Pressure	$P_c - P_a = P_a\left(\frac{T_c}{T_a} - 1\right)$	(b)
Heat and Pressure	$P_c - P_a = P_a\left(\frac{Q_{ac}}{C_v T_a}\right) = \frac{R}{C_v}\left(\frac{Q_{ac}}{V_a}\right)$	(c)
Entropy and Temperature	$\phi_c - \phi_a = C_v \log_e \frac{T_c}{T_a}$	(d)

. . . . (934)

The case presented by the water is peculiar inasmuch as it makes so little difference whether the water be heated at constant pressure or constant volume or at what pressure, its expansion being so small. For the purpose of illustration it is assumed in the diagram that the water is at the same 20 lbs. pressure and that it is kept (*a*) at constant pressure, expanding from A' to D and (*b*) at constant volume, its pressure increasing from A' to E. In ordinary practical work no account is taken of the difference at all, first, because water in use is rarely heated at constant volume, and second, because the work of expansion is vanishingly small.

Constancy of temperature characterizes all changes of state, solid to liquid and liquid to vapor, if the pressure be constant, and also is characteristic of the expansion of gases when the product of pressure and volume is constant, for in this case $PV = RT = Constant$. For melting or boiling, then, the heat added is proportional to the weight of substance changed or the *quality*.

Let L = latent heat of fusion or vaporization per pound;
" w = weight of substance present;
" x = fraction of total weight liquefied from the solid, or vaporized from the liquid, and to be called the quality;
" $1-x$ = fraction of total weight remaining in the solid state in contact with liquid, or in the liquid state in contact with vapor.

Hence for the changes of state at constant temperature and pressure the quantity of heat is the product of weight, quality and latent heat, Eq. (935),

$$Q = wxL. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (935$$

The latent heat L is to be taken from tables of physical properties of substance but will not be the same at different pressures. The representation of th case of vaporization on the PV and $T\Phi$ diagrams, especially the latter, is usuall associated with the heating of liquid which precedes vaporization and wit the line of vapor superheat which succeeds vaporization if the heating is lon continued. Accordingly the successive processes of heating liquid, evapora ing it, and superheating its vapor, are represented by three lines joining eac other, for a single pressure, and by a family of lines if vaporization at a pressures are to be included.

This is illustrated in Fig. 222 where, starting with liquid at 32° F. (A) heatin at constant pressure to the boiling-point, B, will cause a slight expansion, show A to B on the PV diagram, doing the work shown by the area $ABB'A'A$ a requiring the heat shown by $ABB'A'A$ on the $T\Phi$ diagram, as already explaine but repeated here to assure continuity. The formation of vapor at the tem perature represented by $\overline{BB'}$ on the $T\Phi$ and at the corresponding pressu represented by $\overline{BB'}$ on the PV diagram, is represented by the line BC ho zontally on both, until all liquid has been just vaporized which condition denoted by the point D. Therefore, the area under the line of complete vapo

ation $\overline{BD}$ and which is $BDD'B'B$ on the $T\Phi$ diagram, is a graphic representation of the latent heat of vaporization to scale. Similarly, on the PV diagram the corresponding area $BDD'B'B$ is the work of vaporization or $P(V_V - V_L)$ since $\overline{AB}$ is the specific volume of the liquid V_L and $\overline{AD}$ the specific volume of the dry saturated vapor. Should the vaporization be not complete but stop at the point C, there will be present a mixture of liquid and vapor, to be called for want of a better name *wet vapor*. For such wet vapor as is represented by the point C the quality is given by $\frac{\overline{BC}}{\overline{BD}} = x$ on the $T\Phi$ diagram, and the heat used in making it at the boiling-point is represented by the area $BCC'B'B$ and is equal to xL. On the PV diagram the point C for wet vapor is located in a similar way, $\overline{BD}$ corresponding to the quality $x = 1$, so that $\frac{\overline{BC}}{\overline{BD}} = x$, and the wet vapor will occupy a volume $\overline{AC}$ per pound of mixed vapor and liquid.

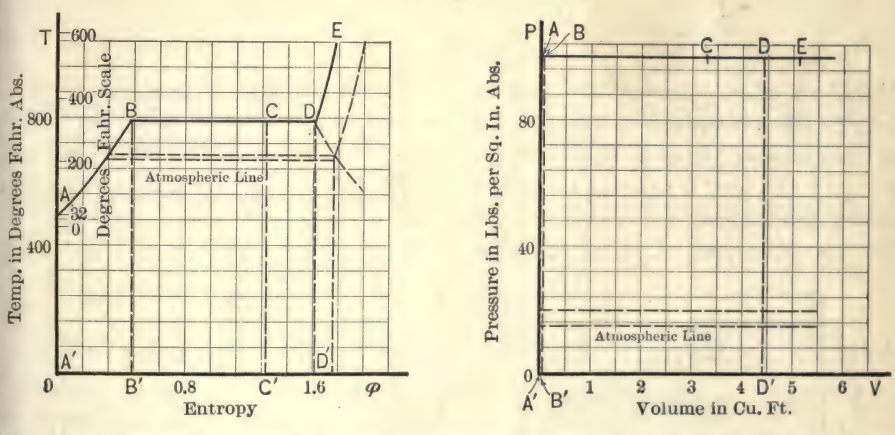


FIG. 222.—Changes in Pressure, Volume, Temperature, Entropy, Quality, Heat Added, and Work, done for *Constant Temperature* Heating of Liquid-vapor with Change of State.

Tabular values for the increase of entropy for vaporization alone or for the entropy of dry saturated steam over liquid at 32°, which includes the liquid part, are very useful in solving problems with wet vapors by the above relation.

Continued heating at the same pressure after the completion of vaporization, of course superheats the vapor along a line DE. If the whole process had taken place at some lower pressure it could be represented as in the dotted lines leading to the family of curves mentioned. It is not possible to express algebraically the relations between heat of vaporization or work and temperature, except empirically and then only by an expression too complicated for use. Problems must, therefore, be solved by the use of the tabular values given at the end of Chapter IV.

Gases receiving heat at constant temperature behave quite differently,

and the relation between the quantities involved can be found directly from the general equation, Eq. (906) for heat and work of gases, here repeated:

$$JQ_{ab} = \left[\frac{\frac{C_p}{C_v} - s}{\frac{C_p}{C_v} - 1}\right] \times (\text{Work of expansion}).$$

If in the equation, s be given the value unity, which it must have for this case, the relation between heat and work for isothermal gas processes follows as given by Eq. (936):

$$\left.\begin{aligned} Q_{ab} &= \frac{1}{J}(\text{Work of expansion}) \quad (a) \\ &= \frac{1}{J} P_a V_a \log_e \frac{V_b}{V_a} \quad (b) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (936)$$

Therefore *when gases expand isothermally the heat added is equal to the work done in the same units or the work done in foot-pounds is equal to the heat added in foot-pounds.* This is equivalent to saying that the internal energy remains constant and heat is converted directly and completely into work as fast as added during isothermal expansion or inversely for isothermal compression. To illustrate this case, the diagrams, Fig. 223, are plotted for the case of 1 lb. of

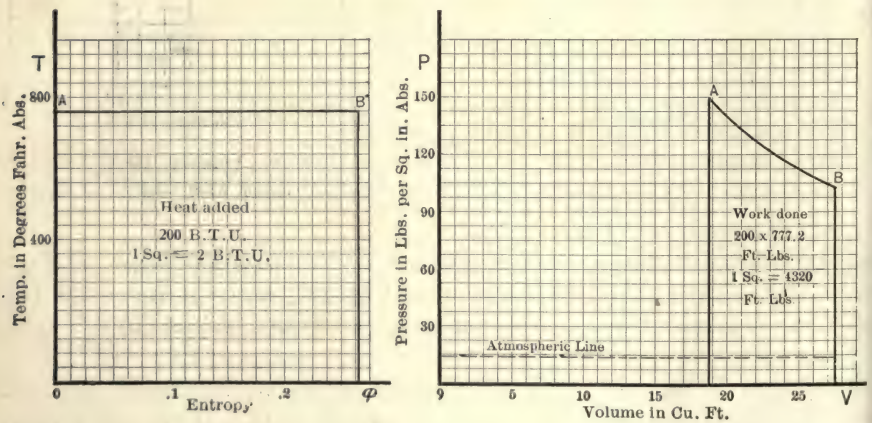


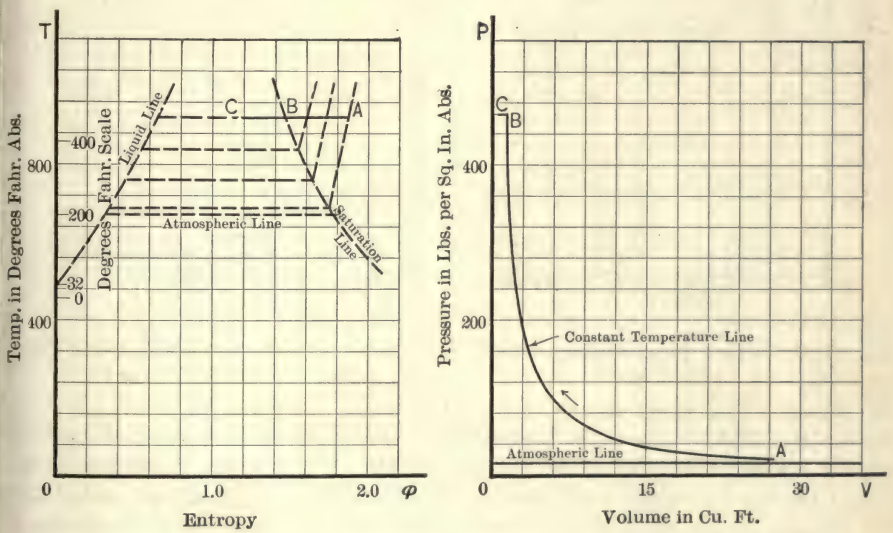
FIG. 223.—Changes in Pressure, Volume, Temperature, Entropy, Heat added, and Work done, for *Constant Temperature* Heating of Gases and Vapors without Change of State.

air at 150 lbs. per square inch absolute and temperature 300° F. expanding isothermally under the reception of 200 B.T.U.'s from A to B. The area under the curve or isothermal AB on the $T\Phi$ diagram is equivalent to 200 B.T.U. and

e area under the hyperbola AB on the PV diagram represents the mechanical uivalent of 200 B.T.U. or 155,504 ft.-lbs. For such cases as this, the following t of relations, Eq. (937), apply:

Related Quantities for Constant Temperature Heating of Gases	Formula	
eat and Volumes	$Q_{ab} = \frac{1}{J} P_a V_a \log_e \frac{V_a}{V_b} = \frac{1R}{J} T_a \log_e \frac{V_a}{V_b}$	(a)
eat and Pressures	$Q_{ab} = \frac{1}{J} P_a V_a \log_e \frac{P_a}{P_b} = \frac{1R}{J} T_a \log_e \frac{P_a}{P_b}$	(b)
olumes and Pressures	$P_a V_a = P_b V_b$	(c)
ork and Heat	$Q_{ab} = \frac{1}{J} W_{ab}$	(d)
ntropy and Temperature	$\phi_b - \phi_a = \frac{Q_{ab}}{T_a} = \frac{W_{ab}}{J T_a}$	(e)

(937)



IG. 224.—*Constant Temperature* or Isothermal Compression of Superheated Steam Reducing Superheat, and Subsequently Causing Partial or Complete Condensation.

hese same equations and general relations apply to superheated vapors provided hey do not approach the region of wetness which they will do on isothermal ompression if it is carried far enough. Condensation of vapors by isothermal ompression is shown by the curve, Fig. 224, for the case of steam, on which he point A represents steam in the condition of high superheat at low pressure. sothermal compression causes the change of state represented by the hyperbola AB on the PV and by the horizontal AB on the $T\Phi$ diagram. When the point

B is reached the vapor has acquired the pressure and volume occupied by dry saturated vapor so that further abstraction of heat at constant temperature results in condensation from B to C without any further pressure change and this would continue until all vapor had so disappeared, after which isothermal abstraction of heat would be impossible, because a liquid cannot give up heat without losing temperature.

Constant-volume cooling of a vapor will cause a change of state just as does isothermal compression, as will also a constant-volume heating of a mixture of vapor and liquid, but constant-volume heating or cooling of a vapor within such limits as keep it always superheated follows the same laws as for gases. The departure from the gas laws for constant-volume changes of vapors is clearly shown by the $T\Phi$ diagram in its relation to the PV as in Fig. 225. On the dia-

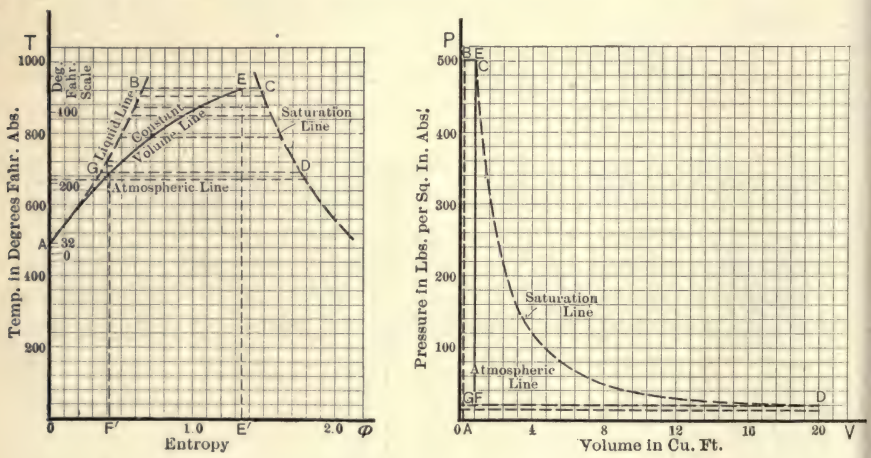


FIG. 225.—*Constant Volume* Changes for Steam Showing Corresponding Changes of Quality

gram AB represents liquid heating to both $T\Phi$ and PV coordinates, BC complete vaporization, so that the length of BC on the $T\Phi$ diagram is proportional to latent heat and on the PV diagram to the specific volume of dry saturated steam for the pressure. Similarly, $\overline{GD}$ represents the latent heat of condensation at the lower pressure to $T\Phi$ coordinates and the specific volume of low-pressure dry saturated steam to PV coordinates. Changes of temperature, pressure and volume for steam continuously dry and saturated is represented by the saturation line CD.

A condition of wetness of the higher pressure steam is indicated by the position of a point E on both diagrams and the following properties are defined by the relations of line lengths:

$$\text{Quality of steam at } E = \left(\frac{\overline{BE}}{\overline{BC}}\right)_{T\Phi} = x;$$

$$\text{Wetness of steam at } E = \left(\frac{\overline{EC}}{\overline{BC}}\right)_{T\Phi} = 1 - x;$$

$$\text{Volume of steam-water mixture} = (\overline{BE})_{PV} = \left(\frac{\overline{BE}}{\overline{BC}}\right)_{T\Phi} \times (\overline{BC})_{PV}.$$

$$\frac{\text{eat of wet steam formation}}{\text{eat of dry steam formation}} = \frac{xL}{L} = \left(\frac{\overline{BE}}{\overline{BC}}\right)_{T\Phi} = \left(\frac{\text{Vol. of wet steam}}{\text{Vol. of dry steam}}\right) = \left(\frac{\overline{BE}}{\overline{BC}}\right)_{PV}$$

hese relations permit of the location of the line representing heat abstraction constant volume and its heat evaluation by the $T\Phi$ area under it. Of course, is phase is represented by a vertical EF to PV coordinates and the position point F on the PV diagram indicates not only the volume the steam will ve after heat abstraction to the low pressure, but the relation it bears to the lume of dry saturated steam at the same pressure. If $(V_V - V_L)_1$ is the high-essure volume increase for dry saturated steam over its liquid, and $(V_V - V_L)_2$ e same at the low pressure, then the original volume of the wet steam is $V_V - V_L)_1 x_1 + (1 - x_1) V_{L1}$ and the final volume of the wet steam is $V_V - V_L)_2 x_2 + (1 - x_2)\ V_{L2}$ and as these are to be equal,

$$(V_V - V_L)_1 x_1 + (1 - x_1) V_{L1} = (V_V - V_L)_2 x_2 + (1 - x_2) V_{L2}.$$

he volume of the liquid is, however, so small as compared to the volume of e vapor that it may be neglected and so will hereafter be neglected.

Calling the specific volume of dry saturated steam V_V to be taken from the bles, there results $V_{V1} x_1 = V_{V2} x_2$, or the product of quality and specific volume dry vapors is constant. Points may be located along $(EF)_{T\Phi}$ by applying e equality between the ratio of partial to total latent heats on $T\Phi$ and ratio saturation to wet volume on PV coordinates. The heat abstracted is, of urse, the area under FE or $(FEE'F'F)$ and is best evaluated graphically by oss-section paper plotting when needed, which is not often.

A series of such constant-volume lines is drawn over a $T\Phi$ chart in Fig. 6, which indicates the change in the character of the line at different degrees wetness and its reversal of curvature from concave down to concave up, as passes into the region of superheat, where it is the same as that for a gas.

Example 1. *Calculation and use of Diagram, Fig.* 226, *giving constant-volume lines steam.* To illustrate the method the location of the line of constant volume of 2 ft. will be traced. Let the first temperature be taken at 800° F. absolute for the st point A, corresponding to 340° F. From the steam tables dry saturated steam 340° F. has a specific volume of 3.787 cu.ft., so that the quality when the volume 2 cu.ft. is $\frac{2}{3.797} = 52.8$ per cent. Therefore the entropy increase in making this

Pressure in Lbs. per Sq. In. Absolute
(Pressure Scale does not apply to Superheated Steam)
Absolute Temperature (°F)
Saturation Curve
Constant Volume Lines
Volumes in Cubic Feet
Liquid Line
A
B
C
D
E
H
K
M
P
Q

steam from 32° F. and at 340° F. is $\phi_a - \phi_{32} = .528 \times 1.0984 + .4902 = 1.0703$. Another point B is located by assuming a temperature $t_b = 440°$ F. or $T_b = 900$, for which $\phi_b - \phi_{32} = 1.5602$ by the same method.

To illustrate the use of the diagram in solving problems, suppose 1 lb. of wet atmospheric pressure steam, occupying 10 cu.ft. be enclosed in tank and heated to raise the pressure to 30 lbs. per square inch absolute, to find the final temperature, entropy and dryness. From 14.7 lbs. per square inch on the pressure scale project to point P on the constant volume line of 10 cu.ft. and follow this line to the point C for 30 lbs. per square inch absolute pressure. Projecting from C to D the absolute temperature is found to be 710° or $t = 250°$ F., and projecting from C to E the entropy $\phi_c - \phi_{32} = 1.332$. The final quality $= \frac{\overline{CM}}{\overline{OM}} = 72.4$ per cent.

Again, if heat be added to raise the temperature to 842° absolute the entropy is found by following the 10 cu.ft. line to the point K opposite the temperature, and projecting down from K to Q the entropy is found $\phi_k - \phi_{32} = 1.724$. The quality may be read off directly from the chart, Fig. 232, of the next section which carries lines of constant quality and which might be superimposed on this constant-volume chart.

Prob. 1. Ten pounds of water are heated from 50° F. to 300° F. without change of state; what is the change of entropy?

Prob. 2. What will be the entropy change for 100 lbs. of the following gases per 1000° temperature rise from 0° F. at constant pressure and at constant volume, assuming a constancy of specific heats as given in the tables of Chapter IV? Hydrogen, air, nitrogen, and oxygen.

Prob. 3. Mallard & LeChatelier give for the specific heat of O_2, $C_p = (.303 + .00003T)$. Compare the entropy change for 100 lbs. of this gas from 0° to 1000° with this value of specific heat with the result of Problem 2.

Prob. 4. A pound of air at 800° F. and a pressure of 150 lbs. per square inch gage expands at constant pressure until the temperature becomes 60° F. Find the volume at the end of expansion, the work done, and the change in entropy. A pound of air at 32° F. and 14.7 lbs. per square inch gage weighs .0807 lb. and the specific heat at constant pressure is .243, assumed constant.

Prob. 5. If the same amount of heat be removed at constant volume from a pound of air at 150 lbs. per square inch gage and 800° F., as was removed in Problem 4 at constant pressure, what will be the pressure, entropy, and temperature change? The specific heat at constant volume is .17.

Prob. 6. Water at 50° F. is changed to steam at 100 lbs. per square inch gage with 5 per cent moisture. What is the entropy change? What would it be if ammonia had been the liquid instead of water? What would be the value if the vapor were superheated 100° instead of being wet?

Prob. 7. Draw a $T\Phi$ and PV diagram for the following case. Ice is heated from 0° F. to 32° F. and then melted, the water being heated to 212° and evaporated. The steam formed is then superheated 200° F. The specific heat of ice is .5 and the latent heat of fusion 142.

Prob. 8. One pound of air receives 1000 B.T.U. while expanding isothermally from 100 lbs. per square inch gage and 60° F. Draw the PV and the $T\Phi$ diagrams for this case.

Prob. 9. A pound of steam at 50 lbs. gage and 200° superheat is compressed isothermally until it becomes 10 per cent wet. Show by a $T\Phi$ diagram the loss in entropy and work done.

Prob. 10. A closed tank contains nothing but superheated steam at 200 lbs. per

square inch absolute at a temperature of 550° F. If the volume of the tank be 10 cu.ft. and it be cooled to 32° F., plot the $T\Phi$ and PV diagrams to scale and find the volumes of liquid and vapor at every 10 lbs. pressure and the corresponding heat abstracted for each 10 lbs. per square inch pressure reduction.

4. Quantitative Relations for Secondary Thermal Phases. Adiabatics for Gases and Vapors. Constant Quality, Constant Total Heat, and Logarithmic Expansion Lines for Steam. Adiabatic expansion and compression for both gases and vapors, whether superheated or wet, are characterized or defined by constancy of entropy, which corresponds to zero heat change, and are represented on the temperature entropy diagram as a straight vertical line for all cases. On the pressure volume diagram it is, of course, an exponential curve, $PV^s = \text{Constant}$. It is possible now to find the value of s for both gases and vapors, including vapor liquid mixtures, best called wet vapors. Making the heat zero, in the general equation,

$$Q_{ab} = \frac{1}{J}\left[\frac{\frac{C_p}{C_v} - s}{\frac{C_p}{C_v} - 1}\right] \times (\text{Work of Expansion}) = 0.$$

It follows that since neither the work of expansion nor the specific heats are themselves zero, the s must be equal to the ratio of specific heats designated by γ,

$$\text{For adiabatic changes in gases, } s = \frac{C_p}{C_v} = \gamma \quad . \quad . \quad . \quad . \quad (938)$$

For exponential expansion of gases in general, and for adiabatic in particular the following relations between the temperatures and pressures, or temperatures and volumes, are derived from the fundamental pressure volume relation,

$$P_a V_a^{\,s} = P_b V_b^{\,s} = \text{Constant} = P_a V_a V_a^{\,s-1} = P_b V_b V_b^{\,s-1}.$$

Putting

$$P_a V_a = RT_a, \text{ and } P_b V_b = RT_b,$$

there results the relation between temperatures and volumes,

$$T_a V_a^{\,s-1} = T_b V_b^{\,s-1}.$$

Also

$$\left(\frac{T_a}{T_b}\right)^{\frac{1}{s-1}} = \frac{V_b}{V_a} = \left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}.$$

Therefore,

$$T_a P_b^{\,\frac{s-1}{s}} = T_b P_a^{\,\frac{s-1}{s}}.$$

Collecting the series for convenience and giving to the adiabatic the special exponent all are given by the tabular equation, Eq. (939),

Related Quantities for Exponential and Adiabatic Changes in Gases and Dry Vapors.	Formula				
	General Exponential Change.		Adiabatic Change.		
P and V	$\frac{P_a}{P_b}=\left(\frac{V_b}{V_a}\right)^s$;	$\frac{V_a}{V_b}=\left(\frac{P_b}{P_a}\right)^{\frac{1}{s}}$	$\frac{P_a}{P_b}=\left(\frac{V_b}{V_a}\right)^\gamma$;	$\frac{V_a}{V_b}=\left(\frac{P_b}{P_a}\right)^{\frac{1}{\gamma}}$	(a)
P and T	$\frac{P_a}{P_b}=\left(\frac{T_a}{T_b}\right)^{\frac{s}{s-1}}$;	$\frac{T_a}{T_b}=\left(\frac{P_a}{P_b}\right)^{\frac{s-1}{s}}$	$\frac{P_a}{P_b}=\left(\frac{T_a}{T_b}\right)^{\frac{\gamma}{\gamma-1}}$;	$\frac{T_a}{T_b}=\left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}$	(b)
V and T	$\frac{V_a}{V_b}=\left(\frac{T_b}{T_a}\right)^{\frac{1}{s-1}}$;	$\frac{T_a}{T_b}=\left(\frac{V_b}{V_a}\right)^{s-1}$	$\frac{V_a}{V_b}=\left(\frac{T_b}{T_a}\right)^{\frac{1}{\gamma-1}}$;	$\frac{T_a}{T_b}=\left(\frac{V_b}{V_a}\right)^{\gamma-1}$	(c)

. . (939)

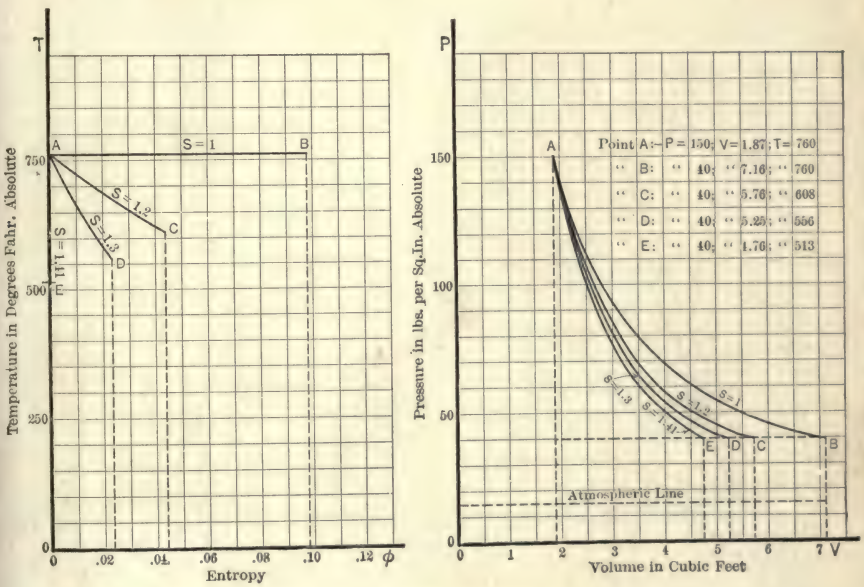


FIG. 227.—*Adiabatic* and *Exponential* Changes for Gases and Comparison with Isothermal.

To illustrate the case of adiabatic processes and to bring out clearly the striking difference between the isothermals and adiabatics for gases on $T\Phi$ diagrams in contrast to the small difference to PV coordinates, there is replotted in Fig. 227 the isothermal example of Fig. 223, and beside it is added an adiabatic for air, $PV^{1.41}=$Constant, from A to E, between the same pressures as well as two general exponentials according to $PV^{1.2}=K$ from A to C, and $PV^{1.3}=K$

from A to D. The PV diagram shows but little variation in work done compared to the variation in heats required as shown on the $T\Phi$ diagram where for

$s=1$ (Heat added) = (Work done);
$s=1.2$ (Heat added) = .51 × (Work done);
$s=1.3$ (Heat added) = .27 × (Work done);
$s=1.41$ (Heat added) = 0.

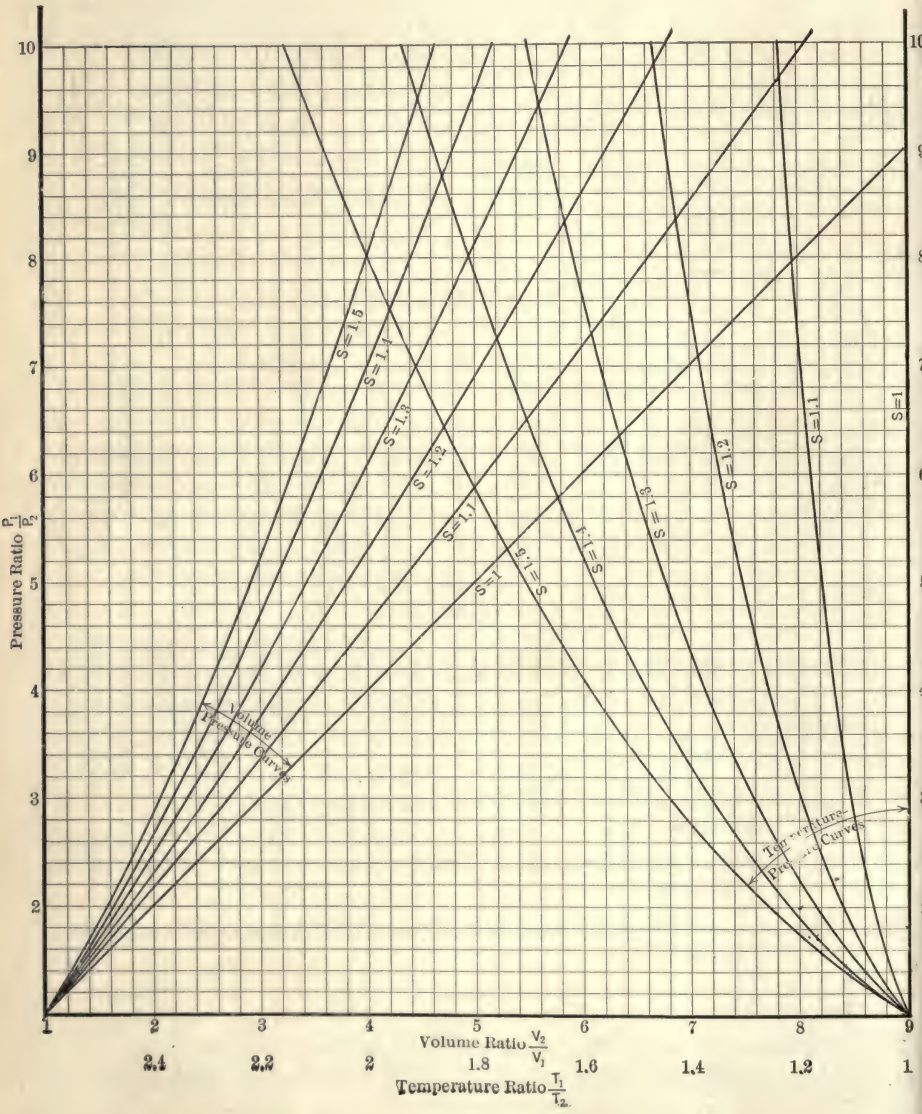


FIG. 228.—Exponential Gas Changes. Relation between Initial and Final Ratios of Pressures Volumes and Temperatures for Small Pressure Ratios.

This heat and work relation is given, of course, by the general equation just used, (Heat added) $=\frac{1}{J}\left(\frac{\gamma-s}{\gamma-1}\right)\times$(Work done), and in which the coefficient has the following values for

$$s=1;\quad 1.2;\quad 1.3;\quad 1.41;$$

$$\left(\frac{\gamma-s}{\gamma-1}\right)=1;\quad .51;\quad .27;\quad 0.$$

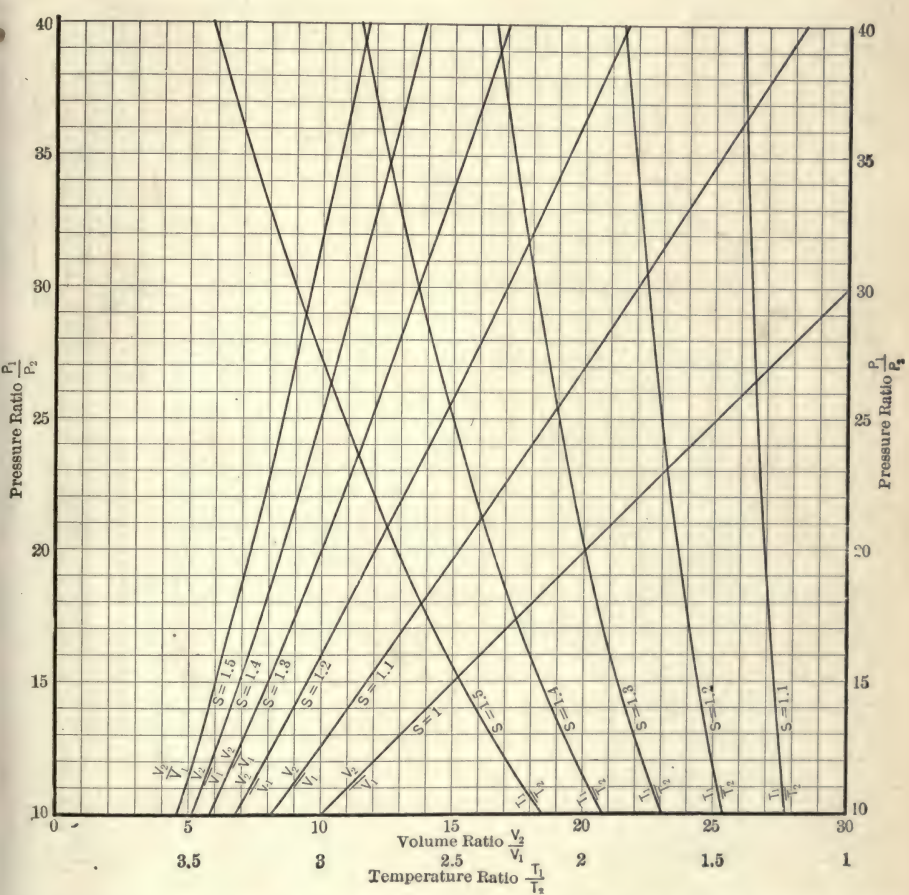


FIG. 229.—Exponential Gas Changes. Relation between Initial and Final Ratios of Pressures, Volumes and Temperatures for Larger Pressure Ratios.

To facilitate calculations of P, V, T relations for expansions and compression having various values of s, the curves, Figs. 228 and 229 have been plotted to a vertical scale of $\left(\frac{P_1}{P_2}\right)$, and with a double horizontal scale for the corresponding $\left(\frac{V_2}{V_1}\right)$ and $\left(\frac{T_1}{T_2}\right)$. Each curve is for a different value of s, as marked on it.

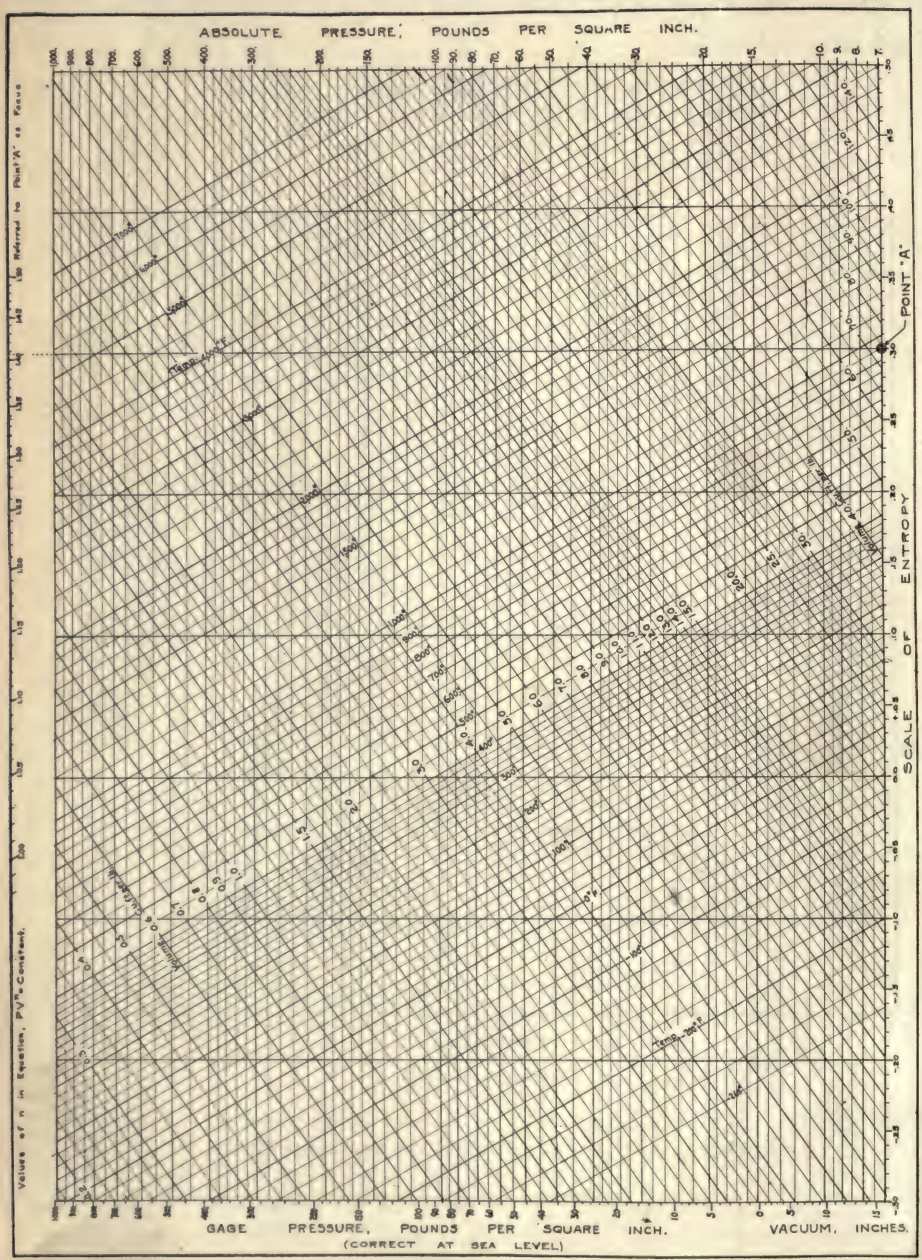


FIG. 230.—Exponential Gas Changes. Relation between Initial and Final Ratios of Pressures, Volumes, Temperatures,

These are also given on logarithmic cross-section paper in Fig. 230 as arranged by Gunn, where all lines become straight, and to which an entropy scale is added.

The simplicity of these relations for adiabatics of gases or superheated vapors not nearing the condensation condition, is in striking contrast to the complexity characterizing the adiabatics for vapors passing into the wet region. It was shown that isothermal compression of a superheated vapor would wet it, or inversely, isothermal expansion dry it or increase the superheat, and it now appears that adiabatic expansion of superheated vapors makes them lose superheat and become wet, or adiabatic compression dries them or increases their superheat, just the inverse of isothermal action. From the diagram, Fig. 231, on which OA is the liquid line and EJ is the saturation line to both $T\Phi$ and PV coordinates it is clear that there are five typical cases of adiabatic change, either compression or expansion between any two pressures, depending on the relation of the adiabatic to the saturation and liquid lines.

First, AB passing through the state of *all liquid* at the high pressure and temperature, indicates that liquid at the boiling-point will evaporate or make vapor, by adiabatic expansion. The weight formed will be, $\left(\frac{\overline{OB}}{\overline{OJ}}\right)_{T\Phi} \times$(weight of substance), while its volume will be likewise, $\left(\frac{\overline{OB}}{\overline{OJ}}\right)_{T\Phi} \times$(volume of dry saturated vapor) $= \left(\frac{\overline{OB}}{\overline{OJ}}\right)_{T\Phi} \times (\overline{OJ})_{PV}$, neglecting the liquid volume.

Second, CD passing through a wet region at the high pressure also passes through the wet region at the low pressure, but the quality will not be the same at all points, though it may at two or more. The quality at the beginning is $\left(\frac{\overline{AC}}{\overline{AE}}\right)_{T\Phi}$, and at the end $\left(\frac{\overline{OD}}{\overline{OJ}}\right)_{T\phi}$, while the initial volume before expansion is $\left(\frac{\overline{AC}}{\overline{AE}}\right)_{T\phi} \times (\overline{AE})_{PV} = (\overline{AC})_{PV}$, and the final volume is $\left(\frac{\overline{OD}}{\overline{OJ}}\right)_{T\Phi} \times (\overline{OJ})_{PV} = (\overline{OD})_{PV}$.

Third, EF passing through the state of dry saturated vapor at the high pressure, enters the region of wet vapor immediately for all lower pressures, proving that dry saturated vapors get wet by adiabatic expansion and, in addition, providing means of evaluating the quality or moisture. Thus, for expansion, the initial quality is 1 and the volume $(\overline{AE})_{PV}$, the final quality $\left(\frac{\overline{OF}}{\overline{OJ}}\right)_{T\Phi}$ and corresponding volume, $\left(\frac{\overline{OF}}{\overline{OJ}}\right)_{T\Phi} \times (\overline{OJ})_{PV} = (\overline{OF})_{PV}$, while the final moisture is $\left(\frac{\overline{FJ}}{\overline{OJ}}\right)_{T\Phi}$.

Fourth, GH passing into the superheat region at high, and into the wet region at low pressures, crossing the saturation line EJ on both diagrams at a point M. The high-pressure superheat is $(T_g - T_e)$ degrees, the low-pressure quality $\left(\frac{\overline{OH}}{\overline{OJ}}\right)_{T\Phi}$ and wetness, $\left(\frac{\overline{HJ}}{\overline{OJ}}\right)_{T\Phi}$. Assuming that superheated volumes

vary as absolute temperatures, then $\frac{V_g}{V_e}=\frac{T_g}{T_e}$ or, $V_g=V_e\frac{T_g}{T_e}$, then the initial volume before expansion is $(\overline{AG})_{PV}=(\overline{AE})_{PV}\times\frac{T_g}{T_e}$, while the final volume is $\left(\frac{\overline{OH}}{\overline{OJ}}\right)_{T\Phi}\times(\overline{OJ})_{PV}=(\overline{OH})_{PV}$.

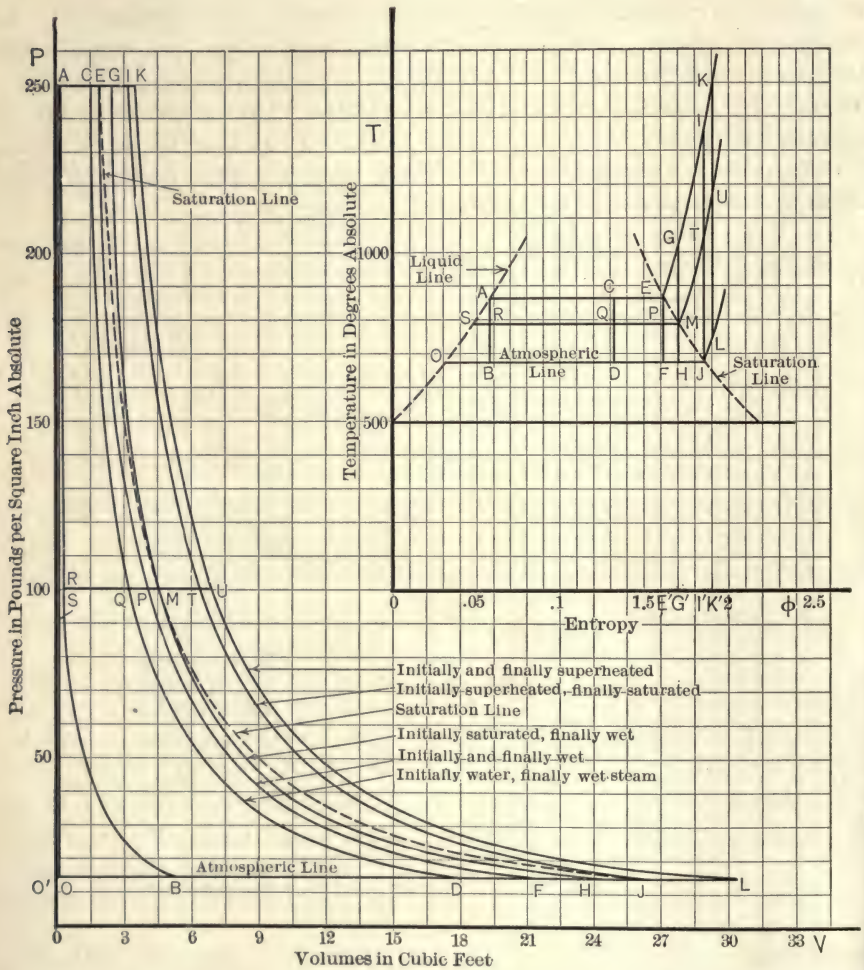


FIG. 231.—*Adiabatics* for Steam showing Changes of Quality and Illustrating a Graphic Method of Finding "*s*" for Wet Steam.

Fifth, IJ passing through the state of dry saturated vapor at the low pressure and indicating that dry saturated vapors immediately superheat on adiabatic compression, or that highly superheated vapors may become dry saturated and then wet by adiabatic expansion.

Another case might have been added is represented by KL for adiabatic expansion entirely in the superheat region, but this is exactly the same as that previously considered for gases and has all its characteristics.

The graphic construction of such diagrams is quite easy when the saturation line is used as a reference, it being plotted to both PV and $T\Phi$ coordinates directly from the steam tables, because in the wet region the volume of wet vapor divided by the volume of dry saturated is equal to the quality of the steam, and in the superheat region the volume of superheated steam divided by the volume of dry saturated is equal to the ratio of actual steam temperature to saturation temperature absolute very nearly; the exact relations due to imperfection of steam gas are given in Marks and Davis Steam Tables. These principles applied to the diagrams give equal line ratios on both, so one can be transformed into the other, or a point fixed on one located correspondingly on the other. Thus for saturated steam

Wet steam compared to dry saturated steam at the *same pressure*, Fig. 231.

$$\left\{ \begin{array}{l} \left(\dfrac{\overline{SR}}{\overline{SM}}\right)_{T\Phi} = \left(\dfrac{\overline{SR}}{\overline{SM}}\right)_{PV}; \\ \left(\dfrac{\overline{SQ}}{\overline{SM}}\right)_{T\Phi} = \left(\dfrac{\overline{SQ}}{\overline{SM}}\right)_{PV}; \\ \left(\dfrac{\overline{SP}}{\overline{SM}}\right)_{T\Phi} = \left(\dfrac{\overline{SP}}{\overline{SM}}\right)_{PV}. \end{array} \right\}$$

Similarly, for the superheated region the ratios are between verticals on the $T\Phi$ and horizontals on the PV since the former represent absolute temperatures and the latter specific volumes; thus for

Superheated steam compared to dry saturated steam at the same *pressure*, Fig. 231.

$$\left\{ \begin{array}{ll} \left(\dfrac{\overline{GG'}}{\overline{EE'}}\right)_{T\Phi} = \left(\dfrac{\overline{AG}}{\overline{AE}}\right)_{PV}; & \left(\dfrac{\overline{II'}}{\overline{EE'}}\right)_{T\Phi} = \left(\dfrac{\overline{AI}}{\overline{AE}}\right)_{PV} \\ \left(\dfrac{\overline{TI'}}{\overline{MG'}}\right)_{T\Phi} = \left(\dfrac{\overline{ST}}{\overline{SM}}\right)_{PV}; & \left(\dfrac{\overline{UK'}}{\overline{MG'}}\right)_{T\Phi} = \left(\dfrac{\overline{SU}}{\overline{SM}}\right)_{PV} \\ \left(\dfrac{\overline{LK'}}{\overline{JI'}}\right)_{T\Phi} = \left(\dfrac{\overline{OL}}{\overline{OJ}}\right)_{PV}. & \end{array} \right\}$$

From any of these adiabatics located on the PV diagram from the vertical line on the $T\Phi$ diagram, the exponent s can be determined by the measurement of simultaneous pressures and volumes at any pair of points and their substitution in Eq. (14), Chapter I. Taking the two points E and F on Fig. 231, the exponent for adiabatic expansion of originally dry saturated steam can be found from

$$s_{ef} = \frac{\log \dfrac{P_e}{P_f}}{\log\left(\dfrac{V_f}{V_e}\right)} = \frac{\log P_e - \log P_f}{\log V_f - \log V_e}.$$

TABLE CXXVI.

VALUES OF "s" FOR ADIABATIC EXPANSION OF STEAM.

A. EXPANSION OF WATER FROM 200 LBS. ABS.

Values of s for 10-lb. Intervals.			Values of s for Whole Range.		
Pressure.	Calculated.	Corrected.	200 Lbs. to	Calculated.	Corrected.
200–190	.0987	.1	190	.0987	.100
190–180	.1435	.141	180	.1175	.118
180–170	.1847	.182	170	.1348	.135
170–160	.2304	.223	160	.1519	.153
160–150	.2671	.264	150	.1682	.168
150–140	.3069	.305	140	.1843	.184
140–130	.3509	.346	130	.2007	.202
130–120	.3911	.387	120	.2172	.218
120–110	.4304	.428	110	.2341	.235
110–100	.4738	.470	100	.2517	.252
100– 90	.5166	.510	90	.2699	.270
90– 80	.5512	.551	80	.2889	.290
80– 70	.5897	.592	70	.3089	.310
70– 60	.6320	.633	60	.3306	.332
60– 50	.6790	.674	50	.3547	.356
50– 40	.7147	.716	40	.3811	.382
40– 30	.7658	.760	30	.4125	.412
30– 20	.8150	.808	20	.4518	.448
20– 10	.8718	.870	10	.5085	.504
10– 1	1.0557	1.042	1	.6381	.638

B. EXPANSION OF DRY SATURATED STEAM FROM 200 LBS. ABS.

Values of s for 10-lb. Intervals.			Values of s for Whole Range.		
Range.	Calculated.	Corrected.	200 Lbs. to	Calculated.	Corrected.
200–190	1.132	1.145	190	1.132	1.143
190–180	1.153	1.145	180	1.143	1.143
180–170	1.142	1.145	170	1.143	1.143
170–160	1.148	1.145	160	1.144	1.143
160–150	1.138	1.144	150	1.143	1.143
150–140	1.128	1.144	140	1.140	1.143
140–130	1.150	1.143	130	1.142	1.142
130–120	1.130	1.143	120	1.140	1.142
120–110	1.135	1.142	110	1.139	1.142
110–100	1.137	1.141	100	1.139	1.141
100– 90	1.148	1.140	90	1.140	1.140
90– 80	1.126	1.138	80	1.138	1.139
80– 70	1.144	1.137	70	1.139	1.139
70– 60	1.138	1.136	60	1.138	1.138
60– 50	1.125	1.135	50	1.137	1.137
50– 40	1.143	1.133	40	1.138	1.136
40– 30	1.131	1.131	30	1.136	1.135
30– 20	1.131	1.130	20	1.135	1.134
20– 10	1.125	1.128	10	1.133	1.131
10– 1	1.124	1.126	1	1.124	1.127

C. EXPANSION OF STEAM. SUPERHEATED THROUGHOUT EXPANSION, FROM 200 LBS. ABS. AND 540° SUPERHEAT.

Values of s for 10-lb. Intervals.			Values of s for Whole Range.		
Pressure.	Calculated.	Corrected.	200 Lbs. to	Calculated.	Corrected.
200–190	1.354	1.342	190	1.354	1.342
190–180	1.314	1.342	180	1.333	1.342
180–170	1.455	1.342	170	1.374	1.342
170–160	1.257	1.342	160	1.340	1.342
160–150	1.403	1.341	150	1.354	1.341
150–140	1.213	1.341	140	1.323	1.341
140–130	1.422	1.341	130	1.340	1.341
130–120	1.343	1.340	120	1.340	1.340
120–110	1.329	1.340	110	1.339	1.339
110–100	1.332	1.339	100	1.338	1.339
100– 90	1.338	1.338	90	1.338	1.338
90– 80	1.287	1.336	80	1.331	1.336
80– 70	1.331	1.335	70	1.331	1.335
70– 60	1.340	1.334	60	1.332	1.334
60– 50	1.315	1.332	50	1.330	1.332
50– 40	1.327	1.330	40	1.329	1.330
40– 30	1.318	1.327	30	1.328	1.327
30– 20	1.328	1.325	20	1.328	1.325
20– 10	1.323	1.322	10	1.327	1.322

D. EXPANSION OF STEAM INITIALLY SUPERHEATED AND FINALLY WET, FROM 20[illegible] LBS. ABS. AND 1[illegible] SUPERHEAT.

(NOTE.—Crosses saturation line at 70 lbs. abs.)

Values of s for 10-lb. Intervals.			Values of s for Whole Range.		
Range.	Calculated.	Corrected.	200 Lbs. to	Calculated.	Corrected.
200–190	1.249	1.334	190	1.249	1.33[illegible]
190–180	1.365	1.332	180	1.306	1.33[illegible]
180–170	1.396	1.330	170	1.336	1.33[illegible]
170–160	1.333	1.327	160	1.336	1.33[illegible]
160–150	1.314	1.324	150	1.331	1.33[illegible]
150–140	1.325	1.321	140	1.330	1.33[illegible]
140–130	1.357	1.316	130	1.334	1.33[illegible]
130–120	1.302	1.312	120	1.329	1.33[illegible]
120–110	1.303	1.306	110	1.325	1.33[illegible]
110–100	1.270	1.300	100	1.317	1.33[illegible]
100– 90	1.396	1.292	90	1.328	1.33[illegible]
90– 80	1.311	1.283	80	1.325	1.33[illegible]
80– 70	1.337	1.272	70	1.327	1.33[illegible]
70– 60	1.230	1.156	60	1.314	1.33[illegible]
60– 50	1.150	1.150	50	1.290	1.2[illegible]
50– 40	1.144	1.146	40	1.268	1.2[illegible]
40– 30	1.138	1.140	30	1.246	1.2[illegible]
30– 20	1.093	1.134	20	1.216	1.2[illegible]
20– 10	1.157	1.127	10	1.202	1.2[illegible]
10– 1	1.116	1.120	1	1.163	1.1[illegible]

NOTE. Irregularities in values of s have been corrected by plotting a smooth curve through calcul[illegible] values, and taking corrected values from this curve.

his gives a sort of overall value for s which is not uniform over the line EF, s other values from

$$s_{ep}=\frac{\log P_e-\log P_p}{\log V_p-\log V_e}, \text{ and } \quad s_{pf}=\frac{\log P_p-\log P_f}{\log V_f-\log V_p},$$

r the same adiabatic using intermediate points, are not the same.

To illustrate this point there have been prepared Tables CXXVI, A to D. ase B shows the results for dry saturated steam expanding in 10-lb. drops om 200 lbs. to 1 lb. per square inch absolute. After the first drop the steam wet and becomes more wet for the next. Case A is for the case of initially l water, the percentage of steam increasing as expansion continues. In case the steam is originally superheated to such a degree that it remains super- ated throughout the range of the expansion. In case D the original super- at is small and at some point during the expansion the steam is momentarily y and saturated and then becomes wet.

Hot water shows by its adiabatic expansion values from, $s=.10$ to $s=1.042$ r the first and last 10 lb. interval, while for dry saturated steam the values e from $s=1.145$ to $s=1.126$. In neither case does the variation for the hole range agree with that for the ten pound interval corresponding. Steam ways superheated shows values slightly decreasing from $s=1.342$, to $s=1.322$ ver the whole range, but when the saturation condition is passed there is a rapid ange, as shown in the last table.

Due to the incompleteness of the necessary data the results as calculated d as given in the second column of the table showed some inconsistencies. hese former values were plotted and the values given in the third column of e table were read from a smooth curve drawn through the calculated results.

Inspection of these tables shows conclusively the variability of s along one pansion line which is due to the condensation that is proceeding at a variable te. This variability of s is also dependent on the pressure at which expansion gins and on the original amount of moisture or quality, and to show this, has been determined for steam expanding from 250, 200, 100 and 50 lbs. to lb., from the initial and final volumes only and for original qualities of 100, , 80, 50, 20 per cent, the results being set down in Table CXXVII.

TABLE CXXVII

VALUES OF "s" FOR ADIABATIC EXPANSION OF STEAM

(Determined from initial and final volumes only)

Pressure Range.	Initial Quality.						
	250° F. Superheat.	100 %.	90%.	80%.	50%.	20%.	Water.
om 250 lbs. to 1 lb....	1.183	1.119	1.110	1.102	1.060	.961	.662
om 200 lbs. to 1 lb...	1.185	1.124	1.117	1.107	1.066	.966	.638
om 100 lbs. to 1 lb. ..	1.192	1.125	1.118	1.109	1.070	.969	.565
om 50 lbs. to 1 lb....	1.206	1.124	1.118	1.111	1.073	.963	.490

If for any possible condition of expansion fixed by pressures and quality, an average value of s were known for the whole range, then work could be calculated by the PV methods of Chapters I and III, but even then the labor would be considerable and there would be necessary an almost infinite number of values of s. If there were no other better way of proceeding this would be satisfactory, but the entropy diagram and steam tables provide methods of *exact* calculation of such simplicity as to command admiration, and warrant the entire abandonment of all other methods of calculation of work of adiabatic expansion. As a matter of fact the thermal method, whether executed graphically or by means of tabular values, involves practically no calculation at all, and it is now possible to read directly from a chart the work done for a complete cycle with adiabatic expansion so that labor is concentrated in the preparation of the chart itself. Indirectly the work of the expansion alone as a single phase may be found from the same data, as will be explained.

For a *complete cycle* of water heating, evaporation at constant pressure, adiabatic expansion, and condensation at constant pressure, the work done is equal to the heat added less the heat abstracted, which, per pound of steam, is the difference between the total heats at the two different conditions before and after adiabatic expansion. Therefore, the work of such complete cycles is to be determined from the total heats of steam. Charts of the total heat to which are added lines of constant pressure, temperature and quality for the same entropy, are graphic means for reading off the work directly.

For expansion alone as a single phase the work can be calculated from the total heats from the general relation,

$$(\text{Heat added}) = (\text{Change in intrinsic energy}) + (\text{Work done}).$$

The expansion being adiabatic the work done will be the difference between the intrinsic energy before and after, with opposite sign, because the heat added algebraically, is zero. Therefore,

Let H_1 and H_2 = Total heat, B.T.U. per pound dry saturated steam initial and final;

" q_1, q_2 and L_1, L_2 = Heats of liquid and latent heats, tabular values;

" $\overline{U}_1$ and $\overline{U}_2$ = Internal energy at initial and final conditions;

" x_1 and x_2 = Quality or dryness fractions;

" V_{V_1} and V_{V_2} = Specific volume of dry saturated steam at high and low temperature, tabular values;

" V_{L_1} and V_{L_2} = Specific volume of liquid at high and low temperature which may generally be neglected.

Then

$$W_{1\,2} = \left\{ \begin{matrix} \text{Work done, ft.-lbs., by} \\ \text{adiabatic expansion} \end{matrix} \right\} = J(\overline{U}_1 - \overline{U}_2). \quad . \quad . \quad . \quad (94$$

But during the formation of steam from water

$$(\text{Change in internal energy}) = (\text{Heat added}) - (\text{Work done}),$$

) that the internal energy of a pound of steam above 32° F. is the amount
' heat it takes to make the steam starting with water at 32° F. less the work
ıe steam does in coming into existence $\frac{1}{J}\times P(V_V-V_L)$. Therefore, the work
)ne in adiabatic expansion is the difference between the total heats before
ıd after, less the difference between the work of steam formation at the two
ates before and after in B.T.U. When steam is wet,

$$\text{Total heat}=q_1+x_1L_1 \text{ B.T.U.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (941)$$

$$\text{Vol. wet steam}=x_1V_{V_1}+(1-x)V_{L_1} \text{ Cu.ft.} \quad . \quad . \quad . \quad . \quad . \quad (942)$$

herefore,

$$\text{(Work of steam formation)}=P_1[x_1V_{V_1}+(1-x_1)V_{L_1}-V_{L_1}]$$

$$=x_1P_1(V_{V_1}-V_{L_1}) \text{ ft.-lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (943)$$

$$\overline{U}_1-\overline{U}_{32}=q_1+x_1L_1-\frac{1}{J}x_1P_1(V_{V_1}-V_{L_1}) \text{ B.T.U.} \quad . \quad (944)$$

$$\overline{U}_2-\overline{U}_{32}=q_2+x_2L_2-\frac{1}{J}x_2P_2(V_{V_2}-V_{L_2}) \text{ B.T.U.} \quad . \quad (945)$$

ubstituting in Eq. (940) the values of the intrinsic energy above 32° F., before
ıd after adiabatic expansion as given by Eqs. (944) and (945) the work of expan-
on is given by Eq. (946),

$$W_{1\,2}=J(q_1-q_2+x_1L_1-x_2L_2)-[x_1P_1(V_{V_1}-V_{L_1})-x_2P_2(V_{V_2}-V_{L_2})] \text{ ft.-lbs.} \quad (946)$$

eglecting liquid volumes, this takes the form of Eq. (947) for the special case
expansion illustrated by the line from C to Q, Fig. 231, which is the case of
)nstantly wet steam,

$$W_{cd}=J(q_a-q_s+x_cL_c-x_qL_m)-(x_cP_cV_{V_c}-x_qP_qV_{V_q}) \quad . \quad . \quad . \quad (947)$$

ut the quality after expansion x_q is a function of the quality before x_c, which
ıay be expressed in terms of entropies or the corresponding heats.

The entropy relations are given by Eq. (948) derived by the following steps:

$$x_c = \frac{\phi_c - \phi_a}{\phi_e - \phi_a}; \quad x_q = \frac{\phi_q - \phi_s}{\phi_m - \phi_s};$$

$$(\phi_q - \phi_s) = (\phi_c - \phi_a) + (\phi_a - \phi_s);$$

hence

$$x_q(\phi_m - \phi_s) = x_c(\phi_e - \phi_a) + (\phi_a - \phi_s).$$

Adding to each side the entropy difference from 32° to the point S,

$$x_q(\phi_m - \phi_s) + (\phi_s - \phi_{32}) = x_c(\phi_e - \phi_a) + (\phi_a - \phi_{32}). \quad . \quad . \quad . \quad (948)$$

In words Eq. (948) may be stated that for points on the same adiabatic *the sum of the entropy of the liquid from* 32° *to any temperature, and the product of the quality into the entropy of complete vaporization at that temperature is a constant.* From Eq. (948) the value of x_q, the final quality after adiabatic expansion can be found for use in Eq. (947) by using the tables of entropy for any initial quality of steam.

A similar relation between condition after and that before adiabatic expansion can be expressed in terms of heats and temperatures. If the specific heat of water be taken as unity then from Eq. (948) this relation will be given by Eq. (949),

$$x_q \frac{L_m}{T_q} + \log_e \frac{T_q}{492} = x_c \frac{L_e}{T_c} + \log_e \frac{T_c}{492};$$

$$x_q \frac{L_m}{T_q} = x_q \frac{L_e}{T_c} + \log_e \frac{T_c}{T_q}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (949$$

In some steam tables, notably Peabody's, the value of x, the quality for a given entropy at any pressure and temperature, is given, and these values are extremely useful in such work as they eliminate the solution of Eq. (946) to Eq (949), constancy of entropy characterizing adiabatic expansion, the tabula values solve such expansion problems directly. If corresponding qualities fo equal entropies are not given directly in tables, which is the case with th Marks and Davis tables, it is possible to estimate without calculation, th quality at one temperature, for a condition defined by an entropy the sam as for some other quality at another temperature, and an example will b given later to illustrate the procedure.

Constant quality lines on the temperature entropy diagram are directl useful in this sort of problem work and permit of direct reading of the qualit

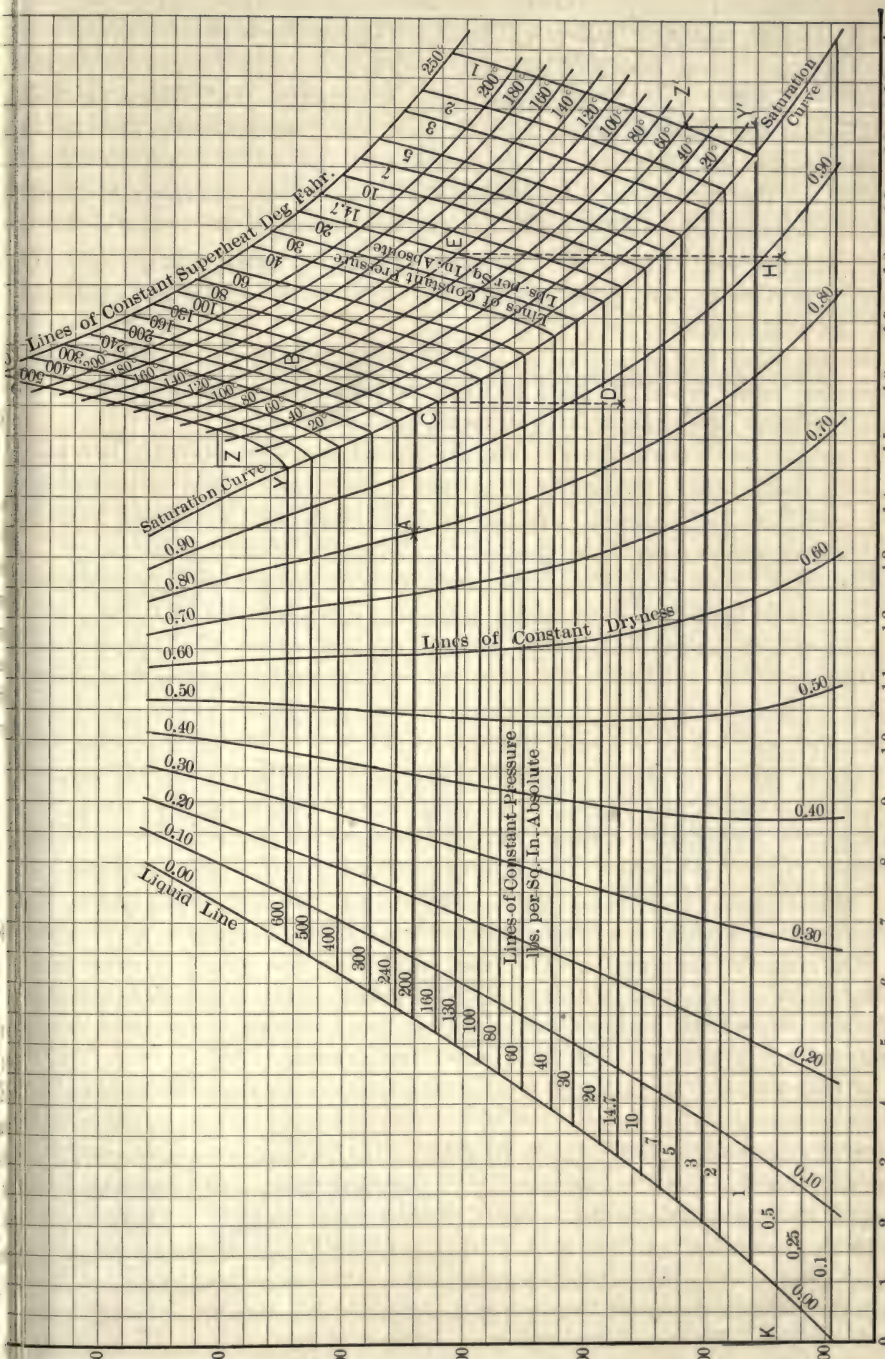


FIG. 232.—Temperature-entropy Diagram with Lines of *Constant Pressure* and *Constant Quality* for Steam.

at any temperature, after adiabatic expansion from another temperature and quality, and are especially useful in the region of superheat or for passing from superheat to wetness, which is the case in nearly all practical steam turbine problems. Such a set of lines is laid out on the chart, Fig. 232, graphically by the dividing each horizontal vaporization line proportionally and joining the one quarter, middle, three-quarter points and others located correspondingly between, in the wet region. In the region of superheat constancy of quality implies constancy of superheat, so that on each pressure line points Z and Z' are so located that $\overline{ZY} = \overline{Z'Y'}$. Changes of quality along adiabatics represented by verticals, are given by the intersection of the vertical adiabatic with the quality curve.

Constant total heat lines may be located directly on such a chart and serve to save the labor of computation of work, which is still appreciable even when quality changes during adiabatic expansion are known.

When the end sought is work done by the whole cycle there are two forms for such charts, first, the ordinary temperature entropy chart with constant total heat lines located on it, and second, the total heat plotted as ordinates and entropy as abscissæ, which latter is known as the Mollier diagram.

The steps by which points are located on the temperature entropy diagram for equal total heats illustrate a very important process in steam turbine working. Steam expanding in a nozzle without friction suffers adiabatic expansion and the work of adiabatic expansion as just determined appears as kinetic energy of the jet of steam. If this steam velocity is reduced to zero, the kinetic energy will all be converted back into heat, which heat will be added to the steam at the low pressure, drying or superheating it in addition. This is precisely what happens in the throttling steam calorimeter for measuring steam quality and was used by Grindley, Griessmann and Peaké in determining the total heat of steam. It is evident that after such an action the steam will have the same total heat at the low as at the high pressure and from such equivalence, with measures of its quality, the heats could be found or at least checked. Between the stages of turbines part of the heat converted into kinetic energy in the nozzles is converted back into work, but the action so far as it goes is the same. These are important matters entirely aside from the more common one of direct reading of adiabatic expansion work from charts.

In Fig. 233, let D represent a condition of wetness from which adiabatic expansion starts and proceeds to E with a loss of energy due to transformation into work, represented by the area $CDEBC$. If this energy be converted back into heat it will reduce the wetness by evaporation from condition E to F, the point F being located so that area $EFGHE$ = area $CDEBC$. Another point K would be located so that area $CDIJC = IKLHI$. In this way a line DI can be drawn, on which the steam has always the same total heat, or along which it suffers no change in heat content, so that it is a constant internal energy line for steam. So long as the heat equivalent of expansion work is all consumed in drying wet steam the construction is comparatively easy, but if the steam is originally dry or nearly so it will become superheated, and if super

eated it will acquire more superheat. In fact for superheated steam, if it ehaved like a perfect gas the internal energy would be a function of tempera- ure only and constant energy lines would be isothermals. It will be observed rom the diagram that this is not the case. To illustrate the location of a point n the superheat region, consider an original condition represented by M to be he beginning of an adiabatic expansion ending at N, then the point O will be ocated so that area $CMNJC$ = area $NROPQN$ and drying has proceeded long NR, followed by superheating RO to the constant total heat line MSO.

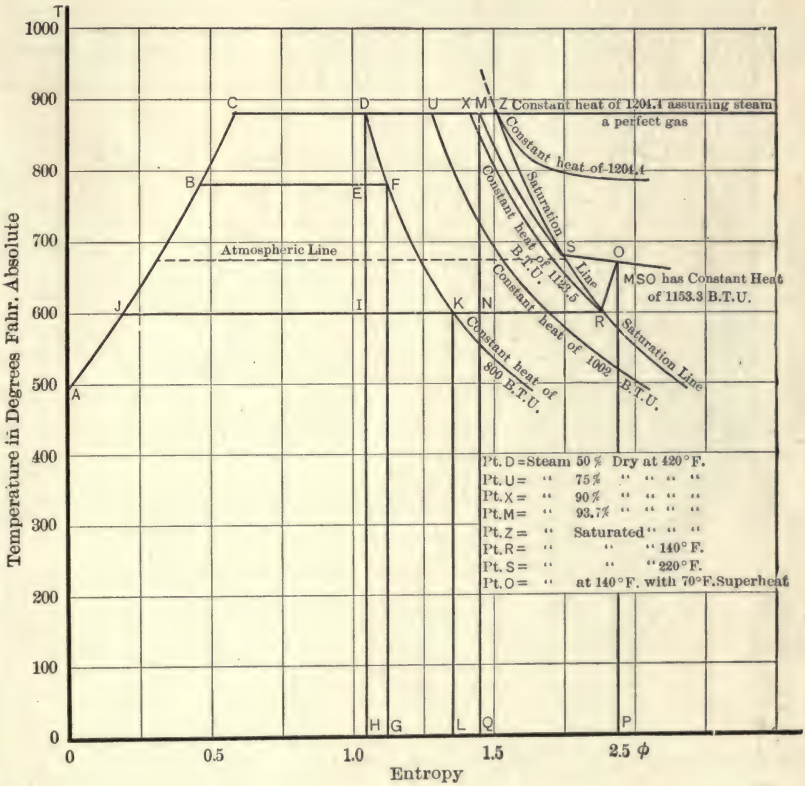


FIG. 233.—*Constant Total Heat* Lines for Steam, Method of Determination.

When the points of such a diagram are replotted so that total heats above 32° are ordinates and entropy from 32° are abscissa there will be a family of curves such as first plotted by Prof. Mollier and as represented in Fig. 234, known by his name as the Mollier diagram. On this chart the vertical distance from any pressure, temperature or quality, to any other, is the work done in heat units, by the whole cycle including an adiabatic expansion and can be marked off on a strip of paper and referred to the scale of heat to permit the work to be read directly, or the ordinate of the low can be subtracted from that

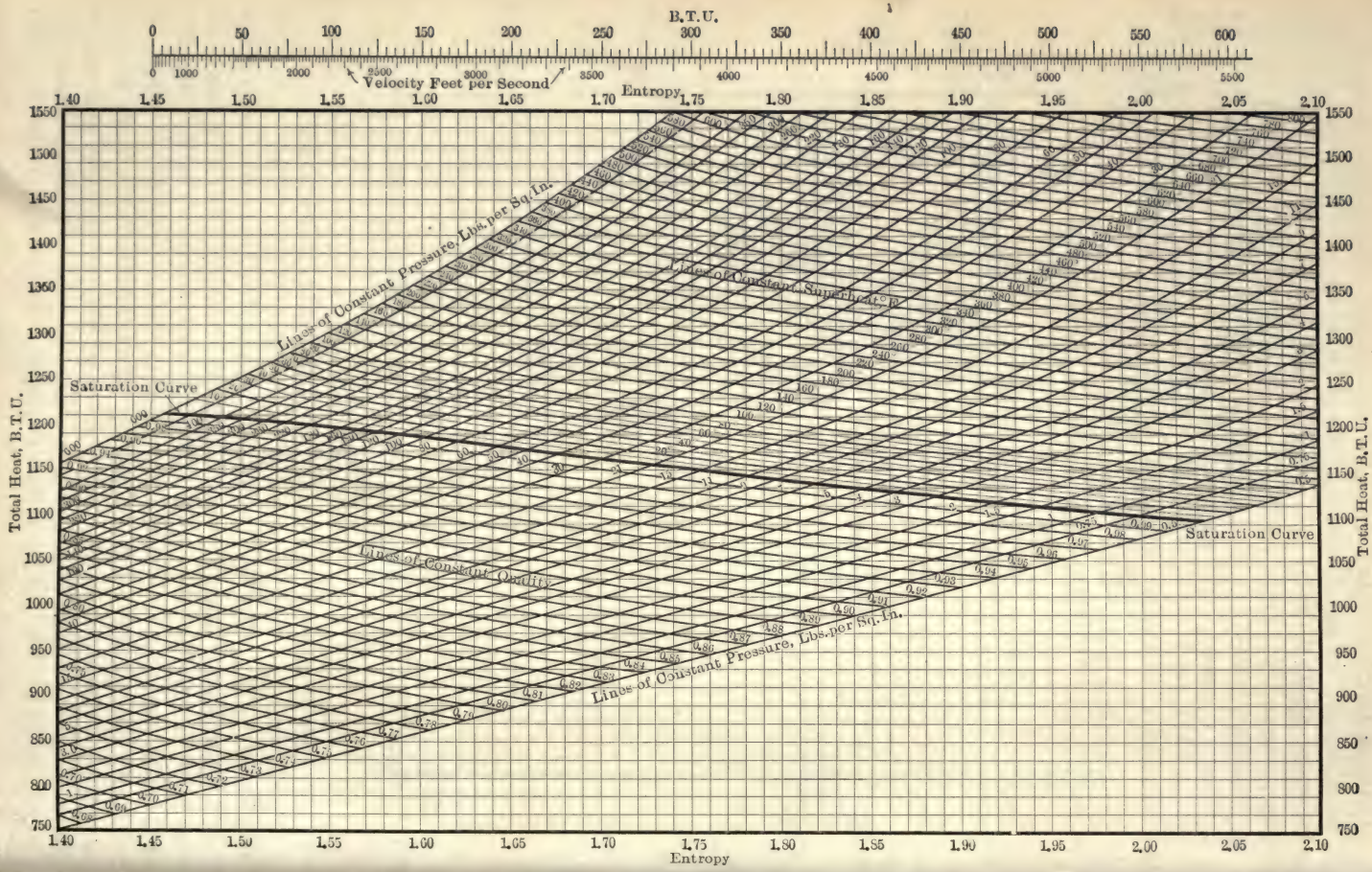


FIG. 224.—The Mollier Total Heat Entropy Diagram for Steam.

f the high point. As this is so convenient for turbine work a scale of correspond-ng steam jet velocities is usually plotted beside that for total heats. A large cale chart of this sort is very necessary when many calculations of this sort re to be made and such may be plotted from the steam tables.

Algebraic evaluation of the heat added during any expansion that is not diabatic is often desirable, especially in dealing with cycles and their efficiency s heat converters into work. Unfortunately this is not always possible with recision, but approximations are possible for some cases, good enough for com-arative purposes. It is necessary for this work that the total heat be expressed s a function of temperature and this is possible only by algebraic expressions f the empiric sort and of complicated form derivable from the general xpression, Eq. (950),

$$(\text{Heat added during expansion}) = \int T d\phi = \int T \frac{d\phi}{dT}\, dT \quad . \quad . \quad . \quad (950)$$

ntegration of Eq. (950) is possible only when the differential coefficient $\frac{d\phi}{dT}$ an be expressed as a function of T alone and by not too complex a function, the final formula is to be of any practical value. Along the saturation curve he total heat of the steam is the sum of heat of liquid and latent heat at any iven temperature above 32° F., so that the abscissa of any point on the $T\phi$ dia-ram from 32° as an origin, will be given by Eq. (951), if C_s is the specific heat of ater, which may be taken as constant and equal to unity or as a function of T,

$$\phi_a - \phi_{32} = \int_{492}^{T_a} C_s \frac{dT}{T} + \frac{L}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (951)$$

ut latent heat can be expressed as a function of T approximately by Eq. (952),

$$L = \alpha + \beta T + \gamma T^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (952)$$

o that the entropy increase above 32° for saturated steam is a function of tem-erature given by Eq. (953),

$$(\phi_a - \phi_{32}) = \int_{492}^{T_a} C_s \frac{dT}{T} + \frac{\alpha}{T} + \beta + \gamma T. \quad . \quad . \quad . \quad (953)$$

Corresponding to this integral expression is a differential form from which he differential coefficient of ϕ with respect to T is given by Eq. (954),

$$\frac{d\phi}{dT} = \frac{C_s}{T} - \frac{\alpha}{T^2} + \gamma. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (954)$$

This takes the two following forms (*a*) and (*b*), Eq. (955), according as $C_s=1$ approximately, or is a series function of the temperature.

$$\left.\begin{aligned} &\text{For specific heat of water } =1, && \frac{d\phi}{dT}=\frac{1}{T}-\frac{\alpha}{T^2}+\gamma && (a)\\ &\text{For specific heat of water } C_s=a+bT+cT^2, && \frac{d\phi}{dT}=\frac{a}{T}+b+cT-\frac{\alpha}{T^2}+\gamma && (b)\end{aligned}\right\} \quad (955)$$

Substituting the simpler form Eq. (955*a*), in Eq. (950), the heat added to keep steam dry and saturated as it expands will be given by Eq. (956),

$$\left\{\begin{array}{l}\text{Heat added during expansion from } B \text{ to } A \text{ along saturation line}\end{array}\right\} = \int_{T_b}^{T_a}\left[1-\frac{\alpha}{T}+\gamma T\right]dT \text{ (approx.)}$$

$$=\alpha \log_e \frac{T_b}{T_a}-(T_b-T_a)-\frac{\gamma}{2}(T_b^2-T_a^2). \quad . \quad (956)$$

A more exact but still approximate result is obtained by substitution of Eq. (955 *b*), in Eq. (950), which gives Eq. (957).

$$\left\{\begin{array}{l}\text{Heat added during expansion from } B \text{ to } A \text{ along saturation line}\end{array}\right\} = \int_{T_b}^{T_a}\left[a+(b+\gamma)T+cT^2-\frac{\alpha}{T}\right]dT$$

$$=\alpha \log_e \frac{T_b}{T_a}-a(T_b-T_a)-\frac{b+\gamma}{2}(T_b^2-T_a^2)-\frac{c}{3}(T_b^3-T_a^3). \quad (957)$$

Numerical problems cannot be solved without evaluation of these constants a, b, c, α, β, γ, which can be done approximately as follows: The Dieterici equation for the specific heat of water is

$$\begin{aligned} C_s &= .9983-.0000288(t-32)+.0000002133(t-32)^2;\\ &= .9994-.0000424t-.000000213t^2;\\ &= .9738+.0001536T-.000000213T^2; \end{aligned}$$

whence

$$\begin{aligned} a &= .9738;\\ b &= .0001536;\\ c &= .000000213. \end{aligned}$$

Similarly, the Davis formula for the total heat of steam above 32° F. is

$$H = 1150.3 + .3745(t-212) - .00055(t-212)^2;$$
$$= 1046.187 + .6077t - .00055t^2.$$

From the specific heat of water the heat of liquid q becomes,

$$q = .9994(t-32) - .0000424t(t-32) - .000000213t^2(t-32);$$
$$= 1.0007t - 31.981 - .0000356t^2 \text{ app.}$$

From these it follows that as $L = H - q$,

$$L = 1078.168 - .393t - .00051t^2;$$
$$= 1151.0 + .076T - .00051T^2;$$

whence

$$\alpha = 1151.0;$$
$$\beta = .076;$$
$$\gamma = -.00051.$$

Should the expansion proceed along any constant quality line in the wet region, the same method will apply, if, to the latent heat or to each term derived from it separately be applied the quality x as a multiplying factor. Hence

$$\left\{\begin{array}{l}\text{Heat added during wet}\\ \text{steam expansion from } B\\ \text{to } A \text{ for constant qual-}\\ \text{ity } x\end{array}\right\} = \alpha x \log_e \frac{T_b}{T_a} - (T_b - T_a) - \frac{\gamma x}{2}(T_b^2 - T_a^2). \quad (958)$$

Or more correctly,

$$\left\{\begin{array}{l}\text{Heat added during wet steam expansion}\\ \text{from } B \text{ to } A \text{ for constant quality } x\end{array}\right\} = \alpha x \log_e \frac{T_b}{T_a} - a(T_b - T_a)$$
$$- \frac{b+\gamma x}{2}(T_b^2 - T_a^2) - \frac{c}{3}(T_b^3 - T_a^3). \quad (959)$$

It is even more difficult to express algebraically the heat added during expansion according to constant superheat, unless the specific heat be assumed constant, which is hardly proper, as it is known to be a function of both pressure and

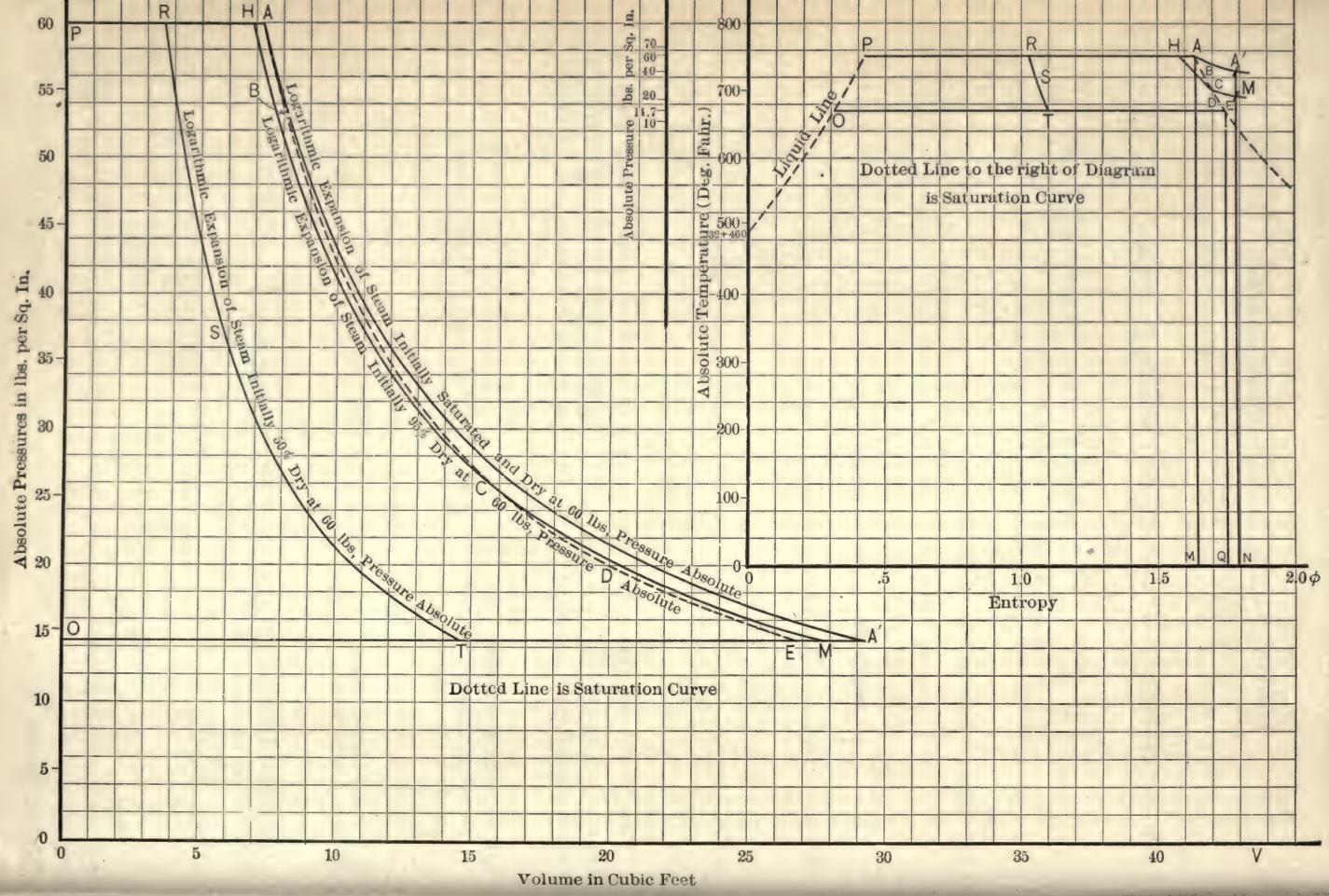


Fig. 225.—Logarithmic Expansion of Steam Showing Changes of Quality

emperature, which function is too complex to use here. If, however, it be lesired to sacrifice accuracy to get a workable result, it may be taken as constant, and if C_p be the mean specific heat of superheated steam for the amount of superheat t_S degrees above the saturation temperature T_e and T_f, and constant for the expansion range, then the initial steam temperature will be T_e+t_S and inal T_f+t_S. For the heat of superheat $C_p t_S$ B.T.U. per pound, the entropy above saturation will be

$$C_p \log_e \frac{T_e+t_S}{T_e} = \int_{T_e}^{T_e+t_S} C_p \frac{dT}{T},$$

which adds another term to the Eq. (954) similar to the first, and which on integration adds a corresponding term. As this case is of so little practical importance the result is omitted.

Logarithmic expansion of steam, it has been stated, is the old assumption for cylinders, and it is of interest to see what changes in quality correspond to it. In Fig. 235 is plotted the saturation line $ABCDE$ to $T\Phi$ and PV coordinates dotted. On the PV diagram are located a number of logarithmic lines for wet and superheated steam which it is found tend to cross the saturation line and so prove that evaporation takes place with logarithmic expansion of wet steam, and superheating follows. In fact for steam initially superheated the logarithmic expansion line is an isothermal and the superheat increases as the difference between saturation temperature for any pressure and the original temperature of the superheated steam. Thus, a logarithmic line AA' through A, a point of original dryness, is an isothermal as nearly as the superheated steam approach to a perfect gas permits. It is located by the ratio

$$\left(\frac{\overline{A'N}}{\overline{EQ}}\right)_{T\Phi} = \left(\frac{\overline{A'O}}{\overline{EO}}\right)_{PV}.$$

Points in the wet region are located by proportionality of horizontal intercepts as for adiabatics. It is of no interest to compute algebraically the heat that must be added during expansion to keep it logarithmic, but for any case where is needed, it can best be evaluated graphically by areas.

Example 1. *Calculation and use of Diagram, Fig.* 232, *lines of constant pressure and quality.* Let it be assumed that the line of quality 80 per cent is to be located, starting with the pressure of 200 lbs. per square inch absolute, point A. From the steam tables $=381.9°$ F. or $T_a=841.9$, the entropy of the liquid is .5437, of evaporation complete, 0019, so that $\phi_a-\phi_{32}=.8\times1.0019+.5437=1.3452$. To locate a point B in the superheat region at the same pressure and for 100° of superheat, the steam tables are found to give directly $\phi_b-\phi_{32}=1.6120$.

As an example of the use of the diagram the following problem will serve. Steam at a pressure of 160 lbs. per square inch absolute, dry and saturated expands adiabatically to atmospheric pressure and to some unknown quality to be found. From the point C representing the initial condition project vertically down to the pressure line 14.7, at point D. By interpolation the quality is found to be 86.5 per cent, as point D lies between the two lines of 80 per cent and 90 per cent quality.

Another example will illustrate the passage into the superheat region. Atmospheric exhaust steam at 20 lbs. per square inch absolute, is superheated 120° by a reheater and then expands adiabatically in an exhaust steam turbine to an absolute pressure of half a pound per square inch absolute, to find the final quality. The initial condition is represented by point E, from which projecting downward to the low-pressure line at H, lying between 80 per cent and 90 per cent, the quality is found by interpolation to be 88.4 per cent and the temperature by projecting to K, is $T = 540°$. The corresponding volumes may be read off the chart Fig. 226, of the last section.

Example 2. *Method of calculating the Diagram, Fig.* 235, *for logarithmic expansion of steam.* Assume for initial data, quality 50 per cent, initial pressure 60 lbs. and final pressure 14.7 lbs. sq.in. absolute. From the steam tables $V_r = 3.585$ cu.ft., $T_r = 752.7°$, $\phi_r = \phi_{32} = 1.0352$. The point T is located as to pressure from the data, as to temperature, from the steam tables and as to volume, from the hypotheses,

$$V_t = \frac{P_t V_t}{P_t} = \frac{60 \times 3.585}{14.7} = 14.62 \text{ cu.ft.}$$

The quality at point $T = \frac{14.63}{26.79} = .546$ and from the steam tables $\phi_o - \phi_{32} = .3118$, $\phi_e - \phi_o = 1.4447$, whence $\phi_t - \phi_{32} = 1.4447 \times .546 + .3118 = 1.1006$.

The curve HCM is drawn between the same pressures, but from an initial quality of 95 per cent. From the steam tables $V_h = 6.812$, $\phi_h - \phi_{32} = 1.5824$ and $V_m = \frac{P_h V_h}{P_m} = \frac{60 \times 6.812}{14.7} = 27.8$ cu.ft. But dry saturated steam at the low pressure has a specific volume of $V_e = 26.79$, so that the expanding steam has become superheated. The amount of superheat may be estimated from the Marks and Davis superheat tables by seeking the steam temperature for this pressure having a specific volume of 27.8, or it may be estimated by the perfect gas law $\frac{T_m}{T_e} = \frac{27.8}{26.79}$. In this way it was found that $T_m = 695°$ and $\phi_m - \phi_{32} = 1.7723$.

Prob. 1. A tank containing 500 cu.ft. of compressed air at 60° F. and twenty atmospheres pressure is drawn upon to supply a compressed-air engine. After the pressure has fallen to five atmospheres without heat exchange, what is (*a*) the air temperature (*b*) the volume of atmospheric air measured at 32° F. withdrawn; (*c*) the work that has been done if all air withdrawn completely expanded; (*d*) the heat necessary to raise the remaining air to the original temperature?

Prob. 2. If the working gases having $\gamma = 1.4$ in a gas-engine cylinder expand according to the law $PV^{1.3} = \text{constant}$, the cylinder having a clearance of 15 per cent of its displacement, 20-inch bore and 30-in. stroke, initial pressure of 400 lbs. per square inch absolute and temperature of 2500° F., find (*a*) the final pressure and temperature, (*b*) the work done by expansion alone, and (*c*) the heat gained or lost during the expansion, (*d*) the net work of the cycle consisting of adiabatic compression from one atmosphere, combined with the above expansion and two constant-volume lines.

Prob. 3. Steam at 200 lbs. per square inch absolute and 150° of superheat adiabatically expands to 100 lbs., then receives 100 B.T.U. at constant pressure and again expands to atmosphere. Find, (*a*) the quality after the first expansion and before and after the second, (*b*) the work done in each expansion, thermally, (*c*) the value of s for each expansion, (*d*) the work done in each expansion by PV methods compared with that found thermally.

Prob. 4. Ammonia is being compressed adiabatically from 30 lbs. to 175 lbs. per square inch absolute. What quality must it have originally to have the final condition of, (*a*) all liquid, (*b*) 50 per cent vapor, (*c*) dry saturated vapor? (*d*) What will be its superheat if originally it were dry saturated?

Prob. 5. Find the value of s for each of the cases of Problem 4 and determine the work of compression alone and of the whole cycle consisting of low-pressure evaporation preceding compression, high-pressure condensation and complete liquid cooling to low temperature, using both PV and $T\phi$ methods.

Prob. 6. Plot to PV and $T\phi$ coordinates compression of 80 per cent quality ammonia and carbon dioxide from 0° F. to 70° F., according to the law $PV = \text{Const.}$, and find (*a*) how much heat must be added or abstracted per pound, by evaluation of $T\phi$ areas and (*b*) the work done by both PV and thermal methods.

Prob. 7. Steam in a cylinder has an initial pressure of 100 lbs. per square inch absolute and a quality of 90 per cent. It expands logarithmically to 30 lbs., is reheated to original temperature and then expands adiabatically in a turbine to 1 lb. per square inch absolute. Plot the $T\phi$ and PV diagrams and find (*a*) the heat added during logarithmic expansion and reheating, (*b*) the quality before and after adiabatic expansion, (*d*) the work done by expansion alone, in each stage checking calculations by diagram areas.

Prob. 8. Using the thermal method entirely compare the work of admission, compression and delivery for CO_2 and NH_3 compressors per pound of vapor originally of quality 75 per cent, between −20° F. and 75° F.

Prob. 9. On a $T\phi$ diagram plot the constant volume and constant quality lines for CO_2 and NH_3, starting with superheated vapor at 300° F. and the pressure corresponding to saturated vapor at 75° F., ending the lines at 0° F.

Prob. 10. For the steam cycle consisting of liquid heating, evaporation at 150 lbs. per square inch absolute, superheating 100°, adiabatic expansion to 2 ins., Hg absolute, and constant pressure condensation, (*a*) plot the PV and $T\phi$ diagrams, (*b*) find the work done by the five thermal methods and compare with the PV area, (*c*) modify the diagram by assuming adiabatic expansion to end at 50 lbs. per square inch absolute, closing with a constant volume line, and find the work so lost by $T\phi$ and PV methods.

Prob. 11. If the steam of the first part of Problem 10 expanded through an orifice and impinged on a plate, what would be its final quality and velocity if (*a*) its final velocity were unchanged by impact, (*b*) reduced to half, (*c*) reduced to zero.

5. Thermal Cycles Representative of Heat-engine Processes. Cyclic Efficiency. A Reference Standard for Engines and Fuel-burning Power Systems. Classification of Steam Cycles. In practice, heat conversion into work has narrowed down to a comparatively few processes out of the infinite number possible, considering all kinds of substances as possible *heat carriers* or work *executors*, and all possible states of each substance and the various modes of heat addition, abstraction, compression and expansion, that may constitute the thermal phases making up complete thermal cycles. The sub-

stances of importance are only two in general, first, water-steam, and second, a substantially perfect gas, which latter may be superheated steam or any other vapor, or a mixture of any number of so called permanent gases behaving as one. The processes through which water steam may pass, will, when studied yield certain principles of heat treatment of the fluid, fundamental to the steam-engine performance, while similarly basic ideas with regard to the gas engine follow from the same study of the processes through which gases may pass. There will result from such analysis of gas and vapor cycles, so called, a definite idea of the limiting possibility of the amount of conversion or maximum possible efficiency, and it will appear that the efficiency depends on how the heat is added and abstracted, how the substance changes state, how compression and expansion are related to each other and to the heating and cooling, how much heat is added per pound of substance, and the total range of variation of pressure, volume and temperature. In short, the efficiency can be found for any substance passing through any cycle and will be found to be not the same for all, sometimes quite high, often very low. Therefore, assuming, as is proper, that the object of engineers dealing with power generation is to convert as much heat into work as is possible or practicable, this cyclic analysis will show what it is good to do, and what thermal actions must be avoided. It will, moreover, establish a numerical limit to the possible conversion or efficiency for any one series of processes that may be embodied in a machine, with which limit, the actual performance of a machine may be compared, thus giving a measure of the perfection of the mechanism as an executor of the cycle and showing how far improvements may be carried. For example, for given conditions of pressure and temperature, it may be found that a given steam engine is converting into work 20 per cent of the heat supplied to it in the form of steam, and this may look very low and great improvements appear to be possible, whereas on examination of the processes being carried out within the pressure and temperature limits imposed, it may be found that not more than 25 per cent efficiency is possible in the most perfect mechanism, in which case the mechanism may be regarded as $\frac{20}{25}=80$ per cent perfect and no amount of ingenuity or expense can make it very much better than it is. This is a good illustration of the fact that the efficiency of a power generator is limited by two things, or is the product of two other efficiencies, first the efficiency of the thermal cycle being carried out by the substance and second, the efficiency of the mechanism as a cycle executor. The former is conveniently termed the *cyclic* or *system efficiency* and the second the *mechanism* efficiency, and this latter must not be confused with mechanical efficiency, which measures only mechanical friction losses, whereas the former measures all energy losses in the mechanism not fundamental or necessary to the cycle itself.

The great contribution of the cyclic analysis to engineering is, therefore, the establishment of *standards* of performance that serve as guides to improvement, telling clearly and unmistakably when one cycle or system must be abandoned in favor of another to attain a desired end, or where and how to

perate in the reduction of losses to improve a given piece of mechanism. With his as a criterion, it is evident that only those cycles that may be carried out by mechanism are worth studying, but a little caution is necessary here because someone may discover to-morrow how a given promising cycle, to-day considered practical impossibility, may really be carried out and so a new power system be created. It is, therefore, of importance that this thermodynamic study of efficiency of conversion be so extended as to show what processes will yield high efficiencies, higher than the processes now considered good practice, so as to point the way for the mechanism designer that he may concentrate his energies in the clothing in metal of a process that promises better results than those in use. Cyclic analysis then has two criterions by which the *worth while* cycles are to be selected, first, their present practicability, and second, the high efficiency promised, realization of which depends on discovery of suitable mechanism to carry them out. No better illustration can be given than is supplied by the development of the gas engine in the last half century. As originally built it was just a novelty, about the possibilities of which no one had any definite ideas: then thermodynamic study of its cycle showed two things: first, that its possible efficiency was very high, compared to the steam systems then in use, and second, that in all gas engines for efficient conversion, the cycle must include a compression of the working gases before heating. Both the higher possible efficiency with respect to steam and the necessity for precompression were known before their realization in gas engines of good construction and competitive size, and this knowledge was the real incentive to practical development. Not only is this so, but as higher efficiency began to be realized in gas engines than had been the rule with steam, so, as a direct consequence, was the improvement of the steam system stimulated, every loss carefully studied as never before, and means taken to reduce each, so that to-day in large-sized plants, especially with high pressure reciprocating and low-pressure turbine units, with superheated steam, feed-water heaters, economizers and high-vacuum condensing apparatus, the efficiency of the steam and gas system on the same coal fuel are substantially the same, and not less than twice as good as was considered satisfactory when the gas engine began its stimulating career.

With this preliminary survey, the cycles for analytical study may be selected from among the great number of possibilities which are, of course, greater for gases than for steam, since in the latter case there is only one practical way of adding heat, that at constant pressure and temperature if the liquid heating and vapor superheating be excepted. It would perhaps be better to say that the bulk of the heat in the steam system or in fact any vapor system must necessarily be added at constant pressure and temperature. Heat abstraction likewise is essentially a constant-pressure and temperature process for vapor systems if the expansion is complete, and in the establishment of standards of possibility there is no reason for dwelling on any but complete expansion because this is always realizable in engines if really desired. Variation in vapor cycles one from the other can come in for consideration mainly as due, not to differences in modes of heat addition or abstraction directly, but rather from the differ-

ences in expansion between primary heating and heat abstraction, and in com pression or its equivalent between heat abstraction and primary heating. Th use of the term primary heating indicates that there may be secondary or minc heating in vapor cycles and this is a convenient way of distinguishing betwee the bulk of the heat received at constant temperature always to be considere as primary, and that received otherwise and to be considered as secondary minor. To illustrate, the primary heat added will, of course, be the late heat of vaporization of so much of the fluid as is vaporized if it is left wet. Th secondary heat will be, (*a*) heat of liquid if liquid heating is part of the cycl (*b*) heat of superheat if the vapor is superheated at the pressure of steam gener tion before expansion begins; (*c*) heat added during expansion when it is n adiabatic, as for example constant quality logarithmic, or constant temperatu for superheated vapor.

With complete condensation of vapor as a result of heat abstraction at co stant pressure and temperature, heating of liquid is necessarily one phase the vapor cycles, but should the heat abstraction or condensation cease befc condensation is complete and the remaining vapor be adiabatically compress so that it just liquefies on reaching the high pressure and temperature whi corresponds on a $T\Phi$ diagram to a vertical line through the condition of hig temperature liquid, then there will be no liquid heating.

All vapor cycles can be divided into two groups with respect to the mode transition from condensation to vaporization phases; the first group will inclu those that completely condense and so involve liquid heating, and the seco those that substitute complete adiabatic compression for liquid heating. course, there may be cycles with part of one and the rest of the other proce but as their characteristics will lie between these two limits they need not separately studied in a general investigation like this.

Expansion may be adiabatic or may not, but if not, it must be accordi to some law for which there are algebraic relations, otherwise algebraic cyc analysis is impossible and without this, principles are difficult to devise, thou concrete problems may be solved. The only case of expansion that is of pr tical interest and yet yields even approximately to algebraic analysis is that constant quality, that is, for steam initially wet, with constant wetness dur expansion, for steam initially dry and saturated, constantly so during expansi and for steam initially superheated, with constant superheat during expansi These cases of expansion for various constant qualities of steam are usually c sidered as nearly representative of the possible action of steam in jacketed cyl ders, and the heat added during expansion is supposedly representative of the h lost by the jacket steam and gained by the cylinder steam at the rate requi for constant quality. As a matter of fact it is very doubtful if jackets co so add heat to cylinder steam, and while they might give to it the total amo equivalent to constant-quality expansion it is pretty sure that the cylin steam even if it received this amount of heat would not receive it at the r corresponding to constant quality. To just about the same degree does the c stant-quality expansion represent the skin friction heating effect of nozz

part of the work developed and appearing as kinetic energy is converted back into heat by velocity reduction. These cycles with constant-quality expansion are not of sufficient practical importance compared to those with adiabatic expansion to warrant equal treatment and while they will be indicated at the end of this section they will not be subjected to analysis.

Adiabatic expansion is by all odds the most important case, because it is typical of heat addition completed before expansion begins, and heat abstraction delayed until expansion is over. This is what would happen in cylinder engines if the walls were non-conductors of heat and did not absorb heat from the steam on steam admission or return it when the steam temperature dropped, and also what would happen in steam turbines if the nozzles were non-conductors of heat and the surfaces in contact with the steam frictionless. Comparison of cycles, otherwise the same, in which expansion is adiabatic with those in which quality is constant will show whether it is better to add all the heat before or continue addition during expansion and this will be taken up later. Of course, the adiabatic expansion may

(*a*) Start with wet and end with wet steam;
(*b*) Start with dry saturated and end with wet steam;
(*c*) Start with superheated and end with wet steam;
(*d*) Start with superheated and end with dry saturated steam;
(*e*) Start with superheated and end with superheated steam;

so there may be five cases of adiabatic expansion cycles, *each differing from the other fundamentally in the amount of heat added per pound of steam at the same pressure.*

From the preceding it appears to be possible to conveniently divide the steam cycles worthy of study into four, each with special cases for various degrees of initial quality or heat added per pound of steam at the high pressure, these four being the possible combinations of, (*a*) zero compression, (*b*) complete adiabatic compression; with (*c*) adiabatic expansion, and (*d*) constant quality expansion defining the typical phases, and represented by the corresponding diagrams of Fig. 236 to both $T\Phi$ and PV coordinates. The relations of these various lines should be clear from the explanations given in the analysis of various phases and to make them clear to the eye, the saturation curve is extended across each diagram as a line of reference.

Referring to the PV diagram of Cycle I, it will be noted that vaporization B to C, or C' whether complete or incomplete, and including superheating C' to C^{IV} if there is any, is represented by the same sort of line as admission of steam to a cylinder, in which the initial volume V_b is that of the liquid from which the amount of steam admitted is formed and which is so small as to be negligible in numerical work but is retained here to make the cycle operation clear. Similarly, the condensation line D''', D'', D', D to A, preceded by loss of superheat D^{IV} to D''' if there is any, is represented to PV coordinates, exactly as would be a constant-pressure exhaust. Therefore, although in cylinders there

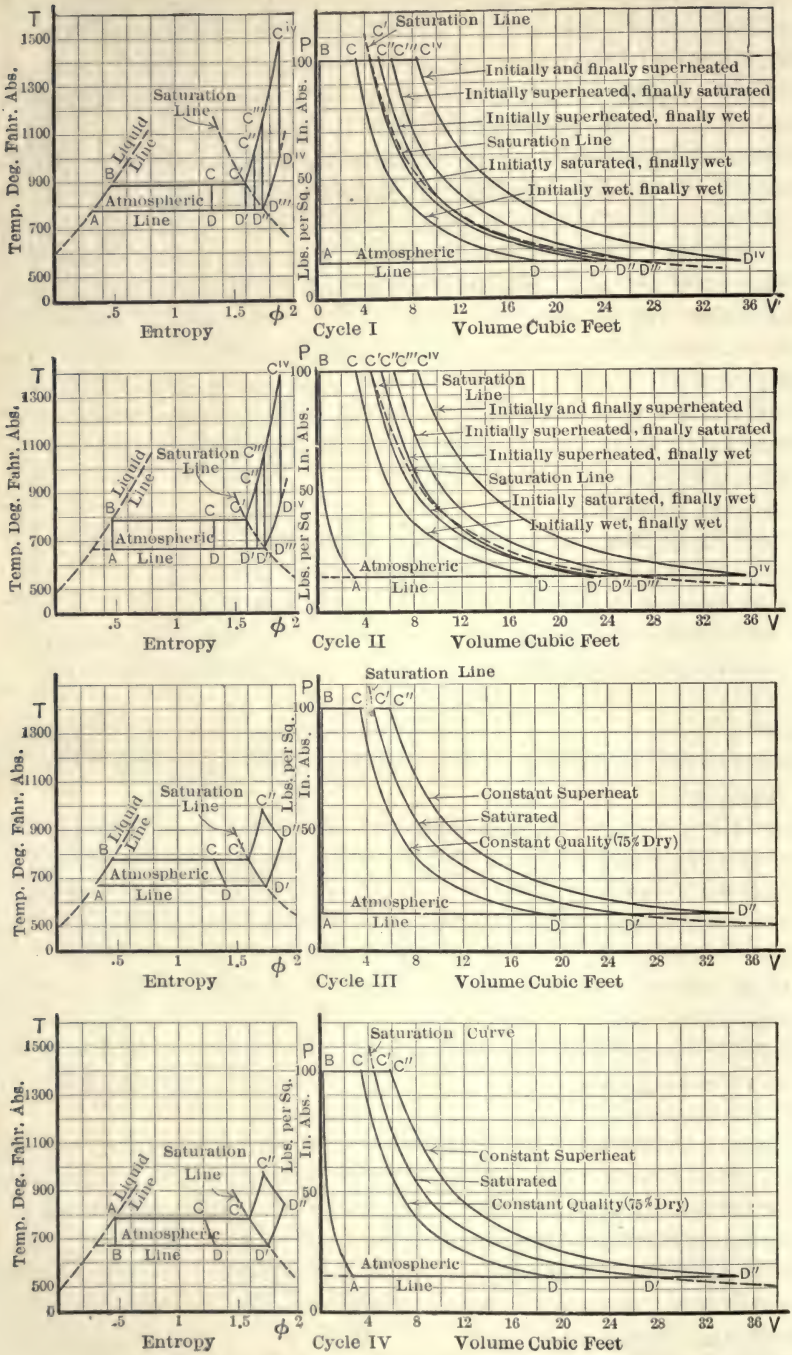


FIG. 236.—Steam Cycles.

ccur admission and exhaust, the efficiency of heat conversion can be studied y imagining as substitutes, constant-pressure heat addition or abstraction, hich is equivalent to imagining the operations which take place in a whole lant from boiler-feed, steam-making, cylinder action, condensation in condens-rs, and return of condensate to boiler, to take place completely in the cylinder. here are no unwarranted assumptions involved in this substitution of equiva-nt thermal phases, as it is obvious that the cylinder will do the same work n either case and there will be necessary the same amount of heat to make possible. However, as ordinarily carried out in separate apparatus these arious processes involve some losses and some compensations, losses for example n pipe lines, and compensations from flue gas and exhaust steam heat. The ompensations do not change the total heat required by the steam to put it n condition, but this amount is not all derived from fresh fuel when waste heats re partially returned.

This same diagram represents in a similar way the action in a steam turbine lant in which the whole process is imagined as concentrated in the nozzles of he turbine, generation and superheating of steam represented by B to C, C', C'', ^{IV} does work represented by the area under this line to PV coordinates, and xpansion does more work represented by the area under the expansion line, ut in escaping from the nozzle this work is done only by pushing away some team at the low pressure, which involves negative work under the low or back-ressure line or its thermal equivalent, the condensation line. The net work s the area enclosed by the PV diagram, as for cylinders, and is equal to the rea enclosed by the $T\Phi$ diagram of the equivalent thermal processes.

All sorts of minor modifications of these cycles can be studied when special ases require it, as could also other cycles, for example, incomplete expansion ollowed by constant-volume release would be illustrated by Fig. 237 to $T\Phi$ nd PV coordinates for various degrees, ranging from no expansion at all to omplete expansion. This diagram otherwise represents the conditions of ycle I for dry steam and is inserted merely to call attention to the possibilities f cyclic modification, of which there are many others that will suggest them-elves to anyone.

Some of these cycles adopted as representative and worthy of analytical tudy have received names associating them with some great man who studied hem either first or most effectively. Thus the name Rankine is applied to the ycle I, which is characterized by adiabatic expansion and no compression, and he name Carnot to those cases of Cycle II, that are without superheat and which re characterized by adiabatic compression and expansion, with heat added all t the high, and abstracted all at the low temperature. To other cases there s either no name applied or there is no agreement as to the name, so where onfusion is likely to result from the application of a name to a cycle which lsewhere is applied to some others, the name will be omitted. There is abso-ute agreement as to the meaning of the Carnot cycle, and fairly general though ot absolute acceptance of the name Rankine as signifying the cycle above efined, though some writers limit this name to the case of initially dry saturated

steam, to which others apply the name Clausius. As the adiabatic-expansion and no-compression cycle is so closely representative of actual steam practice, it must be frequently referred to, and this will be called the Rankine cycle for wet, dry or superheated steam as a better term than Cycle I. In all other cases the cycle will be defined by its phases or number when discussed.

The following analysis of these cycles will all be based on one pound of substance of which varying amounts will be in the vapor and in the liquid states, and *it will be assumed that when there is such a mixture it is of the same temperature throughout or thermally homogeneous.* In cylinders it is certain that moisture

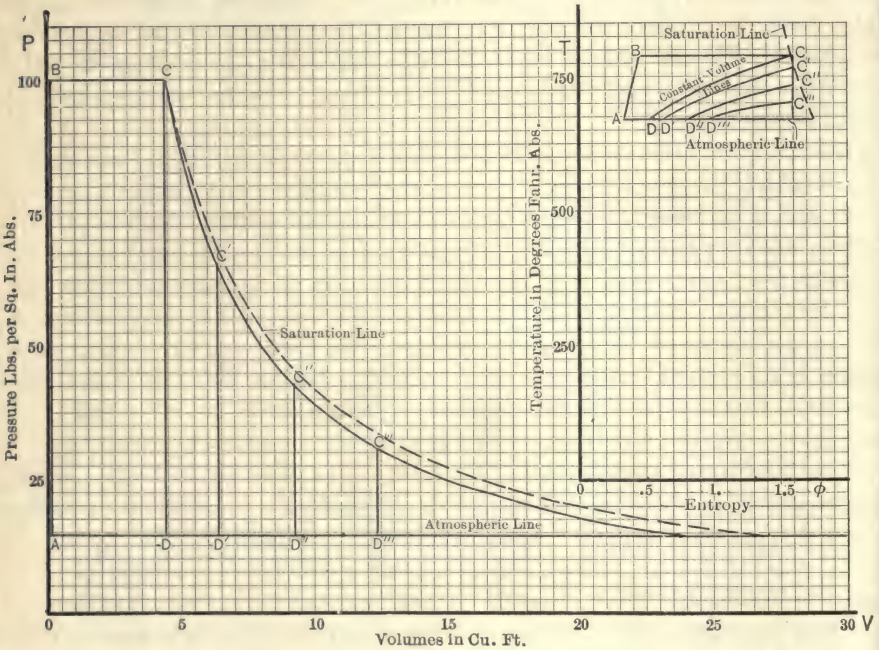


FIG. 237.—Modification of Complete Expansion Steam Cycle by Incomplete Expansion with Constant Volume Closure Lines.

collects next the walls and is cooler than the steam, and if superheated steam or gases are in cylinders that part near the walls will not have the same temperature as the rest of the mass, which is also the case in nozzles, as the heat of friction is imparted first to the skin fluid and may or may not be later communicated to the rest by conduction diffusion or mechanical mixture.

Another assumption that is necessary is that of constancy of specific heat of water at unity, and of superheated steam at a mean value for the temperature range but not necessarily the same for different pressures or temperatures.

STEAM CYCLE I (Rankine).

First phase, from *A* to *B*. Heating liquid (substantially constant volume).

Second phase, from B to C. Heat addition at constant pressure, vaporizing at constant temperature and possibly followed by superheating of vapor at rising temperature.

Third phase, from C to D. Adiabatic expansion.

Fourth phase, from D to A. Heat abstraction at constant pressure, condensation.

EAM CYCLE II (Carnot).

First phase, from A to B. Adiabatic compression of vapor and liquid to all liquid.

Second phase, from B to C. Heat addition at constant pressure, vaporizing at constant temperature and possibly followed by superheating of vapor at rising temperature.

Third phase from C to D. Adiabatic expansion.

Fourth phase from D to A. Heat abstraction at constant pressure condensation.

EAM CYCLE III.

First phase, from A to B. Heating liquid (substantially constant volume).

Second phase, from B to C. Heat addition at constant pressure, vaporizing at constant temperature and possibly followed by superheating of vapor at rising temperature.

Third phase, from C to D. Expansion with constant quality.

Fourth phase, from D to A. Heat abstraction at constant pressure, condensation.

EAM CYCLE IV.

First phase, from A to B. Adiabatic compression of vapor and liquid to all liquid.

Second phase, from B to C. Heat addition at constant pressure, vaporizing at constant temperature and possibly followed by superheating of vapor at rising temperature.

Third phase, from C to D. Expansion with constant quality.

Fourth phase, from D to A. Heat abstraction at constant pressure, condensation.

Prob. 1. Plot to scale to PV and $T\Phi$ coordinates steam Cycle I modified by expan- in two stages, the first extending for half the temperature range, and with eating in the receiver to the initial quality.

Prob. 2. Plot to scale steam Cycle I, modified by logarithmic expansion.

Prob. 3. Plot to scale steam Cycle I modified by logarithmic expansion in two ges as in Problem 1 but with reheating to initial temperature.

Prob. 4. Plot to scale steam Cycle II modified as in Problem 1.

Prob. 5. Plot to scale steam Cycle II modified by logarithmic expansion in the first ge for half the pressure range and cooling in the receiver to a quality of 90 per cent llowed by adiabatic expansion in the last stage.

Prob. 6. Modify Cycle II by two-stage compression adiabatic with intercooling and three-stage adiabatic expansion with reheating to original temperature. Intercooling to begin at half the temperature range and to completely condense the steam reheating to be at $\frac{1}{3}$ and $\frac{2}{3}$ the temperature range and to just dry the steam.

Prob. 7. Modify steam Cycles II and III by two-stage expansion, dividing the temperature range equally, the first to be at constant quality as in a jacketed cylinder and the second adiabatic as in a low-pressure turbine.

Prob. 8. Modify steam Cycles I and II by stopping adiabatic expansion at half the pressure range, closing by a constant-volume line.

Prob. 9. Plot to scale the following cycle: Water heated under pressure sufficient to prevent vaporization at 1000 lbs. per square inch absolute, the pressure being 10 lbs. per square inch higher than saturated vapor pressure. Hot water allowed to escape through a valve to a receiver at 100 lbs. per square inch absolute, there brought to rest and adiabatically expanded to atmospheric pressure. Condensation from and at 200° F. followed by return of water to high-pressure tank and there again heated.

6. The Rankine Cycle. Work, Mean Effective Pressure, Jet Velocity, Water Rate, Heat Consumption and Efficiency of Steam Cycle I. Adiabatic Expansion, Constant-pressure Heat Addition and Abstraction, No Compression.

Let Q_1 = heat added in B.T.U. per lb. for whole cycle;

" Q_2 = heat abstracted in B.T.U. per lb. for whole cycle;

" W = work in ft.-lb. per lb. for whole cycle:

" $\left\{\begin{matrix} P_c \\ V_c \\ T_c \\ x_c \\ (\phi_c - \phi_{32}) \end{matrix}\right\}$ = { pressure volume temperature quality and entropy above 32° F., for steam in condition represented by position of point C and with corresponding subscript for other points. Pressures in lbs. sq. ft., volumes in cu.ft., temperature absolute Fahrenheit, quality is ratio of weight of dry saturated steam to total steam and water weight.

" $\left\{\begin{matrix} q_1 \\ L_1 \\ h_1 \\ H_1 \end{matrix}\right\}$ = { heat of liquid, latent heat, heat of superheat and total heat above 32°, per lb. of dry steam, tabular values at the high temperature and with subscript (2) for low temperature.

" $S_{c'c''}$ = constant mean specific heat of superheated steam from $T_{c'}$ to $T_{c''}$.

Then since the change in intrinsic energy is zero for a complete cycle, the work done is equal to the heat change or the difference between heat put in and that abstracted for the whole cycle, which is algebraically expressed in Eq. (960):

$$W = J(Q_1 - Q_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (960)$$

As the efficiency of conversion of heat into work is the ratio of work done to heat supplied in the same units it is given by Eq. (961):

$$E=\frac{W}{JQ_1}=\frac{Q_1-Q_2}{Q_1}=1-\frac{Q_2}{Q_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (961).$$

Both these expressions, Eq. (960) and (961), are perfectly general and true for any cycle, differences entering only when the heats supplied or abstracted are evaluated in accordance with the particular specifications for the cycle. For this cycle, Fig. 238, there are two forms of expression for the heats

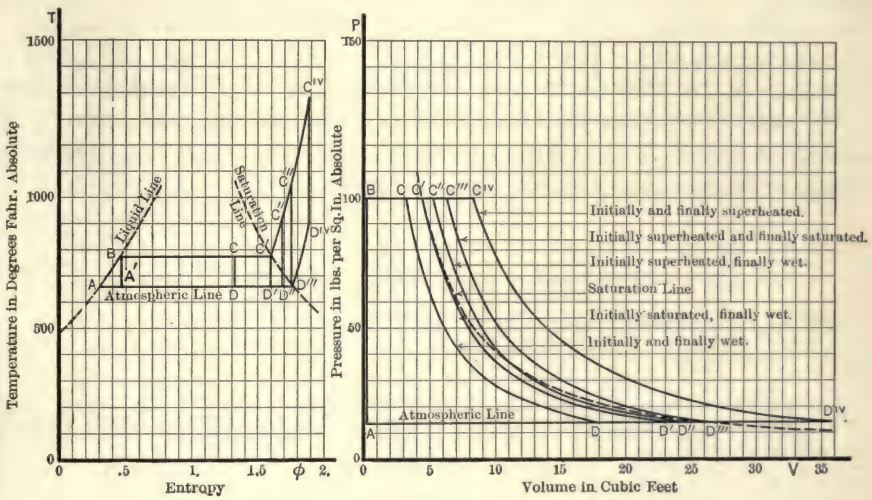


FIG. 238.—Rankine Steam Cycle.

per pound, one, when superheat is initially present and the other when it is not, given as follows by Eq. (962),

$$\left.\begin{array}{ll} Q=q+xL=H-(1-x)L & \text{for saturated steam of quality } x \quad (a) \\ Q=q+L+h=H+h & \text{for superheated steam} \quad (b) \end{array}\right\} \quad . \quad . \quad . \quad (962)$$

These expressions will apply to all the special cases of this cycle which arise from different initial qualities and changes of quality due to adiabatic expansion and lettered (*a*), (*b*), (*c*), (*d*), (*e*) in the last Section.

Case a. Diagram ABCD, Fig. 238, *steam initially wet, finally more wet.*

$$Q_1=q_1-q_2+x_eL_1,$$

$$Q_2=x_dL_2.$$

Therefore

$$W = J(q_1 - q_2 + x_c L_1 - x_d L_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (963)$$

$$E = 1 - \frac{x_d L_2}{q_1 - q_2 + x_c L_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (964)$$

These two expressions Eqs. (963) and (964) for work and efficiency do not contain all independent variables because the quality x_d is a function of the original quality x_c, and of the extent of the adiabatic expansion. If, tables or charts are available, all these quantities may be read off and the answer obtained at this point. It is, however, desirable that this as well as similar expressions for other cycles be reduced to a function of temperature or other fundamental variable so that they may be compared without proceeding to numerical substitution. It has been shown in Section (4), Eq. (948) that for two points on the same adiabatic, the product of low-temperature quality, into the entropy of complete vaporization, is equal to the same for the high temperature added to the entropy change of the liquid from low temperature to high, or for this case, Eq. (965) and Eq. (966),

$$x_d(\phi_{d'''} - \phi_a) = x_c(\phi_{c'} - \phi_b) + (\phi_b - \phi_a). \quad . \quad . \quad . \quad . \quad . \quad (965)$$

$$x_d L_2 = \frac{T_a}{T_b} x_c L_1 + T_a \log_e \frac{T_b}{T_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (966)$$

Substitution of Eq. (966) in Eq. (963) gives two forms, Eq. (967) and Eq. (968) for work,

$$W = J\left[q_1 - q_2 + x_c L_1\left(1 - \frac{T_a}{T_b}\right) - T_a \log_e \frac{T_b}{T_a}\right], \quad . \quad . \quad . \quad . \quad . \quad (967)$$

$$= J\left[(T_b - T_a) + x_c L_1\left(1 - \frac{T_a}{T_b}\right) - T_a \log_e \frac{T_b}{T_a}\right] \text{(approx.)}$$

$$= J\left[(T_b - T_a)\left(1 + \frac{x_c L_1}{T_b}\right) - T_a \log_e \frac{T_b}{T_a}\right] \text{ (approx.)} \quad . \quad . \quad . \quad (968)$$

The first form, Eq. (967), gives work, in terms of heats of liquid, temperatures, initial quality, and corresponding latent heat, and the second form Eq. (968) gives work in terms of the temperatures, the high-pressure latent heat and the initial steam quality on the assumption of unity as the specific heat of water. By putting the latent heat as a series function of temperature as in Eq. (952), Section 4, and substituting it in the last form for work, Eq. (968), a third form results, Eq. (969) in terms of initial steam quality quality and temperatures,

$$W = J\left\{(T_b - T_a)\left[1 + x_c\left(\frac{\alpha}{T_b} + \beta + \gamma T_b\right)\right] - T_a \log_e \frac{T_b}{T_a}\right\} \text{ (app.)} \quad . \quad (969)$$

he efficiency may also be put in terms of temperatures, initial latent heat id quality, or entirely in terms of temperatures and initial quality by similar bstitution. The first form results from the substitution of Eq. (966) in Eq. 64), giving Eq. (970), which is exact, and Eq. (971), which is approximate, cause the specific heat of water is assumed in it, to be unity.

$$E=1-\left[\frac{\frac{T_a}{T_b}x_cL_1+T_a\log_e\frac{T_b}{T_a}}{q_1-q_2+x_cL_1}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (970)$$

$$=1-\left[\frac{\frac{T_a}{T_b}x_cL_1+T_a\log_e\frac{T_b}{T_a}}{(T_b-T_a)+x_cL_1}\right](\text{app.}) \quad . \quad . \quad . \quad . \quad . \quad (971)$$

he first form Eq. (970) gives the efficiency in terms of liquid heats, tempera- ires, initial quality and corresponding latent heat, which reduce in Eq. (971) temperatures, initial quality and corresponding latent heat, and in Eq. (972) a final modification to temperatures and initial quality only, by introducing ie temperature function of latent heat,

$$E=1-\left[\frac{\frac{T_a}{T_b}x_c(\alpha+\beta T_b+\gamma T_b^2)+T_a\log_e\frac{T_b}{T_a}}{(T_b-T_a)+x_c(\alpha+\beta T_b+\gamma T_b^2)}\right](\text{app.}) \quad . \quad . \quad (972)$$

these equations for work and efficiency,

$$\alpha=1151.0;$$
$$\beta=.076;$$
$$\gamma=-.00051.$$

Case b. Diagram ABA', Fig. 238. *No steam initially present.* If the initial ondition is that of water at the boiling-point, $x_c=0$, and the three expressions r work and efficiency take the form Eqs. (973) and (974),

$$\left.\begin{aligned} W&=J\left[q_1-q_2-T_a\log_e\frac{T_b}{T_a}\right] \text{ from Eq. (967)} \quad (a)\\ &=J\left[T_b-T_a-T_a\log_e\frac{T_b}{T_a}\right] \text{ from Eq. (968) or (969)} \quad (b)\end{aligned}\right\} \quad . \quad . \quad (973)$$

$$\left.\begin{aligned} E&=1-\left[\frac{T_a\log_e\frac{T_b}{T_a}}{q_1-q_2}\right] \text{ from Eq. (970)} \quad (a)\\ &=1-\left[\frac{T_a\log_e\frac{T_b}{T_a}}{T_b-T_a}\right] \text{from Eq. (971) or (972)} \quad (b)\end{aligned}\right\} \quad . \quad . \quad (974)$$

Case c. Diagram ABC'D'A, Fig. 238. *Steam initially dry saturated.* If the initial condition is that of dry saturated steam, $x_c = 1$, and the work and efficiency equation take the following special forms, Eqs. (975) and (976),

$$\left.\begin{aligned} W &= J\left[q_1 - q_2 + L_1\left(1 - \frac{T_a}{T_b}\right) - T_a \log_e \frac{T_b}{T_a}\right] \text{ from Eq. (967)} \quad (a)\\ &= J\left[(T_b - T_a)\left(1 + \frac{L_1}{T_b}\right) - T_a \log_e \frac{T_b}{T_a}\right] \text{ from Eq. (968)} \quad (b)\\ &= J\left\{(T_b - T_a)\left[1 + \left(\frac{\alpha}{T_b} + \beta + \gamma T_b\right)\right] - T_a \log_e \frac{T_b}{T_a}\right\} \text{ from Eq. (969)} \quad (c) \end{aligned}\right\} . \quad (975)$$

$$\left.\begin{aligned} E &= 1 - \left[\frac{\frac{T_a}{T_b}L_1 + T_a \log_e \frac{T_b}{T_a}}{q_1 - q_2 + L_1}\right] \text{ from Eq. (970)} \quad (a)\\ &= 1 - \left[\frac{\frac{T_a}{T_b}L_1 + T_a \log_e \frac{T_b}{T_a}}{T_b - T_a + L_1}\right] \text{ from Eq. (971)} \quad (b)\\ &= 1 - \left[\frac{\frac{T_a}{T_b}(\alpha + \beta T_b + \gamma T_b^2) + T_a \log_e \frac{T_b}{T_a}}{(T_b - T_a) + (\alpha + \beta T_b + \gamma T_b^2)}\right] \text{ from Eq. (972)} \quad (c) \end{aligned}\right\} . \quad (976)$$

Case d. Diagram ABC'C''D''A, Fig. 238. *Steam initially superheated and finally wet.* Steam initially superheated to a degree indicated by condition C'', that is, such as will allow it to become wet during expansion, will have different work and efficiency equations, the general form of which is given in Eqs. (977) and (978),

$$W = J(q_1 - q_2 + L_1 + h_1 - x_{d''}L_2). \quad . \quad . \quad . \quad . \quad . \quad (977)$$

$$E = 1 - \frac{x_{d''}L_2}{q_1 - q_2 + L_1 + h_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (978)$$

As before, the final quality $x_{d''}$ is not an independent variable, but related to initial superheat by the equal entropy relations,

$$(\phi_{d''} - \phi_a) = (\phi_{c''} - \phi_a),$$

and

$$x^{d''}(\phi^{d'''} - \phi_a) = (\phi_{c''} - \phi_{c'}) + (\phi_{c'} - \phi_b) + (\phi_b - \phi_a),$$

or

$$x_{d''}\frac{L_2}{T_a} = S_{c'c''} \log_e \frac{T_{c''}}{T_b} + \frac{L_1}{T_b} + \log_e \frac{T_b}{T_a}.$$

$$\therefore \quad x_{d''}L_2 = S_{c'c''}T_a \log_e \frac{T_{c''}}{T_b} + \frac{T_a}{T_b}L_1 + T_a \log_e \frac{T_b}{T_a}. \quad . \quad . \quad . \quad (97$$

This Eq. (979) will on substitution in Eqs. (977) and (978) give the work and the efficiency in terms of completely independent variables which may be reduced entirely to temperatures, as the only two.

Case e. Diagram $ABC'C'''D'''A$, Fig. 238. *Steam initially superheated and finally dry saturated.* Steam with enough initial superheat to become just dry and saturated after expansion will have characteristics similar to the last case, but in which $x_{d''}$ becomes $x_{d'''}$, and $x_{d'''}=1$, which yield the final result of Eqs. (980) and (981).

$$W=J(q_1-q_2+L_1+h_1-L_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (980)$$

$$E=1-\frac{L_2}{q_1-q_2+L_1+h_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (981)$$

In these equations the heat of superheat, $h_1=S_{c'c'''}(T_{c'''}-T_b)$ is not an independent variable, being contingent on dryness without superheat after expansion, therefore, the temperature $T_{c'''}$ is fixed by the entropy relation that leads to Eq. (980) which, making $x_{d'''}=1$, and $T_{c''}=T_{c'''}$ becomes Eq. (982),

$$\frac{L_2}{T_a}=S_{c'c'''}\log_e\frac{T_{c'''}}{T_b}+\frac{L_1}{T_b}+\log_e\frac{T_b}{T_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (982)$$

However, the two cases of small superheat just reviewed will give work per cycle and efficiency values between those for the two cases of, initially no superheat, and initially high superheat high enough to leave the steam superheated after expansion, so that the equations of the two cases of small superheat are omitted and those for high superheat set down.

Case f. Diagram $ABC^{I}C^{IV}D^{IV}D^{III}A$. Fig., 238. *Steam constantly superheated.* When superheat is initially high enough to leave some superheat after expansion, however small the amount, the work and efficiency equations take the form of Eqs. (983) and (984).

$$W=J(q_1-q_2+L_1-L_2+h_1-h_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (983)$$

$$E=1-\frac{L_2+h_2}{q_1-q_2+L_1+h_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (984)$$

In these two equations the heats of superheat h_2 and h_1 are not independent, but related by their temperatures through the equal entropy relations,

$$\phi_{d^{IV}}-\phi_a=(\phi_{c^{IV}}-\phi_{a,})$$

or

$$S_{d'''d^{IV}}\log_e\frac{T_{d^{IV}}}{T_a}+\frac{L_2}{T_a}=S_{c'c^{IV}}\log_e\frac{T_{c^{IV}}}{T_b}+\frac{L_1}{T_b}+\log_e\frac{T_b}{T_a}. \quad . \quad . \quad . \quad (985)$$

From this Eq. (985) the relation between T_c^{IV} and T_d^{IV} can be found, which on substitution in Eq. (986), giving the final heat of superheat, in terms of the initial for substitution in Eqs. (983) and (984), will leave only independent variables.

$$h_2 = h_1\left[\frac{S_d'''_d{}^{IV}(T_a^{IV} - T_a)}{S_{c'}{}_c{}^{IV}(T_c^{IV} - T_b)}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (986)$$

This is, to be sure, a very cumbersome transformation and even so is only approximate, but the exact expression is infinitely worse as a working equation. It helps a little to assume a constant value for specific heat of superheated steam, $S_d'''_d{}^{IV} = Sc'_c{}^{IV}$ which gives Eq. (986), the form Eq. (987).

$$h_2 = h_1\frac{T_a}{T_b}\left[\frac{\frac{T_d^{IV}}{T_a} - 1}{\frac{T_c^{IV}}{T_b} - 1}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (987)$$

In this case it is not necessary to evaluate T_c^{IV} and T_d^{IV} from Eq. (969) but only the ratios $\left(\frac{T_d^{IV}}{T_a}\right)$ and $\left(\frac{T_c^{IV}}{T_b}\right)$, but even this is bad enough as a time consumer and will also be omitted to save space.

These methods of work and efficiency determination furnish a thermal means of deriving some other important quantities, whether W and E are calculated from equations such as have been developed here, or from the shorter, more exact, and, therefore, more practical means, provided by the tables and charts of properties as related to entropy.

Thus

$$\begin{pmatrix}\text{Mean effective pressure in pounds}\\ \text{per square inch}\end{pmatrix} = \left(\frac{W}{144 \times \text{(vol. per lb. at low press.)}}\right) \quad (988)$$

If V_2 be taken as the specific volume (tabular value) of dry saturated steam at the low pressure then this will have the two forms (*a*) and (*b*) of Eq. (989) which may be substituted in Eq. (988) for mean effective pressure. The second form is only approximate, as it assumes the superheated steam to behave as a perfect gas, while it does not, as may be seen by reference to Marks and Davis.

$$\left.\begin{aligned}&\text{(Vol. per lb. at low press.)} = V_2 x_2 \text{ if wet finally} \quad (a)\\ &= V_2\left(\frac{\text{Temp. of superheated}}{\text{Temp. of dry saturated}}\right) \text{if superheated} \quad (b)\end{aligned}\right\} \quad (98$$

Again, since in round numbers the horse-power of 33,000 ft.-lbs. per minute is identically equivalent to 2545 B.T.U. per hour, the hourly heat consumption of the cycle per horse-power is given by Eq. (990).

$$(\text{B.T.U. per hr. per I.H.P.}) = \frac{2545}{E} \quad . \quad . \quad . \quad . \quad . \quad (990)$$

Also if WR is the water rate or pounds steam per hour per horse-power

$$\left.\begin{aligned} (\text{Cyclic water rate}) = WR &= \frac{(\text{B.T.U. supplied per hr. per I.H.P.})}{(\text{B.T.U. per lb. steam})} && (a) \\ &= \frac{(\text{B.T.U. supplied per hr. per I.H.P.})}{\left(\begin{matrix}\text{B.T.U. per lb. st.}\\ \text{at high press.}\\ \text{above } 32^\circ \text{ F.}\end{matrix}\right) - \left(\begin{matrix}\text{Heat of liquid at}\\ \text{low press. above}\\ 32^\circ \text{ F.}\end{matrix}\right)} && (b) \\ &= \frac{2545}{E(H_1 - q_2)}. && (c) \end{aligned}\right\} \quad (991)$$

Steam generated continuously at the high pressure and issuing from a perfect nozzle to a region of the low pressure would have a velocity measured by the work done on it by its own expansion. Since its increase in kinetic energy from rest, is $\frac{1}{2}Mu^2 = \frac{w}{2g}u^2$, the work done per pound of steam w, where $w = 1$, will give to the pound of steam, the velocity in feet per second of Eq. (992).

$$u = \sqrt{2g \times W} = \sqrt{64.4 \times W} \text{ (app.)} \quad . \quad . \quad . \quad . \quad . \quad (992)$$

It is clear, therefore, that this maximum attainable jet velocity is directly proportional to the square root of the work done in foot-pounds or of the difference between the total heats per pound of steam at the conditions before and after adiabatic expansion. This fact prompts the paralleling of the heat scale on the total heat-entropy or Mollier diagram, Fig. 234, by a velocity scale computed as above.

Attention has already been called to the possibility of evaluation of the work of cycles of this sort by means of the temperature-entropy and the total heat-entropy diagrams, and this is the only practical method for engineering calculation, a fact that is appreciated only by solving problems first by the equation method and later by the tabular and chart methods. The Mollier diagram is constructed to read the differences $(Q_1 - Q_2)$ directly so that work in heat units is only a question of reading the scale, and efficiency a mere question of slide-rule division. Moreover, the operations are precisely the same for steam of any initial quality, ranging from liquid water to the highest superheat, and

changes in quality, superheat and volume are just as easily read off as the changes in total heat.

As the Rankine cycle is the only steam cycle that reasonably approximates in representation the action of steam in cylinders and nozzles, and for which work and efficiency can be read off the entropy charts, this is an additional reason why it has become the standard of reference for steam engine and turbine performance among engineers.

To make clear the great difference in labor involved in the solution of problems by tabular and by chart methods compared to the algebraic method, the following numerical examples are worked out, and it is important to

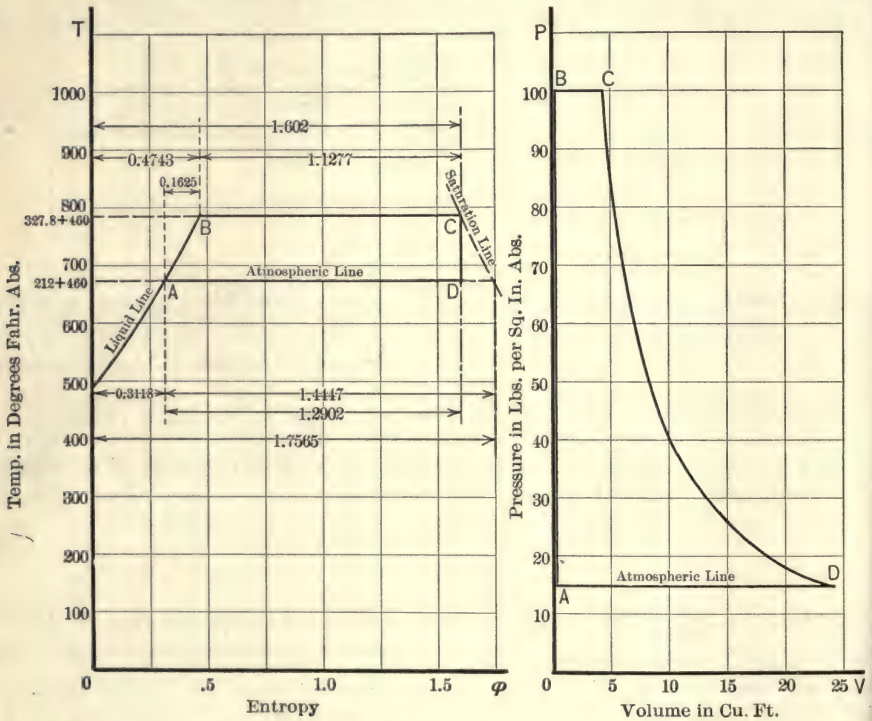


FIG. 239.—Example of the Rankine Steam Cycle.

remember that the table and chart method is more accurate, as well as shorter because it involves no assumption as to constancy of specific heats of wate or steam, nor as to the accuracy of temperature relations with latent heat all of which are necessary to secure even workable equations. In the curve at the end of this section are shown graphically the most important of th derived relations.

Required the work per pound of steam, the thermal efficiency, heat cor sumption, cyclic water rate and mean effective pressure for the Rankine cycl between 100 lbs. per square inch absolute and one atmosphere for steai initially dry. This case is illustrated by Fig. 239.

First Method. *By steam tables alone, without entropy data.*

m the tables

$T_1 = 327.8°$ F. $= 788°$ F. absolute (app.).
$T_2 = 212°$ F. $= 672°$ F. absolute (app.).
Heat of liquid (32° F. to 327.8° F.) = 298 B.T.U.
Heat of liquid (32° F. to 212° F.) = 180 B.T.U.
Heat of liquid (212° F. to 327.8° F.) = 118 B.T.U. A to B, Fig. 239.
Latent heat at 327.8° F. = 888 B.T.U. B to C, Fig. 239.
Latent heat at 212° F. = 970.4 B.T.U., A to X, Fig. 239.
Specific volume at 212° F. = 26.79 cu.ft.

Hence

Q_1 = Heat added = 888 + 118 = 1006 B.T.U.
Q_2 = Heat abstracted = final quality × 970.4 B.T.U.

$$\text{Final quality} = \frac{\overline{AD}}{\overline{AX}} = \frac{\phi_d - \phi_a}{\phi_x - \phi_a} = \frac{\phi_c - \phi_a}{\phi_x - \phi_a},$$

$$\phi_b - \phi_a = \log_e \frac{788}{672} = .1625,$$

$$\phi_c - \phi_b = \frac{888}{788} = 1.1277,$$

$$\phi_c - \phi_a = \phi_d - \phi_a = 1.1277 + .1625 = 1.2902,$$

$$\phi_x - \phi_a = \frac{970.4}{672} = 1.4447.$$

$$\text{Whence final quality} = \frac{1.2902}{1.4447} = .893,$$

d

Q_2 = Heat abstracted = 970.4 × .893 = 866.57, B.T.U.

$W = J(Q_1 - Q_2) = J(1006 - 866.57) = J(139.43) = 108476.5$ ft.-lbs.

$$\text{Efficiency} = \frac{139.43}{1006} = 13.8\%.$$

Final volume of steam = .893 × 26.79 = 23.92 cu.ft.

$$\text{m.e.p.} = \frac{J(139.43)}{144 \times .893 \times 26.79} = \frac{108476.5}{144 \times 23.92} = 31.5 \text{ lbs. sq.in.}$$

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E} = \frac{2545}{.138} = 18442$$

$$\text{Water rate of the cycle} = \frac{2545}{13.8 \times 1006} = 18.33 \text{ lbs.},$$

$$\text{Steam jet velocity} = \sqrt{64.4 \times 108476.5} = 2643 \text{ ft. per second.}$$

Second Method. By ordinary entropy tables.

From 32°F to 328°F entropy change for liquid = .4743
From 32°F to 212°F entropy change for liquid = .3118

From 212°F to 328°F entropy change for liquid = .1625
At 328°F entropy change for vaporization = 1.1277
From liquid at 212°F to dry steam at 328°F entropy change = 1.2902
From liquid at 212°F to dry steam at 212°F entropy change = 1.4447

Difference in entropy = .1545

$$\text{Moisture after expansion} = \frac{.1545}{1.4447} = 10.7\%,$$

$$\text{Quality after expansion} = \frac{1.2902}{1.4447} = 89.3\%,$$

Heat supplied per lb. $= (298 - 180) + 888 = 118 + 888 = 1006$ B.T.U.
Heat abstracted per lb. $= .893 \times 970.4 = 866.6$ B.T.U.

Work = 139.4 B.T.U.

$$\text{Efficiency} = \frac{139.4}{1006} = 13.8\%$$

The other quantities follow in the same way.

Third Method. By Peabody's equal entropy table.

For dry steam at 328° F. find (page 94) for quality = 1, (.9994), the entropy = 1.60, and B.T.U. per pound above 32° F. = 1184.3. For wet steam at the same entropy at 212° F. read directly the quality = 89.15 per cent, and B.T.U. per pound = 1044.9 above 32°.

Hence

Heat supplied above 212° = 1184.5 − 180.3 = 1004.2 B.T.U.
Heat abstracted above 212° = 1044.9 − 180.3 = 864.6 B.T.U.
Work = (1004.2 − 864.6) = 139.6 B.T.U.

$$\text{Efficiency} = \frac{139.6}{1004.2} = 14\%.$$

The other quantities follow in the same way.

Fourth Method. Ordinary temperature entropy diagram with constant volume, quality and total heat lines. This method is essentially the same as Peabody's, as the final quality, initial and final heats are read off directly, work and efficiency are given by difference and ratio of these heats, and are slide-rule determinations.

Fifth Method. Total heat entropy diagram. This method eliminates one step in finding work but is no shorter than the last for efficiencies. Work is given by the length of a vertical line joining initial and final conditions and is known without first finding the total heats.

Sixth Method. By formulas. As this is merely a question of numerical substitution in equations which by inspection are clearly complex it is omitted from the text, but students are advised to do the work as an exercise.

To show more clearly the relations between the various initial conditions and results derived from them, for this Rankine cycle, curves are plotted for some, taken in sets, as it is not convenient to show simultaneously all relations on a single curve. Thus Fig. 240 gives the thermal efficiency and shows how it varies with initial and back pressures and with the corresponding temperatures for steam initially dry and saturated. As heat consumption is in constant inverse proportion to the thermal efficiency, its scale is made to parallel that of thermal efficiency. Water rate being related to heat consumption through the high-pressure total heat per pound of steam above the terminal temperature, for the Rankine cycle, it is necessary to use the family of curves in the left-hand angle to pass from the vertical ordinate of heat consumption, on thermal efficiency, to the horizontal coordinate of corresponding water rate on the left. The resulting chart gives a direct solution of a problem on the relative value of two Rankine steam cycles for any pressure ranges when the steam is initially dry and, moreover, gives a numerical answer for the thermal efficiency, heat consumption and water rate.

It is desirable to compare for given pressures the relative performance of the cycle with steam in various initial conditions of wetness or superheat, and to assist in this the curves of Fig. 241 are plotted according to a system similar to that used in Fig. 240, but with initial quality as the prime variable, but only for steam initially at 200 lbs. per sq. in. gage pressure, which may be regarded as the high limit of present day general practice.

These last two curves, Figs. 240 and 241, show the essential relations between the efficiency heat consumption or cyclic water rate, and the initial pressure or quality, but as capacity for work is just as important as efficiency of performance another pair of curves is plotted in Figs. 242 and 243, showing the work per pound of steam, and the corresponding maximum jet velocity for turbine nozzles or mean effective pressures for cylinder engines. In the curves, Fig. 242, these two results fundamental to capacity for work are shown as functions of initial pressures for steam initially dry, and in the second case as functions of initial quality for 200 lbs. per square inch gage initial pressure, in both

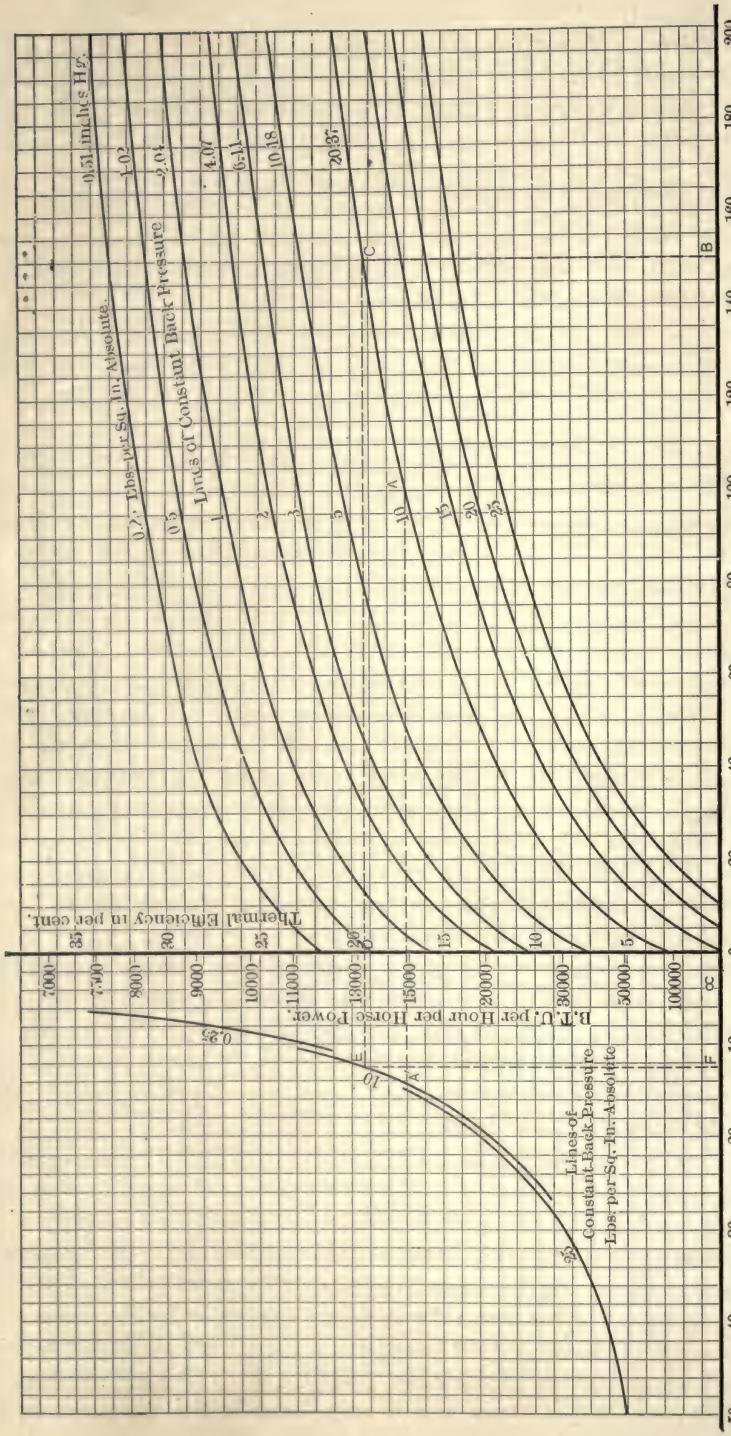


FIG. 240—Rankine Cycle, Thermal Efficiency, Heat Consumption per Hour per Horse-power and Water Rate, for Various Back Pressures and any Initial Pressure. Steam Initially Dry and Saturated.

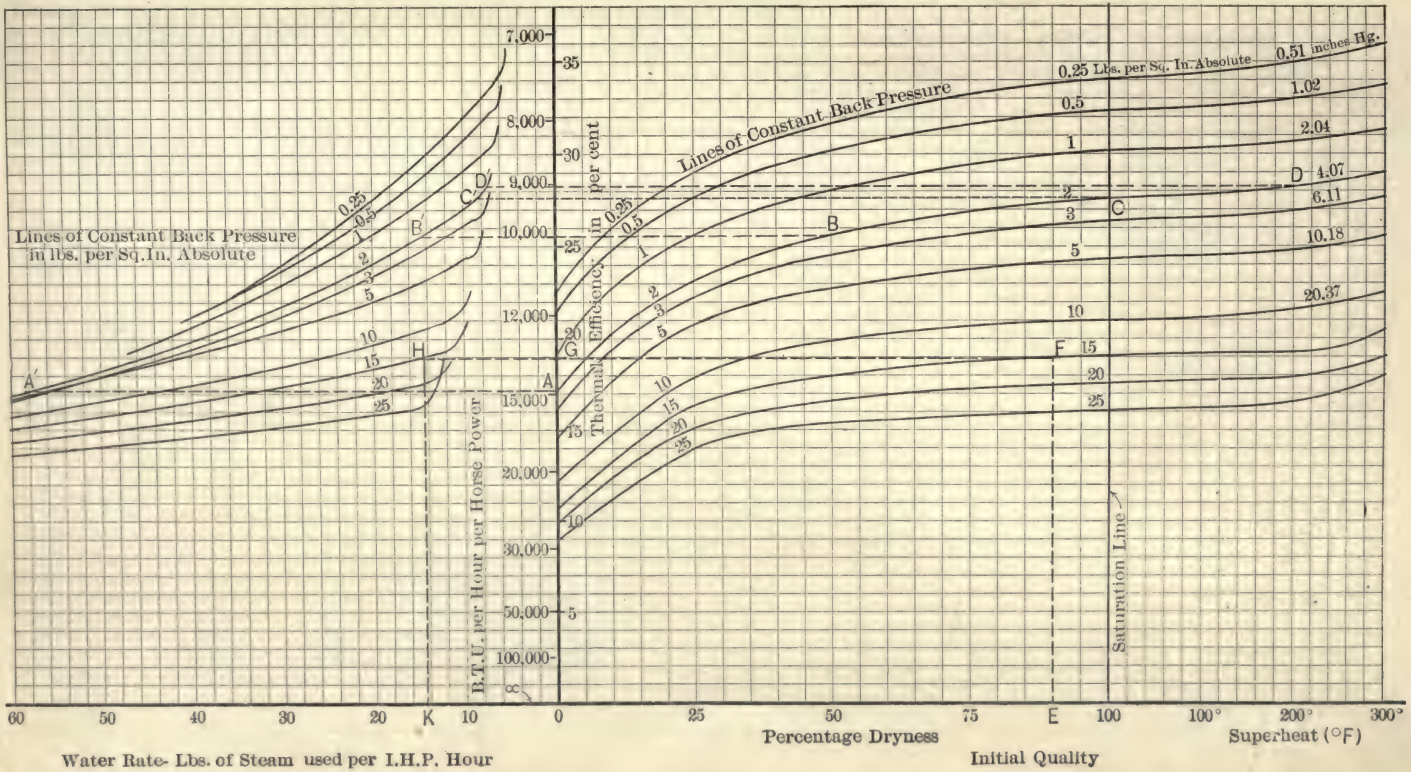


FIG. 241.—Rankine Cycle, Thermal Efficiency, Heat Consumption per Hour per Horse-power and Water Rate, for 200 lbs. per sq.in. gage Initial Pressure and Various Back Pressures, Steam Initially of any Quality.

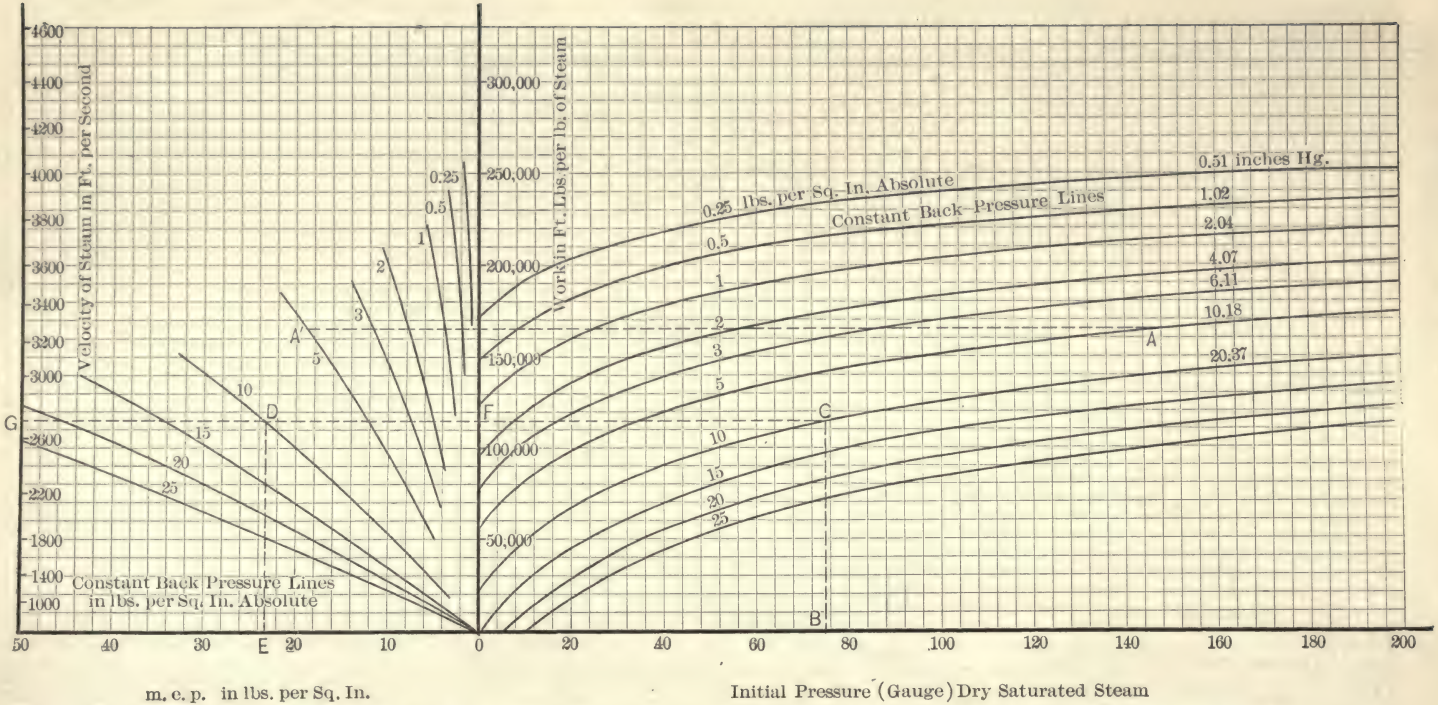


FIG. 242.—Rankine Cycle. Work per lb. of Steam, (m.e.p.), and Jet Velocity for Various Back Pressures, and any Initial Pressure Steam Initially Dry Saturated.

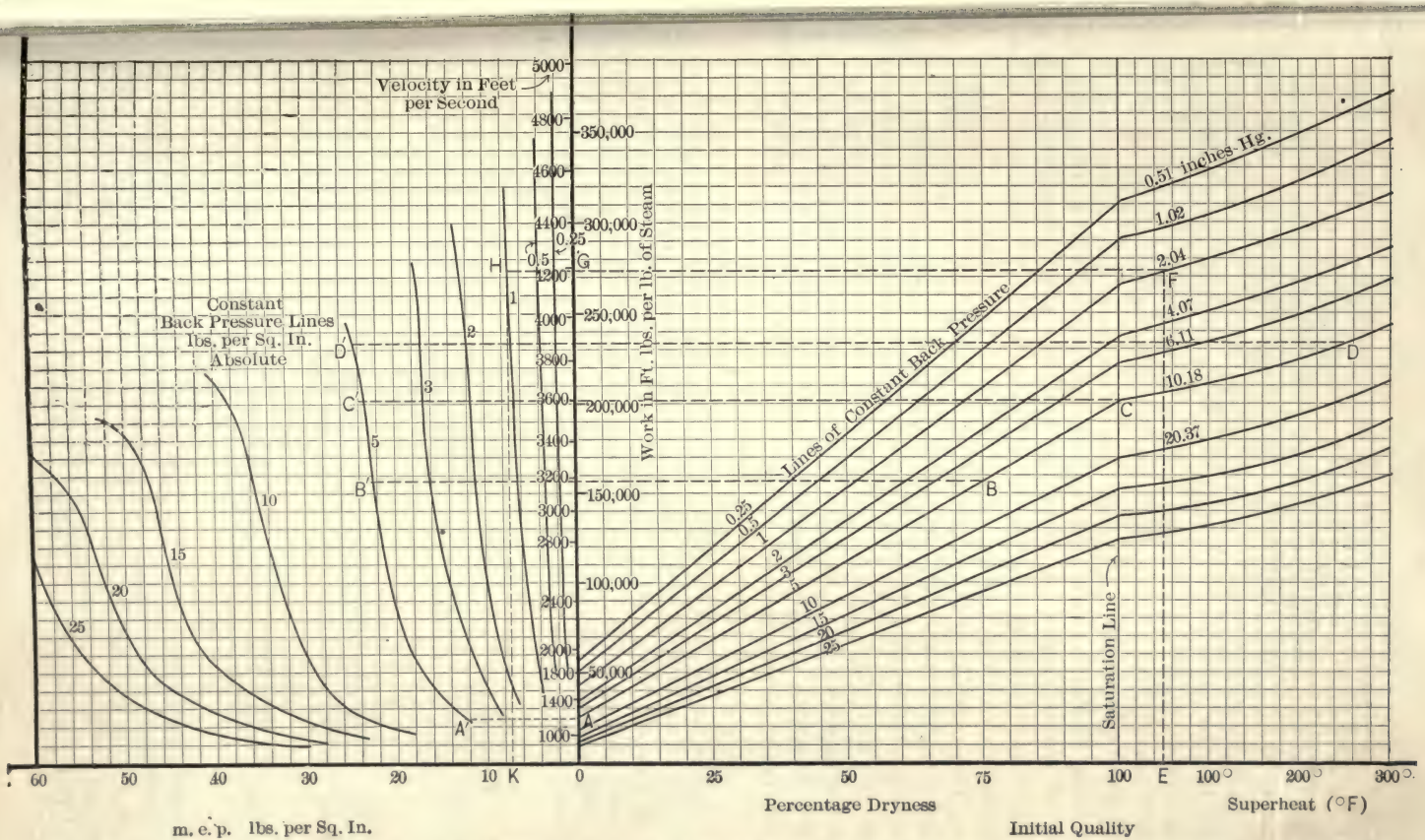


FIG. 243.—Rankine Cycle. Work per lb. of Steam (m.e.p.) and Jet Velocity for 200 lbs. per sq.in. gage Initial Pressure and Various Back Pressures, Steam Initially of any Quality.

cases for various back pressures. The very low mean effective pressure for low initial pressures such for example as in the region of one atmosphere, even with back pressures of only a fraction of a pound, is the very best demonstration of the value of modern low-pressure steam turbines against their only competitor, a low-pressure cylinder. Cylinder sizes must be inversely proportional to mean effective pressures, but there is no similar relation for turbines, the size of which depends on bucket speed and number of stages for a given output, so that the turbine is a logical substitute for cylinders in low-pressure ranges even when its mechanism losses exceed those of cylinders, as is often the case.

Referring to the cyclic efficiency curves, Fig. 240, a fairly rapid rise with increase of initial pressure is found at first, becoming less as the pressure rises, but a most striking increase in efficiency with back-pressure reduction is demonstrated. Thus the gain from $\frac{1}{2}$ lb. to $\frac{1}{4}$ lb. absolute is almost as great as from 15 to 10 lbs., proving the superior value of low back over high back pressures from the efficiency standpoint. Initial quality has most influence when the steam is very wet, more wet than is ever used, as appears from Fig. 241, but the initial quality is of greater importance for low than for high back pressures shown by the slopes of the .25 lb. and 25.00 lb. curves near saturation especially. Initial superheat has a beneficial effect in all cases, but never large at any time.

Example 1. *Calculation and use of Diagram, Fig.* 240, *giving Rankine cycle efficiency water rate, and heat consumption as functions of initial pressure.* The method used in plotting is illustrated by the location of point A, for assumed conditions of initially dry saturated steam at 100 lbs. per square inch gage pressure and 10 lbs. per square inch absolute back pressure. The efficiency is given directly by Eq. (976, *b*) as reference to the general $T\Phi$ diagram, Fig. 238, shows these data to fall under Case *c*. Therefore

$$E_a = 1 - \left(\frac{\frac{T_a}{T_b} L_1 + T_a \log_e \frac{T_b}{T_a}}{T_b - T_a + L_1} \right) = 1 - \left(\frac{\frac{653.2}{797.9} \times 880 + 653.2 \log_e \frac{797.9}{653.2}}{797.9 - 653.2 + 880} \right) = 16.94 \text{ per cent,}$$

and the point A is located opposite this per cent on the efficiency scale. Passing to the left-hand angle of the diagram the point A' is located by Eq. (991, *c*),

$$\text{Water rate} = \frac{2545}{E(H_1 - q_2)} = \frac{2545}{.169(1188.7 - 161.1)} = 14.62 \text{ lbs. per I.H.P. hour,}$$

which fixes the abscissa of point A', the ordinate leing 16.94 per cent efficiency.

To illustrate the use of the diagram, Fig. 240, the following problem will be graphically solved. Find the Rankine cycle efficiency, heat and steam, consumption for an initial pressure of 150 lbs. per square inch gage and dry saturated steam with a back pressure of 10 lbs. per square inch absolute. Starting at the initial pressure point I project up to the 10-lb. back pressure curve point C, and then across to the efficiency

cale point D, reading there a thermal efficiency of 19.3 per cent and a heat consumption of 13,200 B.T.U. per hour per I.H.P. Continuing across horizontally to the back pressure curve of 10 lbs. in the left-hand angle to point E and thence downward to the water-rate scale point F, the value 12.6 lbs. steam per hour per I.H.P. is read off directly.

Example 2. *Calculation and use of Diagram, Fig.* 241, *giving Rankine cycle performance as a function of initial quality.* The determination of he position of the curve $ABCD$ will illustrate the whole series. This represents the case of 2 lbs. per square inch absolute back pressure and 200 lbs. per square inch gage initial pressure. Point A represents an initial condition of all water; B, 50 per cent steam; C, dry saturated steam; D, 200 degrees superheat. Reference is first made to the general $T\Phi$ diagram to find under which case each set of data belongs so as to permit of the selection of the proper formula. The conditions represented by point A are those of Case b, and Eq. (974, b) applies. Therefore

$$E_a = 1 - \left(\frac{T_a \log_e \frac{T_b}{T_a}}{T_b - T_a} \right) = 1 - \frac{586.1 \log_e \frac{847.9}{586.1}}{847.9 - 586.1} = 17.34\%.$$

which is the ordinate of point A, the abscissa of which is zero. Similarly a condition of 50 per cent quality falls under Case a, and Eq. (971) applies. Therefore,

$$E_b = 1 - \left(\frac{\frac{T_a}{T_b} x_c L_1 + T_a \log_e \frac{T}{T_a}}{T_b - T_a + x_c L_1} \right) = 1 - \left(\frac{\left(\frac{586.1}{847.9} \times .5 \times 838 \right) + \left(586.1 \log_e \frac{847.9}{586.1} \right)}{847.9 - 586.1 + (.5 \times 838)} \right) = 25.68\%.$$

Dry saturated steam falls under Case c, and Eq. (976, b) applies, so that

$$E_c = 1 - \left(\frac{\frac{T_a}{T_b} L_1 + T_a \log_e \frac{T_b}{T_a}}{T_b - T_a + L_1} \right) = 1 - \left(\frac{\left(\frac{586.1}{847.9} \times 838 \right) + 586.1 \log_e \frac{847.9}{586.1}}{847.9 - 586.1 + 838} \right) = 27.66\%.$$

An initial superheat of 200° falls under Case d, and Eq. (978) applies, and

$$E_d = 1 - \frac{x_d L_2}{q_1 - q_2 + L_1 + h_1} = 1 - \frac{x_d L_2}{T_b - T_a + L_1 + h_1} \quad \text{(approx.)}$$

Referring to the Mollier diagram, Fig. 234, Section 9, as the easiest way, $x_d = .851$, which could of course have been calculated from Eq. (979). Substituting it,

$$E_d = 1 - \frac{.851 \times 1021}{847.9 - 586.1 + 838 + 110.5} = 28.24\%.$$

In the left-hand angle the corresponding curve to be located is $A'B'C'D'$. The ordinates of these points are the same as for A, B, C, and D, so it is only the abscissa that is to be determined as follows from Eq. (991, c):

$$(WR)_{a'} = \frac{2545}{(T_b - T_a)E} = \frac{2545}{(847.9 - 586.1) \times .1734} = 56.1 \text{ lbs. I.H.P. hr.}$$

$$(WR)_{b'} = \frac{2545}{(T_b - T_a + x_c L_1)E} = \frac{2545}{(847.9 - 586.1 + .5 \times 838) \times .2568} = 14.55 \text{ " "}$$

$$(WR)_{c'} = \frac{2545}{(T_b - T_a + L_1)E} = \frac{2545}{(847.9 - 586.1 + 838) \times .2766} = 8.37 \text{ " "}$$

$$(WR)_{d'} = \frac{2545}{(T_b - T_a + L_1 + h_1)E} = \frac{2545}{(847.9 - 586.1 + 838 + 110.5) \times .2824} = 7.45 \text{ " "}$$

To illustrate the use of the chart, Fig. 241, find the thermal efficiency, heat and steam consumption, for the Rankine cycle, when steam is 90 per cent initially dry at 200 lbs. per square inch gage pressure, and the back pressure 15 lbs. per square inch absolute. From the scale of quality at 90 per cent, point E, project up to point F on 15-lb. curve, and then horizontally to point G at 18.98 per cent thermal efficiency and 13,400 B.T.U. per hour per I.H.P. heat consumption. Continue across to H and down to K, reading the water rate value 14.4 lbs. of steam per hour per I.H.P. on the bottom scale.

Example 3. *Calculation and use of diagram, Fig. 242, giving work, jet velocity, and mean effective pressure, as functions of initial pressure for the Rankine cycle.* Assume for point A an initial condition of dry saturated steam at 150 lbs. per square inch gage and 5 lbs. per square inch absolute back pressure. Then the cycle work will be given by Eq. (975, b) under Case c, so that

$$W_a = J\left[(T_b - T_a)\left(1 + \frac{L_1}{T_b}\right) - T_a \log_e \frac{T_b}{T_a}\right]$$

$$= 778\left[(825.9 - 622.3)\left(1 + \frac{856.9}{825.9}\right) - 622.3 \log_e \frac{825.9}{622.3}\right] = 185{,}700 \text{ ft.-lbs.}$$

Passing to the left the abscissa of point A' is to be found and this is given by Eq. (988) as

$$(\text{m.e.p.}) = \frac{W}{144 V_2} = \frac{185{,}700}{144 \times 60.49} = 21.32 \text{ lbs. per square inch.}$$

In the above the value of V_2 is found from the specific volume of dry saturated steam and final quality, or better, taken directly from the Mollier diagram. The velocity scale paralleling that for work is calculated directly from Eq. (992) and when

$$W = 250{,}000, \quad u = \sqrt{64.14 \times 250{,}000} = 4010 \text{ ft. per second.}$$

To illustrate the use of the diagram, Fig. 243, find the jet velocity, work per pound steam, and mean effective pressure for the Rankine cycle for steam at 75 lbs. initial essure gage, dry and saturated expanding to 10 lbs. absolute. Project up from int B to point C and across to point F where there is read, work done = 121,000 ft.-lbs. r pound of steam. Continuing across to D and down to E, (m.e.p.) = 24.8 lbs. per square ch, or continuing CD across to G the jet velocity is 2790 ft. per second.

Example 4. *Calculation and use of Diagram, Fig.* 242, *giving for the Rankine cycle, rk, jet velocity, and mean effective pressure as functions of initial quality.* Curve $ABCD$ fixed by the four points, A for all water initially; B for quality 75 per cent; C for 100 r cent, and D for 250 degrees superheat, so each point belongs to different case of the nkine cycle and is given by a different though corresponding equation.

From Eq. (973, b), Case b.

$$W_a = J\left(T_b - T_a - T_a \log_e \frac{T_b}{T_a}\right) = 778\left(847.9 - 662.3 - 622.3 \log_e \frac{847.9}{622.3}\right)$$

$$= 25{,}750 \text{ ft.-lbs.}$$

From Eq. (968c), Case a.

$$W_b = J\left[(T_b - T_a)\left(1 + \frac{x_c L_1}{T_b}\right) - T_a \log_e \frac{T_b}{T_a}\right]$$

$$= 778\left[(847.9 - 622.3)\left(1 + \frac{.75 \times 838}{847.9}\right) - 622.3 \log_e \frac{847.9}{622.3}\right] = 156{,}000 \text{ ft.-lbs.}$$

From Eq. (975, b), Case c.

$$W_c = J\left[(T_b - T_a)\left(1 + \frac{L_1}{T_b}\right) - T_a \log_e \frac{T_b}{T_a}\right]$$

$$= 778\left[(847.9 - 622.3)\left(1 + \frac{838}{847.9}\right) - 622.3 \log_e \frac{847.9}{622.3}\right] = 200{,}000 \text{ ft.-lbs.}$$

From Eq. (977), Case d.

$$W_d = J(T_b - T_a + L_1 - x_d L_2 + h_1)$$

$$= 778[847.9 - 622.3 + 838 - (.899 \times 1000.3) + 135.2] = 231{,}200 \text{ ft.-lbs.}$$

To illustrate the use of the diagram, Fig. 243, find work, jet veloicty, and mean effec- ve pressure, for the Rankine cycle when initial pressure is 200 lbs. per square inch ge, 50° superheat and back pressure 1 lb. per square inch absolute. Projecting up m point E to F and across to G, read, work = 272,000 ft.-lbs., velocity = 4190 ft. per cond, and stopping on the 1-lb. curve at point A' the mean pressure 7.4 lbs. per square ch is read directly below at K.

Prob. 1. A locomotive engine with steam at 250 lbs. per square inch absolute initia pressure and 90 per cent quality, exhausting to atmosphere, uses 24 lbs. of steam pe hour per I.H.P. What fraction of the Rankine cycle is being realized?

Prob. 2. A low-pressure steam turbine supplied with atmospheric steam 75 per cen quality, exhausting to a 28-in. vacuum at 3000 ft. elevation, is developing 70 pe cent of the Rankine cycle heat. What is its heat consumption per hour, its wate rate, thermal efficiency, and work per pound of steam?

Prob. 3. A combination low-pressure turbine and high-pressure piston engine operate on 60 per cent of the Rankine cycle efficiency in the former and 70 per cent in th latter. The initial pressure is 190 lbs. per square inch absolute with 100° superheat receiver quality 80 per cent, and pressure 30 lbs., and back pressure $\frac{1}{2}$ lb. per squar inch absolute. What is the combined thermal efficiency and water rate?

Prob. 4. What would be the jet velocity of the turbine of Problem 2 if single stag and all losses took place in the nozzles.

Prob. 5. If a five-pressure stage turbine suffered no heat losses whatever and eac stage were to develop the same energy and jet velocity from 190 lbs. and 100 superheat to 1 lb. absolute, (*a*) what would be the pressure in each stage, (*b*) the je velocity, (*c*) the initial quality? (*d*) Plot a curve through these quality points.

Prob. 6. Compare the thermal efficiencies of two turbines working between th pressures of Problem 5, one with 100 per cent initial quality and the other with 200 superheat, each developing 65 per cent of the Rankine cycle. What will be the differenc in the respective heat consumptions and water rates.

Prob. 7. What mean effective pressure would be developed by a Rankine cycle f the data of Problem 6 and how much cooling water between 45° F. and 70° F. woul be required by a condenser per hour per cyclic horse-power.

Prob. 8. The value of a reheating receiver is to be judged by comparing the Rankin cycle efficiency with and without it for the following data. Initial pressure 200 lb per square inch absolute, 50° superheat, receiver pressure 40 lbs., and back pressu 2 lbs. per square inch absolute. Find the thermal efficiency for (*a*) one-stage adiabat expansion; (*b*) two-stage adiabatic expansion with receiver reheat to original temperatur (*c*) express the value of the reheat in any proper terms.

Prob. 9. Which case of the Rankine cycle corresponds to the following data:

	Initial pressure lbs. sq. in. absolute.	Initial quality.	Back pressure lb. sq. in. absolute.
(*a*)	200	200° superheat	20.0
(*b*)	190	150	10.0
(*c*)	180	100	5.0
(*d*)	170	50	4.0
(*e*)	160	dry saturated	2.0
(*f*)	150	90 per cent	1.0
(*g*)	140	80	2.0
(*h*)	130	70	4.0
(*i*)	120	80	5.0
(*j*)	110	90	10.0
(*k*)	100	dry saturated	20.0

Prob. 10. Find for any one or all cases of Problem 9, the heat added and abstract work per pound, mean effective pressure, thermal efficiency, heat consumption, a cyclic water rate by each of the five methods of the text or any one.

7. The Carnot Steam Cycle and Derivatives. Work, Mean Effective Pressure, ater Rate, Heat Consumption and Efficiency of Steam Cycle II, Adiabatic xpansion and Compression, Constant Pressure Heat Addition and Abstrac-on. To this cycle illustrated in Fig. 244 the same general Eqs. (960) and (961) Sec. 6 apply, and need not be reproduced. It is in the evaluation of the heat pplied and abstracted that all differences between this and the Rankine Cycle ise. In this case there is no heating of the liquid, so the heat supplied to or stracted from the cycle will have a different relation to the total heat per pound steam. Steam in condition C has received from the beginning of the cycle me part of the latent heat at the high pressure, so that the heat supplied is ven by the product of quality into latent heat and this is equal to the total

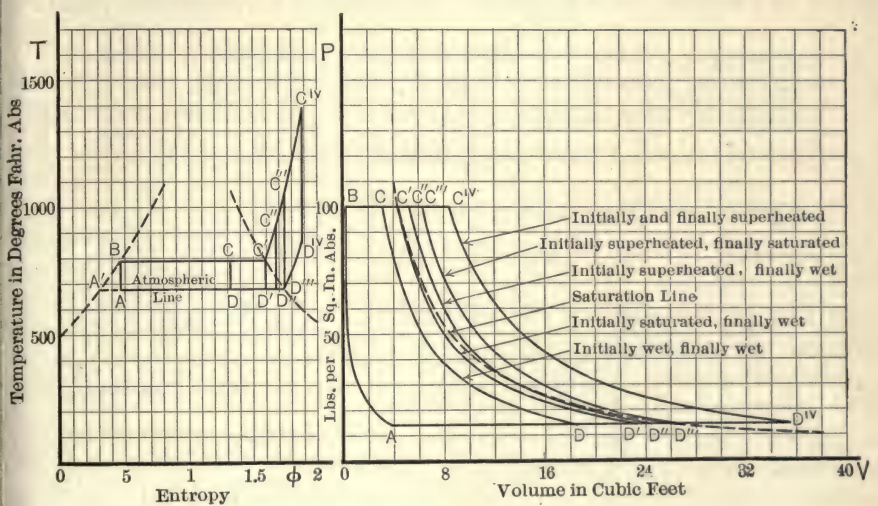


FIG. 244.—Carnot Steam Cycle and Derivatives.

eat of the wet steam above 32° less the heat of the liquid. Hence if H_1 is e total heat of the steam in the initial condition whether wet or dry, it is ven by Eq. (993),

$$\left.\begin{aligned} Q_1 = H_1 - q_1 &= xL_1 \text{ for steam initially wet} && (a)\\ &= L_1 \text{ for steam initially dry saturated} && (b)\\ &= L_1 + h_1 \text{ for steam initially superheated} && (c) \end{aligned}\right\} \quad (993)$$

he heat abstracted is, however, differently related to the total heat per pound, ecause the steam is never fully condensed at the low temperature in this cycle, ut is reduced to water by steam compression at the end, therefore only part of s low-pressure latent heat is ever abstracted. Calling the low-pressure quality which abstraction begins by x_2, and that at which it ends by x_2', then the heat

per pound of steam not abstracted is $x_2'L_2+q_2$, and the amount abstracted is the total heat above 32° for the condition at which abstraction begins less the above quantity, symbolically by Eq. (994).

$$\left.\begin{aligned} Q_2 = H_2 - x_2'L_2 - q_2 &= (x_2 - x_2')L_2 \text{ for wet steam} \quad (a) \\ &= (1 - x_2')L_2 \text{ for dry saturated steam} \quad (b) \\ &= (1 - x_2')L_2 + h_2 \text{ for superheated steam} \quad (c) \end{aligned}\right\} \quad . \quad (994)$$

Each of the cases arising from various positions of the adiabatic expansion line due to different initial quality will be taken up separately for evaluation.

Case a. Steam initially wet, ABCD, Fig. 244, Carnot cycle.

$$W = J(Q_1 - Q_2) = J[x_cL_1 - (x_d - x_a)L_2] \quad . \quad . \quad . \quad . \quad . \quad (995)$$

$$E = 1 - \frac{Q_2}{Q_1} = 1 - \frac{(x_d - x_a)L_2}{x_cL_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (996)$$

In these two equations, Eq. (995) for work, and Eq. (996) for efficiency, there are two dependent variables x_d and x_a which are related high pressure qualities at C and B by conditions of equal entropy, Eq. (997) and Eq. (998):

$$(\phi_a - \phi_{a'}) = (\phi_b - \phi_{a'}), \quad \text{or} \quad \frac{x_aL_2}{T_a} = \log_e \frac{T_b}{T_a}.$$

Therefore

$$x_aL_2 = T_a \log_e \frac{T_b}{T_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (997)$$

Also $(\phi_d - \phi_{a'}) = (\phi_c - \phi_{a'}) = (\phi_c - \phi_b) + (\phi_b - \phi_{a'})$, or $\frac{x_dL_2}{T_a} = \frac{x_cL_1}{T_b} + \log_e \frac{T_b}{T_a}$,

or

$$x_dL_2 = \frac{T_a}{T_b}x_cL_1 + T_a \log_e \frac{T_b}{T_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99$$

Whence combining Eq. (997) with (998) to evaluate the quality differen term of Eq. (995) and Eq. (996), this becomes Eq. (999):

$$(x_d - x_a)L_2 = \frac{T_a}{T_b}x_cL_1 + T_a \log_e \frac{T_b}{T_a} - T_a \log_e \frac{T_b}{T_a} = \frac{T_a}{T_b}x_cL_1. \quad . \quad . \quad (99$$

Substitution of Eq. (999) in (Eq. (995) gives Eq. (1000), which is the work done by the cycle.

$$W = J\left[x_c L_1 - \frac{T_a}{T_b} x_c L_1\right] = J x_c L_1 \left(1 - \frac{T_a}{T_b}\right). \quad . \quad . \quad . \quad . \quad (1000)$$

Substitution in Eq. (996) gives the efficiency by Eq. (1001),

$$E = 1 - \frac{\frac{T_a}{T_b} x_c L_1}{x_c L_1} = \left(1 - \frac{T_a}{T_b}\right) = \left(\frac{T_b - T_a}{T_b}\right) \quad . \quad . \quad . \quad . \quad . \quad (1001)$$

This is a most remarkable case in several ways, and is one example of the famous Carnot cycle. It is characterized by reception of all heat *at* the high temperature and abstraction of all that is abstracted *at* the low temperature, and it will be noted, that the *efficiency depends on the temperatures only and not at all on the quality of the steam.*

It can be demonstrated that between two temperatures, maximum and minimum, it is not possible to convert more heat into work or to get a higher thermal efficiency for any substance than is possible for this Carnot cycle. It is, therefore, a standard of comparison of value, because no system, whether using gases or vapors as media in an engine mechanism, however perfect, could yield a higher efficiency than such a Carnot cycle.

Its value as a standard of comparison is, however often overrated, because it represents an unattainable high value for efficiency, whereas other standards like that of the Rankine for steam are attainable in proportion as the losses are located and reduced to zero. The difference between the attainable maximum for steam efficiency of the Rankine and the unattainable maximum of the Carnot, is not so great as is the corresponding difference for gas engine standards of reference. In the case of gas engine efficiencies there is a very great difference between the efficiency of the Carnot for the temperatures used, and the efficiency of the most closely representative attainable ideal cycle for the particular method of working, and this difference is significant.

The demonstration of the proposition that efficiency of the Carnot cycle is a *maximum for the temperature range* is very simple, by the aid of the $T\Phi$ diagram *ABCD*, Fig. 244, which is a rectangle, whose height is the temperature range, whose base the entropy range, and the enclosed area of which is the work done. It is evident that between two temperatures of working the maximum work area of any cycle will be that of the rectangle embracing and enclosing all points and this is, of course, the Carnot cycle.

Case b. Steam initially dry saturated, ABC′D′, Fig. 244, *Carnot cycle.* For this case x_c becomes $x_{c'} = 1$, and x_d becomes $x_{d'}$, whence

$$W = J(Q_1 - Q_2) = J[L_1 - (x_{d'} - x_a)L_2], \quad . \quad . \quad . \quad . \quad . \quad (1002)$$

and

$$E=1-\left(\frac{x_{d'}-x_a}{L_1}\right)L_2. \quad \ldots \quad (1003)$$

But

$$x_a L_2=T_a \log_e \frac{T_b}{T_a},$$

and

$$x_d L_2=\frac{T_a}{T_b}L_1+T_a \log_e \frac{T_b}{T_a}.$$

Therefore

$$(x_{d'}-x_a)L_2=\frac{T_a}{T_b}L_1. \quad \ldots \quad (1004)$$

Substitution of Eq. (1004) in Eqs. (1002) and (1003) gives work by Eq. (1005), and efficiency by Eq. (1006).

$$W=JL_1\left(1-\frac{T_a}{T_b}\right). \quad \ldots \quad (1005)$$

$$E=\left(1-\frac{T_a}{T_b}\right)=\left(\frac{T_b-T_a}{T_b}\right). \quad \ldots \quad (1006)$$

This is another case of the Carnot cycle, that for dry steam, as the last was that for wet steam, and the cyclic work is of course greater. Comparing the work per pound in the two cases it is found to be directly proportional to the quality of the steam initially.

Case c. Steam initially superheated and finally wet ($ABC'C''D''A$), *Fig.* 244. To the latent heat at the high pressure is now to be added the heat of superheat, to get the heat supplied, while the heat abstracted has the same form as before, so that work is now given by Eq. (1007), and efficiency by Eq. (1008):

$$W=J(Q_1-Q_2)$$

$$=J[L_1+h_1-(x_{d''}-x_a)L_2]=J[L_1+S_{c'c''}(T_{c''}-T_b)-(x_{d''}-x_a)L_2], \quad (1007$$

and

$$E=1-\frac{(x_{d''}-x_a)L_2}{L_1+h_1}=1-\frac{(x_{d''}-x_a)L_2}{L_1+S_{c'c''}(T_{c''}-T_b)}. \quad \ldots \quad (1008$$

n these two equations, the two dependent variables $x_{a''}$ and x_a are related initial quality or superheat by the expansion conditions. As before

$$x_a L_2 = T_a \log_e \frac{T_b}{T_a}.$$

ut

$$(\phi_{a''} - \phi_{a'}) = (\phi_{c''} - \phi_{c'}) + (\phi_{c'} - \phi_b) + (\phi_b - \phi_{a'}),$$

$$\frac{x_{a''} L_2}{T_a} = S_{c'c''} \log_e \frac{T_{c''}}{T_b} + \frac{L_1}{T_b} + \log_e \frac{T_b}{T_a},$$

hence

$$x_{a''} L_2 = S_{c'c''} T_a \log_e \frac{T_{c''}}{T_b} + \frac{T_a}{T_b} L_1 + T_a \log_e \frac{T_b}{T_a}.$$

$$\therefore \quad (x_{a''} - x_a) L_2 = S_{c'c''} T_a \log_e \frac{T_{c''}}{T_b} + \frac{T_a}{T_b} L_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1009)$$

Substitution of Eq. (1009) in Eq. (1007) and (1008) gives Eq. (1010) for ork and Eq. (1011) for efficiency.

$$W = J\left[L_1 + h_1 - S_{c'c''} T_a \log_e \frac{T_{c''}}{T_b} + \frac{T_a}{T_b} L_1\right]. \quad . \quad . \quad . \quad (1010)$$

$$E = 1 - \frac{S_{c'c''} T_a \log_e \frac{T_{c''}}{T_b} + \frac{T_a}{T_b} L_1}{L_1 + h_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1011)$$

These expressions do not reduce to the simple form found for the Carnot ycle, from which this case is a departure. This case is less efficient, indicating hat superheat heat is of less value than latent heat in this cycle when judged y the *temperature limits*, which are now $T_{c''}$ and T_a. However, for the *same ressures*, initial and final, this cycle is more efficient than the Carnot and as practical machines it is pressure limits that control, it is a proof of the perior value of superheat in ordinary working over saturated steam. This as also the case with the Rankine cycle as was shown.

Case d. Steam initially superheated and finally dry saturated, $ABC^{I}C^{III}D^{III}A$, ig. 244. By substituting in the last case $x_{a''} = x_{a'''} = 1$, the work and efficiency qs. (1012) and (1013) for this case follow at once, but h_1, the heat of initial perheat, has a particular value, that which yields after expansion dry turated steam. Hence

$$W = J\left[L_1 + h_1 + S_{c'c'''} T_a \log_e \frac{T_{c'''}}{T_b} + \frac{T_a}{T_b} L_1\right], \quad . \quad . \quad . \quad . \quad (1012)$$

$$E = 1 - \left[\frac{S_{c'c'''} T_a \log_e \frac{T_{c'''}}{T_b} + \frac{T_a}{T_b} L_1}{L_1 + h_1}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1013)$$

Case e. Steam continually superheated, $ABC^{I}C^{IV}D^{IV}D'''A$, Fig. 244. The work and efficiency equations in their first form can be set down by inspection Eqs. (1014) and (1015):

$$W = J[L_1 + h_1 - (1 - x_a)L_2 - h_2], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1014$$

and

$$E = 1 - \frac{(1 - x_a)L_2 + h_2}{L_1 + h_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1015$$

In these equations the heats of superheat are given by $h_1 = S_{c'c^{IV}}(T_{c^{IV}} - T_b$ and, $h_2 = S_{a''' a^{IV}}(T_{a^{IV}} - T_a)$, and the amounts of superheat are related b the necessity for equal entropy. Therefore $(\phi_{a^{IV}} - \phi_a) = (\phi_{c^{IV}} - \phi_b)$, which i equivalent to

$$S_{a'''a^{IV}} \log_e \frac{T_{a^{IV}}}{T_a} + \frac{L_2}{T_a} = S_{c'c^{IV}} \log_e \frac{T_{c^{IV}}}{T_b} + \frac{L_1}{T_b} + \log_e \frac{T_b}{T_a}. \quad . \quad . \quad (101$$

This relation between superheats, Eq. (1016), together with

$$x_a L_2 = T_a \log_e \frac{T_b}{T_a},$$

will, when substituted in Eqs. (1014) and (1015) give expressions for work a efficiency which are somewhat too complex to be very useful.

Unlike the Rankine cycle, this one cannot be solved by the temperatu entropy and Mollier diagram charts, though the tables and diagrams are value in finding individual quantities. For example, on the $T\Phi$ diagra containing the various characteristic lines, the final quality or superheat c be read off directly, thus avoiding the equations; likewise heats of superh for any temperature can be read off by passing along a constant-pressure li from any high temperature and total heat, to the lower temperature of satu tion for the same pressure and its total heat. The difference between th total heats is the heat of superheat. There are many other short-cut ways picking off various quantities from the diagram that cannot be detailed h but must be learned by experience. The really important fact to emphas is that while the $T\Phi$ and Mollier diagrams do help in finding separate quantit they do not yield the answer directly.

Dividing any of these work expressions by the volume range will give mean effective pressure as in all other cases, but it is to be noted that volume range is exactly the same as for the Rankine Cycle I, so that the m

fictive pressure will be given by Eq. (1017), using in it the value of low-ssure volume, Eq. (1018):

$$(\text{m.e.p.}) = \left(\frac{W}{144 \times \text{vol. per lb. at low pressure}}\right) \text{lbs. sq.in.} \quad . \quad . \quad . \quad (1017)$$

$$\left.\begin{array}{l} \text{ol. per} \\ \text{.at low} \\ \text{ressure} \end{array}\right\} = V_2 x_2, \text{ if wet finally} \quad (a)$$

$$= V_2\left(\frac{\text{Temp. of superheated steam}}{\text{Temp. of saturated steam}}\right), \left\{\begin{array}{l}\text{(approx.) if} \\ \text{superheated} \\ \text{finally}\end{array}\right\} (b) \qquad (1018)$$

$$\left.\begin{array}{l} \text{B.T.U.} \\ \text{er hr.} \\ \text{er I.H.P.} \end{array}\right\} = \frac{2545}{E}, \text{ in general} \quad (a)$$

$$= \frac{T_b(2545)}{T_b - T_a}, \text{ for saturated steam wet or dry} \quad (b) \qquad (1019)$$

$$\left.\begin{array}{l} \text{yclic wa-} \\ \text{ter rate} \\ = WR \end{array}\right\} = \frac{2545}{E[(\text{latent heat}) + (\text{heat of superheat})]} \left\{\begin{array}{l}\text{for super-} \\ \text{heated} \\ \text{steam}\end{array}\right\} (a)$$

$$= \frac{2545}{E[(\text{latent heat}) \times (\text{quality})]}, \text{ for wet steam} \quad (b) \qquad (1020)$$

It might seem as if steam jet velocity could be computed for this cycle s or the Rankine, but as a matter of fact such would be at present a purely elemic proceeding because it is difficult to conceive of continuous steam eration necessary to nozzle operation with a cycle that requires adiabatic pression of wet low-pressure steam. This would be regarded to-day as mpracticable procedure though, of course, it is within the range of possi-ity and may be brought to a probability at any time by invention. To meet n a contingency and fill out the set of derived quantities, the jet velocity be set down by the same general expression Eq. (1021):

$$V = \sqrt{2g \times W} = \sqrt{64.4 \times W}, \text{ (app)} \quad . \quad . \quad . \quad . \quad . \quad (1021)$$

There is added a set of four curves similar to those derived for the Rankine e, two, Figs. 245 and 246, giving thermal efficiency and the quantities derived n it, and two more, Figs. 247 and 248, giving the work per pound of steam its derivatives. One curve of each set gives the result as a function of sures for initially dry steam and the other as a function of quality for a n initial pressure. Comparison of this cycle with the Rankine is possible

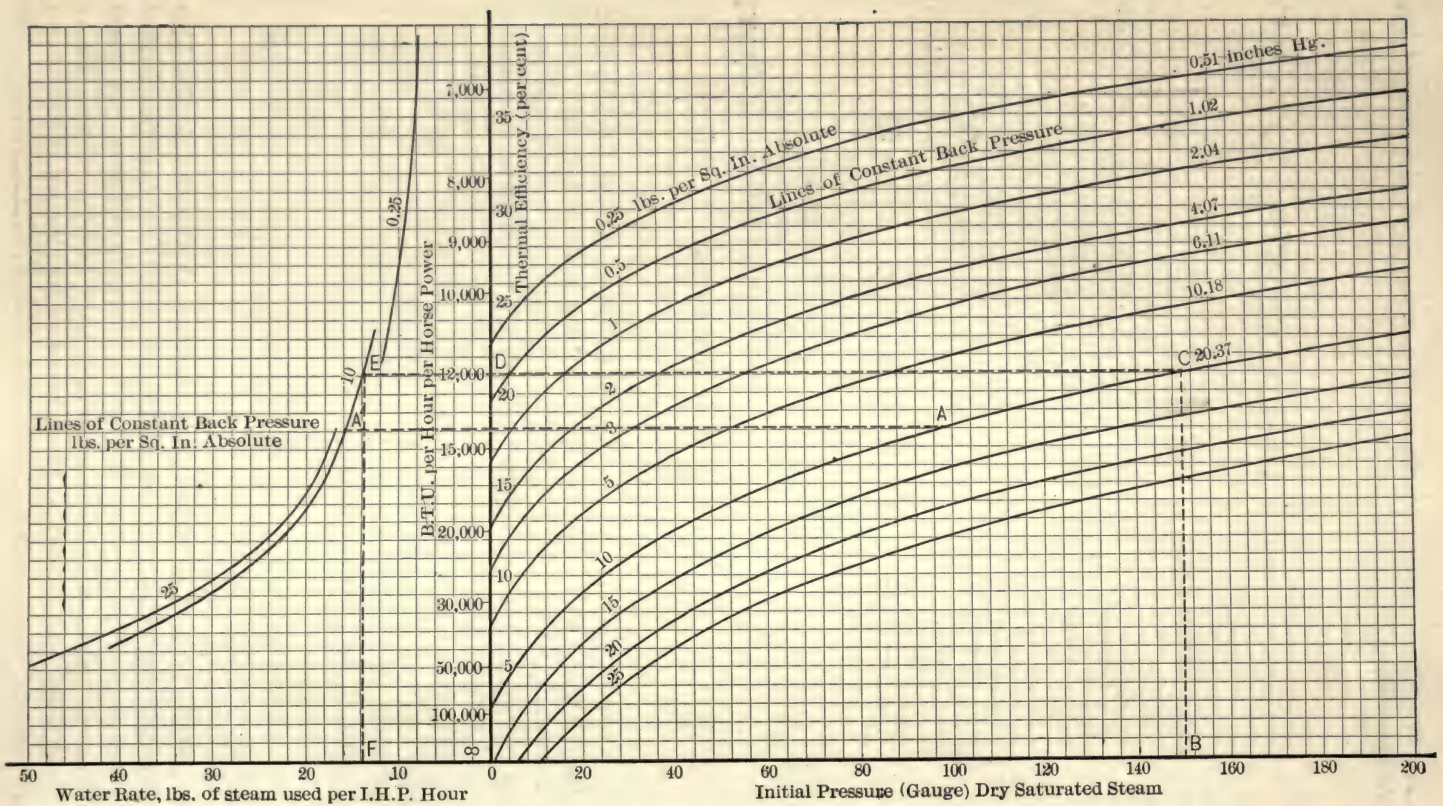


FIG. 245.—Carnot Steam Cycle and Derivatives. Thermal Efficiency, Heat per Hour per Horse-power and Water Rate for Various Back Pressures, and any Initial Pressure, Steam Initially Dry Saturated.

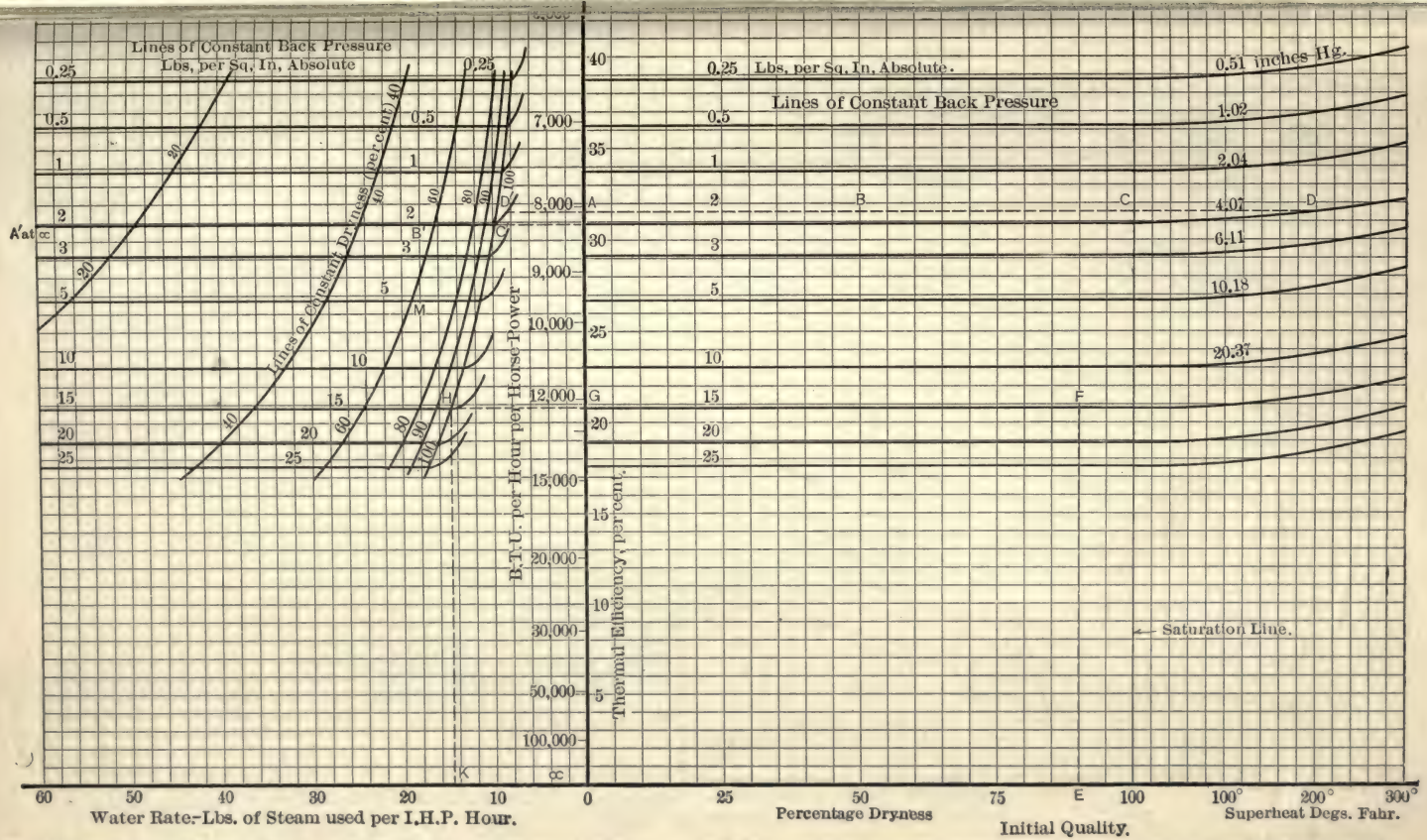


FIG. 246.—Carnot Steam Cycle and Derivatives. Thermal Efficiency, B.T.U. per Hour per Horse-power and Water Rate, for 200 lbs. per sq.in. gage Initial and Various Back Pressures, Steam Initially of any Quality.

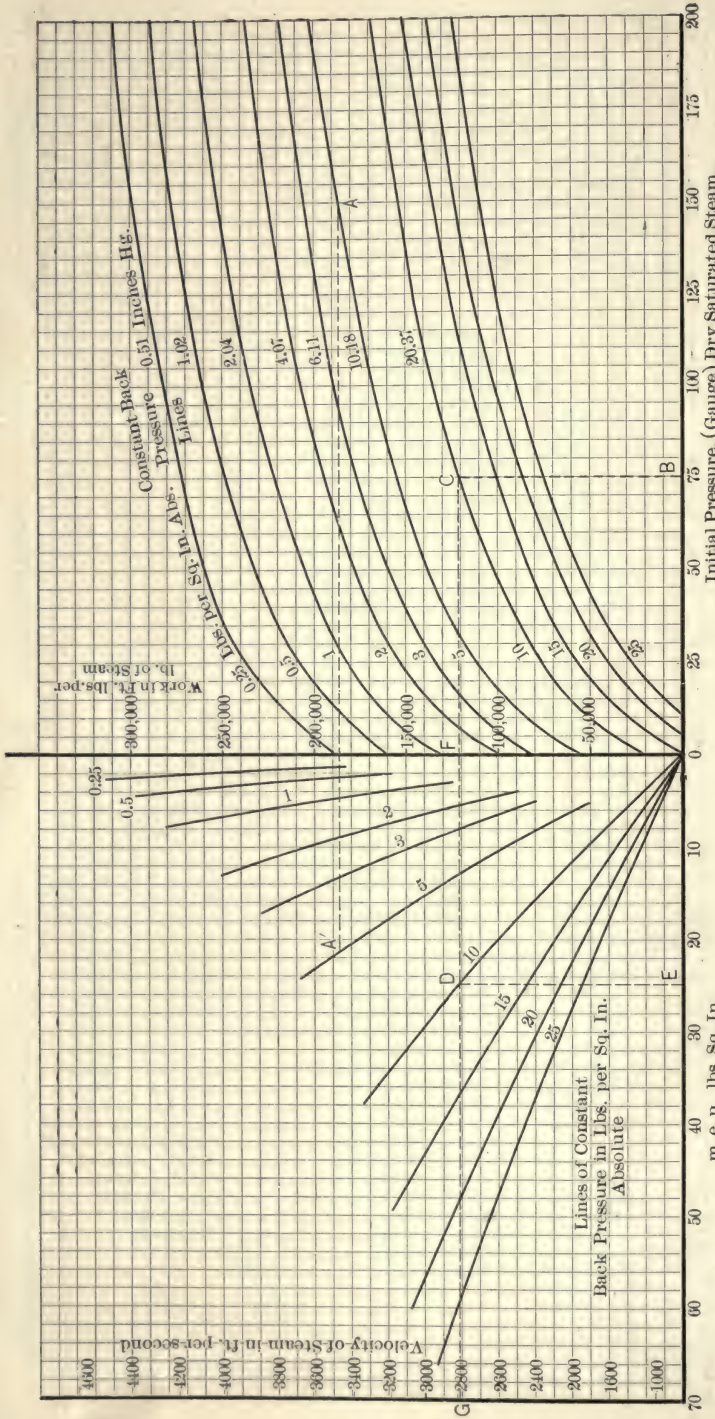


FIG. 247.—Carnot Steam Cycle and Derivatives. Work per lb. of Steam, (m.e.p.) and Jet Velocity for Various Back Pressures and any Initial Pressure, Steam Initially Dry and Saturated.

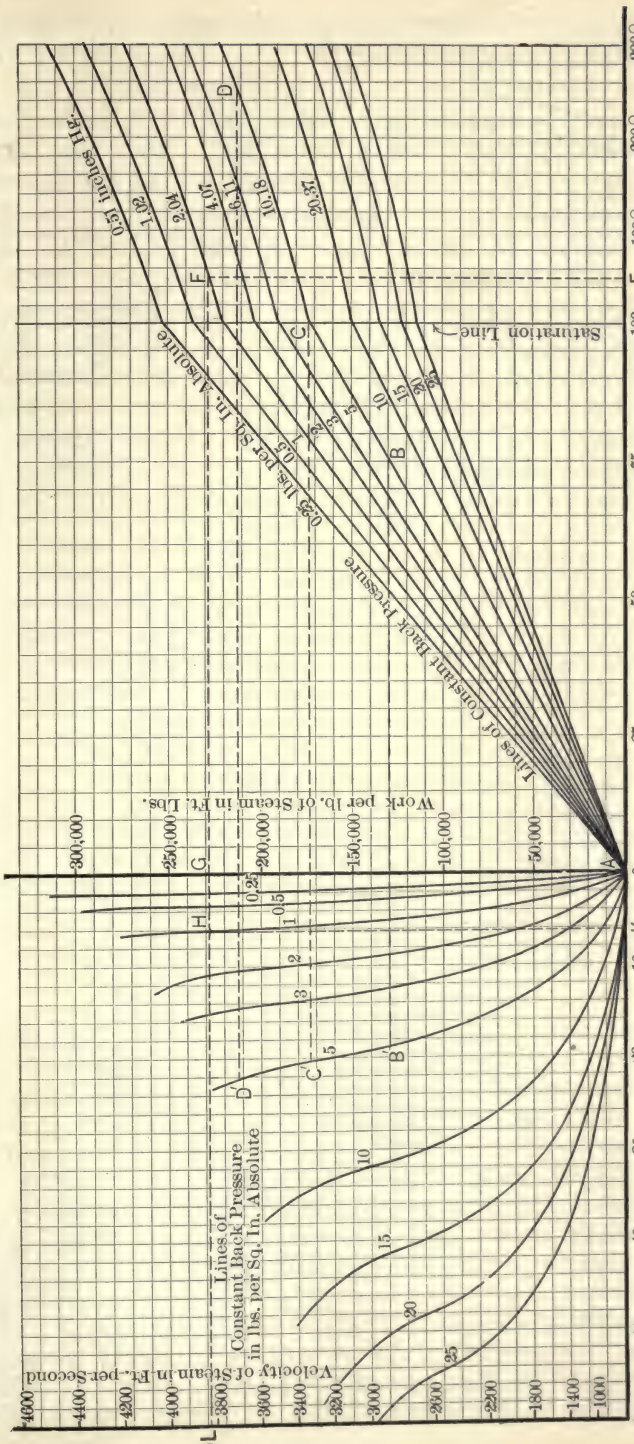


FIG. 248.—Carnot Steam Cycle and Derivatives. Work per lb. of Steam, (m.e.p.) and Jet Velocity, for 200 lbs. per sq.in. gage Initial and

by taking off points from these curves for any given data and solving similarly for the same data by the Rankine curves.

Inspection of the efficiency as a function of initial pressure, Fig. 245, shows the variation of this cycle to be of the same general character as for the Rankine cycle curves, Fig. 240, but usually higher numerically. Thus, dry saturated steam expanding from 100 lbs. per square inch gage to 1 lb. per square inch absolute yields 29.8 per cent for the Carnot, as against 26.8 per cent for the Rankine, and the values for 200 lbs. to 1 lb. are 33.6 per cent for the Carnot and 30.1 per cent for the Rankine cycles. However, efficiency plotted as a function of initial quality or superheat, brings out a striking difference comparing this cycle, Fig. 246, with the Rankine cycle, Fig. 241. For this case the efficiency is independent of quality up to saturation, whereas for the Rankine it increases with quality, so that the Rankine is somewhat less efficient for dry steam and very much less for very wet steam. In the superheat region the character of the curves for the two cycles is the same, both showing a small gain by superheat, and at about the same rates.

Example 1. *Calculation and use of the Diagram, Fig.* 245, *giving for the Carnot cycle the efficiency heat and steam consumption, as functions of initial pressure.* Two points A on the right and A' on the left will be located to show the method used in plotting. Assume initially dry saturated steam at 100 lbs. per square inch gage and 10 lbs. per square inch absolute back pressure. Then by Eq. (1006) as these conditions fall under Case b,

$$E_a = 1 - \frac{T_a}{T_b} = 1 - \frac{653.2}{797.9} = 18.13 \text{ per cent.}$$

Passing across to A' its abscissa is given by Eq. (1020, b),

$$(WR)_{a'} = \frac{2545}{EL_1} = \frac{2545}{.1813 \times 880} = 15.95 \text{ lbs per hour per I.H.P.}$$

To illustrate the use of the diagram, Fig. 245, solve the problem: For the Carnot cycle with dry saturated steam between 150 lbs. per square inch gage and 10 lbs. absolute find the thermal efficiency, heat, and steam consumption. From point B pass up to C and across to D, reading efficiency = 21.1 per cent, and heat consumption 12,060 B.T.U. per hour per I.H.P. Passing horizontally to E and down to F' the water rate of 13.9 lbs. per hour per I.H.P. may be read off directly.

Example 2. *Calculation and use of the Diagram, Fig.* 246, *giving for the Carnot cycle and its derivatives, the efficiency, heat, and steam consumption, as functions of initial quality.* The location of the curve $ABCD$ will illustrate the calculation of the series, point A representing the condition of all water initially; B, 50 per cent steam; C, dry saturated steam; and D, 200° superheat all for 200 lbs. per sq.in. gage initial pressure and 2 lbs. per sq.in. absolute back pressure. Each falls under a special case of the cycle, all apparently has the same thermal efficiency as B and C, but really it is an impossible being Carnot cycles, except the last, where superheat is present. The first point A

condition because without some steam the cycle cannot exist, though with an infinitely small amount it can.

From Eq. (1001), $E_a = E_b = E_c = 1 - \frac{T_a}{T_b} = 1 - \frac{586.1}{847.9} = 30.8$ per cent.

From Eq. (1008), $E_d = 1 - \frac{(x_{d''} - x_a)L_2}{L_1 + h_1} = 1 - \frac{(.851 - .2159) \times 1021}{838 + 110.5} = 31.62$ per cent,

in which $x_{d''}$ and x_a, the two qualities, are determined by the steam tables or from the Mollier diagram. Passing to the left-hand angle the water rates for points A', B', C', and D' are given by the formuals, Eq. (1020):

$$(WR)_{a'} = \frac{2545}{Ex_cL_1} = \frac{2545}{0} \text{ (indeterminate).}$$

$$(WR)_{b'} = \frac{2545}{.3088 \times .5 \times 838} = 19.67 \text{ lbs. per hour per I.H.P.}$$

$$(WR)_{c'} = \frac{2545}{.3088 \times 838} = 9.838 \text{ " " "}$$

$$(WR)_{d'} = \frac{2545}{E(L_1 + h_1)} = \frac{2545}{31.62(838 + 110.5)} = 8.51 \text{ " " "}$$

Lines of constant dryness are drawn across this set and to show how these are found the point M on the 60 per cent quality line will be located for 200 lbs. initial gage and 5 lbs. absolute back pressure. From Eq. (1006) the ordinate or efficiency is found and from Eq. (1020, b) the abscissa or water rate, whence

$$E_m = 1 - \frac{T_a}{T_b} = 1 - \frac{622.3}{847.9} = 26.61 \text{ per cent.}$$

$$(WR)_m = \frac{2545}{Ex_cL_1} = \frac{2545}{.2661 \times .6 \times 838} = 19.02 \text{ lbs. per hour per I.H.P.}$$

The use of the diagram requires no further explanation after what has been given for the Rankine cycle, nor is it necessary to explain the other diagrams completing this set to correspond to the set of four for the Rankine cycle.

Prob. 1. For the data of Problem 1, Section 6, what fraction of the Carnot cycle efficiency is being developed?

Prob. 2. For the same back pressure as in Problem 2, Section 6, what initial pressure for the Carnot cycle would yield the same efficiency?

Prob. 3. Compare the Rankine and Carnot derivative cycle efficiencies for 190 lbs. initial, $\frac{1}{2}$ lb. back pressures per square inch absolute with 100° superheat.

Prob. 4. With the operating data of Problem 3, a steam engine shows by test a thermal efficiency of 18 per cent, what per cent of the Rankine and of the Carnot derivative cycles is being developed.

Prob. 5. Solve for the data of Problem 8, Section 6, the corresponding case with the Carnot derivative as the standard of reference.

Prob. 6. Which case of the Carnot cycle or its derivatives corresponds to each set of data of Problem 9, Section 6.

Prob. 7. Find for the data referred to in the last problem the performance of the corresponding Carnot cycle or its derivative.

8. Gas Cycles Representative of Ideal Processes and Standards of Reference for Gas Engines. While comparatively few thermal processes may be regarded as important as representatives of what may happen in transforming heat into work by steam as a medium, it is not so when perfect or reasonably perfect gases become the medium. Gases may receive their heat at constant pressure, constant volume or constant temperature, whereas steam must receive nearly all of its heat at constant pressure and temperature, as heats of liquid and superheat, when also involved, are both small parts of the total. Gases may, moreover, be expanded isothermally with the product of pressure and volume constant, or adiabatically with the product of pressure and the gamma power of the volume constant; moreover, compression by either of these laws may or may not precede heat addition in various ways peculiar to it. It is possible to construct for gases a very great number of cycles, much greater than for vapors within the range of reasonable possibility of execution, but when the criterion of easy execution by mechanism that shall be very simple is imposed, the number drops at once. In selecting those worth examination there are three conditions to be satisfied by the result. First, the cycles selected shall most closely represent processes now being executed in practical machines so that the cycles may serve as standards of reference for comparison with actual performance, which comparison shall show how perfectly the mechanism is carrying out the imposed process. Second, the selected cycles should include some that might reasonably be carried out especially if they promise high returns in efficiency, controllability, mean effective pressures or any other desired end not yet attained otherwise. Third, they should include those that yield general guiding principles, however academic the cycles may be or however impracticable. As a matter of fact comparatively few cycles, but more than it was found necessary to study when steam was the medium, will serve these ends, the same cycle falling at times in two or all three of the above groups.

Consideration of gas cycles involves differences in the bases of comparison from those that were satisfactory with steam by reason of the differences in limiting conditions. With the steam it is the pressures that constitute the fundamental limits to results attainable, boiler pressures as high and condenser pressures as low as are commercially economical are used and the temperatures that correspond are mere accidental incidents. The one exception is introduced by superheated steam with a rise of temperature at the same

gh initial pressure, and this is of little importance as a departure because ie temperatures used seldom exceed 200° superheat and the heat of super- eat 10 per cent of the total. Thus, with steam the amount of work that will e done per pound, depending as it does on the amount of heat per pound, fixed within fairly narrow limits by the physical properties of steam itself, ır more than by the various phase combinations that may form different eam cycles because these latter are so few.

Gas cycles are not only more numerous even when the limit of practicability imposed than vapor cycles, but the amount of work per pound of gas, depend- ıg on the efficiency of the cycle and the amount of heat added to the gas per ound, is not limited by the physical properties of gases. Gases may be aused to take up as much heat as is convenient to give them, but steam may ot carry more than a definite amount fixed by the safe and economical high ressure. *There is, however, a practical limit to the heat per pound of gas, t least at present, which curiously enough is of about the same order of magni- ude as for steam.* Gases may receive their heat from an external source, ı which case the engines embodying the cycle are termed external combus- ion engines and in this case the amount of heat a pound of gas may receive s limited by the temperature to which it is safe to heat metals, somewhere bout 1100° F. as a high limit and the temperature of cooling water as a low imit, these limits corresponding roughly to about 200 B.T.U. per pound. Thus, external combustion gas cycles may receive only about one-fifth the mount of heat per pound of medium as may steam, which is roundly 1000 3.T.U. for ordinary conditions. This low limit is partly responsible for the bandonment of external combustion gas engines in favor of *internal combustion,* n which explosive mixtures are made with fuel and air, or in which air is caused o support non-explosive combustion of fuel in closed pressure chambers. With internal combustion as a mode of heating the heat per pound of working luid will depend on the thermo-chemical reaction equations, which fix both he weight of air needed to burn a pound of fuel, and the heat of combustion per pound of fuel, which together fix the heat of combustion per pound of working gases. This for air-fuel mixtures is in round numbers about 1000 3.T.U. per pound or about the same as for steam, but for oxygen-fuel mixtures t would be about five times as much.

Gas cycles may receive, therefore, a definite amount of heat per pound, imited at present to about 1000 B.T.U. per pound of gases maximum, so that he problem is, to decide how the greatest part may be transformed into work or which of all the various cycles will give the most work when receiving this amount of heat. This is not the only question, however, because one cycle will require larger *volumes* of gas to do the same work than another, requiring that a piston engine embodying this larger volume cycle be larger. Another cycle will require the gas to rise to a very high pressure to do the given amount of work and its engine will have to be very strong and heavy to resist these high pressures. Therefore, gas cycle efficiency must be studied, not alone, but in conjunction with corresponding pressure and volume ranges which are infinitely

more varied than for steam. The basis of comparison should in all gas cycles be the amount of heat received per pound of working gases rather than the pressures which impose the corresponding limit on steam cycles together with corresponding temperatures of boiling and condensation.

With this preliminary survey of the relation of gas to steam cycle limitations, the next step is to examine some proposals and engine constructions embodying gas cycles to serve as a basis for judging practicability, as a preliminary to the selection of the cycles for analytical study.

One of the first schemes for operating a gas engine was employed by Brown, who was no doubt influenced by the plan of operation used in the very old steam engines, known as atmospheric engines. These old steam engines drew into the cylinder a charge of steam at atmospheric pressure and then injected water to condense it, the resulting vacuum allowing the superior atmospheric pressure to perform the working return stroke. Brown paralleled this by burning gas in a large flame at his suction port, filling the cylinder with hot gases of combustion during the out stroke, at atmospheric pressure. He then injected water which more or less suddenly chilled the gases, causing a pressure drop at constant volume and with this vacuum the return stroke began. This, by reason of the considerable residual volume of the cooled gases, was necessarily a compression stroke and by reason of the slowness of operation, large weights of metal and of injected water, was approximately an isothermal compression, the non-gaseous bodies taking up the heats of compression of the gases as fast as liberated. Thus, the cycle of the Brown engine pretty closely approximates, (*a*) heating at constant pressure; (*b*) cooling at constant volume to original temperature; (*c*) heat abstraction at constant temperature to original volume and pressure. There were many modifications of this, constituting other so called *atmospheric* cycles, but as all were essentially inefficient and required large displacement volumes to do only a little work as measured by low mean effective pressures, they were all abandoned, as were also the atmospheric steam engines in favor of others working at initial pressures above atmosphere.

The next practical proposal embodied in an engine was that of Lenoir about fifty years ago and from which modern gas engines may be said to date, so that steam engines of the practical sort are about one hundred years older than gas engines of the modern kind. Lenoir drew into his cylinder an explosive gaseous mixture and at about one-third stroke exploded it by an electric spark. This caused a rapid rise of pressure, ideally a constant-volume heating, and was followed by adiabatic expansion for the rest of the stroke. If too much mixture had not been taken in, this expansion might be complete and end at atmosphere at the end of the stroke, otherwise there would be a terminal drop at release. Neglecting the charging stroke that ideally neither does or uses up work, the cycle would be, (*a*) constant-volume heating, beginning at atmosphere; (*b*) adiabatic expansion to atmosphere; (*c*) constant-pressure cooling to original volume, this last being the thermal equivalent of exhaust as for steam.

This same series of operations was embodied in the Otto and Langen free-ston engines that succeeded the Lenoir, but with one modification. The ston engaged the shaft by a clutch on the down stroke only, charging, plosion and expansion occupying the up stroke, the cylinder being vertical, ngle acting and open upward. This construction allowed the piston to rise a free projectile till stopped by the vacuum created in the cylinder behind by overexpansion. Returning slowly by its own weight, the low-pressure ses in the water-jacketed cylinder were compressed nearly isothermally atmosphere. Hence the Lenoir cycle is here extended below atmosphere liabatically with an isothermal return. Both these cycles and others of the me general sort are termed non-compression cycles because the gases are t compressed before heating, and are inefficient compared to others that ve preliminary compression, as will be shown, and this is why they were andoned. They did, however, demonstrate the superior advantages of ternal combustion over external, about the same time that external com-stion engines had demonstrated the superiority of compression over non-mpression cycles, though they were not purely such, as some of their heat as exchanged during compression.

These external combustion cycles, involving precompression, are best lustrated by the Stirling and Ericsson engines. The Stirling used an enclosed ass of air so arranged that it could be brought in contact with a hot part the chamber and a cold part alternately, and likewise caused to pass through *regenerator* or large mass of porous or divided solid matter like wire gauze acked together. In passing through this regenerator the air gave up heat hotter itself, which was the case when it flowed from the hot to the cool amber in one direction, and took up heat when it was cooler on the return. he various parts of the mechanism were so arranged that the flow through e regenerator took place at constant gas volume, which could be accom-ished by two pistons moving synchronously in opposite directions, the regen-ator being between the cylinders. Expansion is accomplished by one piston oving out faster than the other and compression inversely. During com-ression the gas is in the cold cylinder which keeps its temperature about nstant; ideally it would be so, and after isothermal compression the air transfer kes place through the regenerator which adds heat to the gas at constant olume. Expansion now takes place in the hot cylinder, the air absorbing heat om the fire, ideally at constant temperature, and this is followed by a nstant-volume return flow through the regenerator, during which the gas cooled to its original temperature and the regenerator is heated. The cycle , therefore, (*a*) constant-temperature compression; (*b*) heating at constant olume; (*c*) expansion at constant temperature; (*d*) cooling at constant volume. he regenerator heat is, however, algebraically equal to zero at the limit nd the only heat derived from the fire is that for isothermal expansion.

Ericsson arranged his pistons differently so far as their relative motions re concerned and secured two constant-pressure phases between the two othermals, by making one piston move out faster than the other during

heating and in one form of engine there was substituted a transfer piston that did no work at all, leaving but one piston, acted on by gas pressures. The Ericsson cycle was ideally, (*a*) constant-temperature compression; (*b*) constant-pressure heating; (*c*) constant-temperature expansion; (*d*) constant-pressure cooling.

Embodying the advantages of internal combustion, which are, (*a*) more heat per pound of gases, and (*b*) faster working because of the removal of conduction limitations to heat absorption by the gas, and in addition introducing the fundamentally necessary precompression before heating and after heating, utilizing the strong transforming power of adiabatic expansion, Otto embodied in his engine a new cycle which has come to be the standard of nearly all modern engines. This Otto cycle is, (*a*) adiabatic compression; (*b*) explosion or heating at constant volume; (*c*) adiabatic expansion to the original volume (*d*) exhaust at constant volume which is equivalent thermally to constant-volume cooling to original pressure. This was modified by Atkinson, who expanded to more than the original volume, and various attempts at compounding were made subsequently to carry the expansion to atmospheric pressure instead of limiting it to the original volume before compression as did Otto.

Contemporaneously with Otto in Germany, Brayton developed an engine in America, using non-explosive but internal combustion which operated on another cycle, that bore to that of Otto the same relation that the Ericsson did to the Stirling. Brayton substituted constant pressure for Otto's constant volume phases between the two adiabatics. His cycle, also known by other names such as Joule, consists of, (*a*) adiabatic compression; (*b*) constant-pressure heating by internal combustion; (*c*) adiabatic expansion to atmosphere if the cylinder is big enough for the amount of heating employed; (*d*) constant-pressure cooling as the thermal equivalent of atmospheric exhaust. Later Diesel utilized the same cycle in a single cylinder, Brayton having used two which structural change necessitated a modification of cycle, as the expansion in one cylinder can proceed only to the original volume, and the Diesel cycle becomes, (*a*) adiabatic compression; (*b*) constant-pressure heating by internal combustion; (*c*) adiabatic expansion to original volume; (*d*) constant-volume cooling as the equivalent of exhaust, to atmospheric pressure.

Following the success of the steam turbine, many proposals have come forward for the operation of gas turbines involving, (*a*) compression of air adiabatically and delivery into and through a combustion chamber for (*b*) heating at constant-pressure followed by, (*c*) adiabatic expansion in turbine nozzles instead of in cylinders, to atmospheric pressure and which must be followed to complete the thermal cycle by, (*d*) constant-pressure cooling at atmosphere, which is, of course, the Brayton cycle as used in cylinders.

This review while decidedly incomplete will serve to indicate the desirability of selecting for study the following types of gas cycles:

1) An atmospheric cycle, to show its essential inefficiency and low mean effective pressure
2) Non-compression cycles, as a basis of comparison with compression cycles to prove the value of precompression.
3) The external combustion compression cycles, to demonstrate the value of precompression for gas cycles, to illustrate the action of a regenerator and as a basis of comparison of isothermal with adiabatic compression cycles.
4) Constant-volume heating, adiabatic precompression cycles, as typical of most modern engines and supplying a standard of reference for them.
5) Constant-pressure heating, adiabatic precompression cycles, as typical of the rest of the modern engines and of the gas turbine proposal, and supplying a standard of reference for them.
6) Constant-temperature heating, adiabatic precompression cycle, to complete the list of possibilities and demonstrate a most valuable general law, viz., that the efficiency of all cycles of four phases in which the compression and expansion lines follow the same law and are included between a pair of similar heating and cooling lines is the same, and equal to that of the general, Carnot, cycle, the standard for both gas and vapor cycles.

In the diagram, Fig. 249, these selected cycles are illustrated to PV and $T\Phi$ coordinates. The PV diagram is of greater value than the $T\Phi$ in this analysis of gas cycles, each point of which must be calculated, before the heats, work and efficiency, the volume and pressure range with quantities derived therefrom, may be determined. In order that there shall be some proportionality between these cyclic diagrams, so necessary for a fair visual comparison, they are all plotted to the same scale on the following assumptions of initial condition, one atmosphere pressure, absolute temperature 492° F. app. = 32° F., volume 12.38 cu.ft. per pound, that for air. To the pound of gas in each cycle is added 1000 B.T.U. after a compression of 7 atmospheres absolute for compression cycles, except for the two external combustion cases, which are allowed 200 B.T.U. and 3 atmospheres compression absolute. Another modification or exception necessary is that of Cycle VIII, which has isothermal heating followed by adiabatic expansion. With 1000 B.T.U. added isothermally at 10 atmospheres pressure initially, the pressure would fall below atmosphere before adiabatic expansion could begin, and the final volume would become several thousand cubic feet. This cycle has accordingly been modified by allowing only 200 B.T.U. per pound.

In all cases the specific heat of the gases is taken as constant and the working fluid is assumed to be air; any other procedure leads to unmanageable formulas, and errors involved in the practice can be allowed for at the same time as errors from other sources, in one or two inclusive correction factors when it is necessary to pass from these ideal to real engine conditions.

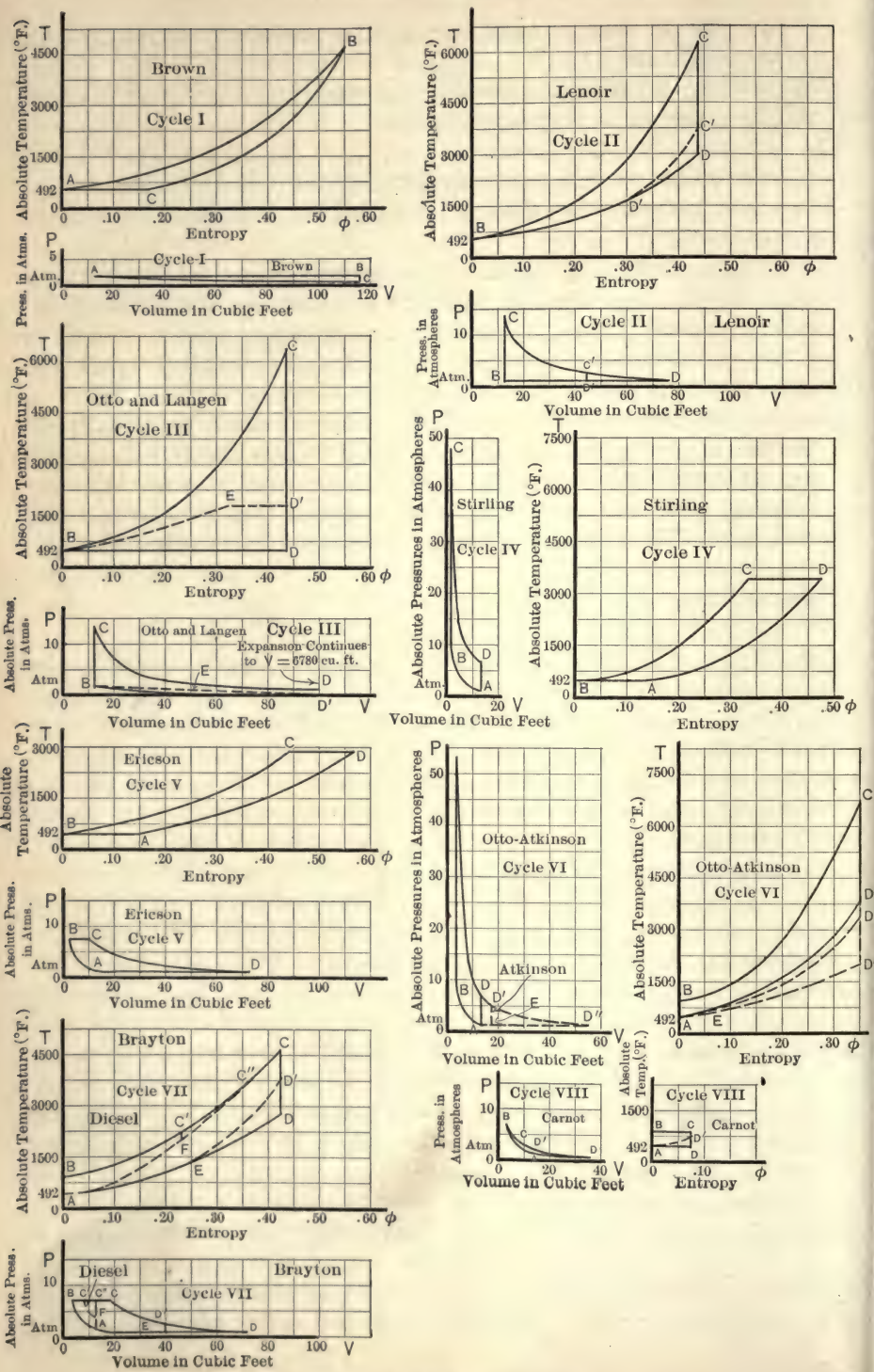


FIG. 249.—Typical Gas Cycles.

The cycles illustrated are described as follows:

ROWN ATMOSPHERIC. CYCLE I.

First phase from A to B. Heating at constant atmospheric pressure.
Second phase from B to C. Cooling at constant volume.
Third phase from C to A. Isothermal compression.

ENOIR. CYCLE II.

First phase from B to C. Heating at constant volume.
Second phase from C to D. Adiabatic expansion.
Third phase from D to B. Cooling at constant pressure.

TTO AND LANGEN. CYCLE III.

First phase from B to C. Heating at constant volume.
Second phase from C to D. Adiabatic expansion.
Third phase from D to B. Isothermal compression.

TIRLING. CYCLE IV.

First phase from A to B. Isothermal compression.
Second phase from B to C. Heating at constant volume.
Third phase from C to D. Isothermal expansion.
Fourth phase from D to A. Cooling at constant volume.

RICSSON. CYCLE V.

First phase from A to B. Isothermal compression.
Second phase fron B to C. Heating at constant pressure.
Third phase from C to D. Isothermal expansion.
Fourth phase from D to A. Cooling at constant pressure.

TTO. CYCLE VI.

First phase from A to B. Adiabatic compression.
Second phase from B to C. Heating at constant volume.
Third phase from C to D. Adiabatic expansion.
Fourth phase from D to A. Cooling at constant volume.

RAYTON. CYCLE VII.

First phase from A to B. Adiabatic compression.
Second phase from B to C. Heating at constant pressure.
Third phase from C to D. Adiabatic expansion.
Fourth phase from D to A. Cooling at constant pressure.

ARNOT. CYCLE VIII.

First phase from A to B. Adiabatic compression.
Second phase from B to C. Isothermal expansion.
Third phase from C to D. Adiabatic expansion.
Fourth phase from D to A. Isothermal compression.

Prob. 1. Modify the Brown atmospheric Cycle I, by substituting adiabatic for isothermal compression and plot between the same temperature limits.

Prob. 2. Plot the modification of the Lenoir Cycle II, introduced by assuming constant-pressure heating to follow constant volume, each equally sharing the whole heat added.

Prob. 3. Modify the Otto and Langen Cycle III by substituting adiabatic compression and isothermal expansion.

Prob. 4. Plot to scale a modification of the Stirling Cycle IV, due to substitution of adiabatic compression and complete isothermal expansion to original pressure.

Prob. 5. Modify the Ericsson Cycle V by substituting adiabatic compression to maximum temperature.

Prob. 6. If the Otto-Atkinson Cycle VI were executed in two cylinders, one carrying out the Otto part and the other the rest of the extended expansion, plot their indicator diagrams to the same stroke, base, and correct pressure scale.

Prob. 7. Modify the Otto cycle for two-stage adiabatic compression with perfect intercooling and both with and without final perfect cooling.

Prob. 8. Assume the Brayton Cycle VII to be carried out in four cylinders, two belonging to a two-stage adiabatic compressor with perfect intercooling and two to a compound steam type of engine with perfect reheating, each without clearance and plot the cycle.

Prob. 9. For all cylinders having 5 per cent clearance each, plot to the same stroke base the indicator cards using correct pressure scales.

Prob. 10. Plot a combination of Diesel and Carnot cycles in which after adiabatic compression half the total heat added is received at constant pressure and the other half at constant temperature.

9. Non-compression Gas Cycles, Brown, Lenoir, Otto and Langen. Work, Mean Effective Pressure, Volume and Pressure Ranges, Efficiency, Heat and Gas Consumption.

Brown. Cycle I

Let Q_1 = heat supplied in B.T.U. per pound gas;

" Q_2 = heat abstracted in B.T.U. per pound gas;

" C_p and C_v = specific heats of gas at constant pressure and volume and assumed constant;

" r_V and r_P = volume and pressure ranges respectively = (maximum) − (minimum);

" H = B.T.U. per cubic foot gas as supplied.

Then referring to the diagram of the cycle, Fig. 250,

$$W = J(Q_1 - Q_2), \quad \ldots \quad (1022)$$

and

$$E = 1 - \frac{Q_2}{Q_1}, \quad \ldots \quad (1023)$$

$$Q_1 = C_p(T_b - T_a), \quad \ldots \quad (1024)$$

$$Q_2 = C_v(T_b - T_c) + \frac{1}{J} P_a V_a \log_e \frac{V_c}{V_a}. \quad \ldots \quad (1025)$$

Evaluation of these heats and determination of all derived quantities requires first the finding of the pressures, volumes and temperatures at each point in terms of the initial conditions, P_a, V_a, T_a, and Q_1, as imposed.

Point A:

$$\left.\begin{aligned} P_b &= P_a, && (a)\\ T_b &= T_a + \frac{Q_1}{C_p} = T_a Y, && (b)\\ V_b &= V_a \frac{T_b}{T_a} = V_a\left(1 + \frac{Q_1}{C_p T_a}\right) = V_a Y && (c) \end{aligned}\right\} \quad . . \quad (1026)$$

in which

$$Y = 1 + \frac{Q_1}{C_p T_a}.$$

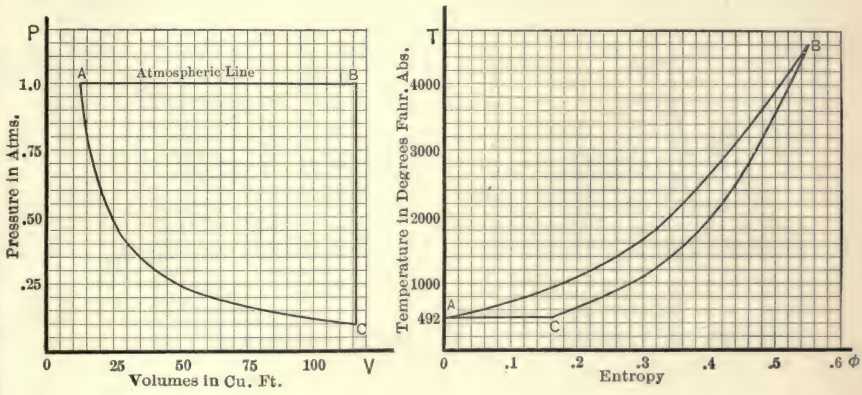


FIG. 250—Brown Gas Cycle. Cycle I.

The point C is located, at the intersection of a constant volume line from B and an isothermal from A, hence $P_a V_a = P_c V_c = P_c V_b$. Hence for

Point C:

$$\left.\begin{aligned} V_c &= V_b = V_a\left(1 + \frac{Q_1}{C_p T_a}\right) = V_a Y, && (a)\\ T_c &= T_a, && (b)\\ P_c &= \frac{P_a V_a}{V_c} = \frac{P_a}{1 + \frac{Q_1}{C_p T_a}} = \frac{P_a}{Y}, && (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1027)$$

Knowing thus the coordinates of all points in terms of the initial conditions and heat supplied, any desired quantities can be set down at once. Thus,

$$Q_2 = C_v\left[T_a + \frac{Q_1}{C_p} - T_a\right] + \frac{1}{J}P_aV_a \log_e\left(1+\frac{Q_1}{C_pT_a}\right),$$

$$= \frac{1}{\gamma}Q_1 + \frac{R}{J}T_a \log_e\left(1+\frac{Q_1}{C_pT_a}\right), \quad \ldots\ldots\ldots \quad (1028)$$

$$W = J(Q_1 - Q_2) = JQ_1\left(1-\frac{1}{\gamma}\right) - RT_a \log_e\left(1+\frac{Q_1}{C_pT_a}\right), \quad (1029)$$

$$E = 1 - \frac{Q_2}{Q_1} = 1 - \frac{1}{\gamma} - \frac{RT_a}{JQ_1}\log_e\left(1+\frac{Q_1}{C_pT_a}\right), \quad \ldots\ldots \quad (1030)$$

Also

$$r_V = V_b - V_a = \frac{V_a}{T_a}\frac{Q_1}{C_p} = \frac{R}{P_a}\frac{Q_1}{C_p} = J\left(1-\frac{1}{\gamma}\right)\frac{Q_1}{P_a}, \quad \ldots\ldots \quad (1031)$$

$$r_P = P_a - P_c = P_a\left[1 - \left(\frac{1}{1+\dfrac{Q_1}{C_pT_a}}\right)\right], \quad \ldots\ldots\ldots \quad (1032)$$

$$(\text{m.e.p.}) = \frac{W}{144r_V} = \frac{P_a}{144}\left[1 - \frac{C_pT_a}{Q_1}\log_e\left(1-\frac{Q_1}{C_pT_a}\right)\right]. \quad \ldots \quad (1033)$$

Heat consumption, B.T.U. per hour per I.H.P. $= \dfrac{2545}{E}$, $\ldots\ldots$ (1034)

Gas consumption, Cu.ft. gas per hour per I.H.P. $= \dfrac{2545}{EH}$, $\ldots\ldots$ (1035)

Lenoir. Cycle II

Referring to the diagram of the cycle *BCD*, Fig. 251, the points are to be evaluated as before.

Case a. Complete expansion to atmospheric pressure.

Point *C*:

$$\left.\begin{aligned} V_c &= V_b && (a)\\ T_c &= T_b + \frac{Q_1}{C_v} = T_bX && (b)\\ P_c &= P_b\frac{T_c}{T_b} = P_b\left(1+\frac{Q_1}{C_vT_b}\right) = P_bX. && (c) \end{aligned}\right\} \quad \ldots \quad (1036)$$

in which

$$X=1+\frac{Q_1}{C_vT_b}.$$

Point D:

$$\left.\begin{aligned} P_d&=P_b, && (a)\\ V_d&=V_c\left(\frac{P_c}{P_b}\right)^{\frac{1}{\gamma}}=V_bX^{\frac{1}{\gamma}} && (b)\\ T_d&=T_c\left(\frac{P_d}{P_c}\right)^{\frac{\gamma-1}{\gamma}}=T_bXX^{\frac{1}{\gamma}-1}=T_bX^{\frac{1}{\gamma}} && (c)\end{aligned}\right\} \quad . \ . \ . \quad (1037)$$

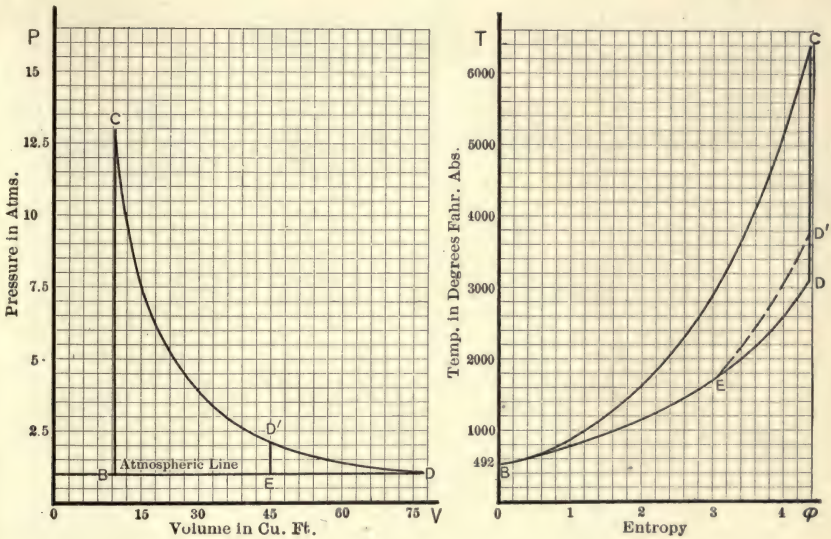


FIG. 251.—Lenoir Gas Cycle. Cycle II.

Whence

$$Q_2=C_p(T_d-T_b)=C_pT_b(X^{\frac{1}{\gamma}}-1). \quad . \ . \ . \ . \ . \ . \quad (1038)$$

$$W=J(Q_1-Q_2)=J[Q_1-C_pT_b(X^{\frac{1}{\gamma}}-1)]. \quad . \ . \ . \ . \quad (1039)$$

$$E=1-\frac{Q_2}{Q_1}=1-\frac{C_pT_b}{Q_1}(X^{\frac{1}{\gamma}}-1), \quad . \ . \ . \ . \ . \ . \quad (1040)$$

$$r_V=V_d-V_c=V_b(X^{\frac{1}{\gamma}}-1). \quad . \ . \ . \ . \ . \ . \ . \ . \quad (1041)$$

$$r_P=P_c-P_b=P_b\ (X-1). \quad . \ . \ . \ . \ . \ . \ . \ . \quad (1042)$$

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{J[Q_1 - C_p T_b (X^{\frac{1}{\gamma}} - 1)]}{144 V_b (X^{\frac{1}{\gamma}} - 1)} \quad . \quad . \quad . \quad . \quad (1043)$$

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1044)$$

$$\text{Cu.ft. gas per hr. per I.H.P.} = \frac{2545}{EH} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1045)$$

Case b. Incomplete expansion with constant-volume cooling to atmosphere Fig. 251, *BCD'E.*

Up to the point C this cycle is the same as the last, but points D' and E are fixed by the limited expansion volumes available.

Let R_V = ratio of final to initial volume, or $\frac{V_e}{V_b} = R_V$.

Point D':

$$\left.\begin{aligned} V_{d'} &= V_b R_V, && (a) \\ P_{d'} &= P_c \left(\frac{V_c}{V_{d'}}\right)^{\gamma} = \frac{P_c}{R_V{}^{\gamma}} = \frac{P_b X}{R_V{}^{\gamma}} && (b) \\ T_{d'} &= T_c \left(\frac{P_{d'}}{P_c}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_b X}{R_V{}^{\gamma-1}} && (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1046)$$

Point E:

$$\left.\begin{aligned} V_e &= V_{d'} = V_b R_V, && (a) \\ P_e &= P_b, && (b) \\ T_e &= T_b \frac{V_{d'}}{V_b} = T_b R_V, && (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1047)$$

Whence

$$\begin{aligned} Q_2 &= C_v (T_{d'} - T_e) + C_p (T_e - T_b) \\ &= C_v T_b R_V \left(\frac{X}{R_V{}^{\gamma}} - 1\right) + C_p T_b (R_V - 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1048) \end{aligned}$$

$$W = J(Q_1 - Q_2) = J\left[Q_1 - C_v T_b R_V \left(\frac{X}{R_V{}^{\gamma}} - 1\right) - C_p T_b (R_V - 1)\right] \quad (1049)$$

$$E = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v T_b R_V}{Q_1}\left(\frac{X}{R_V{}^{\gamma}} - 1\right) - \frac{C_p T_b}{Q_1}(R_V - 1). \quad . \quad . \quad (1050)$$

$$r_V = V_e - V_b = V_b (R_V - 1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1051)$$

$$r_P = P_c - P_b = P_b(X-1). \quad \ldots \quad (1052)$$

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{J\left[Q_1 - C_v T_b R_V\left(\frac{X}{R_V{}^{\gamma}} - 1\right) - C_p T_b (R_V - 1)\right]}{144 V_b (R_V - 1)}. \quad \ldots \quad (1053)$$

$$\left\{\begin{matrix}\text{B.T.U. per hr.} \\ \text{per I.H.P.}\end{matrix}\right\} = \frac{2545}{E}. \quad \ldots \quad (1054)$$

$$\left\{\begin{matrix}\text{Cu.ft. gas per} \\ \text{hr. per I.H.P.}\end{matrix}\right\} = \frac{2545}{EH}. \quad \ldots \quad (1055)$$

Otto and Langen. Cycle III

Case a. Complete expansion to initial temperature. Fig. 252, *BCDB.*

The point C will be as located for the two previous cycles, but point D is at the intersection of an adiabatic through C and an isothermal through B, therefore,

$$P_d V_d = P_b V_b, \text{ or } P_d = P_b \frac{V_b}{V_d},$$

and

$$P_d V_a{}^{\gamma} = P_c V_c{}^{\gamma}, \text{ or } P_d = P_c \left(\frac{V_c}{V_d}\right)^{\gamma}$$

Hence

$$\frac{P_c}{P_b} = \left(\frac{V_d}{V_c}\right)^{\gamma - 1}, \text{ and } V_d = V_c \left(\frac{P_c}{P_b}\right)^{\frac{1}{\gamma - 1}}.$$

Point D:

$$\left.\begin{matrix} T_d = T_b & (a) \\ V_d = V_c X^{\frac{1}{\gamma - 1}} & (b) \\ P_d = \dfrac{P_b}{X^{\frac{1}{\gamma - 1}}} & (c) \end{matrix}\right\} \quad \ldots \quad (1056)$$

$$Q_2 = \frac{1}{J} P_b V_b \log_e \frac{V_d}{V_b} = \frac{R T_b}{J} \log_e (X)^{\frac{1}{\gamma - 1}} = C_v T_b \log_e X, \quad (1057)$$

$$W = J(Q_1 - Q_2) = J(Q_1 - C_v T_b \log_e X), \quad \ldots \quad (1058)$$

$$E = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v T_b \log_e X}{Q_1}, \quad \ldots \quad (1059)$$

$$r_V = V_d - V_b = V_b(X^{\frac{1}{\gamma-1}} - 1), \quad \ldots \quad (1060)$$

$$r_P = P_c - P_d = P_b X - \frac{P_b}{X^{\frac{1}{\gamma-1}}} = P_b\left(X - \frac{1}{X^{\frac{1}{\gamma-1}}}\right), \quad \ldots \quad (1061)$$

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{J(Q_1 - C_v T_b \log_e X)}{144 V_b (X^{\frac{1}{\gamma-1}} - 1)}, \quad \ldots \quad (1062)$$

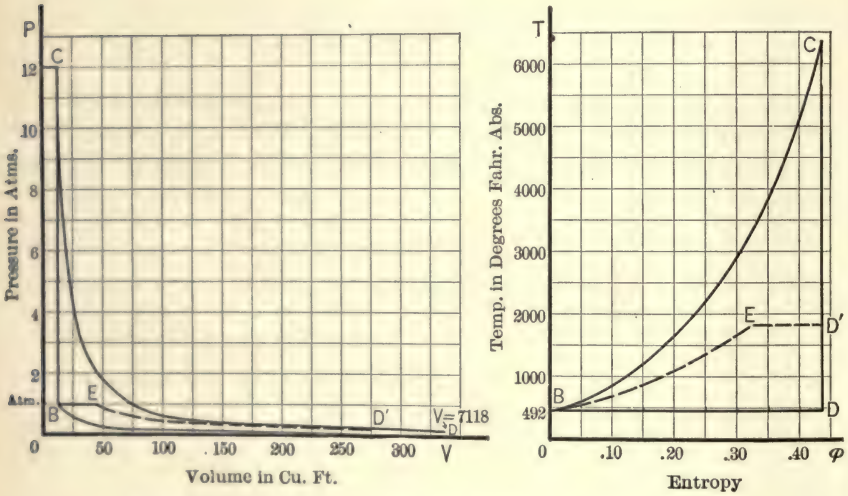


FIG. 252.—Otto and Langen Gas Cycle. Cycle III.

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E}, \quad \ldots \quad (1063)$$

$$\left\{\begin{matrix}\text{Cu.ft. gas per hr.} \\ \text{per I.H.P.}\end{matrix}\right\} = \frac{2545}{EH}, \quad \ldots \quad (1064)$$

Case b. Expansion to temperature lower than that at atmospheric pressure but higher than the initial, with constant-pressure cooling to complete the cycle BCD'EB, Fig. 252.

To locate the point E, which may be anywhere between B and the expansion line, it is most convenient to imagine the expansion proceeding to some fraction of the initial one atmosphere pressure, accordingly let $m = \frac{P_b}{P_{d'}}$, so that if expansion continues to half an atmosphere, $m = 2$.

Point D':

$$\left.\begin{aligned} P_{d'} &= \frac{P_b}{m}, && (a)\\ V_{d'} &= V_c\left(\frac{P_c}{P_{d'}}\right)^{\frac{1}{\gamma}} = V_b(Xm)^{\frac{1}{\gamma}}, && (b)\\ T_{d'} &= T_c\left(\frac{P_{d'}}{P_c}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_b}{m}(Xm)^{\frac{1}{\gamma}} && (c) \end{aligned}\right\} \quad \ldots\ldots \quad (1065)$$

Point E:

$$\left.\begin{aligned} P_e &= P_b && (a)\\ T_e &= T_{d'} = \frac{T_b}{m}(Xm)^{\frac{1}{\gamma}} && (b)\\ V_e &= V_b\frac{T_e}{T_b} = \frac{V_b}{m}(Xm)^{\frac{1}{\gamma}} && (c) \end{aligned}\right\} \quad \ldots\ldots \quad (1066)$$

whence

$$Q_2 = \frac{1}{J}P_eV_e\log_e\frac{P_e}{P_{d'}} + C_p(T_e - T_b)$$

$$= \frac{RT_b}{Jm}(Xm)^{\frac{1}{\gamma}}\log_e(m) + C_pT_b\left[\frac{(Xm)^{\frac{1}{\gamma}}}{m} - 1\right], \quad \ldots\ldots \quad (1067)$$

$$W = J(Q_1 - Q_2) = JQ_1 - \frac{RT_b}{m}(Xm)^{\frac{1}{\gamma}}\log_e(m) + JC_pT_b\left[\frac{(Xm)^{\frac{1}{\gamma}}}{m} - 1\right] \quad (1068)$$

$$E = 1 - \frac{Q_2}{Q_1} = 1 - \frac{RT_b}{JmQ_1}(Xm)^{\frac{1}{\gamma}}\log_e(m) + \frac{C_pT_b}{Q_1}\left[\frac{(Xm)^{\frac{1}{\gamma}}}{m} - 1\right]. \quad (1069)$$

$$r_V = V_{d'} - V_b = V_b[(Xm)^{\frac{1}{\gamma}} - 1], \quad \ldots\ldots \quad (1070)$$

$$r_P = P_c - P_{d'} = P_b\left(X - \frac{1}{m}\right), \quad \ldots\ldots \quad (1071)$$

$$(\text{m.e.p.}) = \frac{W}{144r_V} = \frac{JQ_1 - \frac{RT_b}{m}(Xm)^{\frac{1}{\gamma}}\log_e(m) + JC_pT_b\left[\frac{(Xm)^{\frac{1}{\gamma}}}{m} - 1\right]}{144V_b[(Xm)^{\frac{1}{\gamma}} - 1]}, \quad (1072)$$

$$\left\{\begin{matrix}\text{B.T.U. per}\\ \text{hr. per I.H.P.}\end{matrix}\right\} = \frac{2545}{E}, \quad \ldots\ldots \quad (1073)$$

$$\left\{\begin{matrix}\text{Cu.ft. gas}\\ \text{per hr. per}\\ \text{I.H.P.}\end{matrix}\right\} = \frac{2545}{EH}, \quad \ldots\ldots \quad (1074)$$

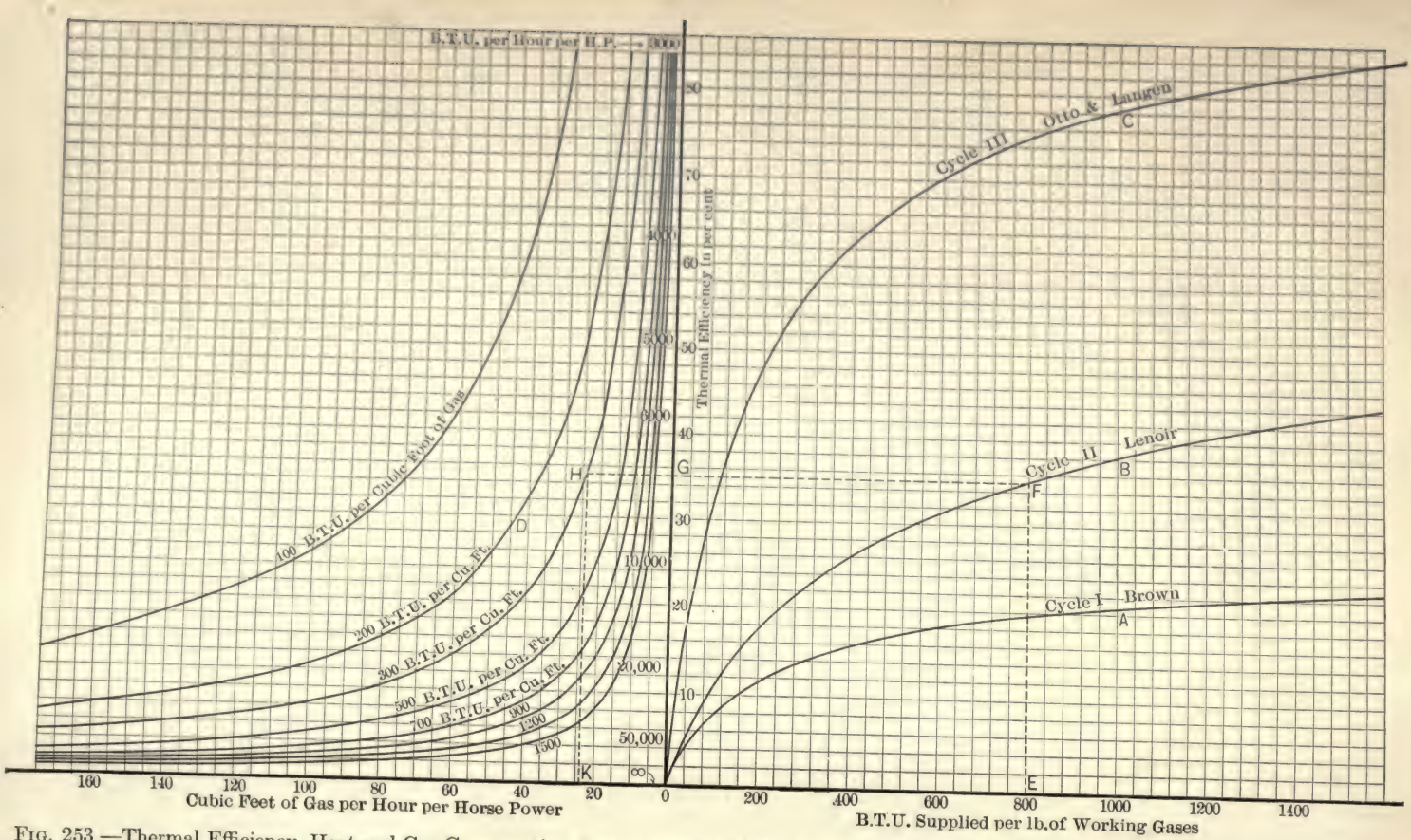
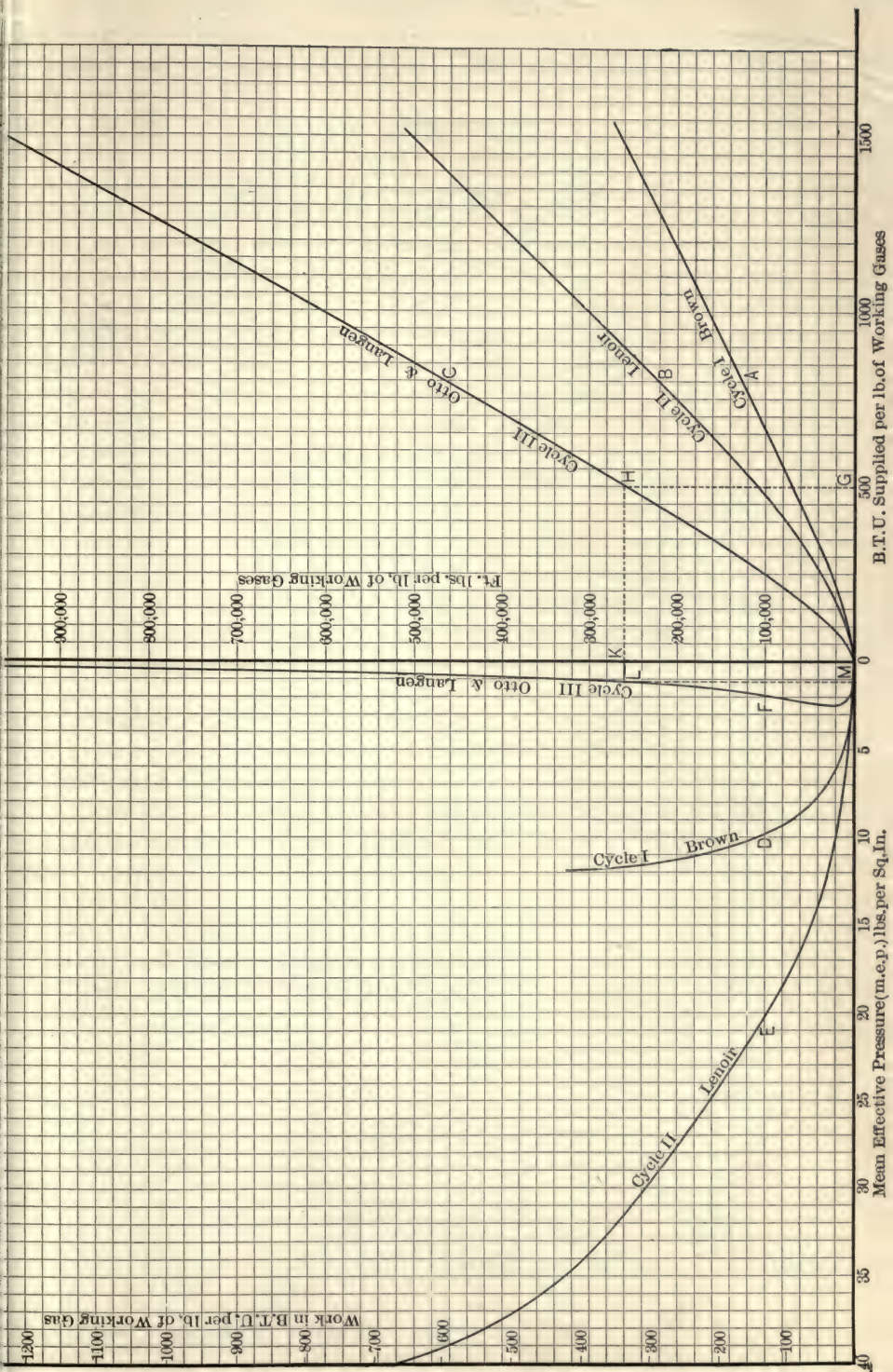


FIG. 253.—Thermal Efficiency, Heat and Gas Consumption for any amount of Heat Supplied per lb. of Working Gases, for the Non-compression and Gas Cycles, Brown, Lenoir, and Otto and Langen.


Mean Effective Pressure (m.e.p.) lbs. per Sq. In.
B.T.U. Supplied per lb. of Working Gases
Ft. lbs. per lb. of Working Gases
Work in B.T.U. per lb. of Working Gas
Cycle I Brown
Cycle II Lenoir
Cycle III Otto & Langen
A
B
C
D
E
F
G
H
J
K
M

The curves, Figs. 253 and 254, give efficiencies and mean effective pressures, and show the relation between the important quantities for the complete cycle in each case. They prove the inferiority of the Brown atmospheric cycle compared to the other two by reason, not only of its low efficiency, but also its small mean effective pressure, which requires large cylinders. For efficiency the Otto and Langen complete expansion cycle is superior to the Lenoir, but its mean effective pressure is very much too low to compete. Its should be noted that while the mean effective pressure of such cycles as the Lenoir is the work divided by the volume range, the mean effective pressure to be expected from the execution of even the perfect cycle in cylinders must be the work divided by the whole or maximum volume, because that part of the displacement up to explosion is used in getting the charge into the cylinder, so that the cylinder mean effective pressures will be smaller than those for the cycles in the ratio of volume range to maximum volume. Comparison of the performance of all these non-compression cycles will show them to be beneath consideration in comparison with compression cycles, to be analyzed next, in both efficiency and mean effective pressure. All these non-compression cycles show an increase of efficiency with heat supplied, so that for small heats supplied which would correspond to weak explosive mixtures or much excess air in any case of internal combustion, the efficiency would be less than for no excess air. This also is in striking contrast to the compression cycles, the efficiency of which is not only better but independent of the amount of heat supplied.

Referring to the efficiencies, Fig. 253, all three cycles show rising efficiency with increase of heat added, the Brown being lowest with the value of 20.8 per cent for 1000 B.T.U. per pound of gases, with Lenoir at 38.4 per cent and Otto and Langen at 78.4 per cent for the same heat supplied. For 200 B.T.U. per pound these three cyclic efficiencies fall off considerably, having the values 10.8 per cent, 18 per cent, and 48.4 per cent. These are all, however, high values and stimulate interest in what will be attainable in cycles with compression, which, it was pointed out, promise still higher results. It must be remembered, however, that in executing these gas cycles in engines realization has fallen far short of cyclic promise, due to excessive heat losses in the mechanism itself.

Example 1. *Calculation of Diagram of Brown gas cycle, Cycle I, Fig.* 250.

Assumed data

$P_a = 1$ atmosphere.
$V_a = 12.39$ cu.ft. (one lb. of air).
$T_a = 492°$ F. absolute.
$Q_1 = 1000$ B.T.U. added per pound of air.

Point B

$$P_b = P_a = 1 \text{ atm.}$$

$$T_b = T_a + \frac{Q_1}{C_p} = 492 + \frac{1000}{.239} = 4766° \text{ F. absolute.}$$

$$V_b = V_a \frac{T_b}{T_a} = \frac{4676}{492} \times 12.39 = 117.8 \text{ cu.ft.}$$

$$\phi_b - \phi_a = C_p \log_e \frac{T_b}{T_a} = .239 \log_e \frac{4676}{492} = .538.$$

Point C

$$\begin{cases} V_c = V_b = 117.8 \text{ cu.ft.} \\ T_c = T_a = 492° \text{ F. absolute.} \\ P_c = \dfrac{P_a V_a}{V_c} = \dfrac{1 \times 12.39}{117.8} = .105 \text{ atm.} \\ \phi_b - \phi_c = C_v \log_e \dfrac{T_b}{T_c} = .17 \log_e \dfrac{4676}{492} = .383. \\ \phi_c - \phi_a = .538 - .383 = .155. \end{cases}$$

Example 2. *Calculation of Diagram of Lenoir gas cycle, Cycle II, Fig.* 251. Data assumed as per Example 1.

Point C

$$\begin{cases} V_c = V_b = 12.39 \text{ cu.ft.} \\ T_c = T_b + \dfrac{Q_1}{C_v} = 492 + \dfrac{1000}{.17} = 6374° \text{ F. absolute.} \\ P_c = P_b \dfrac{T}{T_b} = \dfrac{6374}{492} = 12.96 \text{ atm.} \\ \phi_c - \phi_b = C_v \log_e \dfrac{T_c}{T_b} = .17 \log_e \dfrac{6374}{492} = .436. \end{cases}$$

Point D

$$\begin{cases} P_d = P_b = 1 \text{ atm.} \\ V_d = V_c \left(\dfrac{P_c}{P_d}\right)^{.713} = 12.39(12.96)^{.713} = 77 \text{ cu.ft.} \\ T_d = T_c \left(\dfrac{P_d}{P_c}\right)^{.286} = 6374 \left(\dfrac{1}{12.96}\right)^{.286} = 3056° \text{ F. absolute.} \\ \phi_d - \phi_b = \phi_c - \phi_b = .436. \end{cases}$$

Point D'

$$\begin{cases} V_{d'} = 45 \text{ cu.ft. (assumed)} \\ P_{d'} = P_c \left(\dfrac{V_c}{V_{d'}}\right)^{1.4} = 12.96 \left(\dfrac{12.39}{45}\right)^{1.4} = 2.13 \text{ atm.} \\ T_{d'} = T_c \left(\dfrac{V_c}{V_{d'}}\right)^{.4} = 6374 \left(\dfrac{12.39}{45}\right)^{.4} = 3803° \text{ F. absolute.} \\ \phi_{d'} - \phi_b = \phi_c - \phi_b = .436. \end{cases}$$

Point E

$$\begin{cases} V_e = V_{d'} = 45 \text{ cu.ft.} \\ P_e = P_b = 1 \text{ atm.} \\ T_e = T_{d'} \dfrac{P_e}{P_{d'}} = 3803 \times \dfrac{1}{2.13} = 1785° \text{ F. absolute.} \\ \phi_e - \phi_b = C_p \log_e \dfrac{T_e}{T_b} = .239 \log_e \dfrac{1785}{492} = .308. \end{cases}$$

Example 3. *Calculation of Diagram of Otto and Langen gas cycle, Cycle III, Fig.* 252. Data assumed as before in Example 1 and point C same as for Lenoir cycle of Example 2:

Point D

$$T_d = T_b = 492° \text{ F. absolute.}$$

$$V_d = V_c\left(\frac{T_c}{T_d}\right)^{2.48} = 12.39\left(\frac{6374}{492}\right)^{2.48} = 7118 \text{ cu.ft.}$$

$$P_d = P_b\frac{V_b}{V_d} = \frac{12.39}{7118} = .0017 \text{ atm.}$$

$$\phi_d - \phi_b = \phi_c - \phi_b = .436.$$

Point E

$$V_c = 45 \text{ cu.ft. (assumed).}$$

$$P_e = P_b = 1 \text{ atm.}$$

$$T_e = T_b\frac{V_e}{V_b} = 492 \times \frac{45}{12.39} = 1786° \text{ F. abs.}$$

$$\phi_e - \phi_b = C_p \log_e \frac{T_e}{T_b} = .239 \log_e \frac{1786}{492} = .30^{\circ}.$$

Point D'

$$T_{d'} = T_e = 1786° \text{ F. absolute.}$$

$$V_{d'} = V_c\left(\frac{T_c}{T_{d'}}\right)^{2.48} = 12.39\left(\frac{6374}{1786}\right)^{2.48} = 291.7 \text{ cu.ft.}$$

$$P_{d'} = P_e\frac{V_e}{V_{d'}} = \frac{45}{291.7} = .154 \text{ atm.}$$

$$\phi_{d'} - \phi_b = \phi_c - \phi_b = .436.$$

Example 4. *Calculation and use of Diagram, Fig.* 253, *giving for the non-compression gas cycles Brown, Lenoir, and Otto and Langen, the thermal efficiency, heat, and gas consumption as a function of heat supplied.* One point will be calculated for each, A for Brown, B for Lenoir, and C for Otto and Langen, all for 1000 B.T.U. added per pound of working gases assumed to be air, initially at one atmosphere pressure, and 32° F.

From Eq. (1030), $E_a = 1 - \frac{1}{\gamma} - \frac{RT_a}{JQ_1}\log_e\left(1 + \frac{Q_1}{C_pT_a}\right)$

$$= 1 - \frac{1}{1.4} - \frac{53.3 \times 492}{778 \times 1000}\log_e\left(1 + \frac{1000}{.239 \times 492}\right) = 21.1 \text{ per cent.}$$

From Eq. (1040), $E_b = 1 - \frac{C_pT_b}{Q_1}\left[\left(1 + \frac{Q_1}{C_vT_b}\right)^{\frac{1}{\gamma}} - 1\right]$

$$= 1 - \frac{.239 \times 492}{1000}\left[\left(1 + \frac{1000}{.17 \times 492}\right)^{.713} - 1\right] = 38.3 \text{ per cent.}$$

From Eq. (1059), $E_c = 1 - \frac{C_v T_b}{Q_1} \log_e \left(1 + \frac{Q_1}{C_v T_b}\right)$

$$= 1 - \frac{.17 \times 492}{1000} \log_e \left(1 + \frac{1000}{.17 \times 492}\right) = 78.6 \text{ per cent.}$$

The location of one point D in the left-hand angle will serve to illustrate all the rest. From Eq. (1035) for the Brown cycle, and assuming the fuel gas to have 200 B.T.U. per cubic foot:

$$\text{Cubic feet fuel gas per hour per H.P.} = \frac{2545}{EH} = \frac{2545}{.30 \times 200} = 42.41 \text{ cu.ft.}$$

To illustrate the use of the diagram find for a Lenoir cycle receiving 800 B.T.U. per pound of working gases the thermal efficiency, heat consumption, and cubic feet of 300 B.T.U. per cubic foot fuel gas per hour per I.H.P. From the 800 point E pass vertically to point F on the Lenoir curve and thence horizontally to G on the efficiency scale, reading 35.2 per cent and heat consumption, 7250 B.T.U. per hour per I.H.P. Passing across to the 300 B.T.U. calorific power curve at H and down to K, the gas consumption is found to be 24 cu.ft. per hour per I.H.P.

Example 5. *Calculation and use of the Diagram, Fig.* 254, *giving for the non-compression gas cycles Brown, Lenoir, and Otto and Langen, the work per pound of gases and mean effective pressure.* Three points will be calculated: A, for Brown; B, for Lenoir; and C, for Otto and Langen, all for the initial conditions of Example 4, and 800 B.T.U. supplied.

From Eq. (1029),

$$W_a = JQ_1\left(1 - \frac{1}{\gamma}\right) - RT_a \log_e \left(1 + \frac{Q_1}{C_p T_a}\right)$$

$$= 778 \times 800 \times (1 - .713) - 53.3 \times 492 \log_e \left(1 + \frac{800}{.239 \times 492}\right) = 124{,}400 \text{ ft.-lbs.}$$

From Eq. (1039),

$$W_b = J\left\{Q_1 - C_p T_b \left[\left(1 + \frac{Q_1}{C_v T_b}\right)^{\frac{1}{\gamma}} - 1\right]\right\}$$

$$= 778(800 - .239 \times 492)\left[\left(1 + \frac{800}{.17 \times 492}\right)^{.713} - 1\right] = 226{,}100 \text{ ft.-lbs.}$$

From Eq. (1058),

$$W_c = J\left[Q_1 - C_v T_b \log_e \left(1 + \frac{Q_1}{C_v T_b}\right)\right]$$

$$= 778\left[800 - .17 \times 492 \log_e \left(1 + \frac{800}{.17 \times 492}\right)\right] = 469{,}000 \text{ ft.-lbs.}$$

In the left-hand angle the point D representing by its abscissa the mean effective pressure for the Brown cycle is located by Eq. (1033). Taking a point for 100,000 ft.-lbs. work done,

$$(\text{m.e.p})_d = \frac{W}{144 r_V} = \frac{W}{144J\left(1-\frac{1}{\gamma}\right)\frac{Q_1}{P_a}} = \frac{100000}{144\times 778(1-.713)\frac{675}{215}} = 9.75 \text{ lbs. per square inch.}$$

The corresponding value for the Lenoir cycle point E is located by Eq. (1043):

$$(\text{m.e.p.})_e = \frac{W}{144 r_V} = \frac{W}{144 V_b\left[\left(1+\frac{Q_1}{C_v T_b}\right)^{\frac{1}{\gamma}} - 1\right]}$$

$$= \frac{100000}{144\times 12.3\left[\left(1+\frac{470}{.17\times 492}\right)^{.713} - 1\right]} = 19.68 \text{ lbs. per square inch.}$$

For the Otto and Langen the mean effective pressure for the same cyclic work of 100,000 ft.-lbs. is represented by point F and given by Eq. (1062):

$$(\text{m.e.p.})_f = \frac{W}{144 V_b\left[\left(1+\frac{Q_1}{C_v T_b}\right)^{\frac{1}{\gamma-1}} - 1\right]}$$

$$= \frac{100000}{144\times 12.39\left[\left(1+\frac{247}{.17\times 492}\right)^{.248} - 1\right]} = 1.855 \text{ lbs. per inch.}$$

Illustrating the use of diagram, Fig. 254, the solution of the following problem is offered: Find the work per pound of working gases and the mean effective pressure for an Otto and Langen cycle receiving 500 B.T.U. per pound of gases. Starting at the 500 B.T.U. point G, pass up to the cycle curve at H and then across to the point K on the work scale, reading 260,000 ft.-lbs. Passing horizontally across the point L and thence downward to point M the mean effective pressure is found to be 1.18 lbs. per square inch.

Prob. 1. Find the temperature at which expansion ends for (*a*) the Brown and (*b*) the Lenoir cycles, receiving 300 B.T.U. per pound of gases, at an initial temperature of 160° F.

Prob. 2. How much heat must be abstracted per pound of gases of Problem 1, and what is the corresponding thermal efficiency?

Prob. 3. Compare the maximum and minimum pressures for the Brown, Lenoir, and Otto and Langen cycles receiving 700 B.T.U. per pound from an initial temperature of 200° F.

Prob. 4. What effect will high altitude where the atmospheric pressure is .7 the standard have on the thermal efficiency, work per cycle, and mean effective pressure for these three cycles?

Prob. 5. If a Lenoir cycle is to be carried out in a cylinder, with 500 B.T.U. per pound of working gases, and realizing 30 per cent of the cyclic efficiency, all heat losses taking place during the rise of pressure, what thermal efficiency may be expected and what mean effective pressure?

Prob. 6. Compare the thermal efficiency of a perfectly executed Otto and Langen cycle for 400 B.T.U. supplied per pound of gases with another in which expansion proceeds only to half an atmosphere.

Prob. 7. With the same data, Problem 6, make a comparison of mean effective pressures.

Prob. 8. Compare efficiencies of two Lenoir cycles, one for perfect expansion and the other expanding to half the maximum pressure, in each case for 200 B.T.U. supplied.

Prob. 9. With the same data of Problem 8 compare the mean effective pressures.

Prob. 10. If a Lenoir engine used 40 cu.ft. of 600 B.T.U. gas per hour per I.H.P. requiring seven parts of air to burn it by weight, what fraction of its cyclic efficiency was realized?

10. Stirling and Ericsson Cycles. Work, Efficiency and Derived Quantities for Isothermal Compression Gas Cycles, with and without Regenerators.

Stirling Cycle IV

There are two cases to be considered for this cycle, Fig. 255, first, that dealing with the heats involved as a whole, and second, that dealing with the heat derived from the source of supply, separately considered from the regenerator heat which is alternately added and abstracted and by hypothesis without loss. So far as the diagram points themselves are concerned, there is no difference between the two, and this is also the case for work and mean effective pressure.

Let Q_1 = the whole heat supplied from B to C to D;
" Q_1' = the heat supplied from the fire from C to D;
" Q_1'' = the heat supplied from regenerator from B to C:
" Q_2 = the whole heat abstracted from D to A to B;
" Q_2' = the heat abstracted by water jacket from A to B;
" Q_2'' = heat abstracted by regenerator from D to A;
" C = compression in atmospheres $= \frac{P_b}{P_a}$;
" E = real thermal efficiency of whole cycle;
" E^1 = thermal efficiency of cycle referred to heat from source of supply (C to D).

Point B:

$$\left.\begin{aligned} T_b &= T_a \quad (a) \\ P_b &= P_a C \quad (b) \\ V_b &= V_a \frac{P_a}{P_b} = \frac{V_a}{C} \quad (c) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (1075)$$

Point C:

$$\left.\begin{aligned} V_c &= V_b = \frac{V_a}{C} && (a)\\ T_c &= T_b + \frac{Q_1''}{C_v} = T_a\left(1+\frac{Q_1''}{C_v T_a}\right) = T_a Z && (b)\\ P_c &= P_b\frac{T_c}{T_b} = P_b Z = P_a C Z && (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1076)$$

where

$$Z = 1+\frac{Q_1''}{C_v T_a}.$$

Point D:

$$\left.\begin{aligned} T_d &= T_c = T_a Z && (a)\\ V_d &= V_a && (b)\\ P_d &= P_a\frac{T_d}{T_a} = P_a Z && (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1077)$$

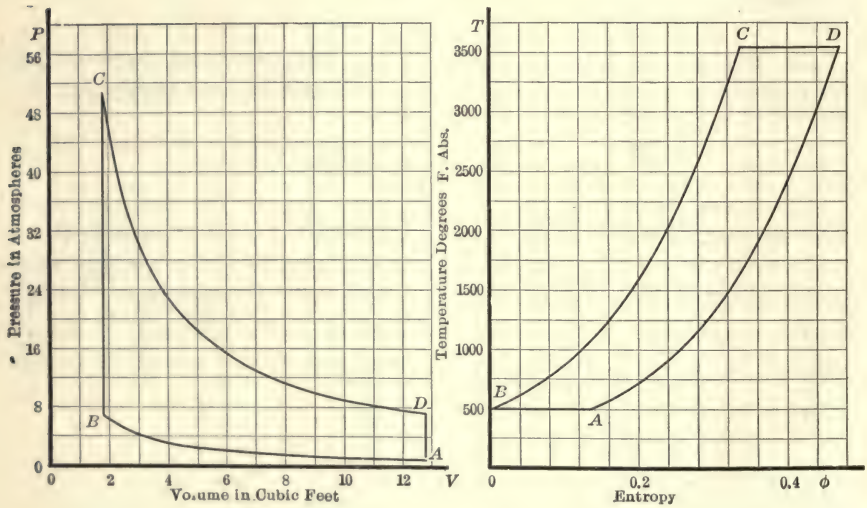


FIG. 255.—Stirling Gas Cycle. Cycle IV.

$$Q_2 = Q_2' + Q_2'' = \frac{1}{J}P_a V_a \log_e \frac{V_a}{V_b} + C_v(T_d - T_a)$$

$$= \frac{R}{J}T_a \log_e C + C_v(T_d - T_a). \quad . \quad . \quad . \quad . \quad (1078)$$

$$Q_1 = Q_1' + Q_1'' = \frac{1}{J}P_c V_c \log_e \frac{V_d}{V_c} + C_v(T_c - T_b)$$

$$= \frac{R}{J}T_d \log_e C + C_v(T_d - T_a). \quad . \quad . \quad . \quad . \quad (1079)$$

$$W = J(Q_1 - Q_2) = R(T_d - T_a) \log_e C. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1080)$$

$$E = \frac{Q_1 - Q_2}{Q_1} = \frac{\frac{1}{J}R(T_d - T_a) \log_e C}{\frac{1}{J}RT_d \log_e C + C_v(T_d - T_a)} \quad . \quad . \quad . \quad . \quad (1081)$$

$$E' = \frac{Q_1 - Q_2}{Q_1'} = \frac{\frac{1}{J}R(T_d - T_a) \log_e C}{\frac{1}{J}RT_d \log_e C}$$

$$= \frac{T_d - T_a}{T_d} = 1 - \frac{T_a}{T_d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1082)$$

This efficiency is therefore equal to the whole temperature range divided by the maximum temperature.

$$r_V = V_a - V_b = V_a\left(1 - \frac{1}{C}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1083)$$

$$r_P = P_c - P_a = P_a(CZ - 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1084)$$

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{R(T_d - T_a) \log_e C}{144 V_a\left(1 - \frac{1}{C}\right)} \quad . \quad . \quad . \quad . \quad (1085)$$

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E'} = \frac{2545 T_d}{T_d - T_a} \quad . \quad . \quad . \quad . \quad . \quad (1086)$$

As this cycle is executed on enclosed air and not by internal combustion of gas mixtures, the gas consumption has not the same significance as in the other cycles. If fuel is burned externally, the consumption per hour per H.P. will be measured only partly by the above value of heat consumption, which is only the heat that reaches the enclosed air, the rest of the generation escaping from the furnace, so that the other factor in fuel consumption will be a furnace efficiency which may be quite low and defined as the ratio of the heat appearing in the cycle along *CD*, Fig. 255, to the amount generated in the fire. While these engines had, as is indicated by Eq. (1082), a high cyclic efficiency, external combustion characteristics gave a higher fuel consumption than ought to correspond. This condition also applies to the following Ericsson cycle.

Ericsson Cycle V

This cycle when executed with regenerators alternately raising the temperature from B to C, Fig. 256, and lowering it from D to A, may, like the Stirling, have two efficiencies, one for the cycle regardless of the source of the heat, and the other considering only the heat received from the fire source.

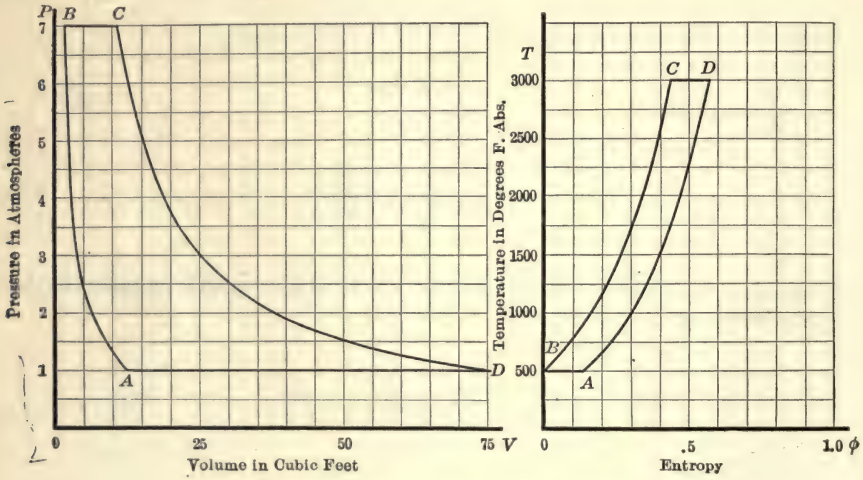


Fig. 256.—Ericsson Gas Cycle. Cycle V.

The position of point B is the same as for the Stirling, and its coordinates need not be set down.

Point C:

$$\left.\begin{aligned} P_c &= P_b = P_a C && (a)\\ T_c &= T_b + \frac{Q_1''}{C_p} = T_a\left(1+\frac{Q_1''}{C_p T_a}\right) = T_a Z' && (b)\\ V_c &= V_b\frac{T_c}{T_b} = \frac{V_a}{C}Z' && (c) \end{aligned}\right\} \quad \ldots\ldots \quad (1087)$$

where

$$Z' = 1+\frac{Q_1''}{C_p T_a}.$$

Point D:

$$\left.\begin{aligned} P_d &= P_a && (a)\\ T_d &= T_c = T_a Z' && (b)\\ V_d &= V_a\frac{T_d}{T_a} = V_a Z' && (c) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (1088)$$

$$Q_2 = Q_2' + Q_2'' = \frac{1}{J} P_a V_a \log_e \frac{V_a}{V_b} + C_p(T_d - T_a)$$

$$= \frac{R}{J} T_a \log_e C + C_p(T_d - T_a). \quad \ldots \ldots \quad (1089)$$

$$Q_1 = Q_1' + Q_1'' = \frac{R}{J} T_d \log_e C + C_p(T_d - T_a) \quad \ldots \ldots \quad (1090)$$

$$W = J(Q_1 - Q_2) = R(T_d - T_a) \log_e C \quad \ldots \ldots \quad (1091)$$

$$E = \frac{Q_1 - Q_2}{Q_1} = \frac{\frac{R}{J}(T_d - T_a) \log_e C}{\frac{1}{J} R T_d \log_e C + C_p(T_d - T_a)} \quad \ldots \quad (1092)$$

$$E' = \frac{Q_1 - Q_2}{Q_1'} = \frac{T_d - T_a}{T_d} = 1 - \frac{T_a}{T_d}. \quad \ldots \ldots \quad (1093)$$

$$r_V = V_d - V_b = V_a\left(Z - \frac{1}{C}\right). \quad \ldots \ldots \quad (1094)$$

$$r_P = P_b - P_a = P_a(C - 1). \quad \ldots \ldots \quad (1095)$$

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{R(T_d - T_a) \log_e C}{144 V_a\left(Z - \frac{1}{C}\right)}. \quad \ldots \ldots \quad (1096)$$

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E'} = \frac{2545 T_d}{T_d - T_a}. \quad \ldots \ldots \quad (1097)$$

To show the important relations between the work done, or the efficiency of transformation, and the two prime variables, quantity of heat supplied per pound of working gases and the amount of compression, requires a series of curves, and four pairs would be necessary, some of which are omitted to save space as the practical value of these cycles is now small though their scientific value is great. The first pair, Fig. 257 and Fig. 258, for the Stirling and Ericsson respectively, show efficiencies and corresponding heat consumption in the working gas, and as supplied in the fire, for various furnace efficiencies, plotted to B.T.U. supplied per pound of working gases as the prime variable, three compressions, 5, 10 and 30 atmospheres each being represented by one curve. As heat supplied to the working gases may be derived from either fire alone, or from fire and regenerator together, a double heat-supplied scale is necessary and each curve must be referred to its appropriate scale. These two sets show a very high possible efficiency when the fire

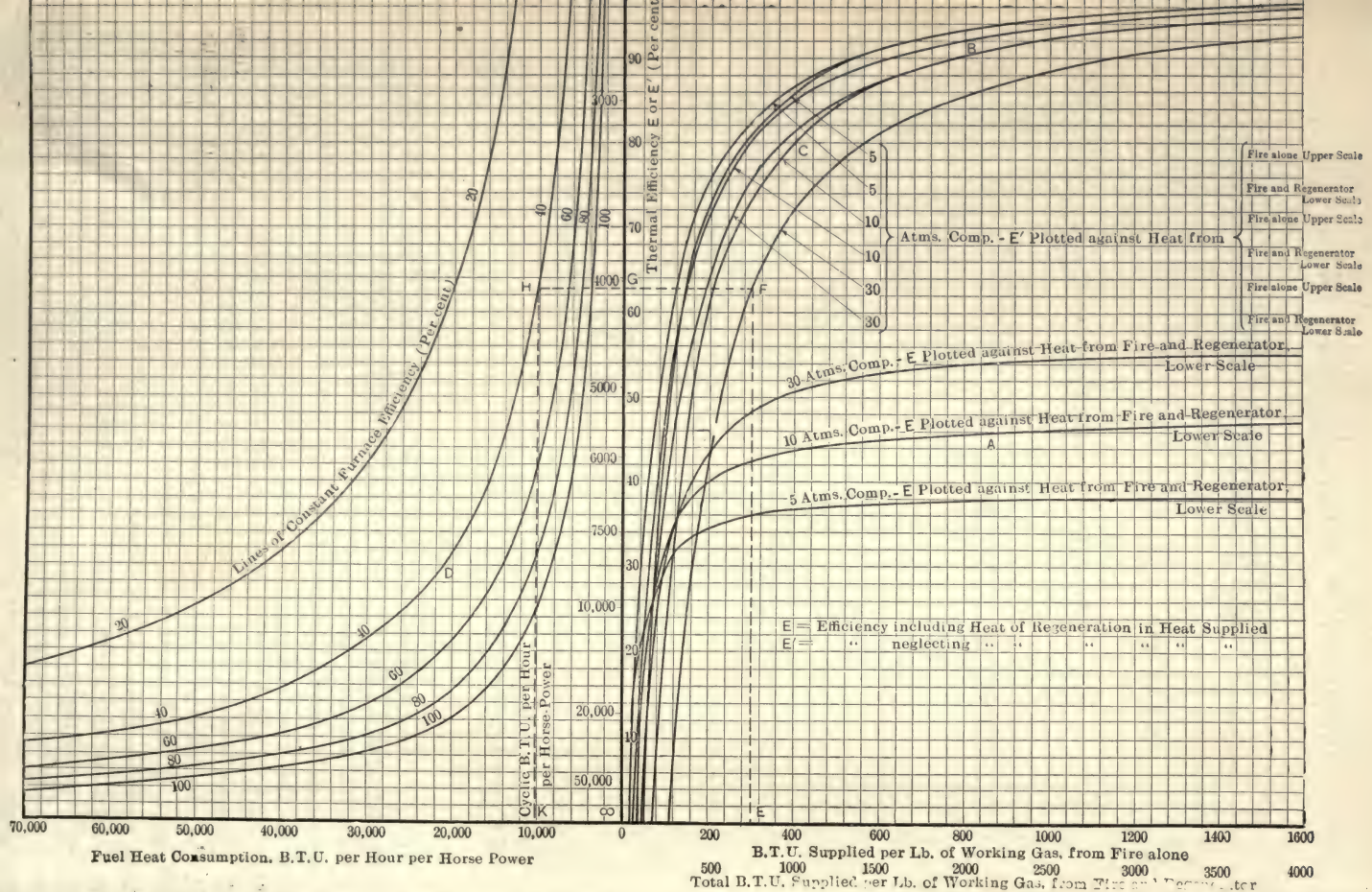
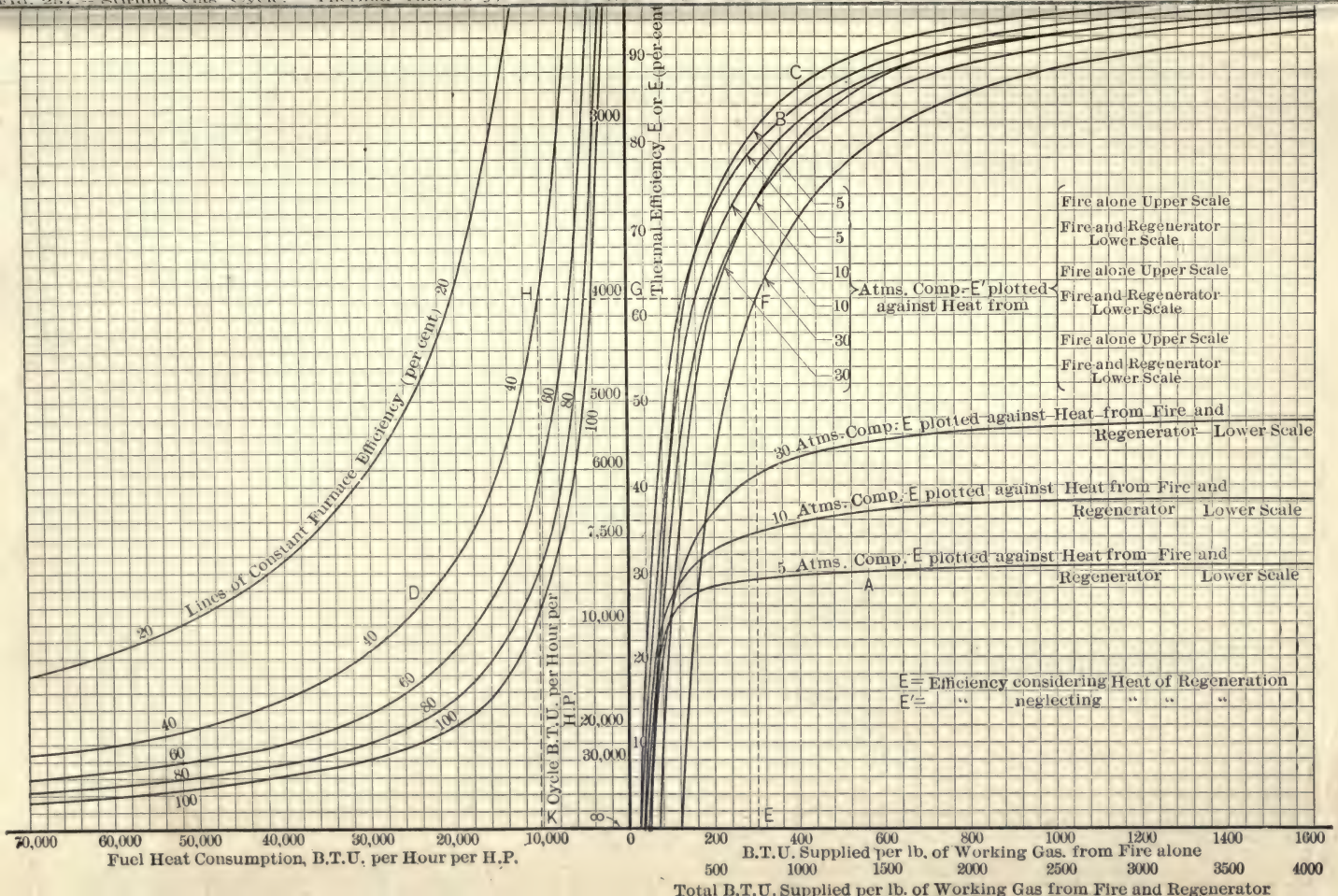
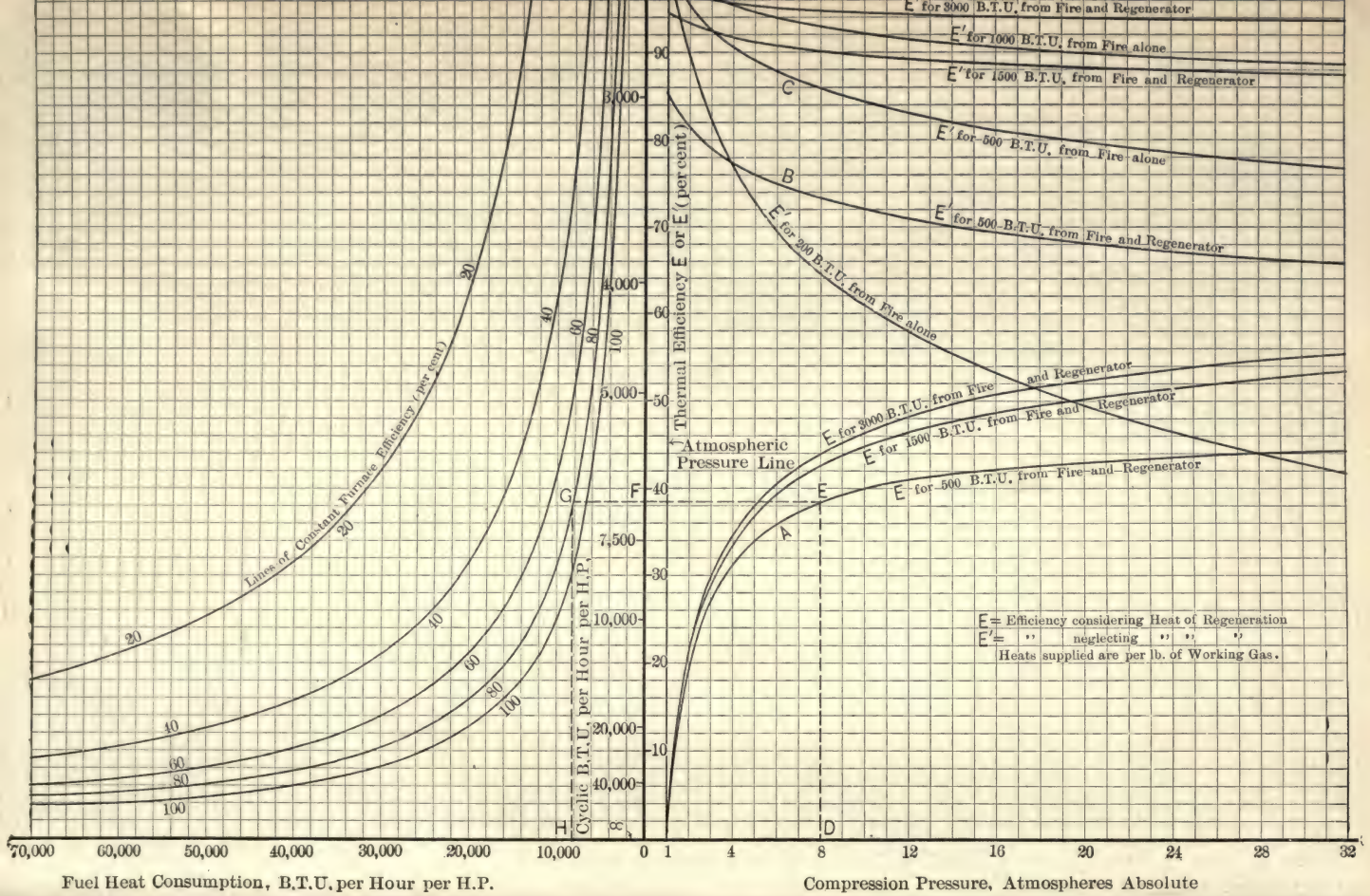
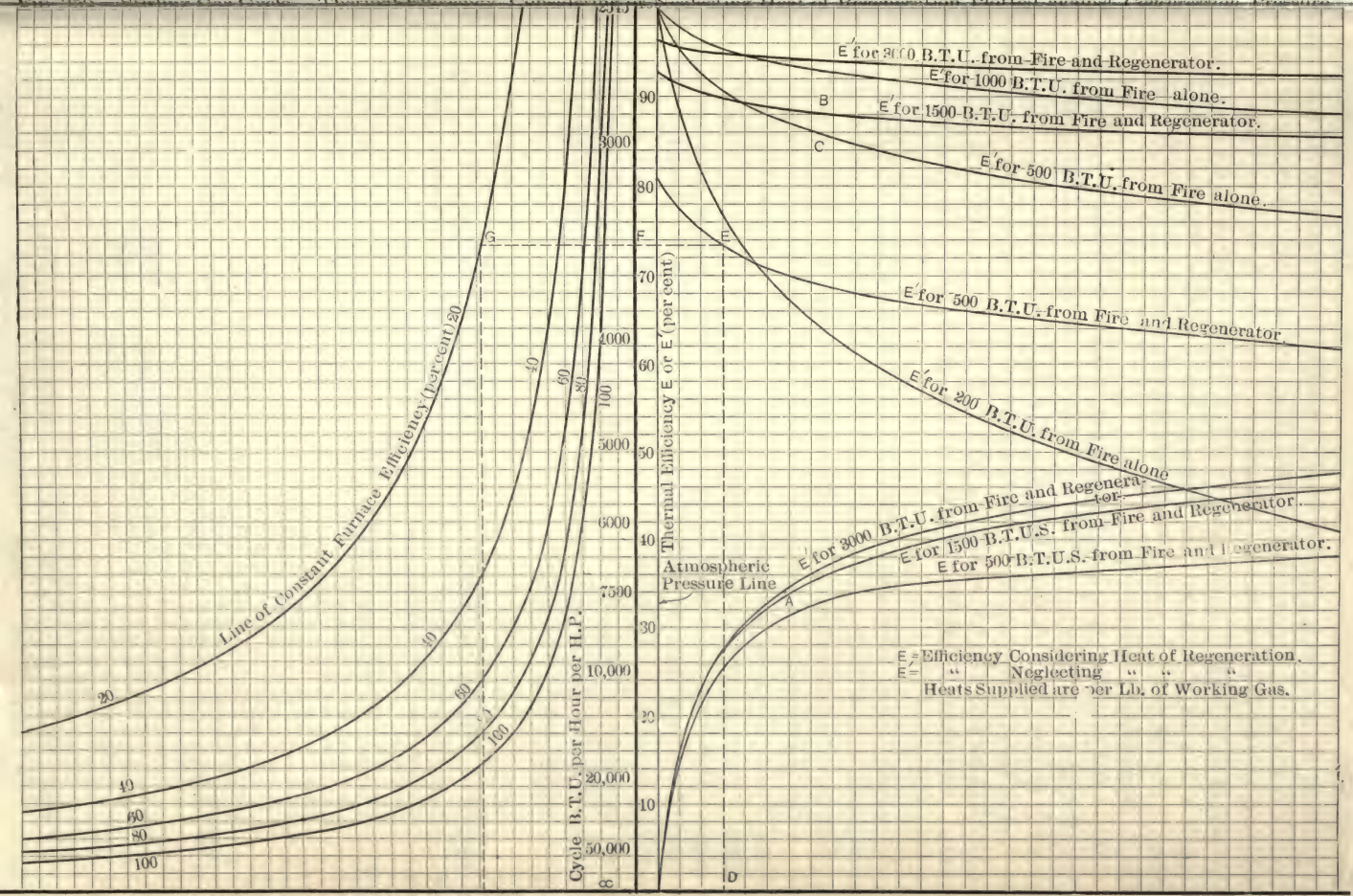


FIG. 257.—Stirling Gas Cycle. Thermal Efficiency, Considering or Neglecting Heat of Regeneration, Plotted against *Heat* from Fire or


Thermal Efficiency E or E' per cent
Lines of Constant Furnace Efficiency (per cent.)
20
40
60
80
100
Cycle B.T.U. per Hour per H.P.
3000
4000
5000
6000
7,500
10,000
20,000
30,000
∞
A
B
C
D
E
F
G
H
5
5
10
10
30
30
Atms. Comp. E' plotted against Heat from
Fire alone Upper Scale
Fire and Regenerator Lower Scale
Fire alone Upper Scale
Fire and Regenerator Lower Scale
Fire alone Upper Scale
Fire and Regenerator Lower Scale
30 Atms. Comp. E plotted against Heat from Fire and Regenerator Lower Scale
10 Atms. Comp. E plotted against Heat from Fire and Regenerator Lower Scale
5 Atms. Comp. E plotted against Heat from Fire and Regenerator Lower Scale
E = Efficiency considering Heat of Regeneration
E' = " neglecting " " "
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
Fuel Heat Consumption, B.T.U. per Hour per H.P.
200
400
600
800
1000
1200
1400
1600
B.T.U. Supplied per lb. of Working Gas. from Fire alone
500
1000
1500
2000
2500
3000
3500
4000
Total B.T.U. Supplied per lb. of Working Gas from Fire and Regenerator


E for 3000 B.T.U. from Fire and Regenerator
E′ for 1000 B.T.U. from Fire alone
E′ for 1500 B.T.U. from Fire and Regenerator
E′ for 500 B.T.U. from Fire alone
E′ for 500 B.T.U. from Fire and Regenerator
E′ for 200 B.T.U. from Fire alone
E for 3000 B.T.U. from Fire and Regenerator
E for 1500 B.T.U. from Fire and Regenerator
E for 500 B.T.U. from Fire and Regenerator
Atmospheric Pressure Line
Thermal Efficiency E or E′ (per cent)
Cyclic B.T.U. per Hour per H.P.
Lines of Constant Furnace Efficiency (per cent)
E = Efficiency considering Heat of Regeneration
E′ = " neglecting " " "
Heats supplied are per lb. of Working Gas.
Fuel Heat Consumption, B.T.U. per Hour per H.P.
Compression Pressure, Atmospheres Absolute


E for 3000 B.T.U. from Fire and Regenerator.
E for 1000 B.T.U. from Fire alone.
E for 1500 B.T.U. from Fire and Regenerator.
E for 500 B.T.U. from Fire alone.
E for 500 B.T.U. from Fire and Regenerator.
E for 200 B.T.U. from Fire alone
E for 3000 B.T.U. from Fire and Regenerator.
E for 1500 B.T.U.S. from Fire and Regenerator.
E for 500 B.T.U.S. from Fire and Regenerator.
Line of Constant Furnace Efficiency (per cent) 20
40
60
80
100
Thermal Efficiency E or E′ (per cent)
Atmospheric Pressure Line
Cycle B.T.U. per Hour per H.P.
E = Efficiency Considering Heat of Regeneration.
E = " Neglecting " " "
Heats Supplied are per Lb. of Working Gas.
A
B
C
D
E
F
G
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
1
4
8
12
16
20
24
28
32
3000
2000
5000
6000
7500
10,000
20,000
50,000
∞
90
80
70
60
50
40
30
20
10

source of heat is considered alone, efficiencies rising to over 90 per cent, whereas half of this value approximately corresponds to charging all the heat supplied to gases. A regenerator of less than 100 per cent efficiency as here considered would yield efficiencies somewhere between these two limits. No difference in the efficiency of the two cycles is found when regenerator heat is not charged, but with it charged the Stirling has a higher efficiency than the Ericsson, the figures being 44 per cent and 36 per cent for 1000 B.T.U. added with 10 atmospheres compression.

The second pair of curves, Fig. 259 and Fig. 260, show efficiencies as functions of compression for various arbitrarily chosen quantities of heat added, as derived from fire alone, or from both fire and regenerator together. The third and fourth pairs necessary for a whole series of curves would give work per pound of gases and the corresponding mean effective pressures, first with heat supplied, and second with compression as the prime variables, but these are here omitted.

As actually used in engines these cycles were operated with very low compression, about three atmospheres and with very small quantities of heat supplied, such as correspond to maximum temperatures not over 800° F., and as the curves show, rather lower but still fair efficiencies are obtainable, though mean effective pressures are hopelessly low compared to modern engines. These curves have been extended beyond the working ranges used in engines that have been built because they show possibilities that are truly remarkable though not yet realizable. It would require a somewhat severe stretch of imagination to picture a steam cycle offering any such efficiencies as are here shown—over 90 per cent—and while these prospects have as yet not been fruitful they certainly point a way for further investigation when high economy must be attained.

Example 1. *Calculation of Diagram, Fig.* 255, *the Stirling gas cycle, Cycle IV.* The original condition assumed for point A is, $P_a = 1$ atm., $V_a = 12.39$ cu.ft. (one pound air), $T_a = 492°$ F. absolute, and in addition it is assumed that there is added to the cycle 1000 B.T.U. and that the compression, $C = \frac{P_b}{P_a} = 7$.

Point B

$$\begin{cases} P_b = 7 \quad \text{(assumed).} \\ V_b = V_a\left(\frac{P_a}{P_b}\right) = 12.39 \times \left(\frac{1}{7}\right) = 1.77 \text{ cu.ft.} \\ T_b = T_a = 492° \text{ F. absolute.} \\ \phi_a - \phi_b = (C_p - C_v) \log_e \left(\frac{V_a}{V_b}\right) = .0686 \log_e \frac{12.38}{1.77} = .133. \end{cases}$$

The whole heat added $Q_1 = 1000$ B.T.U. is divided between the lines BC and CD, only the former part raising temperatures from B to C, so to determine the temperature

at C, this part must be first found by subtracting from 1000 B.T.U. the heat added in the isothermal expansion, which is

$$\frac{R}{J} T_c \log_e \frac{V_d}{V_c}.$$

$$Q_1 = 1000 = C_v(T_c - T_b) + (C_p - C_v) T_c \log_e \frac{V_d}{V_c}$$

$$= .17(T_c - 492) + .0686 T_c \log_e 7, \text{ B.T.U.}$$

Point C

$$\begin{cases} T_c = 3568° \text{ F. absolute.} \\ V_c = V_b = 12.39 \text{ cu.ft.} \\ P_c = P_b \dfrac{T_c}{T_b} = 7 \times \dfrac{3568}{492} = 50.8 \text{ atm.} \\ \phi_c - \phi_b = C_v \log_e \dfrac{T_c}{T_b} = .17 \log_e \dfrac{3568}{492} = \mathbf{.337.} \end{cases}$$

Point D

$$\begin{cases} V_d = V_a = 12.39 \text{ cu.ft.} \\ T_d = T_c = 3568° \text{ F. absolute.} \\ P_d = P_c \dfrac{V_c}{V_d} = \dfrac{50.8}{7} = 7.26 \text{ atm.} \\ \phi_d - \phi_c = (C_p - C_v) \log_e \dfrac{V_d}{V_c} = .133. \end{cases}$$

Example 2. *Calculation of Diagram, Fig.* 256, *the Ericsson gas cycle, Cycle V.* The initial condition of point A, the heat added, and the compression, are taken as for the last example, as is also the condition at point B. The first determination necessary is therefore the temperature at point C, which is found as follows:

$$Q_1 = 1000 = C_p(T_c - T_b) + (C_p - C_v) T_c \log_e C$$

$$= .239(T_c - 492) + .0686 T_c \log_e 7, \text{ B.T.U.}$$

Point C

$$\begin{cases} T_c = 3002° \text{ F. absolute.} \\ P_c = P_b = 7 \text{ atm.} \\ V_c = V_b \dfrac{T_c}{T_b} = 1.77 \times \dfrac{3002}{492} = 10.8 \text{ cu.ft.} \\ \phi_c - \phi_b = C_p \log_e \dfrac{T_c}{T_b} = .239 \log_e \dfrac{3002}{492} = \mathbf{.432.} \end{cases}$$

Point D

$$\begin{cases} P_d = P_a = 1 \text{ atm.} \\ T_d = T_c = 3002° \text{ F. absolute.} \\ V_d = V_c \dfrac{P_c}{P_d} = 10.8 \times 7 = 75.6 \text{ cu.ft.} \\ \phi_d - \phi_c = (C_p - C_v) \log_e C = .565. \end{cases}$$

Example 3. *Calculation and use of Diagrams, Figs.* 257, 258, 259. *and* 260, *giving for the Stirling and the Ericsson cycles, efficiency and heat consumption for any compressions and for any amount of heat added.* For any fixed cycle there may be two thermal efficiencies, so for the same data there will be two efficiency curves, the location of which is illustrated by the determination of one point for each. First, charging against the work done all the heat received by the cycle whether from regenerator, or fire, the efficiency will be as given by the curve carrying point A, for 10 atm. compression, referred to the lower scale of heat supplied. From Eq. (1081),

$$E=\frac{Q_1-Q_2}{Q_1}=\frac{\frac{R}{J}(T_d-T_a)\ \log_e C}{Q_1},$$

but T_d is itself a function of Q_1 given by Eq. (1079),

$$Q_1=\frac{R}{J}T_d\log_e C+C_v(T_d-T_a)$$

whence

$$T_d=\frac{Q_1+C_vT_b}{C_v+(C_p-C_v)\log_e C}.$$

Substituting this in the efficiency expression and solving, for $C=10$ and $Q_1=2000$,

$$E_a=\frac{R}{JQ_1}\left(\frac{Q_1+C_vT_a}{C_v+(C_p-C_v)\log_e C}-T_a\right)\log_e C$$

$$=\frac{53.3}{778\times2000}\left(\frac{2000+.17\times492}{.17+(.239-.17)\log_e 10}-492\right)\log_e 10=46.17\text{ per cent.}$$

Charging against the work done, only such heat as is added directly from the fire during isothermal expansion the efficiency is given by the curve carrying the point B for 10 atm. compression and 2000 B.T.U. total, the value of the efficiency is given by Eq. (1082),

$$E'=\frac{Q_1-Q_2}{Q_1'}=\frac{Q_1-Q_2}{Q_1-Q_1''},$$

whence

$$E'=\frac{\frac{R}{J}(T_d-T_a)\log_e C}{\frac{R}{J}T_d\log_e C}=\frac{\frac{R}{J}\left(\frac{Q_1+C_vT_a}{C_v+(C_p-C_v)\log_e C}-T_a\right)\log_e C}{Q_1-C_v\left(\frac{Q_1+C_vT_a}{C_v+(C_p-C_v)\log_e C}-T_a\right)}$$

$$E_b'=\frac{\frac{53.3}{778}\left(\frac{2000+.17\times492}{.17+.069\log_e 10}-492\right)\log_e 10}{2000-.17\left(\frac{2000+.17\times492}{.17+.069\log_e 10}-492\right)}=91.65\text{ per cent.}$$

This is plotted above the point on total heat supplied lower scale, of 2000 B.T.U. To locate a point on an efficiency curve referred to heat of fire alone, Q_1' upper scale, the same Eq. (1082) is put in the following form, retaining the term Q_1':

$$E' = \frac{Q_1 - Q_2}{Q_1'} = 1 - \frac{R}{JQ_1} T_a \log_e C.$$

Applying this to the point C, for which $Q_1' = 400$ B.T.U. upper scale, and 10 atm. compression it becomes,

$$E_c' = 1 - \frac{53.3}{778 \times 400} \times 492 \log_e 10 = 79.87 \text{ per cent.}$$

Passing into the left-hand angle the location of point D will illustrate the determination of all others. For D a cyclic thermal efficiency of 30 per cent is assumed and a furnace efficiency of 40 per cent, whence from Eq. (1086), by introducing furnace efficiency,

$$\text{B.T.U. per hour per I.H.P.} = \frac{2545}{E_{(\text{cycle})} \times E_{(\text{furnace})}} = \frac{2545}{.3 \times .4} = 21208.$$

To illustrate the use of the diagram, Fig. 257, find the efficiency, cyclic and fuel heat consumption for a Stirling cycle, for 300 B.T.U. supplied from fire per pound of working gases, 30 atm. compression, and a furnace efficiency of 40 per cent. Starting at point E at the value 300 on the upper scale, pass vertically up to point F on the efficiency curve referred to fire heat, and horizontally to G, reading thermal efficiency of 62.8 per cent, and cyclic heat supplied 4050 B.T.U. per hour per I.H.P. Continuing across to point H on the 40 per cent furnace efficiency curve and down to fire heat scale at K, the fire heat supplied is found to be 10,200 B.T.U. per hour per I.H.P.

A similar procedure applies to the curves, Fig. 258, for the Ericsson cycle, using, of course, the appropriate formulas for efficiency and needs no detailed explanation. The determination of points on the diagrams, Fig. 259, for Stirling, and Fig. 260 for Ericsson, is made by the same equations as are used for Fig. 257 and Fig. 258, respectively.

Prob. 1. Derive a formula for the efficiency of the Stirling cycle by the methods of this chapter, for any regenerator efficiency and show that it agrees with the different form found in Rankine's "Steam Engine."

Prob. 2. Derive a similar formula for the Ericsson cycle.

Prob. 3. An Ericsson hot-air engine with two atmospheres compression and a volume increase on constant pressure heating of 20 per cent burns 100 cu.ft. of 600 B.T.U. per cubic foot fuel gas per hour per I.H.P. If the furnace efficiency is 30 per cent and regenerator efficiency 100 per cent, what part of the cyclic efficiency is being realized?

Prob. 4. If in Problem 3 the regenerator were only 70 per cent efficient, what would be the cyclic efficiency and per cent realization?

Prob. 5. Compare the Stirling and Ericsson cycle efficiencies for equal compressions and equal temperature limits.

Prob. 6. What are the numerical values of the efficiencies of Problem 5?

Prob. 7. For the data of Problem 3, find the amount of regenerator surface nece sary for the execution of 50 cycles per minute, if the rate of heat transfer were 3 B.T.U per hour per square foot per degree difference. Use arithmetical mean differences.

Prob. 8. Find the water jacket and furnace surface for any assumed reasonab values of water and fire temperature, corresponding to Problem 7, and for the sam coefficient of heat transfer.

11. Otto, Complete Expansion Otto, Atkinson, Brayton, Diesel, and Carno Cycles. Work, Efficiency and Derived Quantities for Adiabatic Compres sion Gas Cycles.

Otto Cycle. Cycle VI

As this and succeeding cycles include as a first phase, adiabatic com pression to any arbitrarily fixed degree, the amount of this compressio expressed by either the temperature, pressure or volume change which ma characterize it, will constitute an additional independent variable or initi condition.

It is most convenient to express this in terms of pressures; accordingly, le C = compression in atmospheres $= \frac{P_b}{P_a}$. Then the volume and pressure at poin B, Fig. 261, representing the Otto cycle, are given by the ordinary adiabati relation.

Point B:

$$\left.\begin{aligned} P_b &= P_a C, && (a)\\ V_b &= V_a\left(\frac{P_a}{P_b}\right)^{\frac{1}{\gamma}} = \frac{V_a}{C^{\frac{1}{\gamma}}}, && (b)\\ T_b &= T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = T_a C^{\frac{\gamma-1}{\gamma}} && (c) \end{aligned}\right\} \quad \ldots\ldots \quad (109$$

Point C:

$$\left.\begin{aligned} V_c &= V_b = \frac{V_a}{C^{\frac{1}{\gamma}}}, && (a)\\ T_c &= T_b + \frac{Q_1}{C_v} = T_b\left(1+\frac{Q_1}{C_v T_b}\right) = T_b X = T_a C^{\frac{\gamma-1}{\gamma}} X, && (b)\\ P_c &= P_b\frac{T_c}{T_b} = P_b X = P_a C X, && (c) \end{aligned}\right\} \quad \ldots \quad (109$$

Point D:

$$\left.\begin{aligned} V_d &= V_a, && (a)\\ P_d &= P_c\left(\frac{V_c}{V_d}\right)^{\gamma} = P_a C X\left(\frac{1}{C^{\frac{1}{\gamma}}}\right)^{\gamma} = P_a X, && (b)\\ T_d &= T_a\frac{T_c}{T_b} = T_a X, && (c) \end{aligned}\right\} \quad . \quad . \quad (1100)$$

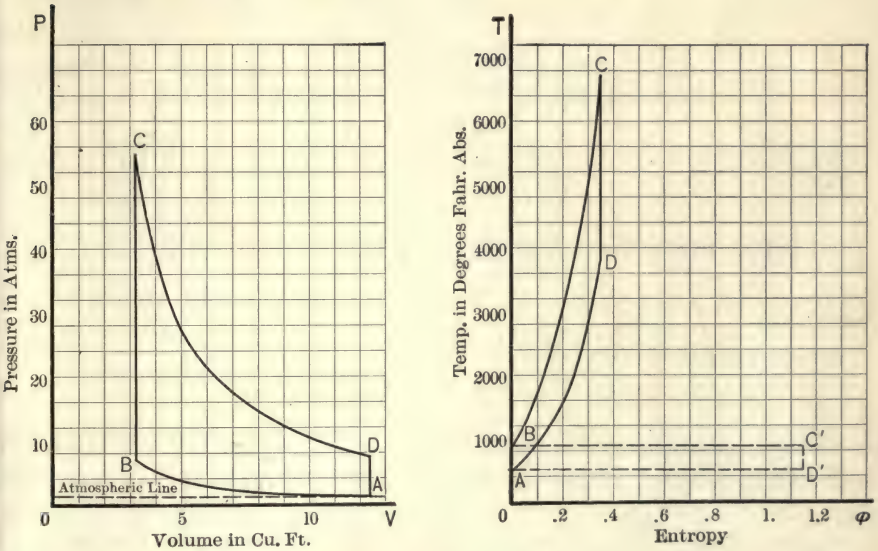


FIG. 261.—Otto Gas Cycle. Cycle VI.

$$Q_2 = C_v(T_d - T_a) = C_v T_a(X-1) = C_v T_a\,\frac{Q_1}{C_v T_b}$$

$$= \frac{T_a}{T_b}Q_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1101)$$

$$\left.\begin{aligned} W = J(Q_1 - Q_2) &= JQ_1\left(1 - \frac{T_a}{T_b}\right) && (a)\\ &= JQ_1\left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}\right] && (b)\\ &= JQ_1\left[1 - \left(\frac{V_b}{V_a}\right)^{\gamma-1}\right] && (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1102)$$

$$\left.\begin{aligned} E = \frac{Q_1 - Q_2}{Q_1} &= 1 - \frac{T_a}{T_b}, && (a)\\ &= 1 - \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}, && (b)\\ &= 1 - \left(\frac{V_b}{V_a}\right)^{\gamma-1}, && (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1103)$$

$$r_V = V_a - V_b = V_a\left(1 - \frac{1}{C^{\frac{1}{\gamma}}}\right) = V_a\left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{1}{\gamma}}\right] \quad . \quad . \quad (110$$

$$r_P = P_c - P_a = P_a(CX - 1), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110$$

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{JQ_1\left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}\right]}{144 V_a\left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{1}{\gamma}}\right]}, \quad . \quad . \quad . \quad . \quad . \quad (110$$

$$\text{B.T.U. per hr. per I.H.P.} = 2545\left(\frac{T_b}{T_b - T_a}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110$$

$$\text{Cu.ft. gas per hr. per I.H.P.} = \frac{2545}{H}\left(\frac{T_b}{T_b - T_a}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110$$

Once more the efficiency of a cycle, this time that of the Otto, is given by a expression similar to that of the Carnot cycle, but in which the temperatu range is not that for the whole cycle but only that for the adiabatic compressio Interpreted thus, the Otto cycle has the same efficiency as a Carnot, for t same amount of adiabatic compression. This is indicated graphically in Fi 261 by the dotted lines $ABC'D'A$ representing the equal Carnot rectangle same height AB, that of the temperature rise in adiabatic compression.

Extended expansion, beyond the original volume as practiced by Atkinso reaches its limit when the pressure falls to atmosphere if all expansion belo atmosphere be excluded as impracticable. This cycle evidently will be mo efficient than the Otto because work area is added without any addition heat supply, but the mean effective pressure will be much lessened as t volumes increase very fast in the lower ranges of pressures. Stopping t expansion at some pressure and volume greater than the initial, will produ a cycle having characteristics midway between the two limiting cases. Th is really the Atkinson cycle. The complete expansion case will be call the Complete Expansion Otto and is of interest because of efforts to produ a perfect compound Otto gas engine for which it is a reference standar These two modifications of the Otto will be considered separately as speci cases of the Otto Cycle, Cycle VI.

ATKINSON CYCLE, EXTENDED EXPANSION OTTO CYCLE. CYCLE VI.

All points up to and including point C are given by the same expressions as for the Otto, but points D and E, Fig. 262, are located at some volume greater than the volume at A, and as this is arbitrary, it adds one more independent variable, accordingly,

Let $$n = \frac{V_e}{V_a} = \frac{\text{Volume after expansion}}{\text{Volume before compression}}.$$

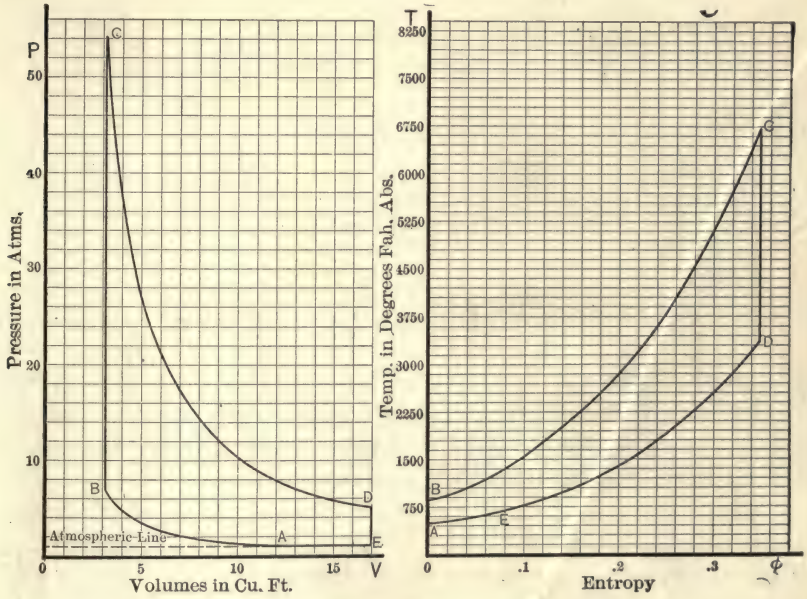


FIG. 262.—Atkinson Gas Cycle. Cycle VI.

Point D:

$$\left.\begin{aligned} V_d &= V_a n && (a)\\ P_d &= P_c\left(\frac{V_c}{V_d}\right)^{\gamma} = P_a C X \left(\frac{\frac{V_a}{1}}{\frac{C^{\frac{1}{\gamma}}}{V_a n}}\right)^{\gamma} = P_a X \left(\frac{1}{n}\right)^{\frac{1}{\gamma}} && (b)\\ T_d &= T_a X \left(\frac{1}{n}\right)^{\gamma - 1}, && (c) \end{aligned}\right\} \quad . \quad . \quad . \quad (1109)$$

Point E:

$$\left.\begin{aligned} P_e &= P_a && (a)\\ V_e &= V_a n && (b)\\ T_e &= T_a \frac{V_e}{V_a} = T_a n && (c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1110)$$

$$Q_2 = C_v(T_d - T_e) + C_p(T_e - T_a)$$

$$= C_v T_a n\left[X\left(\frac{1}{n}\right)^{\gamma} - 1\right] + C_p T_a(n-1). \quad (1111)$$

$$W = J(Q_1 - Q_2) = J\left\{Q_1 - C_v T_a n\left[X\left(\frac{1}{n}\right)^{\gamma} - 1\right] - C_p T_a(n-1)\right\}. \quad (1112)$$

$$E = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T_a}{Q_1}\left\{\left[X\left(\frac{1}{n}\right)^{\gamma} - 1\right] n\, C_v - C_p(n-1)\right\}. \quad (1113)$$

$$r_V = (V_e - V_b) = V_a\left(n - \frac{1}{C^{\frac{1}{\gamma}}}\right) \quad (1114)$$

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{W}{144 V_a\left(n - \frac{1}{C^{\frac{1}{\gamma}}}\right)}. \quad (1115)$$

$$r_P = (P_c - P_a) = P_a(CX - 1). \quad (1116)$$

These expressions are not so simple as for the Otto cycle and will not be carried further; it is, however, clear that efficiency is greater, and now no longer independent of the amount of heat supplied, but on the contrary a function of it, and of the amount of extended expansion.

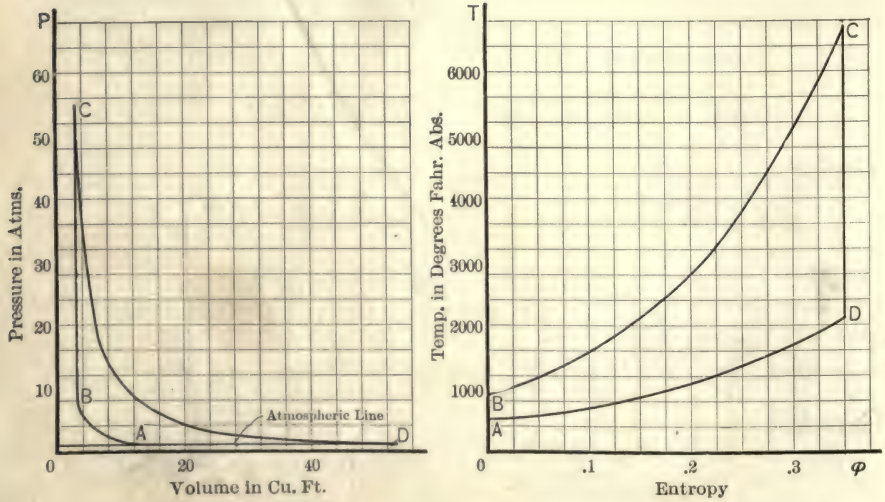


FIG. 263.—Complete Expansion Otto Cycle. Special Case of Atkinson Cycle VI.

Complete Expansion Otto Cycle. Cycle VI.

The only point to be located is that at D, Fig. 263.

Point D:

$$\left.\begin{aligned} P_d &= P_a && (a)\\ V_d &= V_c\left(\frac{P_c}{P_d}\right)^{\frac{1}{\gamma}} = \frac{V_a}{C^{\frac{1}{\gamma}}}(CX)^{\frac{1}{\gamma}} = V_a X^{\frac{1}{\gamma}} && (b)\\ T_d &= T_a\frac{V_d}{V_a} = T_a X^{\frac{1}{\gamma}} && (c) \end{aligned}\right\} \quad \ldots \quad (1117)$$

$$Q_2 = C_p(T_d - T_a) = C_p T_a(X^{\frac{1}{\gamma}} - 1). \quad \ldots \quad (1118)$$

$$W = J(Q_1 - Q_2) = J[Q_1 - C_p T_a(X^{\frac{1}{\gamma}} - 1)]. \quad \ldots \quad (1119)$$

$$E = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{C_p T_a}{Q_1}(X^{\frac{1}{\gamma}} - 1). \quad \ldots \quad (1120)$$

$$r_V = (V_d - V_b) = V_a\left(X^{\frac{1}{\gamma}} - \frac{1}{C^{\frac{1}{\gamma}}}\right). \quad \ldots \quad (1121)$$

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{W}{144 V_a\left(X^{\frac{1}{\gamma}} - \frac{1}{C^{\frac{1}{\gamma}}}\right)}. \quad \ldots \quad (1122)$$

$$r_P = (P_c - P_d) = P_a(CX - 1) \quad \ldots \quad (1123)$$

Brayton Cycle. Cycle VII.

The first data to be determined for this cycle, Fig. 264, are for point C.
Point C:

$$\left.\begin{aligned} P_c &= P_b = P_a C && (a)\\ T_c &= T_b\left(1 + \frac{Q_1}{C_p T_b}\right) = T_a C^{\frac{\gamma-1}{\gamma}} Y && (b)\\ V_c &= V_b\frac{T_c}{T_b} = V_b Y = V_a Y\left(\frac{1}{C}\right)^{\frac{1}{\gamma}} && (c) \end{aligned}\right\} \quad \ldots \quad (1124)$$

Point D:

$$\left.\begin{aligned} P_d &= P_a && (a)\\ T_d &= T_a\frac{T_c}{T_b} = T_aY && (b)\\ V_d &= V_aY && (c) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (1125)$$

$$Q_2 = C_p(T_d - T_a) = C_pT_a(Y-1) = C_pT_a\frac{Q_1}{C_pT_b}$$

$$= \frac{T_a}{T_b}Q_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1126)$$

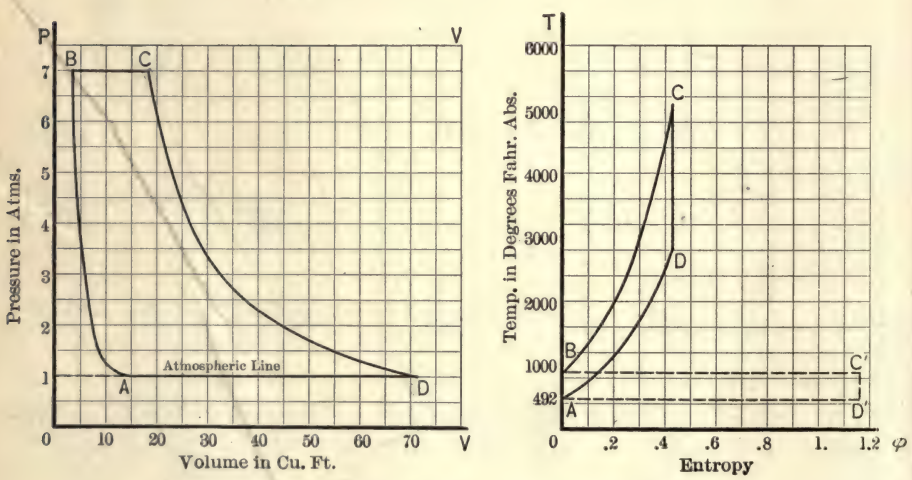


FIG. 264.—Brayton Gas Cycle. Cycle VII.

$$\left.\begin{aligned} W = J(Q_1 - Q_2) &= JQ_1\left(1 - \frac{T_a}{T_b}\right) && (a)\\ &= JQ_1\left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}\right] && (b)\\ &= JQ_1\left[1 - \left(\frac{V_b}{V_a}\right)^{\gamma-1}\right] && (c) \end{aligned}\right\} \quad \ldots \quad (1127)$$

$$\left.\begin{aligned} E &= 1 - \frac{T_a}{T_b}, && (a)\\ &= 1 - \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}} && (b)\\ &= 1 - \left(\frac{V_a}{V_b}\right)^{\gamma-1} && (c) \end{aligned}\right\} \quad \ldots \quad (1128)$$

$$r_V = (V_d - V_b) = V_a\left[Y - \left(\frac{1}{C}\right)^{\frac{1}{\gamma}}\right] \quad \ldots\ldots\ldots \quad (1129)$$

$$r_P=(P_b-P_a)=P_a(C-1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1130)$$

$$(\text{m.e.p.})=\frac{W}{144r_V}=\frac{JQ_1\left[1-\left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}\right]}{144V_a\left[Y-\left(\frac{1}{C}\right)^{\frac{1}{\gamma}}\right]}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1131)$$

$$\left.\begin{matrix}\text{B.T.U. per hr. per}\\ \text{I.H.P.}\end{matrix}\right\}=2545\left(\frac{T_b}{T_b-T_a}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1132)$$

$$\left.\begin{matrix}\text{Cu.ft. gas per hr.}\\ \text{per I.H.P.}\end{matrix}\right\}=\frac{2545}{H}\left(\frac{T_b}{T_b-T_a}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1133)$$

The efficiency and work are the same as for the Otto cycle, and the former s independent of everything but the amount of compression, being the same ıs for the Carnot cycle with the same compression. This Brayton cycle ıas the smallest pressure range for this efficiency, as the Otto has the smallest ʼolume range, and the Carnot the smallest temperature range, the efficiency n each case being measured by the amount of compression. To make the ɔmparison more clear the Carnot cycle rectangle for the same temperature ise in compression, that is, for the same height, is plotted in dotted lines to *T*Φ coordinates *ABC'D'A*.

Diesel or Incomplete Expansion Brayton Cycle. Cycle VII.

A special case of the Brayton cycle is that of the Diesel which, executing ll the processes in a single cylinder, can sweep through no more displacement ʼolume on the working stroke than on the compression stroke, and therefore, utting off the expansion at the initial volume. This is obviously a less fficient cycle as much expansion work is lost. It is represented in Fig. 265, *ABCDA*.

Point *D*:

$$\left.\begin{aligned} V_d&=V_a && (a)\\ P_d&=P_c\left(\frac{V_c}{V_d}\right)^{\gamma}=P_aC\left[\frac{V_aY\left(\frac{1}{C}\right)^{\frac{1}{\gamma}}}{V_a}\right]^{\gamma}=P_aY^{\gamma} && (b)\\ T_d&=T_a\left(\frac{P_d}{P_a}\right)=T_aY^{\gamma} && (c)\end{aligned}\right\} \quad . \quad (1134)$$

$$Q_2=C_v(T_d-T_a)=C_vT_a(Y^{\gamma}-1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (1135)$$

$$W = J(Q_1 - Q_2) = J[Q_1 - C_v T_a(Y^\gamma - 1)]. \quad . \quad . \quad . \quad (1136$$

$$E = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{C_v T_a(Y^\gamma - 1)}{Q_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1137$$

$$r_V = (V_a - V_b) = V_a\left[1 - \left(\frac{1}{C}\right)^{\frac{1}{\gamma}}\right] = V_a\left[1 - \left(\frac{P_a^{\frac{1}{\gamma}}}{P_b}\right)\right] \quad (1138$$

$$r_P = (P_b - P_a) = P_a(C - 1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1139$$

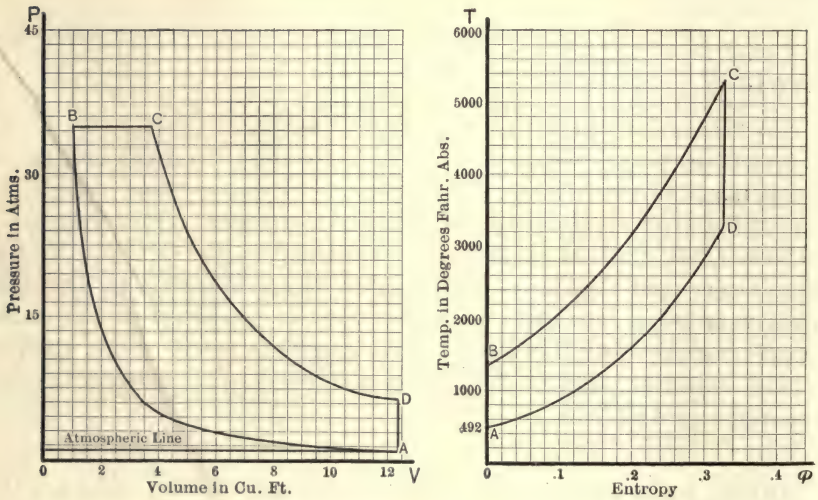


FIG. 265.—Diesel Gas Cycle. Special Case of Brayton Cycle. Cycle VII.

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{J[Q_1 - C_v T_a(Y^\gamma - 1)]}{144\, V_a\left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{1}{\gamma}}\right]} \quad . \quad . \quad . \quad . \quad . \quad . \quad (114$$

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E} \quad . \quad . \quad . \quad . \quad . \quad (114$$

$$\text{Cu.ft. gas per hr. per I.H.P.} = \frac{2545}{HE} \quad . \quad . \quad . \quad . \quad . \quad (114$$

Carnot Cycle for Gases. Cycle VIII.

After adiabatic compression from A to B, Fig. 266, the heat addition takes place isothermally, thus locating the point C by the amount of this addition. When the pressure at B is small and the heat addition large, the point C may easily fall below atmosphere. This is of no importance to the cycle as a whole, but if expansion were to cease at the initial volume, then adiabatic expansion may not only be impossible but only a small amount of heat could be added to the gas isothermally and only a little work done. Moreover, the efficiency would be very low, likewise would the mean effective pressure be also low

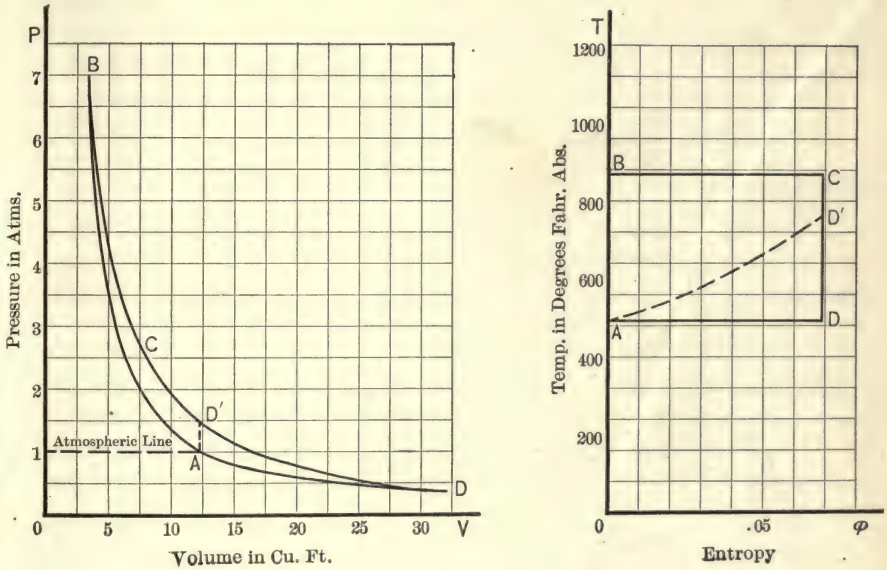


Fig. 266.—Carnot Gas Cycle. Cycle VIII.

because adiabatics lie very close to isothermals, making the work area between them small for very considerable displacements. The original proposal for Diesel engines involved a graduated introduction of the oil fuel into highly compressed air to keep the heating line nearly of this isothermal form, but the small mean effective pressures obtained made the engines too big and there was substituted the modified Brayton cycle, just discussed, which follows by a faster oil injection and combustion, fast enough to keep the pressure from falling till the fuel supply is cut off.

It, therefore, is of no importance to investigate this early Diesel proposal but the Carnot gas cycle will be analyzed not with any idea of its practical importance, but rather to show the extremes of low mean effective pressures and large volumes that it involves in attaining the same thermal efficiency as the Otto and Brayton for equal compressions, they giving the same work with so

little volume range and such small pressure ranges respectively. As there is to be no practical application of this cycle, the several points will not be separately located, but the desired quantities set down by short-cut methods.

$$\left.\begin{aligned} Q_1 &= \frac{1}{J}\left(P_b V_b \log_e \frac{V_c}{V_b}\right) = \frac{R}{J} T_b \log_e \frac{V_c}{V_b} \quad (a) \\ Q_2 &= \frac{R}{J} T_a \log_e \frac{V_d}{V_a} = \frac{R}{J} T_a \log_e \frac{V_c}{V_b} \quad (b) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1143)$$

$$E = \frac{T_b - T_a}{T_b} = 1 - \frac{T_a}{T_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1144)$$

But

$$E = \frac{W}{JQ_1}.$$

Therefore

$$W = JQ_1\left(\frac{T_b - T_a}{T_b}\right) \text{ ft.-lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1145)$$

Also

$$\frac{V_d}{V_a} = \frac{V_c}{V_b} = \log_e{}^{-1}\left(\frac{JQ_1}{RT_b}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1146)$$

$$\therefore \quad V_d = V_a \log_e{}^{-1}\left(\frac{JQ_1}{RT_b}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1147)$$

$$P_d = P_a \frac{V_a}{V_d} = \frac{P_a}{\log_e{}^{-1}\left(\frac{JQ_1}{RT_b}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1148)$$

$$r_V = V_d - V_b = V_a\left[\log_e{}^{-1}\frac{JQ_1}{RT_b} - \left(\frac{1}{C}\right)^{\frac{1}{\gamma}}\right] \quad . \quad . \quad . \quad . \quad . \quad (1149)$$

$$(\text{m.e.p.}) = \frac{W}{144 r_V} = \frac{JQ_1\left(\frac{T_b - T_a}{T_b}\right)}{144 V_a\left[\log_e{}^{-1}\left(\frac{JQ_1}{RT_b}\right) - \left(\frac{1}{C}\right)^{\frac{1}{\gamma}}\right]} \quad . \quad . \quad . \quad (1150)$$

These volumes are very large, and pressures, both final and mean, very low, so that a Carnot cycle for gases even if the apparatus for executing it were available, which it is not, would not be practicable, whereas for steam or other vapors it is not very far from the attainable or actually used processes.

In the curves, Figs. 267 to 272, some of the important relations between the quantities involved in these cycles are more clearly indicated than by the equations they represent.

Referring to Fig. 267, which gives thermal efficiency as a function of heat supplied per pound of working gases, a most wide variation is found for the different cycles. One, the Diesel, shows a decreasing efficiency with increase of heat supply, three, the Otto, Brayton, and Carnot, a constant efficiency, while the complete expansion Otto or perfect Atkinson increases in efficiency. This crossing of efficiency lines makes the relative efficiencies change places with the position chosen on the heat supply scale. Furthermore compression has the same effect, three different compressions, 5, 10 and 30 atmospheres being represented for each cycle, so any question of superiority of one cycle over another will in general depend on conditions. There are, however, certain clear relations, for example, the Diesel is always less and the perfect Atkinson always more efficient for the same compression than the constant value for the three, Otto, Brayton, and Carnot, as is shown by the three curves starting from the origin for a given compression, one rising (perfect Atkinson), one horizontal (Otto, Brayton, Carnot), and one falling (Diesel), the whole set rising with rise of compression. The Diesel, it is interesting to note, is quite impossible beyond a given heat supply where adiabatic expansion ceases, due to constant-pressure heat supply for whole stroke. For 1000 B.T.U. supply per lb. of working gases and 30 atmospheres compression the efficiencies are 71.8 per cent for the complete expansion Otto, 62.2 per cent for Otto, Brayton, and Carnot, and 45.7 per cent for the Diesel. Equal heat supply and 10 atmospheres compression lowers these efficiencies to 63.4 per cent, 48.2 per cent, for the first two and is an impossible condition for the Diesel. Reduction of heat supply lowers the first, leaves the second unchanged, and raises the last, for example with 30 atmospheres compression and 200 B.T.U. added per lb. of gases the efficiencies become 66 per cent, 62.2 per cent, and 58 per cent.

The precise effect of compression alone is shown more clearly in Fig. 268, where all cycles are shown rising in efficiency, the curves of all being nearly parallel each exhibiting at first a rapid rise, which later becomes more gradual. The relative positions for a given heat supply are the same as indicated by the last set of curves.

Mean effective pressure and work per pound working gases as functions of the two prime variables are shown in Fig. 269, with B.T.U. per pound of gases and in Fig. 270 with compression. The most striking thing brought out is the enormous superiority of the Otto and the hopeless smallness of the Carnot values. As the Otto and Diesel the two gas cycles of practical importance are frequent subjects of computation, the two curves of Fig. 271 for the Otto and Fig. 272 for the Diesel are given to separate large scales from which the mean effective pressure may be accurately read off, for 200, 500, and 1000 B.T.U. added. Engineers engaged in this work are advised to reproduce these charts on a large scale, adding more heat supply curves to meet working conditions, as they are great labor savers and may be used in estimating actual

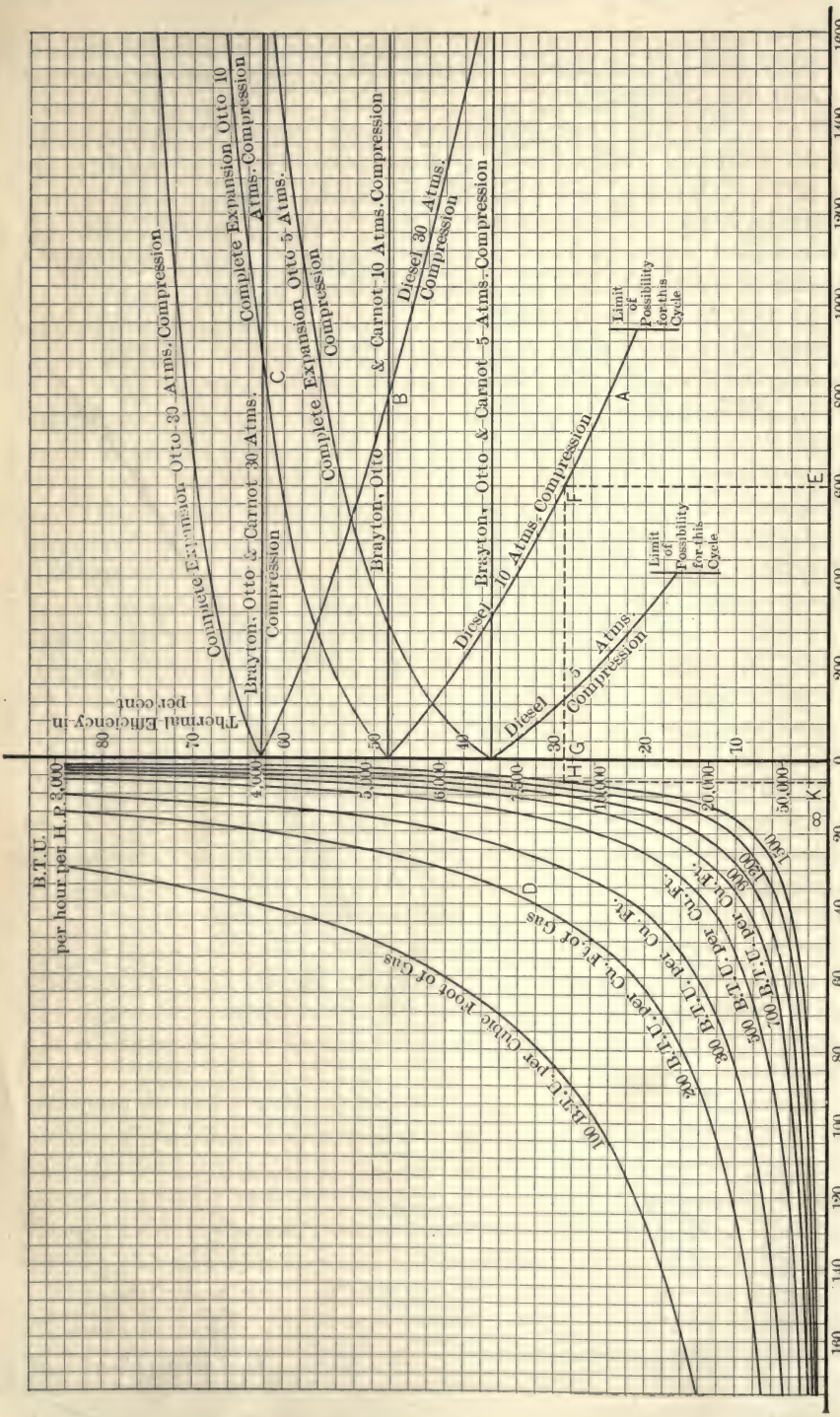


FIG. 267.—Otto, Brayton, Carnot, Diesel, and Complete Expansion Otto Cycles. Thermal Efficiency, Heat, and Gas Consumption, with *Heat Supplied* per lb. of Working Gases, for the Compression Gas Cycles.

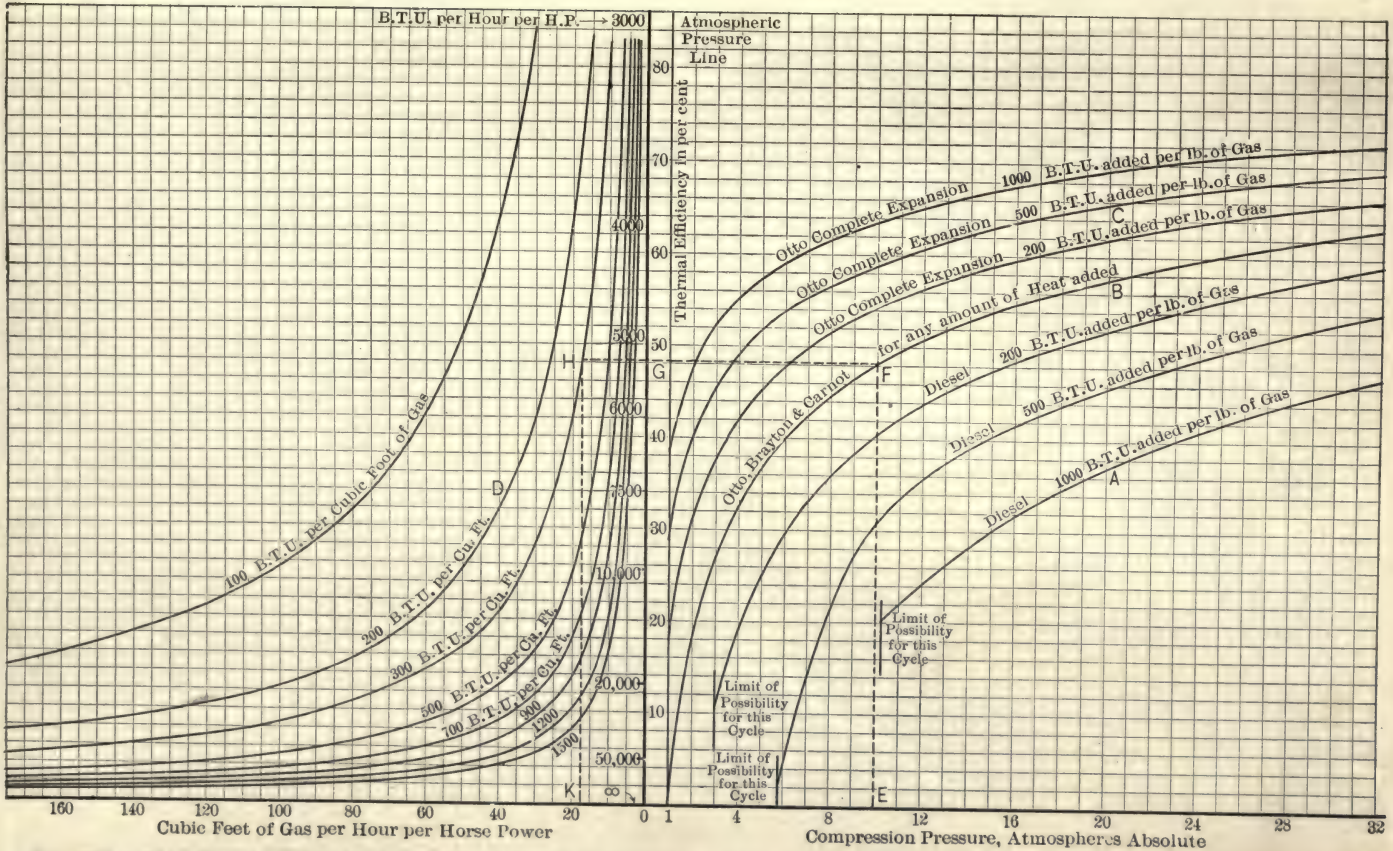


FIG. 268.—Otto, Brayton, Carnot, Diesel, and Complete Expansion Otto Cycles. Thermal Efficiency, Heat, and Gas Consumption, with *Compression*, for the Compression Gas Cycles

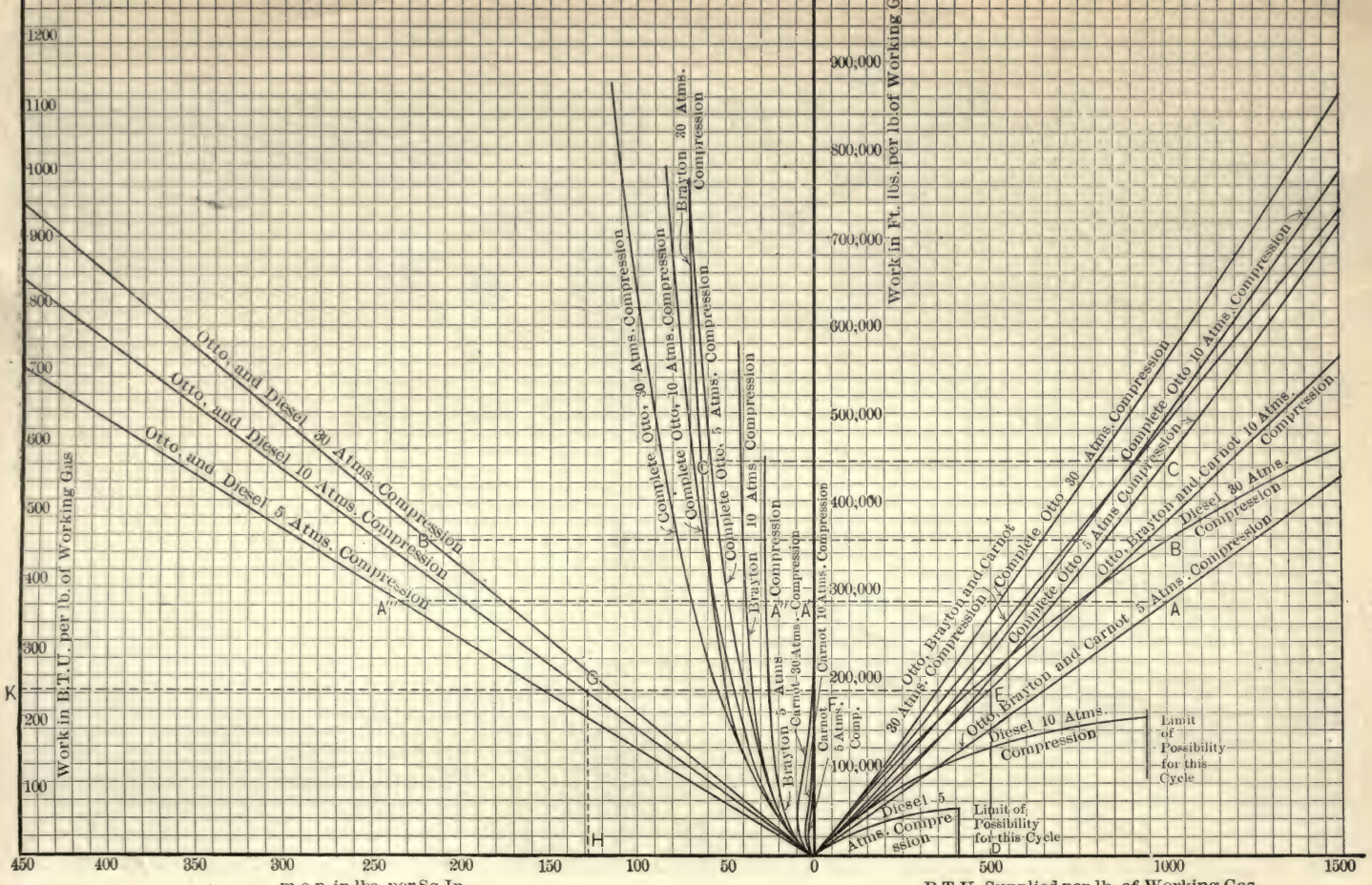


Fig. 369.—Otto, Brayton, Carnot, Diesel and Complete Expansion Otto Cycle Work per lb. of Working Gas and (upper) with

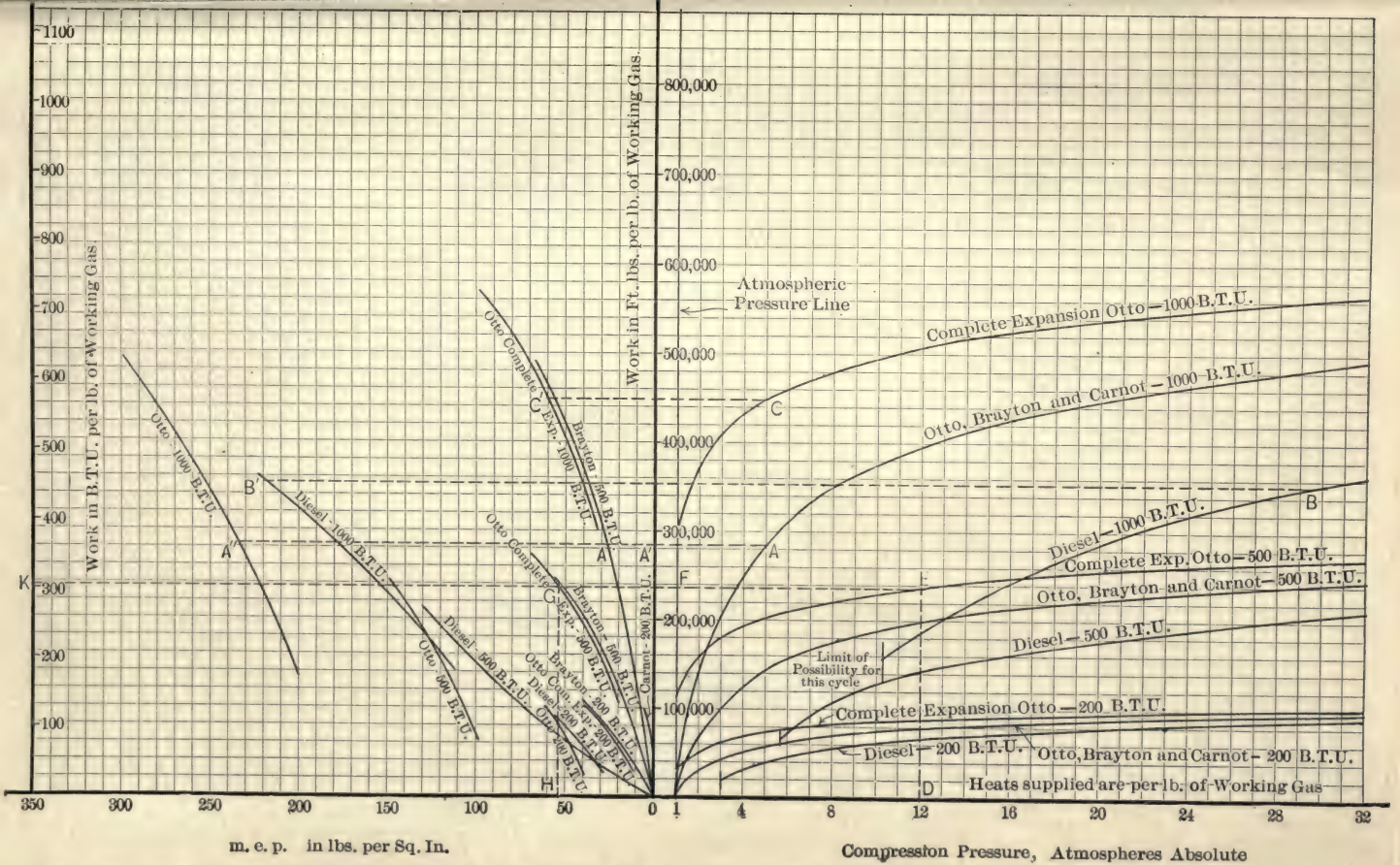
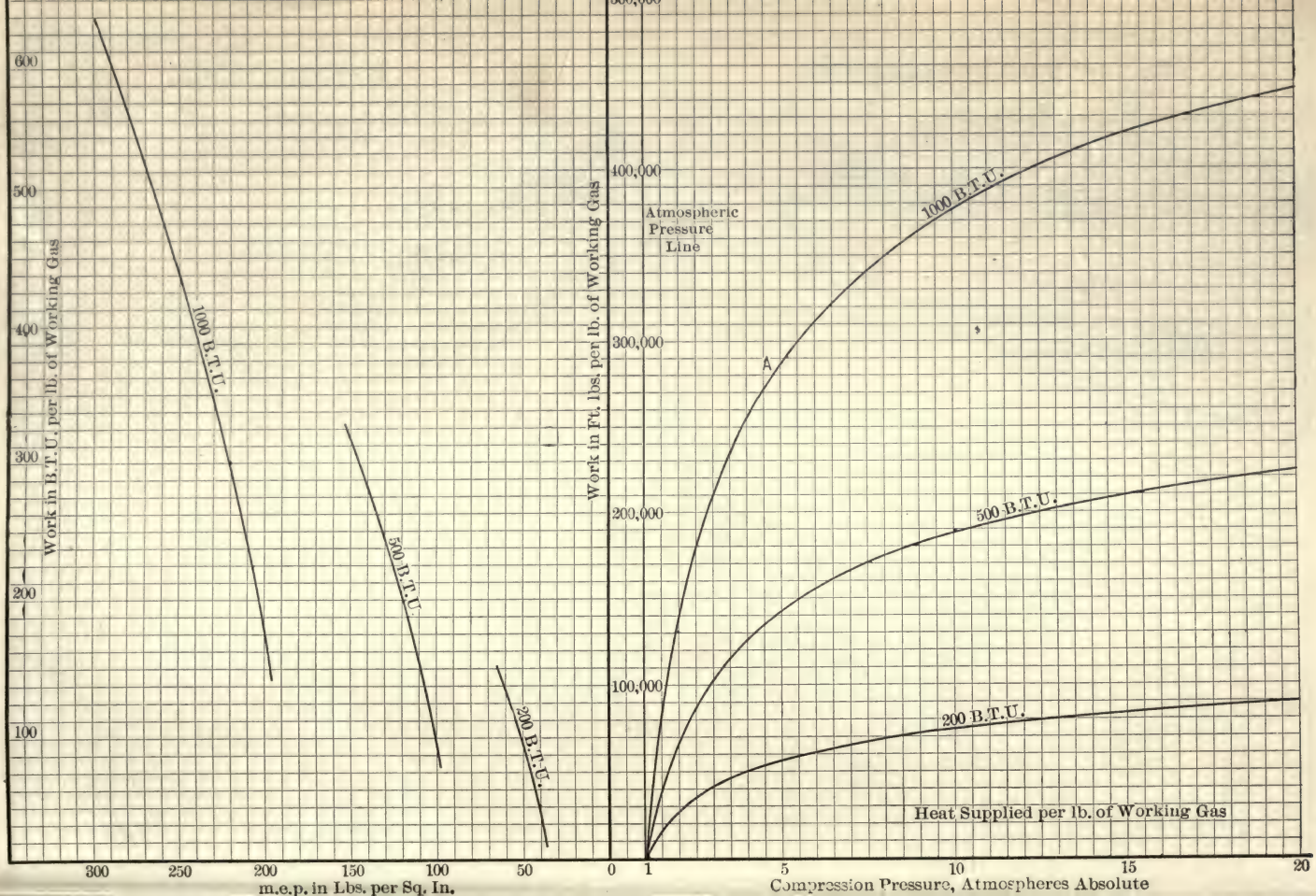


FIG. 270.—Otto Brayton, Carnot, Diesel, and Complete Expansion Otto Cycles. Work per lb. of Working Gases and (m.e.p.) with *Compression*, for the Compression Gas Cycles.


Work in B.T.U. per lb. of Working Gas
600
500
400
300
200
100
1000 B.T.U.
500 B.T.U.
200 B.T.U.
300
250
200
150
100
50
m.e.p. in Lbs. per Sq. In.
Work in Ft. lbs. per lb. of Working Gas
400,000
300,000
200,000
100,000
Atmospheric Pressure Line
A
1000 B.T.U.
500 B.T.U.
200 B.T.U.
Heat Supplied per lb. of Working Gas
0
1
5
10
15
20
Compression Pressure, Atmospheres Absolute

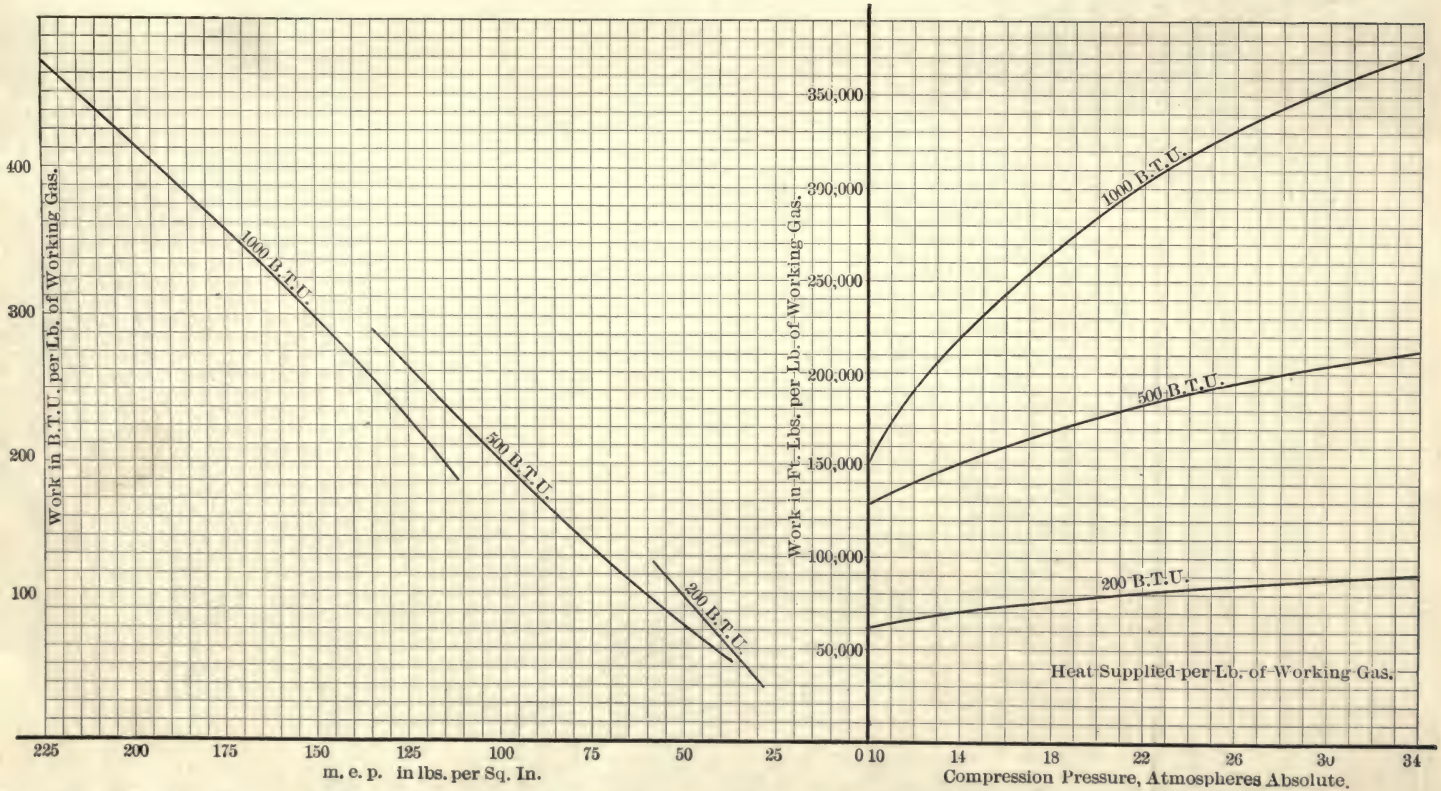


FIG. 272. Diesel Gas Cycle.—Work per lb. of Working Gases and (m.e.p.) for Various Amounts of Heat Added after any Amount of Compression.

horse-power after the determination by tests of performance or diagram factors so called.

Comparison with the curves of the non-compression cycles shows how very much less effective the latter are, and why no engine embodying these non-compression cycles is ever likely to successfully compete with engines executing compression cycles, and also why in the practical construction of gas engines designers are ever on the alert to discover means by which the allowable working compression may be raised. In fact, it is quite possible that one engine mechanism that permits of higher compression than another, even when the former is working on a less efficient cycle than the latter, may give a better actual efficiency. This has for some years been demonstrated by the less efficient Diesel cycle compared with the more efficient Otto, the engines using the former compressing only pure air to about 30 atmospheres without difficulty, while the latter when compressing explosive mixtures, are limited to compressions between four and twelve atmospheres by the temperature of ignition of the mixtures, the low limit for pure kerosene-air mixtures and the high for blast furnace gas. These compressions have resulted in better actual efficiencies in general for Diesel than for Otto engines, reversing the order of cyclic efficiencies.

Another point that must not be forgotten is, that in the execution of all these cycles both steam and gas, realization always falls short of promise, but not to the same degree in the different cases; thus, in round numbers the efficiency actually realizable in steam engines may easily exceed 70 per cent of the most closely representative Rankine cycle for the same pressures while few Otto cycle gas engines have so far been able to realize more than 60 per cent of the Otto cycle efficiency.

Example 1. *Calculation of Diagram, Fig.* 261, *representing the Otto gas cycle, Cycle VI.* The initial condition represented by point A is assumed to be defined by $P_a = 1$ atm., $V_a = 12.39$ cu.ft. (1 lb. air), and $T_a = 492°$ F. absolute, and it is also assumed that the compression is $\frac{P_b}{P_a} = 7$, with 1000 B.T.U. added after compression:

Point B

$$P_b = 7 \text{ atm.} \quad \text{(assumed)}.$$

$$V_b = V_a\left(\frac{P_a}{P_b}\right)^{.713} = 12.39\left(\frac{1}{7}\right)^{.713} = 3.09 \text{ cu.ft.}$$

$$T_b = T_a\left(\frac{P_b}{P_a}\right)^{.286} = 492\left(\frac{1}{7}\right)^{.286} = 859° \text{ F. absolute.}$$

Point C

$$T_c = T_b + \frac{Q_1}{C_v} = 859 + \frac{1000}{.17} = 6741° \text{ F. absolute.}$$

$$P_c = P_b\frac{T_c}{T_b} = 7 \times \frac{6741}{859} = 54.9 \text{ atm.}$$

$$V_c = V_b = 3.09 \text{ cu.ft.}$$

$$\phi_c - \phi_a = C_v \log_e \frac{T_c}{T_b} = .17 \log_e \frac{6741}{859} = .35.$$

Point D

$$\begin{cases} V_d = V_a = 12.39. \\ P_d = P_c\left(\dfrac{V_c}{V_d}\right)^{1.4} = P_c\dfrac{P_a}{P_b} = 54.9 \times \left(\dfrac{1}{7}\right) = 7.85 \text{ atm.} \\ T_d = T_a\left(\dfrac{P_b}{P_a}\right) = 492 \times 7.85 = 3861° \text{ F. absolute.} \end{cases}$$

Example 2. *Calculation of Diagram, Fig.* 262, *representing the Atkinson cycle, Cycle VI or extended but incomplete expansion Otto cycle.* All data are the same as for Example 1, up to and including the point C.

Point D

$$\begin{cases} V_d = 17 \text{ cu.ft.} \quad \text{(assumed)} \\ P_d = P_c\left(\dfrac{V_c}{V_d}\right)^{1.4} = 54.9\left(\dfrac{3.09}{17}\right)^{1.4} = 5.05 \text{ atm.} \\ T_d = T_c\left(\dfrac{V_c}{V_d}\right)^{.4} = 6741\left(\dfrac{3.09}{17}\right)^{.4} = 3408° \text{ F. absolute.} \end{cases}$$

Point E

$$\begin{cases} P_e = P_a = 1 \text{ atm.} \\ V_e = V_d = 17 \text{ cu.ft.} \\ T_e = T_d\dfrac{P_e}{P_d} = 3408 \times \dfrac{1}{5.05} = 675° \text{ F. absolute.} \\ \phi_e - \phi_a = C_p \log_e \dfrac{T_e}{T_a} = .239 \log_e \dfrac{675}{492} = .076. \end{cases}$$

Example 3. *Calculation of Diagram, Fig.* 263, *representing the complete expansion Otto, the limiting case of the Atkinson cycle.* All data of the two preceding examples are identical up to and including point C.

Point D

$$\begin{cases} P_d = P_a = 1 \text{ atm.} \\ V_d = V_c\left(\dfrac{P_c}{P_d}\right)^{.713} = 3.09(54.9)^{.713} = 53.7 \text{ cu.ft.} \\ T_d = T_a\dfrac{V_d}{V_a} = 492 \times \dfrac{53.7}{12.39} = 2132° \text{ F. absolute.} \\ \phi_d - \phi_a = \phi_c - \phi_a = .35. \end{cases}$$

Example 4. *Calculation of Diagram, Fig.* 264, *representing the Brayton gas cycle, Cycle VII.* All data are the same as in the preceding examples through compression and including point B:

Point C

$$\begin{cases} P_c = P_b = 7 \text{ atm.} \\ T_c = T_b + \dfrac{Q_1}{C_p} = 859 + \dfrac{1000}{.239} = 5043° \text{ F. absolute.} \\ V_c = V_b\dfrac{T_c}{T_b} = 3.09 \times \dfrac{5043}{859} = 18.14 \text{ cu.ft.} \\ \phi_c - \phi_a = C_p \log_e \dfrac{T_c}{T_b} = .239 \log_e \dfrac{5043}{859} = .423. \end{cases}$$

Point D

$$P_d = P_a = 1 \text{ atm.}$$

$$V_d = V_c \frac{V_a}{V_b} = 18.14\left(\frac{12.39}{3.09}\right) = 72.7 \text{ cu.ft.}$$

$$T_d = T_a \frac{V_d}{V_a} = 492\left(\frac{72.7}{12.39}\right) = 2890° \text{ F. absolute.}$$

Example 5. *Calculation of Diagram, Fig.* 265, *representing the Diesel gas cycle, a special case of the Brayton cycle, Cycle VII.* All data of Example 4 apply up to, and including point C.

Point D

$$V_d = V_a = 12.39 \text{ cu.ft.}$$

$$P_d = P_c\left(\frac{V_c}{V_d}\right)^{1.4} = 35\left(\frac{3.83}{12.39}\right)^{1.4} = 6.76 \text{ atm.}$$

$$T_d = T_a\left(\frac{P_d}{P_a}\right) = 492 \times 6.72 = 3328° \text{ F. absolute.}$$

Example 6. *Calculation of Diagram, Fig.* 266, *representing the Carnot gas cycle, Cycle VIII.* The data of examples preceding apply through the compression and include point C.

Point C

$$T_c = T_b = 859° \text{ F. absolute.}$$

$$V_c = 8 \text{ cu.ft. (assumed).}$$

$$P_c = \frac{P_b V_b}{V_c} = \frac{7 \times 3.09}{8} = 2.70 \text{ atm.}$$

$$\phi_c - \phi_a = (C_p - C_v) \log_e \frac{V_c}{V_b} = .0686 \log_e \left(\frac{8}{3.09}\right) = .0653.$$

Point D

$$T_d = T_a = 492.$$

$$P_d = P_c\left(\frac{T_d}{T_c}\right)^{3.48} = 2.7\left(\frac{492}{859}\right)^{3.48} = .388 \text{ atm.}$$

$$V_d = \frac{P_a V_a}{P_d} = \frac{12.39}{.388} = 31.93 \text{ cu.ft.}$$

Point D'

$$V_{d'} = V_a = 12.39 \text{ cu.ft.}$$

$$P_{d'} = P_c\left(\frac{V_c}{V_{d'}}\right)^{1.4} = 2.7\left(\frac{8}{12.39}\right)^{1.4} = 1.465 \text{ atm.}$$

$$T_{d'} = T_a \frac{P_{d'}}{P_a} = 4.92 \times 1.465 = 721° \text{ F. absolute.}$$

Example 7. *Calculation and use of Diagrams, Figs.* 267 *and* 268, *giving for the adiabatic compression cycles, Otto, Brayton, complete expansion Otto and Diesel, the efficiency, heat, and fuel gas consumption for any compression and heat supplied per pound of working gases.* The efficiency and its derived quantities are given in Fig. 267 as

unctions of heat supplied and the calculation of these curves will be illustrated by he location of one point on each, A for the Diesel; B for Brayton, Otto, and Carnot, ll having the same efficiency for the same compression, and C for the complete expan- ion Otto. These points are all for 800 B.T.U. supplied per pound of working gases, fter 10 atm. adiabatic compression.

Point A is located by the formula for efficiency of the Diesel cycle given by Eq. 1137).

$$E_a = 1 - \frac{C_v T_a}{Q_1}\left[\left(\frac{Q_1}{C_p T_a C^{\frac{\gamma-1}{\gamma}}} + 1\right)^{\gamma} - 1\right]$$

$$= 1 - \frac{.17 \times 492}{800}\left[\left(\frac{800}{.239 \times 492 \times (10)^{.286}} + 1\right)^{1.4} - 1\right] = 23.68 \text{ per cent.}$$

The limiting case for the efficiency of this cycle is that receiving enough heat to nake $V_c = V_a$ (see Example 5). Whence

$$Q_1 \text{ (limit)} = C_p T_a C^{\frac{\gamma-1}{\gamma}}\left(C^{\frac{1}{\gamma}} - 1\right) = .239 \times 492 \times 10^{.286}(10^{.713} - 1) = 947.3 \text{ B.T.U.}$$

nd this is indicated on the curve carrying the point A by a crossing line and appropriate ettering.

The point B is located from Eq. (1103b) giving,

$$E_b = 1 - \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{1}{C^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(10)^{.286}} = 48.2 \text{ per cent.}$$

By means of Eq. (1120) the efficiency represented by point C is fixed, giving,

$$E_c = 1 - \frac{C_p T_a}{Q_1}\left[\left(\frac{Q_1}{C_v T_a C^{\frac{\gamma-1}{\gamma}}} + 1\right)^{\frac{1}{\gamma}} - 1\right]$$

$$= 1 - \frac{.239 \times 492}{800}\left[\left(\frac{800}{.17 \times 492 \times 10^{.286}} + 1\right)^{.713} - 1\right] = 62 \text{ per cent.}$$

The location of the lines in the left-hand angle needs no explanation in view of he examples for preceding gas cycles.

Illustrating the use of the curve the solution of the following problem is traced raphically on the diagram. Required the thermal efficiency, cyclic heat, and fuel nsumption for the Diesel cycle, supplied with an oil yielding 1500 B.T.U. per cubic ot in its vapor, the cycle receiving 600 B.T.U. per pound of working gases after) atm. compression. From the 600 point E on the heat-supplied scale pass up to the) atm. compression Diesel curve F, and horizontally across to the efficiency scale G

reading 28.6 per cent and 8900 B.T.U. per hour per I.H.P. Continuing across to t fuel calorific power curve of 1500 B.T.U. per cubic foot H, and thence down to K, t fuel consumption is found to be 6 cu.ft.

The second set of efficiency curves, Fig. 268, is obtained and used in exactly t same way as is the first, the only difference between the two being the scales, so th series requires no explanation.

Example 8. *Calculation and use of Diagrams, Figs.* 269, 270, 271, *and* 272, *givi for the adiabatic compression cycles the work per pound of working gases and mean effect pressures.* Explanations of the first, Fig. 269, will apply to all the others as they diff only in scales to facilitate the use of numerical problems.

From Eq. (1102*b*),

$$W = JQ_1\left(1 - \frac{1}{C^{\frac{\gamma-1}{\gamma}}}\right).$$

For 5 atm. compression and 1000 B.T.U. supplied,

$$W_a = 778 \times 1000 \times \left(1 - \frac{1}{5^{.286}}\right) = 287160 \text{ ft.-lbs.}$$

From Eq. (1136),

$$W = J\left\{Q_1 - C_v T_a\left[\left(\frac{Q_1}{C_p T_a C^{\frac{\gamma-1}{\gamma}}} + 1\right)^{\gamma} - 1\right]\right\}.$$

For 30 atm. compression and 1000 B.T.U. supplied,

$$W_b = 778\left\{1000 - .17 \times 492\left[\left(\frac{1000}{.239 \times 492 \times (30)^{.286}} + 1\right)^{1.4} - 1\right]\right\} = 355000 \text{ ft.-lbs.}$$

From Eq. (1119),

$$W = J\left\{Q_1 - C_p T_a\left[\left(\frac{Q_1}{C_v T_a C^{\frac{\gamma-1}{\gamma}}} + 1\right)^{\frac{1}{\gamma}} - 1\right]\right\}.$$

For 5 atm. compression and 1000 B.T.U. supplied,

$$W_c = 778\left\{1000 - .239 \times 492\left[\left(\frac{1000}{.17 \times 492 \times 5^{.286}} + 1\right)^{.713} - 1\right]\right\} = 447300 \text{ ft.-lbs.}$$

Mean effective pressures are derived from these cyclic work values, for the locati of points and curves of the left-hand angle, by the following equations for one poi of each curve A', A'', A''', B', and C'.

From Eq. (1150) for the Carnot cycle,

$$\text{m.e.p.})_{a'} = \frac{W}{144 r_V} = \frac{W}{144 V_a \left[\log_e{}^{-1}\left(\frac{JQ_1}{RT_a C^{\frac{\gamma-1}{\gamma}}} \right) - \left(\frac{1}{C}\right)^{\frac{1}{\gamma}} \right]}$$

$$= \frac{287160}{144 \times 12.39 \left[\log_e{}^{-1}\left(\frac{778 \times 1000}{53.3 \times 492 \times 5^{.286}} \right) - \left(\frac{1}{5}\right)^{.713} \right]} = .0000012 \text{ lb. per sq.in.}$$

From Eq. (1131) for the Brayton cycle,

$$\text{m.e.p.})_{a''} = \frac{W}{144 r_V} = \frac{W}{144 V_a \left[\frac{Q_1}{C_p T_a C^{\frac{\gamma-1}{\gamma}}} + 1 - \left(\frac{1}{C}\right)^{\frac{1}{\gamma}} \right]}$$

$$= \frac{287160}{144 \times 12.39 \left[\frac{1000}{.239 \times 492 \times 5^{.286}} + 1 - \left(\frac{1}{5}\right)^{.713} \right]} = 26.61 \text{ lbs. per sq.in.}$$

From Eq. (1106) for the Otto cycle

$$\text{m.e.p.})_{a'''} = \frac{W}{144 r_V} = \frac{W}{144 V_a \left[1 - \left(\frac{1}{C}\right)^{\frac{1}{\gamma}} \right]}$$

$$= \frac{287160}{144 \times 12.39 \left[1 - \left(\frac{1}{5}\right)^{.713} \right]} = 235.81 \text{ lbs. per sq.in.}$$

From Eq. (1140) for the Diesel cycle,

$$\text{m.e.p.})_{b'} = \frac{W}{144 r_V} = \frac{W}{144 V_a \left[1 - \left(\frac{1}{C}\right)^{\frac{1}{\gamma}} \right]}$$

$$= \frac{355000}{144 \times 12.39 \left[1 - \left(\frac{1}{30}\right)^{.713} \right]} = 218.79 \text{ lbs. per sq.in.}$$

It may be noted for equal works and compressions by the Otto and Diesel cycl the mean effective pressures are equal because the volume range is the same for both.

From Eq. (1122) for the complete expansion Otto cycle,

$$(\text{m.e.p.})_{c'} = \frac{W}{144 r_v} = \frac{W}{144 V_a \left[\left(\frac{Q_1}{C_v T_a C^{\frac{\gamma-1}{\gamma}}} + 1 \right)^{\frac{1}{\gamma}} - \left(\frac{1}{C} \right)^{\frac{1}{\gamma}} \right]}$$

$$= \frac{447300}{144 \times 12.39 \left[\left(\frac{1000}{.17 \times 492 \times 5^{.286}} + 1 \right)^{.713} - \left(\frac{1}{5} \right)^{.713} \right]} = 58.3 \text{ lbs. per square inc}$$

No explanation of the use of the diagram is needed nor is it necessary to give t detailed steps for, or illustrate the use of, the other work and mean effective pressu diagrams, Figs. 270, 271, and 272, for all are similarly derived and all serve to sol graphically, work and mean effective pressure problems for these cycles without calc lation and with sufficient accuracy for general use.

Prob. 1. An Otto cycle engine is supplied with producer gas having 135 B.T. per cubic foot and the compression is 12 atm., heat per pound of working gases 7 B.T.U. Find the cyclic efficiency, cyclic heat consumption, gas consumption, a mean effective pressure.

Prob. 2. For the same data as in Problem 1, solve for the Brayton cycle.

Prob. 3. For the same data as in Problem 1, solve for the complete expansion Ot and from the answers state the value of perfect compounding of Otto gas engines.

Prob. 4. A Diesel engine is operated with 25 atm. compression, on oil havi 19,000 B.T.U. per pound, and the cut-off is such as to add 500 B.T.U. per pound working gases. Find its thermal efficiency, weight of oil per hour per I.H.P., and me effective pressure.

Prob. 5. If 50 per cent of the cyclic efficiency were realized in the Otto cycle engi of Problem 1, all losses taking place during explosion, find the new answers for the sa data.

Prob. 6. If the Diesel engine of Problem 4 realizes 60 per cent of its cyclic efficien and all losses occur during combustion, find the corresponding thermal efficiency, me effective pressure, and oil consumption.

Prob. 7. Otto and Diesel cycle engines are in commercial competition and the b of each have nearly equal thermal efficiency. If this be taken as 35 per cent actual a if each realizes 50 per cent of the cyclic efficiency, all losses taking place during co bustion, and finally if the Otto cycle receives 800 B.T.U. per pound of working gas what are the characteristics of a Diesel engine cycle of equal performance to an O with 15 atm. compression?

Prob. 8. Solve Problem 7 for an Otto engine with 20 atm. compression.

Prob. 9. Find what cut-off volume corresponds to various quantities of heat p pound of working gases for the Diesel cycle at various compressions from 15 to 30 at Plot a curve of relations.

Prob. 10. Compare the terminal pressures and temperatures for all adiabatic co pression cycles receiving 750 B.T.U. per pound of working gases after 15 atm. co pression.

12. Comparison of Steam and Gas Cycles taking the Rankine as Standard for Steam, and the Otto and Diesel as Standard for Gas. Relations of Otto and Diesel to Rankine Cycles. Conditions for Equal Efficiency. Transformation of heat into work is a practical and economic possibility by the use of either a vapor or a gas as the working medium, and, of course, mixtures of vapors and gases as well. Such transformation is effected by a series of thermal processes, the efficiency of which is given by certain equations derived from analysis of the corresponding thermal cycles, which also give the corresponding work of the cycle, mean pressure, and other important quantities. Comparison of these equations for the different cycles establishes principles of practice that are invaluable as working guides in all sorts of ways. For example, the relative value of gases and vapors as the working media fixes corresponding relations between the steam and gas power systems as such, independent of interferences and thermal losses imposed by the mechanisms of the actual machinery. This is the most valuable contribution of such thermal analysis of cycles because such conclusions could not have been obtained in any other way. For steam alone the analysis shows how fast efficiency increases with the rise of initial pressure and temperature and with the lowering of back pressure, and shows the gains to be expected by high boiler pressures or high superheats over low, and by most perfect condensing equipment over a poor equipment, and these gains are to be balanced against the additional cost of equipment involved, in arriving at a good engineering judgment of how far it is worth while to go in any given direction. The situation is the same with gas power efficiency, for the rate of increase of efficiency is fast at first with increase of compression, and slower as compression rises. Higher cylinder compression means more costly construction of engine, so that a means is available for comparing the increased cost with the gain to be derived. However, in this case there is another limit to the compression as has been pointed out and that is the ignition temperature of the explosive mixture when such is the substance being compressed. Similarly, there is another variable in the whole gas power problem and that is the cycle itself, for it has been shown that for the same compressions the maximum pressure which controls the weight and strength to be given to the working parts is not fundamentally related to the conditions for high efficiency but depends rather on the cycle itself, being highest for cycles of the Otto group and lowest for those of the Brayton group.

The most striking general result of the cyclic analysis is the fact that in the abstract, neither steam nor gas as a working medium is always superior to the other as a transformer; both may yield high efficiencies if the proper and special conditions prescribed by the cycle are fulfilled. When, however, the possibility of their easy and cheap fulfillment are examined, then there is noticeable a superiority of gas cycles over steam so far as efficiency is concerned. On the other hand, judging by mean effective pressure standards the differences change again. Although in general the conditions for most efficient working are coincident with low mean effective pressures, which means

that large cylinders are generally needed to fulfill the conditions of high efficiency, this handicap has been removed from the steam system by the substitution of the turbine for the cylinder machine but still remains with the gas system. However, the mean effective pressure for gas systems is not so much a function of the efficiency conditions as it is of the cycle. Nothing could be more convincing in this respect than the comparison of the Otto and Carnot cycle mean effective pressures, those for the Otto extremely high, higher several times over what is possible with efficient steam cycles, while those for the Carnot are vanishingly small. It is sometimes believed that the engine size or mean effective pressure handicap is to be removed in the gas power system by the same turbine method as has succeeded with steam. While this may some day work out, the difficulties of a practical sort are very great, and more important still, the handicap is nowhere near so great as it was for steam and, therefore, the necessity is less, provided the best gas cycle be chosen. This is beyond all question the Otto cycle, because of the high mean pressures and the corresponding high efficiency and from this cycle no departure is permissible without proof of its effectiveness in some practical respect with equally strong proof that a substitute is available, and one that gives a fair return for what is lost by comparison with the Otto. One such is the Diesel, in which the limitation of compression by ignition temperatures is removed by compressing pure air alone and to which fuel is subsequently added, the air compression being carried far enough to itself ignite the fuel on admission and burn it at constant pressure. Though a comparatively inefficient cycle for equal compressions, this may, and does in practice, yield with its 30 atmospheres compression better over-all results in efficiency than the Otto for the lower compressions of about 12 atmospheres, to which the latter has been confined by self-ignition limits even with the least sensitive fuel, in the ratio of about 6 to 5. This is not, however, the case with the corresponding maximum and mean effective pressures, because Diesel cycle engines are approximately twice as costly as Otto cycle engines, due to very high maximum without correspondingly high mean effective pressure. Furthermore, Diesel cycle engines are confined to oil fuel to-day as it is still regarded as impracticable to so highly and separately compress gas fuel for injection into the air at the end of the compression, and gas fuel-air mixture compression is retained in Otto cycle engines in spite of the ignition temperature limitation the practice imposes.

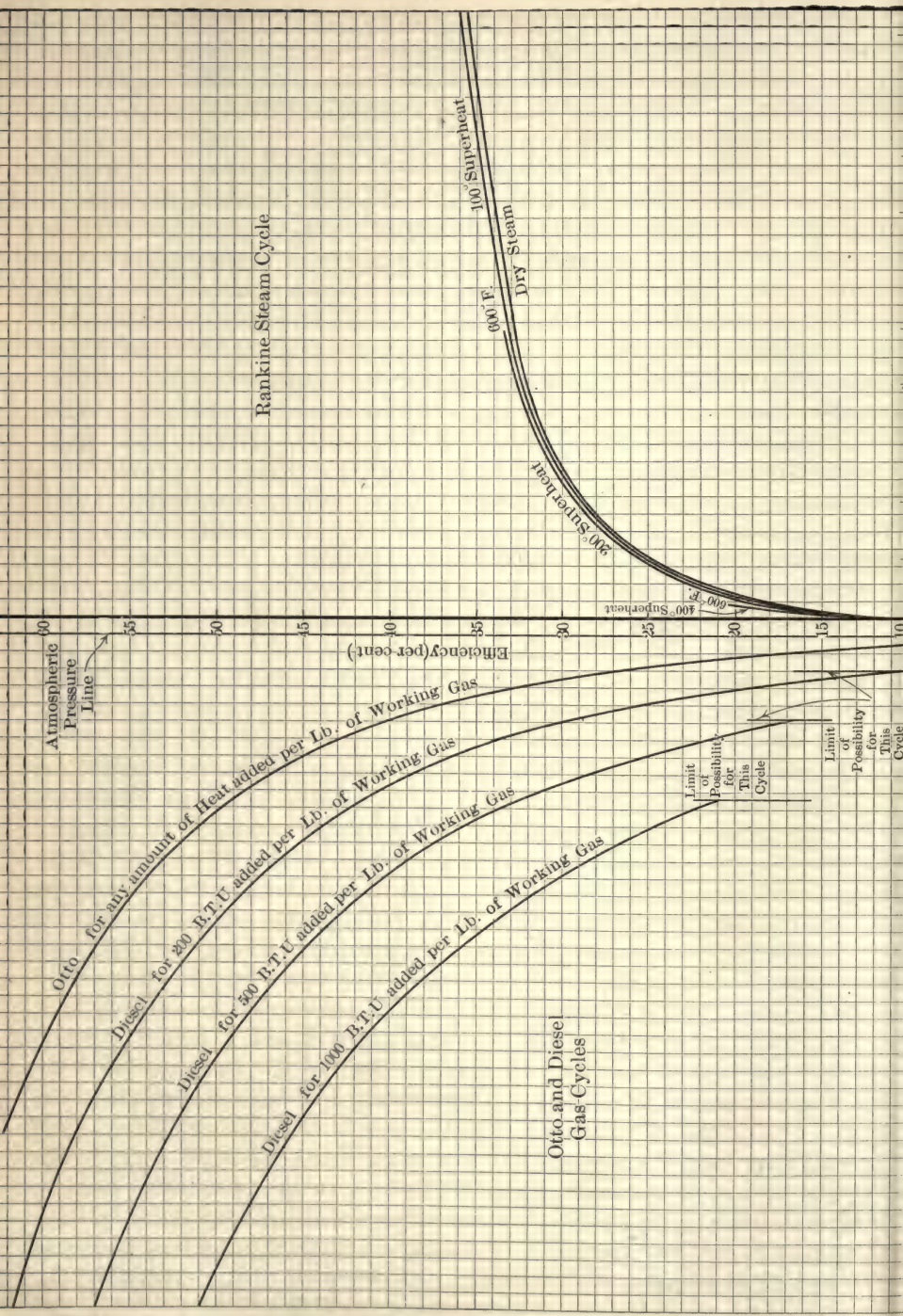
Another illustration of reasonable departure from the standard Otto would be the substitution of a cycle represented by a mechanism that would give speed regulation and reversibility, equal in perfection to that obtainable with steam, and which is impossible with Otto cycle engines, or to cite another case, the substitution of a cycle and mechanism that would permit of safe and direct internal combustion of solid fuel, also now considered impossible. Either of these things would offer advantages enough to offset cyclic deficiency measured by lower thermal efficiency or smaller mean effective pressures as compared with the Otto, if not carried too far.

It appears then that while there are innumerable reasons for not taking

the teachings of the cyclic analyses too seriously, this is not to be regarded as proof of a lack of value. As a matter of fact cyclic analysis, such as has just been briefly developed and whether worked out graphically or algebraically is quite invaluable, but is not to be used or applied without a more or less cultivated engineering judgment.

To close the comparison, it is desirable that the efficiency and work of the cycles, together with the quantities derived from them for the standard steam and gas cycles be brought together so as to show more clearly the conditions to be met in each case for the attainment of equally good results, especially with respect to efficiency, as there are practical ways of meeting the low mean effective pressure handicap, but none for overcoming inherently low efficiencies. The standard cycle for steam is the Rankine and those for gas the Otto and Diesel, so that the comparison need be made only for these; any other steam cycle should and may easily be compared with the Rankine, and the same is true for the gas cycle with respect to the Otto and Diesel. Thus, a given gas cycle is best judged by saying its efficiency is more or less than that of the Otto if it uses constant volume heating, or its mean effective pressure is as great, for either the same amount of heat supplied or for equal compressions, or perhaps for equal maximum pressures. If the gas cycle uses constant-pressure heating it may be compared in performance with the Diesel. This allows of judging all gas cycles by a gas cycle standard and all steam cycles by a steam cycle standard, the comparison of a steam cycle with a gas cycle being made through the relative position of the Rankine with the Otto and Diesel standards. This may at first glance look like a cumbersome and roundabout method, but it really is not.

The comparison may be conveniently made by a diagram of Fig. 273, on which efficiency is plotted vertically, the curves for Otto and Diesel gas cycles to the left, and the Rankine steam cycle to the right, thus making a comparison of conditions easy through the common efficiency scale. Compressions for gas cycles are carried to 40 atmospheres so as to exceed somewhat the working maximum of the Diesel engine, but steam pressures are carried to 500 lbs. though 250 lbs. may be regarded as the corresponding used limit, because the efficiencies of the steam cycles would not otherwise rise to the same values those for gas. For a similar reason wet steam is not considered, back pressures are kept at the lowest practicable limit of half a pound absolute corresponding to 1.03 ins. Hg. and a temperature of 80° F.; superheats are carried in each case to an amount that brings the maximum temperature of the steam up to 600° F., beyond which dissociation, pipe oxidation, and expansion are excessive, though 100° less is a better practicable limit. The diagrams show that it requires some 450 lbs. per square inch initial pressure of steam to reach 35 per cent cyclic steam efficiency, a pressure beyond working practice, whereas less than five atmospheres compression, about the least ever used, gives an equal efficiency to the Otto cycle gas engine standard. For the same 35 per cent efficiency it appears that the Diesel requires $7\frac{1}{2}$, $11\frac{3}{4}$, and $18\frac{1}{2}$ atmospheres compression when there is added to the cycle 200, 500 and 1000



ig. 273.—Comparison of Otto and Diesel Gas Cycles, with Rankine Steam Cycle, for 0.505 pound per square inc
Absolute Back Pressure and Various Initial Pressures and Steam Temperatures. Up to 6000 F.

B.T.U. respectively per pound of working gases. The most striking thing brought out by the comparison is the sustained rapid rise of efficiency for all gas cycles with compression, while the steam cyclic efficiency rises very slowly beyond 200 lbs. initial pressure and by almost a straight line, superheat helping some but not much in proportion.

This chart shows clearly the inherently higher efficiency possibility for gas systems over steam and, as realization has not been in the same proportion, it indicates that the gas engine mechanism as the means of executing the gas cycles must receive very great improvement before it can equal the effectiveness of steam mechanism. It is therefore of great practical importance that gas engine mechanism losses be most carefully studied more than they yet have been, for once brought to equality with steam, the cyclic promise may be realized and the fuel consumption of the world for power purposes cut in half.

Example. Under what operating conditions may a steam engine realizing 60 per cent of the Rankine cycle efficiency be worked, to equal in thermal efficiency an Otto cycle gas engine realizing 40 per cent of the cyclic efficiency on 10 atm. compression. Reference to the diagram, Fig. 273, in the left-hand angle shows that the Otto cycle efficiency is 48.6 per cent. If 40 per cent of this is realized the actual efficiency is 19.44 per cent, and if this is equal to that of the steam engine its cyclic efficiency must be $\frac{19.44}{.60} = 32.4$ per cent. On the right-hand part of the diagram this would be realized for $\frac{1}{2}$ lb. back pressure and absolute initial pressures from 190 to 220 lbs. per square inch approximate with superheats from 200° to zero.

More exact determinations may be made by separate reference to the charts at the end of the sections dealing with these cycles.

Prob. 1. A steam engine operating on 200 lbs. initial pressure and 1 lb. back pressure realizes 55 per cent of the Rankine cycle. Under what operating conditions may an Otto cycle engine realizing 50 per cent of its cyclic efficiency equal the performance?

Prob. 2. Solve Problem 1 for a Diesel cycle realizing the same fraction of its cyclic performance.

Prob. 3. What would be the mean effective pressures for the three cases, the steam, and the O to and Diesel gas cycles?

Prob. 4. Plot to $T\Phi$ coordinates the pure cycles and estimate the probable diagram, accounting for the losses given, and check by areas the efficiencies reported.

Prob. 5. Plot to PV coordinates the pure cycles and estimate the position of the probable diagram, accounting for the losses given, and check by areas the mean effective pressures reported.

13. Gas Cycle Performance as Affected by Variability of the Specific Heats of Gases, Applied to the Otto Cycle. All results so far obtained for gas cycles have been derived on the assumption that the specific heats of gases are constant, but it is known that at high temperatures such as characterize the Otto, for example, and some others, that specific heat varies, so it is necessary for a rigorous treatment to examine the effect of this variability on the conclusion of cyclic performance.

When specific heats vary, it has been assumed in accordance with thermodynamic prediction, that they are functions of the temperature only, and this is true if the gases are *perfect*. Real gases depart from this law and the most notable example is superheated steam, for which the specific heat is fairly well established and now known to be a function of both pressure and temperature. It is, therefore, quite likely that practically all gases are similar in this respect, though no one is yet able to say just what is the true relation, as has several times been pointed out. The usual procedure is to assume a linear relation between specific heats and scale temperature above 32° F., expressing the specific heat *at*, or the mean specific heat *to* a given temperature, as equal to the value at 32°, with a constant multiple of the temperature excess over 32°, added. While this is too simple a relation probably, to correctly state the facts, it is yet too complicated to enable the derivation of cyclic performance equations that shall be simple enough to be useful in engineering practice, as will appear from what follows. Nevertheless, it is highly desirable to go through with the derivation, for the light it will throw on the *nature* of the effect of variability of specific heat, however inadequate the numerical results may prove.

Careful studies of the indicator cards of Otto cycle gas engines by Clerk, Burstall and others has led to some conclusions as to specific heats that may be used for the purpose in hand, though always with doubt as to accuracy. Without in any way implying inferior merit in the work of others the data of Clerk will be adopted and his values are given below:

$$\left.\begin{aligned} C_v &= .194 + .000051t \\ C_p &= .265 + .000051t \end{aligned}\right\}$$ in terms of Centigrade scale temperatures;

$$\left.\begin{aligned} C_v &= .194 + .0000283\{t - 32°\} \\ C_p &= .265 + .0000283\{t - 32°\} \end{aligned}\right\}$$ in terms of Fahrenheit scale temperatures;

$$\left.\begin{aligned} C_v &= .1801 + .0000283T \\ C_p &= .2511 + .0000283T \end{aligned}\right\}$$ in terms of Fahrenheit absolute temperatures.

In general, then,

$$C_v = \alpha + \beta T;$$

$$C_p = \alpha' + \beta T;$$

$$R = J(C_p - C_v) = J(\alpha' - \alpha);$$

$$\gamma' = \frac{\alpha' + \beta T}{\alpha + \beta T}.$$

Referring now to the cycle, Fig. 274, the first phase of which is adiabatic compression, it will appear that the first effect of the above variability of

specific heat relations is to cause the line AB to lie below the corresponding curve for constant specific heats AB', which latter is shown dotted. This is proved algebraically by starting with the general equation for a small increment of heat which for adiabatic changes is put equal to zero.

$$dH = C_v dT + \frac{1}{J} PdV$$

$$= \frac{C_v}{JR}(PdV + VdP) + \frac{1}{J} PdV = 0,$$

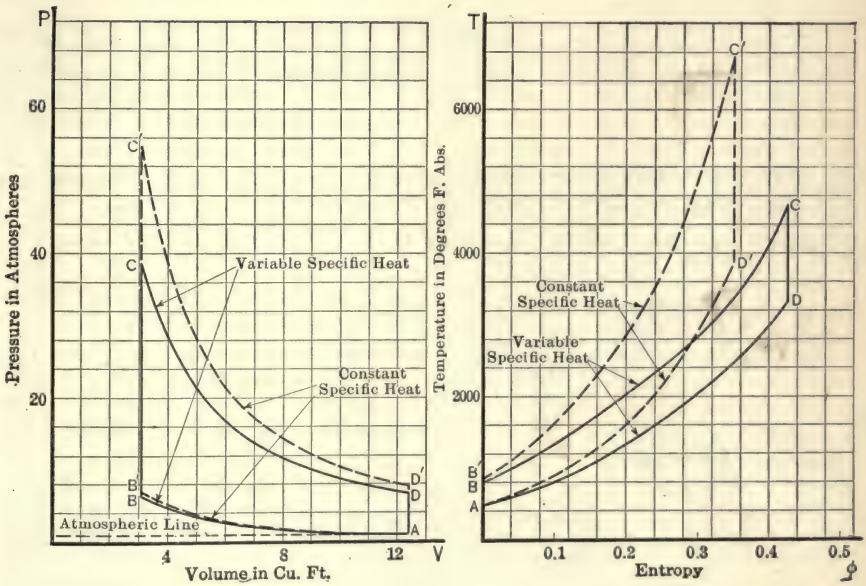


FIG. 274.—Comparisons of Otto Cycles for Constant and for Variable Specific Heat.

whence substituting the new values for C_v and R this becomes

$$(\alpha + \beta T)(PdV + VdP) + (\alpha' - \alpha)PdV = 0,$$

$$\alpha' PdV + \alpha VdP + \beta T(PdV + VdP) = 0,$$

$$\alpha' \frac{dV}{V} + \alpha \frac{dP}{P} + \beta \frac{T}{PV}(PdV + VdP) = 0,$$

$$\alpha' \frac{dV}{V} + \alpha \frac{dP}{P} + \beta dT = 0.$$

Integrating

$$\alpha' \log V + \alpha \log P + \beta T = K_1,$$

$$\frac{\alpha'}{\alpha} \log V + \log P + \frac{\beta}{\alpha} T = K_2,$$

$$P V^{\frac{\alpha'}{\alpha}} e^{\frac{\beta}{\alpha} T} = K.$$

$$P_a V_a^{\gamma} e^{\frac{\beta}{\alpha} T_a} = K = P_b V_b^{\gamma} e^{\frac{\beta}{\alpha} T_b} \quad \ldots\ldots\ldots \quad (1151)$$

or

$$\frac{P_b}{P_a} = \left(\frac{V_a}{V_b}\right)^{\gamma} e^{\frac{\beta}{\alpha}(T_a - T_b)} \quad \ldots\ldots\ldots \quad (1152)$$

This is the new adiabatic relation between pressures and volumes, when specific heats are variable, but it also involves the temperature. By the general gas law, the other relations can be found as follows for T and V:

$$\frac{T_b}{T_a} = \frac{P_b V_b}{P_a V_a} = \frac{V_b V_a^{\gamma} e^{\frac{\beta}{\alpha} T_a}}{V_b^{\gamma} V_a e^{\frac{\beta}{\alpha} T_b}} = \left(\frac{V_a}{V_b}\right)^{\gamma - 1} e^{\frac{\beta}{\alpha}(T_a - T_b)} \quad \ldots \quad (1153)$$

Similarly for T and P

$$\frac{T_b}{T_a} = \left(\frac{P_b}{P_a}\right)^{\frac{\gamma - 1}{\gamma}} e^{\frac{\beta}{\alpha\gamma}(T_a - T_b)} \quad \ldots\ldots\ldots \quad (1154)$$

Finally for P and V

$$P_a V_a^{\gamma} e^{\frac{\beta}{R\alpha} P_a V_a} = K,$$

or

$$\frac{P_b}{P_a} = \left(\frac{V_a}{V_b}\right)^{\gamma} e^{\frac{\beta}{R\alpha}(P_a V_a - P_b V_b)} \quad \ldots\ldots\ldots \quad (1155)$$

By means of these three new adiabatic equations the compression line is completely determined and can be plotted, though the solution is not by any means easy. The next phase is that of heat addition from B or B', to C or C', and as the specific heat increases with temperature rise, the increase in temperature will be less for the variable than for the constant value of the specific

ieat. The relation between temperature rise $(T_c - T_b)$ and the quantity of ieat added is given by Eq. (1156):

$$Q_1 = \int_{T_b} C_v dT = \int_{T_b}^{T_c} (\alpha + \beta T) dT = \alpha(T_c - T_b) + \frac{\beta}{2}(T_c^2 - T_b^2)$$

$$= (T_c - T_b)\left[\alpha + \frac{\beta}{2}(T_c + T_b)\right], \quad \ldots \ldots \ldots \ldots \quad (1156)$$

To express the final temperature T_c in terms of the heat added, Q_1, requires he solution of the second degree equation above and the solution is given by Eq. (1157):

$$T_c = \sqrt{\frac{2}{\beta}\left[Q_1 + \left(\alpha + \frac{\beta}{2}T_b\right)T_b\right] + \left(\frac{\alpha}{\beta}\right)^2} - \frac{\alpha}{\beta}. \quad \ldots \ldots \quad (1157)$$

Of course, as before

$$P_c = P_b \frac{T_c}{T_b}.$$

By means of these equations the point C of the cycle is located and there emains only the fixing of D with reference to C and A. The same adiabatic elations as applied to the compression line AB also apply to the expansion ine CD and in addition, $V_c = V_b$ and $V_d = V_a$.

$$\left.\begin{aligned} \frac{P_c}{P_d} &= \left(\frac{V_a}{V_b}\right)^{\gamma} e^{\frac{\beta}{\alpha}(T_d - T_c)} \quad (a) \\ \frac{T_c}{T_d} &= \left(\frac{V_a}{V_b}\right)^{\gamma} e^{\frac{\beta}{\alpha}(T_d - T_c)} \quad (b) \\ \frac{T_c}{T_d} &= \left(\frac{P_c}{P_d}\right)^{\frac{\gamma-1}{\gamma}} e^{\frac{\beta}{\alpha\gamma}(T_d - T_c)} \quad (c) \end{aligned}\right\} \quad \ldots \ldots \quad (1158)$$

t is possible to find an expression for the pressure, volume, and temperature f points C and D, at the beginning and end of expansion, not merely as has een done in terms of equivalent values at the last previous point, but in terms f initial conditions at A, of the compression and of the heat added, but this

derivation results in very complex equations and is omitted to save spac
The efficiency of the cycle is given by

$$\left.\begin{aligned} E &= \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} && (a)\\ &= \frac{(T_c - T_b)\left[\alpha + \frac{\beta}{2}(T_c + T_e)\right] - (T_d - T_a)\left[\alpha + \frac{\beta}{2}(T_d + T_a)\right]}{(T_c - T_b)\left[\alpha + \frac{\beta}{2}(T_c + T_b)\right]} && (b)\\ &= 1 - \frac{(T_d - T_a)\left[\alpha + \frac{\beta}{2}(T_d + T_a)\right]}{(T_c - T_b)\left[\alpha + \frac{\beta}{2}(T_c + T_b)\right]} && (c) \end{aligned}\right\} \quad . \quad (115$$

When the specific heat is constant, $\beta = 0$, and this expression becomes

$$E' = 1 - \frac{T_d - T_a}{T_c - T_b} = 1 - \frac{T_a\left(\frac{T_d}{T_a} - 1\right)}{T_b\left(\frac{T_c}{T_b} - 1\right)} = 1 - \frac{T_a}{T_b},$$

which is what was previously found for the constant specific heat Otto cyc
but it must be remembered that for equal compressions and heat additio
the temperatures T_b, T_c and T_d for the variable specific heat are not the sa
as those for the constant value $T_{b'}$, $T_{c'}$ and $T_{d'}$.

Substitution of the various temperatures in terms of prime variables
the equation for efficiency gives an expression that cannot be solved at
and so is not worth deriving. Graphic methods are more satisfactory,
the plotting of the cycle shows the relations at a glance, and the degree
accuracy in evaluating work and heat area to get mean effective pressu
and thermal efficiency is entirely a question of the sort of cross-section pap
and the scale used.

However, working on the temperatures as prime variables Wimpe
secures a solution by a series of algebraic approximations that is sufficient
interesting to warrant quoting. Assuming equal temperatures for the poi
B, C, and D, for the two cases of constant and variable specific heat whi
of course, cannot be for equal compressions and heats added, and calli
the efficiency for the former case E', he finds

$$E = E'\left\{1 - \frac{\beta}{2\alpha}[(1 - E')T_c + T_a]\right\} \quad . \quad . \quad . \quad . \quad . \quad (116$$

The efficiency by this equation is plotted in Fig. 275 for two different valu
of the maximum temperature, $T_c = 1000°$ C. $= 1832°$ F., and $T_c = 1600°$

$=2912°$ F., when $T_a=400°$ C. absolute. To this chart a line is added giving Clerk's own calculations of the efficiencies as given in Table CXXVIII compared with those for constant specific heat for the *same temperatures*. These relations

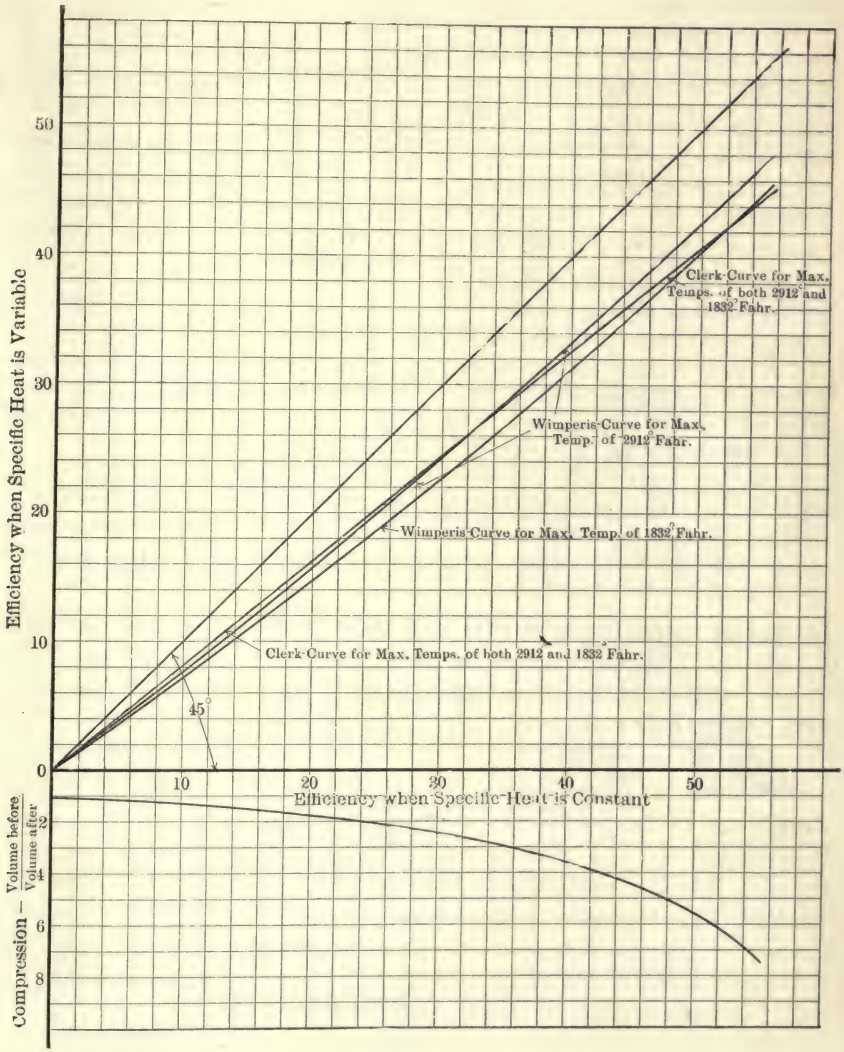


FIG. 275.—Relation of Efficiency of Otto Cycle with Constant Specific Heat, to Efficiency of Same Cycle with Variable Specific Heat.

are clearly of limited value since equal temperatures cannot result from equal compressions and heat additions, nor can any other temperatures for the cycle be equal if the maxima are.

TABLE CXXVIII

CLERK'S COMPARISON OF OTTO CYCLE THERMAL EFFICIENCIES FOR CONSTANT AND VARIABLE SPECIFIC HEATS. EQUAL LIMITING TEMPERATURES

Compression. ($\frac{\text{Volume before}}{\text{Volume after}}$)	Thermal Efficiency.		
	Variable Specific Heat.		Constant Specific Heat.
	Max. temp. =2912° F.	Max. temp. =1832° F.	
2	.197	.200	.242
3	.286	.293	.356
4	.354	.356	.426
5	.384	.394	.473
7	.439	.443	.541

Prob. 1. A gas engine operates on an Otto cycle, and the ratios of volumes before and after compression is 5. What is the cyclic, efficiency, assuming constant specific heats and what according to Wimperis' approximation?

Prob. 2. If in a gas engine the temperature of compression is 500° F. absolute and 1200 B.T.U. are added per pound of gas, what will be the temperature rise if the specific heat is considered constant, and what if it follows the values as given by Clerk? What will be the difference in pressure rise due to the two values of specific heat?

14. Actual Performance of Otto and Diesel Gas Engines, and its Relation to the Cyclic. Diagram Factors for Mean Effective Pressure and Thermal Efficiency. Effect of Load on Efficiency. Heat Balance of Gas Engines Alone and with Gas Producers. All modern gas engines are operated on the internal-combustion method of adding heat to the mass of working gas both explosively and non-explosively, a fact that is responsible for one group of limitations in the results obtainable. There are two direct consequences of importance, first, the charge must be renewed each cycle, fresh air and fuel coming into the working chamber and hot products of combustion leaving alternately, with the other phases between, and second the obtainable temperature rise with its corresponding pressure increase in Otto cycle engines, or volume increase in Diesel cycle engines, is fixed by fuel-air mixture properties. Furthermore, as all these operations take place in cylinders having pistons that require lubrication and which must, therefore, be kept cool enough to prevent destruction of the lubricant, a heat exchange takes place between gases and walls alternately in each direction, that causes corresponding pressure, volume, and temperature changes in the gas, not contemplated by the cycle nor subject to computation or formulation. Besides these influences there are others chargeable to mechanical construction or adjustments that are responsible for further departures in the pressure, volume, and temperature changes from the ideal cyclic ones, as for example, too early an exhaust cur-

tails expansion; too late a closure of admission valve similarly delays the beginning of compression and reduces the total amount. In Otto cycle engines where mixtures are exploded the combustion line may not be vertical and heating fail to be truly at constant volume, while in Diesel cycle engines where the fuel burns as fast as it enters, inaccurately graduated injection may cause the combustion to depart from the truly constant pressure sort, too rapid a feed causing pressure to rise; too slow a feed permitting a fall of pressure.

It is not surprising, therefore, to find that the mean effective pressure, thermal efficiency, or other characteristic performances differ in real gas engines from their cyclic equivalents, but this does not in any way reduce the value of cyclic analysis, in fact it accents the necessity for it. There are two sorts of contributions of practical value in the cyclic analysis taken in conjunction with the performance of real engines. The first is of the fundamental scientific order, giving numerical value to the extent of the possible improvement of real engines of any one class and, pointing out just where the losses occur with the amount of each indicates where any improvement must be made. The second is of more direct practical value to builders and users of engines as it is concerned with the prediction of results, for a given cylinder and fuel just what horse-power and fuel consumption may be expected, or what cylinder size is required for a given output. In these latter cases the cyclic analysis is responsible for the *form* of equations for mean effective pressure and thermal efficiency, actual tests furnishing numerical values for the constants of proportionality.

As cyclic performance is to be the basis of all computations on the approach to perfection of performance in real engines and of their probable power and efficiency, it is necessary to select the standard cyclic equations as a first step. It has been shown that comparatively simple performance equations are derivable for the Otto and Diesel cycles if the specific heats of gases are assumed to be constant, whereas it is known that they are not constant. But with any proposed law of variation in specific heats it has also been shown that the equations for cyclic performance are very difficult of any solution and impossible to exactly solve. This makes it difficult to decide on a course of procedure for practical computations in the first group of comparisons that are concerned with the approach to perfection of real engines. Strictly speaking, they should be compared with cyclic performance as computed on the basis of variability of specific heats, but in view first, of the uncertainty of the law of variation, and second, of the complexity of the cyclic equations, this is not yet a feasible thing as a matter of regular engineering routine, however desirable it may be in exceptional cases.

It is, therefore, generally agreed that the cyclic results based on constancy of specific heats shall be used as a basis of comparison of real engine performance, and in the simple everyday predictions of probable power and efficiency this is quite as good as any other, because a single constant factor can include all cyclic departures as well as the losses in the engine itself. The next step in establishing the cyclic standard, is to fix the physical properties of the working

gases, for it must be remembered that these working gases include some fue some air, and some products of combustion left in the cylinder from a previou explosion. Every change in fuel, or in proportions of fuel, to air, to burn gases, involves a different specific heat for the working mixture and even f a constant mixture, the expansion stroke, being made only with burnt gase will have different specific heats than the compression stroke where the mixtu is yet unburnt. Some authorities attempt to evaluate the specific heats f these various mixtures, but this practice is defensible only in those exception cases where approach to perfection is under study, and here variability of specif heat with temperature and pressure must be taken into consideration as we as variability due to chemical composition, a procedure so far impossibl This has led to the assumption that for the cycle the working gases have th physical properties of *air*, or are air, have specific heats known to apply air at 32° F., and that these do not vary.

On this basis the thermal efficiency of real engines is compared with a s called *air-card standard* which is the cycle most nearly corresponding, an with air as the gas. Compared to this air-card standard, actual performan is found to be surprisingly constant, regardless of the fuel, the compressio or the size of engine, the ratio ranging from .40 to .60 for Otto cyc engines and this is known as the *diagram factor*. In some cases the reaso for an increase or decrease in this number can be found, but in general th is not possible. For example, it is generally higher in large than in sma engines, higher when the cylinder and piston heat absorbing surface per cub foot of working gases is small, but other influences may entirely countera these and the precise effect of each separate influence cannot be evaluated.

According to the above, Otto cycle engines yield efficiencies referred indicated horse-power that are in round numbers somewhere about half th corresponding air-card efficiencies, so that if absolutely perfect they apparentl would be only twice as good as they now are, or would consume on half the fuel they do. A little more precisely, Wimperis has made some com parisons by means of his formula based on Clerk's linear variation of specif heat with temperature, and finds that when the air card with constant specif heat yields efficiencies of 54 per cent for a compression from 7 to 1 volume the air card with variable specific heats yields 44 per cent and 46 per cen for a maximum temperature of 1600° C. = 2912° F. and 1000° C. = 1832° F respectively. If for these cases the diagram factor based on constant specif heats were .50, then based on variable specific heats, it would be .615 and .5 respectively, showing the performance to be really nearer to perfection tha at first appeared.

A similar computation made by Meyer on test data of a Deutz engin 8.67 × 13 ins. with a volumetric compression ratio of 8.9, delivering about 10 H.P showed air-card efficiencies of 39.75 per cent and 40.30 per cent, based on co stant specific heat, and 29.69 per cent and 31.78 per cent, when based on h values of variable specific heat to which correspond the diagram factors .665 and .655 for the former, and .89 and .83 for the latter. From the stand

point of constant specific heat the performance could be half as good again, but if the variability values are right the improvement possible is only about one-ninth better than was obtained, to be quite perfect.

For these Otto cycle engines some little assistance in estimating the value of the diagram factor, is afforded by the determinations and the specific instances in the following Table CXXIX. Calculating from Meyer's test for a four-cycle engine 7.8×11.8 ins., 250 R.P.M., with four different compressions from 40 to 80 lbs. per square inch above atmosphere, Clerk finds the diagram factor practically constant at .58 even when the air-card efficiencies range from 33 per cent to 44 per cent. He also finds from Burstall's tests for the English Engineering Society Gas Engine Research Committee, on a four-cycle engine, 6×12 ins., 200 R.P.M., with compressions ranging from 30 to 105 lbs. per square inch above atmosphere, values of diagram factors from .38 to .59 while air-card efficiencies range between 33 per cent and 47 per cent, though not regularly. He points out, however, a generally higher diagram factor for low maximum temperatures, the values ranging from .50 to .59, when the maximum temperature is about 1100° C. =2012° F., whereas the factor lies between .40 and .50 for maximum temperatures of about 1750° C.=3182° F. Clerk also reports values of the diagram factors of .6 and .66 respectively for a 10-in. National and a 26-in. Crossley engine and while the difference favors the larger engine it is not enough to warrant any conclusion.

For some small gasoline engines, Callendar reports an increase of diagram factor with engine cylinder diameter as follows, .44 for 2.36 ins., .61 for 5.5 ins., .65 for 9.0 ins., .69 for 14.0 ins., but these figures do not apply generally as absolutely contradictory data are available, probably because of different values for other influences than cylinder size alone. Moreover, any values in excess of .6 for diagram factors for Otto engines must be regarded, to say the least, as abnormal.

The load that a constant-speed engine carries will, of course, change its performance because the governing mechanism when in operation always introduces losses greater at one time than at another, and the hand-control mechanism does the same for variable-speed engines. Such variations as these are considered here as incidental rather than fundamental data, and a test series by Hopkinson on a four-cylinder Daimler automobile engine, 3.56×5.11 ins., will serve to illustrate the point. At speeds of 400, 600, 1000, 1100, and 1225 R.P.M., the actual thermal efficiencies were 19.5 per cent, 21.5 per cent, 24.2 per cent, 24.6 per cent, and 22.3 per cent, referred to I.H.P. As the compression ratio by volumes was 3.85, the cyclic efficiency is, $1-\left(\frac{1}{3.85}\right)^{.4}=41.8$ per cent, so that at the successively higher speeds the diagram factors apparently were .48, .53, .58, .59, and .54. Really these are not the diagram factors, because at low speeds the engine was throttled and the cycle changed by the lowered suction line.

TABLE CXXIX

DIAGRAM FACTORS FOR OTTO CYCLE GAS ENGINES

Engine.	Size in Inches.		Test Authority.	Compression.		Efficiencies, Per Cent.		Diagram Factor
	Bore.	Stroke.		Vol. before / Vol. after	Press. after / Press. before	Actual.	Air Card Standard.	
Four cycle.....	7.8	11.8	Meyer	.....	3.73–6.45	25	44	.58
					" "	24.4	42	.58
					" "	21.4	37	.58
					" "	18.8	33	.57
Four cycle.....	6	12	Burstall	.....	3.03–8.13	18.9	33	.57
	6	12	"	.....	3.03–8.13	21.2	36	.59
	6	12	"	.....	3.03–8.13	21.9	43	.51
	6	12	"	.....	3.03–8.13	23.1	47	.49
	6	12	"	.....	3.03–8.13	16.6	33	.50
	6	12	"	.....	3.03–8.13	18.7	36	.52
	6	12	"	.....	3.03–8.13	17.2	43	.40
	6	12	"	.....	3.03–8.13	18.1	47	.38
40 H.P. four cycle	....	....	Hopkinson	6.37	.....	33.5–37.0 depending upon load	52	.64–.71
Cockerill.....	51.18	55.07	Hubert	.....	9.18	22.9	46.9	.49
"	33.465	39.37	"	.....	10.35	25.0	48.7	.514
Delamarre.....	22.64	37.4	Witz	.....	5.8	19.75	39.7	.498
Cockerill.....	23.622	31.5	François	.....	7.28	24.3	43.4	.56
Letombe.....	23.622	31.5	Witz	.....	8.03	27.3	45.0	.606
Winterthur.....	20.47	29.92	Allaire	.....	11.2	25.6	49.9	.514
Cie. Berlin Anhalt	16.92	27.56	Witz	.....	8.17	26.9	45.2	.595
Benz.....	16.73	22	Mathot	.....	13.06	23.8	52.0	.457
Soest.....	15.75	22.83	"	.....	7.35	31.3	43.6	.718
Deutz.....	14.173	22.87	"	.....	11.55	30.4	50.3	.605
Tangye.....	14.5	22	"	.....	4.83	30.6	36.2	.845
Fetu.....	13.78	22	"	.....	9.12	18.0	46.9	.384
Schmitz.....	13.78	21.26	"	.....	9.12	24.2	46.9	.515
Otto-Deutz.....	13	22.83	"	.....	9.4	38.8	47.3	.82
Niel.....	13.78	19	Witz	.....	11.58	31.8	50.4	.63
Winterthur.....	12.2	17.7	Mathot	.....	7.75	31.6	44.5	.71
Schmitz.....	11.85	18	"	.....	11.3	31.3	50.1	.625
Winterthur.....	11.8	17.7	"	.....	10.32	25.2	48.7	.518
Benier.....	11.8	17.3	Witz	.....	4.39	13.75	34.3	.4
Tangye.....	11	20	Mathot	.....	10.64	29.8	49.2	.605
Dudbridge.....	11	18.6	"	.....	4.83	29.2	36.4	.802
Tangye.....	10	19	"	.....	5.81	27.4	39.7	.69
"	10	19	Witz	.....	6.8	30.1	42.4	.71
National.....	10	18	Mathot	.....	5.88	21.2	39.9	.53
Güldner.....	9.85	15.75	Schrotter	.....	10.6	39.0	49.1	.795
"	9.85	15.75	"	.....	10.6	33.9	49.1	.69
Catteau.....	9	18	Witz	.....	12.59	37.2	51.5	.723
Tangye.....	7	16	Hirsch	.....	10.2	25.8	48.6	.53
Four cycle.....	6	12	Burstall	4	.....	21.0	42.8	.49
"	6	12	"	2.44	.....	18.0	29.6	.608
"	6	12	"	4	.....	18.0	42.8	.42
"	6	12	"	2.78	.....	17.6	33.3	.529
"	6	12	"	2.7	.....	16.4	32.7	.502

TABLE CXXIX—*Continued*

DIAGRAM FACTORS FOR OTTO CYCLE GAS ENGINES

Engine.	Size in Inches.		Test Authority.	Compression.		Efficiencies, Per Cent.		Diagram Factor.
	Bore.	Stroke.		Vol. before / Vol. after	Press. after / Press. before	Actual.	Air Card Standard.	
Four cycle.....	6	12	"	2.04	.....	16.2	34.6	.468
"	6	12	"	2.17	.....	15.6	26.2	.595
"	6	12	"	2.04	.....	15.5	24.6	.63
"	6	12	"	2.04	.....	15.4	24.6	.626
"	6	12	"	2.78	.....	15.2	33.3	.456
"	6	12	"	2.7	.....	15.0	32.7	.459
"	6	12	"	2.44	.....	14.4	29.6	.486
"	6	12	"	4.0	.....	13.6	42.8	.318
"	6	12	"	4.0	.....	13.4	42.8	.313
"	6	12	"	1.75	.....	12.6	19.5	.646
"	6	12	"	2.7	.....	11.7	32.7	.358
"	6	12	"	2.22	.....	19.4	26.9	.721
"	6	12	"	2.94	.....	20.0	35.0	.572
"	6	12	"	4.0	.....	22.7	42.8	.53
"	$8\frac{1}{2}$	13	Meyer	3.75	.....	32.7	41.2	.794
"	$8\frac{1}{2}$	13	"	3.6	.....	26.8	40.3	.665
"	$8\frac{1}{2}$	13	"	2.84	.....	20.2	35.2	.574

Compression pressure ratio has been calculated assuming an initial pressure of 14.7 lbs per square inch.

Examination of a very large number of tests showed that for various fuels used in typical Otto engines, the diagram factors lie somewhere within the following limits, for natural gas, coal gas, carburetted water, and producer gas, .45 and .55, for blast furnace gas, gasolene and kerosene vapors previously made and taken in during suction, from .40 to .50, but when injected during compression, the mixture non-homogeneous, the factors may fall quite low or be very high, depending on circumstances difficult, if not impossible, to predict. These figures are for ordinary commercial engines, not for special engines operated under finest test conditions where the factor slightly exceeds .70.

One most interesting and valuable series of tests for diagram factor is that conducted by Strong for the U.S.G. Survey on a 10-H.P. Nash and a 15-H.P. Otto engine, using gasolene and alcohol, the results of which are shown in Fig. 276. In this series the compression was varied from 85 to 215 lbs. per square inch above atmosphere and both fuels were tested in each engine with most unusual accuracy of method. At 6 atmospheres absolute compression, the diagram factors were .65 for gasolene and .716 for alcohol, at 8 atmospheres, .66 and .724, respectively, this being the maximum compression for gasolene, but at compressions of 11 and 15 atm. absolute for alcohol, the factors were .732 and .74. Strong finds that the efficiency is given by Eq. (1161).

$$\left.\begin{aligned} E \text{ (for gasolene)} &= 1-\left(\frac{P_a}{P_b}\right)^{.17} \quad (a) \\ E \text{ (for alcohol)} &= 1-\left(\frac{P_a}{P_b}\right)^{.19} \quad (b) \end{aligned}\right\} \quad \ldots\ldots \quad (1161)$$

Therefore, the diagram factor is as given by Eqs. (1162) and (1163).

$$F \text{ (for gasolene)} = \left[\frac{1-\left(\frac{P_a}{P_b}\right)^{.17}}{1-\left(\frac{P_a}{P_b}\right)^{.29}}\right] \quad \ldots\ldots \quad (1162)$$

$$F \text{ (for alcohol)} = \left[\frac{1-\left(\frac{P_a}{P_b}\right)^{.19}}{1-\left(\frac{P_a}{P_b}\right)^{.29}}\right] \quad \ldots\ldots \quad (1163)$$

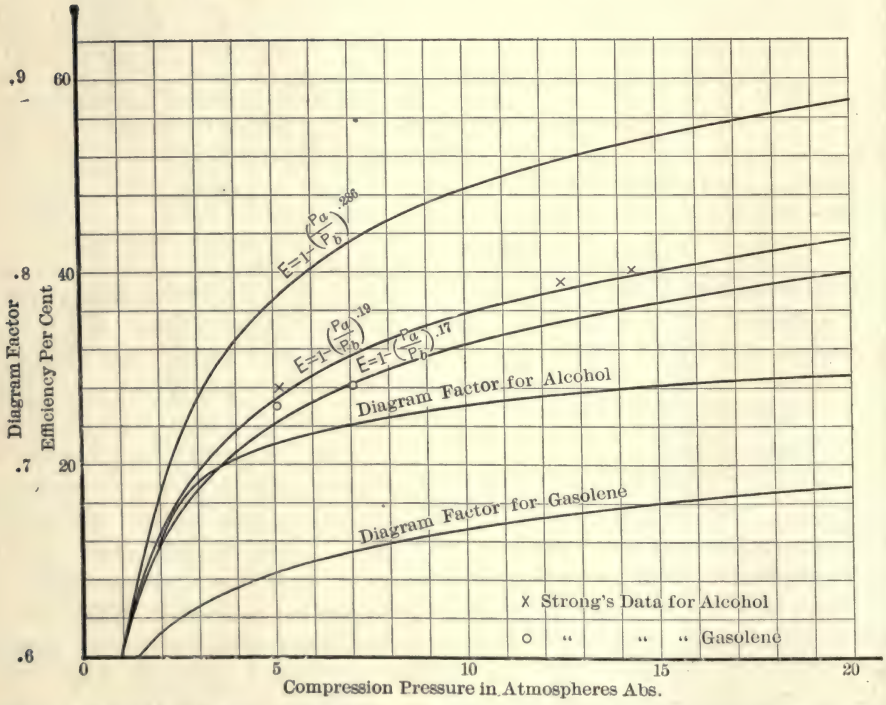


FIG. 276.—Variation of Otto Cycle Gas Engine Diagram Factors with Compression, in Small Gasolene and Alcohol Engines.

Diesel cycle engines have diagram factors at about their rated load, which corresponds to fuel admission for about 10 per cent of the stroke, of very closely, .50, though it must be admitted that there are insufficient data available to make this quite as well established as the factors for Otto engines.

Prediction of thermal efficiency for a given engine is, through the use of established diagram factors reduced to a simple operation of finding the cyclic or air-card efficiency for the allowable compression pressure, and multiplying it by the factor. From this the heat consumption per hour and gas consumption follow directly.

Let E = thermal efficiency of air card;

" E_I = thermal efficiency of engine referred to I.H.P.;

" E_B = thermal efficiency of engine referred to B.H.P.

" E_M = mechanical efficiency = $\frac{\text{B.H.P.}}{\text{I.H.P.}}$;

" F = diagram factor;

" H = B.T.U. per cubic foot gas, standard conditions of 29.92 ins. Hg and 32° F.

Then

$$E_I = EF. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1164)$$

$$E_B = E_I E_M = EFE_M \quad . \quad . \quad . \quad . \quad . \quad . \quad (1165)$$

$$\text{B.T.U. per hr. per I.H.P.} = \frac{2545}{E_I} = \frac{2545}{EF}. \quad . \quad . \quad . \quad . \quad . \quad (1166)$$

$$\text{B.T.U. per hr. per B.H.P.} = \frac{2545}{E_B} = \frac{2545}{E_I E_M} = \frac{2545}{EFE_M}. \quad . \quad . \quad (1167)$$

$$\text{Cu.ft. gas per hr. per I.H.P. (Std.)} = \frac{2545}{E_I H} = \frac{2545}{EFH}. \quad . \quad . \quad . \quad . \quad . \quad (1168)$$

$$\text{Cu.ft. gas per hr. per B.H.P. (Std.)} = \frac{2545}{E_B H} = \frac{2545}{E_I E_M H} = \frac{2545}{EFE_M H}. \quad . \quad (1169)$$

That these equations may be numerically evaluated requires some data on mechanical efficiency and usual compressions in engines, which are given in the following Tables CXXX, and CXXXI, the former showing mechanical efficiency as a function of type of engine structure, and the latter allowable compressions as functions of the fuel class, from 2 atm. to 35 atm. The allowable compressions are always limited by the temperature of ignition of the mixture, when mixtures are under compression and by the effectiveness of the internal wall cooling. When only air is compressed, the compression may be anything, but is generally limited by the cost consideration, high

compressions producing large working forces and requiring heavy and more expensive engine parts. Mechanical efficiencies include all losses, mechanical friction, fluid friction, the work of charging and exhausting, and in addition the precompression or pump work in two-cycle engines.

TABLE CXXX

MECHANICAL EFFICIENCIES OF GAS ENGINES $= E_M = \dfrac{\text{B.H.P.}}{\text{I.H.P. (motor cylinders)}}$

Type of Engine.	Mechanical Efficiency.	
	Four Cycle.	Two Cycle.
Small high-speed automobile multi-cylinder, single acting	.75	
" single-cylinder boat engine, single acting	.85	.68
" or medium single-cylinder stationary, single acting	.87	.7
" " " two cylinder stationary single acting	.84	
" " " three-cylinder stationary, single acting	.82	
" " " four-cylinder stationary, single acting	.80	
Large single-cylinder stationary, single acting	.90	.70
" two-cylinder stationary, single acting	.86	to
" four-cylinder stationary, single acting	.84	.80
Double-acting single cylinder	.83	.75
" " tandem, two cylinder	.81	.73
" " " twin four cylinder	.77	.69

TABLE CXXXI

ALLOWABLE COMPRESSION OF GAS ENGINES IN STANDARD ATMOSPHERES ABSOLUTE

Fuel.	Compression in Standard Atmospheres (Abs.) for Water-cooled Engines.
Gasoline in suction mixture	6 to 8
Kerosene in suction mixture	7 with water injection 3 with hot mixture without water
Alcohol in suction mixture	9 to 13
Oil injection in hot bulb before compression	4
Oil injection after compression	18 in Franchetti-Otto cycle engine 35 in Diesel
Carburetted water gas	6 to 8
Coal gas	6 to 9
Natural gas	8 to 11
Producer gas	8 to 11
Blast-furnace gas	10 to 13
Coke oven gas	8 to 10

Predictions of gas-engine horse-power are best made by establishing values for the mean effective pressure of the whole cycle referred to one stroke, with which the *horse-power of any gas engine, two cycle or four, and of any number*

of cylinders can be found, by multiplying this mean effective pressure by the piston area in square inches, the stroke in feet, the total number of cycles per minute in all working chambers, and dividing the product by 33,000. The prediction of mean effective pressure of gas-engine cycles is, therefore, a matter of considerable practical importance not only as directly fixing the horse-power, but as an indication of good working of the whole mechanism, since any loss in weight of charge by suction resistances, or heating on entrance, as well as any failure, to compress as much as the fuel will stand, or to realize the full

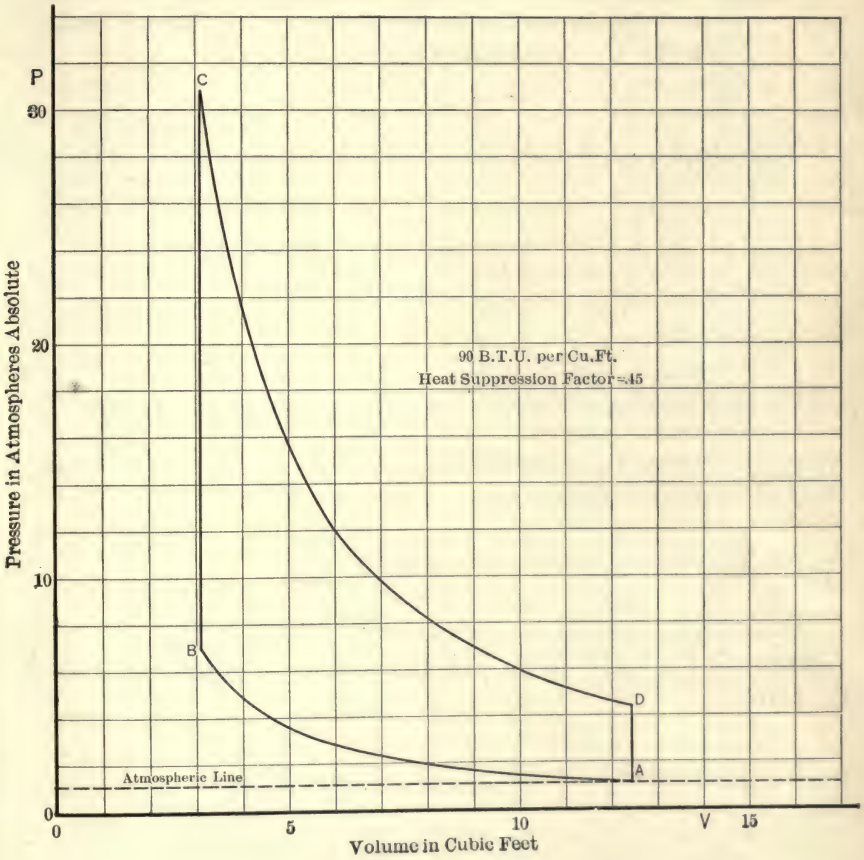


FIG. 277.—Otto Cycle Standard Reference Diagram without Suction or Exhaust Lines.

temperature, pressure or volume increase on combustion, or to completely expand and expel burnt gases with least resistance, all these effects will be shown in reduced mean effective pressures. Unlike the case for steam engines, the mean effective pressure of gas engines is intimately associated with thermal losses and closely associated with thermal efficiency, so a purely thermal analysis of mean effective pressure of gas engines is necessary.

Referring to the ideal diagram of the cycle, Fig. 277, without any losses, except the heat suppression durring combustion,

Let $(V_a - V_b)$ = displacement = volume of fresh charge per cycle;
" V_b = clearance volume = volume of neutral gases added per cycle;
" W = foot-pounds work per cycle;
" E = air-card thermal efficiency;
" E_I = thermal efficiency referred to I.H.P.;
" Q_1 = heat supplied per cycle;
" (m.e.p.)′ = mean effective pressure, pounds per square inch for compression and expansion strokes referred to one stroke;
" H = B.T.U. per cubic foot gas (std.), low value.

Then

$$W = 144(\text{m.e.p.})'(V_a - V_b) = 778EFQ_1 = 7881 - \left[\left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}\right]FQ_1 \quad (1170)$$

whence

$$(\text{m.e.p.})' = 5.4\left(\frac{FQ_1}{V_a - V_b}\right)\left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}\right] \quad . \quad . \quad . \quad . \quad . \quad (1171)$$

But

$$\frac{Q_1}{V_a - V_b} = \frac{\text{Heat liberated per cycle}}{\text{Cu.ft fresh mixture taken in per cycle}} = \frac{\text{Heat per cycle}}{\text{Displacement per cycle}}.$$

Therefore, if the working charge during suction be always at 32° F. and one standard atmosphere pressure, then

$$\frac{Q_1}{V_a - V_b} = \text{B.T.U. per cu.ft. fresh mixture (std.)} = \frac{H}{a+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1172)$$

where a = cubic feet of air per cubic foot of gas for best mixture.

Substituting Eq. (1172) in Eq. (1171), the mean effective pressure is given by Eq. (1173).

$$\left.\begin{aligned}(\text{m.e.p.})' &= 5.4F\frac{H}{a+1}\left[1 - \left(\frac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}}\right] \quad (a)\\ &= 5.4F\frac{H}{a+1}\left[1 - \left(\frac{P_a}{P_b}\right)^{.29}\right] \quad (b)\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1173)$$

It appears from this equation that the same diagram factor F applies to mean effective pressure as to thermal efficiency. For convenience in calculation the values of the bracket are set down in Table CXXXII and and these are to be multiplied by $\frac{H}{a+1}$, values for which have already been calculated in the previous chapter for various typical gases, and by the diagram factor (F) applied to efficiencies previously discussed.

TABLE CXXXII
MEAN EFFECTIVE PRESSURE FACTORS FOR OTTO CYCLE ENGINES

$\frac{Px}{Pa}$ Atmos	$\left(\frac{Pa}{Px}\right)^{\frac{5}{7}}$	$\left[1-\left(\frac{Pa}{Px}\right)^{\frac{2}{7}}\right]$	$5.4\left[1-\left(\frac{Pa}{Px}\right)^{\frac{2}{7}}\right]$
1.0	1	.0000	.000
	1.000		
	.878	.0508	.274
1.2	9.43442−10	8.705522−10	9.438140−10
	.786	.0917	.495
1.4	9.895623−10	8.962180−10	9.694798−10
	.715	.1256	.679
1.6	9.854271−10	9.098990−10	9.831608−10
	.656	.1546	.835
1.8	9.817662−10	9.189181−10	9.921799−10
	.610	.1797	.971
2.0	9.784979−10	9.254451−10	9.987069−10
	.569	.2017	1.090
2.2	9.755412−10	9.304706−10	.037324
	.535	.2213	1.196
2.4	9.728421−10	9.344981−10	.077599
	.505	.2389	1.291
2.6	9.703591−10	9.378234−10	.110842
	.479	.2549	1.377
2.8	9.680601−10	9.406300−10	.138918
	.456	.2694	1.456
3.0	9.659199−10	9.430398−10	.163016
	.436	.2827	1.528
3.2	9.639179−10	9.451403−10	.184021
	.417	.2951	1.594
3.4	9.620372−10	9.469925−10	.202543
	.401	.3065	1.656
3.6	9.602641−10	9.486402−10	.219020
	.385	.3171	1.713
3.8	9.585869−10	9.501223−10	.233841
	.372	.3271	1.767
4.0	9.569957−10	9.514615−10	.247233
	.359	.3364	1.817
4.2	9.554822−10	9.526804−10	.259422
	.347	.3451	1.865
4.4	9.540391−10	9.537983−10	.270601
	.336	.3534	1.909
4.6	9.526601−10	9.548255−10	.280873
	.327	.3612	1.952
4.8	9.513399−10	9.557760−10	.290378
	.3168	.3686	1.991
5.0	9.500736−10	9.566579−10	.299197
	.3080	.3757	2.030
5.2	9.488569−10	9.574794−10	.307413
	.2998	.3623	2.066
5.4	9.476861−10	9.582461−10	.315079
	.2921	.3887	2.100
5.6	9.465580−10	9.589648−10	.322266
	.2849	.3948	2.133
5.8	9.454694−10	9.596410−10	.329028
	.2781	.4007	2.165
6.0	9.444178−10	9.602776−10	.335394
	.2716	.4063	2.195
6.2	9.434006−10	9.608794−10	.341414
	.2656	.4116	2.224
6.4	9.424157−10	9.614486−10	.347104
	.2598	.4168	2.252
6.6	9.414611−10	9.619886−10	.352504
	.2543	.4217	2.278
6.8	9.405351−10	9.625025−10	.357643
	.2491	.4265	2.304
7.0	9.396359−10	9.629909−10	.362527
	.2441	.4311	2.329
7.2	9.387620−10	9.634558−10	.367176
	.2394	.4355	2.353
7.4	9.379120−10	9.639008−10	.371626
	.2349	.4398	2.376
7.6	9.370847−10	9.643265−10	.375883
	.2306	.4439	2.399
7.8	9.362789−10	9.647334−10	.379952
	.2264	.4480	2.440
8.0	9.354936−10	9.651239−10	.383857
	.2225	.4518	2.446
8.2	9.247276−10	9.654984−10	.387602
	.2187	.4556	2.462
8.4	9.339801−10	9.658584−10	.391202
	.2150	.4592	2.481
8.6	9.332501−10	9.662040−10	.394658
	.2115	.4628	2.500
8.8	9.325369−10	9.665384−10	.398002

$\frac{Px}{Pa}$ Atmos	$\left(\frac{Pa}{Px}\right)^{\frac{5}{7}}$	$\left[1-\left(\frac{Pa}{Px}\right)^{\frac{2}{7}}\right]$	$5.4\left[1-\left(\frac{Pa}{Px}\right)^{\frac{2}{7}}\right]$
	.2082	.4662	2.519
9	9.318398−10	9.668591−10	0.401209
	.1931	.4821	2.604
10	9.285714−10	9.683092−10	.415710
	.1804	.4960	2.680
11	9.256148−10	9.695456−10	.428074
	.1695	.5083	2.746
12	9.229156−10	9.706154−10	.438172
	.1601	.5195	2.807
13	9.204328−10	9.715552−10	.448170
	.1518	.5295	2.861
14	9.181337−10	9.723891−10	.456509
	.1445	.5378	2.911
15	9.159935−10	9.731355−10	.463973
	.1380	.5471	2.956
16	9.139914−10	9.738099−10	.470717
	.1322	.5549	2.998
17	9.121108−10	9.744231−10	.476849
	.1269	.5621	3.037
18	9.103376−10	9.749837−10	.482455
	.1221	.5688	3.073
19	9.086604−10	9.754990−10	.487608
	.1177	.5751	3.107
20	9.070693−10	9.799751−10	.492369
	.1136	.5810	3.139
21	9.055556−10	9.764169−10	.496787
	.1099	.5865	3.169
22	9.041126−10	9.768283−10	.500901
	.1065	.5915	3.197
23	9.027337−10	9.772131−10	.504749
	.1033	.5967	3.224
24	9.014135−10	9.775741−10	.508359
	.1003	.6014	3.249
25	9.001471−10	9.779127−10	.511745
	.0976	.6058	3.273
26	8.989305−10	9.782339−10	.514947
	.0950	.6100	3.296
27	8.977597−10	9.785344−10	.517962
	.0925	.6141	3.318
28	8.966316−10	9.788204−10	.520822
	.0902	.6179	3.338
29	8.955430−10	9.790918−10	.523536
	.0881	.6215	3.358
30	8.944914−10	9.793504−10	.526122
	.0860	.6251	3.377
31	8.934741−10	9.795963−10	.528581
	.0841	.6285	3.396
32	8.924893−10	9.798305−10	.530923
	.0823	.6318	3.413
33	9.915347−10	9.800546−10	.533164
	.0806	.6349	3.430
34	8.906086−10	9.802691−10	.535309
	.0789	.6379	3.446
35	8.897094−10	9.804746−10	.537364
	.0773	.6408	3.462
36	8.888355−10	9.807623−10	.539341
	.0758	.6436	3.477
37	8.879056−10	9.808616−10	.541234
	.0744	.6463	3.492
38	8.871583−10	9.810434−10	.543052
	.0730	.6489	3.506
39	8.863525−10	9.812192−10	.544810
	.0717	.6514	3.520
40	8.855671−10	9.813881−10	.546499
	.0705	.6539	3.533
41	8.848011−10	9.815511−10	.548129
	.0693	.6563	3.546
42	8.840733−10	9.817082−10	.549700
	.0681	.6586	3.558
43	8.833147−10	9.818609−10	.551227
	.0670	.6608	3.570
44	8.826105−10	9.820076−10	.552694
	.0659		
45	8.819134−10		
	.0649		
46	8.812316−10		
	.0639		
47	8.805644−10		
	.0630		
48	8.799114−10		
	.0620		

The actual indicator card is subject to losses in suction and exhausting which make it take on the form of Fig. 278, shown to a large scale incomplete and to a small scale complete. The exhaust stroke ends at H, at a pressure greater than atmosphere, so that no new charge at atmospheric pressure can enter till an expansion, actually or in effect, has lowered the pressure to that at point I. Here some volume of atmospheric fresh mixture begins to enter in effect, and this continues till point A has been reached, where compression begins at a pressure less than atmosphere. This resisted suction is characterized by a

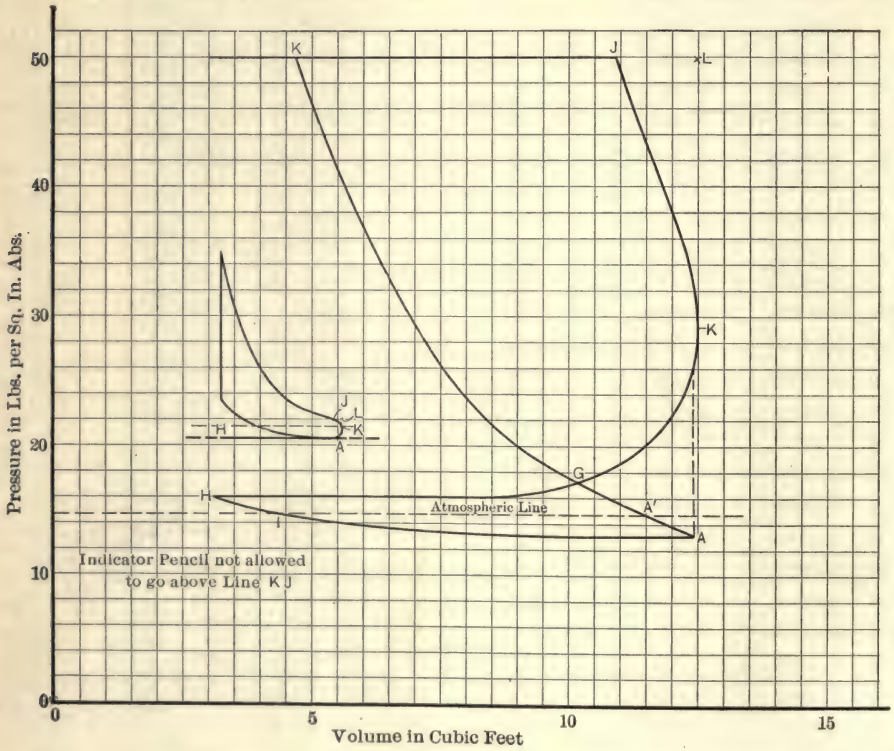


FIG. 278.—Otto Cycle Gas Engine Diagram with Suction and Exhaust Losses.

negative mean suction pressure to be subtracted from the positive mean pressure of the upper loop. There is, however, another suction effect and that is a volume increase of the original fresh cool charge, due to mixture with hot burnt gases in the clearance and to its passage over hot metal surfaces, reducing the weight of the charge, and the real volumetric efficiency. Various tests have shown that the mixture suffers a rise of temperature during suction, ranging from 125° F. to 200° F., and mean suction resistances depending on speed and valve openings from less than 1 lb. square inch to over 5 lbs. These data together lead Güldner to assign values of real volumetric efficiency according to the following Table CXXXIII with values of terminal suction

pressure, p_a, from which the mean suction resistance is estimated at two-thirds of the terminal.

A similar situation exists for the exhaust stroke, where a mean resistance is encountered ranging from half a pound to three or four pounds per square inch, depending on the design and valve setting, and this is added to the suction resistance. The sum is to be subtracted from the mean pressure of the compressing and expanding area *ABCDA* of Fig. 277, to get the mean effective pressure of the whole, referred to one stroke, neglecting the small curved triangular area *JKL* in comparison with the rest.

TABLE CXXXIII

GÜLDNER'S VALUES OF OTTO ENGINE REAL VOLUMETRIC EFFICIENCY WITH ESTIMATED MEAN SUCTION RESISTANCES

Type of Engine.	True Volumetric Efficiency EV.	Terminal Suction Pressure, Lbs. per Sq. In. Abs.	Estimated Mean Suction Pressure, Lbs. per Sq. In.
Slow-speed mechanical valve	.88 to .93	12.8 to 13.7	1.2 to .7
Slow-speed automatic valve	.80 to .87	12.5 to 13.3	4.5 to .9
High-speed mechanical valve	.78 to .85	11.8 to 12.5	1.9 to 1.5
High-speed automatic valve	.65 to .75	11.5 to 12.2	2.1 to 1.3
Very high-speed air cooled automatic valve	.50 to .65	8.8 to 11.0	3.9 to 2.5

If the gas and air mixture exist at a temperature external to the cylinder, other than 32° F. and at a pressure other than one standard atmosphere, then the heat per cubic foot of mixture measured external to the cylinder, will not have the value $\frac{H}{a+1}$ previously assigned, but a correction can be applied.

Let T_m = absolute temperature F of the mixture external to the cylinder;
" h_m = absolute pressure of the mixture external to the cylinder, inches Hg

Then

$$\left.\begin{array}{l} \left\{\begin{array}{l}\text{B.T.U. per cu.ft. of fresh}\\ \text{mixture externally measured}\end{array}\right\} = \left(\frac{H}{a+1}\right)\times\left(\frac{29.92}{h_m}\right)\times\left(\frac{T_m}{492}\right) \quad (a) \\ \qquad\qquad = .06\left(\frac{H}{a+1}\right)\frac{T_m}{P_m} \quad (b) \end{array}\right\} \quad . \quad (1174)$$

It has been shown that the mean pressure of the compression and expansion strokes referred to one stroke is given by an expression containing the term

$$\frac{Q_1}{V_a - V_b} = \frac{\text{Heat per cycle}}{\text{Displacement per cycle}}.$$

But

(Displacement per cycle) $\times E_V$ = cu.ft. externally measured mixture, per cycle

or

$$\frac{Q_1}{V_a - V_b} = E_V \times \left(\frac{\text{Heat per cycle}}{\text{Cu.ft. of externally measured mixture per cycle}} \right)$$

$$= E_V \times (\text{Heat per cu.ft. of mixture externally measured})$$

$$= .06 E_V \left(\frac{H}{a+1} \right) \frac{T_m}{h_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1175$$

Substituting Eq. (1175) in Eq. (1171) giving the mean effective pressur for a compression and expansion stroke, and subtracting the mean resistanc on suction p_s and on exhaust p_e there results for Otto cycle gas engines Eq (1176) and (1177):

$$(\text{m.e.p.}) = 5.4 \times .06 F E_V \left(\frac{H}{a+1} \right) \frac{T_m}{h_m} \left[1 - \left(\frac{P_a}{P_b} \right)^{.29} \right] - p_s - p_e \quad . \quad . \quad (1176$$

$$= .324 F E_V \left(\frac{H}{a+1} \right) \frac{T_m}{h_m} \left[1 - \left(\frac{P_a}{P_b} \right)^{.29} \right] - p_s - p_e. \quad . \quad . \quad . \quad (1177$$

For Diesel cycle engines the following Eq. (1178) will serve:

$$(\text{m.e.p.}) = F E_V \times (\text{m.e.p. of the cycle}) - p_s - p_e. \quad . \quad . \quad . \quad (1178$$

As is the case with steam engines, the efficiency and fuel consumptio of gas engines are not the same at all loads, even when expressed in term of indicated horse-power and the variations are due to the kind of contr used. There is, however, an interesting relation that should be noted, base on the fact that the thermal efficiency referred to I.H.P. would be constar if, (*a*) there were no suction and exhaust resistances; (*b*) the ignition we always adjusted to give vertical explosion lines; (*c*) all the fuel used we burned at the right time. There are three typical methods of control, eac yielding a characteristic fuel or heat-consumption load curve, the qualit control mainly used in oil engines, the hit and miss in cheap engines that nee not regulate closely, and the suction throttle, with sometimes a fourth, suctio cut-off. The first two have always the same compression characteristics ar valve resistances, while with the throttle resistance varies all the time wi load so it would be expected that two characteristic curves would result. F the former, fuel per hour or B.T.U. supplied per hour, gives a straight li when plotted to load, except where, at overload, excess and unburnt fu causes a change of curvature. This is equivalent in steam engines and tu bines to the straight Willans line and the curve which characterizes t throttle type throughout is expressed as a second degree equation similar

that found for cut-off-governed steam engines. Several efficiency curves of each engine type are shown in Fig. 279 on actual and in Fig. 280 on a fractional-load basis to bring all sizes in accord. From these fuel consumption or heat-supplied curves could be found having equations of the first degree if straight or of the second degree otherwise, but this is omitted here to save space.

Any examination of the question of gas-engine efficiency would be incomplete without at least a brief review of the ultimate disposition of the heat supplied, that does not become converted into work. The *heat balance* of gas engines and complete plants is the term applied to a tabular or graphic statement of the disposition of the heat supplied in fuel, in the form of gas or oil for engines, or in the form of coal or coke for gas producer and engine plants together. These balances are, of course, different for different sizes and styles of engines, for different fuels and engine loads, but more widely different for complete plants of many units operating on a fluctuating load. In a recent paper by Andrus and Porter some data were presented showing for average plant working conditions in England that the overall efficiency of complete gas plants averages about 14 per cent from useful power to heat of coal, and over periods ranging from one month to a year. The minimum reported was 12.4 per cent and the maximum 15.05 per cent. They also compared similar data for steam plants which showed average of 6.83 per cent overall efficiency for periods of one to twelve months, but here the lowest was only 3.70 per cent and the highest 9.30 per cent, showing in round numbers about twice as good a return for gas systems as for steam. Comparing engines alone it will be found that the proportion is different, charging heat in steam or in gas as heat supplied, the performances are closer on the average, though the best gas or oil engine performance remains about twice as good as the best steam engine or turbine performance, in efficiency.

One interesting overall estimate of heat balance by Coster for marine plants is as follows, Table CXXXIV.

TABLE CXXXIV

COMPARATIVE HEAT BALANCES OF PRODUCER AND ENGINE PLANTS

	Gas Producer Plant.	Steam Plants.	
		Turbine.	Reciprocating Engines.
B H.P.	25.0	13.0	10.8
Friction loss	1.4	1.0	1.2
Exhaust loss	30.4	56.0	58.0
Gas-engine jacket loss	20.0		
Gas-engine radiation, etc	3.2		
Producer or boiler loss	20.0	30.0	30.0
	100.00	100.0	100.0

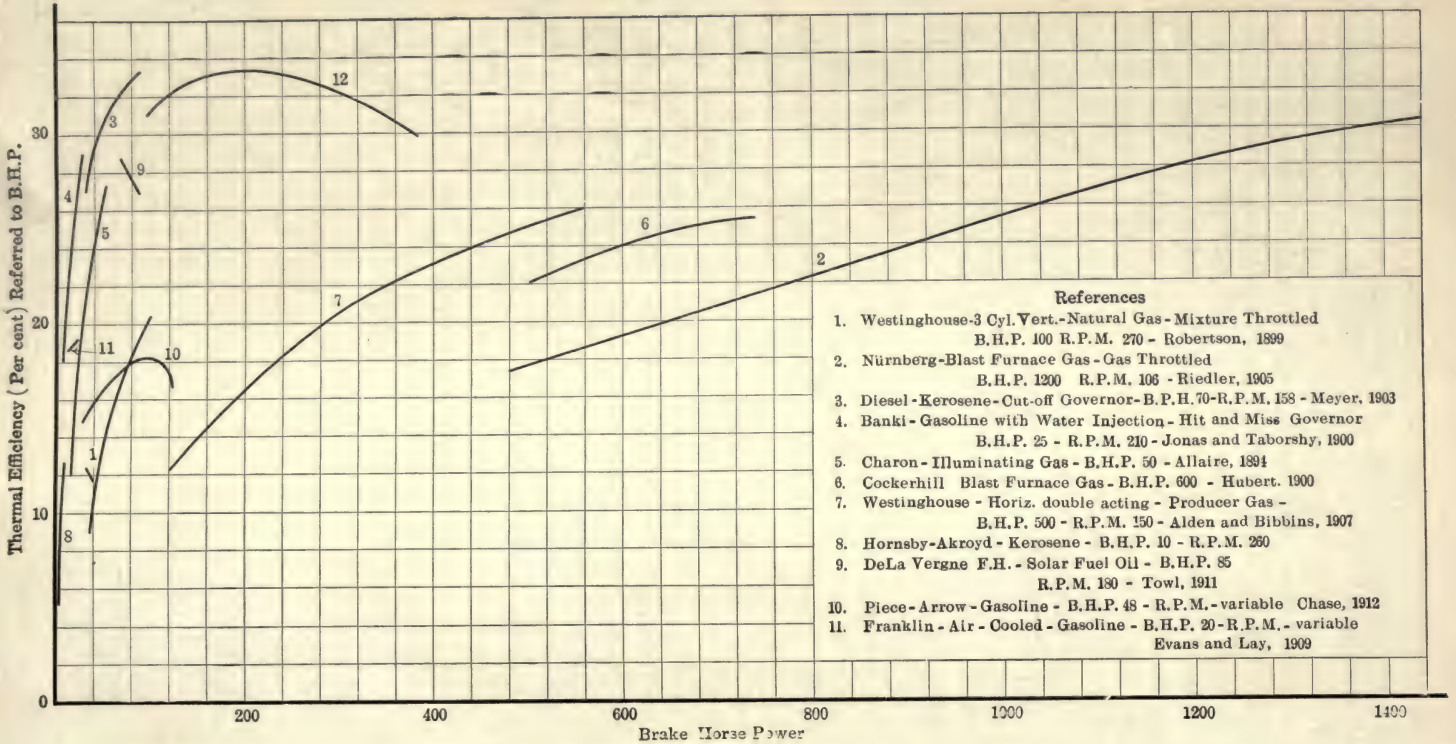


FIG. 279.—Variation of Gas Engine Thermal Efficiency with Actual Load.

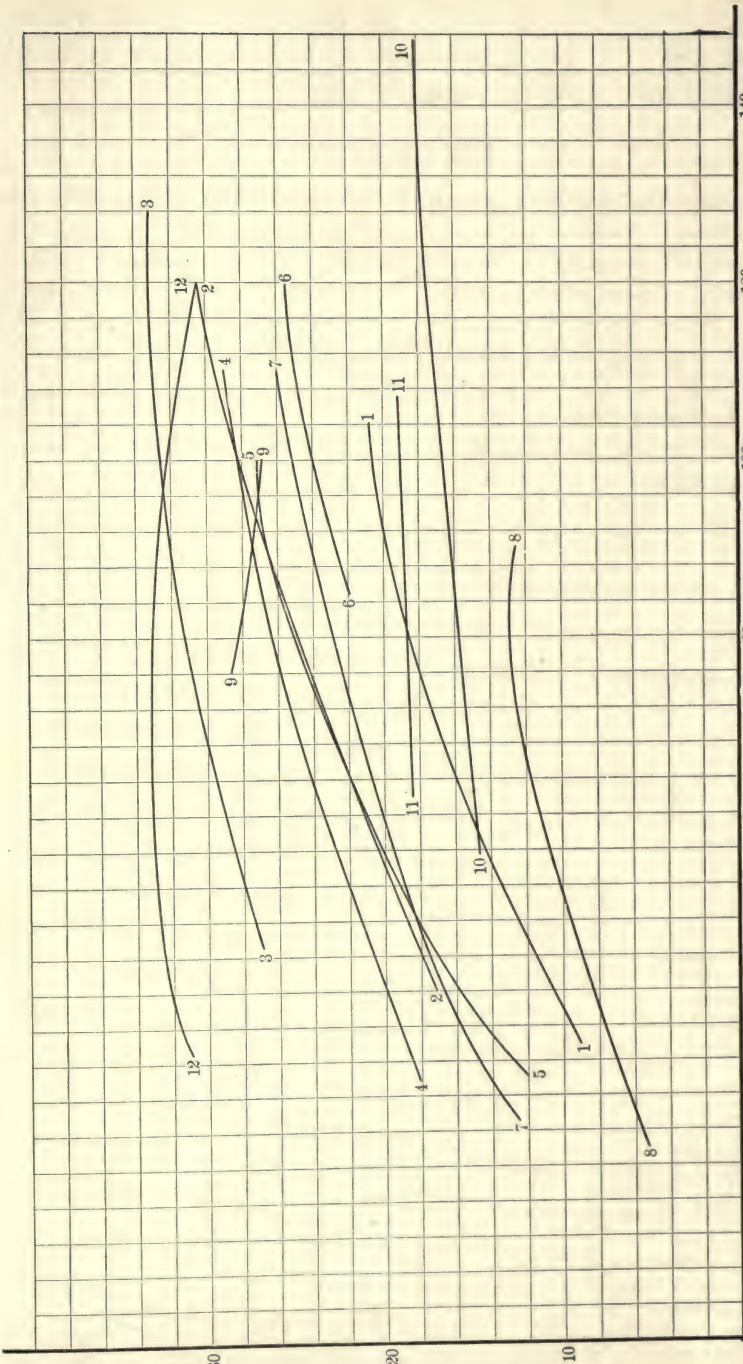


FIG. 280.—Variation of Gas Engine Thermal Efficiency with Fractional Load.

The numbers give the per cent of the heat of coal converted into work, or otherwise disposed. Two balances, one by Bibbins and one by Mathot, of a gas producer plant complete are given below, Table CXXXV.

TABLE CXXXV

HEAT BALANCES OF GAS PRODUCER PLANTS (PER CENT OF COAL HEAT)

Authority.	B.H.P.	Friction and Pumps.	Exhaust and Radiation.	Jacket.	Producer Loss.
Bibbins	18.36	3.37	28.81	25.22	26.22
Mathot	21.00	4.00	23.00	27.00	25.00

In the following Table CXXXVI the heat distribution for several different engines alone are compared to show the range of variations but without any idea of explanation of differences.

TABLE CXXXVI

HEAT BALANCES OF GAS AND OIL ENGINES (PER CENT OF GAS OR OIL HEAT)

Engine and Authority.	I.H.P.	B.H.P.	Friction.	Exhaust.	Jacket.	Radiation and Unaccounted for.
Donkin	22.32	.....	.....	43.29	32.96	1.43
Beck engine, Kennedy	19.4	.....	.....	42.9	33.0	4.7
Griffin engine, Kennedy	21.1	.....	.....	39.8	35.2	3.9
Atkinson engine, Kennedy	25.5	.....	.....	37.9	27.0	9.6
Otto Crossley engine, Kennedy	22.1	.....	.....	35.5	43.2	.8 excess
Comp. Ratio. R.P.M. a/g						
2.67 187 7.11, Slaby	18.0	.....	.....	30.8	51.2	
2.67 247 7.35, Slaby	18.1	.....	.....	36.3	45.6	
4.32 187 7.43, Slaby	24.4	.....	.....	21.8	53.8	
4.32 247 7.40, Slaby	23.7	.....	.....	26.8	49.5	
General, Mathot	33.0	28.0	5.0	31.0*	36.0	
Westinghouse, Bibbins	29.48	24.9	4.58	36.3	34.22	
300 H.P. engine at 197 H.P., Eberly	43.5	33.5	10.0	24.1	34.3	1.9 excess
" " 294 H.P., Eberly	45.8	32.2	13.6 †	23.9	31.8	1.5 "
" " 335 H.P., Eberly	41.5	30.9	10.6	24.8	33.8	.1 "
6 H.P. engine, I.C.E	31.8	26.7	5.1	41.1	27.1 ‡	
24 H.P. engine, I.C.E	33.3	28.3	5	37.1	29.6 ‡	
Deutz 2 H.P., Wimplinger	21.5	16.1	5.4	25	50.4	3.1
Güldner 20 H.P., Schröter	42.7	.....	.....	24.1	33.2	
Walrath 75 H.P., Geer and Yanelain	27.1	21.3	5.8	23.4	49.5	
300 H.P., Goldsmith and Hartwig	24.4	17.1	7.3	50.6	25.0	
Hornsby, Robinson	21	18	3	29	50	
De la Vergne F. H., Towl	40.14	27.52	12.62	20.03	26.50	13.33
Pierce-Arrow, Chase	.....	18	.....	.....	29.4	

* Including radiation. † Including pumps. ‡ Including external radiation.

There is no doubt that one of the prime factors in the failure of the performance of real gas engines, to agree with that of the corresponding cycle, is the combined effect of leakage and heat exchanges between walls and working gases, and it is certain that all jacket loss is derivable from heat transmitted from gases to walls. These exchanges have been much studied, but it must be confessed with as yet little conclusive result. Therefore, these results will be omitted but the most promising mode of attack is given below, with, however, the warning that it is quite useless if any leakage is taking place, and if the physical constants of the gases are not established. Just as investigators have been misled by air-compressor lines into believing leakage to be evidence of cylinder-wall cooling, so have gas engine investigators been finding heat losses from pressure volume measurements, which are nothing more than leaks. One additional cause of difficulty with the gas engine and that which makes the T equivalent of its PV product of, to say the least, doubtful value is the *non-homogeneity* of the charge, both chemically and physically, a fact often forgotten.

If, however, all these precautions are observed, then the gains and losses of heat by the gas may be found by studying the PV lines in accordance with the following algebraic relations:

$$dH = C_v dT + P dV.$$

Therefore,

$$\frac{dH}{dV} = C_v \frac{dT}{dV} + P.$$

But

$$\frac{dT}{dV} = \frac{1}{R}\left(P + V\frac{dP}{dV}\right).$$

Whence, if dH is in foot-pounds,

$$\frac{dH}{dV} = \frac{C_v}{R}\left(P + V\frac{dP}{dV}\right) + P$$

$$= \frac{1}{\gamma - 1}\left(P + V\frac{dP}{dV}\right) + P$$

$$= \frac{1}{\gamma - 1}\left(\gamma P + V\frac{dP}{dV}\right). \quad \text{(1179)}$$

Accordingly, if on any part of the PV diagram or indicator card the pressure P, and volume V, be known, and the change of pressure dP to the change of

volume dV measured a short distance on each side of the point, the heat change for that particular volume change is given by the above equation. It must be noted, however, that this is true for constant specific heats only, but even so, it is of some value and changes in sign indicate whether heat is being received (+), or lost (−), by the gas at that time.

Prob. 1. An engine operates on the Otto cycle and the pressures after and before compression are 85 lbs. and 14 lbs. per square inch gage. The diagram factor being taken as .5 and the mechanical efficiency as .8, what is likely to be the hourly consumption per B.H.P., of gas having a heating value of 600 B.T.U.

Prob. 2. For a large 4-cycle single-cylinder stationary engine operating on natural gas, find the probable consumption of gas per B.H.P. per hour, taking the necessary information from Tables CXXIX and CXXX and assuming 900 B.T.U. as the heating value of the gas.

Prob. 3. An Otto cycle engine works with a ratio of pressures of 5 and uses producer gas No. 6 of the producer gas table of Chapter V. What will be the horse-power if this engine is a single-acting 2-cycle one, running at 150 R.P.M. and having a cylinder 20×30 ins.?

Prob. 4. Estimate the horse-power and fuel consumption for a gas engine of any type and size on any fuel.

15. Actual Performance of Piston Steam Engines and Steam Turbines at Their Best Load and its Relation to the Cyclic. Effect on Efficiency of Initial Pressure, Vacuum, Superheat, Jacketing, and Reheating. Heat Balances of Steam Power Plants. Steam plants including boilers, engines and auxiliaries, are very much more complex both structurally and with reference to thermal changes than are gas plants, so it is not surprising that analysis of thermal losses and heat distribution is more difficult. There are hundreds of thousands of these plants in existence bearing little or no similarity in detail in the stationary class, though the representatives of the locomotives and marine groups do fairly well agree with their respective type forms though less so to-day than a few years ago. It would be a very lengthy and not very profitable procedure to study analytically all existing test data of steam plants and of the separate component units, so the treatment of the subject will be mainly confined to methods of determining the influence of controlling factors with just enough data to serve as illustrations.

The overall heat balance will, of course, be most variable, but the comparison of the balance of a large street railway steam central station, and a locomotive is given in the following Table CXXXVII, together with Gebhardt's estimates of a small non-condensing and a simple condensing stationary plant.

Those losses that occur in boilers have already been examined and plant auxiliary requirements are items that need no separate detailed thermodynamic analysis, but the performance of the engine or turbine does. It is not so much with the actual efficiency of a given steam engine that this analysis is concerned but rather with the best obtainable efficiency under whatever conditions prevailed

Table CXXXVII

STEAM PLANT HEAT BALANCES

	Stationary Plant.						Locomotive, Goss.	
	Simple Non-Condensing, Gebhardt.		Refined Condensing, Gebhardt.		Large Central Station Condensing, Stott.			
Supplied in coal	1.00	....	1.00	....	1.00	....	1.00	
Boiler — Sensible heat of stack	....	.50		.180	....	.227	....	.19
Boiler — Unburned fuel loss	....			.050	....	.024	....	.17
Boiler — Radiation, etc	....			.070	....	.080	....	.07
Piping and auxiliaries — Used for boiler feed	....	....	....	.010	....	.014		
Piping and auxiliaries — Used by condenser pumps	....	....	....	.024	....	.016		
Piping and auxiliaries — Piping and auxiliary losses	....	....	....	.021	....	.023		
Piping and auxiliaries — Returned by feed heater and economizers	....	....	.162	....	.099			
Engine — Discharged by engine exhaust	....	....	....	.624	....	.601		
Engine — Converted into I.H.P.	....	....	....	.183	....	.114	....	.054
Engine — Engine friction	....	....	....	.037	....	.008		
Engine — Converted into B.H.P.	....	.022	....	.146	....	.106		
Totals (I.H.P. excluded)	1.00	....	1.162	1.162	1.099	1.099	1.00	

with a view of making clear a policy or the value of imposing one or another condition of service. It has already been shown that the steam consumption per H.P. hour varies with load according to a characteristic curve always having a minimum point, and since efficiency is directly related, so must every steam engine have a best efficiency at some load. This best efficiency is worthy of study and as was done with the gas engines, the method to be used is that of comparison with the corresponding cyclic efficiency, using for this purpose the Rankine cycle. This comparison will show, first, the approach to perfection of the actual machine, but also it will demonstrate, secondly, the specific value of high initial pressures, low terminal pressures, superheat or initial wetness. Exactly similar methods of comparison will demonstrate the value of reheat in compound engines and throw some light on the desirability or undesirability of using jackets on steam cylinders of piston engines, and on the limiting conditions of staging and vane speed of turbines.

The first step in this analysis is to set down some tests of steam engines, giving their best load thermal efficiency with the initial and final conditions of the steam, and by comparison with the Rankine cycle establish some *cyclic efficiency* factors. A number of these, for both piston and turbine engines are given in Table CXXXVIII.

TABLE CXXXVIII

PISTON STEAM ENGINE AND TURBINE EFFICIENCY FACTORS REFERRED TO THE RANKINE CYCLE AS A STANDARD OF REFERENCE

	Description of Engine.	Initial Condition.		Back Pressure.		Water Rate, Lbs. per Hr. per I.H.P.	Efficiency Per Cent.		Efficiency Factor, Actual Effi. / Rankine Cyc.
		Abs. Press. Lbs. per Sq.in.	Quality.	Lbs. per Sq.in.	Inches Hg Absolute.		Actual.	Rankine Cycle.	
Stationary Piston Engines, General Power	Non-cond. Corliss compound, 12×22×20″, 200 R.P.M	152	Dry sat.	14.7	29.92	19	13.4	17	.78
	Non-cond. four-valve simple, 14×18″, 200 R.P.M.	165	" "	14.7	29.92	23	10.9	17.3	.63
	Manhattan Ry., 7000 KW. cross-compound Corliss, 44×88×60″, 75 R.P.M.	190	98% dry	2	4.23	11.74	19.2	29.5	.65
	Corliss compound, Boston El. Ry., 44×87×60″, 74 R.P.M.	163	70° F. Superheat	2.9	5.91	12.08	18.7	24.9	.75
	Allis-Corliss compound Boston El. Ry., 28×56×60″, 72 R.P.M.	163	Dry sat.	2.0	4.07	14.05	16.8	26.2	.63
	Fleming four-valve, 15×40.5×27″, 150 R.P.M.	164	" "	1.96	4.00	12.33	18.7	26.5	.71
	Corliss simple non-condensing, 250 H.P., 17×16″	115	" "	14.7	29.92	26	9.7	14.8	.66
	Corliss simple non-condensing, 132 H.P., 16×16″	140	" "	14.7	29.92	21.3	11.8	16.2	.73
	Corliss cross-compound, 30×56×72″, 65 R.P.M., Denton	138	14° sup.	2.1	4.28	13.5	17	25.3	.68
	Corliss angle comp. cond., Interboro Power House, N. Y., 42×86×60″, 75 R.P.M.	190	9° sup.	.94	1.91	12.59	17.8	29.6	.60
	As above non-condensing, Stott	199	98.7% dry	16.1	.....	18.06	14	18.2	.77
	Corliss cross-compound, 16×28×42″, 102 R.P.M., Jacobus	157	375° sup.	1.5	3.07	9.56	20.4	29.3	.70
	Willans single valve non-cond., 14×6″, 400 R.P.M.	137	Dry sat.	14.7	29.92	26	9.67	16.0	.60
	McIntosh and Seymour, 1000 H.P., 100 R.P.M., Dean	138	20° sup.	1.39	2.82	12.76	17.75	26.9	.66
	Ball compound non-cond., 12×20×13″, 271 R.P.M.	182	Dry sat.	14.7	29.92	21.14	11.7	17.9	.65
	Sulzer 4-cylinder triple-expansion, 32×47×58×59″, 85 R.P.M.	188	230° sup.	.98	2	8.97	22.6	30.8	.73
	Cole, Marchent & Morley cross-comp. jacketed, 21×36×36″, 101 R.P.M.	129	202° sup.	.85	1.72	8.58	24	30.8	.78
	White automobile, 3×6×4.5″, 850 R.P.M.	441	316° sup.	14.7	29.92	11.96	17.4	25.6	.68
	Westinghouse vertical, 5400 H.P., 76 R.P.M.	200	Dry sat.	1.29	2.62	11.93	19.05	28.8	.66

Stationary Piston Engines, Pumping	Snow pumping engine triple, Louisville, 30×56.5×84×60″, Corliss	170	110° sup.	1.15	2.36	9.65	22.25	28.7	.78
	Snow pumping engine triple, Cleveland, 12.75×24×36×24″,	164	Dry sat.	1.19	2.43	11.51	19.75	27.5	.72
	Leavitt pumping engine, 576 H.P., 11 R.P.M., Miller	191	" "	1.29	2.62	11.20	20.3	28.1	.72
	Holly duplex Comp. pumping engine, 21×42×36″×20 R.P.M. Heck	97	99.1% dry	2.0	4.07	15.63	15.3	23.1	.66
	Allis-Chalmers triple-exp. vert. pumping engine, 28×48×74×60″, 20 R.P.M	136	98.9% "	1.2	2.44	11.8	19.7	27.4	.72
	Snow pumping engine, triple, Indianapolis, 29, 52, 80×60″, 21 R.P.M	169	99% dry	1.6	3.26	11.5	20.7	28.0	.74
	Leavitt pumping engine, Louisville, Ky., 27×54×120″, 19 R.P.M	152	99.4%	1	4.23	12.22	19.4	29.4	.66
	Nordberg pumping engine, 19.5×29×49½×57½×42″, 36 R.P.M	215	Dry sat.	.9	1.83	12.26	22.8	30.7	.74
	Allis triple-exp. pumping engine, Boston, 30×56, 87×66″, 17 R.P.M	200	" "	.85	1.73	10.33	21.63	30.7	.70
	Allis triple-exp. pumping engine, Bissel's Point, 34×94×72″, 17 R.P.M	155	" "	1.2	2.44	10.59	21.06	28.1	.75
	Holly triple-exp. pumping engine, Boston, 22×41×62×60″, 25 R.P.M	165	" "	1.05	2.14	11.01	20.85	29.8	.70
	Worthington triple-exp. pumping engine, Chicago, 650 H.P., 19 R.P.M	162	87.2° sup.	.79	1.6	10.00	21.6	29.8	.72
	Riedler triple-exp. pumping engine, Chicago, 15×29×48×48″, 62 R.P.M	185	166.3° sup.	1.28	2.6	9.73	21.6	31.2	.69
Compressor	Comp. two-stage Nordberg compressor, Tennessee Copper Co., 1905, Channing	164	98.7% dry	2.09	4.26	15.19	15.5	25.7	.60
Locomotive	Locomotive test, Purdue Univ., 16×34″, superheated steam, Goss	215	231° sup.	15	.....	18.9	11.8	20.3	.58
	As above, dry sat. steam	215	Dry sat.	15	.....	25.5	9.8	19.0	.52
	Pennsylvania R.R. cross comp., 2 cyl. consolidation, 23×35×32″, 80 R.P.M	223	98.4% dry	19.4	.....	19.86	12.5	19.3	.65
	Pennsylvania R.R., 4 cyl. Atlantic, 14.2×13.7×25.2″, 160 R.P.M	218	98.8% dry	18.7	.....	20.14	12.4	19.3	.64

TABLE CXXXVIII—*Continued*

PISTON STEAM ENGINE AND TURBINE EFFICIENCY FACTORS REFERRED TO THE RANKINE CYCLE AS A STANDARD OF REFERENCE

	Description of Engine.	Initial Condition.		Back Pressure.		Water Rate, Lbs. per Hr. per I.H.P.	Efficiency Per Cent.		Efficiency Factor, Actual Effi. / Rankine Cyc.
		Abs. Press. Lbs. per Sq.in.	Quality.	Lbs. per Sq.in.	Inches Hg Absolute.		Actual.	Rankine Cycle.	
Locomotive	Pennsylvania R.R. compound locomotive, 14.2×26.1×23.6″, 240 R.P.M	203	91° sup.	17.8	.....	17.61	13.4	19.0	.71
	Locomotive No. 1499, Penn. system, 22×28″, 120 R.P.M.	211	Dry sat.	15	.....	23.4	10.7	18.8	.56
Reciprocating Steamship	Steamship Meteor, triple-exp. engine, 29×44×70×48″, 72 R.P.M	153	98.5% dry	2.7	5.45	14.98	16	25	.64
	Steamship Iona, triple-exp. engine, 22×34×57×39″, 61 R.P.M.	180	98.5% "	.7	1.42	13.35	17.1	30.5	.56
	Steamship Saxonia, quad.-exp. engine, 29×41.5×59″, 84×54″, 78 R.P.M	207	98.5% "	2.3	4.70	13.47	17.4	26.8	.65
	Westinghouse marine compound, 17×27×24″, non-condensing	163	Dry sat.	14.7	29.92	19.3	13.0	17.1	.76
	As above, condensing	162	" "	4	8.14	16.9	14.0	23.4	.60
	Inverted vert. marine cross-compound, 21×36×36″, 101 R.P.M	132	242° sup.	1.47	3	9.26	21	29.2	.72
	Steamship Otaki, triple-exp., 24.5×39.58×39″, and Parsons low-pressure turbine	193	Dry sat.	.93	1.9	13.66	16.5	29.6	.56
Turbine	Curtis turbine, 5000 KW., Port Morris Station, N. Y. C. R. R.	190	150° sup.	.98	2.0*	11.9	17.64	30.1	.59
	As above	190	Dry sat.	.98	2.0	13.4	16.85	29.5	.57
	De Laval 200 KW., Dean	223	81° sup.	1.34	2.72	12.2	17.85	29.4	.61
	Westinghouse 10,000 KW., N. Y. Edison	192	97° "	1.28	1.61	9.57	22.4	29.8	.75
	Westinghouse 10,000 KW., City Electric	182	59° "	1.99	2.02	9.29	23.6	29.5	.80
	Curtiss 10,000 KW., Chicago Edison	191	147° sup.	.22	.45	8.23	24.7	34.6	.71
	Parsons 5000 K.W., Carville	209	95° sup.	.32	.65	8.52	24.5	33.8	.72
	Parsons 3500 KW., Brown-Buveri, Frankfort	157	130° sup.	.50	1.02	8.74	24.0	31.1	.77

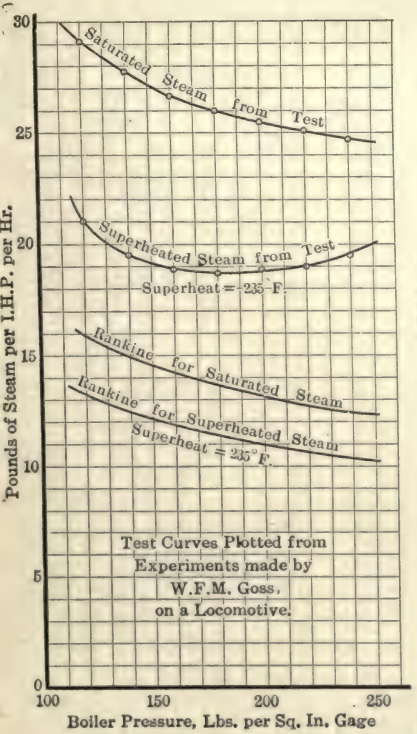
Turbine	Zoelly 3500 KW., Escherwyss, Turin	187	90° sup.	.77	1.57	9.58	22.4	30.4	.74
	Curtis Rateau 4000 KW., Rummelsburg, A.E.G	188	285° sup.	.45	.92	7.59	25.8	33.4	.77
	Westinghouse 1000 KW	169	140° "	94	1.92	10.8	19.5	29.5	.66
	Rateau multicellular, 500 H.P	152	Dry sat.	1.58	3.22	13.42	17.2	26.7	.65
	Westinghouse Parsons 1250 KW	165	" "	14.7	29.92	21.64	11.6	17.3	.67
	De Laval 10 KW	115	" "	14.7	29.92	41.5	6.1	14.7	.41
	Curtis 75 KW	165	" "	14.7	29.92	29.9	8.4	17.3	.48
	Westinghouse Parsons 7500 KW., Int. Rapid Transit, N. Y	198	97% dry	1.12	2.3	10.27	22.1	30.7	.72
	Westinghouse 1000 KW., low-pressure turbine	17.4	Dry sat.	1.97	4.02	25.11	9.6	13.2	.73
	Rateau 1000 KW	179	10° sup.	2.43	4.95	14.0	16.5	26	.63
	Kerr 150 H.P	190	Dry sat.	14.7	29.92	33.6	10.6	18.2	.58
	Elektra 200 KW	201	172° sup.	1.18	2.40	13.42	15.6	29.9	.52
	Reidler Stumpf 1400 KW	205	170° "	2.19	4.46	12.50	17	29	.59
	Zoelly 3500 KW	182	170° "	.53	1.09	8.71	23.15	34.0	.68
	Melms Pfenninger 500 KW	189	230° "	.50	1.02	10.42	19.15	35.7	.54
	Curtis 5000 KW., low-pressure, Stott and Pigott	16.1	92.1% dry	.46	.94	17.8	7.6	13.3	.57
	Rateau 500 KW., low pressure	14.7	97.1% "	.98	1.99	23.1	10.3	18.9	.54

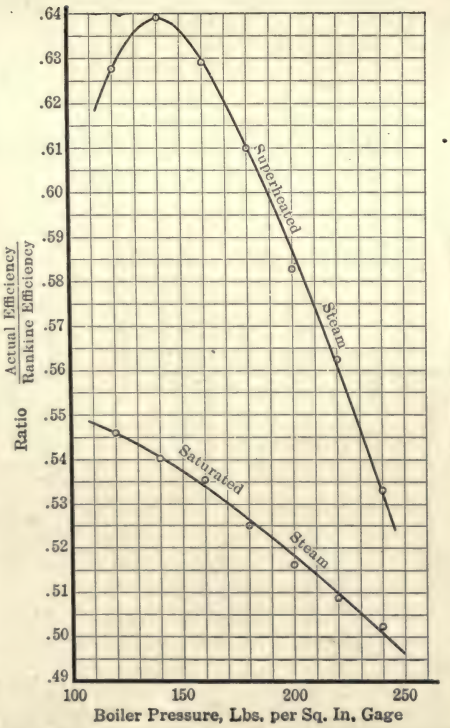
* Water rate in pounds per I.H.P. hr. for turbines has been calculated from data given, by assuming electric generator efficiency =95 per cent, and mechanical efficiency =90 per cent.

Examination of this table brings out clearly two important characteristics of steam engines and turbines, first, that the efficiency factors for both classes are substantially the same and approach 80 per cent as a high limit, which is better by about 15 per cent than that found for gas engines; second, that the factor may be very low, less than 50 per cent. This is due to the fact that piston engines leak, suffer heat exchanges, and expansion is limited by the valve adjustment and cylinder ratios, and in turbines whereas expansion is always complete, the vane and steam jet speed relations may be improper or friction losses occur in the flow path. Where such turbine losses exist they differ not so much in range from the different losses in piston engines even at best load. It is most interesting to note that so far as capabilities of conversion of steam heat into work are concerned, as measured by this factor there is little to choose between these two classes of machines, though piston engines appear to be able to make a little better use of the steam than turbines, when constructed to do so, and allowed to in service.

In all cases where cylinder leakage, heat exchange, and expansion losses, or turbine velocity and flow friction losses, are not intentionally suffered to meet load or first-cost conditions, the factor for steam engines and turbines is consistently and materially higher than in gas engines. This indicates that there is much more room for improvement of efficiency in gas than in steam engines, though it is not yet clear whether the present modes of operation forbid its realization or not. The doubt is, however, all on the side of the gas engine as the nature of the losses in steam machines are pretty well understood and the value of the various factors that control them have been experimentally determined many times.

It takes no elaborate thermal study to see that the higher the initial steam pressure and the lower the terminal pressure the better should be the efficiency and it remains to see whether it is really so. As experimental data to illustrate this point the test results of Goss on locomotives will be used. Several series of runs are available, some with superheated steam and some with saturated, each with boiler pressures from 100 to 250 lbs. per square inch gage, some of which are shown graphically in Fig. 281, on which lines are located to indicate the Rankine cycle efficiency and water rate at the left. These results are replotted in the center, to a scale of efficiency factors, as functions of boiler pressure, but it must be remembered that in no case is expansion complete in a locomotive and the degree must be less for the higher pressures. The corresponding $T\Phi$ diagrams are also given at the right. Examination of these curves indicates clearly that no great improvement in efficiency or water rate follows the pressure rise, and what changes occur are of the order of first decreasing steam consumption and later increasing it, the minimum in one case being at 200 lbs., per sq. inch gage and in the other a little less with the exception of the saturated steam which consistently improves in performance. Of course, the Rankine cycle regularly improves so it appears that as pressures rise the performance departs more and more from that of the Rankine cycle, the factor being highest at 140 lbs., 64 per cent for superheated, and at 100





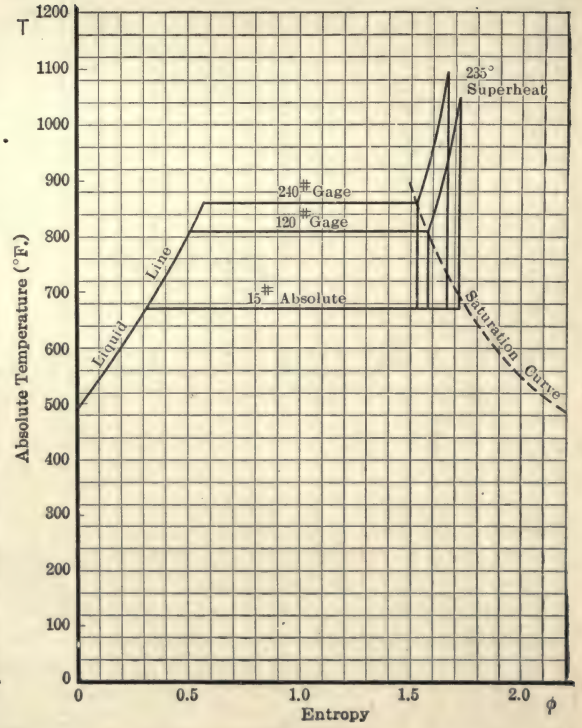


FIG. 281.—Three Diagrams showing the Value of High Initial Pressures with Superheated and Saturated Steam in Locomotive Engines.

lbs., 55 per cent for saturated steam, proving an increase in losses due to leakage, initial condensation, and curtailed expansion. To really prove the value of high initial pressure would require test data of several engines each designed for a successively higher pressure and operated under its own best conditions, but no such data are available. The real limit to high pressures is not thermal, but one of expense, gains due to excess pressures over 175 lbs. for

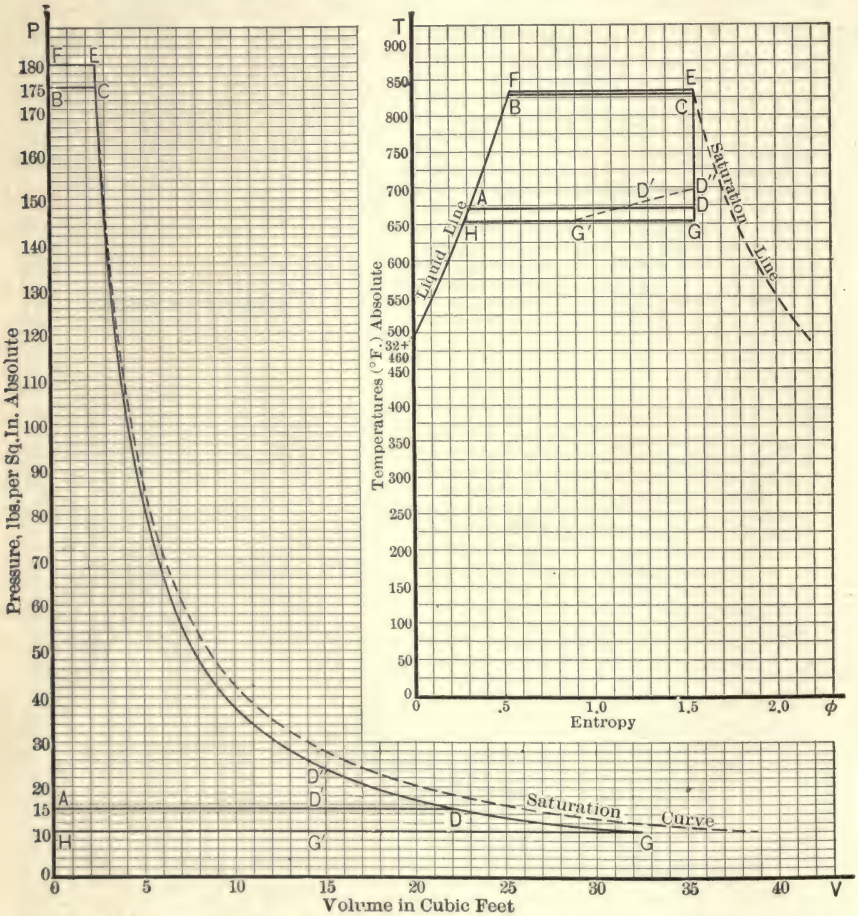


FIG. 282.—Effect on Steam Engine Performance of Raising the Initial Steam Pressure 5 lbs. and Lowering the Back Pressure by 5 lbs. per sq.in.

stationary plants and 200 lbs. per sq.in. gage for locomotives are obtained at too great cost generally to warrant it.

Lowering the back pressure should have the same effect as raising the initial pressure, in improving efficiency, though to a very much greater degree as appears from both the pressure volume and temperature entropy diagrams, Fig. 282. These are drawn for adiabatic expansion from 175 lbs. per square inch initial pressure absolute with dry saturated steam, to one atmosphere, *ABCD*

and for a 5 lb. per square inch rise of initial pressure there is added the area *BCEF*, which is small compared to that obtained from a 5 lb. per square inch reduction of back pressure, given by area *ADGH*. Whether a real engine is capable of taking advantage of this or not depends on its structure and adjustment. For example, if the cylinder volume is represented by *AD'* on the *PV* diagram, the gain due to reduced back pressure will be *AD'G'H* instead of *ADGH*, since expansion must stop at *D''*, so that piston engines that ordinarily do not completely expand the steam, can hardly be expected to utilize all the available gain. The situation is better for turbines where the expansion is always complete, but even here the full advantage is not realizable, unless the vane speeds and steam-jet velocities are correctly related. If so related for the higher back pressure, a reduction of it will result in higher jet speeds and unless vane speeds and R.P.M. correspondingly increase, the steam will leave the vanes with some residual velocity and some kinetic energy not utilized.

All experimental data prove that both piston engines and turbines do gain in efficiency with lowered back pressure and very much more than with corresponding increase of initial pressure, the amounts, of course, depending on the pressures themselves, and it is also experimentally established that turbines are benefited more than piston engines. To illustrate the effect of back pressure on a turbine, the test data of a 1250 K.W. Westinghouse turbine is given below in Table CXXXIX and Fig. 283, and the corresponding Rankine cycle efficiencies and cyclic efficiency ratio calculated and added.

TABLE CXXXIX

STEAM TURBINE EFFICIENCY WITH VARYING BACK PRESSURE WITH STEAM APPROXIMATELY AT CONSTANT INITIAL PRESSURE DRY SATURATED

Initial pressure pounds per square inch absolute	161	162	161	161.6
Back pressure inches Hg absolute	5	4	3	2
Pounds of steam per B.H.P. hour (best)	14.39	14.10	13.59	13.49
Pounds of steam per I.H.P. hour for 94% mech. efficiency	13.53	13.25	12.78	12.68
Actual thermal efficiency referred to I.H.P. per cent	17.22	17.42	17.92	17.84
Rankine cycle efficiency per cent	25.30	26.18	27.08	28.64
Ratio $\left(\frac{\text{Actual efficiency}}{\text{Rankine cycle efficiency}}\right)$	68.05	66.56	66.16	62.29

The curves of actual and Rankine cycle efficiency are not parallel, so the efficiency factor varies from 62 per cent for 2 ins. Hg to 68 per cent for 5 ins. Hg, indicating that not so much use is made of the high velocities of the lower back pressure as of the lesser jet velocities of the higher back pressure.

There are no similar tests available for reciprocating engines but the performance of a large one, 28×54×48 ins., non-condensing and with a 26-inch vacuum, will serve to bring out its ability to utilize as much of the available gain as the turbine, in this particular case, more. This engine required 19.4 and 13.65 lbs. of steam per hour per I.H.P. for non-condensing

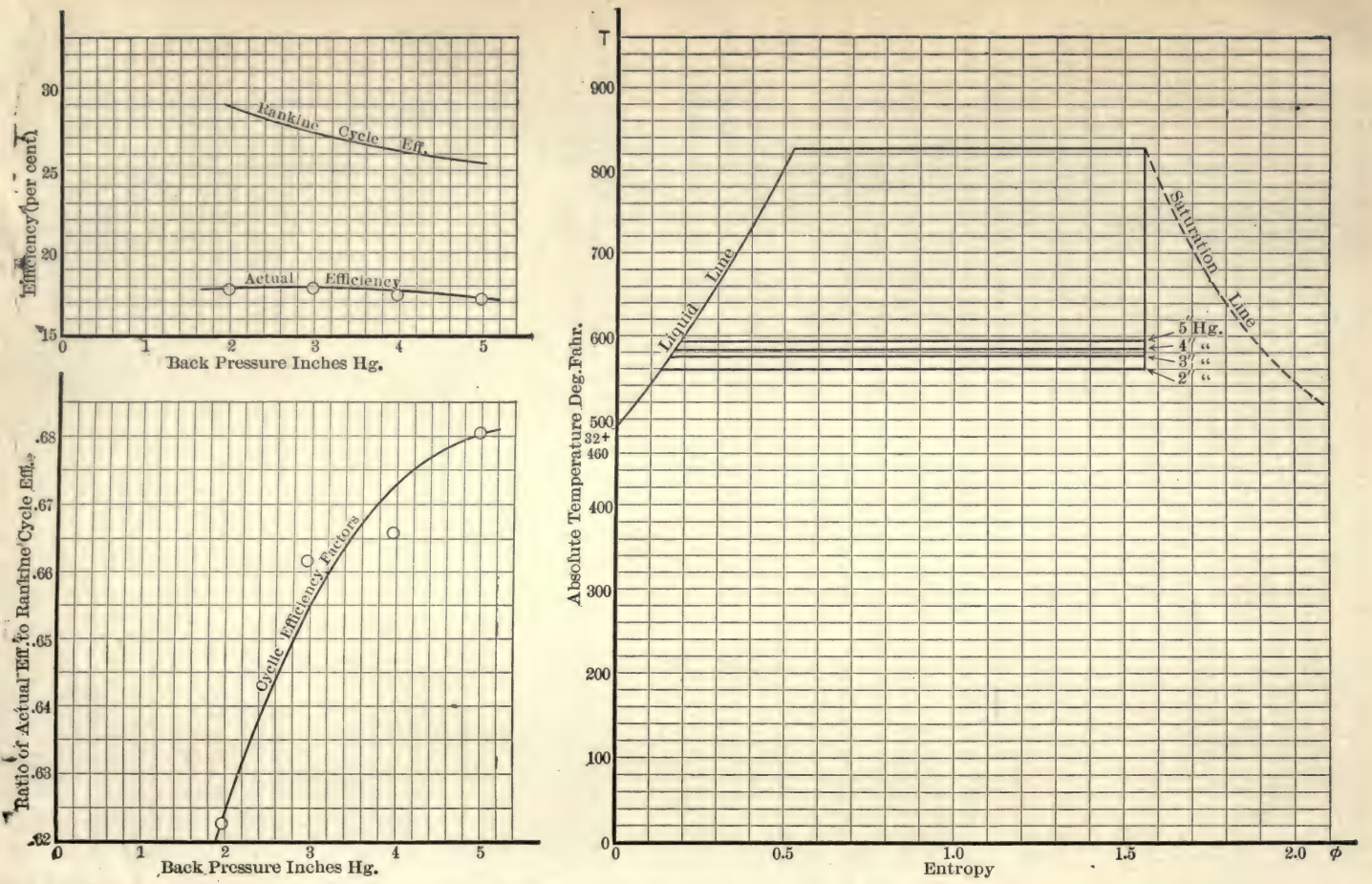


FIG. 283.—Effect of Back Pressure Reduction on Steam Turbine Performance.

and condensing operation with a back pressure of one atmosphere and 4 ins. Hg absolute respectively, and an initial pressure of 165 lbs. per square inch absolute. These water rates for initially dry steam correspond to actual thermal efficiencies of 12.86 per cent, and 16.83 per cent, for which the equivalent Rankine cycle efficiencies are 17.3 per cent, and 26.29 per cent, so that the ratio

$$\left(\frac{\text{Actual thermal efficiency}}{\text{Rankine cycle efficiency}}\right)$$

is equal to .743 for one atmosphere back pressure, and .644 for 4 ins. Hg absolute back pressure.

The turbine is not only able to utilize a fairly large fraction of the available gain due to reduction of back pressure, but is very much cheaper to build than piston engines, especially for very low back pressures which would require in piston engines abnormally big low-pressure cylinders. Thus the practice has been established of using turbines for the low ranges of pressure in preference to piston engines, and of combining reciprocating engines and turbines to work together, the latter taking the exhaust steam from the former at whatever release pressure it is discharged. This practice has been found especially valuable in adding capacity to existing overloaded reciprocating engines, and the most notable example of it is the installation at the 59th Street power station of the Interborough Ry. in New York City. Here the original installation consisted of 15,000 K.W. twin horizontal-vertical compound engines $42\times86\times60$ ins. Need for more power led to an investigation of the relative advantages of installing some new high-pressure turbines, or of adding low-pressure turbines to existing reciprocating engines, and the latter was adopted on a basis of manufacturers' guarantees for both, as it offered 8 per cent better efficiency than the former, and a 7500 K.W. low-pressure turbine was added to one piston engine unit for trial. Most unusually complete tests were conducted which proved beyond question the value of the combination, an average thermal efficiency between loads of 6500 K.W. and 15,500 K.W., of 20.6 per cent was obtained. The engine alone showed a best thermal efficiency of 10.3 per cent on 177.7 lbs. per square inch initial pressure gage, with 8.9° F. superheat, and a back pressure of a little less than 2 ins. Hg absolute, for which the Rankine cycle efficiency is 15.6 per cent, so that the engine realized at best 66.2 per cent of the Rankine cycle. When operated together, however, the rise of back pressure on the engine enabled it to realize a larger per cent of the Rankine cycle, the figure rising to 79 per cent, while the best that was obtained for the low-pressure turbine was about 64 per cent, though not at the same load. Operated together, the overall performance was the best on record, the thermal efficiency being 21.8 per cent, which corresponds to about 78 per cent of the Rankine possibilities, a truly remarkable performance probably not exceeded in service by any single unit of whatever type. At this time the initial pressure was 199.1 lbs. per square inch absolute for the

engine, and 10.35 lbs. per square inch absolute for the turbine with a final back pressure of .46 lb. per square inch absolute, the initial steam being about 1 per cent wet.

The value of a piston engine for steam in the lower pressure ranges has been experimentally established by Carpenter and Sawdon on a Shuman 24×24 ins. simple engine with the following results, Table CXL. Allowing for experimental inaccuracies it is reasonable to assume that the efficiency ratio is constant at about 50 per cent, which is considerably lower than is easily obtainable in low-pressure turbines.

TABLE CXL

USE OF LOW-PRESSURE STEAM IN PISTON ENGINES

	1	2	3	4	5	6
Initial pressure pounds per square inch abs.	8.12	8.15	15.56	15.67	15.8	15.8
Initial steam quality (per cent dry)	89.4	97.7	90.4	95.6	96.9	97.2
Back pressure, inches Hg absolute	1.92	1.90	1.31	1.98	1.90	1.83
R.P.M.	126.6	143.8	129.5	143.7	133	126.9
B.H.P.	11.3	12.1	12.6	18.5	22.2	21.0
Pounds dry steam per hour per B.H.P.	36.8	43.0	35.7	31.0	28.8	29.4
Actual thermal efficiency (per cent)	6.40	5.51	7.57	7.43	8.22	7.97
Ratio $\left(\frac{\text{Actual thermal efficiency}}{\text{Rankine cycle efficiency}}\right)$	.524	.438	.479	.467	.509	.499

Superheat is another important factor in steam-engine efficiency, as examination of the $T\Phi$ diagram will show, though curiously enough and uniquely the actual benefits sometimes exceed the expected. In Fig. 284 is shown a series of Rankine cycles for 190 lbs. per square inch absolute initial, and 2 ins. Hg absolute back pressure, for every 50° of superheat up to 250°, and the value of the superheat is shown at the bottom as computed in two different ways. The curve (1) shows the efficiency of the whole cycle *ABCNA* to *ABCHIKA*, and indicates a slow and regular improvement with increase of superheat from 29½ per cent efficiency for zero superheat to 31 per cent efficiency for 270° superheat. The other curve (2) gives the efficiency of the superheat part of the cycle alone as represented by the areas *CDMNC* to *CHINC*, and this also increases with the amount of superheat and faster than the whole Rankine cycle of which it is a part. According to this, a gain in thermal efficiency with increase of superheat is to be expected very nearly proportional to the superheat as line (1) is nearly straight, but as maintenance costs increase rapidly beyond a moderate superheat practice has fixed on values about 100°, occasionally going as high as 150°, as the economic limit.

These expected gains must now be compared with actual ones and as data some of the tests of locomotives will be used as representing the piston engine, and compared with Hodgkinson's turbine tests. The locomotive tests showed an improvement in efficiency, depending on the boiler pres-

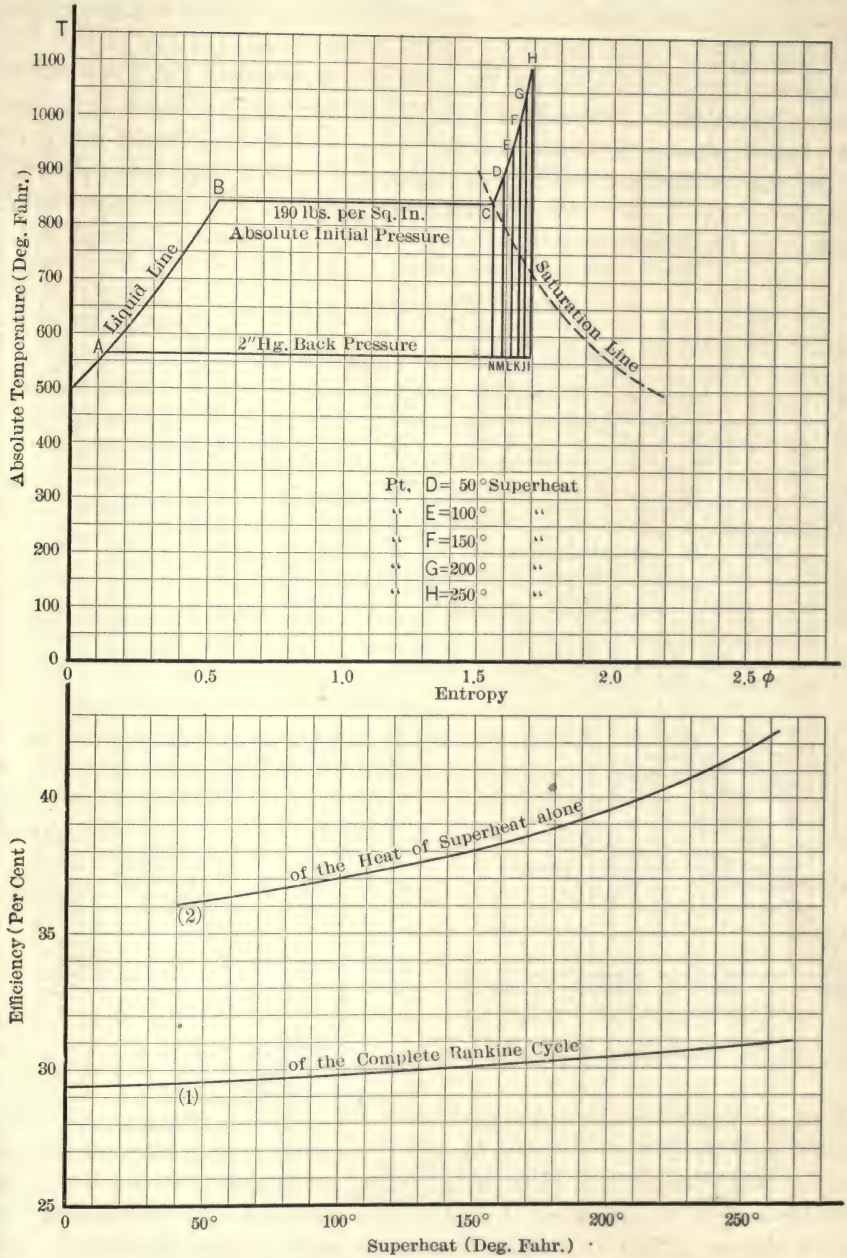


FIG. 284.—Effect of Superheat on Efficiency of Transforming Heat of Steam into Work.

sure, of 2 per cent for 120 lbs. gage pressure, 2.3 per cent for 180 lbs. gage, and only 1.3 per cent for 240 lbs. as would be expected, though another factor of variability of amount of superheat is introduced by the peculiarities of the locomotive structure. All gains are based on the performance with saturated steam as a standard of comparison. The superheat varies in these machines with the boiler pressure and rate of boiler firing and in these tests ran over and under 150°. The net results are given in Fig. 285, and compared with the Rankine cycle. This comparison shows a greater gain by superheat than corresponds to the Rankine cycle expectations as the efficiency factors with

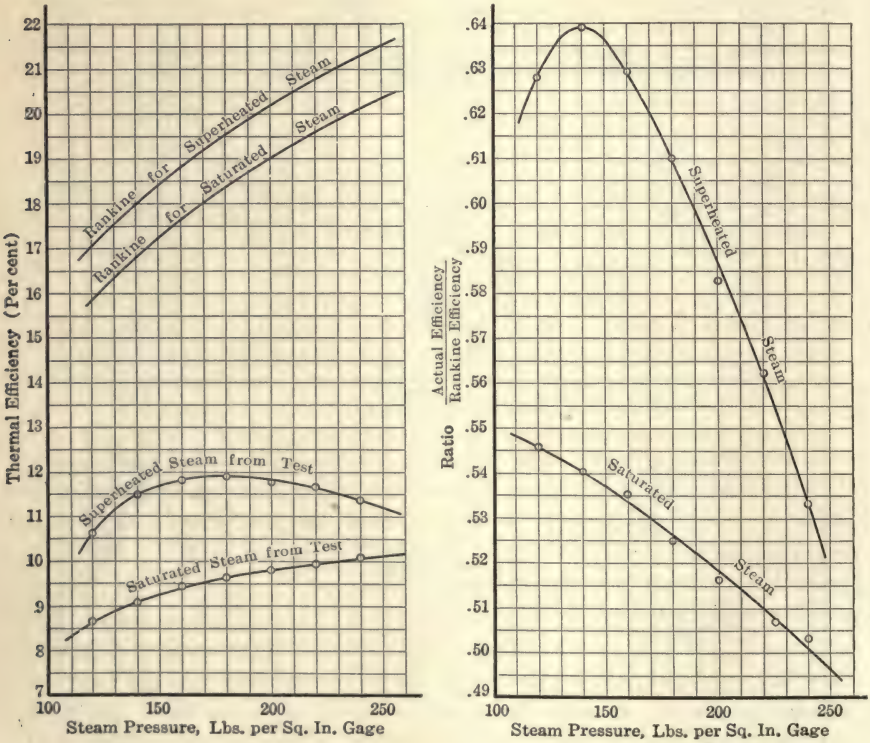


FIG. 285.—Effect of Superheat on Locomotive Engine Efficiency.

superheat are much higher, and also shown by the greater distance between the actual and the Rankine curves for saturated and superheated conditions. This is a most interesting observation, usually explained on the theory of lessened leakage with superheated steam, and less wall heat absorption due to the insulating action of a layer of gaseous steam which is always more effective than a film of water, as has already been established.

Somewhat better data are available for showing the improvement in efficiency due to superheat with turbines, as tests are available at constant pressures with superheat as the only variable and Hodgkinson's results for a

1250 K.W. Westinghouse turbine will illustrate this. With an initial pressure of 162 lbs. per square inch absolute and a back pressure of 2 ins. Hg absolute the water rate per I.H.P. hour on an assumed mechanical efficiency of 94 per cent was for dry saturated steam, 12.7 lbs., 12.4 for 75° superheat, and for 100° superheat 12.05 lbs. These data correspond to actual thermal efficiencies of 17.80 per cent, 17.55 per cent, and 17.86 per cent and the corresponding Rankine cycle efficiencies are 28.68 per cent, 28.90 per cent, and 29.01 per cent, so that the cyclic efficiency ratios are 0.6206, 0.6073 and 0.6156.

The use of superheated steam in small engines according to some tests analyzed by Tolz showed improvement in efficiency with rise of initial temperature according to the curves of Fig. 286 for four engines, though the initial steam temperature instead of the superheat itself is given as the prime variable. Tests on the use of superheated steam in pumping engines has led to the estimates by Foster of a saving over saturated steam, depending on the efficiency of the unit previously, as follows: 6, 10, 20, 40 per cent, when the duty is 150, 100, 50, and 10 million foot-pounds useful pump work per million heat units supplied. These figures are higher than those of the Jacobus tests of a New York pumping unit with and without superheated steam, which showed a gain of 18.5 per cent with 207° superheat and 80 lbs. gage pressure when the duty was about 28 million, based on heat in the steam.

The fact that changes in initial pressure, back pressure and superheat are associated in any one type of engine with a definite and regularly varying change in efficiency has led builders to adopt curves of correction per pound per square inch pressure or per degree superheat change, for their machines. Thus Emmet states with respect to vacuum, that near 27 ins. Hg the change in efficiency per inch is 6.6 per cent, near 28 ins., 7.8 per cent; near 29 ins., 9.5 per cent; and according to Parsons each inch between 23 and 28 ins. affects efficiency 3 per cent in a 100-K.W., 4 per cent in a 500-K.W., and 5 per cent in a 1500-K.W. turbine. If the turbine is correctly designed for best economy the gain per inch of vacuum is almost exactly that for the Rankine cycle, but if not so designed, no prediction can be made. It is pretty well agreed that superheat corrections are properly fixed for prevailing pressures at about 10 per cent per 100° F. superheat, up to about 100° F. superheat as a maximum, but it does fall off in the higher ranges, being about 8 per cent per 100° between 100° and 150°. However, as superheat affects steam jet velocity, just as does high initial or low back pressure, it is clear that the realization must depend on design proportions. With respect to these corrections, therefore, this discussion must be considered merely as pointing out a method for their determination rather than fixing them in general, as this latter can be done only for particular speeds and dimensions.

The next factor in steam engine efficiency is the cylinder jacket of piston engines, the analysis of which cannot be based on the Rankine cycle. It is expected that the jacket will impart heat to the working steam during expansion by condensing some live steam outside the cylinder wall. Exact analysis thermally, of this sort of expansion is quite impossible because the law of heat

gain as expansion proceeds in the working steam depends on laws of transfe
that are not established. Sometimes it is assumed that heat will be receive
at a rate to keep the working steam at constant quality and this will serve t

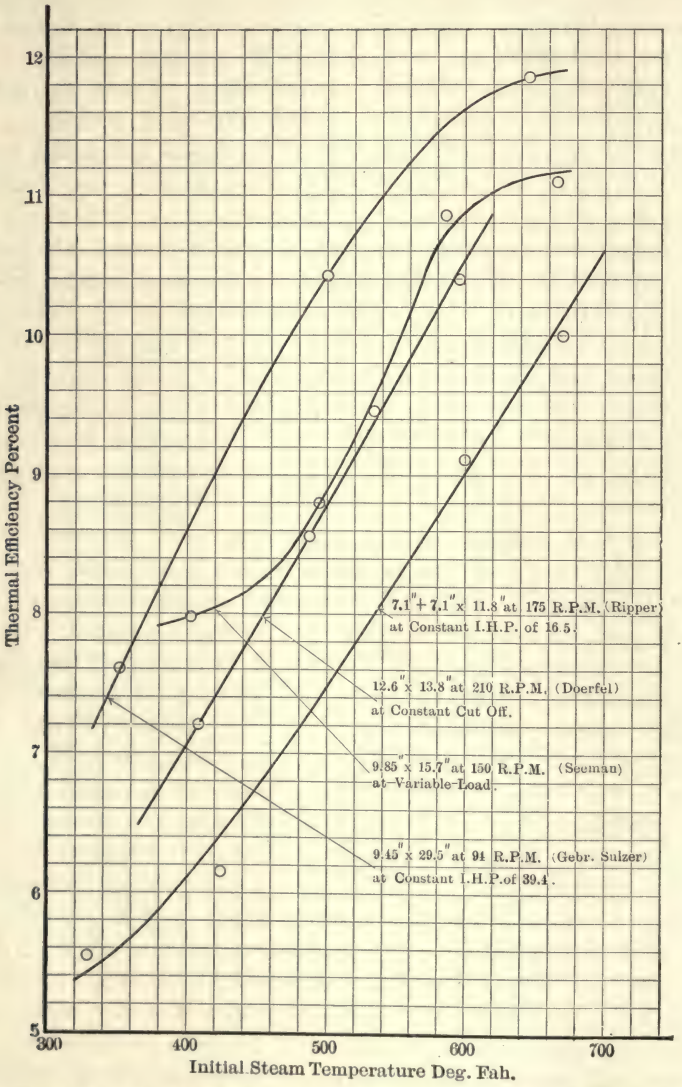


FIG. 286.—Effect of High Initial Temperature of Superheat on the Thermal Efficiency of Steam Engines.

show the *sort of effect* that the process of heat addition during expansion may be expected to produce. Such a case is illustrated in Fig. 287, for initially dry saturated steam at 160 lbs. per square inch absolute pressure, expanding to 2 lbs. absolute, *ABCD* and compared with the corresponding Rankine cycle

ABCE, work being determined graphically by measuring $T\Phi$ areas. According to this the saturated expansion cycle has an efficiency of 24.11 per cent, which is less than the 26.07 per cent for adiabatic expansion, so from purely thermal grounds a loss rather than a gain is to be expected from jackets on this assumption of the cycle. However, this does not prove that in real engines jackets may not be efficiency aids, since in the case of superheat more gain may be realized than expected, but on the other hand with pressure range increases, less was realized so test data are necessary once more. A compound engine 9×16×14 ins. at 265 R.P.M. tested by Carpenter at 112 lbs. per square

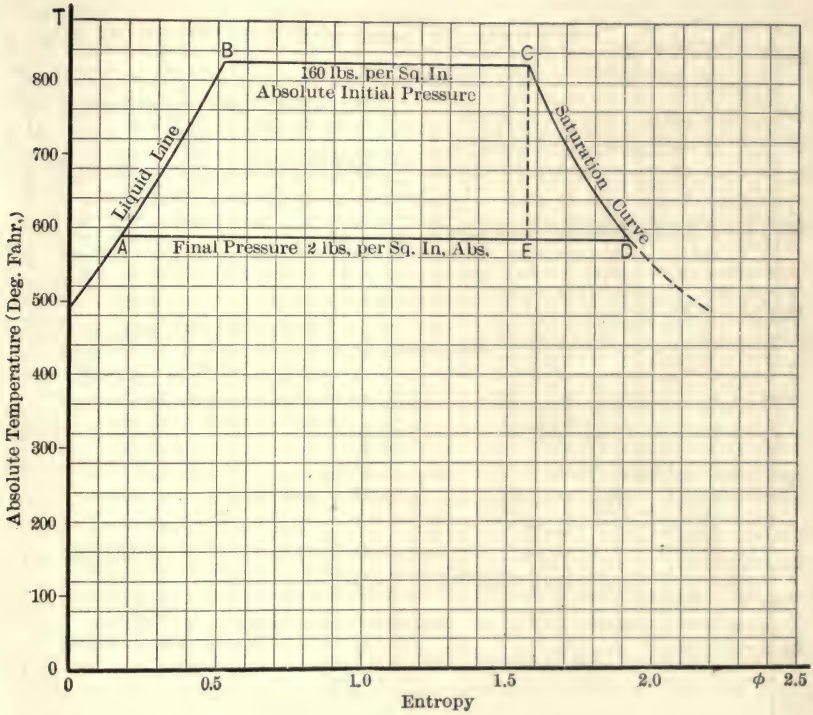


FIG. 287.—Rankine Cycle Modified by Saturated Expansion Assumed to be Typical of Steam in Jacketed Cylinders.

inch gage, at 22-in. vacuum showed a minimum steam consumption per I.H.P. hour of 18.6 lbs. with both cylinders jacketed, and 19.1 lbs. without jackets, on initially dry steam, which is a reduction of $\frac{.5}{19.1}=2.6$ per cent, but at some loads there was an actual loss by the use of jackets. This is true for practically all cases that have been subject to test, and the conclusion is, that a gain of about 2 per cent may be expected from jackets when the engine works always at the load at which the gain is realizable, which is possible for pumping engines,

for example, but when engines have to do work at variable loads the net effect is either a loss or so small as not to warrant the expense of obtaining it.

Reheating the steam between the high- and low-pressure cylinders as a means of improving economy, is in much the same class as jacketing cylinders, in fact it is really a process of jacketing receivers of multiple-expansion engines. What may be expected thermodynamically from perfect reheating,

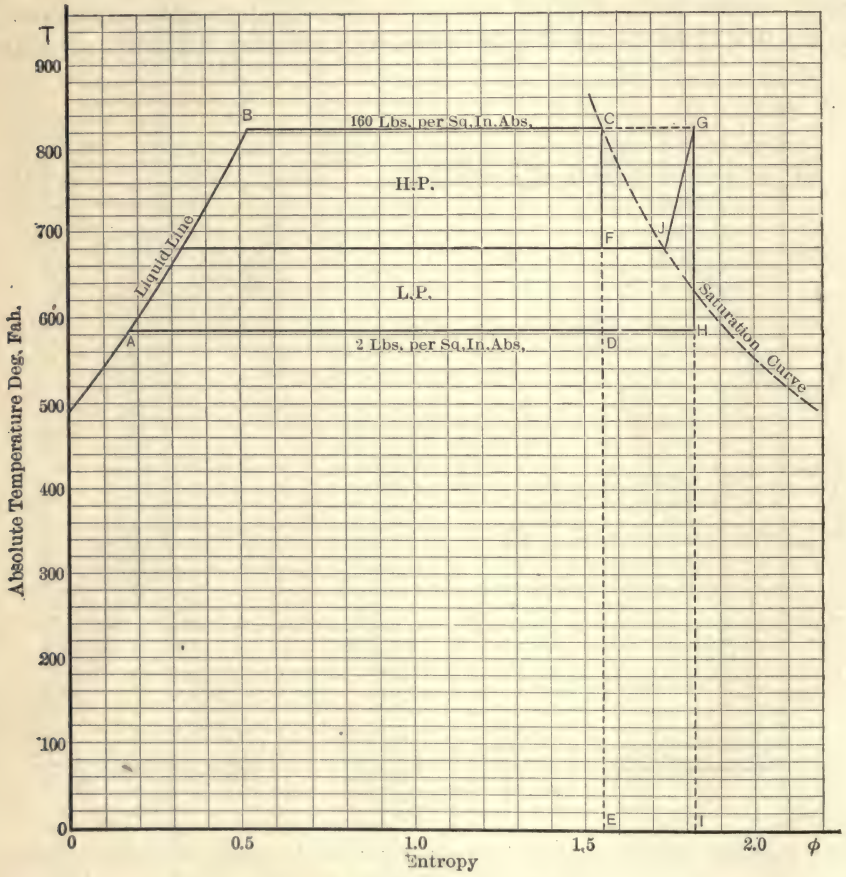


FIG. 288.—Steam Cycle with Perfect Receiver Reheating in Compound Engine Represented by Two Rankine Cycles.

that is, up to the initial temperature, of steam initially dry saturated at 160 lbs. per square inch absolute pressure expanding adiabatically to 2 lbs. absolute, is shown in Fig. 288 for the case of equal division of work without reheating. The work diagram without reheat is bounded by *ABCDA*, the heat of reheat which first dries and then superheats the steam is given by the area *EFJGIE*, the work derived from the heat of reheat by *FJGHDF* and the complete work diagram with reheat is *ABCFJGHA*. Evaluation of these areas shows the

thermal efficiency without reheat to be 26.07 per cent, with reheat 25.03 per cent, which is a slight loss. Tests made by the writer some years ago on a triple expansion Corliss engine of 200 H.P. with and without reheat in the receivers showed a net gain of about 1.5 per cent by reheating at one load and a loss or no measurable difference at other loads, which agrees with practical experience in the use of reheating receivers. These receivers are always troublesome to keep tight and well drained, so the tendency is, therefore, to use simple well-lagged receivers and apply heat in the form of superheat initially which does yield a gain in efficiency at all times.

One interesting test on a large McIntosh & Seymour engine, $29\times60\times56$ ins. in the power station of the Boston Edison Co., by Cooke, shows the combined effect of both reheat and jackets, data being available with heaters both in, and out of service. At full load and with an initial pressure of 160 lbs. gage and 95° of superheat, 26-in. vacuum, the indicated water rate was, for two runs, 10.85 and 11.18, or 11.01 lbs. mean, with both jackets and reheaters in use, and without them 11.55 lbs. Therefore, the gain by the use of both was $\frac{.54}{11.55}=4.7$ per cent at full load. At half load the water rates were 10.33 with reheaters and jackets, and 10.61 without them, or a gain of $\frac{.28}{10.61}=2.64$ per cent. While no data are available for other loads, it is quite probable that the gain would change to a loss.

In nearly all cases it has been found that expected gains in efficiency due to increases of initial, decrease of back pressure, increase of superheat, use of jackets and receiver reheating, do not agree with that realized and the reason must be sought in interferences due to leakage and heat exchanges between walls and working steam. Analysis of these influences, often as it has been attempted and elaborately pursued, has as yet failed to give results worthy of reproduction, so these factors must as yet be classed with the unknown so far as any prediction is possible, just as is the case with the wall heat exchange and leakages in the cylinders of gas engines. One of the most interesting methods of attack yet offered, however, and sufficiently suggestive to warrant reference to it, is that recently used by Clayton, as it deals directly with the detection of missing water and leakage and their separate evaluation, though it has not yet been applied to enough cases to warrant any general conclusions on these troublesome quantities. By making very careful tests of piston engines for the determination of steam consumption and studying the correct indicator cards when the clearance was accurately known, he secured data for the plotting of the pressure volume expansion line and used for the purpose logarithmic cross-section paper on which any equation of the form of $PV^s=\text{Constant}$, becomes a straight line if s is constant. As might be expected from the purely thermal investigation of the values of s for steam, it was found to be substantially constant over the length of one expansion line if not too long, but a variable with respect to the initial wetness or quality of the steam at cut-off. Knowing the quality of the steam supplied to the cylinder, any direct knowledge

of the volume or from it the quality of the steam in the cylinder at cut-off, gives by differences the initial condensation if leakage is absent; this difference is itself the missing water per charge of steam-water mixture supplied. The slope of the expansion lines gives the value of s, and if s is a function of initial wetness only, it follows that the slope of the expansion line is a measure of missing water. Preliminary tests showed s to vary from .70 to 1.34, depending on engine type, size, speed, pressures, and ratio of expansion, but it always increased in any one engine with increase of cut-off and was always above 1.0 and as high as 1.34 for superheated steam, while for wet steam it was usually less than 1.0 and as low as .7. For a 12×24 in. Corliss engine in particular, no value of s below 1.00 was found for values of cut-off quality above 80 per cent, and no value above 1.10 for cut-off quality less than 72 per cent, and in the region of $s = .90$ to $s = 1.00$, cut-off quality ranged from 50 to 70 per cent, the several values corresponding equally to saturated steam with long cut-offs and superheated steam with short cut-offs, indicating an independence of cut-off quality with respect to cut-off and a primary dependence of s on cut-off quality. The average of all results is given by Eq. (1180) and the maximum departure is 4.6 per cent from it.

$$\text{Cut-off quality} = 1.258s - .614. \quad . \quad . \quad . \quad . \quad . \quad (1180)$$

Further tests showed these constants to vary with the pressures and speeds but no general equation was derived from which it could be evaluated. The conclusion is, therefore, that in any one engine or perhaps in all engines of one type, such an equation as the above holds and can be experimentally found, so that once established a single indicator card may serve to determine for missing water as well as the indicated water, provided leakage is absent or its quantity evaluated. Incidentally, *the failure in real engines of the old assumption of logarithmic expansion is well established and renders the use of the exponential equations of Chapter III for horse-power and indicated water rate, more necessary than they have been regarded in the past.*

Prob. 1. Steam is being used in an engine at 100 lbs. per square inch initial pressure and 20 lbs. per square inch absolute back pressure. Show by PV and $T\Phi$ diagrams the efficiency gain, if initial pressure be doubled while back pressure is held constant, and the gain if the initial is held constant and the back pressure lowered to 5 lbs. per square inch absolute. The engine cylinder volume may be taken as being 75 per cent of the volume of 1 lb. of steam at 20 lbs.

Prob. 2. By means of a $T\phi$ diagram show the increase in efficiency due to superheat of 100°, 200°, 300°, 400°, 500°, in a turbine using steam at an initial pressure of 150 lbs. per square inch absolute and running on a 3 in. vacuum.

Prob. 3. An engine is running with an unjacketed cylinder on an initial pressure of 125 lbs. per square inch absolute and a back pressure of 5 lbs. Show by the $T\Phi$ diagram the change in efficiency if there was a jacket imparting sufficient heat to cause the expansion line to be saturated.

Prob. 4. The engine of Problem 3 is run under the same conditions except that there is initially 200° of superheat. Without the jacket adiabatic expansion occurs

and with it the steam is constantly superheated 200°. What will be the difference in efficiencies with and without the jacket in this case?

Prob. 5. A compound engine runs on an initial pressure of 185 lbs. gage and a back pressure of 5 lbs. absolute. The cut-off is so adjusted as to give a receiver pressure of 30 lbs. absolute. Show by $T\Phi$ diagram the efficiency with and without complete reheat for (*a*) no initial superheat; (*b*) 200° superheat and reheat to this temperature; and (*c*) 200° superheat and reheat to saturation temperature of initial pressure.

Prob. 6. Find the efficiency for the engine of the last problem, considering the low-pressure cylinder to be jacketed and low-pressure expansion to follow the saturation law.

Prob. 7. Predict the thermal efficiency, heat consumption, and water rate for any piston steam engine or turbine under any working conditions, at best load.

16. Flow of Hot Water, Steam, and Gases through Orifices and Nozzles. Velocity, Weight per Second, Kinetic Energy, and Force of Reaction of Jets. Nozzle Friction and Reheating. Relative Proportions of Series Nozzles for Turbines for Proper Division of Work of Expansion. Any expansive fluid such as steam, either superheated, dry saturated, wet, or even a liquid at the boiling-point, as well as all the gases, should on suffering a loss of pressure in passing through a nozzle or orifice of any kind, expand adiabatically and transform into work some of its heat content. The amount is given in Fig. 289 by the areas *ABJ* to *ABDEFGA*, depending on the quality, *b* for 1 lb. of steam or hot water. What actually occurs is an approximation to this, as close as interferences permit, and these interferences are of the friction and impact order. In the nozzle itself friction will have the effect of lowering velocity developed previously, and so converting kinetic energy back into heat and as this is a continuous process, the net effect is the same as if heat were added during expansion and the expansion line will take a position somewhat as shown in Fig. 290 instead of being vertical as it would be for an adiabatic change. The usual way of defining this line and the energy change responsible for it is to say that x per cent of the energy developed by a partial expansion, assumed for a small drop in pressure to be adiabatic is, after the expansion is completed, added to the fluid as heat at constant pressure. Thus in Fig. 290 let the initial condition be represented by *B*, *C*, *D*, or *E*, for the cases of water, wet, dry saturated, and superheated steam respectively. If expansion proceeds adiabatically from *D* for example, the work developed in the form of kinetic energy is represented by the area *BDHA*. Adding at constant pressure an amount of heat eqvivalent to $\left(\frac{x}{100}\right) \times$(area *BDHA*), the final condition will be as represented by the position of point H', and a line joining a series of such points, is the representation of expansion with reheat constantly going on. If all the work developed be converted back into heat the expansion line would follow the law of constant energy, and then the kinetic energy of the jet and the velocity would both be zero. Of course, this limiting case is never reached in a nozzle, but is reached when gases and vapors escape through so-called porous plugs or

diaphragms or when a jet once formed, loses velocity by impact, and for these cases the final condition of the fluid may be found by these methods. For

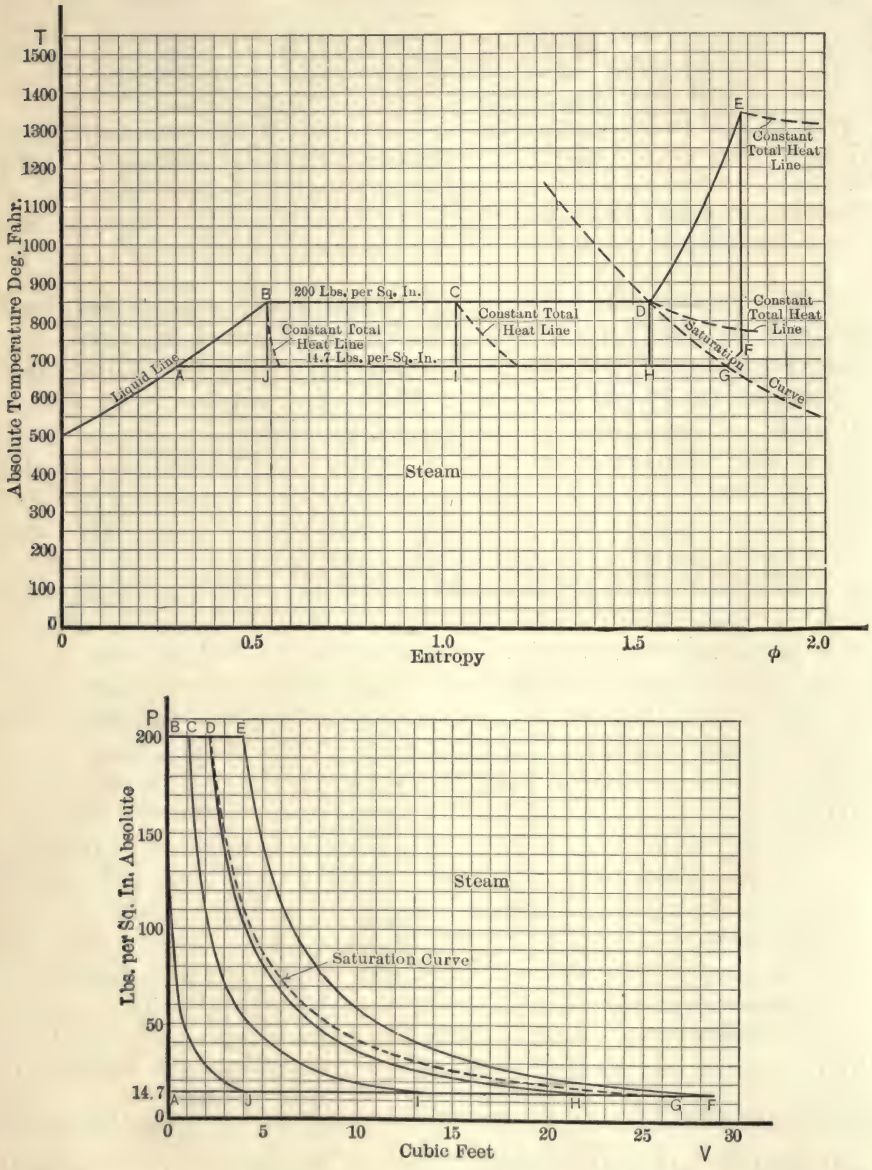


FIG. 289.—Thermal Cycles for Nozzle Expansion Indicating Available Gain in Kinetic Energy.

gases that are perfect, the final temperature would be the same as the initial, since the internal energy is a function of temperature only, but real gases

will always suffer a small temperature change usually to be neglected in engineering computations. Any jet suffering friction reheat less than 100 per cent, and this includes those through all real nozzles and orifices, will have a velocity to be calculated according to an expansion line lying *between*

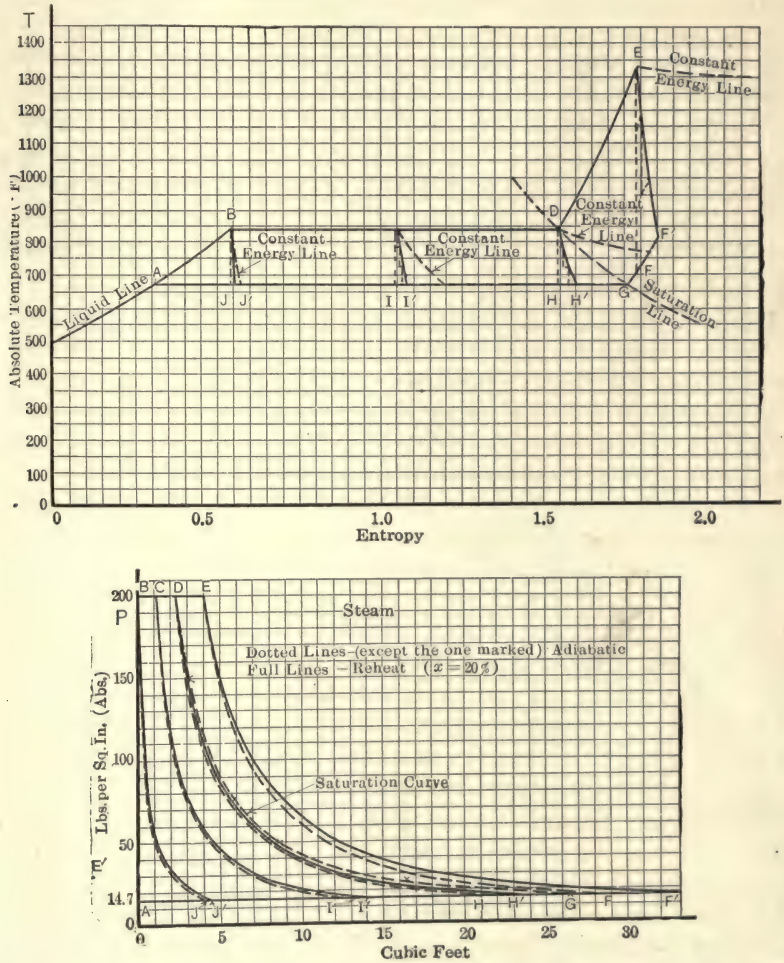


FIG. 290.—Effect of Nozzle Reheat Due to Friction in Nozzle Expansion on Final Condition and Available Gain in Kinetic Energy.

the adiabatic and the constant energy line, just where, depends on the amount of reheat.

After the jet leaves the nozzle with the above velocity and corresponding kinetic energy, it may for a time retain all of it, lose all of it or lose part, depending on subsequent friction and impact. In the throttling steam calorimeter the jet is brought to rest by impact and all kinetic energy converted back into

heat, so the final condition of the steam will be that corresponding to a heat content equal to what it had originally, but at the lower pressure, and likewise the same as if it had escaped through a porous plug emerging with no velocity, continuously expanding along the constant total heat line. Relations between initial and final qualities can be read off on $T\Phi$ or Mollier diagrams by following constant total heat lines. About the same thing happens when steam or air passes through reducing valves—some velocity is developed and subsequently all, or nearly all is lost by impact on walls or on the mass of fluid on the low-pressure side, and experience shows that the loss is practically complete so that perfect gases will not suffer any temperature change ultimately, and steam or hot liquids will have a final quality due to equal total heats. Hot liquids thus will always be partly converted into vapor as in boiler blow-offs and refrigerating expansion valves, while wet steam may be dried wholly, or in part, or more superheated than it was originally.

In steam turbines the jet once formed is brought to moving vanes and there has its direction of motion changed so that it leaves the vanes with a velocity with respect to them, but ideally with no velocity with respect to the casing or nozzle. When this process is perfectly executed the whole kinetic energy of the jet is imparted to the moving vane wheel, but actually there is developed some vane friction which acts as additional reheat giving the steam a new quality higher than it had before it reached the vane, and which is to be measured by an added amount of energy equivalent to the *vane reheat* expressed as y per cent of the energy of expansion. This is very important in those turbines that have a series of successively expanding nozzles, as work to be developed in the second depends in part on the nozzle and vane reheats of the first. Such reheat is, therefore, not as much a loss in such multi-stage turbines as in those of one stage, but always constitutes one of the main energy losses in these machines, the other losses being leakage and windage friction of the rotor in the steam atmosphere, which latter adds more reheat. The whole reheat is commonly taken as 40 per cent, on the average.

To calculate the velocity of a jet two factors are necessary: first, the work that would be done by adiabatic expansion, and second, the reheat factor. This reheat factor is for turbine nozzles about 10 per cent but is not established for other forms, like valve orifices. The determination of velocity without nozzle friction has already been explained; the PV formula of Chapter I is to be used for gases, but for steam the Mollier diagram is the most expeditious, as both the work done and velocity can be read off directly. Determinations of final quality of the fluid jet can also be made for the case of no reheat from the same diagram directly, and by a simple additional step the quality can be found graphically as illustrated in Fig. 291. This is a section of the Mollier diagram for steam, where AB represents adiabatic expansion, during or after which, 20 per cent of reheat occurs. To find the final quality lay off $\overline{BC}=.2\times\overline{AB}$, so that $\overline{AC}$ represents the effective work and $\overline{BC}$ the heat of reheat. A horizontal through C cutting the constant-pressure line through B locates D, the condition of final quality. It is interesting to note that if this heat is constantly

added, that is, if the reheat is a constant fraction in the nozzle, the quality at any pressure can be found by a similar construction used by Roe. With B as a center, an arc CE is drawn, and a tangent to it drawn through A. Similar tangent arcs drawn from other centers give the means of finding the quality at the pressure indicated by that center, by the intersection of hori-

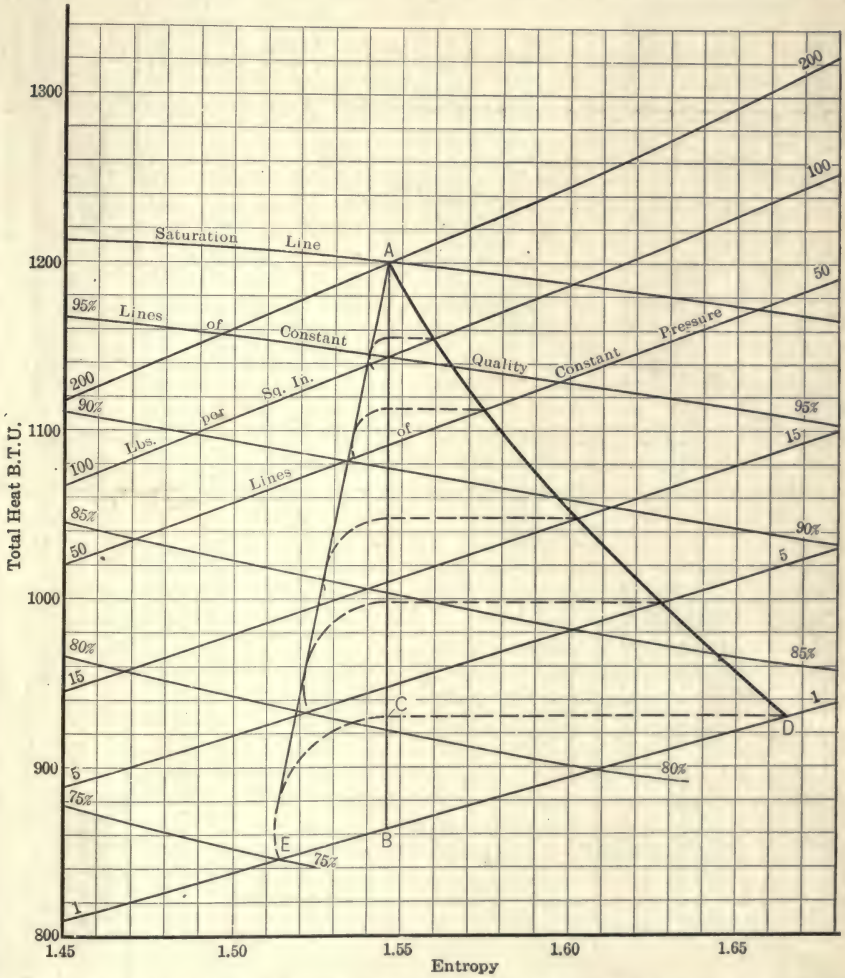
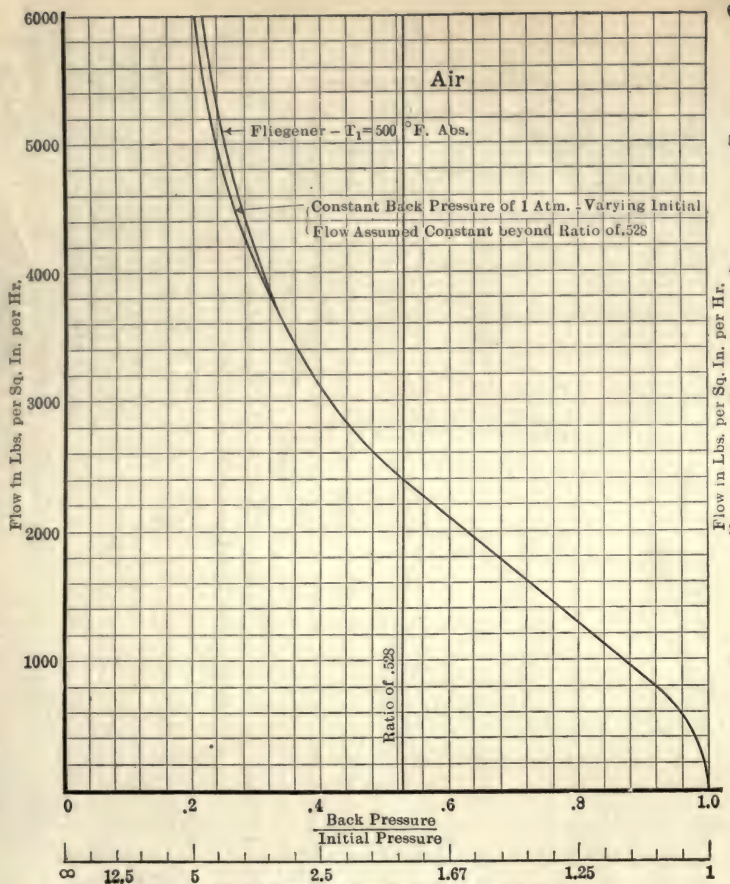
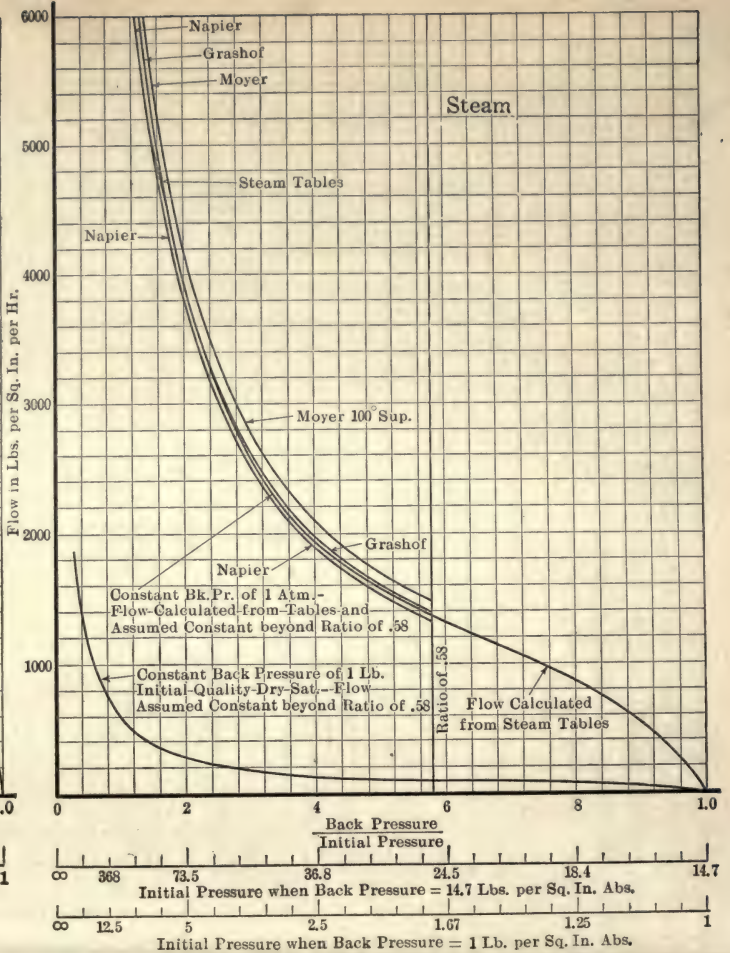


FIG. 291.—Graphic Method of Locating Nozzle Steam Expansion Line and Final Quality with Given Per Cent Friction Reheat.

ontals tangent to the tops of the arcs, with constant-pressure lines through he centers. The line AD then represents the variation of condition of the steam n the nozzle. If it receives vane or other reheat beyond the nozzle the jet vill have a sudden change in quality at the end of this nozzle condition line.

The weight of flow of expansive fluids in pounds per second per square nch of orifice, has a critical value at a given pressure ratio, which divides a


Air
Fliegener – T_1 = 500 °F. Abs.
Constant Back Pressure of 1 Atm. - Varying Initial
Flow Assumed Constant beyond Ratio of .528
Flow in Lbs. per Sq. In. per Hr.
6000
5000
4000
3000
2000
1000
Ratio of .528
0
.2
.4
.6
.8
1.0
Back Pressure / Initial Pressure
∞
12.5
5
2.5
1.67
1.25
1
Initial Pressure when Back Pressure = 1 Atm.


Steam
Napier
Grashof
Moyer
Steam Tables
Napier
Moyer 100° Sup.
Grashof
Napier
Constant Bk.Pr. of 1 Atm.-
Flow Calculated from Tables and
Assumed Constant beyond Ratio of .58
Constant Back Pressure of 1 Lb.
Initial Quality Dry Sat. - Flow
Assumed Constant beyond Ratio of .58
Flow Calculated
from Steam Tables
Flow in Lbs. per Sq. In. per Hr.
6000
5000
4000
3000
2000
1000
Ratio of .58
0
2
4
6
8
1.0
Back Pressure / Initial Pressure
∞
368
73.5
36.8
24.5
18.4
14.7
Initial Pressure when Back Pressure = 14.7 Lbs. per Sq. In. Abs.
∞
12.5
5
2.5
1.67
1.25
1
Initial Pressure when Back Pressure = 1 Lb. per Sq. In. Abs.

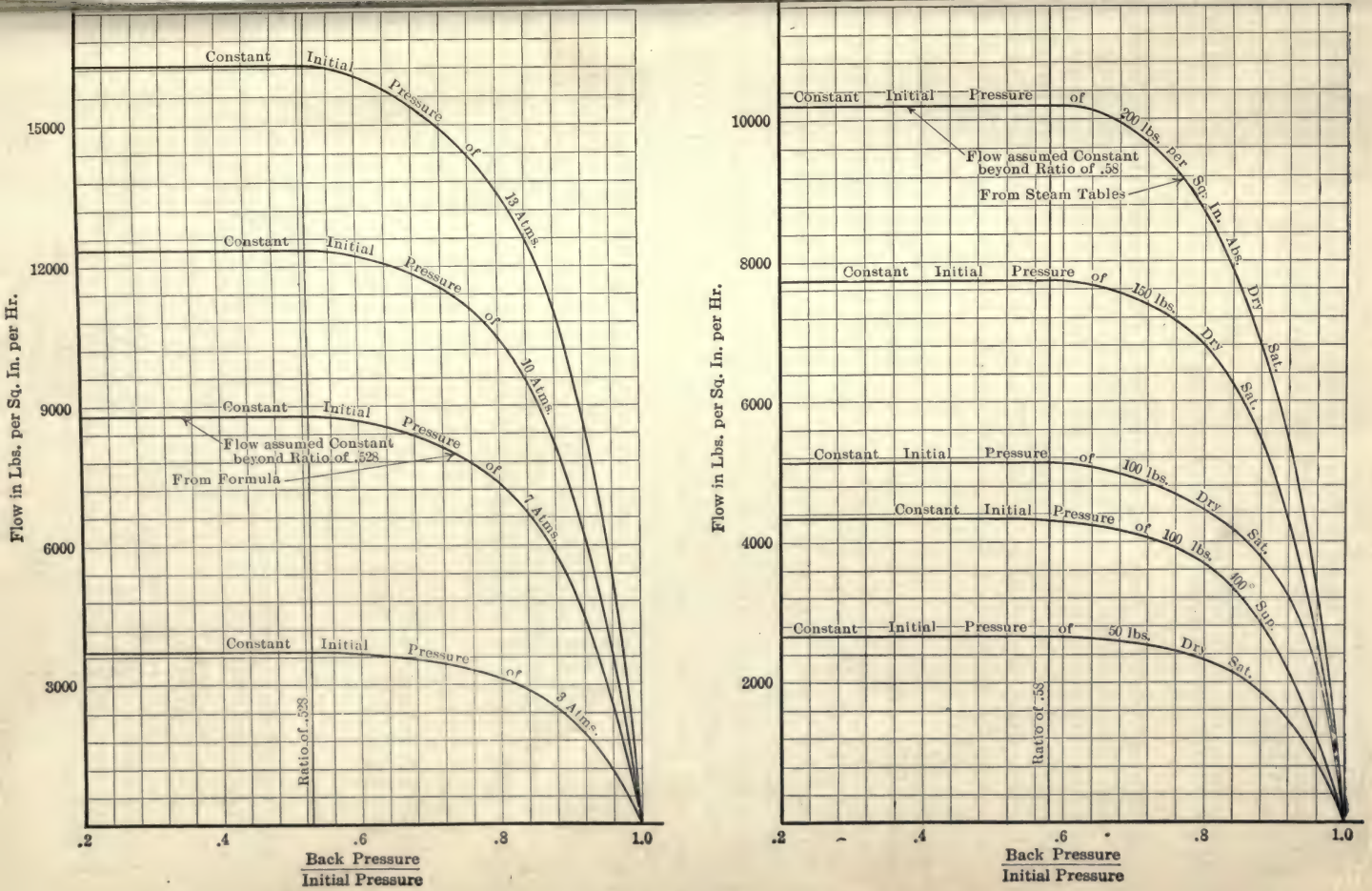


Fig. 202.—Comparison of Rational and Empiric Formulas for Air and Steam Flow with Large Pressure Drops. *Constant Initial* and

range of pressures over which the flow does not change with pressure change, from the other region where it does. In the first Chapter it was shown by PV analysis that this critical pressure was a function of the exponent γ given by Eq. (1181).

$$\text{At the critical point, } \left(\frac{\text{Final pressure}}{\text{Initial pressure}}\right) = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1181)$$

This is easily fixed for gases not suffering wide temperature changes because for them γ is constant, but for vapors the variability of γ makes it not so easy to fix. However, for the conditions in turbine nozzles the ratio is usually taken at the round number of .58 though it depends on the quality, the pressures and temperatures of the steam. From the tables of Section 4 of this Chapter, values of γ for all sorts of steam conditions determined by the $T\Phi$ method, can be used in the above equation. Careful investigation of this critical phenomenon seems to indicate that at some point in every orifice the pressure falls to this critical value of itself, and that the weight of flow is fixed by the pressure at that point regardless of how much lower the pressure may be beyond. Furthermore, it appears that at this critical point of the orifice or throat, each fluid acquires a fixed velocity peculiar to itself about the same, according to Emden, as the velocity of sound in the medium, and for steam this is between 1400 and 1500 ft. per second for such conditions as are met in turbine nozzles. Of course, beyond this critical point, further expansion may take place, increasing the velocity above the critical value but not the weight, *provided the orifice or nozzle is so shaped as to keep the flow axial and prevent sidewise dissipation, in which case the nozzle is an expanding nozzle.*

To make this condition of affairs more clear the curves of Fig. 292 and 293 for steam and for air, have been calculated from Eq. (25), Chapter I, using $\gamma = 1.4$ for air, and by the Mollier diagram for steam. To this diagram are added some curves of experimental flow laws stated symbolically below:

$$\left\{\begin{matrix}\text{Lbs. dry saturated steam} \\ \text{per sq.in. area per hour}\end{matrix}\right\} = \left\{\begin{matrix} 60p^{.97} \text{ (Grashof)} & (a) \\ 51.43p \text{ (Napier)} & (b) \\ 49.6p \text{ (Harter)} & (c) \\ 3.6p\,(16.3 - 96 \log p) \text{ (Rateau)} & (d)\end{matrix}\right\} \quad (1182)$$

$$\left\{\begin{matrix}\text{Lbs. superheated steam per} \\ \text{sq.in. area per hr.}\end{matrix}\right\} = \left\{\begin{matrix} 60p^{.97}\ [1 + .00065 \times (\text{degrees superheat})] \text{ (Moyer)} & (a) \\ 49.6p \text{ to } 45.0p \text{ for superheat } 0° \text{ to } 185° \text{ F. at 160 lbs. sq. in. (Harter)} & (b)\end{matrix}\right\} \quad (1183)$$

$$\left\{\begin{matrix}\text{Lbs. air per} \\ \text{sq.in. area} \\ \text{per hour}\end{matrix}\right\} = \left\{\begin{matrix} 3816\sqrt{\frac{p_2(p_1 - p_2)}{T_1}} \text{ for less than 2 atm. to atm. (Fliegener)} & (a) \\ 1900\frac{p_1}{\sqrt{T_1}} \text{ for more than 2 atm. to atm. (Fliegener)} & (b)\end{matrix}\right\} \quad (1184)$$

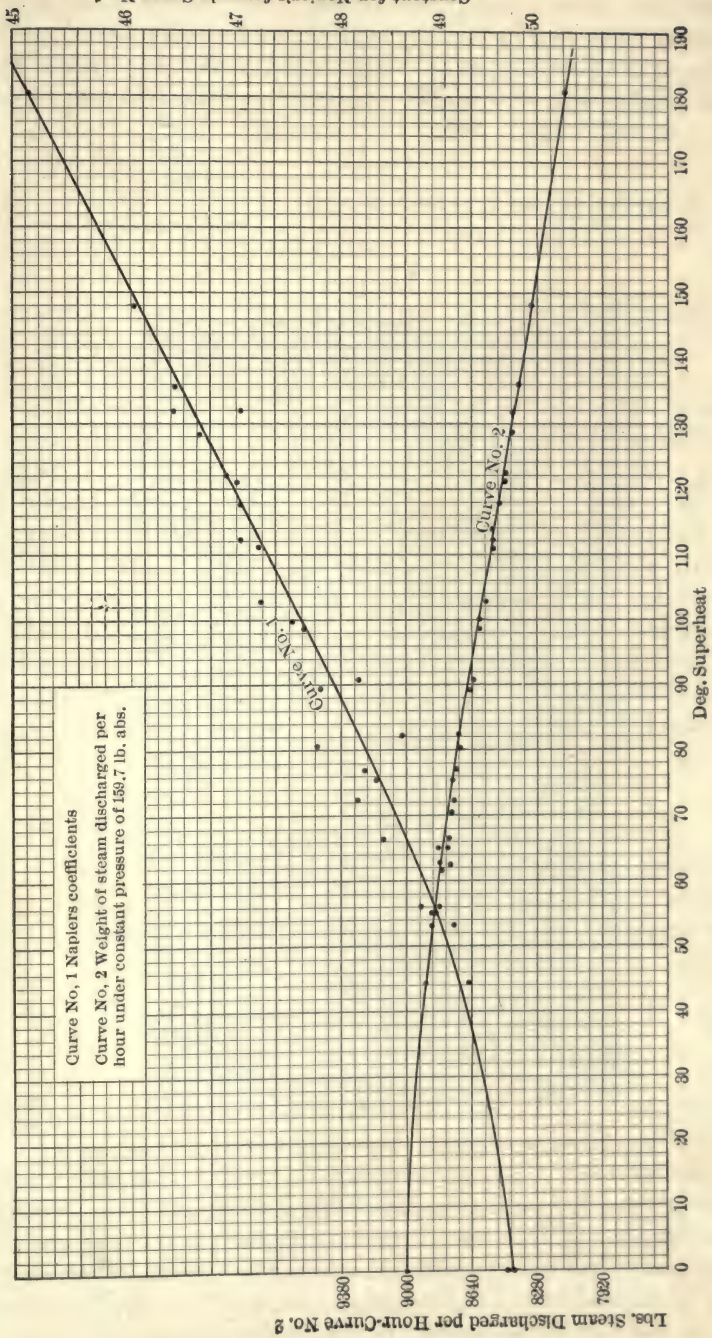


FIG. 294.—Harter's Values of Napier's Coefficient and Weight of Flow for Superheated Steam.

The superheated steam Napier coefficients and corresponding experimenta steam weights are shown in Fig. 294 reproduced from Harter's report.

For steam, the best and most used relation is that of Grashof, which fo wet steam, such as always is found entering low-pressure turbines, becomes

$$(\text{Lbs. wet steam per square inch area per hr.}) = \frac{60\ p^{.97}}{\sqrt{\text{dryness fraction}}}. \quad . \quad (1185$$

By means of these equations the size of nozzle *throat* or narrowest poin actually or, in effect, to pass a given weight of steam or air can be calculated or inversely, the weight that will pass a given area. The form of the nozzl however affects the result by making the actual minimum area differ from tha which is effective, because of stream contraction. The effective area may b only 60 per cent of the actual throat area if sharp corners exist. The product o weight per hour and foot-pounds per pound of steam gives the kinetic energy o the jet in foot-pounds per hour, which has, of course, a horse-power equivalent This jet horse-power will not be realized, however, unless another condition is ful filled and that is a proper shape for the nozzle beyond the throat and a prope ratio of maximum terminal nozzle area, to that of the throat, to enable the ful expansion work to be realized beyond the nozzle throat. The ratio of the larges area of the nozzle that at the mouth, to the least, that at the throat, is the ex pansion ratio of the nozzle and for it, two usable values not in accord, are give

$$\left\{\frac{\left(\begin{matrix}\text{Mouth area}\\ \text{of nozzle}\end{matrix}\right)}{\left(\begin{matrix}\text{Throat area}\\ \text{of nozzle}\end{matrix}\right)}\right\} = \left\{\begin{array}{ll} \dfrac{.1550}{\sqrt{\left(\dfrac{p_2}{p_1}\right)^{1.762} - \left(\dfrac{p_2}{p_1}\right)^{1.881}}} & \text{(Zeuner)} \quad (a) \\ \left[.172\left(\dfrac{p_1}{p_2}\right) + .7\right] \text{ when } \dfrac{p_1}{p_2} < 25 & \text{(Moyer)} \quad (b) \\ \left[.175\left(\dfrac{p_1}{p_2}\right)^{.94} + .7\right] \text{ when } \dfrac{p_1}{p_2} > 25 & \text{(Moyer)} \quad (c) \end{array}\right\} \quad (1186$$

by Eq. (1186); where p_1 and p_2 are the initial and final working pressures for the nozzle. Between throat and mouth all sorts of curves and angles are used, with however, not so much difference in results as might be expected; about the commonest practice is to use an angle of 20° and a straight-line element. Too sudden an expansion due to too wide an angle will cause the steam to bounce from side to side and set up waves that result in increased frictional reheat. Too long a nozzle also brings about increased friction; too large a mouth results in overexpansion, with subsequent compression and more waves; too small a mouth incomplete expansion, all of which is well illustrated by one of Kneass' pictures, Fig. 295. As practically all nozzles must be used under

variable conditions of pressure and quality it is necessary in their design to avoid hair-splitting and use some judgment in estimating which to select for a mean or standard condition.

The reaction force on the nozzle due to the exit of the jet is equal to the weight in pounds per second discharged ÷ 32.2, into the velocity in feet per second, as in Eq. (1187).

$$(\text{Reaction in lbs.}) = \frac{1}{32.2}\,[(\text{lbs. steam per sec.}) \times (\text{jet velocity ft. per sec.})] \quad (1187)$$

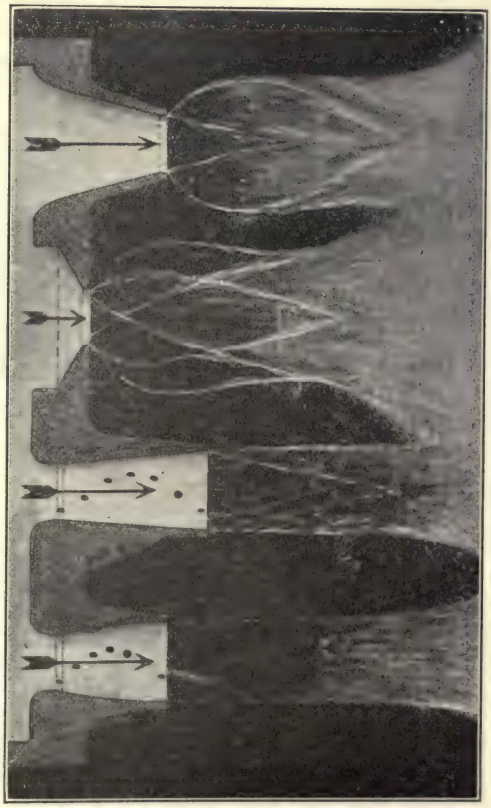


FIG. 295.—Kneass Steam Nozzle Flow Lines.

A series of steam turbine nozzles of different forms were tested for efficiency of conversion of heat into kinetic energy by means of measured weights and reaction forces, by Selby and Kemble and the best results were obtained with a straight taper of 14° 31′, rounded entrance, having a searching tube in the center and a throat diameter of .3949 in. gross, .0734 in. net, a length of $3\frac{1}{64}$ ins. and a mouth diameter of 1.156 ins. gross, 1.0005 ins. net. With initial absolute pressures of 100, 115, 130, and 145 lbs. per square inch, back pressures

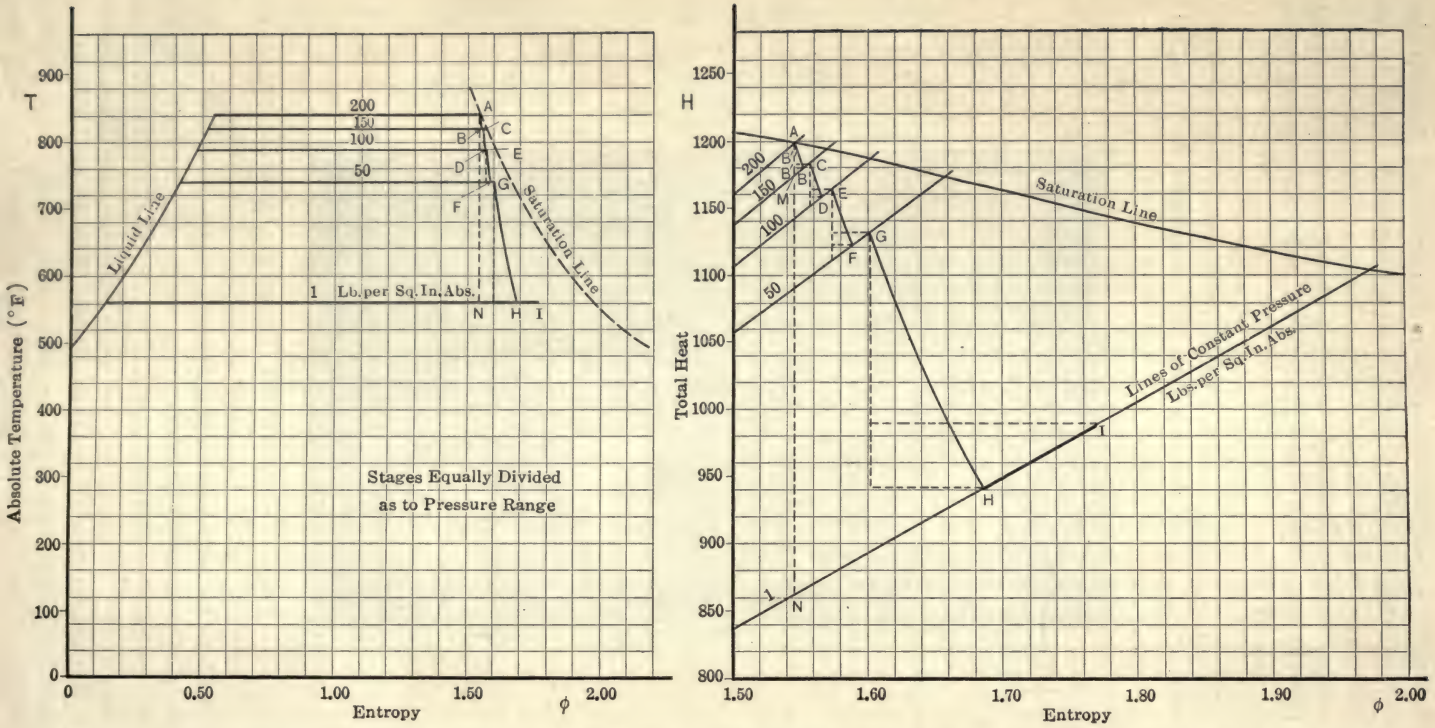


FIG. 296.—Multi-stage Series Nozzle Expansion of Steam, with Nozzle Reheat, $x = 20$ Per Cent, and Vane Reheat, $y = 20$ Per Cent. Stages Equally Divided as to Pressure Range. Work of Stages Unequal.

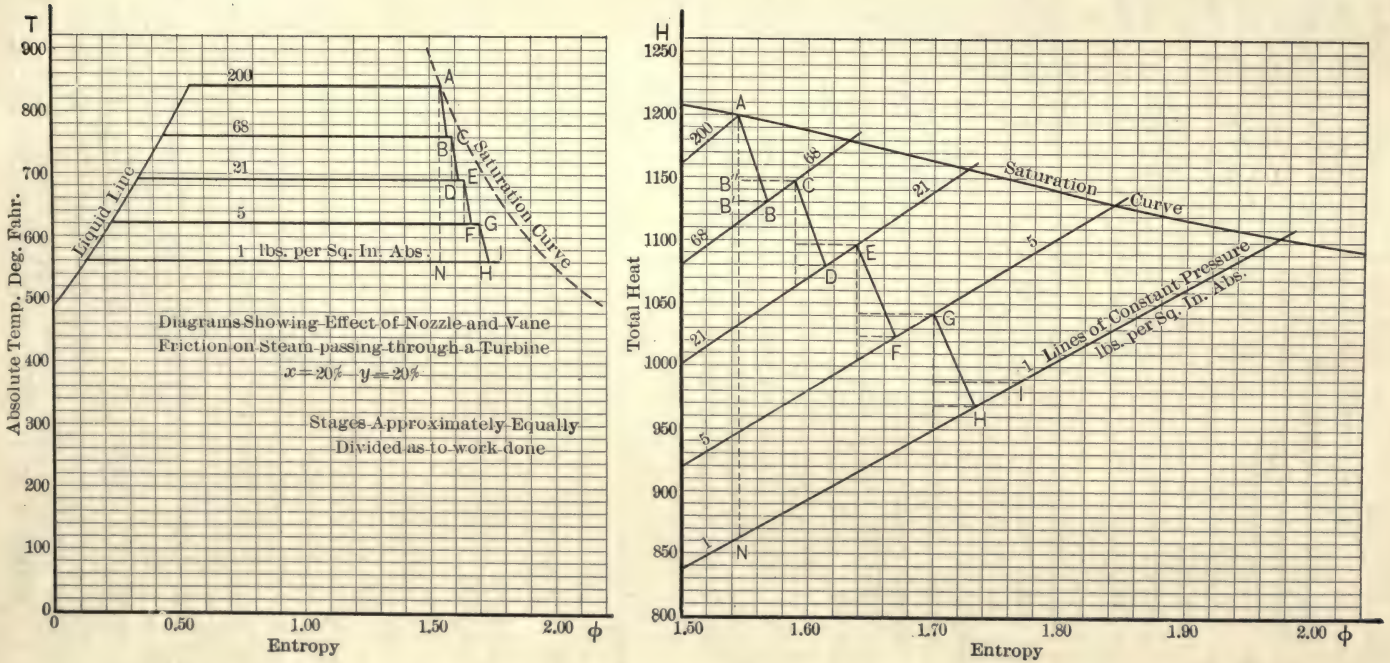


FIG. 297.—Multi-stage Series Nozzle Expansion of Steam, with Nozzle Reheat, $x = 20$ Per Cent, and Vane Reheat, $y = 20$ Per Cent. Stages Equally Dividing the Adiabatic Expansion Work without Reheat.

of 1.116, 1.288, 1.46, and 1.632 lbs. per square inch absolute, the flow was 389, 446, 502, and 558 lbs. per hour and the ratios of the work equivalent to the kinetic energy computed from flow weights and reactions, to the Rankine cycle were, 98.3, 97.9, 97.5, and 97.1 per cent respectively. These figures are given to show how nearly perfect such nozzles may be made, and how simple is the means of experimentally proving their efficiency. Similar experiments made in the laboratory of the writer on both compressed air and steam proved that air could give quite as high efficiency as the steam, in the nozzle expansion, which indirectly proves also the perfect feasibility of carrying out one important step in the development of a gas or mixed gas and steam turbine.

When several nozzles are in series and the same steam weights pass through all, their absolute and relative sizes can be fixed by the previous relations if the pressures in the intermediate chambers are fixed, because each may be treated independently of the other except for initial quality of approach to each, which must be determined. Thus, assume expansion to take place in four pressure stages or four nozzles or sets of nozzles, each with x per cent uniform frictional reheat, and with y per cent vane reheat between nozzles, then will the whole expansion process be according to the line $ABCDEFGHI$ of Fig. 296 to both $T\Phi$ and $H\Phi$ coordinates for equal division of pressures from 200 lbs. to 1 lb. per square inch absolute, and with originally dry saturated steam. This line is determined by drawing the four constant pressure zones, each 50 lbs. per square inch, across the two diagrams, dropping a vertical AM on the $H\Phi$ chart, giving the Rankine cycle work for the first stage pressure range, laying off $\overline{B'M} = \left(\frac{x}{100}\right) \times \overline{AM}$, the nozzle reheat loss, and $\overline{B'B''} = \left(\frac{y}{100}\right) \times \overline{AM}$, the vane reheat loss, referred to the same Rankine cycle basis, so that point B locates the condition of the steam at exit from first nozzle, and C its condition at the entrance to the next. The work of the first stage is, $\left[1 - \left(\frac{x+y}{100}\right) \times (\text{Rankine cycle work})\right]$ for the two pressures, and the first-stage efficiency is, $\left[1 - \left(\frac{x+y}{100}\right)\right]$. A similar construction applies to the other stages, and where graphically carried out will show unequal work and stage efficiencies even with equal per cents of reheat, when the stages are equally divided as to pressure range. It is more usual to divide the work per stage equally and if the stage efficiencies were 100 per cent this would be easily carried out by dividing the vertical AN representing the whole expansion work on the $H\Phi$ diagram into four equal parts and through each drawing the constant pressure lines as in Fig. 297. With any nozzle and vane reheat losses, however, this will not give equal stage work, but is a good starting point for a graphical trial and error method of finding that pressure

division that will give it, after which the nozzle sizes can be found to maintain that pressure relation.

Prob. 1. Steam at 100 lbs. per square inch absolute containing 5 per cent moisture expands in a nozzle to 10 lbs. per square inch absolute with 20 per cent nozzle reheat. What is the final quality, velocity, and horse-power per square inch of nozzle and what fraction is each, of what would result without nozzle friction?

Prob. 2. How would the results of the previous problem differ by changing final pressure to 1 lb., initial pressure to 150 and 200 lbs., and initial quali y to 50 per cent, 100 per cent or 100° superheat, in any combinations?

Prob. 3. Air at 150 lbs. per square inch absolute and 1000° F. expands into a standard atmosphere, where its temperature in the jet is 300° F. If the reheating had been uniform, plot the $T\Phi$ expansion curve, find s and the reheat factor.

Prob. 4. What would be the nozzle reaction per square inch of orifice for Problems 1 and 3 and what dimensions should expanding nozzles have?

Prob. 5. Water from a 150-lb. gage boiler is blown off. What per cent will evaporate? How much will the evaporation change with initial pressure, plot a curve?

Prob. 6. Air flows from a tank where the pressure is 150 lbs. per square inch absolute, to a standard atmosphere through a $\frac{3}{8}$-in. diameter throat nozzle. How many cubic feet of free air must be supplied per hour to maintain the tank pressure and what compressor horse-power will be required if two stage?

Prob. 7. A steam turbine has four $\frac{1}{8}$-in. diameter nozzles in parallel and the back pressure is 2 lbs. per square inch absolute. How much steam per hour and what jet velocities result for, (*a*) 150 lbs. per square inch absolute and 100° superheat initially, (*b*) same pressure and dry saturated steam, (*c*) 100 lbs. per square inch absolute and dry saturated steam, (*d*) same pressure as in (*c*) and 75 per cent quality. Use Mollier diagram.

Prob. 8. Compare results of Problem 7 with those computed by the formulas of Grashof, Napier, Rateau, Harter, Moyer.

Prob. 9. For same initial pressure and temperature and same final pressures compare the preceding results on steam flow with air flow, using rational formula and Fliegener's empiric form.

Prob. 10. If four nozzles of a steam turbine are in series, and the first $\frac{1}{4}$-in. diameter is supplied with dry saturated steam at 180 lbs. per square inch absolute, what must be the sizes of the other three

(*a*) to equally divide the pressure range with no nozzle and no vane reheat
(*b*) " " " " " 20% vane reheat
(*c*) " " " " 20% nozzle and no vane reheat
(*d*) " " " " 20% " 20% "
(*e*) to equally divide the temperature range with no " no "
(*f*) " " " " no " 20% "
(*g*) " " " " 20% " no "
(*h*) " " " " 20% " 20% "

The back pressure is 2 lbs. per square inch absolute.

17. Flow of Expansive Fluids under Small Pressure Drops through Orifices, Valves, and Venturi Tubes. Relation between Loss of Pressure and Flow. Velocity Heads and Quantity of Flow by Pitot Tubes. When gases and vapors flow with a small difference between initial and final pressures, the flow charac-

teristics have a special engineering importance great enough to warrant more precise determinations of equations and constants than those applying to flow in general or with large pressure drop. The loss of pressure through the valves of air compressors, steam and air engines and gas engines, and its relation to velocity and weight of flow per square inch of opening, are examples of one class of application. Others of a similar sort include the flow of gas through burner nozzles, air from ventilating duct openings and through fan passages; and various meter orifices including the Venturi and Pitot tubes involve in their use, similar relations.

As a starting-point for the analysis of all these cases of flow, the flow through an orifice, assumed to be adiabatic, must be adopted. Accordingly, calling P_1 the initial and P_2 the final pressure in pounds per square foot, Eq. (25), of Chapter I, gives the flow when these differ not too much,

$$(\text{Lbs. per sec. per sq.ft.}) = \frac{w}{A} = \sqrt{2g \frac{\gamma}{\gamma - 1} \frac{P_1}{V_1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma+1}{\gamma}} \right]} \quad . \quad . \quad . \quad (1188)$$

Applying this to air passing through a round orifice of d ins. diameter, this becomes for air at 60° F., Eq. (1189).

$$\left\{ \begin{matrix} \text{Lbs. air per sec. through} \\ \text{circle of } d \text{ ins. dia. at } 60° \text{ F.} \end{matrix} \right\} = .0816 d^2 \sqrt{\frac{P_1}{V_1} \left[\frac{1}{R_P^{1.425}} - \frac{1}{R_P^{1.712}} \right]} \quad . \quad . \quad . \quad (1189)$$

Substituting $V_1 = \frac{RT_1}{P_1}$, this becomes

$$\left\{ \begin{matrix} \text{Lbs. air per sec. through} \\ \text{circle of } d \text{ ins. dia. at } 60° \text{ F.} \end{matrix} \right\} = .00049 d^2 P_1 \sqrt{\frac{1}{R_P^{1.425}} - \frac{1}{R_P^{1.712}}} \quad a)$$

$$= .0707 d^2 p_1 \sqrt{\frac{1}{R_P^{1.425}} - \frac{1}{R_P^{1.712}}} \quad (b) \qquad . \quad (1190)$$

With orifice friction and poor orifice forms the actual area may not be effective, so there must be introduced coefficients representing these influences as experimentally established. It is found, however, that the actual flow values agree fairly well with a simpler and more practical formula for very small pressure drops. Such a formula can be developed on the theory that when the drop in pressure is small the work derived by expansion may be neglected, and the fluid may be considered as non-expansive, and of a density corresponding to the mean pressure on the two sides and to the original temperature,

Let h_W = the difference in pressure in inches of water;

" H_A = the head in feet of air corresponding to h_W;

" P = mean pressure, lbs. sq.ft. $= \frac{P_2+P_1}{2}$;

" V_A = cu.ft. per lb. air $= 53.34\frac{T}{P}$;

" A = sq.ft. area of orifice;

" u = velocity in feet per second;

" w = pounds air per second.

Then since 5.2 pounds per square foot is equal to one inch of water column,

$$H_A = V_A(P_2-P_1) = 5.2V_Ah_W, \quad \text{and} \quad \frac{wV_A}{A} = u,$$

so that

$$\left\{\begin{array}{l}\text{Pounds air per sec. per} \\ \text{sq.ft. of orifice}\end{array}\right\} = \frac{w}{A} = \frac{u}{V_A} = \frac{\sqrt{2gH_A}}{V_A} \quad (a)$$

$$= \frac{\sqrt{2g \times 5.2h_W V_A}}{V_A} \quad (b)$$

$$= \sqrt{\frac{64.4 \times 5.2h_W}{V_A}} \quad (c)$$

$$= \sqrt{\frac{64.4 \times 5.2h_W P}{53.34T}} \quad (d)$$

$$= 2.506\sqrt{h_W\frac{P}{T}} \quad (e) \qquad \ldots\ldots (1191)$$

For a round orifice of d ins. diameter this becomes

$$\left\{\begin{array}{l}\text{Lbs. air per sec. through circle} \\ \text{of } d \text{ ins. diam.}\end{array}\right\} = \frac{.7854d^2}{144} \times 2.506\sqrt{h_W\frac{P}{T}} \quad (a)$$

$$= .01366d^2\sqrt{h_W\frac{P}{T}} \quad (b) \qquad . \quad (1192)$$

If the discharge is from a low-pressure chamber into a standard atmosphere, this becomes very nearly

$$\left\{\begin{array}{l}\text{Lbs. air per sec. to atmos. through}\\ \text{circle of } d \text{ ins. diam.}\end{array}\right\} = .6283 d^2 \sqrt{\frac{h_W}{T}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1193)$$

The best determinations of the constants for either the approximate formula, Eq. (1192), or the exact one, Eq. (1190), are those made by Durley, whose calculation on the relation of the results of the two with a 100 per cent coefficient are given in Fig. 298 for pressures up to 100 inches of water for air at 60°

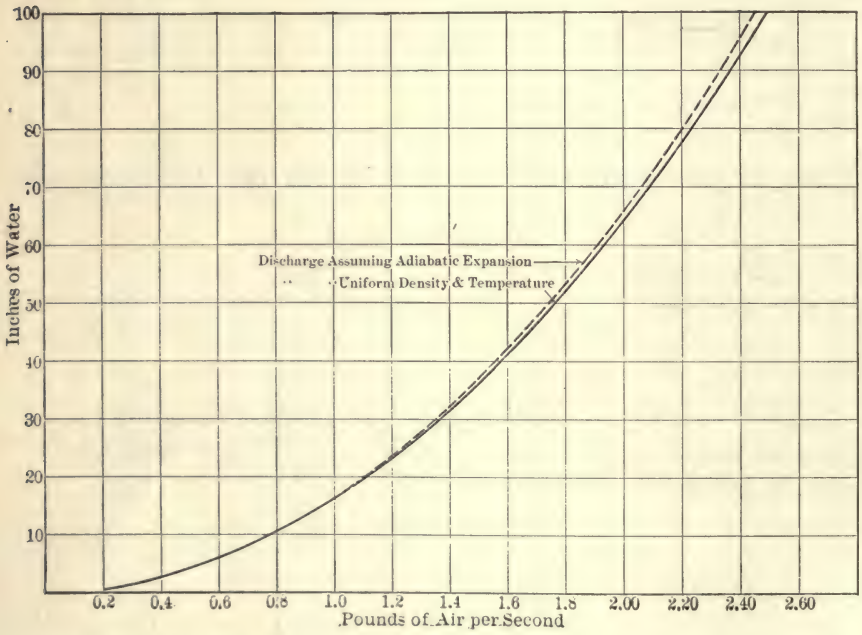


FIG. 298.—Calculated Ideal Frictionless Discharge of Air to Atmosphere from Circular Orifice 3 inches diameter. Initial Temperature 60° F., Barometric Pressure 14.7 pounds per square inch.

F. discharging to atmosphere, 30 ins. barometer, and his experimental coefficients for various orifices in Table CXLI on the opposite page. His results show that the coefficient for small orifices increases with the head but more and more slowly and is constant for a 2-in. diameter orifice, after which it decreases, and more, the larger the orifice. Increase of diameter involves smaller coefficients, but temperture has no effect between 40° F. and 100° F. as proved, and this is probably so also for even wider ranges. The orifice must be not more than $\frac{1}{20}$ the area of the approach chamber for these coefficients to apply. They are all for a plate .057 in. thick and for sharp edges; changing the thickness or rounding the edges will

Table CXLI

COEFFICIENT OF DISCHARGE C FOR VARIOUS WATER HEADS AND DIAMETERS OF ORIFICE (Durley)

Diameter of Orifice. Inches.	1-inch Water.	2-inch Water.	3-inch Water.	4-inch Water.	5-inch Water.
$\frac{5}{16}$	.603	.606	.610	.613	.616
$\frac{1}{2}$	.602	.605	.608	.610	.613
1	.601	.603	.605	.606	.607
$1\frac{1}{2}$	.601	.601	.602	.603	.603
2	.600	.600	.600	.600	.600
$2\frac{1}{2}$	.599	.599	.599	.598	.598
3	.599	.598	.597	.596	.596
$3\frac{1}{2}$	.599	.597	.596	.595	.594
4	.598	.597	.595	.594	.593
$4\frac{1}{2}$	.598	.596	.596	.593	.592

Weight of air discharged per second $= .6283 C d^2 \sqrt{\frac{h_W}{T}}$ lbs. (Barometer at 30 inches.)

change the coefficient, and Moss reports some tests with rounded approach orifices in which the coefficient was as high as $C = 0.942$.

When the thickness of the plate is great in proportion to the diameter of the orifice, the latter becomes a *short tube* for which differences enter, having new experimental values not as numerous as they should be, as this is the case for gas burner nozzles. It appears, however, that the old results of Weisbach on air for both small orifices and short tubes are pretty good and these are reproduced in Table CXLII below.

Table CXLII

VALUES OF C FOR AIR FLOW (Weisbach)

Orifice of diameter = .394 ins.

R_P	1.05	1.09	1.43	1.65	1.89	2.15
C.....	.555	.589	.692	.724	.754	.788

Orifice of diameter = .843 ins.

R_P	1.05	1.09	1.36	1.67	2.01
C.....	.558	.573	.634	.678	.723

Short tube, diameter = .394 ins. and length = 1.181 ins.

R_P	1.05	1.10	1.30
C.....	.730	.771	.830

Short tube, diameter = .557 and length = 1.673 ins.

R_P	1.41	1.69
C.....	.813	.822

Short tube, diameter = .394 ins. and length = .630 ins. rounded entrance

R_P	1.24	1.38	1.59	1.85	2.14
C.....	.979	.986	.965	.971	.978

His general conclusions on the coefficients for these small orifices give them values between .97 and .99 for orifices with rounded entrance, .56 to .79 for sharp edge in thin plates, .81 to .84 short straight tubes, .92 to .93 for short tubes with rounded entrance and .90 to .99 for converging orifices.

When the fluid is not air, the flow is to be determined by the general law of proportionality as a function of density. The velocities due to pressure drop for different gases and vapors in these cases are to each other inversely as the square root of the density; thus, for any gas of specific volume, V_G, or density, $\frac{1}{V_G}$, its velocity is given in terms of that of air by,

$$u_G = u_A \frac{\sqrt{\frac{1}{V_A}}}{\sqrt{\frac{1}{V_G}}} = u_A \sqrt{\frac{V_G}{V_A}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1194)$$

Putting this in terms of weights by the relation, $\frac{wV}{A} = u$,

$$w_G = w_A \frac{V_A}{V_G} \sqrt{\frac{V_G}{V_A}} = w_A \sqrt{\frac{V_A}{V_G}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1195)$$

so that flow weights are to each other directly as the square roots of the densities.

One very common and important case of orifice flow with small pressure drop is that through the valves of gas engines and compressors, valves of the poppet type with both flat and conical seats, the flow being in both directions, that is, toward the cylinder and from it, while the valve always seats away from the cylinder in the direction of the action of the superior pressure. The only series of experimental determinations of the coefficients for such flows is that made under the writer's direction in the laboratories of Columbia University on both mechanically operated and spring-closed valves, all, however, of small size. One valve had a flat seat 1.58 in. diameter, and the other two, conical 45° seats 1.5 and 2.0 ins. diameter respectively. These were supplied with measured air flowing in either direction and with the valves first fixed at known lifts, and later rising and falling with the piston known amounts and at measured rates. From the data obtained the relation between flow and pressure drop up to 30 ins. of water was obtained and compared with the flow-pressure drop relation for no losses for which, $u = \sqrt{2gH_A}$. The ratio of the measured velocity or volume to that computed for no loss, as above, gives a coeffic ent of efflux for the given pressure drop and the results for the more common con cal valve are reproduced graphically in Fig. 299. Double results are given for air measured at meter density and at atmospheric density, the difference being due to the pressure drop experienced when the flow was

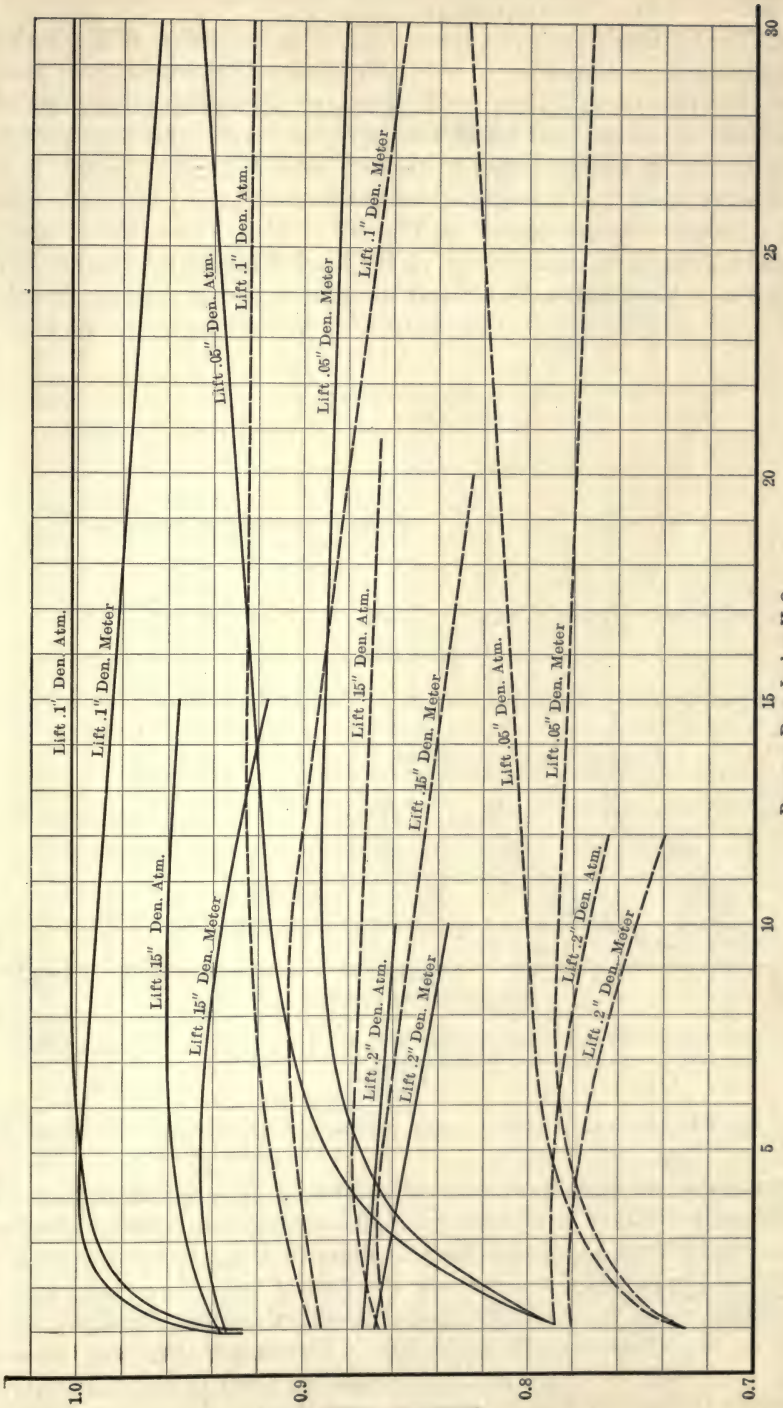


FIG. 299.—Coefficient of Efflux for Fixed Poppet Valve Orifices.

steady and valve fixed, the valve being 2 in. diameter lifting .6 in. maximum. The corresponding coefficients are not easily obtained for intermittent flow, the more direct method being to compare momentary velocities actual and computed, at each point of the rise and fall of the valve for the measured pressure drop, and plot a curve of pressure drop to velocity as in Fig. 300. To give a direct comparison between the steady and the intermittent flow, the corresponding curves for steady flow are plotted in Fig. 301. It is impossible to give any single generalization of these results, as the coefficients varied from 0.2 to 1.0, depending on type of valve, its lift and the rate of flow as well as direction of flow, and the only thing that appears to be sure is, that such flow is very

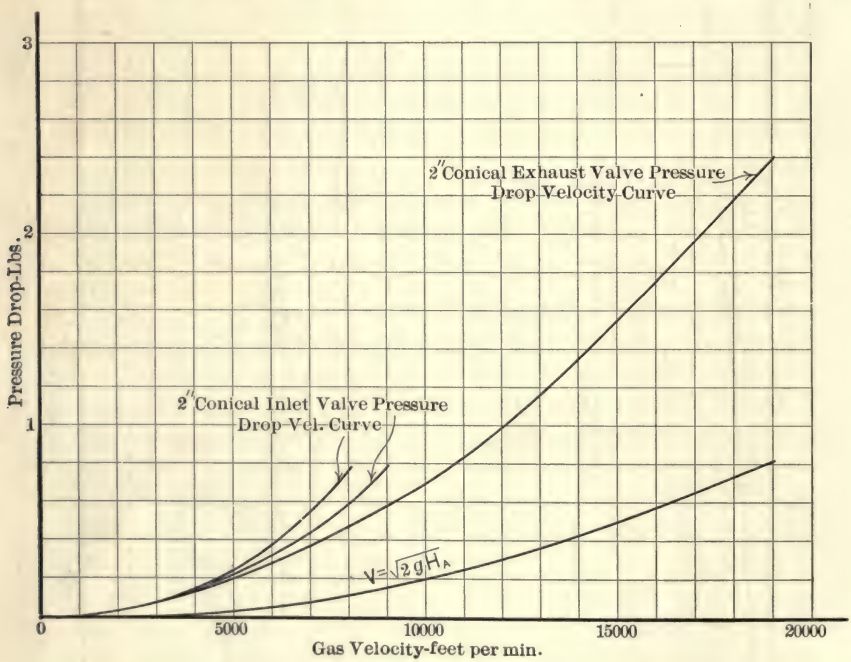
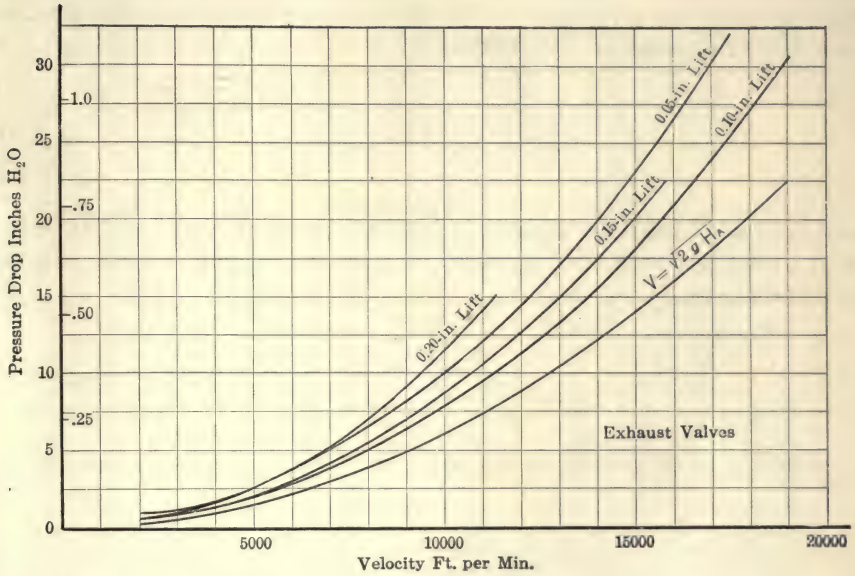
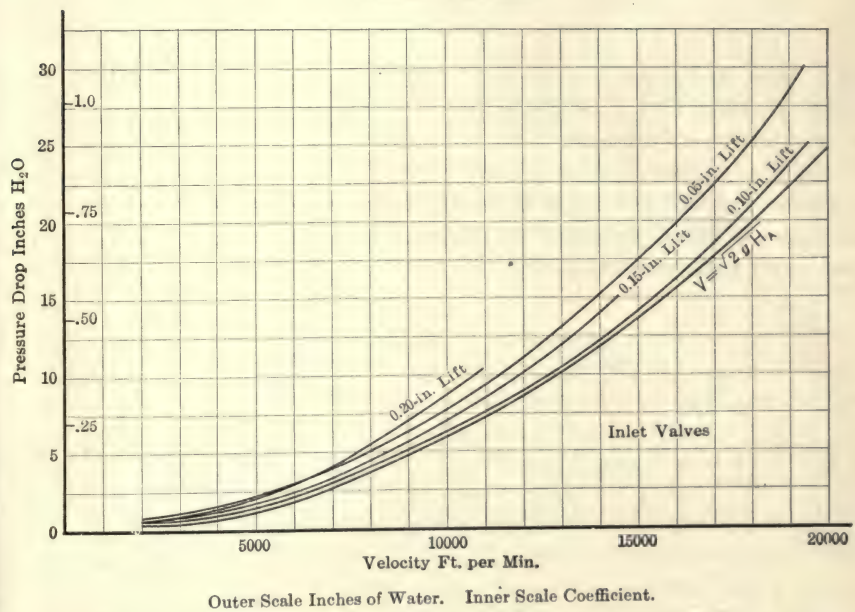


Fig. 300.—Intermittent Flow through Poppet Valves. Relation between Pressure Drop and and Velocity.

complex indeed, and may involve very serious losses, but on the other hand may be very highly efficient.

Venturi tubes are usually made by a converging pipe of straight cone form and small angle, ending in a short small diameter cylinder, which discharges into a diverging cone of smaller angle, ending in the original diameter, and while designed originally for measuring the flow of water have been found of great practical value in measuring gases, the writer having used them for this purpose in diameters from 1 to 16 ins. The larger sizes are especially useful, as gas meters of the volumetric displacement sort of equal capacity, are prohibitively costly. In such a tube the same weight of fluid is passing each point, and neglecting frictional losses and static head differences which apply





Outer Scale Inches of Water. Inner Scale Coefficient.

FIG. 301.—Steady Flow through Poppet Valves. Relation between Pressure Drop and Velocity.

to tubes set vertically, but not to horizontal ones, the sum of the velocity head and pressure head is constant. Therefore calling P_1 and P_2 the upstream and throat pressures lbs. per square foot, and A_1 and A_2 the corresponding areas in square feet, the equation of condition is,

$$P_1+\frac{{u_1}^2}{2g}=P_2+\frac{{u_2}^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1196)$$

If the fluid be of constant density as it passes from the large area to the small one, the velocities must vary inversely as areas, since equal volumes pass each point in this case.

$$\frac{u_1}{u_2}=\frac{A_2}{A_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1197)$$

Therefore the throat velocity u_2 will be given by Eq. (1198), for constant density

$$u_2=\frac{A_1}{\sqrt{{A_1}^2-{A_2}^2}}\sqrt{2g(P_1-P_2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1198)$$

When gases and vapors suffer very small pressure drops Eq. (1198) may be applied to their flow in Venturi tubes, but when the difference between upstream and throat pressures is so small as to warrant the use of this relation, it is too small for the accurate readings needed to determine the quantity flowing. Therefore, for such cases it is necessary to return to the fundamental equation for expansive fluids, equating the increase in kinetic energy to the work of the cycle as for turbine nozzles.

$$\frac{{u_2}^2-{u_1}^2}{2g}=\frac{\gamma}{\gamma-1}\frac{P_1}{\delta_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right], \quad . \quad . \quad . \quad . \quad . \quad (1199)$$

In Eq. (1199), δ_1 is the density of the fluid on the high pressure or upstream side in pounds per cubic foot. As equal weights must pass both the upstream and throat sections, the velocities, densities and areas must be related as in Eq. (1200):

$$w=w_1=w_2=A_1u_1\delta_1=A_2u_2\delta_2=A_2u_2\delta_1\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}. \quad . \quad . \quad . \quad . \quad (1200)$$

Therefore

$$u_1=u_2\frac{A_2}{A_1}\frac{\delta_2}{\delta_1}=u_2\frac{A_2}{A_1}\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1201)$$

Substituting Eq. (1201) in Eq. (1199) and solving for u_2 results in Eq. (1179)

$$u_2 = \sqrt{\left(2g\frac{\gamma}{\gamma-1}\frac{P_1}{\delta_1}\right)\left[\frac{1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}{1-\left(\frac{A_2}{A_1}\right)^2\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}\right]} \quad . \quad . \quad . \quad . \quad (1202)$$

ntroducing this value of u_2 in Eq. (1200) the weight of flow is found as in Eq. 1203):

$$\left.\begin{matrix}\text{Lbs. of gas or vapor} \\ \text{per second}\end{matrix}\right\} = w = A_2\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\sqrt{2g\frac{\gamma}{\gamma-1}P_1\delta_1\left[\frac{1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}{1-\left(\frac{A_2}{A_1}\right)^2\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}\right]} \quad (1203)$$

In the actual use of Venturi tubes the value of $\left(\frac{P_2}{P_1}\right)$ should be between 5 and .995, which values correspond nearly to throat velocities from 300 to 00 ft. per second for low-pressure gases near one atmosphere in pressure, but r higher gas pressures very much higher values may be used. For steam the Iollier diagram may be advantageously used in Eq. (1204):

$$\frac{\gamma}{-1}\frac{P_1}{\delta_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{1}}\right] = 788\times\left\{\begin{matrix}\text{Work of Rankine cycle between} \\ P_1 \text{ and } P_2\text{, in heat units by} \\ \text{Mollier diagram}\end{matrix}\right\} \quad . \quad . \quad (1204)$$

At the works of the Lackawanna Steel Co., Coleman made a check test on ie of these meters, using throat velocities of 572 and 208 ft. per second with eam $\frac{1}{2}$ per cent wet at 110 lbs. per square inch pressure, and weighing the feed ıter, found errors of only +.056 per cent and −.84 per cent, which is sub-antially nothing. Checking similarly another meter on air in a 6-in. line, by easuring velocity heads across both upstream and throat diameters by tot tubes, the weights found by the former were .88289 lbs. per second and by e latter .88986, giving .9922 as the ratio of Venturi to Pitot indications. Other sts with the Pitot tube along one diameter only, gave results that averaged 063, in a 10-in. Venturi tube and .9976, in a 6-in. tube. The Pitot tube, of urse, gives the velocity head of the gas in inches of water directly at the point measurement the center of a small area a, and the average gas velocity over e whole area is, $\frac{\Sigma au}{\Sigma a} = (u)(\text{average})$. In the same plant the writer checked a

16-in. Venturi tube against the displacement of the gas pumps of a 1000 H.P Korting two-cycle gas engine, and against a regular proportional gas meter in the blast-furnace gas pipe, and the Venturi results agreed with the pump displacement within 1 per cent whereas the proportional meter disagreed with all other data.

The Pitot tube carries a double orifice at the end of a double tube that can be inserted in a pipe through stuffing boxes to allow the orifice to be located anywhere along the pipe diameter. One orifice points upstream and measures the velocity head by reducing to zero the velocity of the small portion of the stream that strikes it, to which is added the static gas pressure, while the other measures the gas pressure proper by an orifice tangent to the flow. The difference in pressure between the two orifices is the velocity head only. When the fluid is of constant density with respect to pressure, the velocity is given by Eq. (1205)

$$u = \sqrt{2gH}, \quad \ldots \ldots \ldots \quad (1205)$$

where H is the head in feet of fluid corresponding to the destroyed velocity. Usually water or mercury is used in the Pitot tube giving velocity head in inches of water or mercury. These heads are related to gas heads according to Eq. (1206), where H_W, and H_M, are heads in feet of water and mercury:

$$\left.\begin{aligned} H &= H_W \times \left(\frac{\text{lbs. per cu.ft. of water}}{\text{lbs. per cu.ft. of gas}}\right) \quad (a) \\ &= H_M \times \left(\frac{\text{lbs. per cu.ft. of mercury}}{\text{lbs. per cu.ft. of gas}}\right) \quad (b) \end{aligned}\right\} \quad \ldots \quad (1206)$$

When air is the fluid its density is to be determined from its pressure, temperature, and gas constant, $R = 53.34$, if it is dry, but when moist by the Smithsonian Eq. (1207), where p_V is the partial pressure of the water vapor in the air

$$\left\{\begin{matrix}\text{Lbs. per} \\ \text{cu.ft. of} \\ \text{moist air}\end{matrix}\right\} = \left[\frac{.080723}{1+.0020389(t-32°)}\right]\left(\frac{\text{barometer in ins. Hg} - .378p_V}{29.921}\right). \quad (1207)$$

These relations are proper for such very small velocities, as correspond to no appreciable change of density of the gas for the pressures used, but to get measurable velocity heads, the density variation cannot as a rule be neglected and in such cases the standard nozzle relation must be used as a starting-point

$$\frac{u_1^2 - u_2^2}{2g} = \frac{\gamma}{\gamma - 1}\frac{P_2}{\delta_2}\left[1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right]. \quad \ldots \ldots \quad (1208)$$

P_2 is the pressure measured at the impact orifice, $u_2=0$, and, $u_1=u$.

$$\frac{P_2}{\delta_2}=RT_2=RT_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1209)$$

ıbstituting Eq. (1209) in Eq. (1208) there results the velocity expression q. (1210),

$$\frac{u^2}{2g}=\frac{\gamma}{\gamma-1}RT_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right] . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1210)$$

This is developed into a series by Taylor as follows:

ıt $\frac{P_2}{P_1}=1+K$, and expand, $\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}=(1+K)^{\frac{\gamma-1}{\gamma}}$ into a series, which when ıltiplied by $\frac{\gamma}{\gamma-1}$ gives Eq. (1211).

$$K-\frac{K^2}{2\gamma}+K^3\left(\frac{1+\gamma}{6\gamma^2}\right)-K^4\left[\frac{(1+\gamma)(1+2\gamma)}{24\gamma^3}\right] . \quad . \quad . \quad . \quad . \quad (1211)$$

ıbstituting Eq. (1211) in Eq. (1210), putting $RT_1=\frac{P_1}{\delta_1}$, and solving for u, e velocity is found as in Eq. (1212):

$$u=\sqrt{2gK\frac{P_1}{\delta_1}\left\{1-\frac{K}{2\gamma}+\left(\frac{1+\gamma}{6\gamma^2}\right)K^2-\left[\frac{(1+\gamma)(1+2\gamma)}{24\gamma^3}\right]K^3\right.} \quad . \quad (1212)$$

roducing the readings of P_2 and P_1, in inches of mercury h_{M2} and h_{M1}, and ıltiplying by 60, the velocity in feet per minute for air, is given by Eq. (1213):

$$\left.\begin{matrix}\text{Velocity of}\\ \text{air in feet}\\ \text{per minute}\end{matrix}\right\}=4046.16\sqrt{\left(\frac{h_{M2}-h_{M1}}{\delta_1}\right)(1-.355K+.202K^2-.137K^3)} \quad . \quad (1213)$$

lculation of velocity by this Eq. (1213) and by the equation neglecting den- y changes, gives comparative results plotted in Fig. 302, reproduced from eeland. This shows that at velocities of 23,000 ft. per minute, the difference 5 per cent, at 11,000 ft. per minute, about 6 per cent, which are appreciable, that in using Pitot tubes for gases and vapors high velocities should be used il the formula Eq. (1213) will give the velocity for the observed pressures.

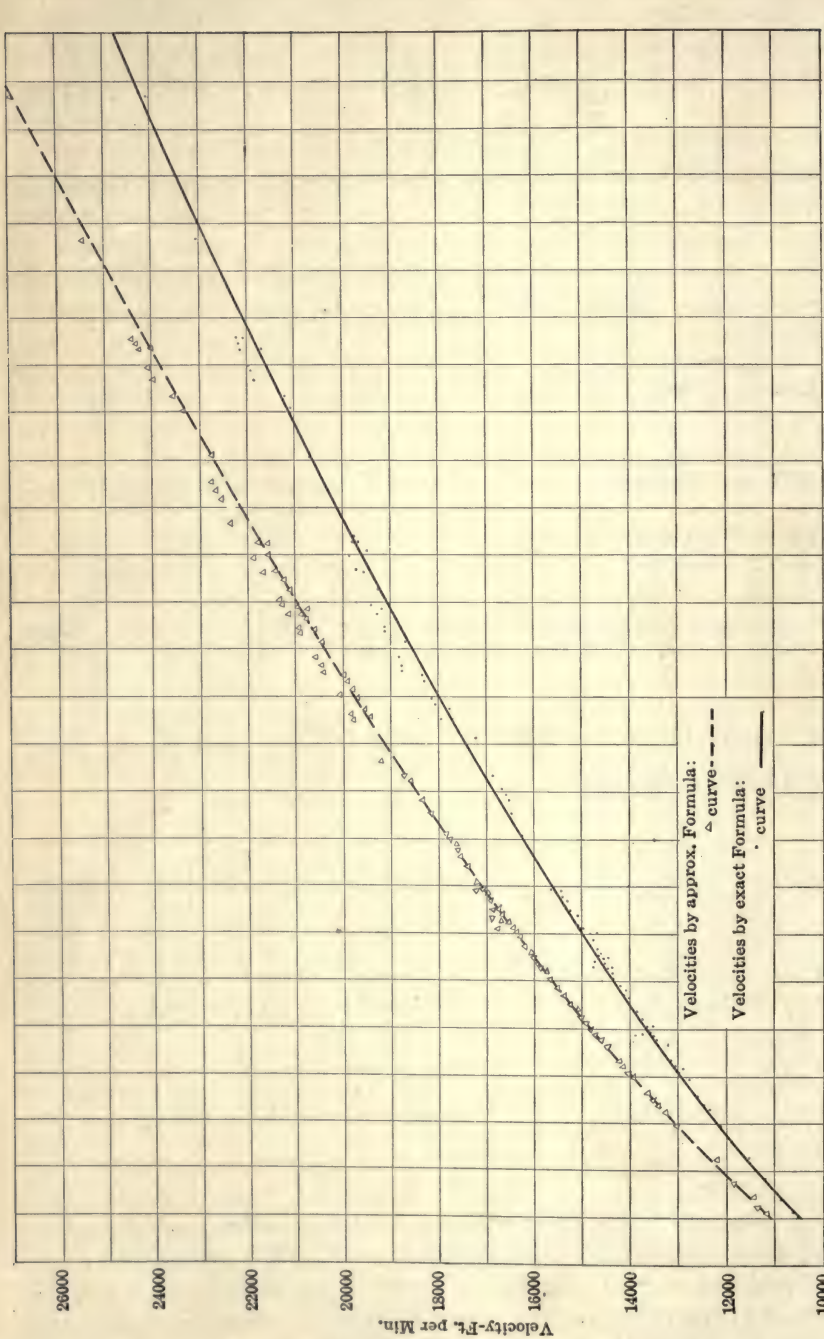


FIG. 302.—Velocity of Air in Pipes in Terms of Pitot Tube Readings, Inches of Mercury, by Approximate and Exact Formulas.

Prob. 1. In a test on air flow sharp-edged orifices of sheet iron were used, to which Durley's coefficients apply. For the following data what was the volume and weight of air flowing? Air temperature = 70° F.

Orifice dia. in.	Head in inches of water.
1	1.4, 1.95, 2.8, 4.1, and 4.8
2	1.25, 1.7, 2.25, " 4.8
3	1.60, 1.95, 2.65, " 5.0

Prob. 2. What size of orifice would pass 200 cu.ft. of air with 5-inch water pressure drop, if (*a*) the coefficient were 100 per cent, (*b*) if Durley's coefficient were used.

Prob. 3. Air flows from a tank into the atmosphere at the rate of 75 cu.ft. per minute. The pressure in the tank is 25 lbs. per square inch absolute, and temperature 60° F. According to Weisbach how big should the hole be?

Prob. 4. A gas burner is supplied at 2-inch water pressure and passes 5 cu.ft. of gas per hour, the density being 60 per cent that of air. What area of burner is needed? Check this by comparing with a standard gas tip.

Prob. 5. A Venturi in an air pipe showed an upstream pressure of 17 lbs. per square inch absolute and a throat pressure $2\frac{1}{2}$ ins. Hg less. The throat diameter is 6 ins., pipe diameter 20 ins. At 60° F., what quantity was flowing?

Prob. 6. If the pipe diameter of Problem 5 had been 12 ins., what would be the difference in pressure between throat and upstream in inches of water when the upstream velocity is 6000 ft. per minute?

Prob. 7. Gas is being measured by the Pitot tube. The pressure is 50 lbs. per square inch gage and the tube reading is $2\frac{1}{2}$ ins. Hg. If the gas density is .8 that of air, pipe 4 ins. diameter, temperature 40° F., what is the flow rate, (*a*) by exact and, (*b*) by approximate formula?

Prob. 8. A steam Venturi tube gave the following readings:

	Dia.	Pressure, lbs. Sq.in. Gage	Quality.
Throat	2 ins.	90	100%
Upstream	4 ins.	100	95%

What weight of steam was flowing and what was the upstream velocity?

18. Flow of Gases and Vapors in Pipes, Flues, Ducts, and Chimneys. Relation between Quantity of Flow and Loss of Pressure. Friction Resistances. Draught and Capacity of Chimneys. Pipe or conduit flow of gases and vapors, is characterized by comparatively small losses in pressure compared to the mean pressure, that is by high values of the ratio of initial to final pressure, also by complete conversion into heat of all the kinetic energy lost by friction, and by appreciable gains or losses of heat by wall conduction and surface radiation, gains when the fluid temperature is lower than the surroundings as it is with refrigerating media, and losses otherwise as for steam. All relations between the rates of flow and the change of fluid conditions corresponding to it can be numerically evaluated, only when friction energy and heat conduction can be evaluated, and these quantities depend so much on structural details as to defy algebraic analysis. Accordingly, while some thermal relations for pipe flow may be set down, the really useful equations for engineering work

are empiric, and will probably always so remain except as to form, which may change as more knowledge becomes available on gas and vapor friction, and on the transmission of heat to or from fluids through pipes and insulating coverings.

Reduction of pressure with no conduction losses in a pipe line would result in the doing of work and the development of kinetic energy equal to that of the Rankine cycle for steam and its equivalent for gases. As, however, neglecting the change of density corresponding to the pressure drop, the velocity is constant in a uniform pipe, the above gain in kinetic energy is the frictional energy, which being converted into heat characterizes the process as one of constant total heat if in a non-conducting container. The velocity attained should, therefore, be that due to attaining the work of a cycle bounded by a constant total heat line between two constant pressure lines, corrected for actual thermal loss by conduction. Equations might be set down giving the velocity or flow per second in terms of initial and final pressures and conduction loss, but this would be of no assistance whatever, in fixing those pressures in terms of pipe dimensions, which is what is needed. This desired relation between pressures lost in a pipe of given dimensions and the corresponding energy was first established on somewhat questionable empiric grounds by Unwin, who, reasoning from experimentally established laws for water, put the loss of pressure in terms of head of fluid as proportional to, (*a*) the velocity head, $H=\left(\frac{u^2}{2g}\right)$, (*b*) the length ($L$) directly; and (*c*) inversely as the mean hydraulic radius, $m=\left(\frac{\text{area}}{\text{perimeter}}\right)$. The name, coefficient of friction, is given to the constant of proportionality and it is usually designated by ζ

$$(\text{Loss of fluid head due to friction})=\zeta\left(\frac{u^2}{2g}\right)\frac{L}{m}=\zeta H\frac{L}{m}. \quad . \quad (1214)$$

With this as a basis and on certain reasonably fair assumptions, the relation between flow velocity and pressure drop can be derived. These assumptions are, that the fluid behaves like a perfect gas according to $PV=wRT$, that the work of friction is the product of friction head in feet of fluid and weight of fluid flowing, that negative work is done by the fall of pressure equal to the change of pressure into the volume, and finally, that the change in kinetic energy in passing from high to low pressure is equal to the sum of the pressure change work, and friction work, neglecting all other influences.

$$\left\{\begin{array}{l}\text{Change in kinetic energy in}\\ \text{passing from high to low}\\ \text{pressure regions in pipes}\end{array}\right\}=\left\{\begin{array}{l}\text{Work done by}\\ \text{by change of}\\ \text{pressure}\end{array}\right\}-\left\{\begin{array}{l}\text{Work of}\\ \text{friction}\end{array}\right\} \quad (1215)$$

These quantities are evaluated algebraically as follows:

Let u = velocity, mean, in feet per second;
" P_1 and P_2 = initial and final pressures, pounds per square foot;
" p_1 and p_2 = initial and final pressures, pounds per square inch;
" V = cubic feet per pound fluid at mean pressure;
" u_m = velocity in feet per minute;
" w = pounds fluid per second;
" A = area pipe in square feet;
" m = mean hydraulic depth;
" L = length of pipe in feet;
" D and d = diameter of pipe in feet and inches respectively.

Then for an infinitely small pipe length dL and a correspondingly small velocity and pressure change du and dP

$$\left\{\begin{array}{l}\text{Change in kinetic energy at ends}\\ \text{of pipe length}\end{array}\right\} = \begin{array}{ll}\dfrac{w}{2g}[(u+du)^2-u^2] = \dfrac{w}{g}u\,du & (a)\\ = w\,dH & (b)\end{array} \quad . \quad (1216)$$

because $H=\dfrac{u^2}{2g}$, and $dH=\dfrac{u\,du}{g}$, and the square of a differential may be neglected in comparison with the differential itself.

Also

$$(\text{Work done by change of pressure}) = -wV\,dP = -\frac{wRT}{P}\,dP. \quad . \quad . \quad . \quad (1217)$$

and

$$(\text{Work of friction}) = (\text{Loss of head in feet}) \times (\text{Pounds of fluid})$$

$$= \zeta H \frac{dL}{m}\,w. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1218)$$

Therefore, substituting these values, Eqs. (1216), (1217), and (1218) in Eq. (1215),

$$w\,dH+\frac{wRT}{P}\,dP+\zeta H\frac{dL}{m}\,w=0;$$

$$\frac{dH}{H}+\frac{RT}{HP}\,dP+\zeta\frac{dL}{m}=0. \quad . \quad . \quad . \quad . \quad . \quad (1219)$$

But

$$\frac{RT}{HP}=\frac{RT}{\frac{u^2}{2g}P}=\frac{2gRT}{\left(\frac{wV}{A}\right)^2P}=\frac{2gA^2RT}{w^2V^2P}=\frac{2gA^2RT}{w^2\frac{R^2T^2}{P^2}P}=\frac{2gA^2P}{w^2RT} \quad . \quad . \quad (1220)$$

Substituting Eq. (1220) in Eq. (1219) gives Eq. (1221),

$$\frac{dH}{H}+\frac{2gA^2}{w^2RT}PdP+\zeta\frac{dL}{m}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1221)$$

Integrating between head limits H_1 and H_2, pressure limits P_1 and P_2, and length limits 0 and L,

$$\log_e\frac{H_2}{H_1}+\frac{gA^2}{w^2RT_1}(P_2^2-P_1^2)+\zeta\frac{L}{m}=0.$$

But

$$\frac{H_2}{H_1}=\frac{P_1}{P_2}, \quad \text{and} \quad w=\frac{Au}{V_1}=\frac{AuP_1}{RT_1};$$

hence

$$\frac{gA^2}{w^2RT_1}=\frac{gRT_1}{u^2P_1^2}.$$

Therefore,

$$\log_e\frac{P_1}{P_2}+\frac{gRT_1}{u^2P_1^2}(P_2^2-P_1^2)+\zeta\frac{L}{m}=0. \quad . \quad . \quad . \quad . \quad (1222)$$

This gives for the velocity at the beginning of the L foot section

$$u=\sqrt{\frac{gRT_1(P_1^2-P_2^2)}{P_1^2\left(\zeta\frac{L}{m}+\log_e\frac{P_1}{P_2}\right)}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1223)$$

For pipe lengths of usual size and the small allowable pressure drops, or small $\left(\frac{P_1}{P_2}\right)$, it is usually fair to drop $\left(\log_e\frac{P_1}{P_2}\right)$, in comparison with the quantity to which it is added, and doing this the velocity becomes as in Eq. (1224)

$$u=\sqrt{\frac{gmRT_1(P_1^2-P_2^2)}{\zeta LP_1^2}}=\sqrt{\frac{gm(P_1^2-P_2^2)}{\zeta L\delta P_1}} \text{ feet per second} \quad . \quad (1224)$$

When the pipe is of circular section

$$m=\frac{\text{area}}{\text{perimeter}}=\frac{\pi D^2}{4\times\pi D}=\frac{D}{4}=\frac{d}{12\times4}=\frac{d}{48},$$

$$p=\frac{P}{144}, \text{ and } u_m \text{ (feet per minute)}=60u \text{ (feet per second)};$$

$$u_m=(\text{Const.})\times\sqrt{\frac{dRT(p_1^2-p_2^2)}{\zeta Lp_1^2}}=(\text{Const.})\times\sqrt{\frac{d(p_1^2-p_2^2)}{\zeta L\delta p_1}} \text{ ft. per minute} \quad . \quad (1225)$$

The corresponding weight is given by, $w_m = A u_m \delta = \frac{\pi D^2}{4} u_m \delta = .00545 d^2 u_m \delta$.

$$\left.\begin{aligned}\text{(Pounds of fluid per minute)} &= \text{(Const.)} \times \sqrt{\frac{d^5 \delta^2 RT(p_1^2 - p_2^2)}{\zeta L p_1^2}} \quad (a)\\ &= \text{(Const.)} \times \sqrt{\frac{d^5 \delta (p_1^2 - p_2^2)}{\zeta L p_1}} \quad (b)\end{aligned}\right\} \quad (1226)$$

In all ordinary cases of pipe flow the ratio of the absolute pressure at the end of a length to that at the beginning is sensibly equal to 1, at least to greater limits of accuracy than the coefficient of friction can be established or the premises of the above formula can be accepted as sound. When this is so the ratio $\frac{p_1^2 - p_2^2}{p_1}$ becomes

$$\frac{(p_1 - p_2)(p_1 + p_2)}{p_1} = (p_1 - p_2)\left(1 + \frac{p_2}{p_1}\right) = 2(p_1 - p_2), \text{ (approx.)}$$

which on substitution gives the second form of Eq. (1226) the following value Eq. (1227):

$$w_m = \text{(Constant)} \times \sqrt{\frac{d^5 \delta (p_1 - p_2)}{\zeta L}}. \quad \ldots\ldots \quad (1227)$$

The first determination of the value of the coefficient of friction based on experiments of some accuracy is that made by Unwin on data obtained from tests of the compressed-air mains of Paris, all being less than one foot in diameter and his value is given by Eq. (1228),

$$\zeta = .003\left(1 + \frac{3.6}{d}\right). \quad \ldots\ldots \quad (1228)$$

This indicates that the velocities vary as the square root of diameter divided by the above fraction of diameter, whereas for liquids, velocities by some formulas are given as functions of the square root of diameter, and by others generally believed to be better, as increasing with diameter faster than the square root. No better values appearing for many years, engineers have adopted and used the above results with practical success, though it must be confessed without scientific accuracy, and experience has led to various values of the constants, the most used of which is the Babcock value in Eq. (1229). Carefu

$$w = 87 \sqrt{\frac{d^5 \delta (p_1 - p_2)}{L\left(1 + \frac{3.6}{d}\right)}}. \quad \ldots\ldots \quad (1229)$$

analysis of existing information by Martin in 1903 led to the adoption by him of a different constant $\sqrt{7000}$ giving the following series of formulas, all equivalent, but put in convenient form for solution of any desired quantity.

$$w=\sqrt{\frac{7000d^5\delta(p_1-p_2)}{L\left(1+\frac{3.6}{d}\right)}}=V\delta=\frac{u\delta d^2}{3.056}; \quad \ldots \quad (1230)$$

$$V=\frac{w}{\delta}=\sqrt{\frac{7000d^5(p_1-p_2)}{L\delta\left(1+\frac{3.6}{d}\right)}}=\frac{ud^2}{3.056}; \quad \ldots \quad (1231)$$

$$u=\frac{3.056w}{\delta d^2}=\frac{3.056V}{d^2}=256\sqrt{\frac{(p_1-p_2)d}{\delta L\left(1+\frac{3.6}{d}\right)}}=\sqrt{\frac{454dH_G}{L\left(1+\frac{3.6}{d}\right)}}; \quad \ldots \quad (1232)$$

$$(p_2-p_1)=\frac{w^2L\left(1+\frac{3.6}{d}\right)}{7000\delta d^5}=\frac{V^2\delta L\left(1+\frac{3.6}{d}\right)}{7000d^5}=\frac{u^2\delta L\left(1+\frac{3.6}{d}\right)}{65360d}; \quad \ldots \quad (1233)$$

$$H_G=\frac{w^2L\left(1+\frac{3.6}{d}\right)}{49\delta^2d^5}=\frac{V^2L\left(1+\frac{3.6}{d}\right)}{49d^5}=\frac{u^2L\left(1+\frac{3.6}{d}\right)}{454d}=\frac{144(p_2-p_1)}{\delta}; \quad \ldots \quad (1234)$$

$$d=\sqrt{\frac{.0002w^2L}{(p_2-p_1)\delta}}=\sqrt{\frac{.0002V^2\delta L}{(p_2-p_1)}} \text{ (approx.)}; \quad \ldots \quad (1235)$$

$$L=\frac{7000(p_2-p_1)\delta d^5}{\left(1+\frac{3.6}{d}\right)w^2}=\frac{7000(p_2-p_1)d^5}{\left(1+\frac{3.6}{d}\right)V^2\delta}; \quad \ldots \quad (1236)$$

where (p_2-p_1) = loss of pressure in pounds per square inch;
" H_G = loss of pressure in feet of fluid;
" V = cubic feet of fluid per minute;
" u = velocity in feet per second;
" δ = density of fluid in pounds per cubic foot;
" w = weight of fluid flow pounds per minute;
" d = diameter of pipe in inches;
" L = length of pipe in feet;
" $27.7(p_2-p_1)$ = loss of pressure in inches of water;
" $2.04(p_2-p_1)$ = loss of pressure in inches mercury.

These formulas are applicable to all fluids, steam, both saturated and superheated, compressed air, ammonia, carbon dioxide and to the combustible gases, but not with equal degrees of accuracy over the whole range of kinds of fluid, rates of flow and density. All sorts of special tests are available, giving data to check the formula for one case under test and a review of these tests would be of value here, did it lead to any suggestion for a better formula, but no such suggestion is available, so cases of individual departure from the best general law ever proposed will be omitted with the warning that engineers working with some one fluid over a definite range of conditions of service determine for themselves the correct numerical constant to be used in the general equations to meet those conditions.

One general investigation for steam recently conducted by Eberle under the auspices of the German Society of Engineers has led to an important and useful conclusion, inasmuch as it established that the coefficient of friction is independent of pipe diameter, steam pressures, quality and velocity for very considerable ranges. The velocities varied from 2000 to 15,000 ft. per minute, pressures from three to ten atmospheres, with steam saturated as well as superheated to 180° F., and in pipes from 3 ins. to 6 ins. diameter approximately, while the coefficients varied some 20 per cent. Gutermuth had used a constant coefficient of friction, which is 1.43 times mean of Eberle's, and while the latter's value is most accurate for high pressures, the Gutermuth value is recommended by Rietschel for low pressures such as are found in steam-heating systems and steam exhaust pipes. This simplified formula is given by Eq. (1237),

$$(p_2-p_1)=2.667\times10^{-5}\times\frac{u^2L}{d}\delta \text{ (Eberle)} \quad . \quad . \quad . \quad . \quad (1237)$$

A number of these pipe flow formulas have been analyzed and reduced to common terms by Gebhardt and the constant used in the following expressions are those found by him set together to show the variations:

(Velocity feet per minute)

$$\left.\begin{aligned} &=A\sqrt{\frac{(p_2-p_1)d}{\delta L}}, \quad \text{where } A=\begin{cases} 9240 & \text{Geipel \& Kilgour} & (a)\\ 9976 & \text{Hawksley} & (b)\\ 10350 & \text{Martin} & (c)\\ 10360 & \text{Hurst} & (d)\end{cases}\\ &=B\sqrt{\frac{(p_2-p_1)d}{\delta L\left(1+\frac{3.6}{d}\right)}}, \quad \text{where } B=\begin{cases} 15950 & \text{Babcock} & (e)\\ 16050 & \text{Unwin} & (f)\end{cases}\\ &=442\sqrt{\frac{(p_1^{1.94}-p_2^{1.94})d}{\delta^2 L}} \qquad\qquad \text{Ledoux} \quad (g)\end{aligned}\right\} \quad . \quad . \quad (1238)$$

(Flow in pounds per minute)

$$= C\sqrt{\frac{(p_2-p_1)\delta d^5}{L}}, \quad \text{where } C = \begin{cases} 50.2 \text{ Geipel \& Kilgour} & (a) \\ 54.4 \text{ Hawksley} & (b) \\ 56.5 \text{ Martin} & (c) \\ 56.5 \text{ Hurst} & (d) \end{cases}$$

$$= D\sqrt{\frac{(p_2-p_1)\delta d^5}{L\left(1+\frac{3.6}{d}\right)}}, \quad \text{where } D = \begin{cases} 87 \text{ Babcock} & (e) \\ 87.5 \text{ Unwin} & (f) \end{cases} \qquad \text{. . (1239)}$$

$$= 2.44\sqrt{\frac{(p_1^{1.\,4}-p_2^{1.94})d^5}{L}} \qquad \text{Ledoux} \quad (g)$$

(Pressure drop pounds per square inch)

$$= (p_2-p_1) = E\frac{w^2L}{\delta d^5}, \quad \text{where } E = \begin{cases} .0003960 \text{ Geipel \& Kilgour} & (a) \\ .0003370 \text{ Hawksley} & (b) \\ .0003133 \text{ Martin} & (c) \\ .0003126 \text{ Hurst} & (d) \end{cases}$$

$$= F\frac{w^2L\left(1+\frac{3.6}{d}\right)}{\delta d^5}, \quad \text{where } F = \begin{cases} .0001321 \text{ Babcock} & (e) \\ .0010306 \text{ Unwin} & (f) \end{cases} \qquad \text{. . (1240)}$$

$$(p_1^{1.94} - p_2^{1.94}) = .1669\,\frac{w^2L}{d^5} \qquad \text{Ledoux} \quad (g)$$

(Pipe diameter, inches)

$$= d = G\sqrt{\frac{w^2L}{(p_2-p_1)\delta}}, \text{ where } G = \begin{cases} .2087 & \text{Geipel \& Kilgour} & (a) \\ .2010 & \text{Hawksley} & (b) \\ .1990 & \text{Martin} & (c) \\ .1990 & \text{Hurst} & (d) \end{cases} \qquad \text{. . (1241)}$$

$$= .699\sqrt{\frac{w^2L}{(p_1^{1.94}-p_2^{1.94})}} \qquad \text{Ledoux} \quad (e)$$

Actual pipe lines have bends, branches and valves, together with enlargements and reductions of cross-section, all of which introduce resistances as well as the skin friction of the pipe and which are very much more difficult to evaluate. The general method of procedure is to give the resistance in terms of the equivalent length of straight pipe; thus, Hunt gives the following for 90° elbows.

Diameter of pipe.....	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20	22	24
Elbow resistance in feet of straight pipe.....	1.5	4.9	9.4	14.5	20	25.9	32	38	44.4	50.7	63.7	76.7	90.1	104	117	130	144

These are of little practical value, as the proportions of elbows differ in every make of fittings, and are given here to indicate the appreciable quantities involved and the necessity for accurate tables of data in commercial work relating to some one line of fittings to be used.

When low-pressure gases like air or furnace products flow in ducts, flues, and chimneys, there arises a special case of flow conditions characterized by a possibility and often a probability that the whole skin frictional resistance due to straight pipe may be entirely overbalanced by the aggregate of the special resistances due to bends, branches and changes of cross-sectional areas. There are two typical cases of such low-pressure gas flow, first, that in which the flow is all horizontal or if vertical the whole system at constant temperature, and this is a typical duct-flow condition, second, that in which there is a vertical flow not necessarily straight upward and in which the gases are hotter than other gases communicating with them, and this is a typical chimney-flow condition. Each case must be treated by itself in order that workable equations and constants may be developed and the former will be taken up first.

Usually direct flow takes place from, or to, the free atmosphere, or a place where at least the pressure is that of the atmosphere whether the temperature is the same as it or not, and the problem always takes the form of fixing the excess pressure over atmosphere that will deliver a required quantity of air or gases through ducts of a given size, or inversely the size for a given capacity, when the whole resistance is that due to the system of passages. The whole resistance is, therefore, to be evaluated and this consists of various frictional resistances due to straight duct walls, to changes of direction of flow, to changes of cross-section of duct, to inlet and to outlet orifice contractions. To these resistances in terms of feet of head of gas, or pounds per square foot, ounces per square foot, or inches of water, there is to be added the velocity head to get the total equivalent head or pressure that must be applied to dead air, to establish flow at the required velocity against the imposed resistances.

Let $H_G = \frac{u^2}{2g}$ = head of gas equivalent to velocity of flow;

" $P_V = \frac{u^2 \delta}{2g}$ = pressure pounds per square foot equivalent to velocity head;

" δ = density of gas in pounds per cubic foot;

" P_F = pounds per square foot static pressure lost in wall friction.

" P_R = pounds per square foot static pressure lost in changes of flow direction, cross-section, inlets and outlets.

" P = whole static pressure equivalent to velocity and all resistances;

" F_F = friction factor, which, applied to velocity head gives the frictional loss of pressure;

" F_R = resistance factor, which applied to velocity head gives the loss of pressure due to resistances;

" ζ = coefficient of friction for straight walls;

" C = circumference or perimeter of duct in feet;

" A = area of duct in square feet;

" L = length of duct in feet.

Then

$$P = \left\{ \begin{array}{l} \text{Velocity pressure} \quad P_V \\ +\text{Frictional pressure} \; P_F \\ +\text{Resistance pressure} \; P_R \end{array} \right\} \text{ in pounds per square foot.} \quad . \; . \; . \quad (1242)$$

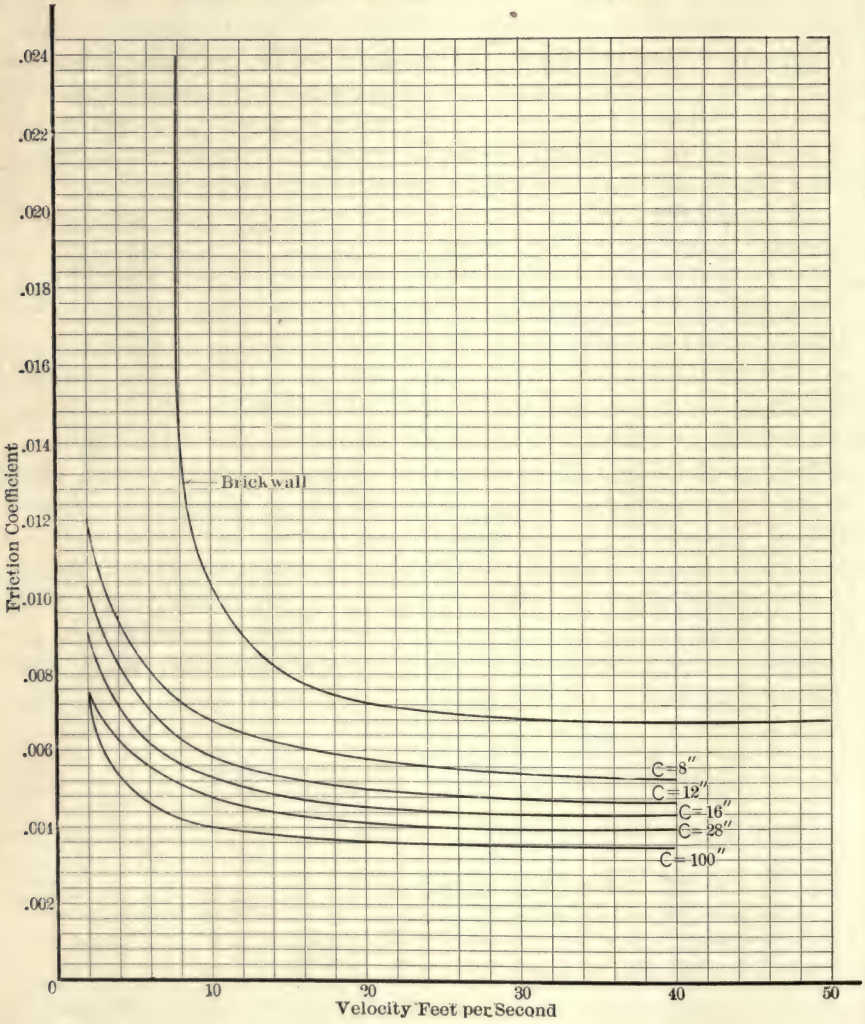


FIG. 303.—Coefficients of Friction ζ for Air in Ducts.

$$P = \frac{u^2 \delta}{2g}(1 + F_F + F_R). \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1243)$$

But $$F_F = \zeta L \frac{C}{A}.$$

Hence

$$P=\frac{u^2\delta}{2g}\left(1+\zeta L\frac{C}{A}+F_R\right)=\left\{\begin{array}{l}\text{Velocity pressure } \dfrac{u^2\delta}{2g}\\ +\text{Maintained resistance } \dfrac{u^2\delta}{2g}\left(\zeta L\dfrac{C}{A}+F_R\right)\end{array}\right\} \quad (1244)$$

This can easily be evaluated numerically, when there are available experimental values of the coefficient of friction for straight ducts, and of the flow change resistance F_R, but as it seldom if ever happens that a duct is uniform, and as these values depend on the dimensions, there must be a summation of the pressures lost at each point of change and in each straight section. Hence,

$$\text{Maintained resistance}=\frac{\delta}{2g}\left[\Sigma\left(u^2\zeta\frac{C}{A}\right)+\Sigma u^2F_R\right]. \quad . \quad . \quad (1245)$$

Values of the coefficient of friction are given graphically, Fig. 303, for straight ducts of brick or iron for velocities up to 50 ft. per second, and for iron ducts different values are given for perimeters or circumferences from 8 to 100 ins. These values as well as the flow-change factors in the following Table CXLIII are those recommended by Ohmes from Rietschel for ventilating ducts, and are intended for the usual velocities therein attained, 6 to 24 ft. per second, when served by fans, and 3 to 8 ft. per second when the flow is due to natural or convection draft.

TABLE CXLIII

FLOW CHANGE RESISTANCE FACTORS F_R (REITSCHEL)

Condition.	Resistance Factor F_R
Sharp 90° elbow	1.1
" 135° elbow	.3
Long bend: r = width of duct	.25
" r = 2 to 4 duct widths	.15
" r = 5 to 6 duct widths	.07
Long bend 135°	.15
Long double offset	.4 to .1
Outlet register with valves $\frac{2}{3}$ free area and 2 × flue area	.6
" " face at $\frac{2}{3}$ free area	.4
" wire screens 1.5 × flue area	0.0
Entrance for square corners	1.0
" rounded corners	.5 to .2
" flue extending into header as short pipe	1.5
Enlargement of area from A_1 to A_2, sharp corners	$\left(\frac{A_2}{A_1}-1\right)^2$
Reduction of area from A_2 to A_1, sharp corners	$\left(1+\frac{A_1}{A_2}\right)^2$
Free discharge into room when velocity becomes zero	1.0

These pressures that are lost, due to the resistances, are convertible from pounds per square foot to inches of water by the usual factor, but in this branch of engineering practice they are usually given in ounces per square foot and this value follows directly from the formulas, if the density is used not in terms of pounds per cubic foot but in ounces per cubic foot. Of course, there may be other resistances in the flow path such as filter screens or pipe coils serving to heat or cool the air, and any such resistance is to be separately evaluated and added, but as such things are special details they are omitted here and if needed are to be looked up in tables of data prepared for engineers practicing this specialty.

When the flow of gases in the system depends on convection, which follows the heating of a rising column, which is replaced by a falling cold column, the flow characteristics are dependent on the temperatures and vertical heights involved and the most important practical case of such flow is that in chimneys, though in some ventilating systems a corresponding case is found.

Let H = vertical distance in feet equal to the difference in level between inlet and outlet of the heated column of gases;

" P = difference in pressure at base of hot column between it and the atmosphere, to be called the draft and here given in pounds per square foot;

" h_W = draft in inches of water;

" H_G = equivalent head of hot gas;

" T_H = temperature of hot column assumed constant throughout;

" T_C = temperature of outside air;

" .084 = density of products of combustion at 32° F. and 29.92 ins. Hg;

" δ_H = density of hot gases, pounds per cubic foot $= \frac{492 \times .084}{T_H} = \frac{41.43}{T_H}$ for products of combustion, or $\frac{398}{T_C}$ for outside air;

" .0807 = density of pure dry air at 32° F. and 29.92 ins. Hg;

" δ_C = density of pure cold air at 32° F. and 29.92 ins. Hg $= \frac{492 \times .0807}{T_C} = \frac{39.8}{T_C}$;

" C = circumference or perimeter of stack in feet;

" A area of stack in square feet;

" w = weight of gases per second.

Then

$$\left\{\begin{matrix}\text{Weight per cubic foot of} \\ \text{hot or cold gases}\end{matrix}\right\} \times H = \left\{\begin{matrix}\text{Pressure at base of height } H \\ \text{in pounds per square foot}\end{matrix}\right\} . \quad (1246)$$

Hence for pure air in the chimney the difference in pressures at the base is given by Eq. (1247),

$$P = H(\delta_C - \delta_H) = 39.8H\left(\frac{1}{T_C} - \frac{1}{T_H}\right). \quad . \quad . \quad . \quad . \quad . \quad (1247)$$

For products of combustion of assumed density in the chimney the corresponding pressure difference is Eq. (1248),

$$P = H\left(\frac{39.8}{T_C} - \frac{41.43}{T_H}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1248)$$

The equivalent head of hot gases is given by Eq. (1249):

$$H_G = \frac{P}{\delta_H} = \frac{P}{\frac{41.43}{T_H}} = H\left[\frac{\frac{39.8}{T_C} - \frac{41.43}{T_H}}{\frac{41.43}{T_H}} = H\right]\left(.961\frac{T_H}{T_C} - 1\right). \quad . \quad . \quad (1249)$$

This is in turn equivalent to a draft in inches of water, the usual unit of measurement, given by Eq. (1250),

$$\left.\begin{aligned} h_W = \frac{P}{5.21} &= H\left(\frac{7.64}{T_C} - \frac{7.95}{T_H}\right) \text{ for chimney gases;} \quad (a) \\ &= 7.64H\left(\frac{1}{T_C} - \frac{1}{T_H}\right) \text{ for air.} \quad (b) \end{aligned}\right\} . \quad . \quad (1250)$$

Solving for a chimney temperature, $t_H = 600°$ F., or $T_H = 1060°$ F., this gives $\frac{3}{4}$ in. of draft per 100 ft. of height. However, these are the full static pressures or their equivalents and after flow is established some pressure will be changed to velocity, and some more used up in overcoming the maintained resistances in coal beds, boiler tube banks, and economizers in furnace-chimney flow, which correspond to tempering coils and filters in heating and ventilating duct flow. Calling these special resistances F_S the general relation becomes that of Eq. (1251),

$$P = \frac{\delta_H}{2g}\left[u^2 + \Sigma\left(u^2\zeta\,\frac{C}{A}\right) + \Sigma(u^2F_R) + \Sigma(u^2F_S)\right]. \quad . \quad . \quad . \quad . \quad (1251)$$

All these resistances due to flue-wall friction, entrances, outlets, bends or changes of cross-section can be evaluated as in ventilating-duct work, but fuel-bed,

conomizer, and boiler-tube resistances cannot be so evaluated, because in hem resistance conditions are constantly changing, as is also the velocity and emperature of gases due to changes of boiler load. It is, therefore, necessary dealing with furnace chimney draft to proceed even more empirically than r air ducts, and the usual method is to apply a general factor inclusive of all terferences and resistances and yielding a numerical result near enough to the uth to be of practical value. Thus, Stirling suggests that as an overall cor-ction factor is to be applied there is nothing to be gained in assuming the ases to be different in composition from air, so that the static maintained raft after flow is established, as measured in the base of the stack will be that r no flow, less something to account for velocity and friction resistances all gether, derived somewhat as follows:

The flue friction loss in head is given by $\frac{u^2\delta}{2g}\zeta\frac{C}{A}$ for ducts, but $u\delta=\left(\frac{w}{A}\right)$, nd for a given weight or volume per second flowing, the velocity is inversely roportional to the area, so that $u^2\delta=(\text{Constant})\times\frac{w}{A^2}$. Therefore, the draft loss ould be

$$(\text{Constant})\times\frac{\zeta C}{2g}\frac{w}{A^3}=f\frac{wC}{A^3},$$

here f is given the empiric values of Eq. (1252) and the maintained draft is ven by Eq. (1253),

$$f=\left\{\begin{array}{l}.0015 \text{ for steel stacks, gas temperature}=600^\circ \text{ F. or } T_H=1060\\ .0011 \text{ for steel stacks, gas temperature}=350^\circ \text{ F. or } T_H=\ 810\\ .0020 \text{ for brick stacks, gas temperature}=500^\circ \text{ F. or } T_H=\ 960\end{array}\right\}. \quad (1252)$$

$$h_W=7.64H\left[\left(\frac{1}{T_C}-\frac{1}{T_H}\right)\right]-fw\frac{C}{A^3}. \quad . \quad . \quad . \quad . \quad . \quad (1253)$$

s a matter of fact the actual maintained draft, while it is a prime factor in he rate of combustion of coal, is not of such great importnace in the problem flow through chimneys as the capacity of chimneys. This capacity is pri-arily given in terms of weight of gases per hour, but for practical application it common to express it in terms of fuel-burning capacity, or boiler horse-power quivalent to the weight of gases per hour, by assuming values for the con-ersion factors.

Let G = weight of gases per hour;
" G_m = maximum weight of gases per hour;
" K = proportionality coefficient.

Then

$$G=3600w=3600Au\delta_H=3600AK\sqrt{2gH_G\delta_H}$$

$$=3600AK\sqrt{2gH\left(.961\frac{T_H}{T_C}-1\right)}\times\frac{41.43}{T_H}$$

$$=149148K\frac{A}{T_H}\sqrt{2gH\left(.961\frac{T_H}{T_C}-1\right)}. \quad . \quad . \quad . \quad . \quad . \quad (1254)$$

It is clear that as T_H becomes greater the draft increases and the velocity corresponding to it, but the density becomes less, so that there would appear to be some temperature at which the weight would be a maximum, and this is to be found by differentiation. Accordingly, put the above in the form

$$G=149148KA\sqrt{2gH\left(.961\frac{1}{T_CT_H}-\frac{1}{T_H{}^2}\right)}$$

and making $\frac{dG}{dT_H}=0$, gives

$$-.961\frac{1}{T_H{}^2T_C}+\frac{2}{T_H{}^3}=0,$$

or

$$T_H=\frac{2T_C}{.961}=2.081T_C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1255)$$

This shows that the capacity will be a maximum when the absolute temperature of the hot column is, in round numbers, twice that of the cold surrounding air. The capacity at this time is found by substituting this internal temperature in Eq. (1254), which gives

$$G_m=149148K\frac{A}{T_H}\sqrt{2gH\ (2-1)}$$

$$=1196250K\frac{A}{T_H}\sqrt{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1256)$$

When the problem is that of stack design, it is this maximum capacity that is the important quantity, but it must be remembered that unless the gas temperature is right, the stack will work at a lesser capacity and the actual or probable capacity of a stack at a temperature other than that giving maximum capacity is a determination quite frequently necessary. In stack designing the

gas temperatures are never known, though they may be approximated, but instead of making approximations for each quantity separately as involved in the stack capacity, this is set down most commonly in terms of coal-burning capacity or boiler horse-power, as formulas in the derivation of which general assumptions are made. Some of these common assumptions are

(*a*) The stack will operate at maximum capacity;
(*b*) The air temperature is 50°F., or $T_C = 510$, and $T_H = 2.08 \times 510 = 1061$, or $t_H = 601°$ F.
(*c*) Twenty-four pounds of air will be required per pound of coal yielding 25 lbs. of gases;
(*d*) Five pounds of coal per hour will be equivalent to one boiler horse-power.

The justice of these assumptions need not be discussed as they are purely arbitrary, and the user of these equations may modify them at will, starting from the fundamental relations, but it is on such assumptions that most of the existing stacks have been designed and the coefficient K determined to apply to them. Introducing the assumptions it follows that

$$\left(\begin{matrix}\text{Max. lbs. coal per hr.} \\ \text{for the stack}\end{matrix}\right) = \frac{G_m}{25} = \frac{1196250}{25 \times 2.08 \times 510} KA\sqrt{H} = 45KA\sqrt{H}. \quad . \quad (1257)$$

$$\left(\begin{matrix}\text{Boiler horse-power} \\ \text{for the stack}\end{matrix}\right) = \frac{1}{5} \times \left(\frac{G_m}{25}\right) = 9KA\sqrt{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1258)$$

The value of the coefficient K, which here may be regarded as a general correction factor, though it is often called a coefficient of discharge, is most commonly taken as, $K = .3$, but Kent has used another value combined with the assumption that next the walls there will be a dead layer of gas so that the effective area (KA) will be the actual area less the cross-section of this dead gas. This layer is arbitrarily taken as being 2 ins. thick, so that it will subtract about, $\frac{2}{12}\pi D$, and, $\frac{2}{12} \times 4D$ sq.ft., from the area A for round and square stacks respectively, leaving the effective area as, $(A - .59\sqrt{A})$ for round, $(A - .666\sqrt{A})$ for square, or approximately, $(A - .6\sqrt{A})$ for both. This substituted in Eq. (1258) yields the Kent formula Eq. (1259),

$$\text{(Boiler horse-power for the stack)} = .333(A - .6\sqrt{A})\sqrt{H} \quad . \quad . \quad (1259)$$

A great many empiric chimney formulas are available, all derivable from the same fundamental expressions here developed but with different values for the constants, and as they are all empiric beyond the first steps they are omitted, and

reference is made to an excellent comparison in Gebhardt's Steam Power Plant Engineering. Chimneys intended to carry off the products of combustion of gas and oil fires must be proportioned on the same principles except one, as control those for coal fires, and that is due to the general absence of anything in the former commensurate with the fuel bed resistance in the latter, and which is there the main resistance. The general procedure with coal fires is to first fix the height that will give the draft needed to burn the coal at the desired rate on the grates, and then to select an area sufficient to carry off the gases produced. In the latter case the draft is of minor or zero consequence so that a shorter, wider stack may be just as serviceable for a gas or oil fire as a narrow high one of equal capacity, whereas with coal fires the height is needed for, and controlled by the bed resistance or rate of combustion.

Prob. 1. Simultaneous readings showed the pressure at the ends of a 6-inch steam pipe line, 650 ft. long, to be 120 lbs. gage and 105 lbs. gage respectively. What was the flow as found by Babcock formula? By the Eberle?

Prob. 2. It is desired to transmit 1000 cu.ft. of compressed air per minute a distance of 2 miles and with pressure drop not to exceed 20 lbs. If the air is initially under a pressure of 100 lbs. per square inch gage, what size pipe will be needed?

Prob. 3. Natural gas is being transmitted at a pressure of 150 lbs. per square inch gage through a 10-in. pipe. What will be the amount which may be transmitted with a loss of 5 lbs. per sq. inch per mile. What size pipe would be required to carry 1000 cu.ft. (std.) per minute with this drop?

Prob. 4. A 4-in. pipe 500 ft. long is carrying dry saturated steam at a pressure of 100 lbs. per square inch gage initially and with a drop of 5 lbs. Check the weight flowing, by the formula of each of the authorities.

Prob. 5. For the same pipe find the pressure drop should the quantity flowing be doubled.

Prob. 6. A furnace burning 1000 lbs. of coal per hour has a stack 4 ft. in diameter and 150 ft. high. If pounds of air per pound of coal is 25, outside temperature 80° F., and stack temperature 400° F., what will be the draft?

Prob. 7. What will be the maximum weight of coal which can be burned with this stack for the given amount of air per pound of coal?

Prob. 8. A stack in Glasgow has a height of 435 ft., a diameter of 13 ft. 6 ins. and is stated to have a capacity of 10,000 H.P. or 50,000 lbs. of coal per hour. Check these figures.

19. Thermal Efficiency of Compressed Air Engines Alone and in Combination with Air Compressors. Effect of Preheating and Reheating. Compressor Suction Heating and Volumetric Efficiency. Wall Action. For certain special service conditions compressed air supplied to piston engines is the most desirable method of developing the necessary power, and for this reason a brief review of the thermal characteristics not only of the engines but of the whole system including the compressors is of value. It is, however, important for another reason, and that is the possibility of securing ever so much better results than have yet been obtained with air engines, which would have the effect of bringing the system into wider use than at present, when it

is a sort of last resort. In mines, however, where exhaust steam or gas-engine exhaust gases would foul the air that must be breathed by the workers, or even set fire to combustible gases in coal workings, compressed-air engines are the only available sources of power except electric motors, and are now used as haulage locomotives, hoists, pumps, drills and cutters. Practically all excavation work must be conducted under conditions that are served by compressed-air power and the use of very small units like hammers, riveters, drills and clippers is almost universal in shops. Probably nothing has called attention to the possibilities of wider large scale use of compressed air, than the recent installation of a whole system of compressed-air deep-mine hoists in the Butte, Mont., mines. This is especially remarkable in view of the fact that the compressors are operated electrically from transmitted water power, and it was found better to use electricity to operate compressors discharging into storage reservoirs from which the air passed through heaters on the way to hoisting engines, than to operate the hoists electrically. One factor in this conclusion is the special construction of the engine which as the cage descends operates as a compressor storing air by the energy of the falling hoist, and acting as a brake at the same time. Another similar development has recently been accomplished in another direction by the perfection of the Porter compound compressed-air locomotive for use in mines, lumbering operations, or other places where fire is much feared and in hauling cars about the yards of large manufacturing plants where cleanliness as well as avoidance of fire risk is important, and where the service is so intermittent that a steam locomotive might burn more coal to keep up pressure standing, than in hauling loads.

It seems a very simple operation at first glance to admit to a cylinder of ordinary steam engine construction, a charge of compressed air instead of steam, but two important difficulties or objections appear, peculiar to the substitution of the air for steam. In the first place unless there is considerable expansion the economy will be very poor and the work obtained only a small fraction of that expended in compressing the air, and second, as a consequence of the expansion the air will become very cold, even a moderate amount of expansion lowering its temperature much below the freezing-point and filling cylinder and ports with ice from the moisture of the air unless it be dried previously. If, however, the air be heated either by internal combustion, as it can well support a fire itself, or by external heat application as it passes through chambers or tubes heated by steam or the hot products of coal, oil or gas fires, it will increase in volume thereby and one cubic foot of cool compressed air will become two or three cubic feet of hot compressed air, if its absolute temperature be doubled or trebled, and so capable of doing two or three times the work of the cool air. By this means, more power can be obtained than is required to run the compressor, and in fact if compressor and engine be coupled together there will be possible a sufficient net output of power to warrant calling the combined unit a form of gas engine. Heat so added to compressed air is termed preheat and the heat of preheat is converted into work in part, whether the compressor and engine work separately at a distance or are coupled together as a single power-

generating unit. It will be shown later that if high pressures are used, the efficiency of transformation of the heat of preheat into work may be very high, in fact the same as a Brayton gas cycle, which in turn is the same as the modern standard Otto gas-engine cycle for equal compressions. This preheat, moreover, removes any difficulties of freezing and in some cases only sufficient heating is used to accomplish this, and it is right here that a good prospect has been neglected.

Reheating the air between the cylinders of compound compressed-air engines in the receiver, is exactly the same in kind and effect as preheating and yields efficiencies that correspond to the pressure at which it is taken up; the higher the pressure the more return will be available for the heat used, but as practiced other factors than thermal efficiency may control the operation. One very remarkable case of this sort is that of the Porter locomotive in which the air is stored at a pressure of about 800 lbs. per square inch in the tank, which gives the tank the large compressed-air weight and low pressure volume capacity, needed for a long haul, and also dries the air. This air then passes through a reducing valve where it develops kinetic energy of expansion, immediately losing it by impact and so not suffering any temperature drop, when losing pressure from 800 to 250 lbs. per square inch. At this latter pressure and at approximately atmospheric temperature, it is admitted to and expanded in the high-pressure cylinder, exhausting to the receiver at a temperature as much below that of the atmosphere as practicable, about 30° below zero F. This permits it to be reheated from *the atmospheric air itself*, which is blasted through the receiver tubes, the working air so warmed, subsequentlyexpanding in the low-pressure cylinder *converts some of the heat of the atmosphere into work.* This process is therefore, peculiarly interesting, inasmuch as it is a means of utilizing for useful work some of the heat of the atmosphere, incidently improving economy in the use of the compressed air about 30 per cent, and avoiding at the same time cylinder freezing or interference with lubrication.

It has been shown that so far as economic performance independent of cylinder capacity is concerned, compressors give the same result with, as without clearance, therefore, in considering the efficiency of combined air engines and air compressors, it will be assumed that the compressor has no clearance and this assumption materially simplifies the work. With respect to the engine, the same would be true if the compression were complete and as it can be made so, this assumption will also be made, and the air engine created as if it had zero clearance. If in any case it has clearance without complete compression it will require more compressed air per unit of work done, which is an avoidable waste, most easily eliminated by making compression complete.

Consider Fig. 304, as representing a zero clearance single-stage adiabatic compressor card $ABCG$ on which the isothermal is dotted, AD. Then, if the air after compression be cooled to its original temperature in the receiver or pipe line, which usually happens, the volume of cool compressed air available for work is $\overline{CD}$. This may be assumed to be admitted to an air engine without

clearance loss and there adiabatically expanded along *DE* completely, or used non-expansively along *DF*. These two cases represent limiting economic performances, best and worst. The efficiency of the whole process is given by

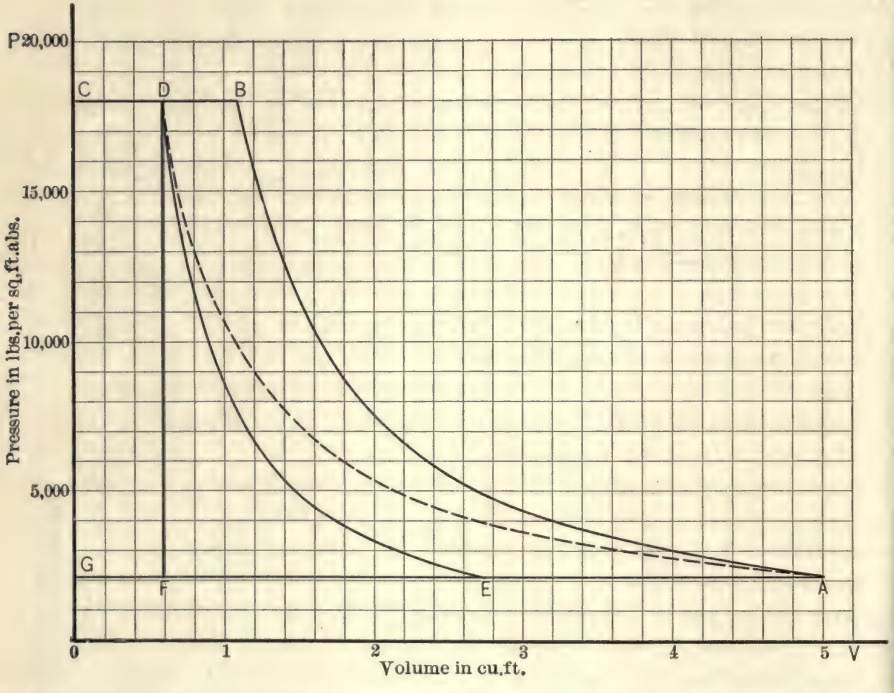


FIG. 304.—Simple Compressed Air Engine, and Single-stage Compressor, Combined Diagram.

the ratio of the engine work area, to the compressor work area, whence, *for single-stage adiabatic compression and simple compressed-air engines,*

$$\text{Efficiency} = \left\{ \begin{array}{ll} E_1 = \dfrac{\text{area } CDFG}{\text{area } CBAG} & \text{for zero engine expansion} \quad (a) \\ E_2 = \dfrac{\text{area } CDEG}{\text{area } CBAG} & \text{for complete engine expansion} \quad (b) \end{array} \right\}. \quad (1260)$$

These areas can be evaluated algebraically.

Let P_b = highest pressure, pounds per square foot;

" P_a = lowest pressure, pounds per square foot;

" $\dfrac{P_b}{P_a} = R_P$ = pressure ratio;

" T_a = atmospheric temperature absolute;

" T_b = maximum delivery temperature of compressor.

Then according to Eq. (48) making $s=\gamma$,

$$\text{Area } CBAG=\frac{\gamma}{\gamma-1}P_aV_a\left(R_P^{\frac{\gamma-1}{\gamma}}-1\right).$$

$$\text{Area } CDFG=(P_b-P_a)V_d=(P_b-P_a)V_a\frac{P_a}{P_b}=V_aP_a(R_P-1)\frac{1}{R_P}$$

$$=P_aV_a\left(\frac{R_P-1}{R_P}\right).$$

$$\text{Area } CDEG=\frac{\gamma}{\gamma-1}P_eV_e\left(R_P^{\frac{\gamma-1}{\gamma}}-1\right).$$

But
$$V_e=V_d\left(\frac{P_d}{P_e}\right)^{\frac{1}{\gamma}}=V_a\frac{P_a}{P_b}\left(\frac{P_b}{P_a}\right)^{\frac{1}{\gamma}}=V_aR_P^{\frac{1}{\gamma}-1}=\frac{V_a}{R_P^{\frac{\gamma-1}{\gamma}}}.$$

Therefore,

$$\text{Area } CDEG=\frac{\gamma}{\gamma-1}P_aV_a\left[\frac{R_P^{\frac{\gamma-1}{\gamma}}-1}{R_P^{\frac{\gamma-1}{\gamma}}}\right].$$

Substituting these algebraic values for the areas in Eq. (1260) the efficiency
lts as in Eq. (1261),

$$\left.\begin{aligned}E_1&=\frac{P_aV_a\left(\frac{R_P-1}{R_P}\right)}{\frac{\gamma}{\gamma-1}P_aV_a\left(R_P^{\frac{\gamma-1}{\gamma}}-1\right)}=\frac{\gamma-1}{\gamma}\left[\frac{R_P-1}{R_P\left(R_P^{\frac{\gamma-1}{\gamma}}-1\right)}\right] &(a)\\ E_2&=\frac{\frac{\gamma}{\gamma-1}P_aV_a\left[\frac{R_P^{\frac{\gamma-1}{\gamma}}-1}{R_P^{\frac{\gamma-1}{\gamma}}}\right]}{\frac{\gamma}{\gamma-1}P_aV_a\left(R_P^{\frac{\gamma-1}{\gamma}}-1\right)}=\frac{1}{R_P^{\frac{\gamma-1}{\gamma}}} &(b)\end{aligned}\right\}. \quad (1261)$$

If the compressor were two stage and the engine simple, the diagrams would
ear as in Fig. 305. The results will be different, as the denominator in the
iency equation, which is the work of the compressor, has a different expres-
given by Eq. (1262).

$$\text{rk of compressor, two-stage, no clearance})=\frac{2\gamma}{\gamma-1}P_aV_a(R_P^{\frac{\gamma-1}{2\gamma}}-1). \quad (1262)$$

Calling the new efficiencies E_3 and E_4 for two-stage compression compar with zero engine expansion and complete engine expansion respectively,

$$E_3 = \frac{\gamma-1}{2\gamma}\left[\frac{R_P-1}{R_P(R_P^{\frac{\gamma-1}{2\gamma}}-1}\right] \qquad (a)$$

and

$$E_4 = \tfrac{1}{2}\left[\frac{R_P^{\frac{\gamma-1}{\gamma}}-1}{R_P^{\frac{\gamma-1}{\gamma}}-1}\right] = \tfrac{1}{2}\left[\frac{R_P^{\frac{\gamma-1}{2\gamma}}+1}{R_P^{\frac{\gamma-1}{\gamma}}}\right] \qquad (b)$$

. . . (12

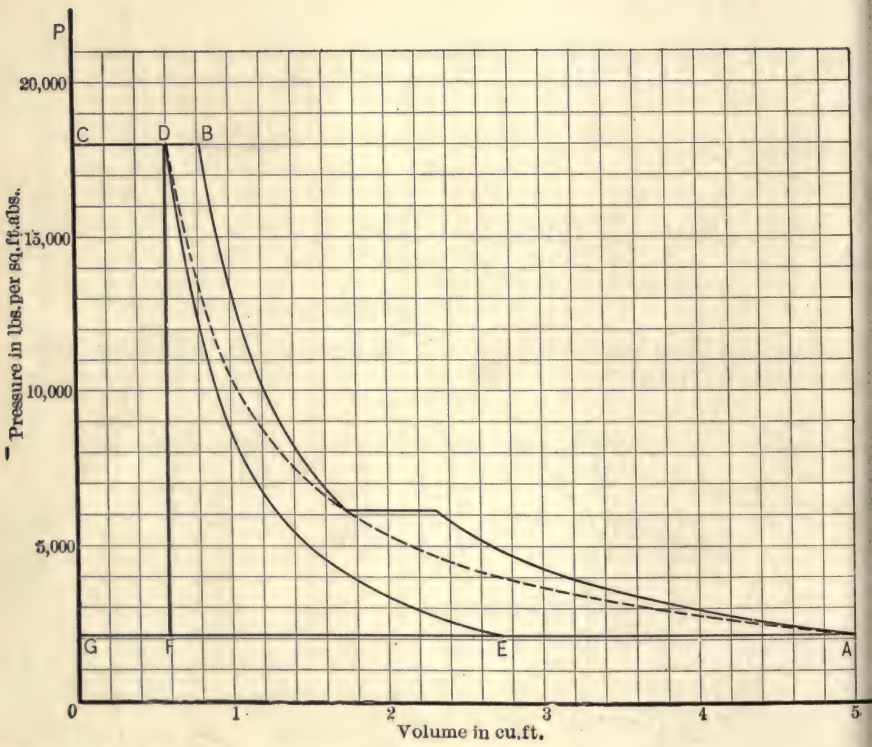


FIG. 305.—Simple Compressed Air Engine, and Two-stage Compressor, Combined Diagr

It is clear that practically any combination of compressor, single, two-st and three-stage, with engine simple, compound, triple, having an infinite finite or no receiver, and any phase relations, may be made, so a compl investigation of all cases would be very lengthy. To avoid spreading out work of analysis, no more combinations will be examined, but the meth used can be applied by any one to other combinations, by the aid of the eq

ns of Chapter II for compressors, and Chapter III for engines with expo-
ntial laws for the expansion.

The effect of preheating the air between compressor and engine may be
wn in two different ways. According to the first method the fraction of heat,
supplied that is ultimately converted into work, can be determined by divid-
the net work area between the compression and expansion lines by the foot-
nd equivalent of the heat. This will give the efficiency of the combination
a transformer of heat of preheat into work, and is the thermal efficiency of
as-engine cycle consisting of phases made up by the lines in question. The
er way compares the whole work of the air engine taken as useful output,
he sum of the compressor work and the foot-pound equivalent of the heat
plied, and this gives the efficiency of the whole combination as it is. Both
lts are of value, each, however, for what it shows.

In Fig. 306, the single compressor with simple air engine is reproduced with
ious amounts of preheat, and the $T\Phi$ diagram drawn to correspond to the
rmal cycle thus created. The no-expansion case of the air engine is omitted
ause the primary purpose is to show how good an effect of reheat may be
ained and obviously, failure to expand after preheating involves pure avoid-
e waste. The preheat is added from D to D', D to B, or D to D'', the first
ng small in amount, insufficient to bring the temperature and volume of the
led air to the values corresponding to compressor delivery, the second just
roduces these values, the third makes the volumes and temperatures
ger.

Preheat to compressor delivery temperature will, of course, result in an
ine exhaust temperature equal to that of the atmosphere or compressor
tion. Less preheat than this will cause the exhaust to be cooler than the
osphere and the minimum allowable amount should be that which will pre-
t freezing or interference with lubrication. If the air be very dry such as
ld be the case if compressed to 1000 lbs. per square inch or even less, the
aust may be 20 to 30° F. below zero without making much trouble, but with
e moisture, such as would be carried by air at 100 to 200 lbs. per square inch
ssure, the final temperature may be as low as 10 to 15° F., since the high
ial temperatures will prevent ice sticking to the walls. While in many cases
preheat used is just sufficient to prevent freezing, that is, the initial tem-
ature absolute, at the beginning of expansion, high enough for the ratio
expansion to give the above final temperatures, the real economic
e of preheat is obtained only with larger amounts. There is no reason why
peratures as high as the hottest superheated steam should not be used,
° to 600° F. in engines of exactly the same structure, or in excess of this to
degree if gas-engine structures are used as a model. These large preheats
no higher per cent thermal return than the lower ones, but make it possible
operate larger engines with smaller compressors, materially reducing the
cost of the equipment.

The efficiency of the heat of preheat alone is that of a cycle bounded by the
ximum and minimum pressure lines and two adiabatics, the first adiabatic

corresponding to the work that would be obtained without preheat, and second that corresponding to expansion from the temperature and volu produced by the preheat. On the diagram, Fig. 306, this cycle is represen for the three different amounts of preheat by, $EDD'E'E$, $EDBAE$, $EDD''E''E$ to both PV, and $T\Phi$ coordinates. These are, of course, all Bray gas cycles, having established efficiencies given by Eq. (1264) below.

$$(\text{Efficiency of heat of preheat}) = 1 - \frac{1}{R_P^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{T_b}{T_a}. \quad . \quad . \quad (12$$

This is true no matter how much heat is added and no matter what the tem atures. It is exactly the same as an Otto gas cycle efficiency for equal c

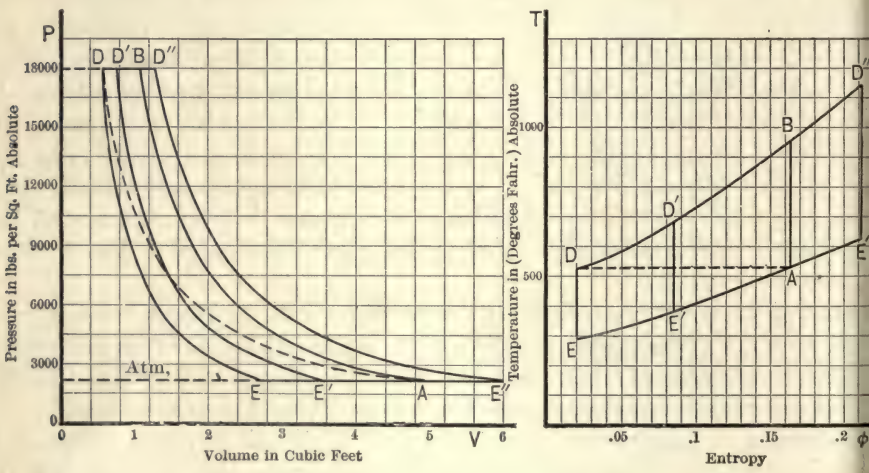


Fig. 306.—Effect of Preheat on Compressed Air Engine Performance.

pressions, and is very high, yielding not less than twice as high a return for heat, as is obtained from the heat of steam in a steam-driven compressor cylin and may be three or four times as high, depending entirely on the condition steam and air pressures. It proves the desirability of using very high air p sures for high thermal per cents of return from the heat of preheat, and compressed air at 13 atmospheres or 191 lbs. per square inch absolute f 14.7 lbs. standard atmosphere, the thermal efficiency would be, ideally,

$$E = 1 - \frac{1}{(13)^{.29}} = 52 \text{ per cent.}$$

If the diagram factors for gas engines applied to these cases the realiz amount would be about half f this, or 26 per cent, but without water jac

t is likely that fully $\frac{3}{4}$ would be realized, so the probable thermal efficiency of he heat of preheat would be 39 per cent, a truly high value that will bear experimental investigation.

The other standard of efficiency, that for the whole system, is given by Eq. 1265):

$$\left\{\begin{array}{l}\text{Efficiency of compressor and}\\ \text{engine with preheat}\end{array}\right\} = \left[\frac{\text{Work of the engine}}{\begin{array}{l}(\text{Work of compressor})\\ +(\text{Ft. Lb. equiv. to heat of preheat})\end{array}}\right] \quad (1265)$$

The foot pound equivalent of the heat of preheat is, of course, equal to, $778\times$(the heat of preheat) $=778\times wC_p$ (Temperature after preheat $-$ Temperature before). If the temperature before preheat be taken as equal to that of the atmosphere, which corresponds to complete cooling of the compressed air in receiver and pipe line, this becomes, calling the final temperature T_H and air temperature T_A and w the weight of air

$$(\text{Ft. Lb. equivalent to heat of preheat}) = 778wC_p(T_H - T_A). \quad . \quad . \quad . \quad . \quad (1266)$$

The work of the engine with preheat is equal to that without, plus that gained. The work without preheat was given by

$$\frac{\gamma}{\gamma-1}\,P_aV_a\left[\frac{R_P^{\frac{\gamma-1}{\gamma}}-1}{R_P^{\frac{\gamma-1}{\gamma}}}\right],$$

while the work done by the preheat is

$$(\text{Work done by preheat}) = 778\ (\text{Efficiency of preheat})\times(\text{Heat of preheat})$$

$$= \left[1-\frac{1}{R_P^{\frac{\gamma-1}{\gamma}}}\right]\times 778wC_p(T_H-T_A).$$

Hence

$$\left\{\begin{array}{l}\text{Work of engine}\\ \text{with preheat}\end{array}\right\}$$

$$= \frac{\gamma}{\gamma-1}\,P_aV_a\left[\frac{R_P^{\frac{\gamma-1}{\gamma}}-1}{R_P^{\frac{\gamma-1}{\gamma}}}\right] + \left[\frac{R_P^{\frac{\gamma-1}{\gamma}}-1}{R_P^{\frac{\gamma-1}{\gamma}}}\right]\times 778wC_p(T_H-T_A). \quad (1267)$$

But

$$P_a V_a = wRT_a = 778w(C_p - C_v)T_A,$$

and

$$\frac{\gamma}{\gamma - 1} = \frac{\frac{C_p}{C_v}}{\frac{C_p}{C_v} - 1} = \frac{C_p}{C_p - C_v},$$

so that $\frac{\gamma}{\gamma - 1} P_a V_a = 778wC_pT_A$, whence

$$\text{(Work of engine with preheat)} = 778wC_p\left[\frac{R_P^{\frac{\gamma-1}{\gamma}} - 1}{R_P}\right](T_A + T_H - T_A)$$

$$= 778wC_pT_H\left[\frac{R_P^{\frac{\gamma-1}{\gamma}} - 1}{R_P}\right]. \quad . \quad . \quad . \quad . \quad . \quad (1268$$

$$\left\{\begin{matrix}\text{Efficiency of compressor and} \\ \text{engine with preheat}\end{matrix}\right\} = \frac{778wC_pT_H\left[\frac{R_P^{\frac{\gamma-1}{\gamma}} - 1}{R_P}\right]}{\frac{\gamma}{\gamma - 1}P_aV_a\left[R_P^{\frac{\gamma-1}{\gamma}} - 1\right] + 778wC_p(T_H - T_A)}$$

$$= \frac{R_P^{\frac{\gamma-1}{\gamma}} - 1}{\left[R_P\frac{T_A}{T_H}\left[R_P^{\frac{\gamma-1}{\gamma}} - 1\right] + \left(1 - \frac{T_A}{T_H}\right)\right]}. \quad . \quad . \quad (1269$$

The same methods apply, but of course, different equations result, for the mo complex cases of any number of compressor stages, combined with any numb of expansions with both preheat and receiver reheat. To illustrate, one ca will be worked out, that of two-stage compression with compound engine, fir with reheat without preheat, and later with preheat also, as in Fig. 307. Th first case is that for two-stage compression with perfect intercooling *ABC* followed by engine expansion *E* to *F* after the delivered air has reacquired th atmospheric temperature by cooling from *D* to *E*. After this high-pressu cylinder expansion the air is reheated to the atmospheric or original temper ture *F* to *C* in the engine receiver, which may be done by the atmosphere itse warming the working air as is done in the Porter air locomotive, before su sequent expansion *C* to *G* in the low-pressure cylinder. The other case has th same compression line *ABCD*, without final cooling after delivery, followed b

F to G and low-pressure cylinder expansion G to H. In every case the efficiency of the use of the heats, whether of preheat or reheat, is given by Eq. (1270):

$$\begin{pmatrix}\text{Efficiency of preheat or}\\ \text{reheat}\end{pmatrix} = \begin{pmatrix}\text{Work done per unit}\\ \text{of heat added}\end{pmatrix} = 1 - \frac{1}{(R_P)^{\frac{\gamma-1}{\gamma}}}, \quad . \quad . \quad (1270)$$

or

$$\begin{pmatrix}\text{Work added by preheat}\\ \text{or reheat}\end{pmatrix} = 778\left[1 - \frac{1}{R_P^{\frac{\gamma-1}{\gamma}}}\right] \times \begin{bmatrix}\text{Heat added at con-}\\ \text{stant pressure}\end{bmatrix}. \quad (1271)$$

Of course, this is true only for complete expansion, but as complete expansion is attainable and as this is the best possible result it is the only one worth spending time to analyze.

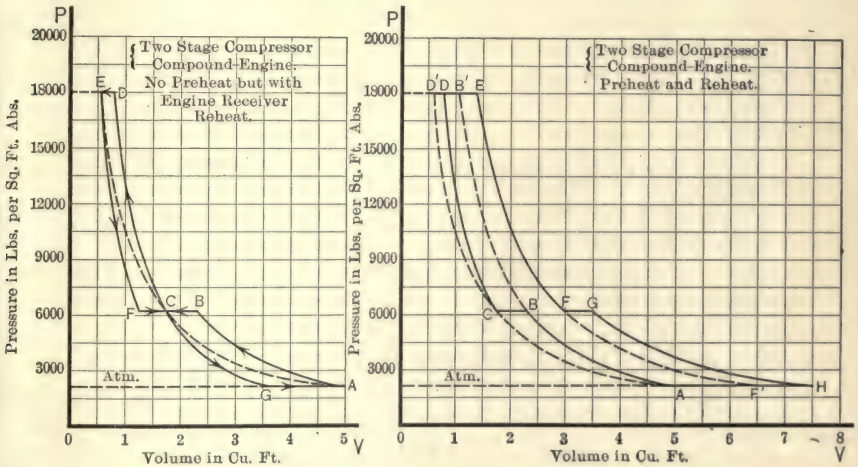


FIG. 307.—Two-stage Compressor, Compound Compressed Air Engine with and without Preheat, and with Receiver Reheat.

Evaluation of the work of the engine is best done by the pressure-volume methods, assuming no preheat or reheat, and to it may be added the work derived from preheat and reheat as thermally determined. The net work of engine and compressor together represented by the enclosed areas is best evaluated thermally, because any such area is made up of a number of Brayton cycles for which the pressure ranges and temperatures and quantities of heat are known. Thus taking the right-hand diagram, Fig. 307, its work by areas is given by Eq. (1272):

$$\begin{pmatrix}\text{Total work area}\\ ABCDEFGHA\end{pmatrix} = \left\{\begin{array}{l}\text{Work of Brayton cycle, area } BCDB'B, \text{ I}\\ +\text{Work of Brayton cycle } ABB'EFF'A, \text{ II}\\ +\text{Work of Brayton cycle } F'FGHF', \quad \text{III}\end{array}\right\} . \quad . \quad (1272)$$

For both compressor and engine the receiver pressure should be

$$P_c = P_b = P_f = P_g = \sqrt{(\text{max. pressure}) \times (\text{min. pressure})},$$

so that

$$\frac{P_d}{P_c} = \frac{P_d}{\sqrt{P_d P_a}} = R_P^{\frac{1}{2}} = \frac{P_f}{P_{f'}}.$$

As the efficiency is a function of these pressure ratios only

$$\text{Work I} = (\text{Heat added } D \text{ to } B') \times \left(1 - \frac{1}{R_P^{\frac{\gamma-1}{2\gamma}}}\right)$$

$$\left.\begin{aligned} &= C_p w (T_{b'} - T_d)\left(1 - \frac{1}{R_P^{\frac{\gamma-1}{2\gamma}}}\right) \qquad (a) \\ &= C_p w (T_{b'} - T_d)\left(1 - \frac{T_c}{T_d}\right) \qquad (b) \end{aligned}\right\} \quad . \; . \; . \; (1273)$$

$$\left.\begin{aligned} \text{Work II} &= C_p w (T_e - T_{b'})\left(1 - \frac{1}{R_P^{\frac{\gamma-1}{\gamma}}}\right) \qquad (a) \\ &= C_p w (T_e - T_{b'})\left(1 - \frac{T_a}{T_{b'}}\right) \qquad (b) \end{aligned}\right\} \quad . \; . \; (1274)$$

$$\left.\begin{aligned} \text{Work III} &= C_p w (T_g - T_f)\left(1 - \frac{1}{R_P^{\frac{\gamma-1}{2\gamma}}}\right) \qquad (a) \\ &= C_p w (T_g - T_f)\left(1 - \frac{T_{f'}}{T_f}\right) \qquad (b) \end{aligned}\right\} \quad . \; . \; (1275)$$

$$\text{The heat of preheat} = C_p w (T_e - T_d). \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1276)$$

$$\text{The heat of reheat} = C_p w (T_g - T_f). \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1277)$$

These results are not carried further, to save space, and they are introduced only to illustrate a method of procedure that is useful and easily applied to any special case that may arise. It should be noted, however, that if the air after compressor delivery be cooled down to its original temperature before preheating, it will have the volume $V_{d'}$, and the heat so lost must be made up by preheating before the net work evaluation as above is made. If it is just made up and no more, the line EF will fall on DC and no net work results in the high-pressure range, if it is not all made up, EF will fall to the left of DC and a negative work area will result.

Just as in steam and gas engines there is a heat exchange between walls and gases due to the difference in temperature, sometimes in one and sometimes in the other direction, so also is there a heat exchange between air and cylinder walls in both an engine and an air compressor, which must be taken into account in any discussion of their performance, when passing from the cyclic possibilities to the probabilities for real machines. Two general methods of attack are available, but have been used to only a small extent however, in evaluating the effect or fixing its nature. The first is that developed for gas engines and is concerned with heat exchanges during expansion and compression. It was shown that the rate of heat gain or loss by the gas per unit of volume change given by, $\left(\frac{dH}{dV}\right)$, could be expressed as a function of pressures and volumes measured directly from an indicator card if the clearance is accurately known and if the cylinder and valves are tight. The only net effect of this exchange that is of any practical value is to change the value of the exponent " s " giving it some value other than the adiabatic. While for compressors and air engines this sort of analysis shows heat exchanges that are alternately positive and negative, the net effect is not serious enough to warrant the use of any exponent other than the adiabatic value, $s = \gamma = 1.4$ for air. Other values have been frequently reported, but with the exception of cases confined to small cylinders at low piston speeds, all are open to suspicion as they are in the direction that the effects of leakage would be. It was shown in Fig. 5 that for expansion lines, the smaller the value of " s " the higher the curve will lie, the adiabatic being the lowest, and gain of heat such as would be expected as a net result in cold compressed air engines, if anything, would be shown by a lesser value of " s " than 1.4. Conversely, for compression lines, Fig. 6, the smaller " s," the lower the curve, and heat loss as would be expected as the net result in compressors if anything would also lower the curve. Therefore, in both cold-air engines and compressors such net heat changes as might be expected would lower the value of " s," but it is also found that leakage may affect the diagrams as much as a considerable heat exchange. It requires only a moment's thought in view of the very low conductivity of air and the quiescent condition of the layers of air next the walls to realize that, in the short time available for heat transfer, the amount that could be exchanged during expansion or compression must be very small and negligibly so in normal cases. The discussion of heat transfer in Chapter IV also bears this out, for there it was shown how very low and consistently low, was the rate of heat transfer whenever a gas was involved as giving or receiving the heat. Furthermore, it was shown that increased rates were only obtainable with very vigorous surface scrubbing such as corresponds to high velocity of flow and with gases divided up into thin layers or streams. In cylinders the air is in about the best condition that could by found for prevention of heat transfer, so it is difficult to conceive of " s " having any materially different value than the adiabatic $s = \gamma = 1.4$.

There is, however, another period in compressors and air engines where a material heat exchange occurs, and that is during entrance, corresponding in

engines to admission at the high pressure up to cut-off, and in compressors to suction at the low pressure. The effect in air engines is to cool the admitted air, so that for each cubic foot measured at conditions external to the cylinder and admitted, there will be in the cylinder less than a cubic foot before expansion begins, and at a temperature lower than that of the air supply. This is similar in kind to the initial condensation of steam engines and has the effect of increasing the consumption per hour per I.H.P. over the indicated value, but in air engines it has the additional effect of bringing the exhaust air to a lower temperature because expansion starts with air colder than the supply. In compressors the effect is to prevent the cylinder taking in as much as corresponds to the product of its apparent volumetric efficiency and displacement, measuring volumes at pressures and temperatures external to the cylinder.

Determination of the effect of both compressor suction heating and engine initial cooling have been made by measuring the volume supplied or delivered at pressures and temperatures external to the cylinder and comparing these volumes with those indicated. The results can be expressed in terms of volumes or in terms of the temperature changes that correspond to the volume differences, and as cylinders work at such widely different temperatures and the effect is one of temperature, the latter is to be preferred. The problem may then be stated, how much will the volume change during admission to air compressor or engine cylinders, when the difference between initial and final temperatures of the air in passing through has a known value? Some general conclusions are available and some specific examples may be cited to support them, that will make it possible to pretty closely approximate this effect. The effect is generally to change the volume from 10 to 30 per cent, reducing it for engine admission and increasing it for compressor suction, or better still, the temperature rise is from 25 to 50 per cent of the difference between the temperatures of supply and delivery air, entering and leaving the cylinders. One series of long-run tests of an ammonia compressor using weighed anhydrous gave the following results in round numbers, which show the rise in temperature during admission to average about 33 per cent of the difference between maximum and minimum fluid temperature, excluding an abnormal case:

Temperature before suction, degrees F ...	15	14	15	14	18
Temperature after delivery, degrees F	216	218	246	253	243
Temp. diff. between delivery and suction..	201	204	231	239	225
Temperature rise during suction	68	67	76	59	71
$\left(\frac{\text{Temperature rise during admission}}{\text{Temperature difference in fluid}}\right)$	.34	.33	.33	.25	.32

A small air engine working on an atmospheric temperature supply, of compressed air, showed a volumetric reduction of 10 per cent on entrance, which is equivalent to a reduction of temperature of about 30 per cent of the difference between supply and exhaust-air temperatures. One valuable contribution, that of Trinks, that the extent of the surface exposed to the suction heating has no material influence on the heating effect, was established by measuring air

an 8×8 in. compressor fitted with different clearance surfaces but with nstant clearance volume. The whole range of results show an increase of to 25 per cent in the volume of air while entering. Many examples of separate ta of the same sort all lead to the conclusion that the volumes will be affected much as corresponds to a temperature rise during entrance of 25 to 50 per nt of the difference between initial and final temperature, but it is impossible relate the figures to speed, cylinder size, construction, kind of cylinder cooling, mperature range or any other factor, that might seem to affect it but as perimentally proved frequently fails to produce an anticipated effect.

This being the case the volume relations may be set down as follows for air gines:

$$\left(\frac{\text{Cubic feet air indicated at cut-off}}{\text{Cubic feet air supplied}}\right) = \left(\frac{\text{Absolute temp. air at cut-off}}{\text{Absolute temp. air supplied}}\right) \quad . \quad (1278)$$

nt if the temperature after entrance, is equal to that at supply, less 25 to 50 r cent of the difference between discharge and supply air, then

$$\text{Temp. at cut-off}) = (\text{Temp. supply}) - (.25 \text{ to } .50) \times \left(\begin{matrix}(\text{Temp. supply}) \\ -(\text{Temp. exhaust})\end{matrix}\right) \quad (1279)$$

$$\left(\frac{\text{Cu.ft. air indicated}}{\text{Cu. ft. air supplied}}\right) = 1 - (.25 \text{ to } .50) \times \left[1 - \left(\frac{\text{Abs. temp. exhaust}}{\text{Abs. temp. supply}}\right)\right] \quad . \quad (1280)$$

Similarly, for compressors,

$$\left(\frac{\text{Cu.ft. air indicated on suction line}}{\text{Cu.ft. air supplied}}\right) = \left(\frac{\text{Abs. temp. air after suction}}{\text{Abs. temp. air supplied}}\right) \quad . \quad . \quad (1281)$$

But

$$\text{Temp. after suction}) = (\text{Temp. supply}) + (.25 \text{ to } .50) \times \left(\begin{matrix}(\text{Temp. del.}) \\ -(\text{Temp. sup.})\end{matrix}\right) \quad (1282)$$

Hence

$$\left(\frac{\text{Cu.ft. air indicated}}{\text{Cu.ft. air supplied}}\right) = 1 + \left[(.25 \text{ to } .50)\left(\frac{\text{Abs. temp. air del.}}{\text{Abs. temp. air sup.}}\right) - 1\right] \quad . \quad . \quad (1283)$$

It is customary in compressors to use the term volumetric efficiency defined by Eqs. (1284) and (1285):

$$\text{Apparent volumetric efficiency} = E_V = \frac{\text{Cu.ft. air indicated on suction line}}{\text{Displacement}} \quad . \quad (1284)$$

$$\text{Real volumetric efficiency} = E^1_v = \frac{\text{Cubic feet air supplied}}{\text{Displacement}}$$

$$= \frac{E_V}{1 + (.25 \text{ to } .50)\left[\left(\frac{\text{Abs. temp. air del.}}{\text{Abs.temp. air sup.}}\right) - 1\right]} \quad (128$$

Prob. 1. Air at 135 lbs. pressure per square inch gage and 70° F. is admitted a cylinder in which the cut-off is $\frac{1}{2}$. What will be the temperature after expansio What will it be for $\frac{1}{4}$ cut-off, with, and without considering initial cooling?

Prob. 2. At what temperature must air be initially, to prevent freezing of exhaus if the expansion is from 80 lbs. per inch gage to atmosphere?

Prob. 3. Air is compressed adiabatically in one stage from atmosphere to 10 atmo pheres absolute, and then used in a single-cylinder steam without preheat expansion. What will be the efficiency of the system, and to what value would it raised by using complete expansion in the engine? The compressor may be assumed have zero clearance and the engine to have complete compression.

Prob. 4. Should the high pressure be only 5 atmospheres instead of 10, how wou the results differ?

Prob. 5. Should the compressor of Problem 3 be two-stage would the results changed, and if so, to what extent?

Prob. 6. What would be the efficiency of the heat of preheat in a case where air w used at a pressure of 120 lbs. per square inch gage, the low pressure being zero gag the air being preheated 100° F. above atmosphere. Would the result be different the preheating were doubled and if so, how much?

Prob. 7. What would be the efficiency of compressor and engine when the hi pressure is 140 lbs. per square inch absolute, the low pressure 14.1 lbs. per square in absolute, and the air preheated 300° F. above that of the original temperature? Wou the result be different if the amount of preheat were doubled? If halved? If so, how much? What is the efficiency of the heat of preheat alone?

Prob. 8. In an air engine it was found that the temperature of the exhaust w 10° F. and that of the supply air 350° F. What would be the likely limit of the ra of cubic feet of air indicated to cubic feet supplied, and the horse-power expected of t air engine per horse-power of compressor.

Prob. 9. Air is being adiabatically compressed from atmosphere to 100 lbs. per squ inch gage. The atmospheric temperature is 50° F. and apparent volumetric efficien is 95 per cent; what will be a fair value for the real volumetric efficiency?

20. Mechanical Refrigeration, General Description of Processes a Structures. Thermal Cycles and Refrigerating Fluids. Limiting Temper tures and Pressures. It is just as important that some substances cooled to temperatures lower than the surroundings and kept cool for lo periods of time, as is the heating and keeping hot of other substances, and un the thermal processes here discussed were understood this could be done o to the limited degree corresponding to the use of natural ice with or witho salts to lower the melting-point. The preservation of foods including me fish, fruits, poultry, eggs and butter to prevent destruction between the perio

of production and consumption, the cooling of beverages, waters, beers and wines to render them more palatable, the manufacture of ice in places where natural ice is too costly, impure or impossible, are all illustrations of important cooling processes, involving the abstraction of heat from a great variety of substances and such as would suggest themselves to anyone. The perfection of apparatus, however, to accomplish these obviously desirable ends has resulted in a very much wider application of that same equipment with suitable modifications to other equally important, though more purely industrial purposes, the refrigerating processes becoming one step in the general scheme of production of some result not itself a cooled body. For example, the drying of air is best accomplished by cooling it to as low a temperature as will reduce the moisture in saturated air to the desired low value, and dry air is very necessary in some places; just as moist air is necessary in cotton spinning, for example, so is dry air in the manufacture of photographic films. Dried air is useful also in the drying and dry storing of wood, fabrics, crystals and powders, and in the blowing of blast furnaces, where each pound of moisture by dissociation into hydrogen and oxygen consumes heat of coke combustion that by its removal becomes unavailable for reducing iron from the ore. Some other examples of the use of refrigeration as a process are found in the manufacture of chocolates, the storage of furs against moths, the working of tobacco, the recovery of paraffine from oil, shaft sinking through soft earth by freezing the whole mass to permit of the removal of the frozen material by the ordinary methods of excavation, and finally the control of the rate of fermentation in various liquors and the rate of germination of seeds or growth of bacteria.

To accomplish the cooling of any given substance and to maintain it at a low temperature continuously, requires primarily, that some other still colder substance to be termed the *refrigerating fluid* be brought close to it, so heat may flow by reason of the temperature difference to the colder refrigerating fluid through plates or pipes separating them. It is evident, therefore, that any sort of contact conditions that may be suitable or convenient will serve to cool anything, if only a still colder refrigerating fluid be available, so that thermally considered, the problem of refrigeration really consists in the discovering of means for making some fluid as cold as may be necessary at the lowest cost, and without danger to operators or injury to the apparatus. As the absorption of heat by any cold refrigerating fluid would ultimately raise its temperature as high as the body giving up heat to it, continuity of the refrigerating process involves as another essential element, the circulation of the refrigerating fluid, removing continuously that which has taken up some heat and replacing it with fresh quantities. As a result, there would be accumulating somewhere beyond the refrigerator large amounts of refrigerating fluid colder than the cooled substances and, of course, colder than the surrounding atmosphere and earth, so the disposition of this fluid must be provided for. Such refrigerants as have been found suitable are expensive so they cannot be thrown away after once performing a refrigerating function, but must

be restored to their original condition to be used again and again, without end.

These thermal and work processes on the refrigerating fluid itself which make possible the absorption by it of heat in the low-temperature ranges, and its ultimate restoration to a condition which will permit a repetition, constitute the so-called refrigerating cycles, and as the fluid may be either a gas, or a liquid-vapor, there may be a very great variety of refrigerating cycles, at least as many as there are of cycles for transforming heat into work by both vapor and gas systems. It is not, however, necessary or desirable here to analyze all possible refrigerating cycles, as the conclusions of the past fifty years may be accepted as proving the superiority of a few survivors from the numerous proposals.

In general, whether the fluid be a gas or a vapor, it may be imagined as passing through a cycle consisting of heat absorption in low ranges of temperature as one phase, and if the original condition is to be restored, heat absorbed must be given up to surroundings which will constitute another phase; between absorption of heat at low temperatures and its discharge to air or water later at temperatures higher than their own, the fluid itself must be raised in temperature and this can be done by doing work on it, so a third phase will be one of compression. At least one more phase is needed to close the cycle, but there may be more than one. If the fluid be a gas, the absorption of heat by it in the refrigerator as it circulates is most feasibly accomplished at constant pressure, so heating at constant pressure, A to B, would be the first phase, Fig. 308, the next, adiabatic compression, B to C, followed by heat abstraction or cooling, C to D, again at constant pressure, the fuel temperature always above that of the available water which is to carry it away, and closing the cycle by adiabatic expansion D to A to lower the temperature to the original value.

It will be noted that this is a Brayton gas-engine cycle executed in the reverse direction, and therefore all the thermal relations established for that cycle in Section 11 of this Chapter may be used with appropriate modifications for this reverse refrigerating cycle when its analysis is taken up later.

Should a liquid-vapor be used, absorption of heat by it at a temperature lower than in the refrigerator will evaporate liquid, and this is the first phase; compression adiabatically will raise temperature and pressure to such values as will permit condensation as heat is given up to the circulating water, and these steps constitute the second and third phases, while liquid cooling accompanied by reduction of pressure closes the cycle, though there are modifications to be noted depending on quality of the vapor at various points and the manner in which the drop in pressure after condensation takes place. If after condensation is complete at D, Fig. 309, the liquid adiabatically expands D to A, it will have the quality at A at the low temperature. If, however, it merely escapes through a valve it will follow the line of constant total heat D to A' and be more wet, and this is more nearly the process as practiced. From the point A or A', evaporation proceeds at a constant temperature lower than that in the refrigerator until no more liquid remains, B'', after

vhich it may be superheated, $B''B'''$. At any time, that is when the quality ıas any value, such for example as corresponds to points B, B', B'' or B''', he heat absorption may be stopped by removal of the vapor-liquid from the efrigerator to the compressor, for adiabatic compression to condenser pres-ure. Here, four different compression lines are shown, the first, BC, illustrating ıdiabatic compression from wet vapor to less wet vapor; $B'C'$, from wet to dry aturated vapor; $B''C''$, from dry saturated to superheated; $B''C'''$, from

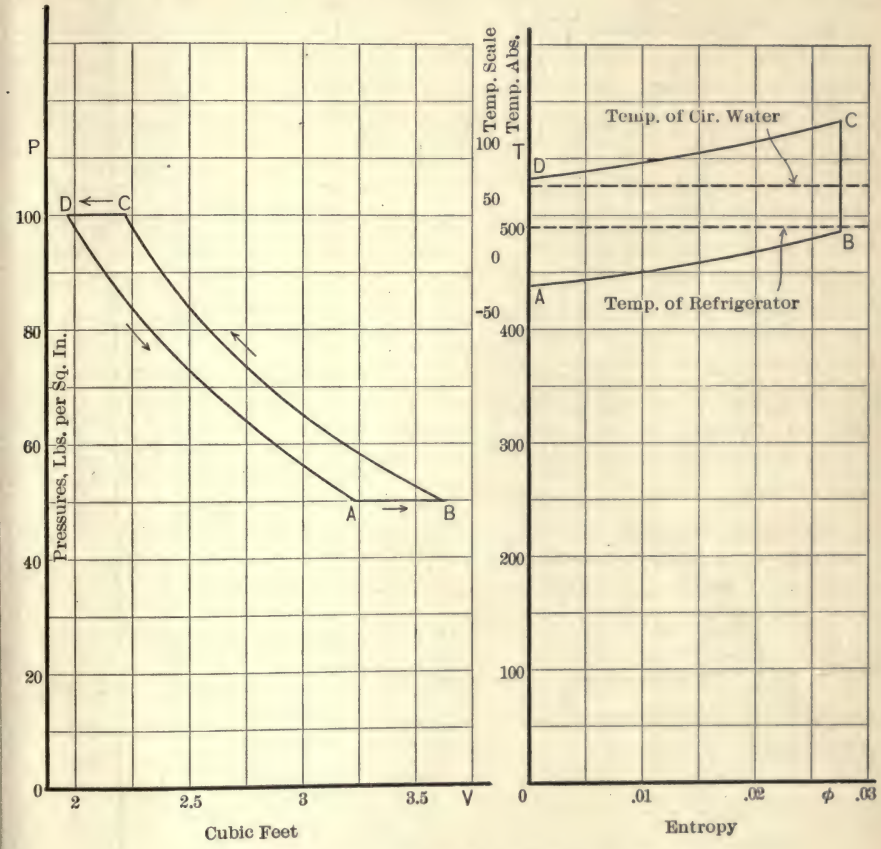


Fig. 308.—Refrigeration Cycle for a Gaseous Fluid.

ıperheated to more superheated states. When the vapor after adiabatic ɔmpression is superheated the first action in the condenser is not condensation, ut a sort of precooling for the removal of superheat, $C'''C''C'$, after which ·ue condensation proceeds C' to D, at which it is complete.

As the constant total heat line DA' for the liquid passage from condenser ɔ refrigerating evaporator is fairly close to the liquid line DA'' for the common ·frigerating fluids, it is customary to consider the heats involved as equal in nu-ıerical value for the small temperature ranges involved, and this procedure is

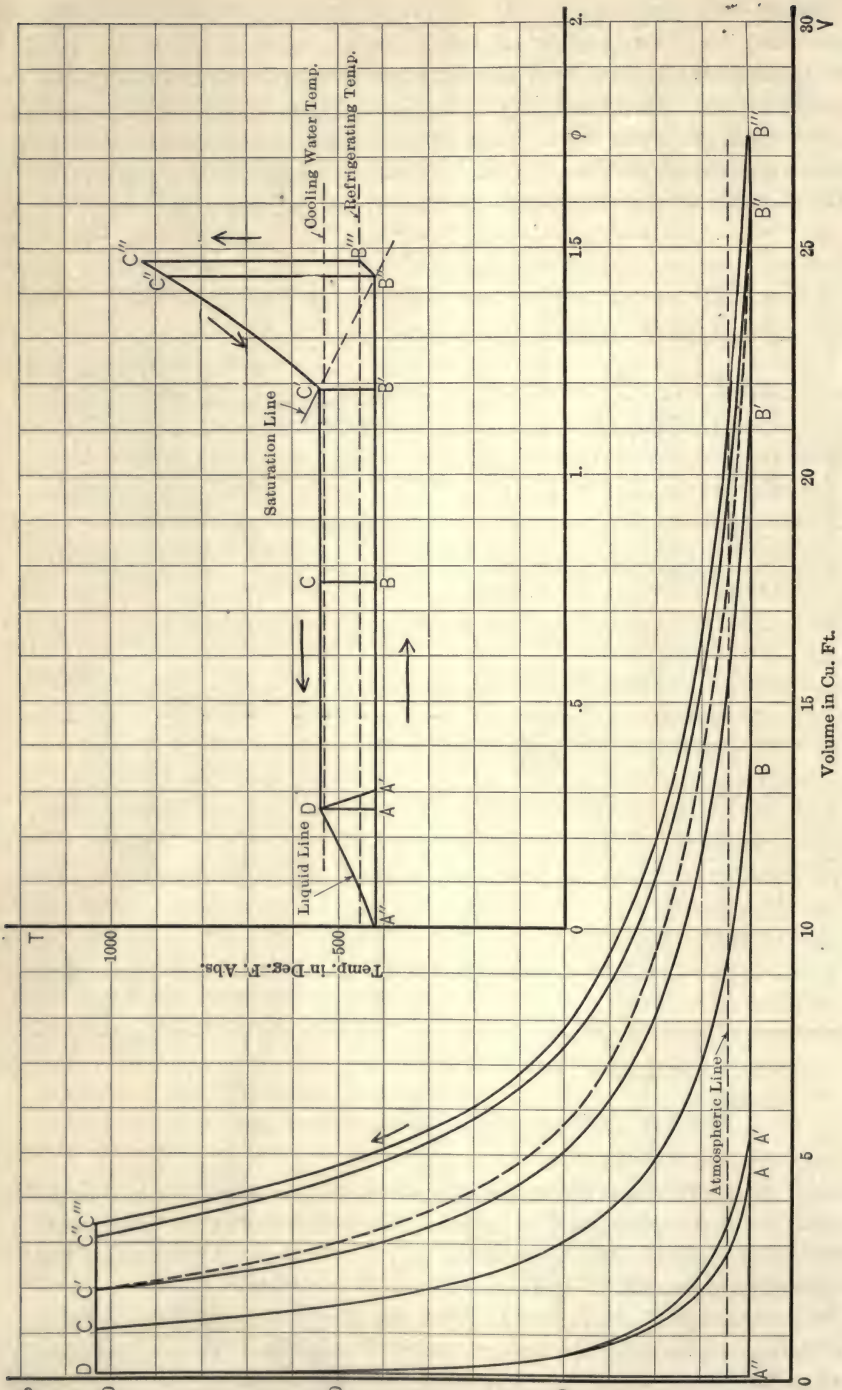


FIG. 309.—Vapor Liquid Fluid, Refrigerating Cycle.

justified by the fact that the error is very small, smaller than the lines are correctly located or the properties of these fluids accurately known. Therefore, the refrigerating cycle involving the use of a vapor-liquid fluid may be taken as the Rankine cycle reversed and all the methods and conclusions developed for this cycle as a heat to work transforming process, are available with suitable modifications when analyzing it as a refrigerating cycle.

It is clear from the preceding discussion that any gas or any liquid may be used as a refrigerating fluid when subjected to these cycles of operation with only one limitation, and that is they must not pass through the critical state. This does not mean, however, that all are equally available, for they are not, and the first consideration of importance that reduces the number to a very few is the pressure necessary to cause evaporation at such low temperatures as refrigerants must assume, a high limit for which may be taken at 0° F. and a low limit at −50° F. If the pressures corresponding to the saturated vapor at such temperatures are too low, there would be needed very large pipes and compressors to handle the necessary quantity. This consideration with others considered later, limits the available fluids to the following, having the boiling-points at atmospheric pressure corresponding and in order: sulphuric ether, C_2H_5O, 34° C. = 93° F.; sulphur dioxide, SO_2, −10° C. = 14° F.; methyl chloride, CH_3Cl, −22° C. = −8°F.; methyl ether, C_2H_6O, −24° C. = −11° F.; ammonia, NH_3, −33° C. = −27° F.; carbon dioxide, CO_2, −79° C. = −110° F.; nitrous acid, N_2O, −88° C. = −126° F.; ethylene, C_2H_4, −103° C. = −152° F.; oxygen, O_2, −181° C. = −294° F.; air, −192° C. = −314° F.; nitrogen, N_2, −198° C. = −324° F.; hydrogen, H_2, −243° C. = −416° F.

The other items in the elimination of the less desirable fluids are; (*a*) the condenser pressure corresponding to ordinary circulating water temperatures which, if too high, require thick and expensive pipes and unduly expensive compressors; (*b*) the latent heat of evaporation at the refrigerator or evaporating pressures which, if too small, require the circulation of excessive quantities of fluid to produce a given amount of refrigeration; (*c*) the volume of vapor per pound at the low pressure which, if too large, requires too large a compressor to circulate the required quantity, the compressor size being a function of the cubic feet of vapor per B.T.U. of latent heat taken by it from the refrigerator; (*d*) the cost per lb. of the fluids which must be replaced to supply leakage; (*e*) the corrosiveness of the fluid on the metal parts. All these things carefully studied have resulted in the placing of anhydrous ammonia in the first rank of refrigerating fluids, it being used in the production of over 90 per cent of all the mechanical refrigeration now in service, with carbonic acid and sulphur dioxide next in order, and practically all others eliminated except air, which is used entirely as a gas and which is the only gas so used.

These fluids as the media for the execution of the refrigerating cycles must be treated in mechanical apparatus of suitable form, and this apparatus in type at least is now pretty well standardized and falls in one of three so-called systems of mechanical refrigeration which must be understood in principle before

attempting any analysis or calculations of performance. These three systems are:

A. The dense-air system, so called because the air which is the medium is never allowed to fall to atmospheric pressure, so as to reduce the size of the cylinders and pipes through which a given weight is circulating.

B. The compression system, using ammonia, carbon dioxide and sulphur dioxide, and so called to distinguish it from the third system, because a compressor is used to raise the pressure of the vapor and deliver it to the condenser after removing it from the evaporator.

C. The absorption system, using ammonia, and so called because a weak water solution removes vapor from the evaporator by absorption, the richer aqua ammonia so formed being pumped into a high-pressure chamber called a generator in communication with the condenser, where the ammonia is discharged from the liquid solution to the condenser by heating the generator, to which the solution is delivered by the pump.

No matter what the system may be, it will require circulating water to receive its heat, and the temperature of the water determines the highest temperature allowable in the system and indirectly the highest pressure, so one structural element is a cooler or condenser or both or several of each, to which circulating water is supplied. In some cases it will be found that the same water may be used in series over two different parts when the temperature in one is higher than in the other. Another structural element common to all is the refrigerator proper, in which the air of the air system is warmed and the ammonia or carbon dioxide evaporated, and its most common form is a coil of pipe. This pipe may be placed directly in a room to be kept cool, in which case the arrangement is called direct expansion, but more usually the coil or an equivalent structure forms part of a brine-cooling chamber, the cold brine from which is circulated by pumps through room coils or through ice-making tanks, holding cans that in turn contain the water to be frozen, though a great variety of combinations of direct cooling and brine circulation are in use. It will be assumed in what follows that in all cases a brine cooler will be used with the understanding that the cooling coils may instead, cool anything other than brine directly.

The air system is illustrated diagrammatically in Fig. 310, for a closed system, in which air previously dried of moisture is continuously circulated, showing a compressing cylinder delivering air through a water cooler to an expansion cylinder, which in turn sends the air through refrigerating coils back to the compressing cylinder. As the work done by the expansion of the cooled compressed air is less than that needed to compress the air from the refrigerating coils through the same pressure range, there will be necessary a source of power to supply the difference, so a steam or gas-engine cylinder is added.

While any range of temperatures desired may easily be obtained with reasonable pressures, some standard is desirable for purposes of comparison, so it will be assumed that the air must leave the refrigerating coils at 0° F. and enter at −80° F., also that the cooling water available will permit of an initial temperature in the expansion cylinder of 70° F. If expansion in this cylinder is complete, as it may be if the cut-off is suitably adjusted, the pressure ratio is given by Eq. (1286):

$$R_P = \frac{P_H}{P_L} = \left(\frac{T_H}{T_L}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{530}{380}\right)^{3.46} = (1.4)^{3.46} = 3.2 \text{ (approx.)} \quad . \quad . \quad (1286)$$

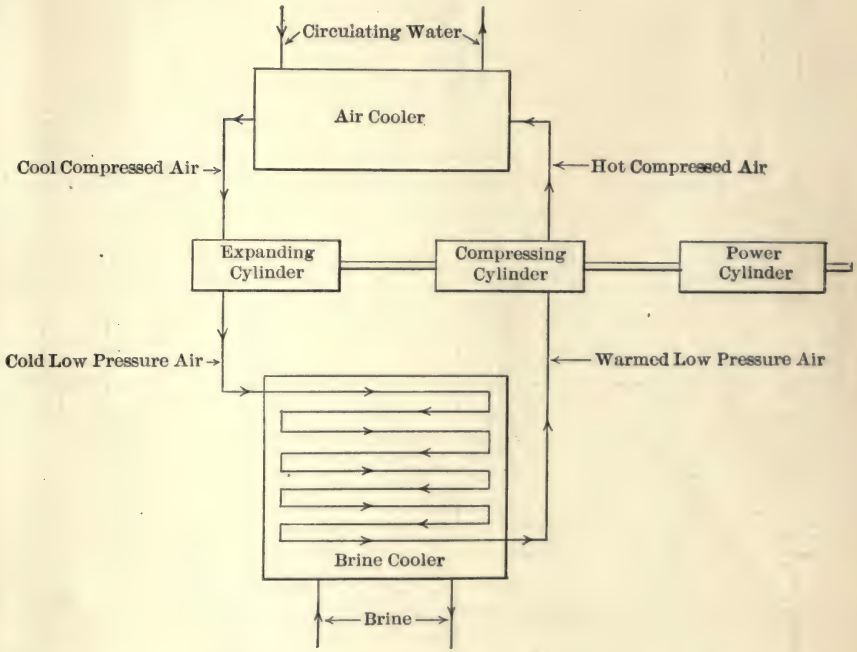


FIG. 310.—Diagrammatic Arrangement of the Air System of Refrigeration.

If the pressure in the refrigerating coils is one atmosphere that in the water-cooling coils must be 3.2 atmospheres absolute, but if high pressures are carried in the refrigerating coils to reduce the size of equipment, four atmospheres, for example, the high pressure will be thirteen atmospheres absolute. The indicator diagram of the compressor without losses is shown for this latter case by $ABCD$ of Fig. 311, on which $\overline{BC}$ represents the volume of hot air actually delivered by the compressor due to the clearance volume V_c. If this be cooled and admitted to an expansion cylinder having a clearance volume V_h, and working with complete compression GH and expansion EF, the admission volume $\overline{HE}$, is to the compressor delivery volume $\overline{BC}$, as the absolute temperatures after

and before cooling. These two indicator cards when combined become equivalent to the PV cycle of Fig. 312, on which $\overline{CB}$ represents the delivered volume of hot compressed air, and $\overline{CE}$, the admitted volume of resultant cooled air, or $\overline{EB}$ the reduction in volume due to cooling, EF the expansion, FA the refrigerator heating of the air and AB the compression, substantially, as described previously. Of course, compressor suction and delivery valve losses, and suction heating, as well as incomplete or overexpansion and com-

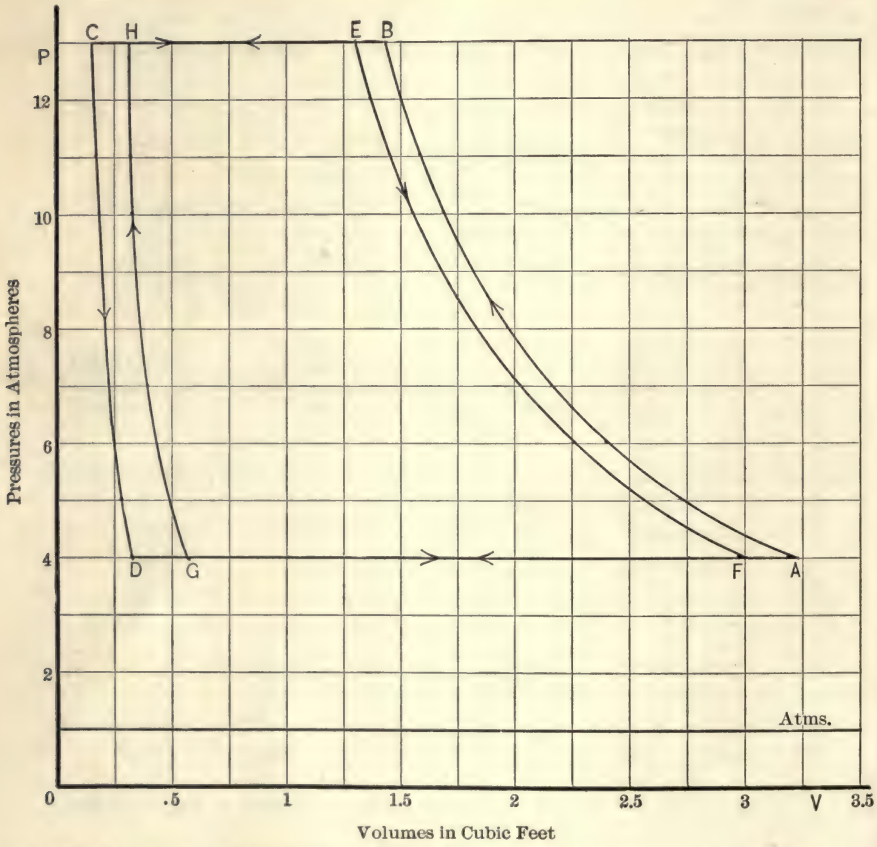


FIG. 311.—Dense Air Refrigeration, Compressor and Engine Combined Indicator Cards.

pression in the expansion cylinder, with admission cooling of the air therein, will distort the diagram of actual machines, but the effects of such distortion are easily understood and need not be subjected to analysis, the cycle above representing the perfect conditions will give best possible results that are to be determined analytically.

The compression system for ammonia, carbon dioxide or other similar condensible vapors is illustrated in Fig. 313, diagrammatically, the arrangement of equivalent parts being maintained as in the air system, Fig. 310, to bring out

differences more clearly. Here the condenser replaces the air cooler, and the vapor delivered to it by the compressor is changed therein to liquid at the same pressure, which liquid escapes through a throttle, generally termed an expansion valve, to the refrigerating coils where the pressure is low and evaporation thereby induced, the discharge vapor from the coils being regularly drawn off by the compressor suction. Of course, other elements are added in practice, for example, a bypass piping, so the condenser may be pumped out into the refrigerator for cleaning or repairs, a charging connection for taking in fresh ammonia, a forecooler at the condenser to remove superheat and which may use warmer water than the condenser itself, a liquid receiver beyond the condenser to store

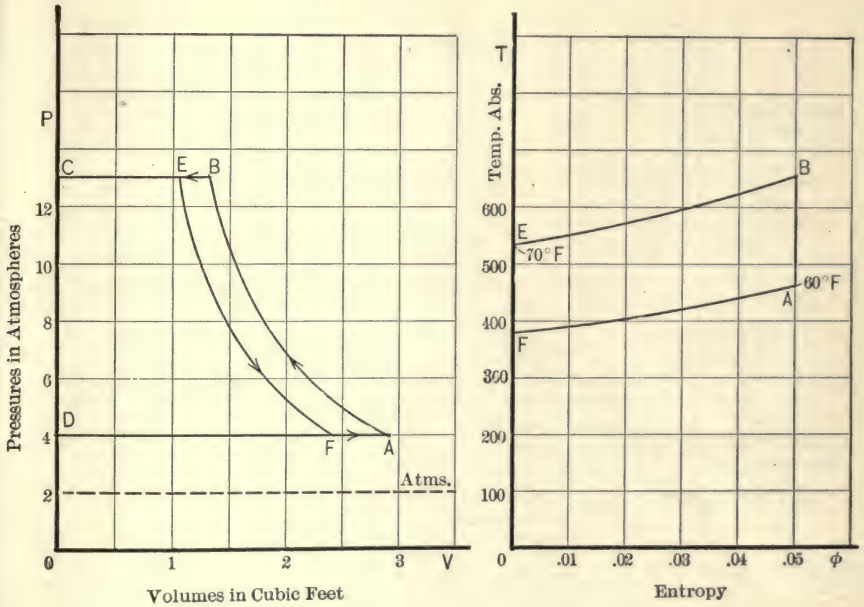


FIG. 312.—Dense Air Refrigerating Thermal Cycle.

a quantity of fluid to meet sudden fluctuations of demand while vapor returns slowly, a purge valve at the highest point of the condenser to blow off non-condensible gases from the system, a liquid separator on the compressor suction to prevent the liquid of wet vapors entering the cylinder, oil or glycerine traps between compressor and condenser to prevent entrance into the latter of any such matter used in the cylinder and discharged from it, and other similar elements often in large numbers, none of which, however, affect the fundamental thermal process.

For equal temperatures maintained at delivery from condenser and refrigerator coils as were assumed reasonable for the air system, that is 70° F. and 0° F., the other temperatures and pressures will be different for this system and likewise different for each fluid used in it. Only two fluids, ammonia and

carbon dioxide, will be here considered, and it will be assumed for convenience that the vapor leaving the coils in which pressure is assumed constant throughout, is dry saturated, and that the condenser is so constructed that the pressure therein corresponds to liquid leaving at 70° F.; actually it will be higher. Under these circumstances the compressor suction pressure will be 29.7 lbs. per square inch absolute for ammonia, and 313.7 lbs. per square inch for carbon

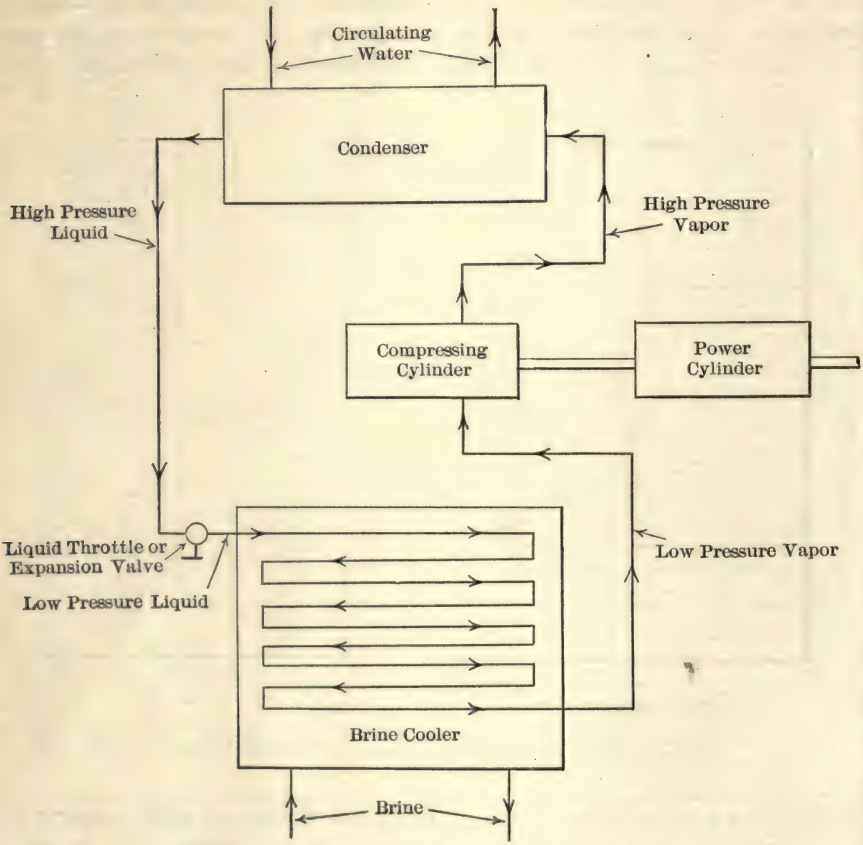


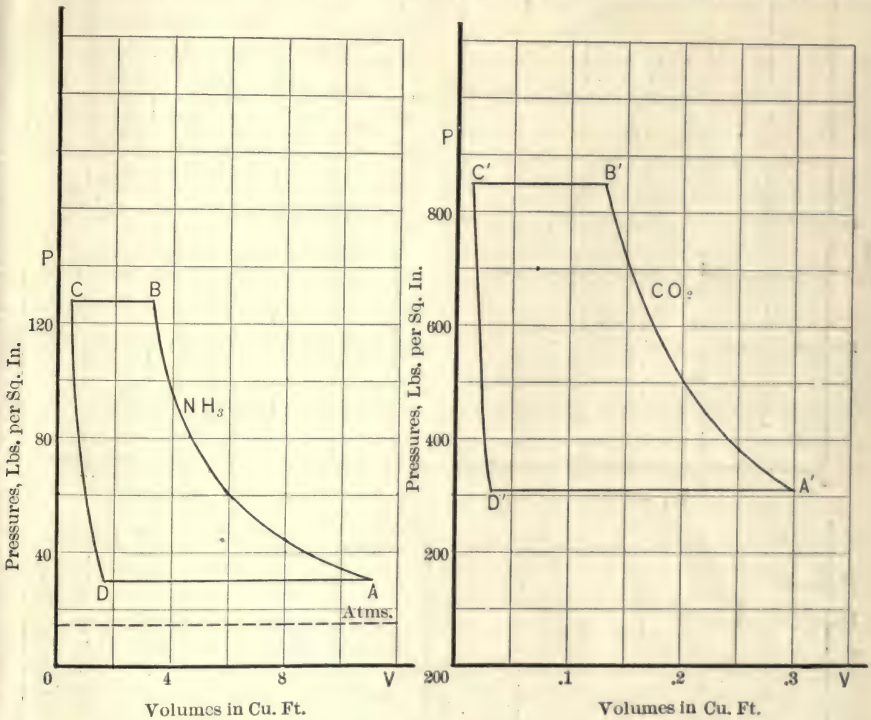
FIG. 313.—Diagrammatic Arrangement of Ammonia or Carbon Dioxide Compression System of Refrigeration.

dioxide, while the corresponding condenser pressures will be 128.4 lbs. per square inch for ammonia and 851.5 lbs. per square inch for carbon dioxide, all absolute and taken directly from Tables XLIX, and L, of the properties of saturated vapors, Chapter IV. Accordingly, the pressure ratios are, $\frac{P_H}{P_L}=4.32$ for ammonia, and $\frac{P_H}{P_L}=2.72$ for carbon dioxide, and neglecting wall action in the

ompressor as well as pipe heating or cooling, the temperature of the gas after elivery from compressor to condenser will be as given by Eq. (1287), taking $\gamma = 1.3$ for both vapors superheated.

$$T_H = T_L\left(\frac{P_H}{P_L}\right)^{\frac{\gamma-1}{\gamma}} = \left\{\begin{array}{ll} 460(4.32)^{.231} = 645^\circ \text{ F. abs. for ammonia} & (a) \\ 460(2.72)^{.231} = 580^\circ \text{ F. abs. for carbon dioxide} & (b) \end{array}\right\}. \quad (1287)$$

For these pressures the two compressor indicator cards are drawn, Fig. 314, or the assumption of no losses and for equal per cents of clearance, $ABCD$



IG. 314.—Ammonia and Carbon Dioxide Compressor Diagrams between Equal Temperature Limits.

epresenting ammonia and $A'B'C'D'$ carbon dioxide. On each, the suction olume is $\overline{DA}$ or $\overline{D'A'}$, and the delivery volume $\overline{BC}$ or $\overline{B'C'}$, so they are equivlent to no clearance cards with these bottom and top lengths. As the funcion of the condenser is that of volume reduction at constant pressure, its action ay be equally well represented by the delivery line $\overline{BC}$ and $\overline{B'C'}$, Fig. 315, nd similarly, the coil evaporation at constant pressure is also represented by he suction line $\overline{DA}$ and $\overline{D'A'}$, so that the whole process is represented by the PV cycles, $ABCD$ and $A'B'C'D'$, for which the corresponding $T\Phi$ cycles are hown at the right, each for one pound of substance.

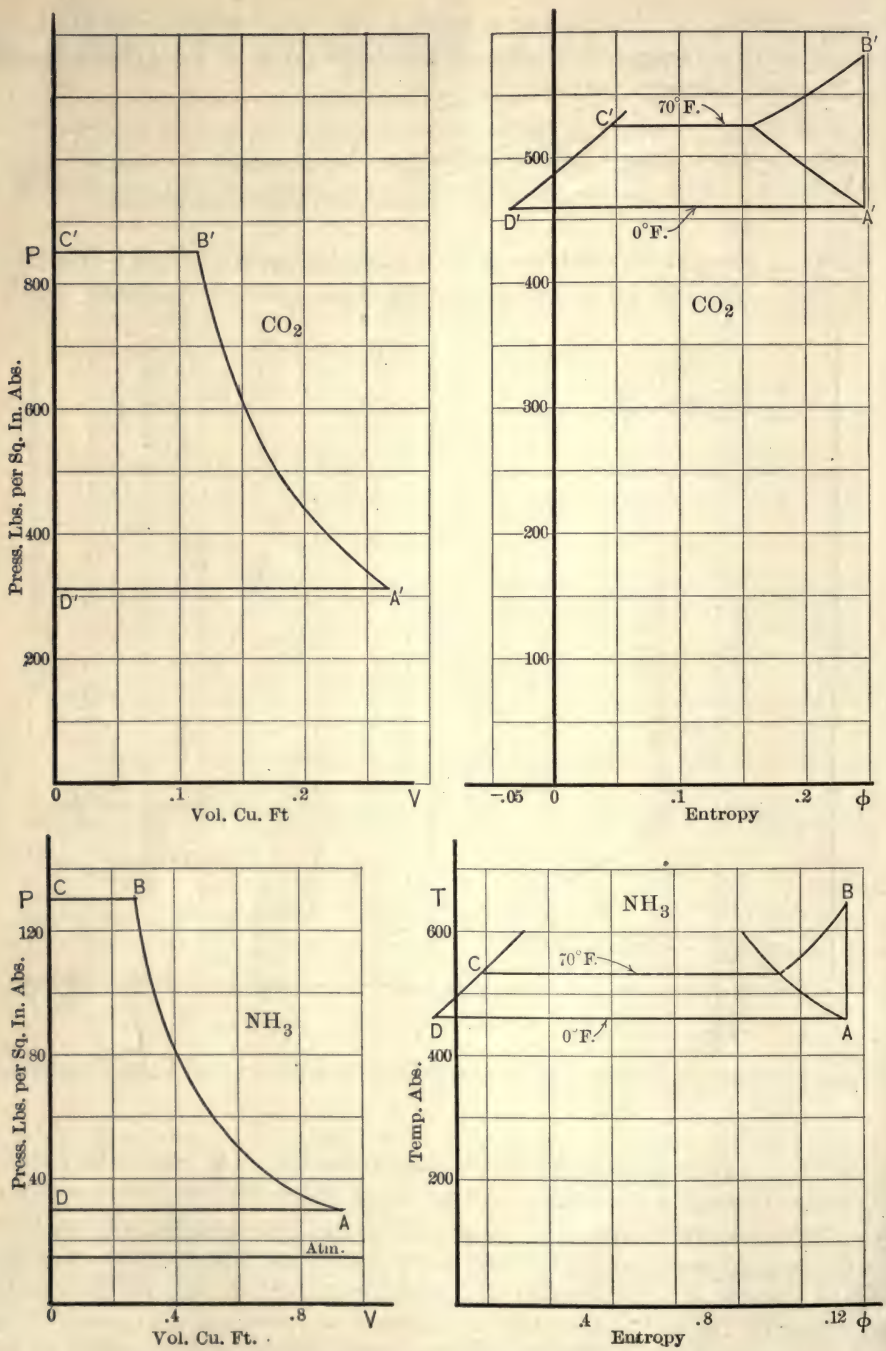


FIG. 315.—Ammonia and Carbon Dioxide Refrigerating Cycles.

The absorption system of apparatus for ammonia is represented in Fig. 316 iagrammatically, which illustrates only one of several possible arrangements, ut a common one that serves as a basis for analysis. Here the condenser, nhydrous ammonia liquid line, expansion valve, evaporating coils and brine ooler are the same as before, all differences being concerned with the drawing ff of the vapor produced in the refrigerating coils and its delivery at the higher ressure to the condenser. This is accomplished by treating an aqueous solu- ion of ammonia, alternately allowing it to absorb ammonia from the evaporat- ng coils in an absorber at a rate determined by a regulating valve and the tem- erature and concentration of the solution, and later heating this richer solution n a generator by means of steam coils. Thus, referring to the diagram, the ow-pressure vapor passes from the evaporating coils through the suction- egulating valve to the absorber chamber where it meets a weaker aqueous olution, being absorbed and so developing heat, which is carried off by the irculating water coil and which latter controls the temperature of absorption. A more concentrated and cool solution of strong liquor is then pumped from this ow-pressure absorption chamber to the high-pressure heating chamber or enerator, passing on the way through a heat exchanger where it gets warmer y cooling weak liquor flowing in the other direction. The heating in the enerator will drive off both ammonia and water vapor but mainly the former, nd this vapor mixture rises through a double cooling chamber or analyzer nd rectifier. It is first cooled a little by coming into contact with the incoming trong liquor in the analyzer, and later more cooled in the rectifier by circulating vater that has been used in the condenser. At each cooling step some water apor is thrown down and returns with such ammonia as it absorbs on the way o the generator, while the ammonia not so reabsorbed passes to the con- enser, as the cooling has not been carried far enough in analyzer and rectifier o condense it. Of course, a little water vapor will pass over to the condenser, naking its delivery consist not of pure anhydrous ammonia, but rather of an queous solution, mainly NH_3, which behaves nearly enough like anhydrous or practical purposes, unless by mismanagement it finally attains too much vater, which must be removed before continuing the refrigeration effectively.

It will be assumed that the condenser discharge is pure anhydrous and in his case the coil and condenser pressures will be the same as for the compressing ystem. Obviously, a considerable range of liquor strengths may be used and still wider range of temperatures maintained in both absorber and generator, o for the purpose of calculation some common values must be assigned to these uantities for preliminary calculation. Accordingly, it will be assumed that the enerator has the uniform temperature of 250° F., which corresponds to aturated steam at 29.82 lbs. per square in absolute or 15 lbs. gage pressure n round numbers, receiving rich liquor 35 per cent ammonia by weight, at 15° F. It will also be assumed that the weak liquor, 25 per cent, enters the bsorber at 125° F. so that the exchange has reduced it 125°, while the 35 per ent liquor leaves the absorber at 100° F. and is warmed in the exchanger 0° less than the weak is cooled because of superior specific heat, so that

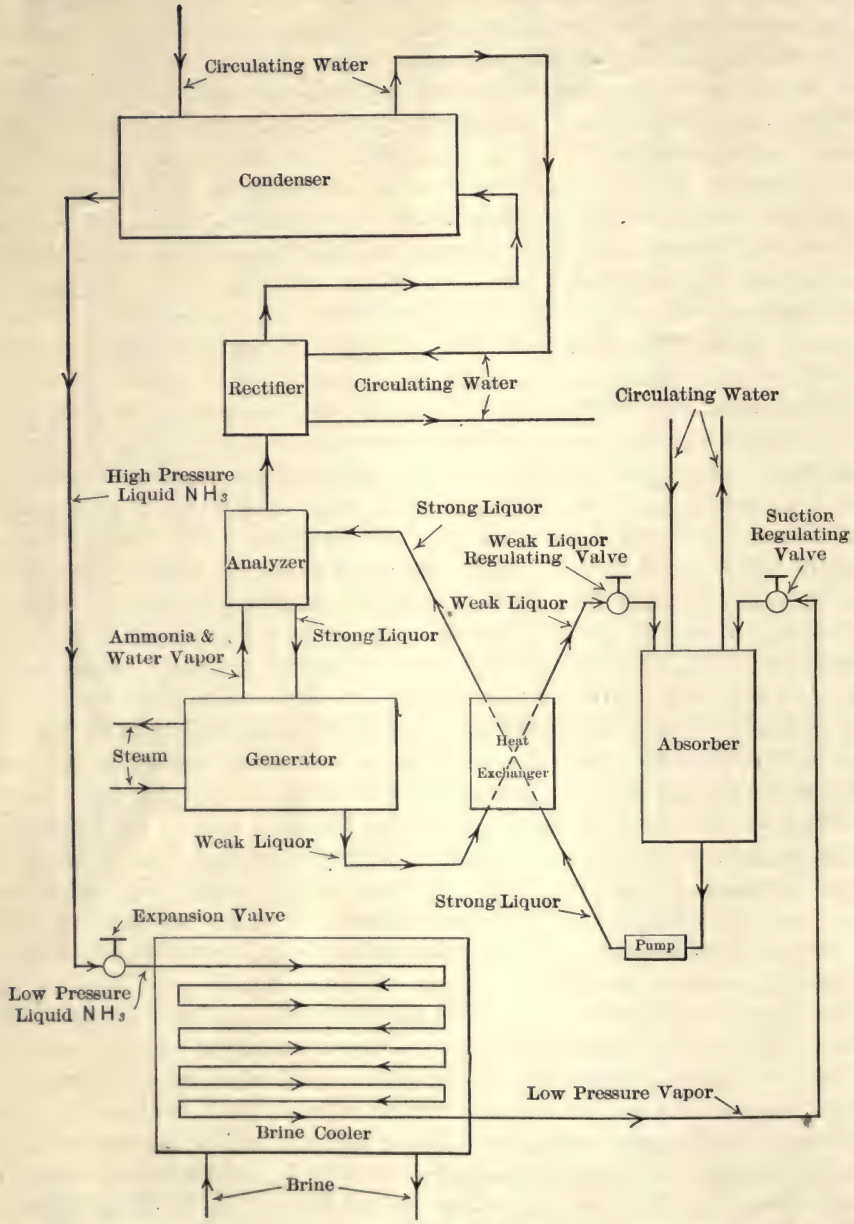


FIG. 316.—Diagrammatic Arrangement of the Ammonia Absorption System of Refrigeration.

its rise of temperature is $125°-10°=115°$, entering the generator at 100° F. $+115°=215°$ F. Finally, it will be assumed that the pressure in the absorber is 5 lbs. per square inch gage lower than in the evaporating coils by reason of the action of the regulating valve, which makes the absorber pressure $29.7-5=24.7$ lbs. per square inch absolute. Referring to the table of properties of ammonia water solutions, Chapter IV, and the corresponding charts, it appears that a solution at the pressure 24.7 lbs. per square inch absolute in the absorber and at a temperature of 100° F. would if saturated contain 00 per cent by weight, so that the 35 per cent solution which leaves is not saturated. Similarly, in the generator the condenser pressure of 128.4 lbs. per square inch is maintained and at the temperature of the 25 per cent solution, leaving it 250° F., would if saturated contain 00 per cent by weight.

These quantities are all subject to almost infinite variation, yet something must be assumed approximately corresponding to practice as a basis of comparative computation, and in this case it is especially important, as no thermal diagrams or cycles can be conveniently established to represent in general terms the whole process as is so easy for the other systems.

This general description of the methods and the functions of the structural elements of those mechanical refrigerating systems as are to-day reduced to standard engineering practice, will serve as a basis of both absolute and comparative calculations concerning them, in which the important quantities to be determined are, the quantity of refrigerating fluid that must be circulated per minute to produce a unit of refrigerating effect, the size and displacement of cylinders necessary, the work that must be done per unit of refrigeration and the general relations between heats gained and lost by the fluid and the work that must be done on it.

21. Performance of Mechanical Refrigerating Cycles and Systems. Quantity of Fluid Circulated per Minute per Ton Refrigeration, Horse-power, and Heat Supplied per Ton. Refrigeration per Unit of Work Done and its Relation to Thermal Efficiency of the System. As in the cases of the steam boiler and gas producer where capacity is to be measured in terms of a quantity of heat per minute delivered, a capacity unit has been established and given the arbitrary term "horse-power," with a different meaning in each case, so in dealing with refrigerating systems engineering practice has developed and adopted a somewhat similar term but with a more rational basis as the unit of capacity, and this is the *ton refrigeration*. If the latent heat of fusion of ice be taken as 144 B.T.U., which is nearly correct, and the ton as 2000 lbs., then one ton of ice in melting would absorb from the surroundings $2000\times144=288{,}000$ B.T.U., and if this were accomplished in a day of 24 hours, heat would be flowing at the rate of 288,000 B.T.U. per 24 hours, or 12,000 B.T.U. per hour, or 200 B.T.U. per minute. Whenever the refrigerating fluid absorbs heat at these rates it is said to be developing a ton refrigeration, or when brine absorbs heat at the same rate it likewise is said to be developing a ton refrigeration, so that capacity may be measured, as in the refrigerating fluid, as in the brine, and also as from the body giving up heat such as beef, or water ice. When at one point of the

system, heat is being transferred at the ton refrigeration rate it will not be necessarily, at other points in the system, so it is necessary to establish a standard that shall be invariable and that is in the fluid itself. Therefore, *a system has one ton refrigerating capacity when the refrigerating fluid is absorbing heat in the primary refrigerating coils at the rate of* 288,000 *B.T.U. per* 24 *hours,* 12,000 *B.T.U. per hour, or* 200 *B.T.U. per minute though the useful effect in cooling or freezing will be less if there are any gains or losses between.*

Each pound of fluid is capable of taking up in the refrigerating coils a definite amount of heat to be called refrigeration per pound of fluid, so that to produce the ton refrigeration a definite weight of fluid in pounds per minute must be circulated according to the general formula, Eq. (1288):

$$(\text{Lbs. of fluid per minute per ton}) = \frac{200}{(\text{Refrigeration per lb. of fluid})}. \quad . \quad (1288)$$

This is a general expression for which there is a volumetric equivalent given by Eq. (1289), where the volume is that of delivery from the refrigerating coils.

$$\left.\begin{aligned} \left\{\begin{matrix}\text{Cu.ft. fluid delivered from refrigerating coils per min. per ton}\end{matrix}\right\} &= 200\left(\frac{\text{Cu. ft. per lb. fluid at coil delivery}}{\text{Refrigeration per lb. of fluid}}\right) \quad (a) \\ &= \left(\frac{200}{\text{Refrigeration per cu.ft. fluid at coil del.}}\right) \quad (b) \end{aligned}\right\} \quad (1289)$$

Assuming no heating or cooling of the fluid between the points of coil delivery and compressor entrance, the same volume will approach the compressor in compression systems, as left the coils, but on entering the compressor it will be heated and expanded; it will also suffer a loss of pressure, and reexpansion in the compressor will delay entrance. Defining the true volumetric efficiency, E_V', as the ratio of the volume drawn from the suction measured at conditions of pressure and temperature external to the cylinder, to the displacement volume, the compressor displacement per ton capacity may be set down as in Eq. (1290), for the air and compression systems.

$$\left.\begin{aligned} \left\{\begin{matrix}\text{Compressor displacement cu.ft. per min. per ton}\end{matrix}\right\} &= \left[\frac{200}{E_V'}\left(\frac{\text{Cu.ft. per lb. fluid at coil delivery}}{\text{Refrigeration per lb. of fluid}}\right)\right] (a) \\ &= \left[\frac{200}{E_V'(\text{Refrig. per cu.ft. fluid at coil del.})}\right] \quad (b) \end{aligned}\right\} \quad (1290)$$

That which corresponds in the absorption system to compressor displacement in the other systems, is the displacement of the pump which is equal to the

cu.ft. of strong liquor per minute that is capable of carrying off the quantity of ammonia circulating, if its volumetric efficiency is 100 per cent, which it may be substantially if of proper type and properly run.

Let w_R = pounds rich liquor to be circulated per pound of NH_3 taken up in absorber;

" C_R = per cent NH_3 in rich liquor;

" C_W = per cent NH_3 in weak liquor.

Then $\left(\frac{C_W}{100}\right)$, and $\left(\frac{C_R}{100}\right)$ = pounds NH_3 per pound solution weak and rich;

" $\left(\frac{100-C_W}{100}\right)$, and $\left(\frac{100-C_R}{100}\right)$ = pounds water per pound solution weak and rich;

" $\left(\frac{C_W}{100-C_W}\right)$, and $\left(\frac{C_R}{100-C_R}\right)$ = pounds NH_3 per pound water in weak and rich solution.

Therefore, the weight of ammonia gained per pound of water when the solution passes from the weak to the rich state is, $\left[\left(\frac{C_R}{100-C_R}\right)-\left(\frac{C_W}{100-C_W}\right)\right]$. But 1 lb. of water is associated with $\left(\frac{100}{100-C_R}\right)$ pounds of rich solution, therefore, the pounds of NH_3 taken up per pound of rich liquor formed, is equal to the pounds NH_3 gained per pound of water, divided by pounds of rich liquor per pound of water; and the pounds rich liquor formed per pound of NH_3 taken up is the reciprocal of this and given by Eq. (1291):

$$w_R=\frac{\left(\frac{100}{100-C_R}\right)}{\left(\frac{C_R}{100-C_R}\right)-\left(\frac{C_W}{100-C_W}\right)}=\frac{100(100-C_W)}{C_R(100-C_W)-C_W(100-C_R)}=\frac{100-C_W}{C_R-C_W}. \quad (1291)$$

The displacement of the pump per ton will, therefore, be the product of the weight of rich liquor per pound of NH_3 absorbed, into the cu.ft. of liquor per pound, and the pounds of NH_3 per minute per ton.

$$\left\{\begin{matrix}\text{Rich liquor pump}\\ \text{displacement per}\\ \text{minute per ton}\end{matrix}\right\}=\left(\frac{100-C_W}{C_R-C_W}\right)\times\left(\begin{matrix}\text{Cu.ft. rich}\\ \text{liquor per lb.}\end{matrix}\right)\times\left[\frac{200}{\begin{matrix}\text{Refrigeration}\\ \text{per lb. } NH_3\end{matrix}}\right]. \quad (1292)$$

Solution of all these equations for weights and volumes of fluids and the corresponding displacements per minute per ton, depends on the evaluation of the term, (Refrigeration per pound of fluid). In the case of the air system this is, of course, nothing more than the product of specific heat at constant pressure into the temperature rise in the coils. For the liquid vapor system it is made up of parts depending on the final quality of vapor discharge which may be set down as in Eq. (1293):

$$\left\{\begin{array}{l}\text{Refrig-}\\ \text{eration}\\ \text{per lb.}\\ \text{fluid}\end{array}\right\} = \left\{\begin{array}{ll} C_p\left[\begin{array}{l}\text{(Temp. air leaving coils)} -\\ \text{(Temp. air entering coils)}\end{array}\right] \text{For air system} & (a)\\ \left[\begin{array}{l}\text{(Latent heat)} \times \text{(Quality}\\ \text{of vapor leaving coils)}\\ -\text{(Heat of liquid bet.}\\ \text{supply and coil temp.)}\end{array}\right] \left\{\begin{array}{l}\text{Final wet vapor}\\ \text{for vapor sys-}\\ \text{tems}\end{array}\right\} & (b)\\ \left[\begin{array}{l}\text{(Latent heat)}\\ -\text{(Heat of liquid bet.}\\ \text{supply and coil temp.)}\end{array}\right] \left\{\begin{array}{l}\text{Final dry sat.}\\ \text{vapor for va-}\\ \text{por systems}\end{array}\right\} & (c)\\ \left[\begin{array}{l}\text{(Latent heat)}\\ -\text{(Heat of liquid bet.}\\ \text{supply and coil temp.)}\\ +C_p\,[\text{(Final vapor}\\ \text{temp.)} - \text{(Sat. vapor}\\ \text{temp. in coils)}]\end{array}\right] \left\{\begin{array}{l}\text{Final super-}\\ \text{heated vapor}\\ \text{for vapor sys-}\\ \text{tems}\end{array}\right\} & (d)\end{array}\right\} \quad (1293)$$

These are all tabular quantities except the heat of air and of vapor superheat, which are easily evaluated, but to facilitate determinations the charts of Figs. 317 for ammonia and 318 for carbon dioxide have been calculated and are somewhat similar to those given for boiler capacity, where B.T.U. per pound of steam for boilers and pounds evaporation per hour per boiler horse-power, correspond to B.T.U. refrigeration per pound of fluid and pounds of fluid per minute per ton here.

From the data of these charts applied to the equations above the displacements of compressors and pumps can be computed by the slide rule, introducing the chart quantities in the equations. When superheated vapor densities are to be evaluated either vapor may be assumed to behave as a perfect gas, volumes being directly, and density inversely proportional to absolute temperatures. The volume per pound of ammonia solutions is to be evaluated from that of water in the steam tables, dividing the water value by the specific gravity of the solution table, at the end of Chapter IV, or read off directly on the additional chart, Fig. 319.

With a given weight and volume of fluid per minute per ton passing through the cylinders, the maximum and minimum pressures for which are determined by the temperature desired in the coils and that available by the cooling water in coolers or condensers, it follows that a definite horse-power per ton refrigeration will result for each system and each operative condition of any one. The cycle

of operations to PV or $T\Phi$ coordinates offers a means of evaluating the work in foot-pounds per pound of fluid which, when multiplied by the pounds per minute per ton, gives work in foot-pounds per minute per ton and this divided by 33,000 is the horse-power indicated, per ton refrigeration. These same cycles, but more particularly those drawn to $T\Phi$ coordinates, also offer means of directly evaluating the work per B.T.U. absorbed by the fluid in the refrig-

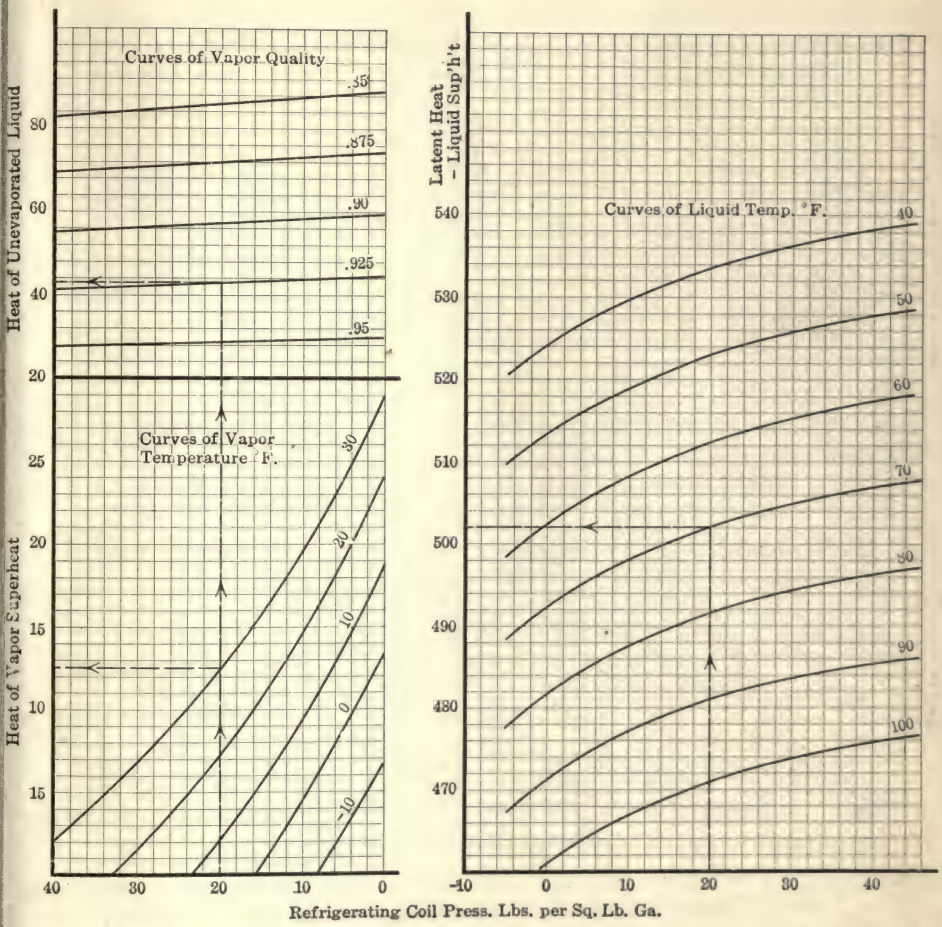


Fig. 317.—Chart to Determine Available Refrigerating Effect per Pound of *Ammonia* for Any Refrigerator Pressure and Any Refrigerator or Liquid Temperature.

erator, the procedure being similar to that used with engine cycles in obtaining the ratio of work to heat supplied which was the efficiency of the work developing cycle. Both of these determinations are worth while.

As compression is always actually or substantially adiabatic and as vapors may be wet, dry saturated or superheated and as the value of "s" may and

usually is a variable as quality changes, making the PV method uncertain, the $T\Phi$ method of evaluating the work of compression for wet vapors is the only one that can be relied upon for accurate results. The Mollier diagram for NH_3 and CO_2 gives these results directly, so in Figs. 320 and 322 are given the $T\phi$ diagrams for NH_3 and CO_2, and in Figs. 321 and 323 the corresponding Mollier diagrams. It must be remembered that the results from these Mollier diagrams

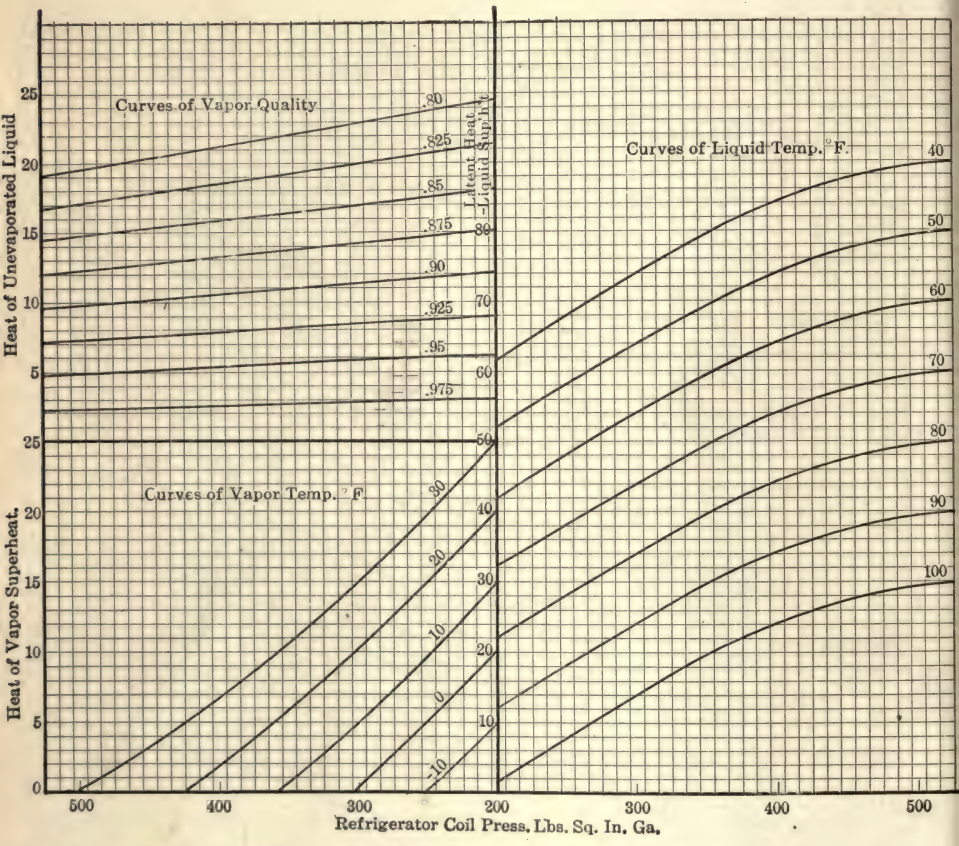


FIG. 318.—Chart to Determine Available Refrigerating Effect per Pound of *Carbon Dioxide* for any Refrigerator Pressure and any Refrigerator or Liquid Temperature.

give the work in B.T.U. for the whole cycle, which is similar in form to the Rankine for steam.

The diagrams give also the B.T.U. absorbed per pound at the lower temperature, so the ratio of the work to refrigeration can be taken directly from the diagrams. This is the measure of the goodness of the process for refrigeration, sometimes though improperly termed efficiency, improper in that it is a different meaning for the word than that heretofore used. To facilitate

calculation still further but also to show most clearly the influence of the several factors that control it, this ratio,

$$\frac{\text{B.T.U. work of refrigerating cycle}}{\text{B.T.U. absorbed at low temp.}}$$

is plotted, Figs. 324 and 325 for ammonia and Fig. 326 and 327 for carbon dioxide, to coordinates similar to those used for the Rankine cycle efficiency, which is a similar ratio, the work there, however, being divided by the heat taken in at the high temperature.

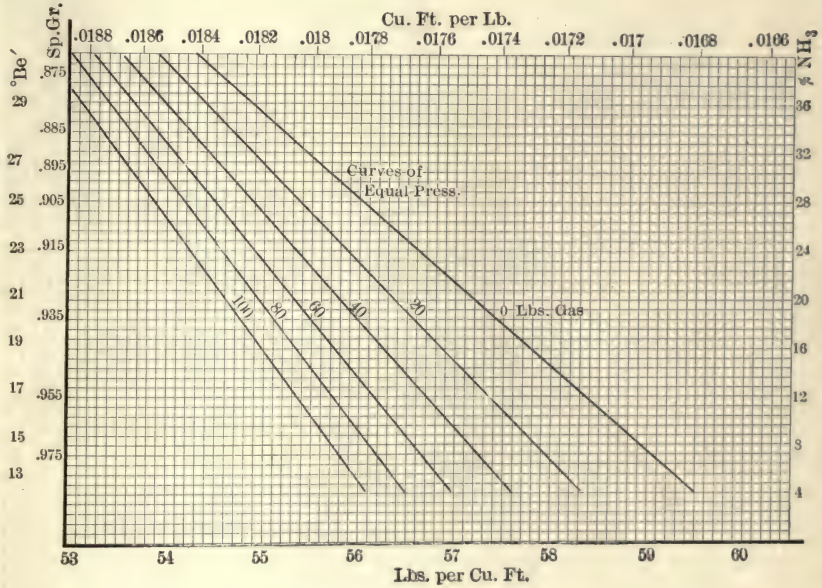
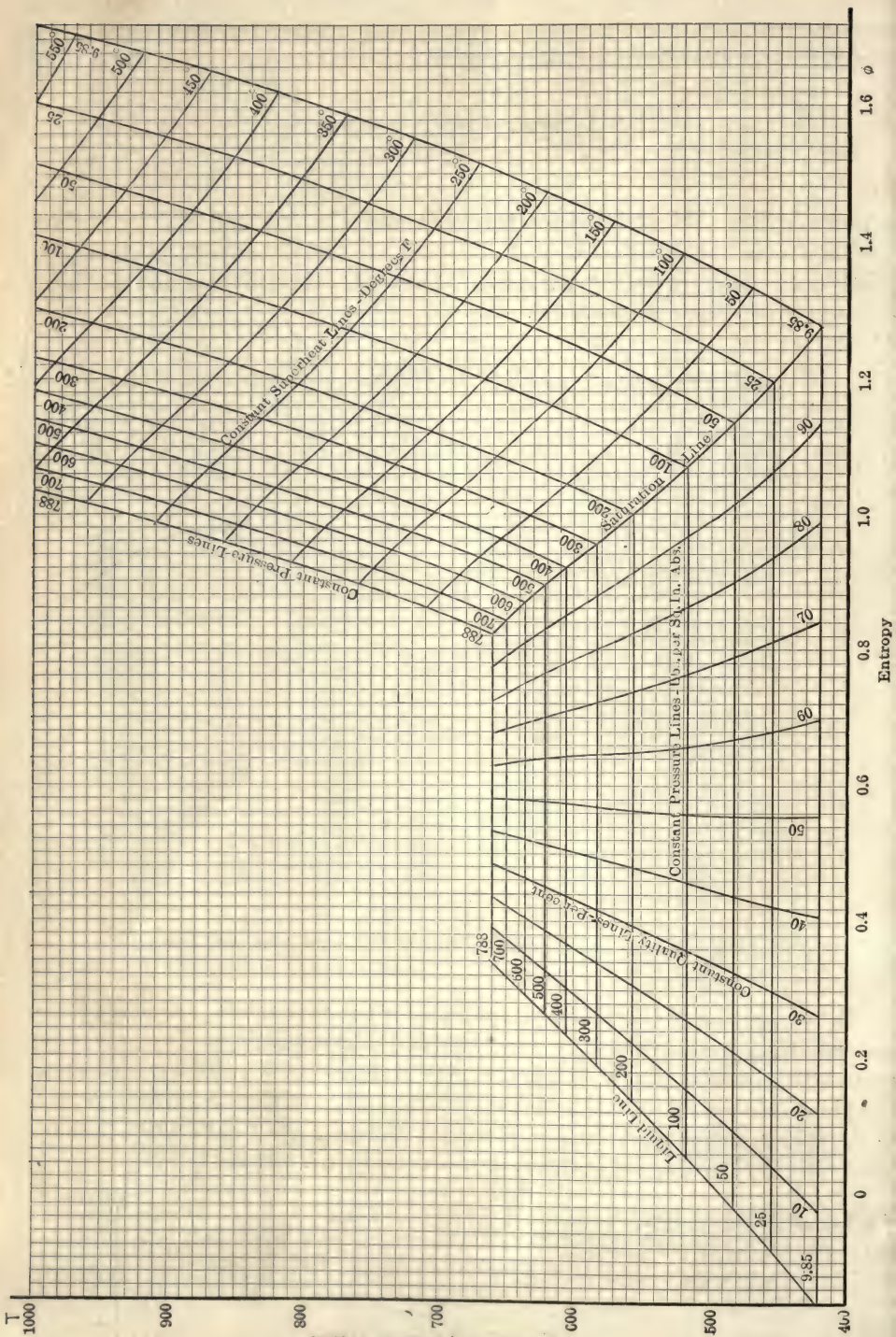


FIG. 319.—Density and Specific Volume of Ammonia-water Solutions.

The evaluation of this performance ratio for the gas cycle is best made algebraically by methods similar to those used in the analysis of the Brayton work cycle. This cycle is shown in Fig. 328, $ABCD$, to both PV and $T\Phi$ coordinates. Here heat is taken up from D to A in the refrigerator and abstracted from B to C in the cooler, so the work done between the two adiabatics is equal to the difference on the $T\Phi$ diagram between the areas under CB and DA:

$$\left.\begin{aligned}\left\{\begin{matrix}\text{Work of air refrigerating}\\ \text{cycle in B.T.U.}\\ \text{per pound of air}\end{matrix}\right\} &= \left(\begin{matrix}\text{Heat given up to}\\ \text{the water cooler}\end{matrix}\right) - \left(\begin{matrix}\text{Heat taken in at}\\ \text{the refrigerator}\end{matrix}\right) && (a)\\ &= C_p(T_b - T_c) - C_p(T_a - T_d) && (b)\\ &= C_p(T_b - T_c - T_a + T_d) && (c)\end{aligned}\right\} \quad (1294)$$


Constant Superheat Lines - Degrees F.
Constant Pressure Lines
Saturation Line
Constant Pressure Lines - Lbs. per Sq. In. Abs.
Constant Quality Lines - Per Cent
Liquid Line
Temperature, Degrees F. Abs.
Entropy
T
ϕ

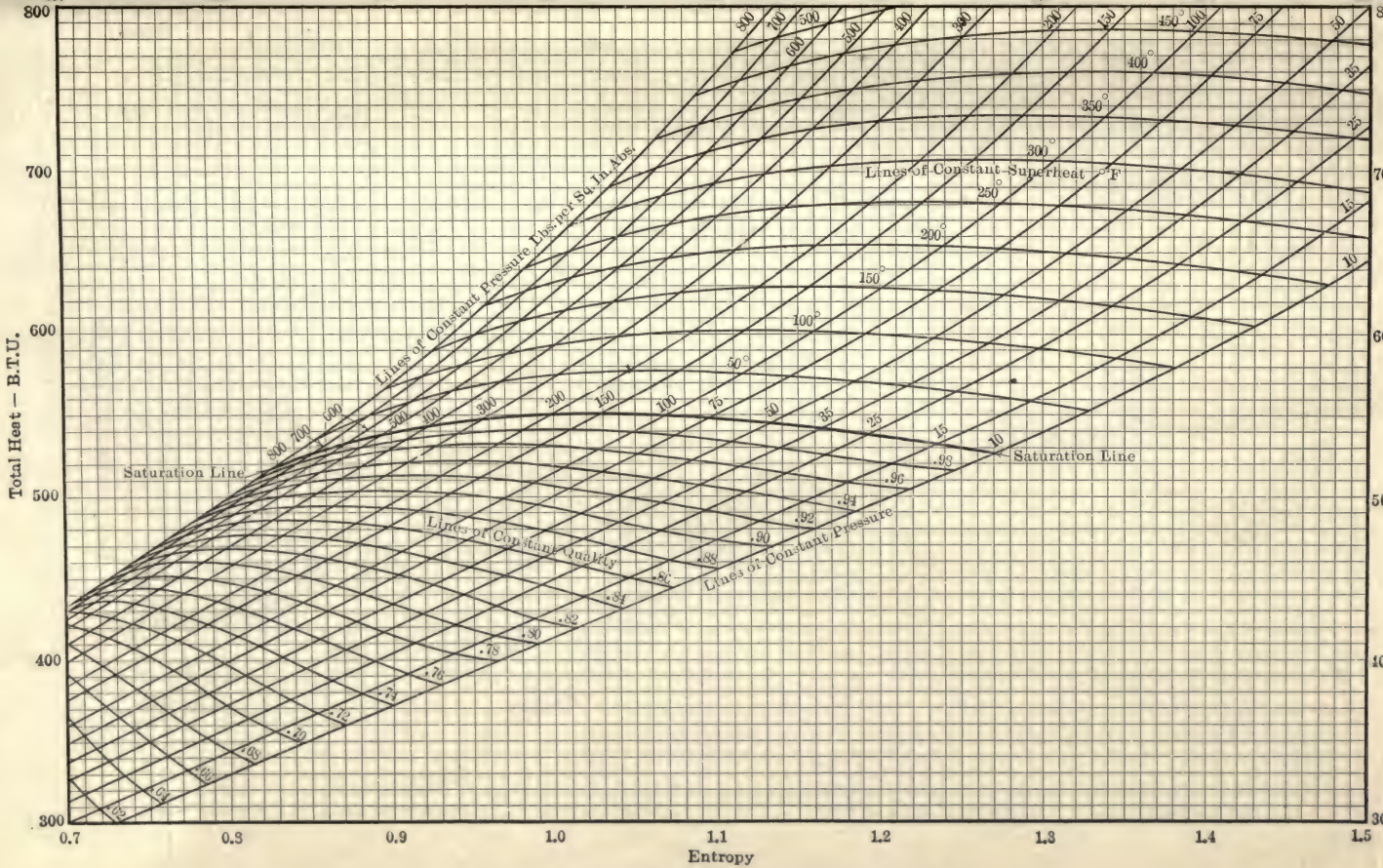
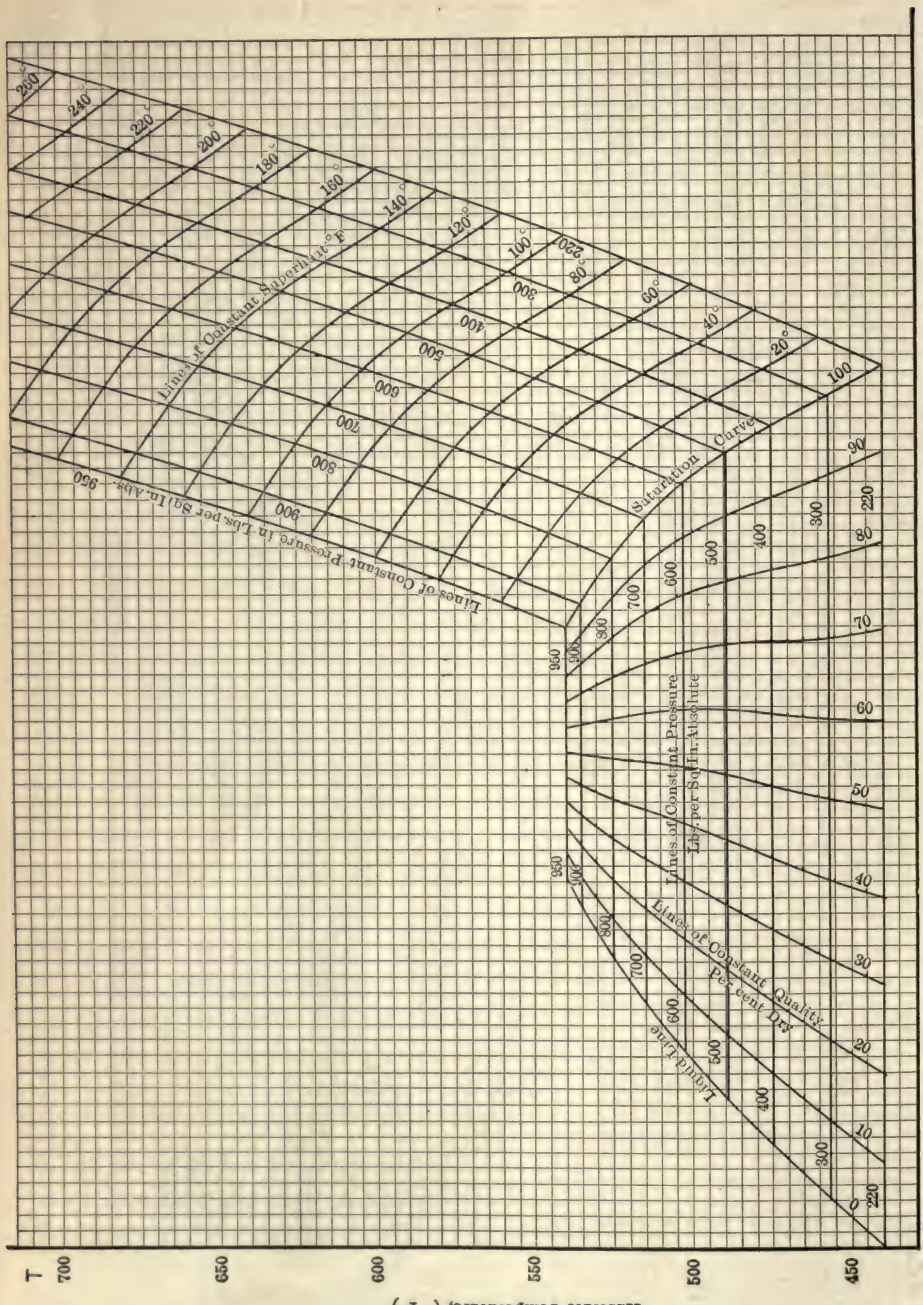


FIG. 321.—Mollier Diagram for Ammonia (NH_3)


Lines of Constant Superheat, °F
Lines of Constant Pressure, In Lbs. per Sq. In. Abs.
Saturation Curve
Lines of Constant Pressure
Lbs. per Sq. In. Absolute
Lines of Constant Quality
Per cent Dry
Liquid Line
Absolute Temperature, (° F)
T
ϕ

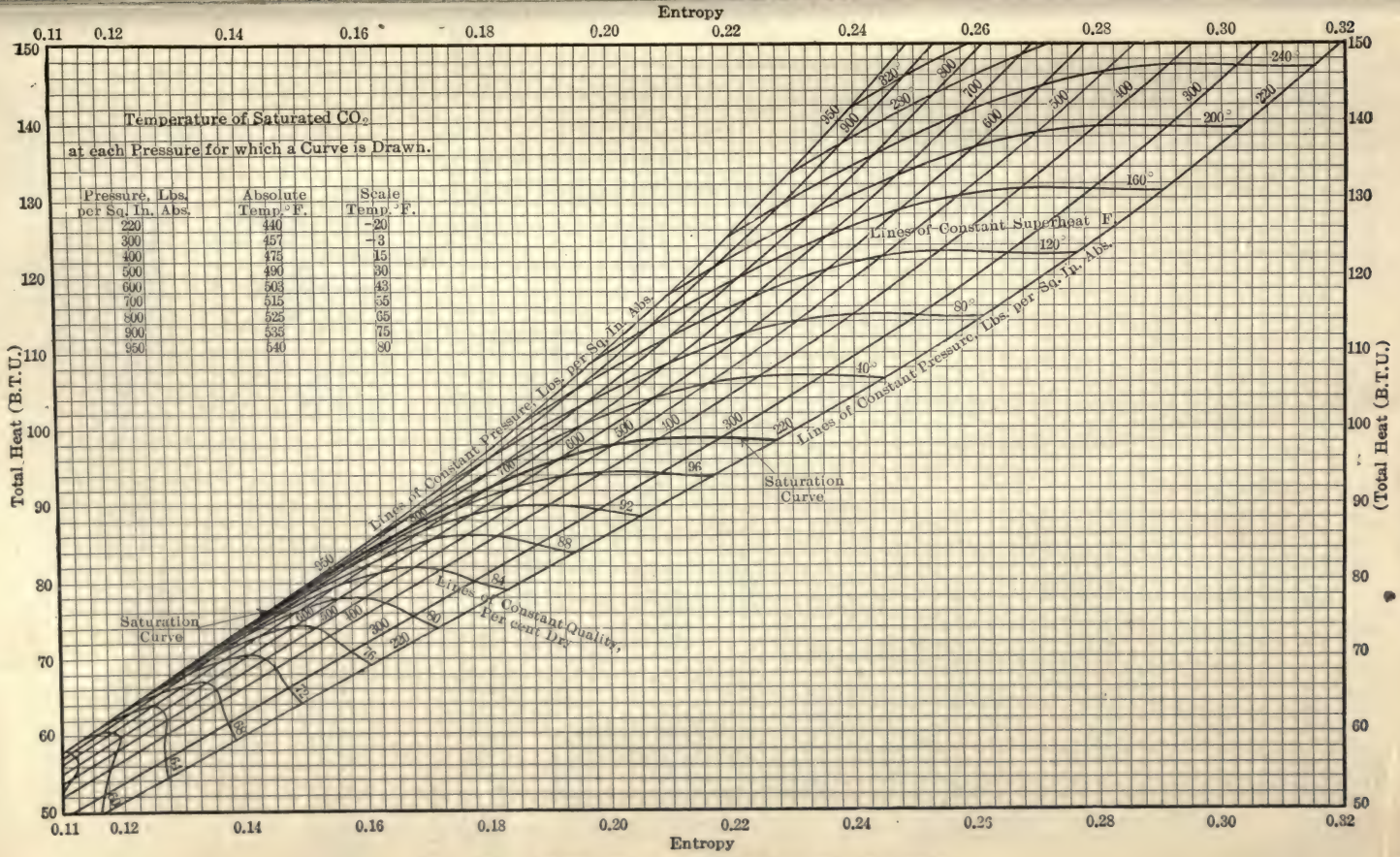


FIG. 323.—Mollier Diagram for Carbon Dioxide (CO_2).

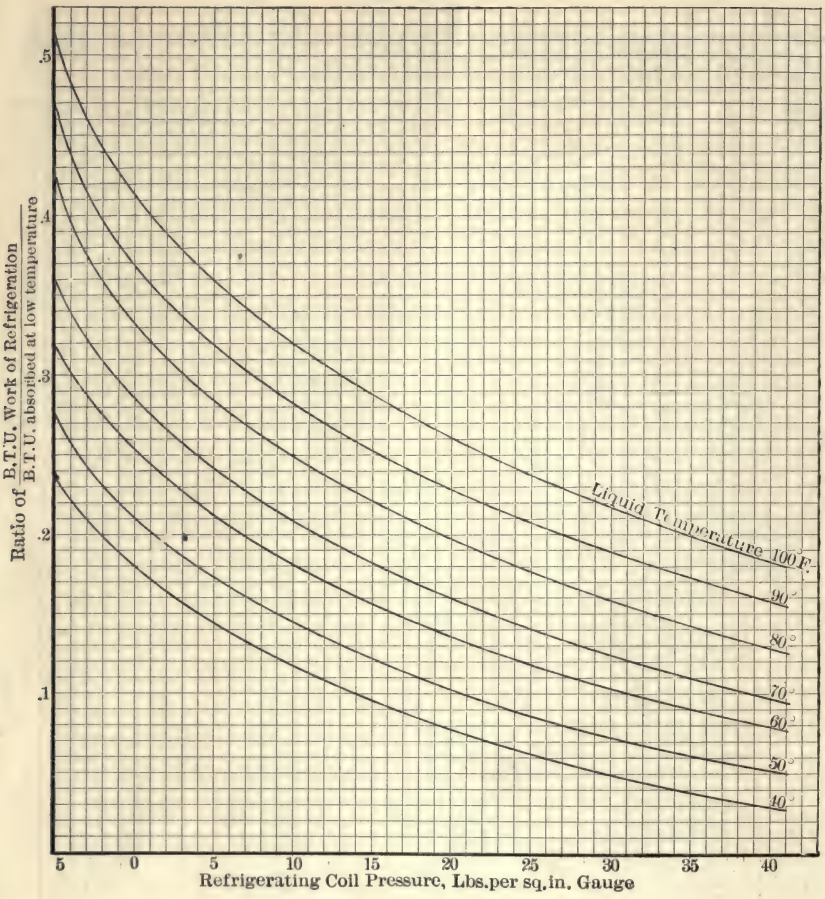


FIG. 324.—Work in B.T.U., per B.T.U. Absorbed in Refrigeration, by Ammonia Supplied as Liquid at any Temperature and Vaporizing at any Coil Pressure to Dry Saturated Vapor.

$$\left.\frac{\text{Work of air refrigerating cycle}}{\text{Heat given to water cooler}}\right) = 1 - \frac{T_a - T_d}{T_b - T_c} = 1 - \frac{T_a\left(1 - \frac{T_d}{T_a}\right)}{T_b\left(1 - \frac{T_c}{T_b}\right)}$$

$$= \left\{ \begin{array}{l} 1 - \dfrac{T_a}{T_b} = 1 - \dfrac{T_d}{T_c} \\ 1 - \left(\dfrac{P_d}{P_c}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\dfrac{P_a}{P_b}\right)^{\frac{\gamma-1}{\gamma}} \\ 1 - \left(\dfrac{V_c}{V_d}\right)^{\gamma-1} = 1 - \left(\dfrac{V_b}{V_a}\right)^{\gamma-1} \end{array} \right\} = E_B = \left\{ \begin{array}{l} \text{Efficiency of Brayton work cycle} \end{array} \right\} \left\{ \begin{array}{l} (a) \\ (b) \\ (c) \end{array} \right\} \quad (1295)$$

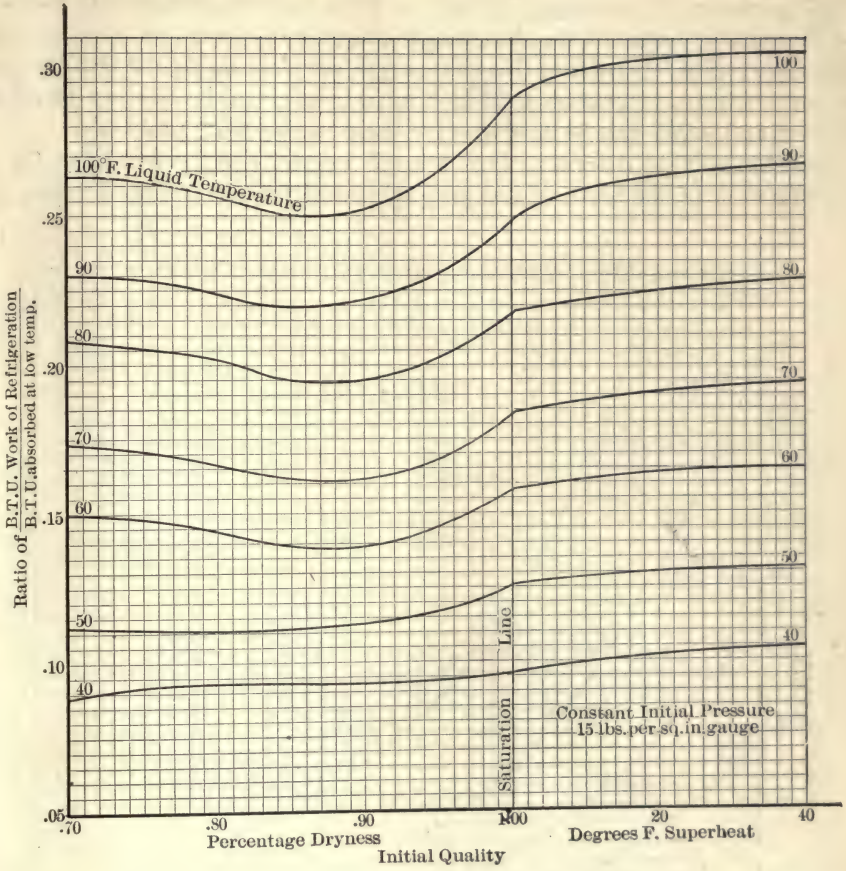


FIG. 325.—Work in B.T.U., per B.T.U. Absorbed in Refrigeration, by Ammonia Supplied as Liquid at any Temperature and Vaporizing to any Quality or Superheat at 15 Pounds per Square Inch Gage.

$$\left(\frac{\text{Work of air refrigerating cycle}}{\text{Heat absorbed in refrigeration}}\right) = \frac{T_b - T_c}{T_a - T_d} - 1 = \frac{T_b\left(1 - \frac{T_c}{T_b}\right)}{T_a\left(1 - \frac{T_d}{T_a}\right)} - 1$$

$$= \left\{ \begin{array}{l} \dfrac{T_b}{T_a} - 1 = \dfrac{T_c}{T_d} - 1 \\ \left(\dfrac{P_b}{P_a}\right)^{\frac{\gamma-1}{\gamma}} - 1 = \left(\dfrac{P_c}{P_d}\right)^{\frac{\gamma-1}{\gamma}} - 1 \\ \left(\dfrac{V_a}{V_b}\right)^{\gamma-1} - 1 = \left(\dfrac{P_d}{P_c}\right)^{\gamma-1} - 1 \end{array} \right\} = \left(\frac{E_B}{1 - E_B}\right) \qquad \left\{ \begin{array}{l} (a) \\ (b) \\ (c) \end{array} \right\} \qquad (1296)$$

The importance of expressing the ratio of work done to refrigeration effect fo the air cycle in terms of the Brayton work-cycle efficiency, is due purely t convenience of calculation as this efficiency is the same as for the Otto cycle fo the same compression, curves for which have been given in Section 11 of thi Chapter and which can be used to solve these refrigerating problems.

Pressure-volume determinations are, of course, equally available for th determination of the work of the compressors of the air and the compression

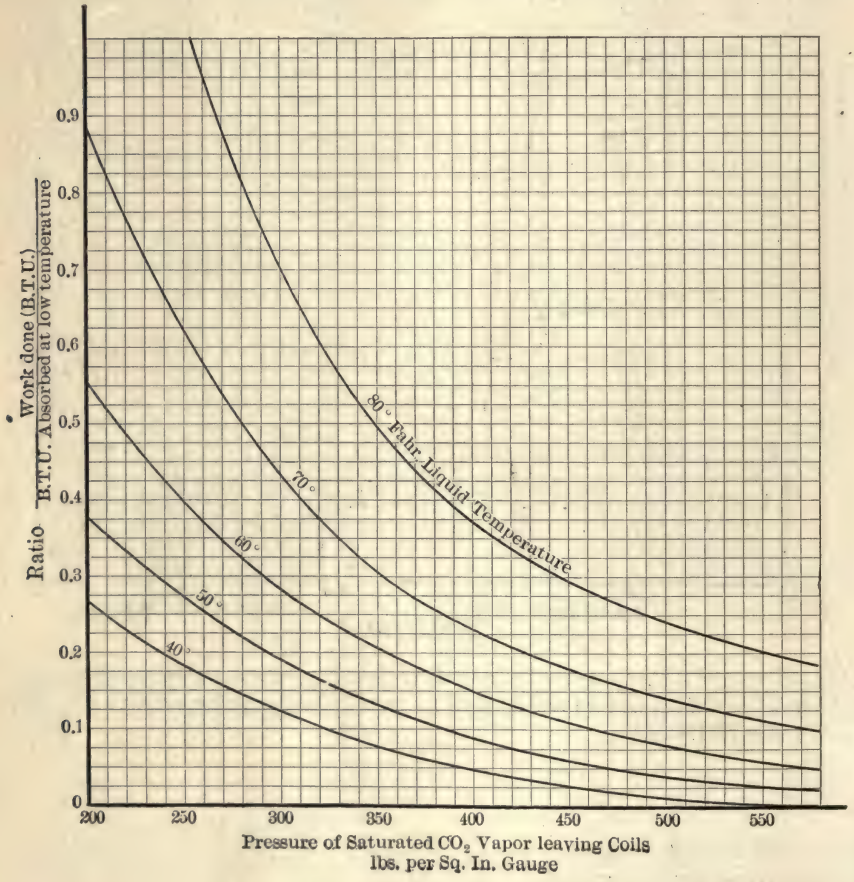


FIG. 326.—Work in B.T.U., per B.T.U. Absorbed in Refrigeration, by Carbon Dioxide Supplie as Liquid at any Temperature and Vaporizing at any Coil Pressure to Dry Saturated Vapor

vapor systems, of the engine of the air system, and of the pump of the absorp tion system, but these methods need not be repeated here as they are completely presented in Chapters II and III, except that for pump work, which is to b evaluated as for expansive fluids with no cut-off or compression, but without an reexpansion of clearance fluid. It will be found for the vapors even if they are superheated, that the work as determined by PV methods does not check the

work as determined by $T\Phi$ methods and the difference is a measure of the uncertainty of the physical properties, specific heat of liquid and of superheated vapor, latent heat and densities of liquid and vapor. Of course, these data might be manipulated to give checking results for work, but this is an improper pro-

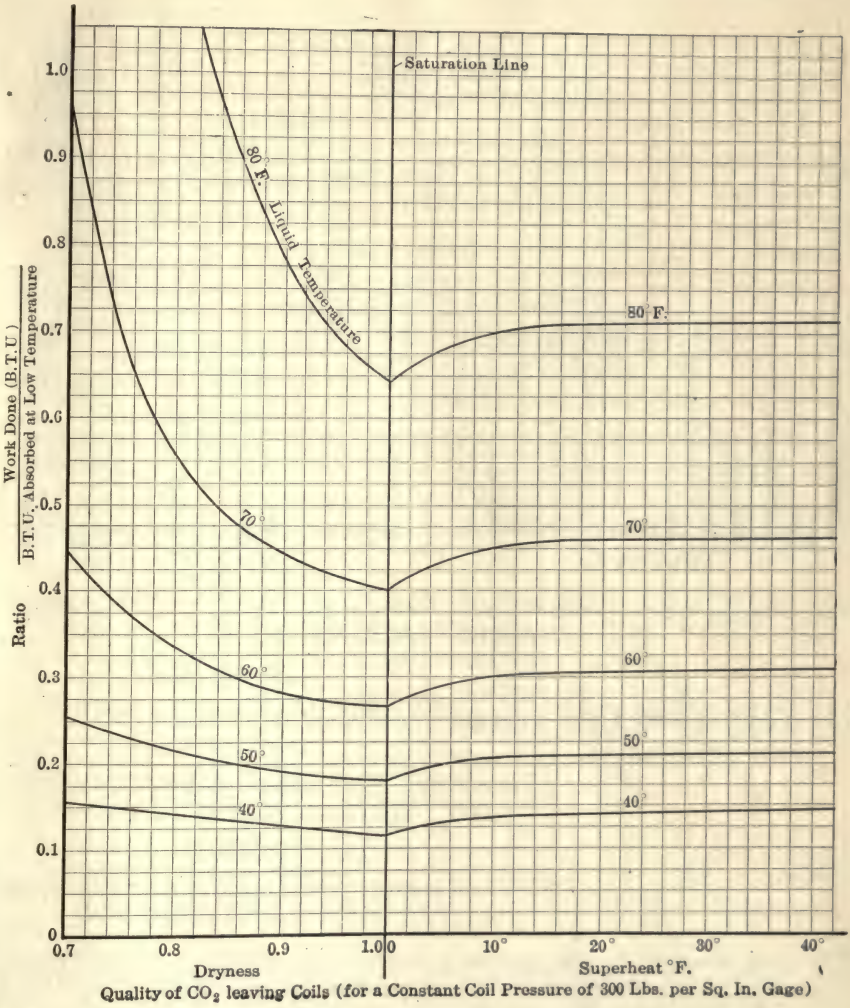


FIG. 327.—Work in B.T.U., per B.T.U. Absorbed in Refrigeration, by Carbon Dioxide Supplied as Liquid at any Temperature and Vaporizing to any Quality or Superheat at 300 Pounds per Square Inch Gage.

cedure; the best practice is to use both methods and assume the correct result to be somewhere between but nevertheless indeterminate with precision, until a redetermination of properties experimentally has been made, such as is about to be undertaken by the U. S. Bureau of Standards.

From the values of the work in heat units, per unit of heat absorbed in refrigeration the horse-power per ton refrigeration follows by the simple transformations below, which show a direct proportionality, for the compression systems.

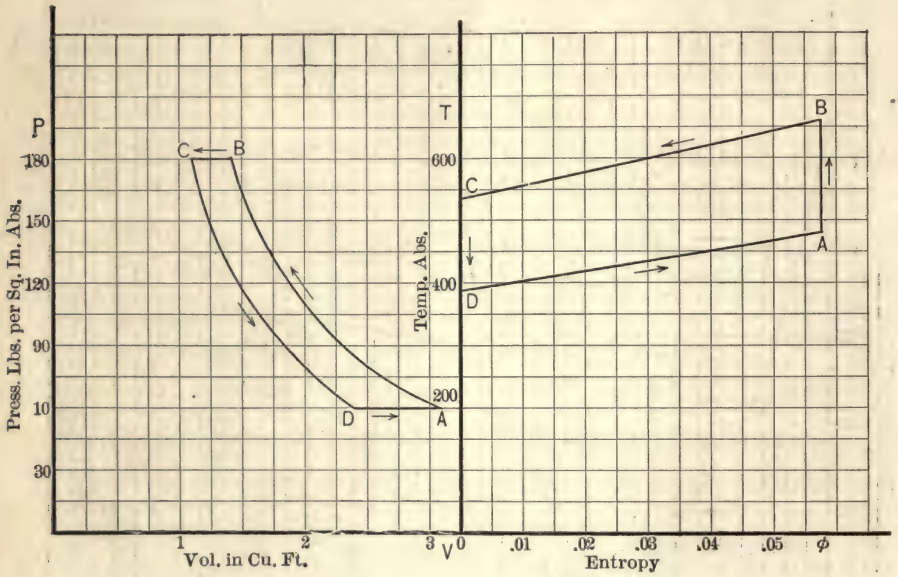


FIG. 328.—Air System. Diagram for Evaluating Work in B.T.U., per Unit of Heat Absorbed in Refrigeration.

$$\text{(I.H.P. per ton refrigeration)} = \frac{778}{33000} \times \text{(Work in B.T.U. per min. per ton)}$$

$$= \frac{778 \times 200}{33000} \times \left(\frac{\text{Work in B.T.U. per minute}}{\text{B.T.U. absorbed per minute}} \right)$$

$$= 4.71 \times \left(\frac{\text{Work in B.T.U.}}{\text{B.T.U. absorbed at low temp.}} \right) \quad (1297)$$

The heat supplied to the system per ton refrigeration is as important a determination as the work or horse-power per ton, and in the case of the absorption system more so, as the rich liquor pump work is very small while the heat supplied to the generator is large, both together representing the heat equivalent of the energy necessary to get the low-pressure vapor into the high-pressure condition. Such heat as is supplied to develop power in steam or gas engines is given by the following Eq. (1298):

$$\left(\begin{matrix} \text{B.T.U. per hour per} \\ \text{I.H.P. of engine} \end{matrix} \right) = \frac{2545}{\text{Engine thermal efficiency.}}$$

But

$$\left(\begin{matrix}\text{Engine thermal}\\ \text{efficiency}\end{matrix}\right) = (\text{Efficiency of engine cycle}) \times (\text{Efficiency factor}).$$

Hence

$$\left(\begin{matrix}\text{B.T.U. per hour per}\\ \text{I.H.P. of engine}\end{matrix}\right) = \frac{2545}{(\text{Efficiency of engine cycle}) \times (\text{Efficiency factor})}. \quad (1298)$$

ntroducing the ratio of, I.H.P. of compressor to I.H.P. of engine, which is less han unity, the heat consumption per I.H.P. of compressor follows by Eq. 1299):

$$\left.\begin{matrix}\text{B.T.U. supplied per hr. to engine per I.H.P. of compressor}\end{matrix}\right\} = \left[\frac{2545}{\left(\begin{matrix}\text{Efficiency of}\\ \text{engine cycle}\end{matrix}\right) \times \left(\begin{matrix}\text{Efficiency}\\ \text{factor}\end{matrix}\right)}\right] \times \left[\frac{\text{I.H.P. of engine}}{\text{I.H.P. of comp.}}\right]. \quad (1299)$$

he actual I.H.P. of the compressor per ton refrigeration will be the cycle H.P. er ton divided by the diagram factor of the refrigerating cycle, which includes ll losses, but which is nearly 100 per cent, certainly in any good compression ot less than 95 per cent.

herefore,

B.T.U. supplied per hour to engine, per ton refrig.)

$$= \left[\frac{2545}{\left(\begin{matrix}\text{Efficiency of}\\ \text{engine cycle}\end{matrix}\right) \times \left(\begin{matrix}\text{Efficiency}\\ \text{factor}\end{matrix}\right)}\right] \times \left[\frac{\text{I.H.P. eng.}}{\text{I.H.P. comp.}}\right] \times \left[\frac{\text{Cycl.H.P. per ton}}{\text{Refrig. diag. fact.}}\right] \quad (1300)$$

hese quantities have all been evaluated except the ratio of compressor to ngine horse-power, which is the mechanical efficiency of the drive and which epends on mechanical construction, speed, size, and must be estimated by ood judgment.

For the absorption system the heat supplied to the pump is to be evaluated n a manner equivalent to the above, but the heat of steam supplied to the enerator is a very much larger amount indeed, though not necessarily at so igh a temperature as is necessary for economical power generation, and must e determined separately from the properties of aqueous solutions of ammonia. t is a little surprising to find that this cannot be done with precision ecause of lack of data on the thermal properties of the solutions, but estimates ıay be made on various assumptions and often these estimates are taken by ngineers as representing the truth when they really do not. The most com-

mon assumption made in evaluating the heat supplied to the generator per to refrigeration or per pound of anhydrous ammonia circulating, to which it i directly related, is that nothing but NH_3 vapor will be discharged and tha the heat of its liberation is equal to the heat of absorption of the same amoun under inverse circumstances. For example, the curves and equations of Sectio 12, Chapter IV, give the heat of absorption per pound of ammonia absorbed when a solution of one strength becomes of higher ammonia content. Inversely if a rich liquor becomes weaker through the loss of the same amoun as was absorbed as above, there will be required an equal amount of heat It is, however, quite impossible to discharge ammonia vapor from a solutio without also discharging water vapor, and the difficulty involved here lie in the fixing of the amount of water vapor so carried off and the evaluation o heat of its evaporation, which must be added to that of the ammonia discharged With an analyzer above the generator, condensing and returning some wate vapor with its heat by incoming rich liquor as in the diagram, Fig. 316, it i necessary only to consider the water content of the NH_3 vapor beyond thi point and this is less than at generator discharge, so the error of neglecting th heat of vaporization of the water vapor is less when an analyzer is presen than when it is not, and appreciably less. If data were available to give th amount of water vapor in a saturated mixture beyond the analyzer and th temperature of vapor evolution at a given pressure in the generator itself, th heat equivalent of the water-vapor discharge could be found, but neither o these things are known. The law of molecular rise of boiling-point does no apply to ammonia water solutions, so the boiling-point or temperature o vapor evolution for a given liquor cannot be calculated for a given pressure The best that can be done at present is to neglect the water vapor entirely an to remember that the heat required by the generator will be somethin (x) more than thus found. On this basis

$$\begin{pmatrix}\text{B.T.U. supplied to generator}\\ \text{per lb. } NH_3 \text{ circulating}\end{pmatrix} = \begin{pmatrix}\text{Heat of absorption per pound } NH_3\\ \text{between } C_W \text{ and } C_R\end{pmatrix} + x.$$

$$\begin{bmatrix}\text{B.T.U. supplied to generator per hour per ton refrigeration}\end{bmatrix} = \left[\begin{pmatrix}\text{Heat of absorption per lb. } NH_3 \text{ between } C_W \text{ and } C_R\end{pmatrix} + x\right] \times \begin{bmatrix}\text{Lbs. } NH_3 \text{ per hr. per ton}\end{bmatrix} \quad (130$$

The heat balance for refrigerating systems might be set down as for powe generating systems equating energy taken in to that given out but as such hea balances are long and complicated without the introduction of vitiating assump tions they will be omitted. For any particular case in which they may b required the methods discussed in previous sections provide the necessar material as far as it is at present possible to do.

Example 1. Construction and use of Diagrams, Figs. 317 and 318. These diagram are for the purpose of finding the refrigerating effect per pound of fluid, which is made u of the latent heat, or as much of it as is available, less the heat necessary to cool th

liquid from its original temperature to that due to the pressure in the coils, plus the heat absorbed in superheating the vapor.

A horizontal scale of pressures is laid off in both directions for a vertical axis carrying a B.T.U. scale. In the section to the right of the center axis curves are drawn representing various temperatures of the liquid before entering the refrigerator coils. These are so drawn that the vertical scale opposite the intersection of a vertical from any pressure with any curve gives the latent heat for that pressure less the heat required to cool the liquid. This is the available heat for refrigerating if the vapor leaves the coils dry and saturated. In the section to the left of the center axis are two sets of curves, the lower, representing temperatures of the vapor leaving the coils, is so drawn that the value of the left-hand vertical scale opposite a point of intersection of a vertical from any pressure with any curve, gives the heat absorbed in superheating the vapor. The sum of this and the value found in the first section gives the total refrigerating effect for the case when the vapor leaves the coils in a superheated state. The upper curves in this section represent quality of the vapor if the liquid has not been entirely evaporated and are so drawn that the value on the vertical scale opposite the point of intersection of a vertical from any pressure with any curve, shows the heat unavailable for refrigerating, due to incomplete evaporation of the liquid, and the difference between this value and that found in the first section gives the total refrigerating effect for the case of wet vapor leaving the coils.

As an example let it be required to find the refrigerating effect per pound of ammonia when the pressure in the coils is 20 lbs. gage, the temperature of the liquid before entering the coil is 70° F. and

(*a*) Vapor leaves dry and saturated;
(*b*) Vapor leaves 90 per cent dry;
(*c*) Vapor leaves at a temperature of 30° F.

From 20 in the right-hand section project up to curve 70°. The value on the vertical scale at this point is 502 B.T.U., which is the value for case (*a*). From 20 in the left-hand section project to curve 90 per cent; the value on the left-hand vertical scale is 57, therefore, for case (*b*) the result is $502-57=445$ B.T.U. For case (*c*), project from 20 to curve 30°, the value on the vertical scale corresponding to which is 12.5, hence the result for this case is $502+12.5=514.5$.

Prob. 1. In a cold-storage room it is estimated that 350 lbs. of ice melt every minute. What would be the capacity of a machine to do an equal amount of refrigeration? What will be the capacity of a machine required to make 100 tons of ice per day from water at 60° F., the ice being at a temperature of 25° F.?

Prob. 2. Three types of machines are under consideration, (*a*) dense air; (*b*) ammonia compression; (*c*) carbon dioxide compression. In every case the temperature of the gas or vapor leaving the coils is 20° F. In the air system the air enters the coils at a temperature of −50° F. In the case of the ammonia system the pressure in the coils is 15 lbs. per square inch absolute, the liquid temperature before entering is 60° F., and in the carbon dioxide system the pressure is 300 lbs. per square inch gage, and the temperature before entering the coils is 80° F. What must be the compressor displacement for each case for a common true volumetric efficiency of 80 per cent?

Prob. 3. If the ammonia should leave the coils in Prob. 2 with 5 per cent unevaporated, what effect would there be on the temperature of the vapor and on the compressor size? If the carbon dioxide should leave dry and saturated what would be the change in these two quantities.

Prob. 4. Should an ammonia absorption system work so that the temperature of t anhydrous liquid entering the coils was 70° F. and that of the vapor leaving the coils w 30° F. with a pressure of 1 atmosphere, what would the displacement of the rich liqu pump, the rich liquor being 30 per cent NH_3, and the weak liquor 20 per cent by weigh

Prob. 5. In a refrigerating system it is desired to maintain a temperature of 10° in the coils, and there is available for condensing purposes water having a temperature 70° F. What will be the required cylinder displacement per minute of the compress and the horse-power per ton refrigeration for an ammonia-compression system, and similar system using CO_2? What would be the relative worth of the two systems?

Prob. 6. Air is used for refrigeration in a system where the highest pressure whi is desirable to carry is 150 lbs. gage, the cooling water temperature is 60° F., and t cold air is desired to have a temperature of −50°. What will be the temperature of t air leaving the refrigerator if each pound absorbs 15 B.T.U.? What will be the lo pressure, work done, ratio of work to heat absorbed in refrigerator and given to coole Draw the PV and $T\phi$ diagrams.

Prob. 7. In an ice-making plant making 100 tons of ice per day from water at 40° the ice is at 20° F. when removed from the cans; the cooling water available has temperature of 60° F. and the pressure in the coils is 10 lbs. gage. If the engine drivi the compressor has a thermal efficiency of 12 per cent, the work required to overcon friction in the engine and compressor is 15 per cent of the engine horse-power, and t compressor-diagram factor is 90 per cent, how many B.T.U. must be supplied to t engine per hour, for, (*a*) an ammonia plant; (*b*) a carbon dioxide plant?

Prob. 8. An absorption system is in operation so as to produce 50 tons refriger tion. The cooling water for the condenser is at 50° F. and the liquid NH_3 is cool to within 10° of this value. The temperature of the vapor leaving the coils is 20° F. a the pressure in them is 10 lbs. gage. The strength of the ammonia liquor varies betwe 15 and 30 per cent. Assuming that 25 per cent more heat must be supplied to the ge erator than is needed to liberate the ammonia, what will be the B.T.U. supplied to ge erator per hour per ton refrigeration?

INDEX

INDEX

Absolute pressure, 4
temperature, 412
Absorption of gases by liquids and, solids 493
table, 634
of air in water, table, 635
Acceleration, work of, 6
Adiabatic changes in gases, value of exponent, 904
expansion of steam, value of 's' for, 912
processes, 21
Air, absorption of in water, table, 635
composition of, 513
Air diagram, pressure, volume, temperature, entropy, 908
flow of, Fliegener's formula for, 1090
Air-gas and blast-furnace gas, composition of, 853
Air-gas, characteristics of, diagram, 708
heats for, weight and volume proportions, 702
heats of reaction and combustion for, 705
Air and gas mixture, explosion pressures, 748
limits of proportion, 869
best, calorific properties of, 867
Air-system, work absorbed in refrigeration, 1172
Alcohols, vapor pressure of, 487
Altitudes, barometric heights and pressures, table for, 65
Ammonia, anhydrous liquid, specific heat, 425
P-T relations, 461
aqueous solution, specific heat, 427
compression system of refrigeration, 1148
density of liquid and vapor of, 474
heats of complete dilution of, 500
heat of the liquid, curve, 613
heats of partial dilution of, 501
latent heat of, 476
curve, 614
pressure-temperature, curve, 612
solutions of, pressure, temperature and per cent., table, 629
specific volume and density of liquid, 616
specific volume and density of vapor, 617
temperature-entropy and Mollier diagram for, 1164
total heat, curve, 615
Ammonia-water solutions, chart, 495
density and specific volume of, 1163
vapor, 632
Ammonia, work in refrigeration by, 1168
Analysis, cellulose and wood, 650
of coals, 818
of cokes, 683
boiler flue gas, 756.
proximate, 650
ultimate, 650
Anthracite coal, sizes and ash contents, 659
Area, conversion table of units of, 62
Ash and clinker formation, 661
Atmospheric air, composition of, 513
moisture, chart, 491
Atmospheres, table for conversion, 63
Atomic weights, international, 586

Baumé specific gravity scale, 577
Barometric heights, altitudes and pressures, table of, 65
Best load, steam engines and turbines, actual performance 1062
receiver pressure for two-stage compressor, 106, 123
two receiver pressures for three-stage compressor, 127
Blast-furnace gas and air-gas, composition of, 853
density and calorific power of, 707
Blowing engines, 73
Boiling points defined, 459
point, rise of, for solutions, 466
points, table, 588
Boilers, commercial rating of, 776
Boiler, constants for capacity, locomotive and marine, 788
efficiency of, 798
curves between quantities entering into, 804
influence of various factors on, curves 809
Boilers, heat balance for, table, 800
Boiler horse-power definition, 775
heat equations of, 791
law relating heat absorbed and available in, 782
test, Goss locomotive, 780
tests of the U. S. Geological Survey, 781
Boudouard's equilibrium relations CO and CO_2, 697
Brake horse-power, 47, 369
Brayton, Otto, Carnot, Diesel cycles, diagrams of properties, 1018
cycle, 974, 1011
Brines, freezing-point for, 465
Brine, specific heat of, 427
British thermal unit conversion table, 573
Brown cycle, 978
B. T. U. or British thermal unit, 416

Calcium chloride brine, specific heat of, 426
chloride, freezing-point of, table, 425
Calorie, 416
conversion table, 573
Calorific power and density of blast-furnace gas, 707
Calorific power of coals, formulæ, 662
and composition of characteristic coals, 818
and density of coke oven gas, 684
high and low values for hydrogen, 517
of hydrocarbon oils, 837
of gasolenes and kerosenes, 691
of mineral oils, 672, 836
of oil gas, 693
and density of oil gas, 694
of hypothetical producer gas, 733
and density of producer gas, 737
of retort coal gas, 685
of water gas, 718
properties of air and gas mixtures, best, 867
of best air and gas mixtures, 867
value of hydrocarbons, Slaby's formula for, 526
Capacity of isothermal compressor without clearance, 83
with clearance, 88
of single stage exponential compressor, no clearance, 94
single-stage, exponential compressor with clearance, 98, 99
two-stage compressor, no clearance, best receiver pressure, 104
with clearance, best receiver pressure, 111
with clearance, any receiver pressure, 115
of three-stage compressor, no clearance, best receiver pressure, 127
with clearance, any receiver pressure, 139
best receiver pressure, 145
Carbon dioxide and monoxide, rate of reactions, 701
relations of equilibrium, 697
rate of reactions, 855
Carbon dioxide compression system of refrigeration, 1148
density of liquid and vapor of, 475
heat of the liquid, curves, 623
latent heats of, 476
latent heat, curves, 624
P-T relations, 461
pressure temperature, curves, 622
properties of saturated vapor, 618
specific volume and density of the liquid, curves, 626
of the vapor, curves, 627
total heat, curves, 625
temperature-entropy and Mollier diagram for, 1166
work in refrigeration by, 1170
Carnot steam cycle and derivatives, 957
diagram of properties, 964
Carnot, Otto, Brayton, Diesel cycles; diagrams of properties, 1018
Cellulose and wood analysis, 650
Centigrade and Fahrenheit temperature chart, 410
table, 571
thermometer scales, 410
Centigrade heat unit, 415
conversion table, 573
Cheval-Vapeur, 2
conversion table, 64
Change of state, mathematical equation for, 889
Chimneys, pipes, flues and ducts, flow of gases and vapors in, 1111
Clapeyron's equation for change of state, 889
for latent heat, 468
Clearance, methods for finding, 37
Clinker and ash formation, 661
Coal, anthracite sizes and ash content, 659
distillation, bituminous, products of, 679
gas, coke oven and retort, composition of, 842
impurities in, 660
calorific power of, formulæ, 662
of the volatile of, diagram, 668
classification by composition, 652
of, by gas and coke qualities, 654
composition and calorific power of, 81
lignites and peats, calorific power of, diagram, 665
rates of combustion of, 769
Coefficient of expansion of liquids, cubical, 581
of heat transmission, variation of, 555
transfer, steam condensers, 564
transfer, 550
of linear expansion of solids, 580
Coefficients of expansion, 435
for gases, 437
of radiation, table, 535
Coking or non-coking coals, 653
Coke oven gas, density and calorific power of, 684
United States, composition of, 846
Cokes, analysis of, 683
Combined diagrams of compound engine, 238
Combustible and volatile of coals, etc., 82
Combustion, air required for, table, 515
and related reactions, 506
chamber temperature, 806
effects, 740
of coal, rates of, 769
of hexane, factors for weights in, 51
rate of, effect of draft on, diagram, 77
rate of propagation of, 768
temperature due to, 743
weight relations in, 511
Commercial cut-off, 369
rating of boilers, 776
Composition of air, 513
of air-gas and blast-furnace gas, 853
of flue gas, 868
of oil producer gas, 866
of producer gas, 858

Compressed air engines with and without preheat, 1128
Compressibility of gases by their isothermals, table, 584
Compression commonly used in gas engines, 763, 1050
exponential, 76
system of refrigeration, 1148
work of, 13
Compressor, characteristics of actual, 153
chart for mean effective pressure, 175
Compressor chart comparing single, two-stage, three-stage and isothermal, 181
for cylinder displacement for given capacity, 183
for single-stage, 169
for two-stage, 171
for three-stage, 172
for volumetric efficiency, 174
economy of, 148, 181
graphic solution of problems, 168
imperfect intercooling, 159
maximum work of, 151, 177
PV cycles for, 75
structure and processes, 73
two and three-stage compared to single stage, 179
true low-pressure capacity, 157
of variable capacity, 160
isothermal, no clearance, 81
with clearance, 85
in terms of dimensions, 87
single-stage, no clearance (exponential), 91
with clearance (exponential law), 96
dimensions of cylinder, single-stage (exponential), 98
two-stage, no clearance, 103
with clearance, 109
with best receiver pressure in terms of dimensions, 120
any receiver pressure in terms of dimensions, 113
three-stage, no clearance, 125
with clearance, 131
with best receiver pressure, in terms of dimensions, 143
any receiver pressure, 135
Compressors, limiting pressures for single, two, three or four-stage, 160
Conductivity, internal thermal, 639
relative thermal, 642
solids and liquids, 530
through joints, 532
Constant temperature phase for gases and vapors, 898
Constant-volume phase for a vapor, 900
Consumption, per hour per I. H. P., general expressions, 51
of steam engine, variation of, 371
of steam engine, actual, 376
variation of, with load of engine, 381
simple engine, no clearance, logarithmic law, 200
with clearance, logarithmic law, 212
with clearance, exponential law, 221
Consumption, compound engine, infinite receiver, no clearance, logarithmic law, 260
infinite receiver, no clearance, exponential law, 269
finite receiver, no clearance, logarithmic law, 277
no clearance, exponential law, 289
no receiver, no clearance, logarithmic law, 296
exponential law, 302
infinite receiver, with clearance, logarithmic law, 300
exponential law, 321
finite receiver, with clearance, logarithmic law, 329
exponential law, 337
no receiver, with clearance, logarithmic law, 342
exponential law, 348
triple expansion engine, no clearance, logarithmic law, 352
Conversion tables, 62
Corresponding states, the principle of, 460
Crank angle, piston positions for, table, 395
Critical point, 453
Crude oils, 688
Cut-off best in actual engine, 378
Cycle, 24
Brown, 978
Brayton, 974, 1011
Diesel, 1013
Ericsson, 996
Lenoir, 980
Otto, 1006
Otto and Langen, 983
Rankine, 936
Stirling, 993
Cycles for simple engines, 192
for steam, heat diagram, 932
Cyclic heat for Brown cycle, 980
for Brayton cycle, 1011
for Diesel cycle, 1014
for Ericsson cycle, 997
for Lenoir cycle, 981
for Otto cycle, 1007
for Otto and Langen cycle, 984
for Rankine cycle, 939
for Stirling cycle, 994
or system efficiency, 928
Cylinder ratio, compound engine, finite receiver, no clearance, logarithmic law, 283
no clearance, no receiver, logarithmic law, 296
no receiver, no clearance, logarithmic law, 302
infinite receiver, with clearance, logarithmic law, 312

Dense air system of refrigeration, 1148
Density and calorific power of blast-furnace gas, 707
of coke oven gas, 684
of oil gas, 694
of producer gas, 737
of retort coal gas, 685

Density and calorific power of water gas, 718
and specific volume of ammonia-water solutions, 1163
of gases, 585
of liquid, for water, 601
and vapor of ammonia, 474
of carbon dioxide, 475
Detonating mixtures of gas and air, 764
or explosive waves, velocity of, 766
Dew point, chart, 489, 490
Diagram factor for engines, 369
factors for Otto cycle gas engines, 1046
for small gasolene engines, variation with compression, 1048
of refrigeration systems, 1149
Diesel and Otto gas engines, actual performance of, 1042
cycle, 1013
diagrams of properties, 1023
Otto, Brayton, Carnot cycles, diagrams of properties, 1018
Dissociation, 519
Distance, conversion table of, units of, 62
Distillation of gasolenes, fractional, 690
of kerosene and petroleums, fractional, 689
products of crude mineral oils, 686
Draft, effect of on rate of combustion, diagram, 770
rate of combustion with, 870
Dry vacuum pumps, 73
Ducts, chimneys, pipes and flues, flow of gases and vapors in, 1111
Dulong's formula for heat of combustion, 663

Economy of compressors, 148, 181
Effective horse-power, 47
Efficiency, boiler, 798
mechanical, 47, 370
of compressors, 148
of hypothetical gas producer, 732
of steam engines, variation due to initial pressure or back pressure, 1068
of transmission, 47
thermal, 47
of Brown cycle, 980
of Brayton cycle, 1011
of Diesel cycle, 1014
of Ericsson cycle, 997
of Lenoir cycle, 981
of Ottocycle, 1007
of Otto and Langen cycle, 984
of Rankin cycle, 937
of Stirling cycle, 994
Endothermic reaction, 507
Energy, intrinsic, 882
Energy equation for substances, general, 883
Engine, combined diagrams of, 238
compound, chart for equal division of work, 390
consumption of steam, 371
cut-off for best economy, 378
cycle, events of, 190
cylinder ratio compound with finite receiver, 283
diagram factor, 369
Engine, diagram factor, relations of hypothetical to actual, 367
dimensions, miscellaneous actual, 396
equal work division, compound finite receiver, 279
mean effective pressure chart for, 388
multiple expansion, structure and processes, 225
general case, by graphic methods, 357
reference cycles, compound, 235
steam, water rate—load curves, 383
single cylinder without clearance, logarithmic law, 197
structure and processes, 187
single cylinder without clearance, exponential law, 204
with clearance, logarithmic law, 208
exponential law, 219
compound with infinite receivers, logarithmic law, no clearance, 256
exponential law, no clearance, 268
logarithmic law, clearance and compression, 306
exponential law, clearance and compression, 319
finite receiver, logarithmic law, no clearance, 274
exponential law, no clearance, 287
logarithmic law, clearance and compression, 325
exponential law, clearance and compression, 335
without receiver, 249
logarithmic law, no clearance, 292
exponential law, 301
logarithmic law, clearance and compression, 339
exponential law, clearance and compression, 346
triple expansion with infinite receiver, logarithmic law, no clearance, 349
Entropy change for heat gains or losses, 893
defined, 878
diagram for steam, Mollier, 920
Entropy-pressure-volume-temperature, diagram for 1 lb. air, 908
Equilibrium relations CO and CO_2, Boudouard's, 697
Equivalent gas densities, diagram, 447
Ericsson cycle, 996
Ethylenes and naphthalenes from Russian petroleum, 835
Events of engine cycle, 190
of the indicator card, 367
Exothermic reactions, 507
Expansion, coefficients of, 435
for gases, 437
of gases and vapors at constant pressure, table, 582
of liquids, cubical coefficient of, 581
of solids, coefficient of linear, 580
work of, 13
Explosion pressures, air and gas mixture, 748
Explosive mixtures of Pintsch oil gas and air, 747

Exponential compression, 76
Exponent, s, table of values for, 67
value of, 20, 54
External latent heat, 479

Fahrenheit and Centigrade temperatures, tables, 571
First law of thermo dynamics, 883
Fliegener's formula for flow of air, 1090
Flow of air, Fliegener's formula for, 1090
in nozzles, Moyer's formula for, 1090
Napier's formula for, 1090
Rateau's formula for, 1090
nozzle, with large pressure drop, 1083
with small pressure drop, 1098
of gases and vapors in pipes, flues, ducts, and chimneys, 1111
through nozzles, weight of, 55
in valves, 1102
with large pressure drop, orifices, 1083
Flue gas analysis, boiler, 756
composition of, 868
temperature, 806
Flues, pipes, ducts and chimneys, flow of gases and vapors in, 1111
Foot-pound, 1
Foot-pounds, conversion table, 64
Foot tons, conversion table, 64
Force, conversion table of units of, 63
Fractionation tests of gasolenes, 851
of kerosenes and petroleums, 847
Free expansion by PV method, velocity due to, 52
Freezing or melting points, 586
effect of pressure on, 890
points, lowering of, solutions, 465
Frictional resistance in pipe flow, 1118
Friction horse-power, 47
Fuel elements and chemical compounds, heats of combustion of, 636
mineral oil and natural gas, 670
Fuels, natural solid, 649
Furnace temperatures, 751
Fusion. latent heats of, 591
pressure and temperature of ice, 456

Gases, absorption of by liquids and solids, 493
adiabatic changes in, value of exponent, 904
and vapors at constant pressure, volumetric expansion of, table, 582
and vapors at constant volume, pressure rise of, 583
Gas consumption, Brown cycle, 980
Brayton, cycle, 1011
Diesel cycle, 1014
Ericsson cycle, 997
Lenoir cycle, 981
Otto cycle, 1007
Otto and Langen cycle, 984
Stirling cycle, 994
Gases, coefficients of expansion, 437
comparison of coke oven and retort, 682
compressibility of, by their isothermals, table, 584
perfect, intrinsic energy for, 884
Gases, pressure, volume and temperature relations for, 438
Gas and steam cycles, comparison of, 1031
and vapor mixtures, 481
constant and molecular weight, 446
constants and specific heats, 579
constant and the two specific heats, 884
R, values of, 584
cycles, typical, 970, 976
densities of, 585
density and molecular weight, 446
engines, compression commonly used in, 762, 1050
(and oil) engines, heat balances of, 1060
gas cycles representative of, 970
mean effective pressure factors for, 1053
mechanical efficiencies of, 1050
Otto and Diesel, actual performance of, 1042
Otto cycles, diagram factors for, 1046
engine performance curves, actual, 1058
engines, preignition in, 761
producer, efficiency of hypothetical, 732
producers, classification, 720
producer tests, 864
yield of cannel coal, 682
Gases, specific heats of; effect of variability on cycles, 1035
Gas-vapor mixtures, 481
Gases and vapors, specific heats of, 430, 578
Gasification of fixed carbon and coke, 695
Gasolene engines, variation of diagram factors with compression, 1048
Gasolenes and kerosenes, calorific power of, 691
fractional distillation of, 690, 851
Gasolene, vapor pressure of, 485
General energy equation for substances, 883
Goss locomotive, boiler test. 780
Graphic methods for multiple expansion engine, 357
Graphic method of finding value of exponent 's', 21
Graphical representation of work, 8
Grashoff's formula for flow in nozzles, 1090

Harter's formula for flow in nozzles, 1090
Heat and power conversion table, 573
balance for boilers, table, 800
for locomotive boiler, curves, 808
of gas and oil engines, 1060
of producer plants, 1057
steam plant, 1063
diagram defined, 878
equations of boiler, 791
flow by radiation, 534
gains or losses, entropy change for, 893
mechanical equivalent of, 417
of combustion of air-gas, weight and volume proportions, 702
of fuel elements and chemical compounds, 636
of formation, 521

Heat of reaction, 516
for hypothetical producer gas, diagram, 729
of the liquid, ammonia, 613
carbon dioxide, 623
steam, 598
curves for steam, 598
origin of and transformation to useful form, 644
radiation, Stefan and Bolzmann law, 536
suppression, 746
transfer, classification of apparatus, 541
transfer, 528
coefficients of, 550
coefficient, steam condensers, 564
transmission between separated fluids, 538
variation of coefficient of, 555
Heating process, classification of, 401
Heats, specific, defined mathematically, 888
Heck's formula, value of x, 396
for missing water, 377
High pressure capacity, see capacity
hot, 93
cold, 93
Horse-power, defined, 2
boiler, definition, 775
brake, 47
conversion table, 64, 573
friction, 47
indicated, 44
effective, 47
of nozzles and jets, 55
per pound mean effective pressure, 45, 63
Humidity chart, 491
Hydrocarbons, curves for heat of combustion of, 522
heat of combustion of, 522
Slaby's formula for calorific value of, 526
vapor pressure of, 485

Ice, fusion pressure and temperature, 456
vapor pressure of, 456
Ignition temperature for H_2 and CO mixed with O_2, 759
temperatures, table, 763
for producer gas, 761
Indicated consumption, 373
horse-power (I. H. P.), 44
I. H. P. compressor, isothermal, 89
of compressor, single stage, exponential law, 100
two-stage with clearance, any receiver pressure, 116
best receiver, pressure, 123
three-stage, with clearance, any receiver pressure, 139
best receiver pressure, 146
of engine, simple, no clearance, logarithmic law, 200
with clearance, logarithmic law, 210
of engines, various types, see mean effective pressure
Indicated water rate, 373
Indicated water rate, see also consumption
Indicator, 34
card, events and periods, 367
cards combined, compound engine, 223
triple expansion engine, 234
compound engine, 236
from Vauclain locomotive, 230
relation of pressure volume diagrams to, 34
steam engine, 191
triple expansion engine, 237
Initial pressure, effect of, locomotive, 1069
Intercooling in compressors, imperfect, 152, 159
perfect, 79
Intrinsic energy, 882
Iron above 500°, specific heat of, curves, 422
Isothermal compression, 75
compressor, in terms of dimensions, 87
processes, 22

Jacketed cylinders for steam engine, 1079
Juptner's formula for heat of combustion, 664

Kerosene class, vapor pressure of, 486
Kerosenes and gasolenes, calorific power of, 691
and petroleums, fractional distillation of, 689, 847
Kilogramme, table for conversion, 63
Kilogrammeter, 1
conversion table, 64

Langen and Otto cycle, 983
Latent heat, Clapeyron's equation for, 468
for steam, 599
external, 479
of ammonia, 614
of carbon dioxide, 476
of vaporization, table, 590
heats of fusion, 591
Law of thermodynamics, first, 883
second, 887
Lenoir cycle, 980
Length, conversion table of units of, 62
Lignites, composition of, 656
Limits of proportion, air and gas mixtures, 869
Liquids, specific heats of, table, 576
Litres, table for conversion, 62
Load curves for gas engines, 1059
division for two engines, 386
Locomotives, effect of initial pressure, 1069
Locomotive, Goss, boiler test, 780
Vauclain, indicator cards from, 230
Logarithmic expansion of steam, changes of quality, 924
temperature difference, 549
Low pressure capacity, 81, 157
of compressors, see capacity

Mark's formula, pressure-temperature of steam, 460
Maximum work of compressors, 151, 177
Mean back pressure, 32

Mean effective pressure, work determination by, 31
variation of, 363
chart for engines, 388
factors for gas engines, 1053
isothermal compressor, 88
single-stage exponential compressor, 100
of two-stage compressor, with clearance and any receiver pressure, 115
two-stage compressor, best receiver pressure, 122
of the three-stage compressor, 138
three-stage compressor with clearance and best receiver pressure, 144
one-, two-, and three-stage by isothermal standard, 150, 181
simple engine, no clearance, logarithmic law, 200
exponential law, 205
with clearance, logarithmic law, 210
exponential law, 220
compound engine, infinite receiver, no clearance, logarithmic law, 259
exponential law, 269
finite receiver, no clearance, logarithmic law, 277
exponential law, 288
no receiver, no clearance, logarithmic law, 296
exponential law, 302
infinite receiver, with clearance, logarithmic law, 309
exponential law, 321
finite receiver, with clearance, logarithmic law, 329
exponential law, 337
no receiver, with clearance, logarithmic law, 342
exponential law, 348
triple expansion engine, infinite receiver, no clearance, logarithmic law, 351
Brown cycle, 980
Brayton cycle, 1011
Diesel cycle, 1014
Ericsson cycle, 997
Lenoir cycle, 981
Otto cycle, 1007
Otto and Langen cycle, 984
Stirling cycle, 994
Mean forward pressure, 32
temperature difference, 540
Mechanical efficiency, 47, 370
efficiencies of gas engines, 1050
equivalent of heat, 417
refrigeration, description of processes and structures, 1142
Melting or freezing points, 459, 586
point, effect of pressure on, 890
Mercury column, pressures measured by, 63
Meters, table for conversion, 62
Metric tons, table for conversion, 63
Mineral oils, calorific power of, 836
properties of, 838
Missing water or steam, 376
Perry's formula for, 377
Heck's formula for, 377
Mixtures, gas and vapor, 481
Moisture, atmospheric, chart, 491
Molecular lowering of freezing point, 465
rise of boiling point, 466
Mollier's total heat-entropy diagram for steam, 920
diagram for ammonia, 1165
for carbon dioxide, 1167
used for nozzle flow, 1087
pressure staging of nozzle flow, 1094
Moyer's formula for flow in nozzles, 1090

Naphthalenes and ethylenes from Russian petroleum, 835
Napier's formula for flow in nozzles, 1090
Natural gases, composition of, 841
gas, calorific power of, 673
Nozzle flow with large pressure drop, 55, 57, 1083
with small pressure drop, 1098
proportions, 1092
Nozzles, Moyer's formula for flow in, 1090
Napier's formula for flow in, 1090
Rateau's formula for flow in, 1090

Occlusion of liquids by solids, 493
Oil gas, calorific power of, 693
density and calorific power of, 694
properties of, 693
yield of, 694
Oils, hydrocarbon, calorific power of, 837
mineral, calorific power of, 672
Oil producer gas, composition of, 866
products, American, 687
Orifices, flow with large pressure drop, 1083
Otto cycle, 974, 1006
diagrams of properties, 1022
gas engines, diagram factors for, 1046
Brayton, Carnot, Diesel cycles, diagrams of properties, 1018
and Diesel gas engines, actual performance of, 1042
and Langen cycle, 983
diagrams of properties of, 986

Paraffines from Pennsylvania petroleum, 835
Partial pressures in gas mixtures, 482
in gas-vapor mixtures, 484
Peats, composition of, 655
Peat distillation, products of, 677
Perfect gases, intrinsic energy for, 884
Performance of Otto and Diesel gas engines, 1042
of refrigerating systems, 1157
Periods of the indicator card, 367
Perry's formula for missing water, 377
Petroleums and kerosenes, fractionation tests of, 847
Petroleum, Pennsylvania, paraffines from, 835
Russian, ethylenes and naphthalenes from, 835
Phase, 24
Phases, thermal, 895

Pintsch oil gas, 747
Pipes, flues, ducts and chimneys, flow in, 1111
Piston positions for any crank angle, table, 395
Poppet valves, flow through, 1104
Powdered coal, producer gas, composition of, 866
Power and heat conversion table, 573
conversion table of units of, 64
defined, 2
required for refrigeration, 1168
Preignition in gas-engines, 761
Pressure, absolute, 4
Pressures, barometric heights and altitudes, table of, 65
Pressure, conversion table of units of, 63
Pressure-entropy-volume-temperature, diagram for 1 lb. air, 908
Pressure effect on freezing or melting point, 890
Pressures measured by the mercury column, 63
partial, in gas mixtures, 482
in gas-vapor mixtures, 484
Pressure rise of gases and vapors at constant volume, 583
staging of nozzle flow, use of Mollier diagram, 1094
temperature, curves for steam, 597
Pressure-volume calculations, summary of general formulæ, 70
Pressure, work in terms of, 3
Producer gas, calorific power of hypothetical, 733
composition of hypothetical, 725
composition of, 858
density and calorific power of, 737
heats of reaction for hypothetical, diagram, 729
ignition temperatures for, 761
plants, heat balances of, 1057
Properties of saturated steam, 592
of carbon dioxide vapor, 618
of superheated steam, 596
Proximate analysis, 650
Psychrometric chart, 491

Quality for steam, lines of constant, 917

Radiation coefficients, table, 535
heat flow by, 534
Rankine cycle, 936
diagrams of properties of, 949
efficiency of steam engines and turbines, 1064
steam cycle, example, 944
Rates of combustion of coal, 769
Rate of combustion with draft, 770, 870
of formation of CO from CO_2 and carbon, 701, 855
of propagation of combustion, 768
Rateau's formula for flow in nozzles, 1090
Ratios of pressures, exponential gas changes, 906
Reactions, endothermic, 507
exothermic, 507
Receiver pressure, compound engine, infinite receiver, 261, 269, 311 and 323
finite receiver, 278, 331
no receiver, 344
triple expansion engine, infinite receiver, 354
two-stage compressor, with clearance, 115
three-stage compressor, with clearance, 137
Refrigerating effect per pound of ammonia, chart, 1161
of carbon dioxide, 1162
systems, 1148, 1157
Refrigeration, mechanical, description of processes and structures, 1142
Reheating receivers for the compound engine, 1080
Reheat in nozzle flow, 1085
Realtive humidity, 488
Resistance to heat flow, 532
Retort coal gas, density and calorific power of, 685
Russian petroleum, ethylenes and naphthalenes from, 835

Saturated carbon dioxide vapor, properties of, 618
steam, properties of, 592
vapor defined, 459
mixture of vapor and gas, 486
Saturation law, expansion or compression according to, 22
Second law of thermodynamics, 887
Selby and Kemble experiments on steam nozzles, 1093
Simple engines, standard reference cycles, 192
Slaby's formula for calorific value of hydrocarbons, 526
Sodium chloride brine, specific heat of, 427
Solids, specific heats of, table, 574
Solutions, ammonia-water, density and specific volume of, 1163
aqueous, freezing point of, 465
rise of boiling point for, 466
Soot, composition of from bituminous coal, 753
Specific displacement, 49
gravity scale, Baumé, 577
heat, 419
constants, gases, table, 431
heats defined mathematically, 888
of gases and vapors, 430, 578
effect of variability on cycles, 1035
heat of iron above 500°, curves, 422
heats of liquids, table, 576
of solids, table, 574
heat of superheated steam, curves, 433
of water, curve, 423
volume and molecular weight, 446
of vapor, for steam, 602
Speed table, 64
Steam, adiabatic expansion of, value of 's' for, 912
engine indicator cards, 191

Steam and gas cycles, comparison of, 1031
consumption, actual, 376
adiabatic expansion of, value of 's' for, 912
cycle, Carnot and derivatives, diagram of properties, 964
engine efficiency, referred to the Rankine cycle, 1064
engines and turbines, best load, actual performance of, 1062
heat cycles for, 932
of the liquid, curves for, 598
latent heat, curves for, 599
lines of constant quantity for, 917
logarithmic expansion of, changes of quality, 924
plant heat balances, 1063
pressure, initial, effect on efficiency, 1069
pressure-temperature, curves, 597
specific heat of superheated, curves, 433
volume and density of the vapor, 602
and density of the liquid, 601
total heat, 471, 600
Stefan and Bolzmann law, heat radiation, 536
Stirling cycle, 993
diagrams of properties of, 998
Sublimation, 456
Sulphur dioxide, compression system of refrigeration, 1148
Superheat defined, 459
Effect of on efficiency, 1075
Superheated steam, properties of, 596
Specific heat of, curves, 433
Superheat conversion table of units of, 62
Table of, XIX

Surface and heat for gases, diagram, 745
Temperatures, Centigrade and Fahrenheit, 410, 571
Temperature, combustion chamber, 806
due to combustion, 743
Temperature-entropy and Mollier diagram for ammonia, 1164
for carbon-dioxide, 1166
Temperature, flue gas, 806
Temperatures, ignition, table, 763
Temperature of ignition for H_2 and CO, mixed with O_2, 759
scales, 407
Temperature-volume-entropy-pressure, diagram for 1 lb. air, 908
Temperature-entropy and Mollier diagram for steam, 920
Thermal efficiency definitions, 47, 877
of compressed air systems, 1126
phases, 895
Thermodynamic function, 878
Thermodynamics, laws of, 887
Thermometer and absolute temperature scales, 407
Total heat, curves for steam, 600
heat-entropy diagram for steam, Mollier, 920
heat of steam, 471
lines, constant, 918
Transmission, efficiency of, 47
Triple expansion engine indicator cards, 237
Triple point, 458
Trouton's law, 469
Turbine efficiency referred to the Rankine cycle, 1064
Turbines and steam engines at best load, actual performance of, 1062

Ultimate analysis, 650
Units of distance, conversion table of, 62
of power, conversion table of, 64
of pressure, conversion table of, 63
of surface, conversion table of, 62
of velocity, 64
of volume, conversion table of, 62
of weight and force, conversion table of, 63
of work, conversion table of, 64

Valves, flow in, 1102
Van't Hoff's law of movable equilibrium, 509
Vapor, formation from liquid, equations, 889
Vapors and gases, specific heats of, 430, 578
Vapor pressure of alcohols, 487
of gasolene, 485
of hydrocarbons, 485
of ice, 456
of kerosene class, 486
tension, 452
Vaporization, latent heat of, 467, 590
work done during, 468
Vauclain locomotive, indicator cards from, 230
Velocity due to acceleration, 6
due to free expansion by PV method, 52
units of, 64
Volatile and combustible of coals, etc., 826
Volume, conversion table of units of, 62
Volumetric efficiency, isothermal compressor with clearance, 88, 99
exponential compressor, with clearance, 99
two-stage compressor, any receiver pressure, 113
three-stage compressor, any receiver pressure, 139
two-stage compressor, with best receiver pressure, 121
three-stage compressor, best receiver pressure, 146
actual, 157
values for Otto engine, 1055
Volume-temperature-entropy-pressure, diagram for 1 lb. air, 908
Volume and pressure, work in terms of, 3

Water, specific heat of, curve, 423
Water-gas, characteristics, 710
composition of, 857
density and calorific power of, 718
heats of reaction for, 716
Water-rate-load curves, for steam engines, 383
vapor-P-T relations, 454, 460
Water-vapor-ice, combined curves, 458

Weight and force, conversion table of units of, 63
of flow in nozzles, 1090
relations in combustion, 511
Willans' line, 381
Wood and cellulose analysis, 650
distillation, products of, 675
Work by pressure-volume change, 10
conversion table of units of, 64
defined, 1
determination by mean effective pressure, 31
in refrigeration, air-system, 1172
by ammonia, 1168
by carbon dioxide, 1170
in terms of pressure and volume, 3
graphical representation of, 8
of expansion and compression, 13
phases and cycles, 24
Working fluid, 398
Work of isothermal compressor without clearance, 83
with clearance, 88
of single-stage compressor, no clearance, exponential law, 93
with clearance, exponential law, 100
of two-stage compressor, no clearance, best receiver pressure, 104
with clearance, any receiver pressure, 115
best receiver pressure, 122
of three-stage compressor, no clearance, best receiver pressures, 128
with clearance, any receiver pressures, 136
Work of three-stage compressor with best receiver pressures, 144
of simple engine, no clearance, exponential law, 205
logarithmic law, 200
with clearance, logarithmic law, 210
exponential law, 219
of compound engine, infinite receiver, no clearance, logarithmic law, 259
exponential law, 268
finite receiver, no clearance, logarithmic law, 276
exponential law, 288
no receiver, no clearance, logarithmic law, 294
exponential law, 302
infinite receiver, with clearance, logarithmic law, 308
exponential law, 321
finite receiver, with clearance, logarithmic law, 329
exponential law, 337
no receiver, with clearance, logarithmic law, 341
exponential law, 348
of triple expansion engine, infinite receiver, no clearance, 351
of Brown cycle, 980
of Brayton cycle, 1011
of Diesel cycle, 1014
of Ericsson cycle, 997
of Lenoir cycle, 981
of Otto cycle, 1007
of Otto and Langen cycle, 984
of Rankine cycle, 939
of Stirling cycle, 994



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